(b) Constructing a Mathematical Model: The next step is the construction of a suitable mathematical model. Model construction means translating the problem definition into mathematical relationships. A model should include the following three important things:

(i) Decision variables.

(ii) Constraints or restrictions.

(iii) Objective function.

(c) Model Solution: This phase involves the computation of those values of decision variables that optimise the objective function. An optimal solution is one which maximise or minimise the performance of any measure in a model subject to the conditions and constraints represented in the model.

(d) Model Testing: It involves checking as if the model can reliably predict the actual system's performance. It also includes comparing the performance of the model with past data available for the actual system. A model must be applicable for a reasonable time period and should be updated from time to time, taking into consideration the past, present and future aspects of the problem.

(e) Establishing control over the solution: After testing the model, the next step is to establish control over the solution, by proper feedback of the information on variables. In other words, if one or more of the controlled variables change, the model may be modified accordingly.

(f) Implementation of the final results: The optimal solution obtained from the model should be applied in practice to improve the performance of the system and the validity of the solution be verified under changing conditions.

Allocation Models: Allocation models are used to allocate available resources to activities in such a way to minimize or maximize subject to given conditions.

(a) Waiting Line (or queuing models): It deals with the situation in which queue is formed. These models have been developed to establish a trade-off between cost of providing service and the waiting time of a customer in the queuing system. **It involves the following:**

(i) How much average time will be spent by the customer in a queue?

(ii) What will be an average length of a queue?

(iii) Average time spent by a customer in the queue.

(b) Inventory Models: Inventory control model helps in minimizing the sum of three conflicting inventory costs; the cost of holding or carrying extra inventory, the cost of shortage or delay in the delivery of items when it is needed and cost of ordering.

(c) Network Models: These models are used to plan, schedule and monitor large projects having complexities and inter-dependence of activities. PERT and CPM techniques helps in identifying trouble spots in project through the identification of the critical path.

(d) Sequencing Models: These models involve the selection of such a sequence of performing a series of jobs to be done on service facilities that optimize the performance of the system.

(e) Replacement Models: These models are related to finding an optimal replacement policy to replace the machines, plant, automobiles, etc.

(f) Assignment Models: It involves the assignment of a number of jobs to the same number of men.

SEQUENCIN G PROBLEM

- Sequencing problem arises when a choice has to be made as to the order in which a number of tasks can be performed.

- Selection of an appropriate order for a series of jobs to be done on a finite number of service facilities, in some pre-assigned order, is called sequencing.

- Sequencing is the determination of the order in which tasks are performed and scheduling is the determination of times for each task.

- Sequencing problem is the problem in which it is necessary to determine the order or sequence of jobs in which they should be performed so as to minimize the total costs.

- **Different types of Sequencing Problems**:

(a) Processing ‘n’ jobs through two machines.

(b) Processing ‘n’ jobs through three machines.

(c) Processing ‘n’ jobs through ‘k’ machines.

(d) Processing 2 jobs through ‘k’ machines.

Notations:

(a) **tij** = time required for processing *i*th job on the *j*th machine.

(b) **T** = Total elapsed time for processing all the jobs, including idle time.

(c) **Xij** = Idle time on machine *j* from the end of job (*i*- 1) to the start of job *i*.

Terminology:

(a) Number of Machines: It represents the number of service facilities through which a job must pass before it is assumed to be completed.

(b) Processing time: It indicates the time required by a job on at each machine.

(c) Processing Order: It is the sequence in which given machines are required to

(d) Total elapsed time: It is the time interval between starting the first job and completing the last job including the idle time.

(e) Idle time on a machine: It is the time for which a machine does not have a job to process.

(f) No passing rule: It is the rule of maintaining the order in which jobs are to be processed on given machines.

<u>Processing N Jobs through Two machines</u>:

- Here the objective is to determine the sequence S in which the ‘n’ jobs may be performed so that T (total elapsed time) is minimized.

- Total elapsed time is determined by the point of time at which the first job goes on machine A and the point of time on which the last job comes off machine B.

- As passing is not allowed, thus machine A will remain busy in processing all the n jobs one by one while machine B may remain idle after completion of one job and before starting of another job.

- At any moment of time, machine B may be idle. Let **x2j** be the time for which machine **M2** remains idle after finishing (j-1)th job and before starting processing the *i*th job.

Johnson's Rule:

- It is a method of sequencing which was propounded by American Mathematician Johnson in 1954.
- It provides the various jobs along with their processing time on machine A and Machine B.
- Examine the columns for processing times on machine A and B and find the smallest processing time in each column.
- If the minimum is for machine A, then place the corresponding job in the first available position in the sequence and if it is for machine B, then place the corresponding job in the last available position in the sequence.

- **If there is a tie in selecting the minimum of all the processing times, then there may be three situations:**

(a) Minimum among all processing times is same for A and B machines, then process job on Machine A first and Machine B last.

(b) If the tie for minimum occurs among processing times on machine A only, then select arbitrarily the job to process first.

(c) If the tie for the minimum occurs among processing times on machine B only, then select arbitrarily the job to process last.

- Cross off the jobs already assigned and repeat steps II through IV.

- Calculate idle time for machine A and B

- Calculate idle time for machine A and B:

(a) Idle time for Machine A = Total Elapsed time – time when the last job in a sequence finishes on A.

(b) Idle time for Machine B = Time at which the first job in a sequence starts on A + (time when the job in a sequence starts on B – time when the last job in a sequence finishes on B).

- The total elapsed time to process all jobs through two machines is given by:

Total Elapsed Time= Time when the last job in a sequence finishes on Machine B.

Q1: There are five jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs that will minimise the total elapsed time.

Job No.	J1	J2	J3	J4	J5
Processing time A	5	1	9	3	10
Processing time B	2	6	7	8	4

Q2: There are six jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A and Machine B.

Job No.	J1	J2	J3	J4	J5	J6
Processing time A	1	3	8	5	6	3
Processing time B	5	6	3	2	2	10

Q3 : There are seven jobs, each of which must go through the two machines A and B in the order AB. Processing time are given in the table below:

Determine a sequence for the given jobs and determine the value of T and idle time on machine A and Machine B.

Job No.	J1	J2	J3	J4	J5	J6	J7
Processing time A	6	24	30	12	20	22	18
Processing time B	16	20	20	12	24	2	6

Processing “N” jobs through 3 Machines:

- Here the case is similar to that of processing n jobs through 2 machines except that three machines are involved.
- Each job requires the same sequence of operations and no passing is allowed and if either or both the following condition/s are satisfied:

(a) The minimum time on machine A is greater than or equal to the maximum time on machine B.

(b) The minimum time on machine C is greater than or equal to the maximum time on machine B.

i.e. if either minimum of A $\geq$ maximum of B, or

minimum of C $\geq$ maximum of B, then the following can be applied:

- Replace the problem with an equivalent problem involving n jobs and two machines. Denote the fictitious machines by G and H and define the processing time **Gi** and **Hi** by:

$$\mathbf{G_i = A_i + B_i}, \qquad \mathbf{H_i = B_i + C_i}$$

- Now, workout the new problem, two machines and n jobs, with the prescribed order GH, by the same method of processing “n” jobs through 2 machines.