Q7: A dealer supplies you the following information with regard to a product dealt by him:

Annual demand:10,000 units; **Ordering cost:** Rs. 10/- order;

Price=Rs. 20/- unit; Inventory carrying cost: 20% of the value of inventory/- year.

The dealer is considering the possibility of allowing some back order to occur. He has estimated that the annual cost of backordering will be 25% of the value of inventory.

(a) What should be the optimal number of units of product he should buy in one lot?

(b) What quantity of the product should be allowed to be back-ordered, if any?

(c) What would be the maximum quantity of inventory at any time of the year?

(d) Would you recommend to allow back-ordering? If so, what would be the annual cost saving by adopting the policy of back-ordering.

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Transportation Problem

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- A transportation problem has a number of origins and a number of destinations. The transportation problem indicates the amount of consignment to be transported from various origins to different destinations so that the total transportation cost is minimized without disturbing the availability constraints and the requirement constraints.

- It is a special type of LPP where the objective is to minimise the cost of distributing a product to a number of destinations.

- In a transportation problem, the total amount of supply available at the origin and the total quantity demanded by the destinations are given in the problem. The cost of shipping a unit of goods from a known origin to a known destination is also given.

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Types of Transportation Problem:

(a) **Balanced Transportation Problem:** When the total availability at the origin is equal to the total requirments at the destinations, it is called a balanced transportation problem.

(b) **Unbalanced Transportation Problem:** When the total availability at the origins is not equal to the total requirements at the destinations, it is called unbalanced transportation problem.

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Basic Concepts:

(a) **Feasible Solution:** A feasible solution is that solution which satisfies the requirements of demand and supply. This is the solution which simultaneously removes all the existing surpluses and satisfies all the existing deficiencies.

(b) **Initial Basic Feasible Solution:** An initial basic feasible solution with an allocation of (m+n-1) number of variables, is called a basic feasible solution, i.e. "1" less than the number of rows and columns in the transportation table.

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(c) Optimal Solution: A feasible solution is said to be optimal if it minimizes the total transportation cost.

(d) When the total capacity equals total requirement, the problem is called as balanced transportation problem, otherwise it is known as unbalanced transportation problem.

(e) Degenerate Problem: When the number of positive allocations at any stage of the feasible solution is less than the required number (n+m–1), the solution is said to be degenerate solution, otherwise non-degenerate solution.

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Steps in solving transportation problem:

(a) First check whether total demand is equal to total supply. If yes, it is a balanced problem. If not, introduce a imaginary origin/destinations, to make the problem balanced one.

(b) Find an initial basic feasible solution.

(c) After obtaining the IBFS, check whether number of allocations are equal to (m+n-1); if yes proceed for optimal solution otherwise treat it as degeneracy.

(d) Calculate the total minimum cost.

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Method of Finding an Initial Basic Feasible Solution:

(a) North – West Corner Method.

(b) Lowest Cost Entry Method.

(c) Vogel’s Approximation Method (VAM)

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North-West Corner Method:

(a) Firstly select the upper left hand corner cell which is in the North-West corner of the table, and allocate units equal to the minimum of S1 and D1 against the supply and demand quantities in the respective rows and columns.

(b) If the quantity of supply for the first row is exhausted then proceed down to the cell in the second row and first column and adjust as required.

(c) If the quantity of demand for the first column is exhausted then move horizontally to the next cell in the second column and second row and adjust as required.

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(d) If, at any cell, the supply-quantity equals the demand-quantity then the next allocation has to be made in the cell either in the next row or column.

(e) The procedure can be continued until the available supply and demand quantities are allocated properly to the concerned cells as required.

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Q: Solve the following transportation problem by NWC method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

Least Cost Method (Matrix Minima Method): According to this method, we take into consideration the lowest cost for the purpose of appropriate allocation. The following steps are being followed:

(a) Select the cell with the lowest cost amongst all the figures of cost in all the rows and columns of the given data.

(b) To this selected cell in (a) allocate all the possible number of units either supply or demand as the case may be.

(c) According to step (b) when the demand gets satisfied or the supply gets exhausted, we eliminate the concerned row or column. Steps (a) and (b) is repeated till the supply at various plants gets exhausted according to the demand from the concerned warehouses.

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Steps in solving transportation problem:

(a) First check whether total demand is equal to total supply. If yes, it is a balanced problem. If not, introduce a imaginary origin/destinations, to make the problem balanced one.

(b) Find an initial basic feasible solution.

(c) After obtaining the IBFS, check whether number of allocations are equal to (m+n-1); if yes proceed for optimal solution otherwise treat it as degeneracy.

(d) Calculate the total minimum cost.

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Q: Solve the following transportation problem with the help of least cost method.

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

VOGEL'S APPROXIMATION METHOD (VAM):

(a) For each row and column, find the difference between the smallest cost and the next smallest cost in the concerned row or column. Each such difference is called a penalty.

(b) Find the row or column with the largest penalty and in this row or column select the cell having the smallest cost and allocate the maximum possible quantity to this cell. The row or column for which the supply gets exhausted or the demand gets satisfied, becomes a deleted row or column.

(c) The process stated in (a) and (b) is repeated till the entire supply at the different plants gets exhausted by satisfying the demand at the various wareshouses.

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Q: Solve the following transportation problem with the help of Vogels' Approximation method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	10	**7**
F2	70	30	40	60	**9**
F3	40	8	70	20	**18**
Requirement (demand)	**5**	**8**	**7**	**14**	

Methods of finding the Optimum Solution:

- Find an initial basic feasible solution using any of the three methods.
- Test the initial basic feasible solution for optimality using any of the following methods:

(a) Stepping Stone Method.

(b) Modified Distribution method (MODI)

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STEPPING STONE METHOD:

(a) Find the initial basic feasible solution by using any of the methods.

(b) Identify each unoccupied cell and follow its closed path, so as to determine its net cost change. If all the net cost changes have zero or positive sign, then the solution becomes optimal. If any negative net change or changes is found, then find out the unoccupied cell with the largest negative value and carry out allocation process.

(c) After finding the quantity to be appropriately allocated to the selected unoccupied cell, follow the closed path for this cell and with reference to the negative sign in this path identify the minimum quantity. By allocating this quantity find the new solution.

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Q: Solve the following transportation problem and find the optimal solution:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	30	25	40	20	**100**
F2	29	26	35	40	**250**
F3	31	33	37	30	**150**
Requirement (demand)	**90**	**160**	**200**	**50**	

MODIFIED DISTRIBUTION METHOD (MODI):

In the MODI method, the improvement cost of all the unoccupied cells are calculated, without drawing their respective closed paths. Only one closed path is drawn after the unused square with the highest negative value is identified.

Various steps of the MODI method are:

(a) Determine the IBFS using any of the three methods.

(b) For each occupied cell in the current solution, solve the system of (m+n-1) equations:

$$\mathbf{Ui + Vj = Cij}$$

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Since the number of unknown **Ui** and **Vj** are (m + n), we can assign an arbitrary value to **Vj = 0** or **Ui = 0** for each unoccupied basic cell and enter them in the upper right corner of the corresponding cell.

$$\Delta ij = \mathbf{Cij} - \mathbf{(Ui + Vj)}.$$

(c) Examine the sign of each opportunity cost. If $\Delta ij > 0$, the given solution is an optimal one. If atleast one $\Delta ij < 0$, the given basic feasible solution is not an optimum one and further savings in the transportation cost are possible.

(d) Select the unoccupied cell with the largest negative Δij as the cell to be included in the next solution.

(e) Trace a closed path for the unoccupied cell selected in above step.

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(f) Assign alternate plus and minus sign.

(g) Determine the maximum number of units that should be shipped to this unoccupied cell.

(h) This whole procedure is repeated till we obtain an optimal solution.

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Q: Find the optimal solution of the following transportation problem using VAM method:

Factory	WAREHOUSES				Capacity (Supply)
	W1	W2	W3	W4	
F1	19	30	50	12	**7**
F2	70	30	40	60	**10**
F3	40	10	60	20	**18**
Requirement (demand)	**5**	**8**	**7**	**15**	

Q: **Solve the following transportation problem and find the optimal solution using Vogel's Approximation method:**

Factory	WAREHOUSES					Capacity (Supply)
	W1	W2	W3	W4	W5	
F1	4	1	3	4	4	**60**
F2	2	3	2	2	3	**35**
F3	3	5	2	4	4	**40**
Requirement (demand)	**22**	**45**	**20**	**18**	**30**	

Variations in Transportation Problem:

(a) **Unbalanced transportation problem:** The transportation problem in which the total availability is not equal to the total requirement is called unbalanced transportation problem.

STEPS:

(b) Add one more imaginary origin or warehouse where availability is more as compared to requirement.

(c) Add one more imaginary destination where requirement is more as compared to availability. The main condition of adding a new destination is that the true transportation cost of the units should be kept zero.

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Q: Consider the following transportation problem and find out the optimum answer.

Factory	WAREHOUSES			Capacity (Supply)
	W1	W2	W3	
F1	8	16	16	**152**
F2	32	48	32	**164**
F3	16	32	48	**154**
Requirement (demand)	**144**	**204**	**82**	

(b) **Degeneracy:** If the number of occupied cells is less than (m + n – 1) at any stage of the solution, then the problem is said to have a degenerate solution. Degeneracy can occur at 2 stages: (1) at the initial solution (2) During testing of optimal solution.

(i) **Degeneracy occurs at the initial solution:** To resolve degeneracy at the initial stage, an artificial quantity denoted by the greek letter (€) is used in one or more of the unoccupied cells so that the number of occupied cells is (m + n – 1)

The quantity of € is so small that it does not affect the supply and demand constraints.

The € value is assumed to be zero when actually used in the movement of the goods from one cell to another.

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Once an "€" is introduced into the solution, it will remain there until degeneracy is removed or a final solution is arrived at.

Q: Determine an optimal distribution for the company in order to minimize the total cost.

Factory	WAREHOUSES					Capacity (Supply)
	W1	W2	W3	W4	W5	
F1	5	8	6	6	3	**80**
F2	4	7	7	6	6	**50**
F3	8	4	6	6	3	**90**
Requirement (demand)	**40**	**40**	**50**	**40**	**80**	

Q: Solve the following transportation problem:

Factory	WAREHOUSES			Capacity (Supply)
	W1	W2	W3	
F1	50	30	220	**1**
F2	90	45	170	**3**
F3	250	200	50	**4**
Requirement (demand)	**4**	**2**	**2**	

(b) <u>Degeneracy occurs during the testing of the Optimal Solution:</u>

Degeneracy during the solution stage occurs when the inclusion of the unoccupied cell with maximum negative opportunity costs results in vacating of 2 or more occupied cells. This problem is resolved by allocating € to one or more of the vacated cells to complete the required (m + n – 1) conditions.

Q: Solve the transportation Problem:

	W	X	Y	SUPPLY
A	7	3	6	5
B	4	6	8	10
C	5	8	4	7
D	8	4	3	3
DEMAND	5	8	10	

Transportation problems of Maximum Profit:

In some transportation problems, information is given in the form of per unit profit, to solve such problems highest profit value from the profit matrix is selected and all values are subtracted from this highest value. After obtaining this matrix, same procedure is applied.

Q: Solve the following transportation problem for maximum profit:

	A	B	C	D	SUPPLY
X	12	18	6	25	200
Y	8	7	10	18	500
Z	14	3	11	20	300
DEMAND	180	320	100	400	

REPLACEMENT THEORY

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- Replacement theory is concerned with the problem of replacement of machines due to their deteriorating efficiency, failure or breakdown.
- **Replacement theory deals with:**

(a) When existing items have outlived their effective lives and it may not be economical to continue with them.

(b) Items which might have been destroyed either by accident or otherwise.

(c) Replacement of items that deteriorate with time.

(d) Replacement of an equipment that becomes out of date due to new developments.

(e) Improved technology has given access to much better and technically superior products.

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Causes of Replacement:

(a)Deterioration: It involves reduction in the value of an asset due to wear & tear.

(b)Obsolescence: It may occur due to advancement of technology.

(c)Inadequacy: The capacity of an equipment may be inadequate to meet the demand or to increase the production to a desired level.

(d)Working conditions: Old equipment may become noisy, smoking or unsafe for the workers.

(e)Economy: Existing equipment may outlived their effective life and it is not viable to continue with them.

(f)Sudden failure: The existing equipment may destroy all of a sudden.

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Characteristics of Replacement:

(a)Replacement reduces maintenance cost but it involves a high average capital cost.

(b)Equipment must be constantly reviewed and updated at certain intervals to avoid its obsolescence rather than watching until it is physically out of order.

Replacement models and their solutions: It involves comparison of alternative replacement policies and the factors relevant to the replacement are as follows:

(a)Technical: It includes deterioration, obsolescence and inadequacy.

(b)Financial: It includes initial cost, running cost, labour cost, operating cost, salvage value and insurance.

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<u>Various types of Replacement Problems</u>:

(a)Replacement policy for equipment that deteriorates gradually.

(b)Replacement policy for items that fail suddenly.

(c)Staff replacement problems.

<u>Failure Mechanism of Items</u>: There are two types of failure:

(a) Gradual Failure: The failure mechanism under gradual failure is progressive, i.e. as the life of an item increases, its efficiency deteriorates resulting in:

(i) increased expenditure for operating costs.

(ii) decreased productivity of the equipment.

(iii) decrease in the value of equipment, i.e. resale value decreases.

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(b) Sudden Failure: This class of failure is applicable to those items that do not deteriorate with service but which ultimately fail after a period of use. Sudden failures can be progressive, retrogressive or random.

(i) Progressive Failure: Under this, the probability of failure increases with increase in the life of an item.

(ii) Retrogressive Failure: Under this, items have more probability of failure in the beginning of their life and as time passes, the chances of failure become less.

(iii) Random Failure: Under this, constant probability of failure is associated with items that fail from random causes such as physical shocks, not related to age.

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Methodology of solving replacement problems:

(a)Identify the items to be replaced and also their failure mechanism which can be sudden or gradual.

(b) Collect the data relating to the depreciation cost and the maintenance cost over a time period for the items which follow gradual failure. In case of sudden failure, collect the data for failure rates and cost of preventive replacement.

(c) By using the above data, develop a suitable model in OT for determining the exact time of replacing the equipment.

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Replacement Policy for Equipment/Asset ;which deteriorates gradually:

•It is economical to replace the equipment with a new one when operational efficiency of the equipment deteriorate by comparing the number of alternative choices available on the basis of average maintenance and operating cost involved.

(a) Replacement policy for items whose running cost increases with time without change in the value of money during a period: Cost of an equipment over a given period of time has the following three elements:

(i) C = Purchase price of the equipment.

(ii) S = Scrap value of the equipment taken to be same over n years.

(iii) g(t) = Maintenance/running cost of the equipment at time t.

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Thus, **the annual cost of the machine at any time t**

= Capital cost – scrap value + maintenance (or running) cost at time t.

<u>Now g(t) can be discrete or continuous function of time.</u>

- If g(t) is a discrete function, then summation sign is used to find the total maintenance cost in a given period.

$$g(t) = \sum_{t=0}^{n} \mathrm{g(t)}$$

- If g(t) is a continuous function, then integrals can be used.

$$\mathrm{g(t)} = \int_0^n g(t)dt$$

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Average Cost, A(n) would be defined as:

$$A(n) = \frac{1}{n}[c - s + \sum g(t)]$$

- It is advisable to replace the item when A(n) is maximum. However, A(n) will be minimum when:

  $g(n) < A(n-1)$ and

  $A(n) < g(n+1)$

- Replace the item in the year in which the average cost is minimum.

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Q1: A firm is using a machine whose purchase price is Rs. 13,000. The installation charges amount to Rs. 3600 and the machine has a scrap value of only 1600. The maintenance cost in various years is given in the following table. The firm wants to determine after how many years should the machine be replaced on economic considerations, assuming that the machine replacement can be done only at the year ends.

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Cost (Rs)	250	750	1000	1500	2100	2900	4000	4800	6000

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Q2: A company has a machine whose purchase price is Rs. 80,000. The expected maintenance costs and resale price in different years are as given below. After what time interval, in your opinion, should the machine be replaced?

Year	**1**	**2**	**3**	**4**	**5**	**6**	**7**
Maintenance cost	1000	1200	1600	2400	3000	3900	5000
Resale value (‘000)	75	72	70	65	58	50	45

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Q3: (a) Machine A costs Rs. 9,000. Annual operating costs are Rs. 200 for the first year, and then increase by Rs. 2,000 every year. Determine the best age at which to replace the machine. If the optimum replacement policy is followed, what will be the average yearly cost of owning and operating the machine?

(b) Machine B costs Rs. 10,000. Annual operating costs are Rs. 400 for the first year, and then increase by Rs. 800 every year. You now have a machine of type A which is one year old. Should you replace it with B; if so when?

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Replacement Policy of Equipment/Asset whose running cost increases with time but value of money changes at constant rate:

•In this case, we have to discount the future payments on maintenance cost so as to express them all in terms of present value. In this, a machine has to be replaced when weighted average annual cost is minimum.

•**Notations:**

(a) C = initial cost (or purchase price) of the item to be replaced.

(b) g(t) = Running (or maintenance) cost in *t*th year.

(c) r = rate of interest

(d) v = 1/(1+r) is the present worth of a rupee to be spend a year hence.

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Steps to be followed:

(a)In first column, write maintenance costs of machine for different years.

(b)In second column, write discounting factor indicating the present value.

(c)In third column, calculate discounted running cost by multiplying column (a) and (b).

(d)In fourth column, calculate cumulative discounted running cost.

(e)Then, calculate cumulative discounted factor shown in column (b).

(f)Finally, in the last column calculate weighted average annual cost by dividing the cumulative discounted running cost (column d) by cumulative discounted factor (column e).

(g)Replace the machine in the year in which the weighted average annual cost is minimum.

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Q4: The initial cost of a machine is Rs. 30,000 and running or operating expenditure which increases with age of the machine is given below. What is the replacement policy? When this machine should be replaced? It is given that the rate of interest is 10% and scrap value is nil.

Year	1	2	3	4	5	6	7
Running Cost (Rs.)	5,000	6,000	8,000	10,000	13,000	16,000	20,000

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Q5: The initial cost of an item is Rs. 15,000 and maintenance or running costs for different years are given below. What is the replacement policy to be adopted if the capital is worth 10% and there is no salvage value?

Year	1	2	3	4	5	6	7
Running Cost (Rs.)	2500	3000	4000	5000	6500	8000	10000

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Q6: An engineering company is offered two types of material handling equipments A and B. A is priced at Rs. 60,000 including cost of installation and the costs for the operation and maintenance are estimated to be Rs. 10,000 for each of the first five years, increasing every year by Rs. 3,000 per year in the sixth and subsequent years.

Equipment B, with rated capacity same as A, requires an initial investment of Rs. 30,000 but in terms of operation and maintenance costs more than A. These costs for B are estimated to be Rs. 13,000 per year for the first six years, increasing every year by Rs. 4,000 per year from seventh year onwards. The company expects a return of 10% on all its investments. Determine which equipment the company should buy.

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Replacement of items that fail completely:

•There are situations where the failure of a certain item occurs all of a sudden instead of gradual deterioration e.g. electric light bulb, which result in complete breakdown of a system.

•This breakdown implies loss of production and idle inventory labor.

•It is very difficult to determine the probability of failure of any item in the system and it can be done by assigning the probability distribution of failures.

•It is assumed that failures occur only at the end of the period.

•Thus, the main objective is to find the time period ‘t’ which minimizes the total cost involved for the replacement.

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There are two types of replacement policies:

(a)Individual Replacement Policy: Under this policy an item is replaced immediately whenever it fails.

(b)Group Replacement Policy: Under this policy, decision is taken as to when all the items must be replaced irrespective of the fact that items have failed or not, with the provision that if any item fails before the optimal time, it may be replaced individually. It requires twofold considerations:

(i) the rate of individual replacement during the period, and

(ii) the total cost incurred for individual and group replacements during the selected interval.

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- The period for which the total cost is minimum will be the optimal period for replacement.

- For group replacement policy one should know the probability of failure, loss incurred due to these failures, cost of individual replacements and costs of group replacements.

- Replace the group of items at the end of the period if the cost of individual replacement for that period is greater than the average cost per period.

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Q: The management is considering the periodic replacement of light bulbs fitted in its rooms. There are 500 rooms in the hotel and each room has 6 bulbs. The management is now following the policy of replacing the bulbs as they fail at a total of Rs. 30 per bulb. The management feels that this cost can be reduced to Rs. 10 by adopting replacement method. On the basis of the information given below, evaluate the alternative and make a recommendation to the management:

Months of use	**1**	**2**	**3**	**4**	**5**
% of bulbs failing by that month	10	25	50	80	100

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Q: The following failure rates have been observed for a certain type of transistors in a digital computer. The cost of replacing an individual failed transistor is Rs. 1.25. The decision is made to replace all these transistors simultaneously at fixed intervals, and to replace the individual transistors as they fail in service. If the cost of group replacement is 30 paise per transistor, what is the best interval between group replacement? At what group replacement price per transistor would a policy of strictly individual replacement become preferable to the adopted policy?

End of the week	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Prob. of failure to date	0.05	.13	.25	.43	.68	.88	.96	1.00

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