ASSIGNMENT PROBLEM

The assignment problem refers to another special class of linear programming problem where the objective is to assign a number of resources to an equal number of activities on a one-to-one basis so as to minimize total costs of performing the tasks at hand or maximize total profit.

Mathematical Model of Assignment Problem:

Given ‘n’ facilities (resources) and ‘n’ jobs (activities) and effectiveness (in terms of cost, time, profit) of each facility for each job, the problem lies in assigning each resource to one and only one activity so that the given measure of effectiveness is optimized.

The cost matrix is same as that of transportation cost matrix except that capacity at each of the resources and the requirements at each of the destinations is taken to be unity because assignments are made on a one to one basis.

Let Xij denote the assignment of ‘ith’ facility to ‘jth’ activity such that

Xij = [1, if ‘ith’ facility is assigned to ‘jth’ activity]

[0, otherwise]

Balanced Assignment Problem: An assignment problem is said to be balanced if the number of persons is equal to the number of jobs.

Unbalanced Assignment Problem: An assignment problem is said to be unbalanced if the number of persons is not equal to the number of jobs. To make unbalanced assignment problem, a balanced one, a dummy person or a dummy job is introduced with zero cost or time.

Assignment Algorithm (The Hungarian Method): The computational procedure for solving assignment problem can be summarized in the following steps:

I:- Develop the Opportunity Cost Table:

(a) **Row Reduction:** Subtract the minimum entry of each row from all the entries of the respective row in the cost matrix.

(b) **Column Reduction:** After completion of row reduction, subtract the minimum entry of each column from all the entries of the respective column.

II: Make assignments in the opportunity cost matrix:

(a) Starting with Ist row of the matrix received in first step, examine the rows successively one by one until a row containing exactly one zero is found. Now cross all the zeros in the column in which the assignment is made. This procedure should be adopted for each row assignment.

(b) When the set of rows has been completely examined, an identical procedure is applied successively to columns. Starting with column 1, examine all columns until a column containing exactly one zero is found.

(c) Repeat the operations until one of the following situation arises:

(III) <u>Optimality Criterion:</u>

(a) If all the zeros in rows/columns are either marked cross and there is exactly one assignment in each row and in each column. In such a case optimal assignment policy for the given problem is obtained.

(b) There may be some row (or columns) without assignment i.e. the total number of marked zeros is less than the order of the matrix. In such a case, go to the next step.

(IV) Revise the opportunity cost matrix.

(V) Develop the new revised opportunity cost matrix.

Q1: The distance between the cities are given in the following distance matrix:

	I	II	III	IV
A	11	17	8	16
B	9	7	12	6
C	13	16	15	12
D	14	10	12	11

Q2: Five machines are available to do 5 different jobs. Find the assignment of machines to jobs that will minimize the total time taken:

	I	II	III	IV	V
A	8	4	2	6	1
B	0	9	5	5	4
C	3	8	9	2	6
D	4	3	1	0	3
E	9	5	8	9	5

Q3: The number of man-hours needed to complete a job for each job-man combination are given below:

	A	B	C	D	E
I	12	8	7	15	4
II	7	9	17	14	10
III	9	6	12	6	7
IV	7	6	14	6	10
V	9	6	12	10	6

Q4: The following table shows the annual sales that can be generated by each salesman in each territory. Find the optimum assignment.

	T1	T2	T3	T4	T5
S1	32	38	40	28	40
S2	40	24	28	21	36
S3	41	27	33	30	37
S4	22	38	41	36	36
S5	29	33	40	35	39

(2) Unbalanced Assignment Problem:

When the cost matrix of an assignment problem is not a square matrix, i.e. number of rows are not equal to number of columns, then the assignment problem is called an unbalanced transportation problem. In such cases, dummy rows/columns are added in the matrix with zero costs. Then, the usual Hungarian method is used to solve the resulting balanced problem.

Q5: solve the following unbalanced assignment problem:

	I	II	III	IV	V
A	6	2	5	3	6
B	2	5	8	7	7
C	7	8	6	9	8
D	6	2	3	4	5
E	9	3	8	9	7
F	4	7	4	6	8

Q6: A company has 6 machines and 5 jobs. Each job can be assigned to one and only one machine. The cost of each job on each machine is given in the following table:

	1	2	3	4	5	6
A	12	10	15	22	18	8
B	10	18	25	15	16	12
C	11	10	3	8	5	9
D	6	14	10	13	13	12
E	8	12	11	7	13	10

Q7: Solve the adjoining unbalanced assignment problem of minimizing total time for doing all the jobs:

	1	2	3	4	5
1	6	2	5	3	6
2	2	5	8	7	7
3	7	8	6	9	8
4	6	2	3	4	5
5	9	3	8	9	7
6	4	7	4	6	8

Special Variations in the assignment Problem:

(1) **Maximisation case in assignment problem:** Sometimes, the assignment problem may relate to the maximisation of profit by assigning people to tasks or jobs to machines. Such problems are first reduced to minimization problem before applying the hungarian method. This is achieved by subtracting from the highest element in the assignment matrix, all other elements of the matrix.

Q: A captain of a cricket team has to allot five middle order batting positions to five batsmen. The average runs scored by each batsmen at these positions are given in the table:

	I	II	III	IV	V
A	40	40	35	25	50
B	42	30	16	25	27
C	50	48	40	60	50
D	20	19	20	18	25
E	48	60	59	55	53

Introduction of Optimization Techniques

- **According to Churchman, Arnoff,** "Operations Research is the application of scientific methods, techniques and tools to problems involving the operations of a system so as to provide those in control of the system with optimum solutions to the problem."

- Operations Research consists of techniques which were designed for solving assorted problems relating to transportation, assignment of tasks to persons or salesmen in various cities, allocation of resources for minimizing loss or maximising profits and reducing wastages.

Methodology of Operations Research:

(a) Formulating the Problem: The primary phase of OR is to develop a clear and concise statement of the problem. The problem must be formulated in the form of an appropriate model. For this, the following components should be taken into consideration:

(i) What are the objectives?

(ii) What are the uncontrollable variables that may effect the possible solutions?

(iii) What are the restrictions on the variables?