

UNIT-1

Review of Number Systems & codes

①

There are four types of number systems in digital electronics

- ① Decimal number system (Base 10)
- ② Binary number system (Base 2)
- ③ Octal number system (Base 8)
- ④ Hexa-decimal number system (Base 16)

Base is also known as radix.

If the base of any number system describes the no. of distinct symbols and the highest digit/number in that number system.

Ex: In hexa decimal number system, there are 16 symbols they are

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F

* List the first 20 numbers in hexadecimal number system

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F, 10, 11, 12, 13

* List the first 15 numbers in Base 12 system

0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, 10, 11, 12

* List the first 10 numbers in octal number system

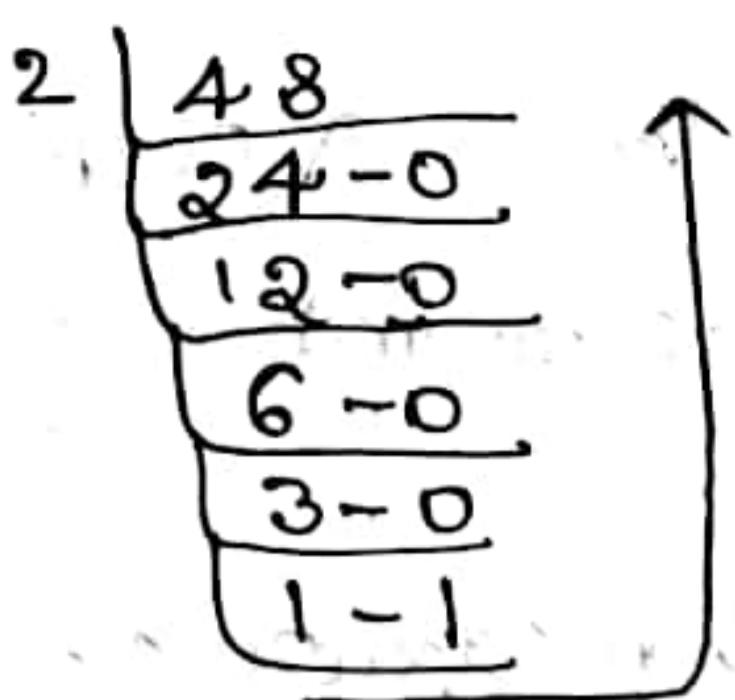
0, 1, 2, 3, 4, 5, 6, 7, 10, 11

Base Conversions:

To convert $()_{10}$ to $()_8$ perform the successive division of the given decimal number with required base upto the coefficient is less than the base.

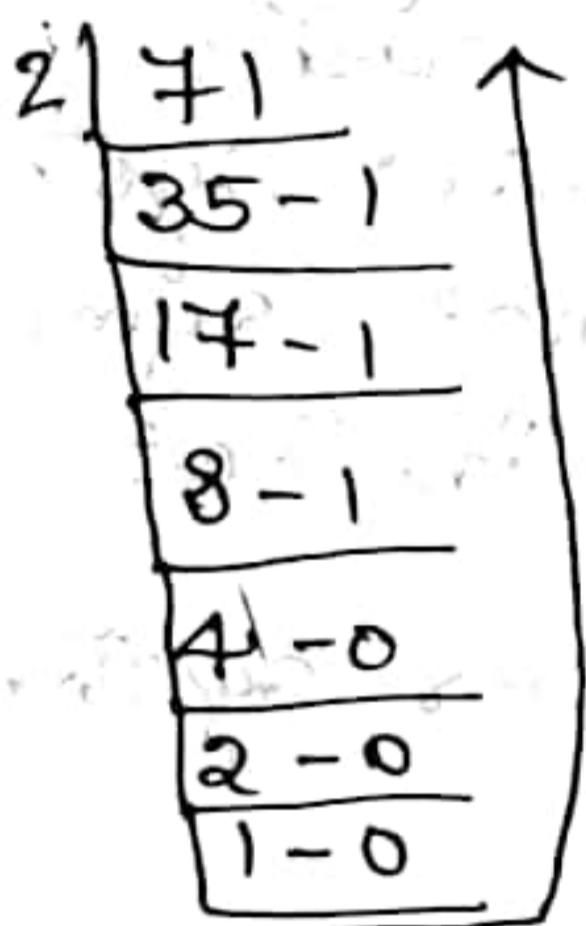
Ex: Convert $(48)_{10} = (?)_2$

(2)

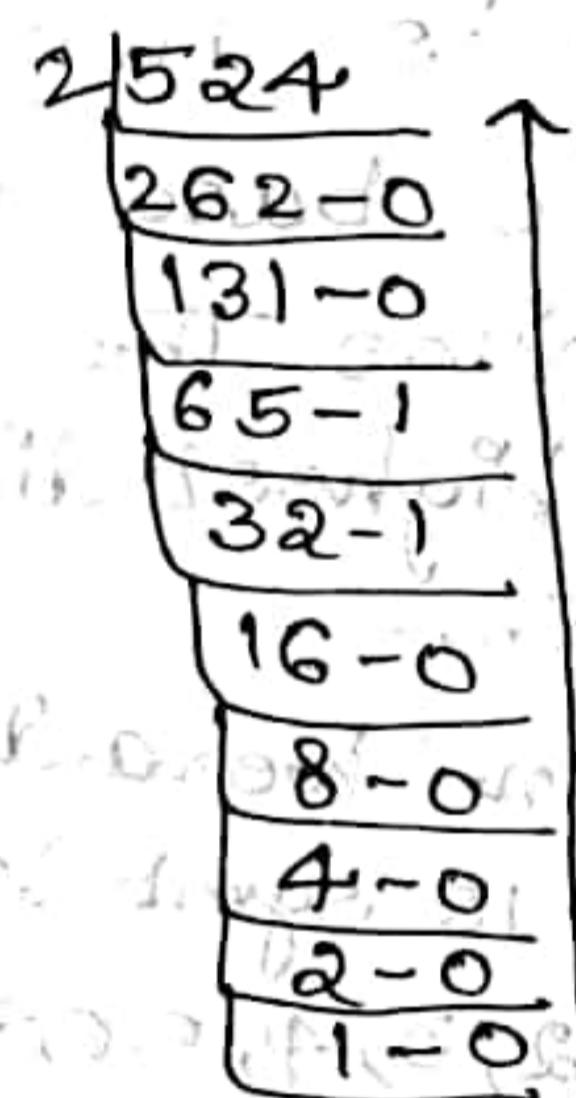


$$\rightarrow (48)_{10} = (110000)_2$$

*Convert $(71)_{10} = (?)_2$ *convert $(524)_{10} = (?)_2$

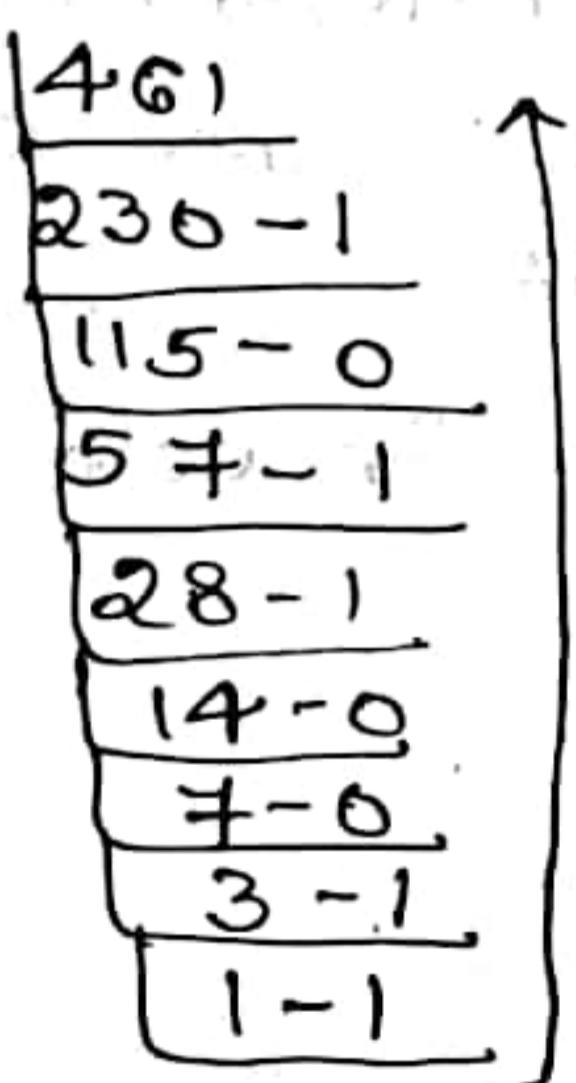


$$(71)_{10} = (1000111)_2$$



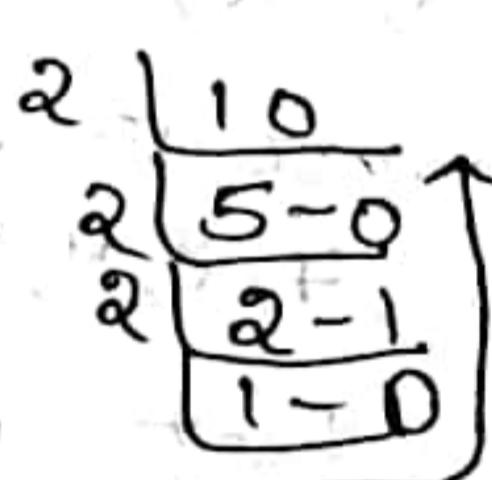
$$(524)_{10} = (1000001100)_2$$

*Convert $(461)_{10} = (?)_2$



$$(461)_{10} = (111001101)_2$$

*Convert $(10.125)_{10} = (?)_2$



$$\begin{aligned}
 125 \times 2 &= 0.250 \\
 250 \times 2 &= 0.500 \\
 500 \times 2 &= 1.000
 \end{aligned}$$

$$(10.125)_{10} = (1010.001)_2$$

* Convert $(39.225)_{10} = (?)_2$

$$\begin{array}{r} 2 | 39 \\ 2 | 19 - 1 \\ 2 | 9 - 1 \\ 2 | 4 - 1 \\ 2 | 2 - 0 \\ \hline 1 - 0 \end{array}$$

(3)

- $225 \times 2 = 0.450$
- $450 \times 2 = 0.900$
- $900 \times 2 = 1.800$
- $800 \times 2 = 1.600$

$$(39.225)_{10} = (100111.0011)_2$$

* Convert $(63.525)_{10} = (?)_2$

$$\begin{array}{r} 2 | 63 \\ 2 | 31 - 1 \\ 2 | 15 - 1 \\ 2 | 7 - 1 \\ 2 | 3 - 1 \\ \hline 1 - 1 \end{array}$$

- $525 \times 2 = 1.050$
- $050 \times 2 = 0.100$
- $100 \times 2 = 0.200$
- $200 \times 2 = 0.400$
- $400 \times 2 = 0.800$

$$(63.525)_{10} = (111111.1000)_2$$

Conversion of decimal to octal

* Convert $(25)_{10} = (?)_8$

$$\begin{array}{r} 8 | 25 \\ 8 | 3 - 1 \\ \hline \end{array}$$

$$(25)_{10} = (31)_8$$

* Convert $(125)_{10} = (?)_8$

$$\begin{array}{r} 8 | 125 \\ 8 | 15 - 5 \\ \hline 1 - 7 \end{array}$$

$$(125)_{10} = (175)_8$$

* Convert $(93)_{10} = (?)_8$

$$\begin{array}{r} 8 | 93 \\ 8 | 11 - 5 \\ \hline 1 - 3 \end{array}$$

$$(93)_{10} = (135)_8$$

* Convert $(154.125)_{10} = (?)_8$

$$\begin{array}{r} 8 | 154 \\ 8 | 19 - 2 \\ \hline 2 - 3 \end{array}$$

$125 \times 8 = 1.000$

$$(154.125)_{10} = (232.1)_8$$

* Convert $(23.225)_{10} = (?)_8$

$$\begin{array}{r} 8 | 23 \\ 8 | 2 - 7 \\ \hline \end{array}$$

$$(23.225)_{10} = (27.1681)_8$$

- $225 \times 8 = 1.800$
- $800 \times 8 = 6.400$
- $400 \times 8 = 3.200$
- $200 \times 8 = 1.600$

⇒ Conversion from decimal to hexadecimal

* Convert $(31)_{10} = (?)_{16}$ * Convert $(49)_{10} = (?)_{16}$ ④

$$\begin{array}{r} 16 \mid 31 \\ \hline 1-F \end{array}$$

$(31)_{10} = (1F)_{16}$

$$\begin{array}{r} 16 \mid 49 \\ \hline 3-1 \end{array}$$

$(49)_{10} = (31)_{16}$

* Convert $(62)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \mid 62 \\ \hline 3-E \end{array}$$

$(62)_{10} = (3E)_{16}$

* Convert $(74)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \mid 74 \\ \hline 4-A \end{array}$$

$(74)_{10} = (4A)_{16}$

* Convert $(70.250)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \mid 70 \\ \hline 4-6 \end{array}$$

$(70.250)_{10} = (46.4)_{16}$

$250 \times 16 = 4000$

* $(92.150)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \mid 92 \\ \hline 5-6 \end{array}$$

$(92.150)_{10} = (5C.2666)_{16}$

$150 \times 16 = 2400$

$400 \times 16 = 6400$

$400 \times 16 = 6400$

$400 \times 16 = 6400$

* Convert $(10.125)_{10} = (?)_4$

$$\begin{array}{r} 4 \mid 10 \\ \hline 2-2 \end{array}$$

$(10.125)_{10} = (22.02)_4$

$125 \times 4 = 0.500$

$500 \times 4 = 2.000$

(5)

8 to 10 conversions

Binary to decimal conversion

$$\rightarrow (1100000)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$$

$$\Rightarrow 32 + 16$$

$$\rightarrow \boxed{(48)_{10}}$$

$$\rightarrow (1000001100)_2$$

$$1 \times 2^9 + 0 + 1 \times 2^3 + 1 \times 2^2$$

$$512 + 8 + 4$$

$$\boxed{(524)_{10}}$$

$$\rightarrow (100011)_2$$

$$1 \times 2^6 + 0 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0$$

$$64 + 4 + 2 + 1$$

$$68 + 3$$

$$\boxed{(71)_{10}}$$

$$\rightarrow (111001101)_2$$

$$1 \times 2^8 + 1 \times 2^7 + 1 \times 2^6 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^0$$

$$256 + 128 + 64 + 8 + 4 + 2 + 1$$

$$\boxed{(461)_{10}}$$

$$\rightarrow (1010.001)_2 = (?)_{10}$$

$$1 \times 2^3 + 1 \times 2^1 + 0 \times 2^{-1} + 1 \times 2^{-3}$$

$$8 + 2 + \frac{1}{2^3} \Rightarrow 10 + 0.125$$

$$\boxed{(10.125)_{10}}$$

$$\rightarrow (100111.0011)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^4 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-3} + 1 \times 2^{-4}$$

$$\Rightarrow 32 + 16 + 8 + 1 + \frac{1}{2^3} + \frac{1}{2^4}$$

$$\Rightarrow 39 + 0.125 + 0.0625 \Rightarrow (39.1875)_{10} \approx$$

$$\rightarrow (111111.1000)_2 = (?)_{10}$$

$$1 \times 2^5 + 1 \times 2^4 + 1 \times 2^3 + 1 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1}$$

$$32 + 16 + 8 + 4 + 2 + 1 + 0.5 \Rightarrow \boxed{(63.5)_{10}}$$

Octal to decimal conversion

$$*(31)_8 = (?)_{10}$$

$$8^1 \times 3 + 1 \times 8^0$$

$$24 + 1$$

$$(25)_{10}$$

$$*(135)_8 = (?)_{10}$$

$$1 \times 8^2 + 3 \times 8^1 + 5 \times 8^0$$

$$64 + 24 + 5$$

$$(93)_{10}$$

$$*(232.1)_8 = (?)_{10}$$

$$2 \times 8^2 + 3 \times 8^1 + 2 \times 8^0 + 1 \times 8^{-1}$$

$$128 + 24 + 2 \times \frac{1}{8}$$

$$154 + 0.125$$

$$154.125$$

$$*(175)_8 = (?)_{10}$$

$$1 \times 8^2 + 7 \times 8^1 + 5 \times 8^0$$

$$64 + 56 + 5$$

$$(125)_{10}$$

$$*(24.1631)_8 = (?)_{10}$$

$$2 \times 8^1 + 7 \times 8^0 + 1 \times 8^{-1} + 6 \times 8^{-2} + 3 \times 8^{-3} + 8^{-4} \times 1$$

$$16 + 7 + 0.125 + 0.09375 + 0.00585 + 0.00024$$

$$(24.2248)_{10}$$

Hexadecimal to decimal

$$\rightarrow (1F)_{16} = (?)_{10}$$

$$1 \times 16^1 + 15 \times 16^0$$

$$16 + 15 \Rightarrow (31)_{10}$$

$$\rightarrow (31)_{16} = (?)_{10}$$

$$16^1 \times 3 + 16^0 \times 1$$

$$48 + 1 = (49)_{10}$$

$$*(5C.2666)_{16} = (?)_{10}$$

$$5 \times 16^1 + 12 \times 16^0 + 2 \times 16^{-1} + 6 \times 16^{-2} + 6 \times 16^{-3} + 6 \times 16^{-4}$$

$$80 + 12 + 0.125 + 0.0234 + 0.0014 + 0.00009$$

$$\Rightarrow (92.1498)_{10} \approx (92.150)_{10}$$

$$\rightarrow (3E)_{16} = (?)_{10}$$

$$3 \times 16^1 + E \times 16^0$$

$$48 + 14 \times 1$$

$$(62)_{10}$$

$$\rightarrow (46.4)_{16}$$

$$4 \times 16^1 + 6 \times 16^0 + 4 \times 16^{-1}$$

$$64 + 6 + 0.250 = (70.250)_{10}$$

$$(74)_{10}$$

$$(46.4)_{10}$$

$$(70.250)_{10}$$

Base 4 to Base 10

$$*(22.02)_4$$

$$2 \times 4^1 + 2 \times 4^0 + 0 + 2 \times 4^{-2}$$

$$8 + 2 + 0 + 2 \times \frac{1}{16}$$

$$10 + 0.125$$

$$(10.125)_{10}$$

(7)

8-8 Conversions:

In binary to octal conversion segment the given binary number as group and each group should contain 3 bit. Replace each group with its octal equivalent.

3-bit binary code

2	1	0
4	2	1
0	0	0 - 0
0	0	1 - 1
0	1	0 - 2
0	1	1 - 3
1	0	0 - 4
1	0	1 - 5
1	1	0 - 6
1	1	1 - 7

$$2^3 = 8$$

* Convert $(1011010)_2$ to $(?)_8$

sof
001|011|010
1 3 2

$$(132)_8$$

* $(1110101101101)_2$ to $(?)_8$

001|110|101|101||101
1 6 5 5 5

$$(16555)_8$$

$$*(1110111110110 \cdot 0110111)_2 = (?)_8$$

(8)

$$\begin{array}{r}
 111|011|111|110|110 \cdot 011|011|100 \\
 + 3 \quad + 6 \quad 6 \cdot 3 \quad 3 \quad 4 \\
 \hline
 (73+66 \cdot 334)_8
 \end{array}$$

Binary to hexadecimal :-

4 bit
binary
code

2^3	2^2	2^1	2^0	
8	4	2	1	
0	0	0	0	-0
0	0	0	1	-1
0	0	1	0	-2
0	0	1	1	-3
0	1	0	0	-4
0	1	0	1	-5
0	1	1	0	-6
0	1	1	1	-7
1	0	0	0	-8
1	0	0	1	-9
1	0	1	0	-10 - A
1	0	1	1	-11 - B
1	1	0	0	-12 - C
1	1	0	1	-13 - D
1	1	1	0	-14 - E
1	1	1	1	-15 - F

$$2^4 = 16$$

$$*(11001010)_2 = (?)_{16} \quad * (10)$$

(9)

$\begin{array}{r} 11001010 \\ \text{C} \quad \text{A} \end{array}$

$(\text{CA})_{16}$

$$*(10101110101011)_2 = (?)_{16}$$

$\begin{array}{r} 0010101110101011 \\ 2 \quad \text{B} \quad \text{A} \quad \text{B} \end{array}$

$(2BAB)_{16}$

$$*(101101010.101010111)_2 = (?)_{16}$$

$\begin{array}{r} 000101101010.101010111 \\ 1 \quad 6 \quad \text{A} \quad \text{A} \quad \text{B} \quad 8 \end{array}$

$(16AAB8)_{16}$

00010000000010001

Octal to Binary conversion

* In octal to binary conversion each digit of octal number should be replaced with its 3-bit binary equivalent

$$*(132)_8$$

$\begin{array}{r} 1 \mid 3 \mid 2 \\ 001 \mid 011 \mid 010 \end{array}$

$(001011010)_2$

$$*(16555)_8$$

$\begin{array}{r} 1 \mid 6 \mid 5 \mid 5 \mid 5 \\ 001 \mid 110 \mid 101 \mid 101 \mid 101 \end{array}$

$$*(73766.334)_8$$

$\begin{array}{r} 7 \mid 3 \mid 7 \mid 6 \mid 6 \mid 3 \mid 3 \mid 4 \\ 111 \mid 011 \mid 111 \mid 110 \mid 110 \cdot 011 \mid 011 \mid 100 \end{array}$

$(1110111110110 \cdot 0110111)_2$

Hexadecimal to binary:

(10)

* $(1011)_{16} = (?)_2$

$$\begin{array}{r} 1 \mid 0 \mid 1 \mid 1 \\ 0001000000010001 \end{array}$$

$$(0001000000010001)_2$$

* $(CA)_{16} = (?)_2$

$$\begin{array}{r} C \mid A \\ 1100 \mid 1010 \end{array}$$

$$(11001010)_2$$

Octal to hexadecimal conversions:

* to convert Octal to hexadecimal convert the octal number to binary and then to hexadecimal.

→ convert $(145)_8 = (?)_{16}$

(a) $(145)_8 = (?)_2$

$$\begin{array}{r} 1 \mid 4 \mid 5 \\ 001 \mid 100 \mid 101 \end{array}$$

$$(001100101)_2$$

(b) $0000 \mid 0110 \mid 0101$

0 6 5

$$(065)_{16} = (65)_{16}$$

$$\rightarrow (1372)_8 = (?)_{16}$$

(11)

(a) $(1372)_8 = (?)_2$.

$$\begin{array}{r} 001 \\ | \quad | \\ 1372 \\ | \quad | \\ 0111010 \end{array}$$

$$(00101111010)_2$$

(b) $(00101111010)_2 = (?)_{16}$.

$$\begin{array}{r} 0010 \\ | \quad | \quad | \quad | \\ 1111 \quad 1010 \\ 2 \quad F \quad A \end{array}$$

$$(2FA)_{16}$$

hexadecimal to octal

$$\rightarrow (DAC)_{16} = (?)_8$$

(a) $(DAC)_{16} = (?)_2$.

$$\begin{array}{r} D \quad | \quad A \quad | \quad C \\ 1101 \quad 1010 \quad 1100 \end{array}$$

$$(110110101100)_2$$

(b) $(110|110|101|100)_2 = (?)_8$

$$(6654)_8$$

*Note

10 → successive division

Y → powers multiplication

Y → if powers relation exists then grouping and use bit code else

use intermediate (decimal / binary) to convert

$$\rightarrow (1A8)_{16} = (?)_8$$

(a) $(1A8)_{16} = (?)_2$

$$\begin{array}{r} 1 \quad | \quad A \quad | \quad 8 \\ 0001 \quad 1010 \quad 1000 \end{array}$$

$$(000110101000)_2$$

(b) $000|110|101|000$

$$(650)_8$$

Binary addition:

$$\begin{array}{r} \rightarrow 8 \rightarrow 1 \ 0 \ 0 \ 0 \\ \rightarrow 4 \rightarrow 0 \ 1 \ 1 \ 1 \\ \hline 15 \rightarrow \underline{1 \ 1 \ 1 \ 1} \end{array}$$

Here $2 \rightarrow 10$

$$\begin{array}{r} \rightarrow 7 \ 0 \ 1 \ 1 \ 1 \\ 7 \ 0 \ 1 \ 1 \ 1 \\ \hline 14 \ \underline{1 \ 1 \ 1 \ 0} \\ \rightarrow 15 \ 1 \ 1 \ 1 \ 1 \\ 15 \ 1 \ 1 \ 1 \ 1 \\ \hline \underline{1 \ 1 \ 1 \ 1 \ 0} \end{array}$$

$$\begin{array}{r} \rightarrow 5 \ 0 \ 1 \ 0 \ 1 \\ 5 \ 0 \ 1 \ 0 \ 1 \\ \hline 10 \ \underline{1 \ 0 \ 1 \ 0} \end{array}$$

$$\begin{array}{r} \rightarrow 110 \ 10 \ 1 \\ 1 \ 0 \ 1 \ 0 \\ 1 \ 1 \ 1 \ 1 \\ 0 \ 1 \ 1 \ 1 \\ \hline \underline{1 \ 0 \ 0 \ 0 \ 0 \ 0} \end{array}$$

Binary subtraction:

$$\begin{array}{r} \rightarrow 8 \ 1 \ 0 \\ - 4 \ \underline{\underline{4}} \\ \rightarrow 4 \ 0 \ 0 \\ 1 \ 0 \ 1 \ 0 \\ - 0 \ 1 \ 0 \ 1 \\ \hline \underline{0 \ 1 \ 0 \ 1} \end{array}$$

$$\begin{array}{r} 2 \ 5 \\ + 7 \\ \hline \underline{1 \ 8} \end{array}$$

$$\begin{array}{r} 0 \ 1 \ 1 \ 2 \\ 0 \ 1 \ 2 \\ \hline 6 \ 0 \ 1 \ 1 \ 0 \ 8 \\ - 2 \ 0 \ 1 \ 0 \ 7 \\ \hline 4 \ 1 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \end{array}$$

$$\begin{array}{r}
 & 0 & 1 & 2 & 2 \\
 * & 1 & 6 & 1 & 0 & 0 & 0 & 0 \\
 1 & 4 & 0 & 1 & 1 & 1 & 0 \\
 \hline
 2 & 0 & 0 & 0 & 1 & 0
 \end{array}$$

$$\begin{array}{r}
 & 1 & 8 & 0 & 2 & 0 & 2 \\
 * & 1 & 3 & 0 & 1 & 1 & 0 & 1 \\
 5 & \hline
 0 & 0 & 1 & 0 & 1
 \end{array}$$

Complements:

For any base "r" system there exists two complements

1. r's complement
2. (r-1)'s complement

* To find r's complement the following formula is used i.e., $r^n - N$

where r - Base of number system

n - no. of integer digits in the given number

N - The number for which r's complement to be calculated

* To find (r-1)'s complement the following formula is used i.e., $(r^n - 1) - N$

Complements in binary number system.

1's complement

→ Find 1's complement of 101

$$N = 101$$

$$n = 3, r = 2$$

$$\text{Formula: } (r^n - 1) - N$$

$$(2^3 - 1) - N$$

$$(8 - 1) - 101$$

$$7(111) - 101$$

$$111 - 101$$

$$\boxed{010}$$

$$\begin{array}{r}
 111 \\
 101 \\
 \hline
 010
 \end{array}$$

1's complement of 101 is 010

→ Find 1's complement of 1010

1's complement of 1010 is 0101

→ 100011

1's complement is 011100

→ 2's complement

2's complement can be calculated by adding 1 to 1's complement of number.

* Find the 2's complement of following number

→ 1101

1's complement = 0010

Add 1 = 1

2's complement 0011

→ 1000

1's complement = 0111

Add 1 = 1

2's complement 1000

→ 10000

1's complement = 01111

Add 1 = 1

10000

→ 10110

1's complement = 01001

01010

By directly 01010

→ 10000

1's complement = 01111

11111
10000

Simply replace
1's with 0 & 0 with
1.

(14)

use
not in exams

* To find 2's complement directly there are two cases.

(i) if unit place have zero then from right to left do not change the number until 1 is reached then next numbers are interchanged by 0's as 1 & 1's as 0

(ii) if unit place have 1 then no exchange kept it as 1 & then next numbers are interchanged by 0 as 1 & 1 as 0

(1) 1010
2's comp Ans 01010

(2) 111101
2's comp 000011

1's Complement Subtraction

(15)

If $A - B$ is the required operation take the 1's complement of B & add it to A. After adding there are two cases

case(i): Carry generated.

If carry generated after the addition of A and 1's complement of B end-around the carry i.e., add carry to the LSB (Least significant bit)

case(ii): Carry not generated

If carry not generated after the addition, take the 1's complement of result again and put a '-' sign before the result.

Ex:

$$\begin{array}{r} 9 \rightarrow 1001 \\ (-) 4 \\ \hline 5 \end{array}$$

case(i)

$$\begin{array}{r} 9 \rightarrow 1001 \\ 1's\ complement \rightarrow 1011 \\ (+) \hline \text{carry } 1 \\ \hline 5 \rightarrow 0101 \end{array}$$

$$\begin{array}{r} * -8 \rightarrow 1000 \\ 3 \rightarrow 0011 \\ (-) \hline 5 \rightarrow 0101 \end{array}$$

case(ii):

$$\begin{array}{r} 8 \quad 1000 \\ -3 \quad (+) 1100 \\ \hline 5 \quad \text{carry } + \\ \hline 0101 \end{array}$$

case(i):

$$\begin{array}{r} 4 \quad 0100 \\ 9 \quad 0110 \\ (+) \hline 1010 \\ -0101 \rightarrow 1's\ complement \end{array}$$

case(ii):

$$\begin{array}{r} 3 \quad 0011 \\ 8(1's\ complement) \quad 0111 \\ (+) \hline 1010 \\ -0101 \rightarrow \text{carry not generated} \end{array}$$

* $10 \rightarrow 1010$

$\begin{array}{r} (-) 7 \rightarrow 0111 \\ \hline 3 \end{array}$

(16)

$10 \rightarrow 1010$

is comp of 7
 $\begin{array}{r} (+) 000 \\ \hline 10010 \\ (+) \quad \quad \quad 1 \\ \hline 0011 \end{array}$

$7 \rightarrow 0110$

$\begin{array}{r} 10 \quad 0110 \\ -3 \quad \hline 1100 \\ -0011 \end{array}$

* $15 \rightarrow 1111$

$\begin{array}{r} (-) 11 \\ \hline 4 \end{array} \rightarrow 1011$

is comp of 11
 $\begin{array}{r} 11 \quad 1011 \\ 0000 \\ \hline -4 \quad 1011 \\ -0100 \end{array}$

* $\begin{array}{r} 101101 \\ 010110 \\ \hline \end{array}$

case (i)
 $\begin{array}{r} 010110 \\ (+) 010010 \\ \hline 101000 \\ (-) 010110 \end{array}$

$\begin{array}{r} 101101 \\ (+) 101001 \\ \hline 1010110 \\ +1 \\ \hline 010111 \end{array}$

2's Complement Subtraction

(17)

If "A-B" is the required operation, takes the Two's Complement of "B" and Add it to "A". After addition there are two cases.

Case (i):

Carry generated. After adding "A + 2's complement of B" if carry is generated neglect carry and discard the

Case (ii):

Carry is not generated. After adding "A + 2's complement of B" if carry is not generated take the 2's complement of result again and put a (-) (minus) sign before it.

Ex:

$$* 8 \rightarrow 1000 \rightarrow 1000$$

$$\begin{array}{r} (-) 5 \rightarrow 0101 \rightarrow \\ \underline{-3} \end{array}$$

(+) 1011

0011

Ans. 3

$$\begin{array}{r} 5 \rightarrow 0101 \rightarrow \\ (-) 8 \rightarrow 1000 \rightarrow \\ \underline{-3} \end{array}$$

(+) 1000

1101

$$* 9 \rightarrow 1001 \rightarrow 1001$$

$$\begin{array}{r} (-) 4 \rightarrow 0100 \rightarrow \\ \underline{5} \end{array}$$

(+) 1100

1010

$$4 \rightarrow 0100 \rightarrow 0101 \rightarrow 5$$

$$\begin{array}{r} (-) 9 \rightarrow 1001 \rightarrow 0100 \rightarrow \\ \underline{-5} \end{array}$$

(+) 0111

1011

2's (-0101) → -5

* 23 1 0 111 → 1 0 111 (8)

$$\begin{array}{r} \text{(+)} \\ \text{(-)} \underline{0 1 0 1 0} \\ \text{(+)} \begin{array}{r} 1 \\ 0 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \\ \text{discard } \boxed{0 1 1 0 1} \end{array}$$

$$\begin{array}{r} 10 0 1 0 1 0 \rightarrow 0 1 0 1 0 \\ \text{(+)} \begin{array}{r} 1 \\ 0 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \\ \text{(+)} \begin{array}{r} 0 \\ 1 \end{array} \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \\ \hline 1 0 0 1 1 \\ \hline (-0 1 1 0 1) \end{array}$$

* 23 16 842
 $\begin{array}{r} 0 0 1 1 0 1 \rightarrow 0 0 1 1 0 1 \\ 1 1 1 1 0 1 \\ \text{(+)} \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 0 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \\ \hline 0 1 0 0 0 0 \\ \hline -1 1 0 0 0 0 \end{array}$

$$\begin{array}{r} 1 1 1 1 0 1 \rightarrow 1 1 1 1 0 1 \\ 0 0 1 1 0 1 \\ \text{(+)} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 0 \\ 0 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \begin{array}{r} 1 \\ 1 \end{array} \\ \hline 1 1 0 0 0 0 \\ \text{discard } \boxed{1 1 0 0 0 0} \end{array}$$

9's Compliment Subtraction:

If 'A-B' is the required operation take the 9's complement of B and add it to A. After adding there are two cases

case 1: Carry generated.

If carry generated after the addition of A and 9's complement of B. End around the carry i.e., add carry to LSB

case 2: Carry not generated

If carry not generated after the addition take the 9's complement of result again and put a '-' sign before the result.

$$* \begin{array}{r} 8 \rightarrow 8 \\ -5 \xrightarrow{9's \text{ comp}} \begin{array}{r} 4 \\ 12 \\ \hline 3 \end{array} \end{array} \quad \begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array}$$

$\uparrow f_1$ → end around carry

19

$$\begin{array}{r} 5 \rightarrow 5 \\ -8 \xrightarrow{9's \text{ comp}} \begin{array}{r} 1 \\ 6 \\ \hline -3 \end{array} \end{array} \quad \begin{array}{r} 9 \\ -8 \\ \hline 1 \end{array} \quad \begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array}$$

9's of 6

$$* \begin{array}{r} 9 \rightarrow 9 \\ -4 \xrightarrow{9's \text{ comp}} \begin{array}{r} (+) 5 \\ 14 \\ \hline 5 \end{array} \end{array}$$

$$\begin{array}{r} 9 \\ 4 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 4 \rightarrow 4 \\ -9 \xrightarrow{9's} \begin{array}{r} (+) 0 \\ 4 \\ \hline -5 \end{array} \end{array}$$

$$\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$$

*

$$* \begin{array}{r} 25 \rightarrow 25 \\ -19 \xrightarrow{9's \text{ comp}} \begin{array}{r} (+) 80 \\ 105 \\ \hline 6 \end{array} \end{array}$$

$$\begin{array}{r} 99 \\ 19 \\ \hline 80 \end{array}$$

$$\begin{array}{r} 19 \rightarrow 19 \\ -25 \xrightarrow{9's \text{ comp}} \begin{array}{r} (+) 74 \\ 953 \\ \hline -6 \end{array} \end{array}$$

$$\begin{array}{r} 99 \\ 25 \\ \hline 74 \\ -93 \\ \hline 6 \end{array}$$

$$* \begin{array}{r} 84 \rightarrow 84 \\ -27 \xrightarrow{9's \text{ comp}} \begin{array}{r} (+) 72 \\ 156 \\ \hline 57 \end{array} \end{array}$$

$$\begin{array}{r} 99 \\ 27 \\ \hline 72 \end{array}$$

$$27 \rightarrow 24$$

$$\begin{array}{r} 99 \\ 84 \\ \hline 15 \\ 99 \\ 42 \\ \hline 57 \end{array}$$

$$\begin{array}{r} 999 \\ 362 \\ \hline 634 \end{array}$$

$$(-) 84 \rightarrow (+) 15$$

$$9's \text{ of } 42 \quad \boxed{-57}$$

$$\begin{array}{r} 999 \\ 459 \\ \hline 540 \end{array}$$

$$* \begin{array}{r} 459 \rightarrow 459 \\ (-) 362 \xrightarrow{9's \text{ of } 6} \begin{array}{r} (+) 634 \\ 1096 \\ \hline 1097 \end{array} \end{array}$$

$$\begin{array}{r} 362 \rightarrow 362 \\ -(459) \rightarrow (+) 540 \\ \hline 902 \end{array}$$

$$\begin{array}{r} 999 \\ 902 \\ \hline 97 \end{array}$$

10's Complement Subtraction

(20)

If "A - B" is the required operation, takes 10's complement of B and add it to A. After addition there are two cases:

Case(i): Carry generated

If carry generated after adding A & 10's complement of B, if carry generated then discard the carry.

Case(ii): Carry not generated.

After adding A & 10's complement of B if carry not generated take 10's complement of result again and put a (-) minus sign before it.

$$\begin{array}{r}
 \text{PP} \\
 \text{PB} \\
 * \quad \begin{array}{r}
 9 \rightarrow 9 \\
 (-1) 3 \xrightarrow{\text{10's of } 3} 7 \\
 (+) \cancel{6} \\
 \hline \boxed{7} \quad \text{discard.}
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 9 \\
 (-) 3 \\
 \hline 6
 \end{array} \\
 \begin{array}{r}
 3 \rightarrow 3 \\
 - 9 \xrightarrow{\text{10's of } 9} 1 \\
 \hline 4
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 9 \\
 - 9 \\
 \hline 0
 \end{array} \\
 \begin{array}{r}
 10's \text{ of } 4 \quad \boxed{-6}
 \end{array} \quad \begin{array}{r}
 +1 \\
 \hline 1
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{PP} \\
 \text{PB} \\
 * \quad \begin{array}{r}
 8 \rightarrow 8 \\
 (-) 4 \xrightarrow{\text{10's of } 4} 6 \\
 (+) \cancel{4} \\
 \hline \boxed{4}
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 9 \\
 (-) 4 \\
 \hline 5
 \end{array} \\
 \begin{array}{r}
 4 \rightarrow 4 \\
 - 8 \xrightarrow{\text{10's of } 8} 6 \\
 \hline 2
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 9 \\
 - 8 \\
 \hline 1
 \end{array} \\
 \begin{array}{r}
 10's \text{ of } 6 \quad \boxed{-4}
 \end{array} \quad \begin{array}{r}
 +1 \\
 \hline 4
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \text{PP} \\
 \text{PB} \\
 * \quad \begin{array}{r}
 56 \rightarrow 56 \\
 - 43 \xrightarrow{\text{10's of } 43} 57 \\
 (+) \cancel{1} \\
 \hline \boxed{13}
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 99 \\
 - 43 \\
 \hline 56
 \end{array} \\
 \begin{array}{r}
 43 \rightarrow 43 \\
 - (56) \xrightarrow{\text{10's of } 56} 44 \\
 (+) \cancel{8} \\
 \hline 7
 \end{array} \quad \begin{array}{r}
 10's \text{ comp} \\
 99 \\
 - 56 \\
 \hline 43
 \end{array} \\
 \begin{array}{r}
 10's \text{ of } 7 \rightarrow \boxed{-13}
 \end{array} \quad \begin{array}{r}
 +1 \\
 \hline 44
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 99 \\
 - 87 \\
 \hline 12 \\
 + 1 \\
 \hline 13
 \end{array}$$

$$\begin{array}{r}
 * - 546 \rightarrow 546 \\
 - 307 \xrightarrow{\text{1's comp}} 693 \\
 \hline
 \end{array}
 \quad
 \begin{array}{r}
 999 \\
 307 \\
 \hline
 692 \\
 + 1 \\
 \hline
 693
 \end{array}
 \quad
 \begin{array}{r}
 307 \rightarrow 307 \\
 - 546 \rightarrow 454 \\
 \hline
 761
 \end{array}
 \quad
 \begin{array}{r}
 999 \\
 546 \\
 \hline
 453 \\
 + 1 \\
 \hline
 454
 \end{array}$$

10's comp 61 → 239

Binary codes:

Binary codes are classified into 6 types

1. Weighted Codes
2. Non-Weighted Codes
3. Sequential codes
4. Reflective codes
5. Alphanumeric codes
6. Error Correcting & detecting codes

Weighted Codes:

In weighted codes each place value is having a specific weight. Based on the weights, the codes are derived.

Ex: 8421, 2421, 5411 etc....

Non-Weighted codes:

These are the codes derived from weighted codes and no specific weight for place value

Ex: Gray code, Parity, excess 3 code etc.

Sequential codes:

In sequential codes the weights are continuously increasing from LSB to MSB

Ex: 8421.

Reflective codes:

In these codes zero is the 1's complement of 9, 1 is the 1's complement of 8

(P.T.O)

2 is the complement of 8 and so on.

Ex: 2 4 2 1

(22)

→ Write the 2421 code

2 4 2 1

0 0 0 0 0

1 0 0 0 1

2 0 0 1 0

3 0 0 1 1

4 0 1 0 0

5 1 1 0 1

6 1 1 0 0

7 1 0 0 1

8 1 1 1 0

9 1 1 1 1

Alphanumeric codes:

In Alphanumeric codes, each alphabet (Both upper case & lower case), numbers and symbols are having specific binary representation.

Ex: ASCII code

American Standard Code for
Information Interchange

Error Correcting & Error detecting codes

By using some of the binary codes errors can be detected but not corrected they are called error detecting codes.

By using some of the binary codes errors can be detected & corrected they are called error correcting code.

Ex² Hamming code.

BCD Codes (Binary Coded decimal)

In BCD code all the decimal digits from 0 to 9 are coded with binary numbers as shown below

	8	4	2	1
0-0	0	0	0	0
1-0	0	0	1	0
2-0	0	1	0	0
3-0	0	1	1	0
4-0	1	0	0	0
5-0	1	0	1	0
6-0	1	1	0	0
7-0	1	1	1	0
8-1	0	0	0	0
9-1	0	0	1	0

BCD addition

(24)

In BCD addition, after adding two BCD numbers, the result should be valid BCD number. If it is invalid add correction factor 6 (0110) to the invalid BCD number.

$$\begin{array}{r} * 5 \rightarrow 0101 \\ 3 \rightarrow 0011 \\ \hline 8 \rightarrow 1000 \end{array}$$

$$\begin{array}{r} * 5 \rightarrow 0101 \\ 5 \rightarrow 0101 \\ \hline 10 \rightarrow 1010 \end{array} \text{ Invalid BCD.}$$

$$\begin{array}{r} * 15 \rightarrow 00010101 \\ 12 \rightarrow 00010010 \\ \hline 27 \rightarrow 00100111 \end{array}$$

$$\begin{array}{r} * 39 \rightarrow 00111001 \\ 23 \rightarrow 00100011 \\ \hline 62 \rightarrow 01011100 \end{array} \text{ Invalid BCD.}$$

$$\begin{array}{r} 0110 0010 \\ 6 2 \\ + 6 \\ \hline 0110 0010 \\ 6 2 \end{array}$$

$$\begin{array}{r} * 8 \rightarrow 1000 \\ + 6 \rightarrow 0110 \\ \hline 14 \rightarrow 1110 \end{array} \text{ Invalid BCD.}$$

$$\begin{array}{r} + 6 \rightarrow 0110 \\ \hline 00010100 \\ 1 \quad 4 \end{array}$$

$$\begin{array}{r}
 * 28 \rightarrow 0010 \quad 1000 \\
 + 26 \rightarrow \begin{array}{r} 0010 \\ + 1 \\ \hline 0100 \end{array} \quad 0110 \\
 \hline 0101 \quad 0100 \quad + 6 \quad \begin{array}{r} 1110 \\ \hline 0110 \end{array} \rightarrow \text{Invalid BCD} \\
 \hline 5 \quad 4 \quad \begin{array}{r} 10110 \\ \hline 0101 \quad 0100 \\ \hline 5 \quad 4 \end{array}
 \end{array}$$

$$\begin{array}{r}
 * 45 \rightarrow 0100 \quad 0101 \\
 + 38 \rightarrow \begin{array}{r} 0011 \quad 1000 \\ + 110 \quad \hline 0111 \end{array} \rightarrow \text{Invalid BCD} \\
 \hline 83 \quad \begin{array}{r} 0110 \\ \hline 1000 \quad 0011 \\ \hline 8 \quad 3 \end{array}
 \end{array}$$

$$\begin{array}{r}
 * 157 \rightarrow 0001 \quad 0101 \quad 0111 \\
 + 138 \rightarrow \begin{array}{r} 0001 \quad 0011 \quad 1000 \\ + 111 \quad \hline 0010 \quad 1000 \quad 1111 \end{array} \\
 \hline 295 \quad \begin{array}{r} 0110 \\ \hline 0010 \quad 10010101 \\ \hline 2 \quad 9 \quad 5 \end{array}
 \end{array}$$

$$\begin{array}{r}
 * 867 \rightarrow 1000 \quad 0110 \quad 0111 \\
 + 574 \rightarrow \begin{array}{r} 0101 \quad 0111 \quad 0100 \\ + 1101 \quad 1101 \quad 1011 \end{array} \\
 \hline 1441 \quad \begin{array}{r} 0110 \\ \hline 0001 \quad 0100 \quad 0100 \quad 0001 \\ \hline 1101 \quad 1110 \quad 0001 \end{array}
 \end{array}$$

$$\begin{array}{r}
 + 6 \quad \begin{array}{r} 110110 \\ \hline 11100100 \quad 0001 \end{array} \\
 \hline \begin{array}{r} 00010100 \quad 0100 \quad 0001 \\ \hline 1 \quad 4 \quad 4 \end{array}
 \end{array}$$

(25)

BCD Subtraction:

(26)

If $(A - B)$ is required operation, take the 9's complement of B and add it to A . After adding there are two cases.

Case 1: Invalid BCD

If the sum is invalid BCD number validate it by adding 6 (0110). After addition of 6, the carry has to be end rounded.

Case 2: Valid BCD

After addition, if the result is valid BCD, take the 9's complement of result once again & put (-)minus sign before it.

Ex:

①

$$\begin{array}{r}
 8 \rightarrow 1000 \\
 (-) \cancel{4} \xrightarrow{\text{9's comp}} 0101 \\
 \hline
 4 \rightarrow \text{invalid BCD.}
 \end{array}$$

$$\begin{array}{r}
 1000 \\
 + 0101 \\
 \hline
 1101
 \end{array}$$

$$\begin{array}{r}
 1000 \\
 + 0101 \\
 \hline
 1001 \\
 + 1 \\
 \hline
 0100
 \end{array}$$

$$\begin{array}{r}
 4 \rightarrow 0100 \\
 (-) \cancel{8} \xrightarrow{\text{9's comp}} 0001 \\
 \hline
 -0100
 \end{array}$$

$$\begin{array}{r}
 0100 \\
 + 0101 \\
 \hline
 0101
 \end{array}$$

valid BCD

9's comp result \rightarrow 0100

②

$$\begin{array}{r}
 9 \rightarrow 1001 \\
 (-) \cancel{3} \xrightarrow{\text{9's comp}} 0110 \\
 \hline
 6 \rightarrow \text{invalid BCD.}
 \end{array}$$

$$\begin{array}{r}
 1001 \\
 + 0110 \\
 \hline
 1111
 \end{array}$$

$$\begin{array}{r}
 1001 \\
 + 1 \\
 \hline
 0110
 \end{array}$$

$$\begin{array}{r}
 3 \rightarrow 0011 \\
 (-) \cancel{9} \xrightarrow{\text{9's comp}} 0000 \\
 \hline
 -0011
 \end{array}$$

9's comp result is -0110

$$\begin{array}{r}
 * 25 \\
 - 13 \\
 \hline
 12 \\
 0001 \ 0010
 \end{array}
 \xrightarrow{\text{q's of 13}}
 \begin{array}{r}
 0010 \ 0101 \\
 (+) 1000 \ 0110 \\
 \hline
 1010 \ 1011 \\
 + 6 \\
 \hline
 1110 \\
 \hline
 1011 \ 0001
 \end{array}
 \quad
 \begin{array}{r}
 99 \\
 - 13 \\
 \hline
 86 \\
 27
 \end{array}$$

$$\begin{array}{r}
 + 9 \ 0110 \\
 \hline
 100010000 \\
 \downarrow \quad + 1 \\
 \hline
 00010010
 \end{array}
 \quad
 \begin{array}{r}
 99 \\
 - 87 \\
 \hline
 12
 \end{array}$$

$$\begin{array}{r}
 13 \\
 - 25 \\
 \hline
 -12
 \end{array}
 \rightarrow
 \begin{array}{r}
 0001 \ 0011 \\
 0011 \ 0100 \\
 + 0111 \ 0100 \\
 \hline
 10000111
 \end{array}
 \quad
 \begin{array}{r}
 99 \\
 - 25 \\
 \hline
 44
 \end{array}$$

$$\begin{array}{r}
 * 257 \\
 - 143 \\
 \hline
 114 \\
 0001 \ 0001 \ 0100
 \end{array}
 \rightarrow
 \begin{array}{r}
 0010 \ 0101 \ 0111 \\
 (+) 0001 \ 0100 \ 0011 \\
 \hline
 1010 \ 1010 \ 1101
 \end{array}
 \quad
 \begin{array}{r}
 999 \\
 - 143 \\
 \hline
 856
 \end{array}$$

$$\begin{array}{r}
 + 6 \\
 \hline
 101010110011
 \end{array}
 \quad
 \begin{array}{r}
 + 6 \\
 \hline
 101100010011
 \end{array}
 \quad
 \begin{array}{r}
 + 6 \\
 \hline
 100100010100
 \end{array}$$

$$\begin{array}{r}
 143 \rightarrow 0001 \quad 0100 \quad 0011 \\
 -257 \rightarrow 0111 \quad 0100 \quad 0010 \\
 \hline
 -114 \quad + \quad \hline
 \end{array}$$

$$\begin{array}{r}
 999 \\
 257 \\
 \hline
 742
 \end{array}$$

$$\begin{array}{r}
 999 \\
 885 \\
 \hline
 114
 \end{array}$$

q's of 885 is $\boxed{-0001 \quad 0001 \quad 0100}$

Gray Code:

Gray Code is a non weighted code, derived from binary code. In gray code there is only one bit change from one number to the immediate next number.

Q) Generate 4-bit gray code from single bit gray code.

Ans

$$\begin{array}{r}
 Q = 0 \quad 0 \quad 0 \quad 0 \\
 \hline
 Q = 1 \quad 0 \quad 0 \quad 1 \\
 \hline
 * 0 \rightarrow 0
 \end{array}$$

0 - 0	0 - 0 0	0 - 0 0 0
1 - 1	1 - 0 1	1 - 0 0 1
	$\frac{1-01}{2-11}$	2 - 0 1 1
3 - 1 0	3 - 0 1 0	3 - 0 0 1 0
	$\frac{3-01}{4-11}$	4 - 0 1 1 0
5 - 1 1 1		5 - 0 1 1 1
6 - 1 0 1		6 - 0 1 0 1
7 - 1 0 0		7 - 0 1 0 0
		$\frac{7-01}{8-11}$
		8 - 1 1 0 0
		9 - 1 1 0 1
		10 - 1 1 1 1
		11 - 1 1 1 0
		12 - 1 0 1 0
		13 - 1 0 1 1
		14 - 1 0 0 1
		15 - 1 0 0 0

Binary to Gray Code

(29)

To convert given 4-bit binary to its equivalent 4-bit gray code, the following formula is used.

Let the ^{given} 4-bit binary number is $(B_3 B_2 B_1 B_0)$ and its equivalent 4-bit gray code is $(G_3 G_2 G_1 G_0)$ then

$$G_3 = B_3$$

$$G_2 = B_3 \oplus B_2$$

$$G_1 = B_2 \oplus B_1$$

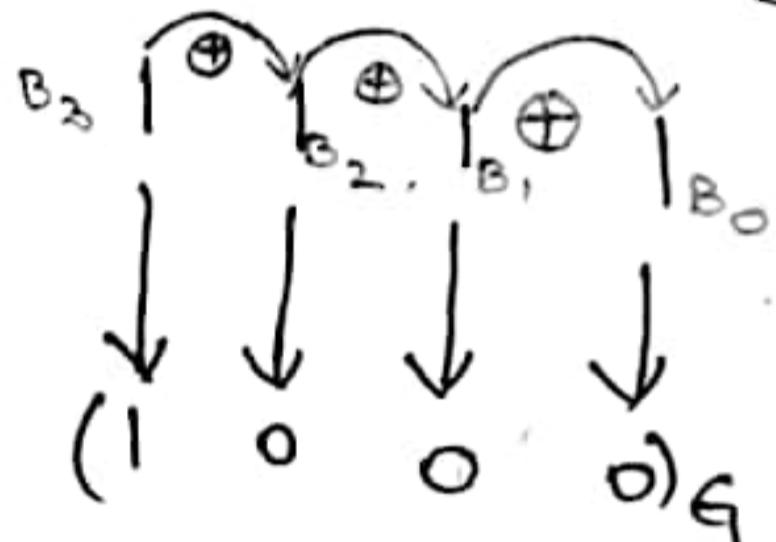
$$G_0 = B_1 \oplus B_0$$

Ex-OR truth table

A	B	y
0	0	0
0	1	1
1	0	1
1	1	0

→ Convert $(111)_B = (?)_G$

Ans:



$(101110)_B = (?)_G$

$\begin{array}{cccccc} 1 & 0 & 1 & 1 & 1 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 1 & 0 & 0 & 1 \end{array}$

$(111001)_G$

→ $(0111)_B = (?)_G$

Ans:

$\begin{array}{cccc} 0 & 1 & 1 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 0 & 1 & 0 & 0 \end{array}$

→ $(1001)_B = (?)_G$

$\begin{array}{cccc} 1 & 0 & 0 & 1 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 1 & 0 & 1 \end{array}$

→ $(1110)_B = (?)_G$

$\begin{array}{cccc} 1 & 1 & 1 & 0 \\ \downarrow & \downarrow & \downarrow & \downarrow \\ 1 & 0 & 0 & 1 \end{array}$

Gray code to Binary code Conversion: (30)

Let the given 4-bit gray code is $(G_3 G_2 G_1 G_0)$ and its equivalent binary is $(B_3 B_2 B_1 B_0)$. Then

$$B_3 = G_3$$

$$B_2 = G_3 \oplus G_2, \quad B_3 \oplus G_2.$$

$$B_1 = B_2 \oplus G_1$$

$$B_0 = B_1 \oplus G_0$$

→ Convert $(1000)_G = (?)_B$

$$\begin{array}{cccc} G_3 & G_2 & G_1 & G_0 \\ 1 & 0 & 0 & 0 \\ \downarrow & \nearrow & \downarrow & \nearrow \\ B_3 & B_2 & B_1 & B_0 \\ (111) & B \end{array}$$

→ $(1011)_G = (?)_B$

$$\begin{array}{cccc} 1 & 0 & 1 & 1 \\ \downarrow & \nearrow & \downarrow & \nearrow \\ 1 & 1 & 0 & 1 \\ (1101) & B \end{array}$$

→ $(0111)_G = (?)_B$

$$\begin{array}{cccc} 0 & 1 & 1 & 1 \\ \downarrow & \nearrow & \downarrow & \nearrow \\ 0 & 1 & 0 & 1 \\ (0101) & B \end{array}$$

Excess-3 code

Excess-3 code is a non weighted code delivered from BCD code for every BCD number, 3 is added to get excess-3 code

BCD	Excess-3
0 - 0000	$\rightarrow 0011$
1 - 0001	$\rightarrow 0100$
2 - 0010	$\rightarrow 0101$
3 - 0011	$\rightarrow 0110$
4 - 0100	$\rightarrow 0111$
5 - 0101	$\rightarrow 1000$
6 - 0110	$\rightarrow 1001$
7 - 0111	$\rightarrow 1010$
8 - 1000	$\rightarrow 1011$
9 - 1001	$\rightarrow 1100$

Excess-3 addition:

In excess-3 addition after adding two excess-3 numbers there are two cases

Case(i): No carry

If there is no carry after excess-3 addition subtract (0011) 3 from the result to get final result.

Case(ii): Carry generated

After excess-3 addition if carry generated add 3 (0011) to both sum & carry.

Ex:

$$\begin{array}{r}
 3 \rightarrow 0110 \\
 (+) 4 \rightarrow (+) 0111 \\
 \hline
 7 = 1010 \\
 (1010) \quad 0011 \text{ (sub-3)} \\
 \hline
 1010
 \end{array}$$

$$2 \rightarrow 0101$$

$$+ 3 \rightarrow 0110$$

$$\hline 85 \quad 1011$$

$$1000 \quad 0011 \text{ (Sub 3)}$$

$$\hline 1000$$

$$6 \rightarrow 1001$$

$$+ 7 \rightarrow 1010$$

$$\hline 13 \quad 0011$$

$$0011 \quad 0011 \text{ Add 3}$$

$$\hline 0100 \quad 0110$$

$$\hline 1 \quad 3$$

(32)

$$13 \rightarrow 0100 \quad 0110$$

$$+ 2 \rightarrow 0100 \quad 0101$$

$$\hline 25 \quad 1011$$

$$\frac{0101}{2} \quad \frac{1000}{5}$$

$$- 0011 \text{ Sub(3)}$$

$$\hline 1000 \quad 1000$$

$$\begin{array}{r} \text{Sub 3} \\ - 0011 \\ \hline 0101 \quad 1000 \end{array}$$

$$\hline \frac{2}{5}$$

$$38 \rightarrow 0110 \quad 1011$$

$$+ 25 \rightarrow 0101 \quad 1000$$

$$\hline 63 \quad 1100 \quad 0011$$

$$1001 \quad 0110$$

$$- 0011 \quad 0011 \text{ Sub 3}$$

$$\hline 1001 \quad 0110$$

$$45 \rightarrow 0111 \quad 1000$$

$$36 \rightarrow 0110 \quad 1001$$

$$\hline 81 \quad 1110 \quad 0001$$

$$\frac{1011}{8} \quad \frac{0100}{1}$$

$$0011 \cdot \text{Add 3}$$

$$\hline 1110 \quad 0100$$

Sub 3 0011

$$\begin{array}{r} 1011 \quad 0100 \\ \hline 1011 \quad 0100 \end{array}$$

$$93 \rightarrow 1100 \quad 0110$$

$$48 \rightarrow 0111 \quad 1011$$

$$\hline 141 \quad 0001 \quad 0001$$

$$0100 \quad 0111 \quad 0100$$

$$0011 \quad 0011 \quad 0011 \text{ Add (3)}$$

$$\hline 0100 \quad 0111 \quad 0100$$

Excess-3 Subtraction

(33)

In excess-3 subtraction if $A - B$ is the required operation then take the q's complement of B and add it to A. After addition there are two cases.

Case 1: Carry generated.

After addition if carry generated, end-around the carry and add to the sum to get the final answer.

Case 2: Carry not generated.

After addition if carry is not generated subtract 3 from the sum and once again take the q's complement of the result and put (-) sign before the result.

* Excess-3 is a selfcomplement code, because q's complement of q's complement is same.

Ex:

$$\begin{array}{r}
 6 \rightarrow 100\ 1 \\
 - 3 \xrightarrow{\text{q's} \oplus 3} 100\ 1 \\
 \hline
 3 \quad 100\ 1 \\
 0110 \quad \boxed{+} \quad \text{(end-around)} \\
 \hline
 00110
 \end{array}$$

$\frac{00110}{0110}$ Add 3(0011)

$$\begin{array}{r}
 3 \rightarrow 011\ 0 \\
 - 6 \xrightarrow{\text{q's} \oplus 6} 011\ 0 \\
 \hline
 -3 \quad 110\ 0 \\
 -0110 \quad \frac{0011}{\text{Sub } 3(0011)} \\
 \hline
 \text{q's result} \quad \frac{1001}{-0110}
 \end{array}$$

$$\begin{array}{r} *5 \\ -4 \xrightarrow{\text{9's comp}} \\ \hline 1 \end{array}$$

(34)

$$\begin{array}{r} 1000 \\ (+) 1000 \\ \hline 0000 \end{array}$$

end-around carry.

$$\begin{array}{r} 0001 \\ 0011 \\ \hline 0100 \end{array}$$

Add 3 (0011)

$$\begin{array}{r} 4 \\ -5 \\ \hline -1 \end{array}$$

$$\begin{array}{r} 0111 \\ (+) 1111 \\ \hline 1011 \end{array}$$

Sub 3 (0011)

$$\begin{array}{r} 0011 \\ (+) 1011 \\ \hline 0100 \end{array}$$

9's comp out (0100),

$$\begin{array}{r} * 26 \rightarrow 0101 \\ -14 \xrightarrow{\text{9's comp}} 0100 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 1001 \rightarrow 0101 \\ 0111 \xrightarrow{\text{1's comp}} (+) 1111 \\ \hline 0001 \end{array}$$

$$\begin{array}{r} 1001 \\ (+) 1111 \\ \hline 0001 \end{array}$$

$$\begin{array}{r} * 14 \rightarrow 0100 \\ -26 \xrightarrow{\text{9's comp}} 1010 \\ \hline 12 \end{array}$$

$$\begin{array}{r} 0111 \\ (+) 1101 \\ \hline 0011 \end{array}$$

(Sub 3)

$$\begin{array}{r} 1010 \\ (+) 1101 \\ \hline 0011 \end{array}$$

Sub 3

$$\begin{array}{r} 1011010 \\ (+) 0011 \\ \hline 1011010 \end{array}$$

9's comp out (0100 0101)

35

$$\begin{array}{r}
 * \quad 2 \ 5 \ 5 \rightarrow 0101 \ 1000 \ 1000 \rightarrow 0101 \ 1000 \ 1000 \\
 (-) \ 1 \ 6 \ 3 \rightarrow 0100 \ 1001 \ 0110 \xrightarrow{(+)} 1011 \ 0110 \ 1001 \\
 \hline
 0 \ 9 \ 2 \qquad\qquad\qquad 1 \ 0 \ 0 \ 0 \ 0 \ 1 \ 1 \ 1 \ 0 0 \ 0 1
 \end{array}$$

0011 1100 0101
~~0000 1111 0010~~
~~0011 0011 0011~~
~~0010 0101~~
 Add3 (+)
~~(0011)~~
~~Add3~~
 0000 1111 0010
 0010 0101
 0011
~~0000 1111 0101~~

$$\begin{array}{r}
 \text{(Add 3)} \quad 0011 \quad 0011 \\
 & (-) \downarrow \downarrow \downarrow \\
 \hline
 0011 & \underline{1} \cancel{\underline{1}} 0^0 & \underline{0} \underline{1} 0 \\
 & \underline{\quad\quad\quad} & \underline{\quad\quad\quad} \\
 & 0 & 2
 \end{array}$$

$$\begin{array}{r}
 \rightarrow 163 \xrightarrow{\quad} 0100 & 1001 & 0110 \\
 (-) 255 \xrightarrow{1's} \xrightarrow{(+)} 1010 & 0111 & 0111 \\
 \hline -092 & \hline 1111 & 111 & 11 \\
 & \hline 0000 & 0000 & 1101
 \end{array}$$

$$(c) \begin{array}{r} 0011 \\ + 0011 \\ \hline 1100 \end{array} \quad \begin{array}{r} 0011 \\ + 0011 \\ \hline 0000 \end{array} \quad \begin{array}{r} 0011 \\ + 0011 \\ \hline 1010 \end{array}$$

1's object

$$\begin{pmatrix} -0.011 & 0.00 & 0.101 \end{pmatrix}_i$$

$$\begin{array}{r}
 * \quad \begin{matrix} 2 \\ 3 \\ - 1 \end{matrix} \quad \begin{matrix} 17 \\ 88 \\ 15 \end{matrix} \rightarrow \begin{matrix} 0110 \\ 1011 \\ 1000 \end{matrix} \\
 - \quad \begin{matrix} 1 \\ 9 \\ 6 \end{matrix} \rightarrow \begin{matrix} 1011 \\ 0011 \\ 0110 \end{matrix} \\
 \hline
 \begin{matrix} 1 \\ 8 \\ 9 \end{matrix} \quad \begin{matrix} 10001 \\ 1110 \\ 1110 \end{matrix} \quad \begin{matrix} 999 \\ 196 \\ 803 \end{matrix}
 \end{array}$$

$$\begin{array}{r}
 0100 \quad 1011 \quad 1100 \\
 \hline
 & 000 & 110 & 1020^{\textcircled{2}} & 111 & @1 \\
 (+) 001' & & (-) 001' & & (-) 011' & \\
 \hline
 0100 & 1011 & 1100 \\
 \hline
 & 1 & 8 & 9
 \end{array}$$

36

$$\begin{array}{r}
 196 \rightarrow 0100 \quad 1100 \quad 1001 \\
 -385 \\
 \hline
 -189 \\
 \end{array}$$

q's of result

9 9 9

(1) 2000 0 1000 0 1000

Signed Binary numbers

37

In Signed binary numbers, one sign bit is allocated for representation of +ve numbers and -ve numbers. For +ve numbers sign bit is 0. For -ve numbers the sign bit is 1.

For Ex:

sign bit
0 0001 = +1
1 0001 = -1

- * The signed binary numbers can be represented in three ways.

(1) Sign magnitude representation,

(2) 1's complement ..

(3) 2's complement ..

Note:

*** In all three representations, +ve numbers are having unique representation, where as -ve numbers having different representations.

Ex:

$$\begin{array}{r} \text{+5} \\ \hline \end{array}$$

Sign magnitude 0 0000101

1's Complement 0 0000101

2's Complement 0 0000101

$$\begin{array}{r} -5 \\ \hline \end{array}$$

1 0000101

1 1111010

1 1111011

Ex' using 2's Complement

$$+7 = 00000111$$

$$+5 = 00000101$$

$$-5 = 11111011$$

$$-7 = 11111001$$

$$+2 = 00000010$$

$$-2 = 11111110$$

$$+12 = 00001100$$

$$-12 = 11110100$$

$$\begin{array}{r} +7 \rightarrow 00000111 \\ +5 \rightarrow 00000101 \\ \hline +12 \rightarrow 00001100 \end{array}$$

$$\begin{array}{r} +7 \rightarrow 00000111 \\ -5 \rightarrow 11111011 \\ +2 \rightarrow 00000010 \\ \hline \text{discard } 1 \rightarrow 00000010 \end{array}$$

$$\begin{array}{r} -7 \rightarrow 11111001 \\ +5 \rightarrow 00000101 \\ -2 \rightarrow 11111110 \\ \hline \end{array}$$

$$\begin{array}{r}
 +7 \rightarrow 11111001 \\
 -5 \rightarrow \\
 \hline
 -12 \xrightarrow{(+)} 11111011 \\
 \text{discard } \times 11110100
 \end{array}$$

solve above question using 1's complement
 * Using 1's Complement.

$$\begin{array}{l}
 +7 \rightarrow 00000111 \\
 +5 \rightarrow 00000101 \\
 -7 \rightarrow 11111000 \\
 -5 \rightarrow 11111010 \\
 +2 \rightarrow 00000010 \\
 -2 \Rightarrow 11111101 \\
 +12 \rightarrow 00001100 \\
 -12 \rightarrow 11110011
 \end{array}$$

$$\begin{array}{r}
 +7 \rightarrow 00000111 \\
 +5 \rightarrow \\
 \hline
 +12 \xrightarrow{(+)} 00001100
 \end{array}$$

$$\begin{array}{r}
 +7 \rightarrow 00000111 \\
 -5 \rightarrow \\
 \hline
 +2 \xrightarrow{(+)} 10000000 \\
 \hline
 00000001
 \end{array}$$

$$\begin{array}{r}
 -7 \rightarrow 11111000 \\
 +5 \rightarrow \\
 \hline
 -2 \xrightarrow{(+)} 11111101
 \end{array}$$

$$\begin{array}{r}
 -7 \rightarrow 11111000 \\
 -5 \rightarrow \\
 \hline
 -12 \xrightarrow{(+)} 11110010 \\
 \hline
 11110011
 \end{array}$$

L

⇒ ERROR DETECTING CODES

(39)

Parity bit is an extra bit which is added to the original bit. Parity is of two types
1. Even Parity
2. Odd Parity.

Even Parity: For even parity the no. of 1's in the information should be even including parity bit.

Odd Parity: For odd parity the no. of 1's in information is odd including parity bit.
Ex: Generate Even & odd parity for given message.

Given → 0 0 1 0 1 1 0

0 0 1 0 1 1 0 1 → even parity
0 0 1 0 1 1 0 0 → odd parity.

Block Parity: In block parity the parity is taken for both rows & columns for group of messages.

Ex:

Even parity

0 0 1 0 1 0	0
0 0 1 0 1 0	0
0 0 1 0 1 1	1
1 0 1 0 1 0	1
<hr/>	
1 0 0 , 0 0 1	

Error detection & correction codes

(40)

In the error detection & correction codes Hamming code is the best example. By using hamming code, error can be detected and it can be corrected.

Hamming code generation:

To generate hamming code for 'm' no. of message bits the following formula is used

$$2^P \geq m + p + 1$$

where $P = \text{no. of parity bits}$

Generate Hamming code for 1011.

Ans:

$$\begin{matrix} 1 & 0 & 1 & 1 \\ m_1 & m_2 & m_3 & m_4 \end{matrix}$$

$m = 4$

$$2^P \geq m + p + 1$$

$$2^P \geq 4 + p + 1 \quad [\because m = 4]$$

if $p = 0$

$$2^0 \geq 4 + 0 + 1$$

$$1 \geq 5$$

if $p = 1$

$$2^1 \geq 4 + 1 + 1$$

$$2^1 \geq 6$$

if $p = 2$

$$2^2 \geq 4 + 2 + 1$$

$$4 \geq 7$$

if $p = 3$

$$2^3 \geq 4 + 3 + 1$$

$$8 \geq 8$$

$$m = 4$$

$$p = 3$$

$$\text{Total} = \underline{7}$$

<u>1001!</u>	1	2	3	4	5	6	7
P_1	P_2	m_1	P_3	m_2	m_3	m_4	
			1		0	0	1
1,3,5,7	0		1		0	1	
2,3,6,7		0	1		0	1	
4,5,6,7			1	0	0	0	1

1010: 1 2 3 4 5 6 7 (42)

P₁ P₂ m₁ P₃ m₂ m₃ m₄

1 0 1 0

1,3,5,7

1	1	0	0
0	1	1	0
1	0	1	0

(101010)

1110:

1 2 3 4 5 6 7

P₁ P₂ m₁ P₃ m₂ m₃ m₄

1 1 1 0

1,3,5,7

0 1 1 0

2,3,6,7

0 1 1 0

4,5,6,7

0 1 1 0

(0010110)

Error detection and correction:

→ original: 0101010 $\xrightarrow{\text{err}}$ 0100010

1 2 3 4 5 6 7

0 1 0 0 0 1 0

C₁ → 1,3,5,7

0 0 0 0 → 0

C₂ → 2,3,6,7

1 0 0 0 → 0

C₃ → 4,5,6,7

0 0 1 0 → 1

C₃ C₂ C₁

1 0 0 → 1

∴ There is an error in 4th position.
the correct information is 0101010

$$\Rightarrow 0101010 \xrightarrow{\text{error}} 0101000$$

(43)

1	2	3	4	5	6	7
0	1	0	1	0	0	0
$c_1 \rightarrow 1, 3, 5, 7 \rightarrow 0$	0	0	0	0	0	$0 \rightarrow 0$
$c_2 \rightarrow 2, 4, 6, 7 \rightarrow 1$	0	0	0	0	0	$0 \rightarrow 1$

$c_3 \rightarrow 4, 5, 6, 7 \rightarrow$

1 0 0 0 $\rightarrow 1$

$c_3 c_2 c_1$

1 1 0 $\rightarrow 0$

∴ There is an error in 6th position.
The correct information is 0101010

$$\Rightarrow 0101010 \xrightarrow{\text{error}} 0001010$$

1	2	3	4	5	6	7
0	0	0	1	0	1	0
$c_1 \rightarrow 1, 3, 5, 7 \rightarrow 0$	0	0	0	0	0	$0 \rightarrow 0$
$c_2 \rightarrow 2, 3, 6, 7 \rightarrow 0$	0	0	0	0	1	$0 \rightarrow 1$

$c_3 \rightarrow 4, 5, 6, 7 \rightarrow$

1 0 0 1 0 $\rightarrow 0$

$c_3 c_2 c_1$

0 1 0 $\rightarrow 2$

∴ There is an error in 2nd position, The
Correct information is 0101010.

$$\Rightarrow 0101010$$

1	2	3	4	5	6	7
0	1	0	1	0	1	0

$c_1 \rightarrow 1, 3, 5, 7 \rightarrow 0$

0 0 0 0 $\rightarrow 0$

$c_2 \rightarrow 2, 3, 6, 7 \rightarrow 1$

1 0 0 1 0 $\rightarrow 0$

$c_3 \rightarrow 4, 5, 6, 7 \rightarrow$

1 0 1 0 $\rightarrow 0$

$c_3 c_2 c_1$

0 0 0

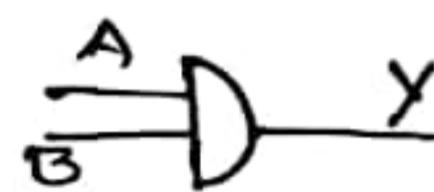
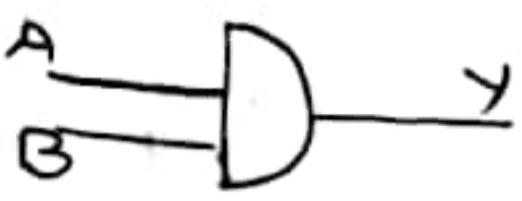
No error.

Logic Gates:

44

- (1) Basic gates \rightarrow AND, OR, NOT
- (2) Universal gates \rightarrow NAND, NOR
- (3) Special gates \rightarrow EX-OR, EX-NOR

AND Gate



A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$Y = A \cdot B \\ = AB$$

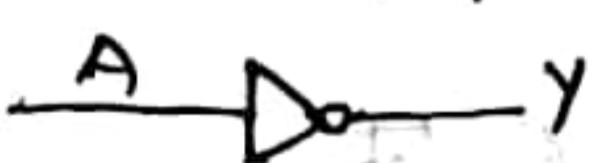
OR Gate



$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

NOT Gate



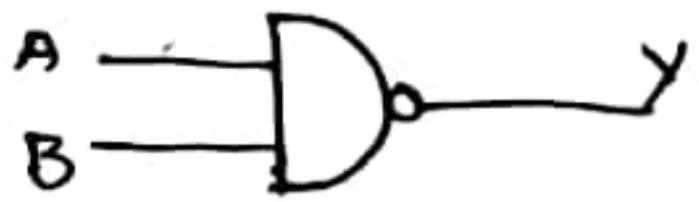
$$Y = \overline{A} = A'$$

A	Y
0	1
1	0

NAND Gate :

(45)

NAND gate is AND gate followed by NOT gate



A	B	y
0	0	1
0	1	1
1	0	1
1	1	0

$$y = \overline{A \cdot B}$$

$$= \overline{AB}$$

Any boolean function can be purely implemented with these two gates (NAND, NOR)

NOR Gate :

NOR gate is OR gate followed by NOT gate.



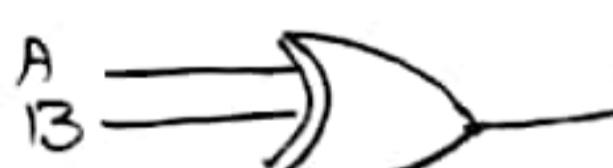
A	B	y
0	0	1
0	1	0
1	0	0
1	1	0

$$y = \overline{(A+B)}$$



Special gates:

→ Ex-OR: (Inequality gate)



A	B	y
0	0	0
0	1	1
1	0	1
1	1	0

$$Y = A \oplus B$$

$$= A\bar{B} + \bar{A}B$$

Important:
Property: $A \oplus 0 = A$
 $A \oplus 1 = \bar{A}$

Ex-NOR (equality gate)

46



A	B	y
0	0	1
0	1	0
1	0	0
1	1	1

$$\begin{aligned}
 y &= A \oplus B \\
 &= AB + \overline{A}\overline{B} \\
 &= AB + \overline{A}\overline{B}
 \end{aligned}$$

To find complement of AND & OR gates

$$\begin{array}{ccc}
 1 & 0 & 0 \\
 1 & 1 & 0 \\
 0 & 1 & 0 \\
 0 & 0 & 1
 \end{array}$$

Stop today

See you tomorrow