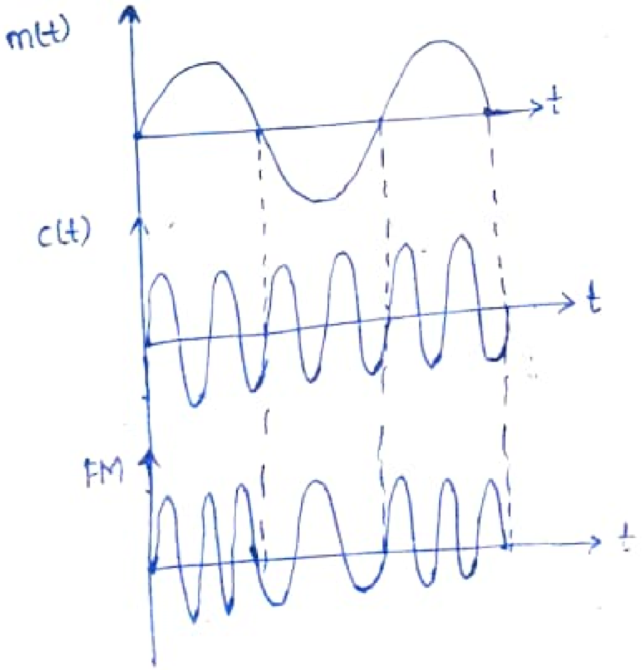
Unit-2

Angle Modulation — Basic Definition
↓
Frequency modulation — Narrow band FM / Wide band FM
↓
Transmission bandwidth of FM signals
↓
Generation of FM signals
↓
Demodulation of FM signals
↓
FM stereo Multiplexing

Definition of Angle Modulation →

* It is the process in which the angle of c(t) is varied in accordance with the instantaneous amplitude values of m(t), Keeping the amplitude of c(t) constant.
* There are two types of Angle modulation
    1. Frequency Modulation
    2. Phase Modulation

Frequency Modulation →



* The frequency of carrier wave changes in according to message signal (or) modulating signal.
* It is a form of angle modulation in which instantaneous frequency, $f_i(t)$ is varied linearly with $m(t)$ & is given by

$$\boxed{f_i(t) = \underbrace{f_c}_{\text{frequency of unmodulated carrier}} + \underbrace{k_f}_{\text{frequency sensitivity Hz/volt}} \cdot \underbrace{m(t)}_{\text{modulating signal}}} \rightarrow (1)$$

W.K.T $f_i(t) = \frac{1}{2\pi} \cdot \frac{d\theta}{dt}$

$$\frac{d\theta}{dt} = 2\pi f_i(t) \rightarrow (2)$$

Integrating on both sides w.r.t 't'.

$$\theta(t) = \int_0^t 2\pi f_i(t)\, dt \rightarrow (3)$$

$$= \int_0^t 2\pi (f_c + k_f m(t))\, dt$$

$$= \int_0^t 2\pi f_c\, dt + \int_0^t 2\pi k_f\, m(t)\, dt$$

$$= 2\pi f_c \int_0^t 1\, dt + 2\pi k_f \int_0^t m(t)\, dt$$

$$\boxed{\theta(t) = 2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt} \rightarrow (4)$$

* AM wave in time domain can be written as,

$$s(t) = A_c \cos[\theta(t)] \rightarrow (5)$$

Sub 4 in 5 we get

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\, dt\right]$$

$$\therefore m(t) = A_m \cos 2\pi f_m t$$

$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t A_m \cos 2\pi f_m t\, dt\right]$$

$$= A_c \cos\left[2\pi f_c t + \cancel{2\pi} k_f \cdot \frac{\sin 2\pi f_m t}{\cancel{2\pi} f_m}\right]$$

$k_f = \Delta f \rightarrow$ frequency deviation

$$s(t) = A_c \cos\left[2\pi f_c t + \frac{\Delta f}{f_m} \sin 2\pi f_m t\right]$$

$$\boxed{s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]}$$

where $\beta \to$ modulation index of AM i.e, $\beta = \frac{\Delta f}{f_m}$

Types of FM $\to$

* Based on β FM is of two types
  1. Narrow Band FM
  2. Wide Band FM

Narrow Band FM $\to$

* The time domain expression for FM is

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin(2\pi f_m t)] \to ①$$

Using trignometric identity,

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$A = 2\pi f_c t \ ; \ B = \beta \sin 2\pi f_m t$$

$$s(t) = A_c[\cos(2\pi f_c t)\cos(\beta \sin 2\pi f_m t) - \sin(2\pi f_c t) \sin(\beta \sin 2\pi f_m t)] \to ②$$

In NBFM, β is small, hence it is possible to approximate

$$\cos(\beta \sin 2\pi f_m) \simeq 1 \qquad \to ③$$
$$\sin(\beta \sin 2\pi f_m) \simeq \beta \sin 2\pi f_m t$$

Sub 3 in 2, we get

$$s(t) = A_c \cos(2\pi f_c t) \cdot 1 - A_c \sin 2\pi f_c t\,(\beta \sin 2\pi f_m t)\}$$

$$s(t) = A_c \cos 2\pi f_c t - \beta A_c \sin 2\pi f_c t \sin 2\pi f_m t \to ④$$

WKT $\sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$

$$\therefore s(t) = A_c \cos 2\pi f_c t - \frac{\beta A_c}{2} \cos 2\pi(f_c - f_m)t + \frac{\beta A_c}{2} \cos 2\pi(f_c + f_m)t \to ⑤$$

The FM wave consists of

i. carrier with amplitude 'A_c' & frequency 'f_c'

ii. USB with amplitude $\frac{\beta A_c}{2}$ & frequency "$f_c + f_m$"

iii, LSB with amplitude $-\frac{\beta A_c}{2}$ with frequency "$f_c - f_m$"

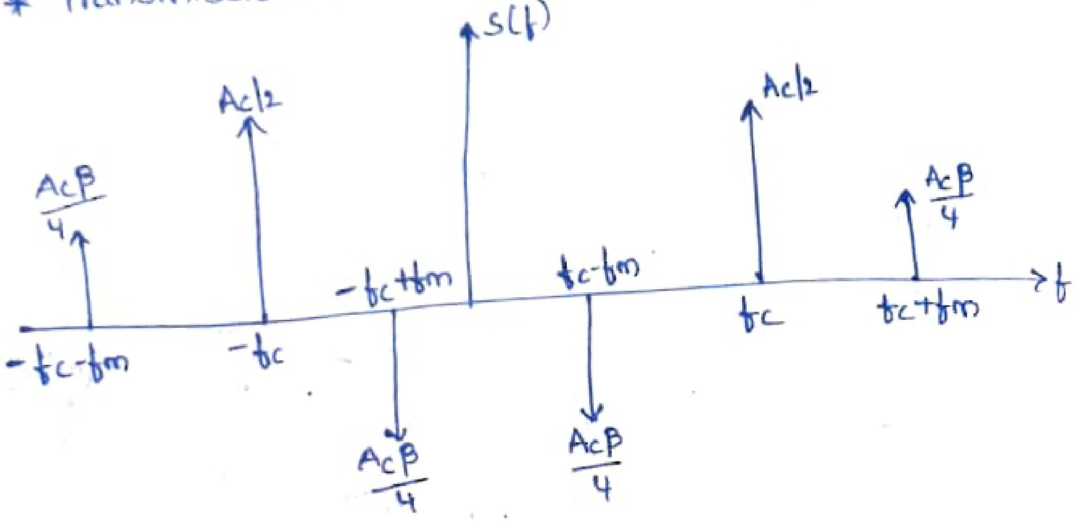
* The narrow band FM requires same band width as that of AM

Taking Fourier transform on both sides of eqn 5

$$S(f) = \frac{A_c}{2}\left[\delta(f+f_c) + \delta(f-f_c)\right] - \frac{\beta A_c}{4}\left[\delta(f-(f_c-f_m)) + \delta(f+(f_c-f_m))\right]$$

$$+ \frac{\beta A_c}{4}\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right]$$

* transmission Bandwidth of Narrow Band FM is $2f_m$



Total power

$$P_t = \frac{A_c^2}{2} + \frac{A_c^2\beta^2}{8} + \frac{A_c^2\beta^2}{8}$$

$$= \frac{A_c^2}{2}\left[1 + \frac{\beta^2}{2}\right]$$

$$P_t = P_c\left[1 + \frac{\beta^2}{2}\right]$$

* Whenever, magnitude of spectrum is same, power is also same as it depends only on magnitude but not on sign.

Wide Band FM →

* It has a much larger value of β, which is theoritic -ally infinite.

* For larger value of β, the FM wave ideally contains the carrier & an infinite no of side bands located symmetrically around the carrier

* Such a FM wave has infinite Bandwidth and hence called wide band FM

* FM wave for sinusoidal modulation is given by

$$s(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t] \rightarrow ①$$

$$\theta = 2\pi f_c t + \beta \sin 2\pi f_m t$$

$$s(t) = Re[A_c \cdot e^{j\theta}]$$

$$s(t) = Re[A_c \, e^{j(2\pi f_c t + \beta \sin 2\pi f_m t)}]$$

$$= Re[A_c \cdot e^{j2\pi f_c t} \, e^{j\beta \sin 2\pi f_m t}]$$

$$= Re[e^{j2\pi f_c t} A_c \cdot e^{j\beta \sin 2\pi f_m t}]$$

$$s(t) = Re[e^{j2\pi f_c t} \cdot \hat{s}(t)] \rightarrow ②$$

Where $\hat{s}(t) = A_c \cdot e^{j\beta \sin 2\pi f_m t} \rightarrow ③$

* $\hat{s}(t)$ is a periodic time function with a fundamental frequency, "f_m"

* This can be expressed using complex fourier series as,

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} C_n \, e^{j2\pi n f_m t} \rightarrow ④$$

Where C_n is a complex fourier co-efficient given by

$$C_n = f_m \int_{-1/2f_m}^{1/2f_m} \hat{s}(t) \, e^{-j2\pi n f_m t} \, dt \rightarrow ⑤$$

Sub 3 in 5 we get

$$C_n = f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} A_c \cdot e^{j\beta \sin 2\pi f_m t} \, e^{-j2\pi n f_m t} \, dt \rightarrow ⑥$$

$$= A_c \cdot f_m \int_{-\frac{1}{2}f_m}^{\frac{1}{2}f_m} e^{j[\beta \sin 2\pi f_m t - 2\pi n f_m t]} \, dt$$

let $x = 2\pi f_m t \rightarrow ⑦$

Differentiating eqn 7 wrt 't'

$$\frac{dx}{dt} = 2\pi f_m (1)$$

$$dt = \frac{dx}{2\pi f_m}$$

Giving the limits

WKT $x = 2\pi f_m t$

when $t = \frac{-1}{2f_m}$

$$x = 2\pi f_m \times \frac{-1}{2f_m}$$

$$x = -\pi$$

when $t = \frac{1}{2f_m}$

$$x = 2\pi f_m \times \frac{1}{2f_m}$$

$$x = \pi$$

$$C_n = A_c \cdot f_m \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} \cdot \frac{dx}{2\pi f_m}$$

$$C_n = \frac{A_c \cdot f_m}{2\pi f_m} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

$$= \frac{A_c}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$$

$$\boxed{C_n = A_c \cdot J_n(\beta)} \rightarrow (8)$$

Where, $J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)} dx$

* $J_n(\beta)$ is a bessel function of 1st kind, nth sides with an argument, β.

Sub eqn 8 in 4

$$\hat{s}(t) = \sum_{n=-\infty}^{\infty} A_c J_n(\beta) e^{j2\pi n f_m t} \longrightarrow (9)$$

Sub 9 in 2

$$s(t) = Re\left[A_c \cdot \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t} e^{j2\pi f_c t} \right]$$

$$S(t) = Re\left[A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\, e^{j2\pi(f_c+nf_m)t}\right]$$

WKT $e^{j\theta} = \cos\theta + j\sin\theta$

$Re[e^{j\theta}] = \cos\theta$ & $\theta = 2\pi(f_c+nf_m)t$

Similarly,

$$\boxed{s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta)\cos[2\pi(f_c+nf_m)t]} \rightarrow (10)$$

Giving the values for n, between $-\infty$ to $+\infty$

i.e., $n = 0, 1, -1, +2, -2, \ldots -\infty$ to $+\infty$

$$S(t) = A_c\Big[[J_0(\beta)\cos 2\pi f_c t + J_1(\beta)\cos 2\pi(f_c+f_m)t + J_{-1}(\beta)\cos 2\pi(f_c-f_m)t] + J_2(\beta)\cos[2\pi(f_c+2f_m)t] + J_{-2}(\beta)\cos[2\pi(f_c-2f_m)t] + J_3(\beta)\cos[2\pi(f_c+3f_m)t] + J_{-3}(\beta)\cos[2\pi(f_c-3f_m)t]\Big] \rightarrow (11)$$

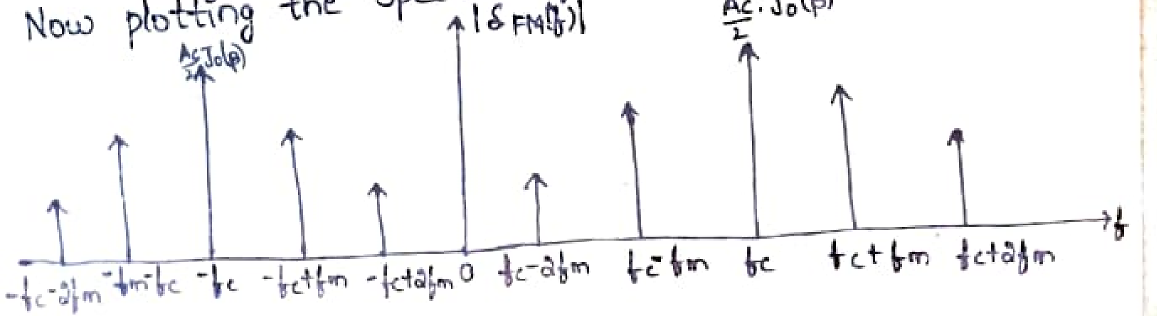
$$s(t) = A_c\Big[J_0(\beta)\cos 2\pi f_c t + J_1(\beta)[\cos 2\pi(f_c+f_m)t - \cos 2\pi(f_c-f_m)t] + J_2(\beta)[\cos 2\pi(f_c+2f_m)t - \cos 2\pi(f_c-2f_m)t] + J_3(\beta)[\cos 2\pi(f_c+3f_m)t - \cos 2\pi(f_c-3f_m)t] + \ldots\Big]$$

* Thus modulated signal has a carrier component & an infinite number of side frequencies $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$, $f_c \pm nf_m$

Taking fourier transform on both sides of eqn 11 we get,

$$S(f) = \frac{A_c}{2} J_0(\beta)\left[\delta(f+f_c) + \delta(f-f_c)\right] + \frac{A_c}{2} J_1(\beta)\left[\delta(f-(f_c+f_m)) + \delta(f+(f_c+f_m))\right] + \frac{A_c}{2} J_{-1}(\beta)\{\delta(f-(f_c-f_m)) + \delta(f+(f_c+f_m))\} + \ldots + \frac{A_c}{2} J_n(\beta)\{\delta(f-(f_c+nf_m)) + \delta(f+(f_c+nf_m))\} + \frac{A_c}{2} J_{-n}(\beta)\{\delta(f-(f_c-nf_m)) + \delta(f+(f_c-nf_m))\} \rightarrow (12)$$

Now plotting the spectrum of the above equation



Transmission Bandwidth of FM →

* Theoritically, FM has infinite no. of side bands, so the bandwidth required for transmission is also infinite.
* "Carson" generalized the Bandwidth formula for an FM wave
* According to him, the approximate formula for computing the bandwidth of an FM signal generated by a single tone modulating signal frequency 'f_m' is

$$B_T \simeq 2(1+\beta) f_m \rightarrow ①$$

* the above formula holds good for all values of 'β'.
* The transmission bandwidth 'B_T' can also be expressed in terms of frequency deviation 'Δf'.

W.K.T,
$$\beta = \frac{\Delta f}{f_m}$$

$$\Delta f = \beta \cdot f_m$$

From eqn ①,

$$B_T = 2(1+\beta) f_m$$

$$B_T = 2f_m + 2\beta f_m$$

$$= 2f_m + 2\Delta f$$

$$= 2\Delta f\left(1 + \frac{f_m}{\Delta f}\right)$$

$$B_T = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

Comparision of FM & AM →

FM	AM
1. Eqn for FM wave is $s(t) = A_c[\cancel{\sin}\ \cos(2\pi f_c t + \beta \sin 2\pi f_m t)]$	1. Eqn for AM wave is $s(t) = A_c[1+\mu \cos 2\pi f_m t]\cos 2\pi f_c t$
2. Modulation index can have any value i.e., <1 or >1 i.e., $\beta < 1$ or $\beta > 1$	2. Modulation index is always in between 0 and 1 $0 < \mu < 1$

3. Transmitted power is useful	3. carrier & one side band power are useless
4. $P_c = \frac{A_c^2}{2R}$	4. $P_t = P_c\left[1 + \frac{\mu^2}{2}\right]$
6. the modulation index determines the no. of side bands in an FM signal.	5. In AM, only two side bands are produced, irrespective of modulation index
6. $B.W = 2(\Delta f + f_m)$ Bandwidth depends on modulation index	6. $B.W = 2f_m$ B.W doesn't depend on modulation index.
7. % modulation index = $\frac{\text{Actual freq deviation}}{\text{max. allowed freq deviation}}$	7. % modulation index = $\frac{A_m \times}{A_c}$
8. Advantage of FM over AM is the noise immunity.	8. AM is more susceptible to noise & more effected by noise than FM.
9. FM transmission and reception equipment are more complex	9. AM equipments are less complex
10. FM transmission is expensive than AM.	10. AM transmission is cheaper than FM
11. Used for short distance communication.	11. Used for long distance communication.

Generation of FM Signals→

* There are essentially two basic methods of generating frequency modulated waves
  1. Direct FM (or) parameter variation method
  2. Indirect FM (or) Armstrong method.

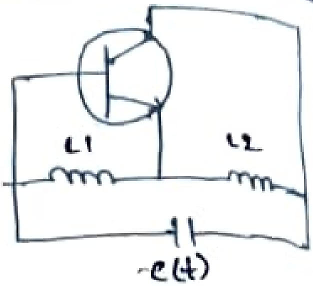
1. Direct Method (parameter variation method):-

* In this method, the instantaneous freq of the carrier wave is varied according to the modulating signal by

device called a Voltage-controlled oscillator (VCO).

* To produce FM, we use a circuit that converts a modulating voltage into a corresponding change in capacitance (or) inductance to the oscillator tank.

* Here, the FM is obtained by the variation of any element or parameter on which the freq depends is called "parameter variation method" of FM generation.



* The freq of oscillation of the Hartley oscillator is given by,

$$f_1(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot c(t)}} \rightarrow ①$$

two inductance in tuned circuit

total capacitance of fixed capacitor & variable voltage capacitor

* For modulating wave, m(t) the capacitance c(t) is given by,

$$c(t) = c_0 - K\cdot m(t) \rightarrow ②$$

c_0 → total capacitance in absence of modulation

K → variable capacitor sensitivity to voltage change

Sub ② in ①

$$f_1(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0-Km(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1-\frac{K}{C_0}m(t)\right)}}$$

$$f_1(t) = f_0\left[1-\frac{K}{C_0}m(t)\right]^{-1/2} \rightarrow ③$$

f_0 → unmodulated freq of oscillation

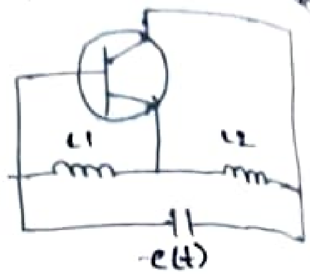
$$f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot C_0}} \rightarrow ④$$

$$f_1(t) \simeq f_0\left[1+\frac{K}{2C_0}m(t)\right] \rightarrow ⑤$$

$$f_1(t) = f_0 + K_f\, m(t)$$

device called a voltage controlled oscillator (VCO).

* To produce FM, we use a circuit that converts a modulating voltage into a corresponding change in capacitance (or) inductance to the oscillator tank.



* Here, the FM is obtained by the variation of any element or parameter on which the freq depends is called "parameter variation method" of FM generation.

* The freq of oscillation of the Hartley oscillator is given by,

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot c(t)}} \rightarrow ①$$

(L_1+L_2) → two inductance in tuned circuit

$c(t)$ → total capacitance of fixed capacitor & variable voltage capacitor

* For modulating wave, m(t) the capacitance c(t) is given by

$$c(t) = C_0 - K\cdot m(t) \rightarrow ②$$

C_0 → total capacitance in absence of modulation

K → variable capacitor sensitivity to voltage change

Sub 2 in 1

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1+L_2)(C_0-Km(t))}}$$

$$= \frac{1}{2\pi\sqrt{(L_1+L_2)C_0\left(1-\frac{K}{C_0}m(t)\right)}}$$

$$f_i(t) = f_0\left[1-\frac{K}{C_0}m(t)\right]^{-1/2} \rightarrow ③$$

f_0 → unmodulated freq of oscillation

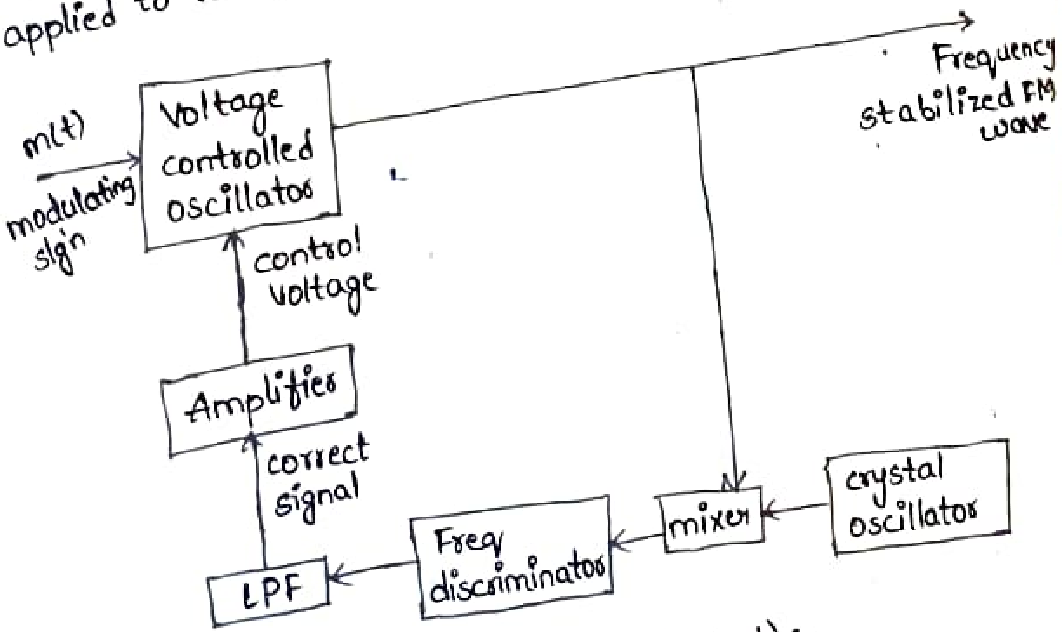
$$f_0 = \frac{1}{2\pi\sqrt{(L_1+L_2)\cdot C_0}} \rightarrow ④$$

$$f_i(t) \simeq f_0\left[1+\frac{K}{2C_0}m(t)\right] \rightarrow ⑤$$

$$f_i(t) = f_0 + k_f m(t)$$

Where $K_f = \frac{f_0 K}{2C_0}$ & A is the resultant freq sensitivity of the modulator

* The direct method has an disadvantage, that carrier freq is not obtained from a highly stable oscillator
* To, overcome this we use freq stabilization method, here carrier is generated by a crystal oscillator which is very stable.
* In this method, the o/p of FM generator is applied to a mixer along with the o/p of crystal oscillator.
* Mixer produces the difference freq term & applied to freq discriminator, it produces o/p voltage proportional to the freq of the FM wave applied to its i/p.
* The o/p of freq discriminator is applied to a LPF. Then for correct carrier freq, the o/p of LPF is zero
* It gives dc o/p voltage proportional to carrier frequency deviation from the assigned value of the carrier frequency. And it is applied to VCO to modify freq of oscillator.



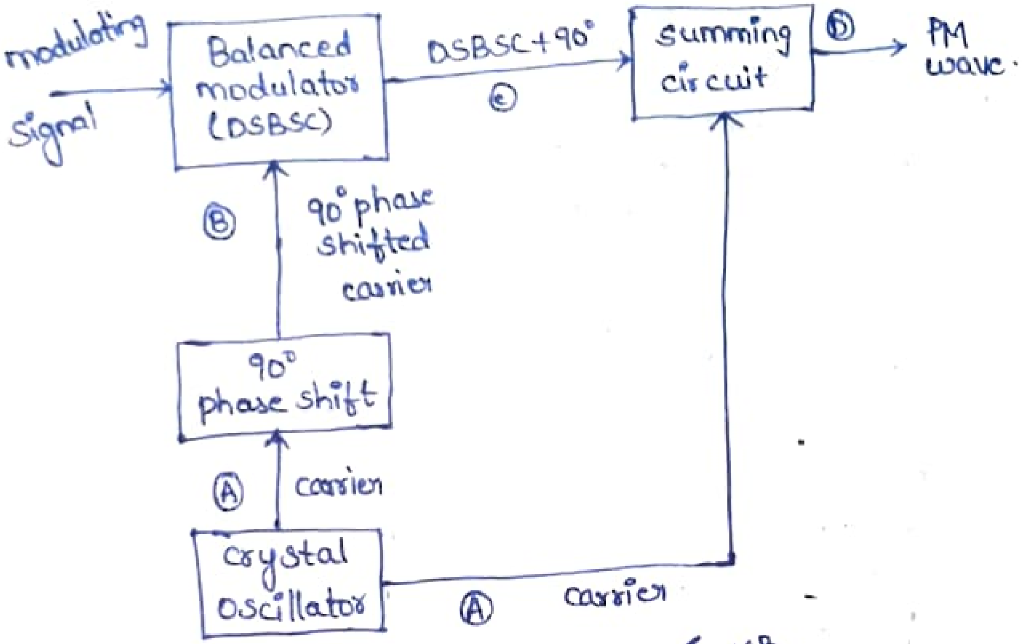
2. Indirect Method (Armstrong method):-

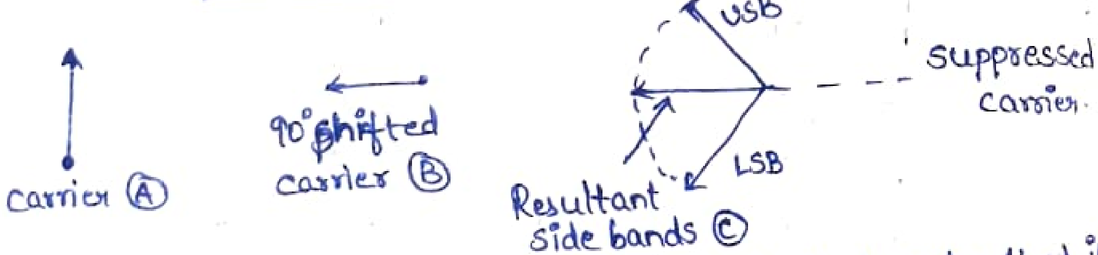
* FM generation using Armstrong method is achieved by following steps:
  a. Generation of PM wave
  b. Generation of NBFM from PM wave
  c. Generation of WBFM from NBFM

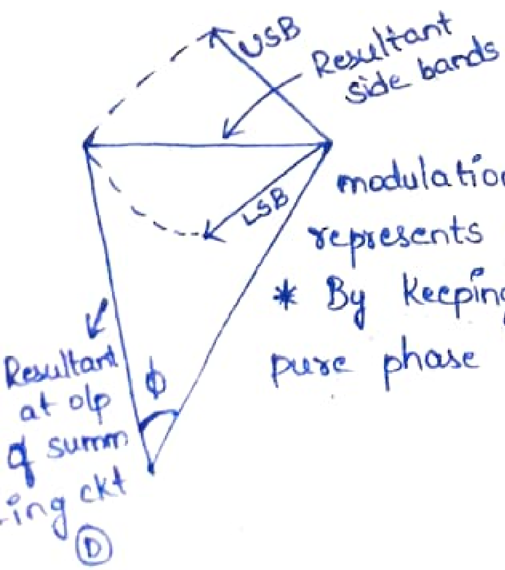
a. Generation of PM wave :-

PM - phase modulator.

* Crystal oscillator is used to generate a stable unmodulated carrier which is applied to 90° phase shifter & the summing circuit.
* The 90° phase shifted carrier is applied to the Balanced modulator along with the modulating signal. Thus the, shifted carrier is DSBSC modulated in the balanced modulator giving us only two side bands with their resultant in phase with 90° shifted carrier
* The two side bands & the unshifted carrier are applied to a summing circuit to get the resultant of vector addition of the carrier & two side bands

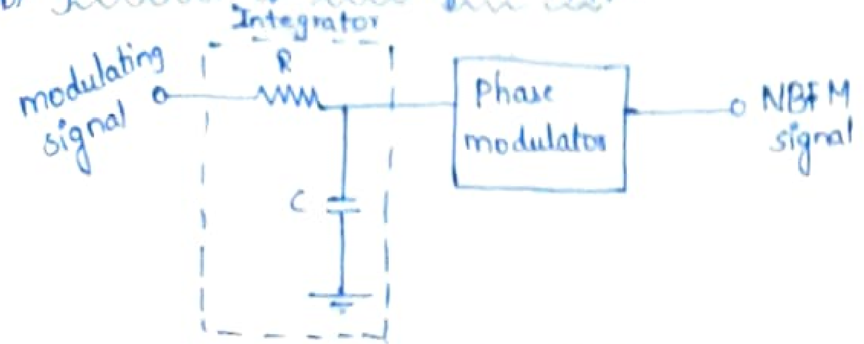






* It is important to note that if the phase shift of the resultant carrier exceeds 30°, then along with phase modulation, we have amplitude modulation. This represents distortion.
* By keeping the phase shift within limits [30°], pure phase modulation is obtained.

b) Generation of NBFM from FM:-



the expression for a PM wave,

$$s(t) = A_c \cos[\omega_c t + m_p \sin\omega_m t] \rightarrow ①$$

Let $s(t) = A_c \cos\theta$

where $\theta = \omega_c t + m_p \sin\omega_m t$

* Instantaneous angular frequency of the PM wave is defined as,

$$\omega_i = \frac{d\theta}{dt} = \frac{d}{dt}[\omega_c t + m_p \sin\omega_m t]$$

$$= \omega_c + m_p \omega_m \cos\omega_m t$$

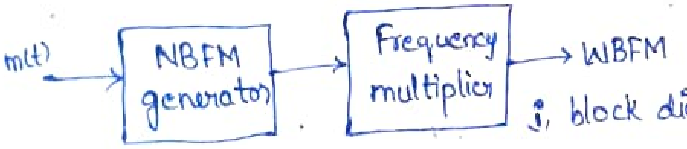
$$\boxed{f_i = f_c + m_p f_m \cos\omega_m t} \rightarrow ②$$

$\Delta f_{max} = m_p \cdot f_m \rightarrow$ frequency deviation.

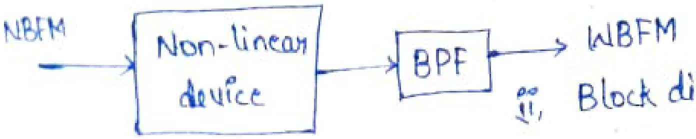
* since m_p is proportional to the modulating voltage, the frequency f_i will vary in proportion with the modulating voltage thus FM can be obtained using PM.

c) Generation of WBFM from NBFM:-

* In Indirect method of FM, the modulating signal first generates NBFM & therefore the freq deviation is increased by converting NBFM into WBFM. the NBFM is converted to WBFM by using frequency multipliers



i, block diagram of FM wave generator



ii, Block diagram of freq multiplier

* The o/p of frequency multiplier contains, frequency mod.. wave with carrier frequency $f_1, 2f_1 \ldots nf_1$ & deviati.. $\Delta f_1, 2\Delta f_1, \ldots n\Delta f_1$

* The BPF following non-linear device is used:

i, To pass the FM wave centered at the carrier freq nf_1 with freq deviation of $n \cdot \Delta f_1$

ii, To attenuate completely all remaining FM spectra.

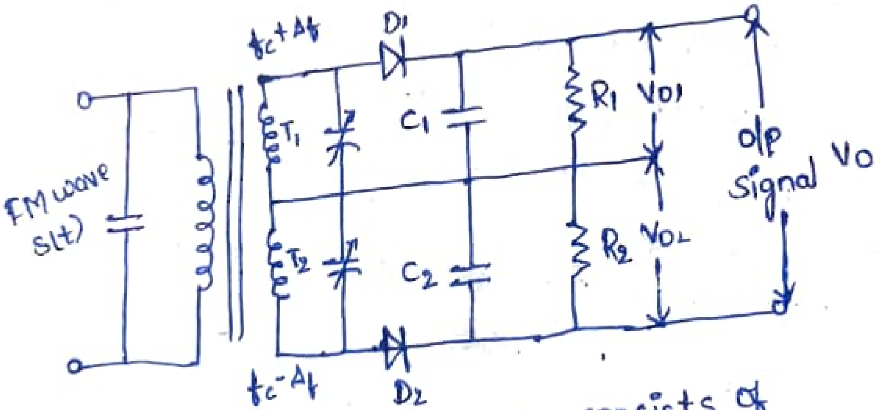
Detection of FM waves:-

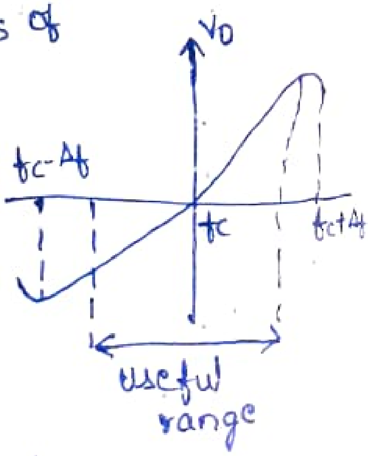
* The original modulating signal is recovered from the FM wave

1. Direct method
   a) Frequency discriminator (or) Balanced slope detector
   b) Zero-crossing detector
   c) Phase discriminator (or) Foster seeley discriminator
   d) Radio detector
2. Indirect method
   a) Phase-locked loop.

Balanced slope detector →

Frequency discriminator (or) Balanced slope





* The Balanced slope detector consists of two slope detector circuits.

* The i/p transformer has a centre tapped secondary. Hence the i/p voltages to the two slope detectors are 180° out of phase.

* There are 3 tuned circuits

i, The primary is tuned to IF i.e., f_c

ii, the upper tuned circuit of the secondary (T_1) is tuned above 'f_c' by Δf i.e., its resonant frequency is "$f_c + \Delta f$"

iii, the lower tuned circuit of the secondary (T_2) is tuned below 'f_c' by Δf i.e., its resonant frequency is "$f_c - \Delta f$".

* R_1C_1 & R_2C_2 are the filter circuits
* V_{01}, V_{02} are the o/p voltages of the two slope detectors
* The final o/p voltage V_0 is obtained by taking the difference of the individual o/p voltages V_{01} & V_{02}

$$\boxed{V_0 = V_{01} - V_{02}}$$

Operation:-

* We can understand the operation by dividing the i/p frequency into ranges as follows:-

i $f_{in} = f_c$:- when i/p frequency is equal to carrier frequency "f_c", the induced voltage in the "T_1" winding of secondary is exactly equal to that induced in the winding 'T_2'.

* Thus the i/p voltages to both the diodes D_1 & D_2 will be same
* The dc o/p voltages V_{01} & V_{02} will also be identical but they have opposite polarities, hence $V_0 = 0V$.

ii, $f_{in} > f_c$:- when i/p frequency is greater than "f_c", the induced voltage in "T_1" winding is higher than induced in 'T_2'.

* The i/p to D_1 is higher than D_2. So '+ve' o/p V_{01} (of D_1) is higher than the -ve o/p V_{02} (of D_2). Thus o/p voltage, V_0 is +ve.

iii, $f_{in} < f_c$:- when i/p frequency is less than "f_c" [i.e., $f_{in} \simeq f_c - \Delta f$] the induced voltage in 'T_2' winding is higher than in 'T_1'. So i/p voltage to diode, D_2 is higher than that of D_1. Hence -ve o/p 'V_{02}' is greater than V_{01}.

* The o/p voltage of the Balanced slope detector is -ve in this frequency range.

$$\therefore V_0 = \begin{cases} 0 ; & f_{in} = f_c \\ +ve ; & f_{in} > f_c \\ -ve ; & f_{in} < f_c \end{cases}$$

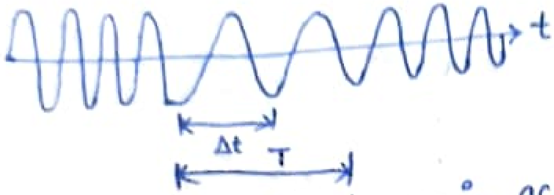
Zero crossing detector →

* The zero crossing detector operates on the principle that the instantaneous frequency of an FM wave is approximately given by,

$$f_i \simeq \frac{1}{2\Delta t}$$

where,

Δt is the time difference between adjacent zero crossings of the FM wave



* The time interval 'T' is chosen in accordance with the following two conditions.

i. the interval 'T' is small compared to the reciprocal of the message Bandwidth 'W' i.e., $\frac{1}{W}$

ii. the interval 'T' is large compared to the reciprocal of the carrier frequency 'f_c' of the FM wave, i.e., $1/f_c$.

* Let 'n_0' denote the number of zero crossings inside the interval 'T'. Hence, Δt is the time between the adjacent zero crossing points given by,

$$\boxed{\Delta t = \frac{T}{n_0}}$$

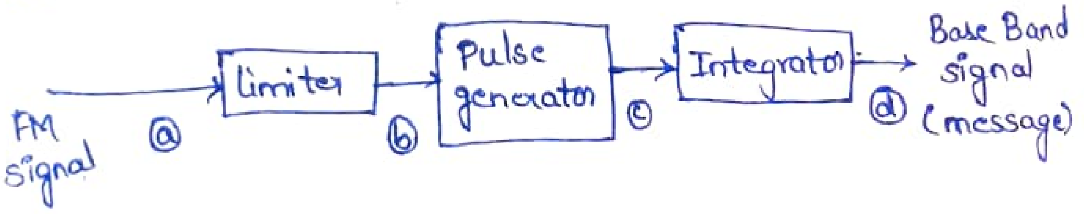
∴ Instantaneous frequency is given by.

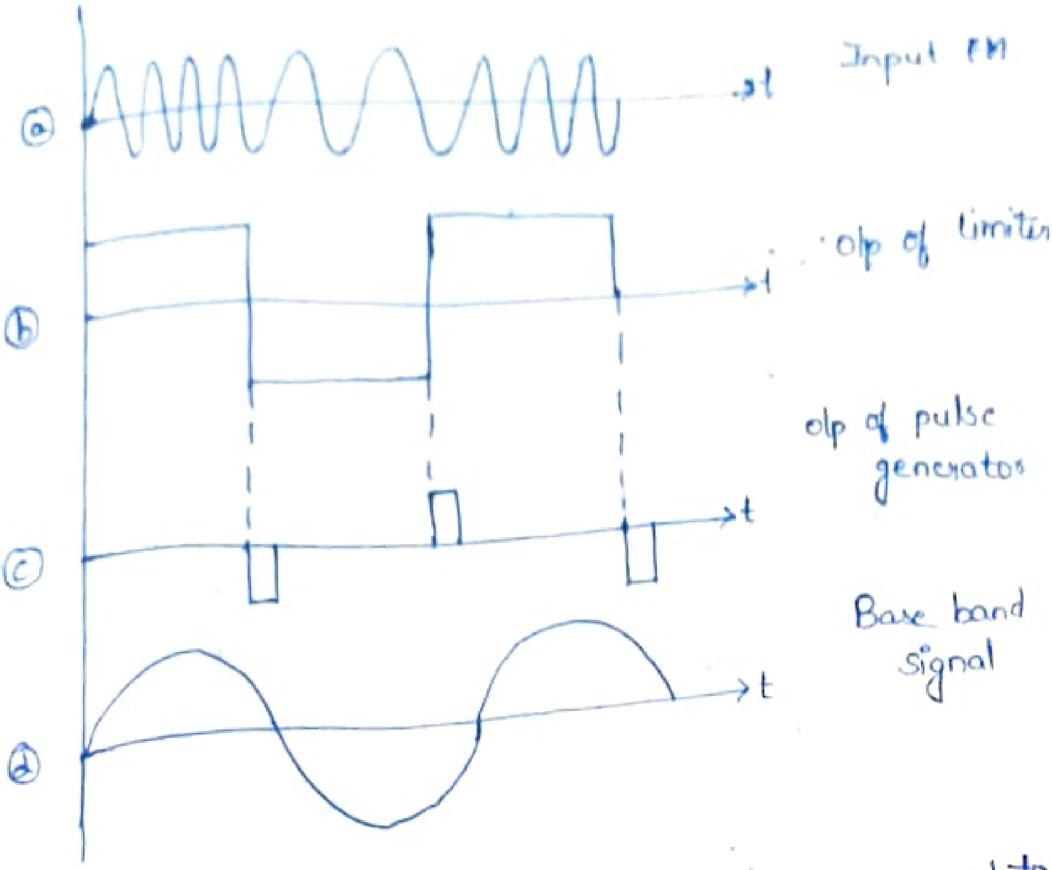
$$f_i \simeq \frac{1}{2\Delta t}$$

$$f_i \simeq \frac{1}{2\times\frac{T}{n_0}}$$

$$\boxed{f_i \simeq \frac{n_0}{2T}}$$

* By the definition of Instantaneous frequency, w.k.T there is a linear relation between f_i & message signal m(t). Hence, we can recover m(t) if "n_0" is known.

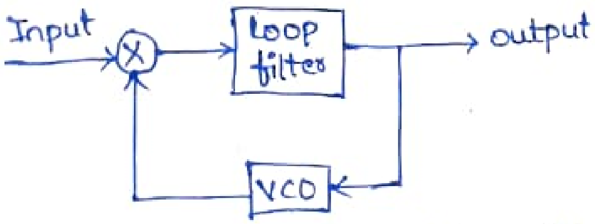




Phase locked loop →

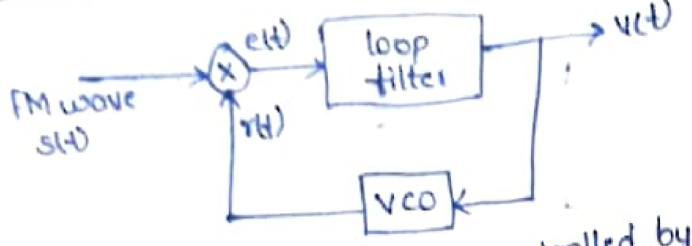
* The PLL is a -ve feed back system. It can be used to track the phase & the frequency of the carrier component of an incoming signal.
* It is mainly used for the detection of FM, when signal-to-noise ratio is poor.
* PLL has three basic components:-

1. A voltage controlled oscillator (VCO)
2. A multiplier serving as a phase detector or a phase comparator.
3. A loop filter, which is a low pass filter.

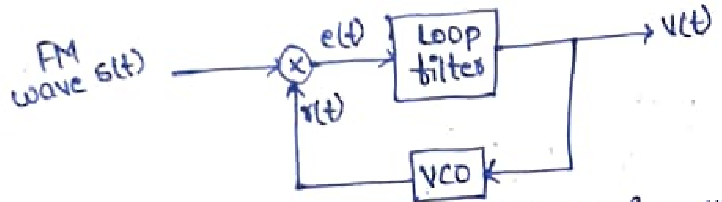


* The frequency of VCO can be controlled by the external voltage. In a VCO, the oscillation frequency varies linearly with its i/p voltage.
* The o/p of the multiplier is passed through a LPF (or) loop

filter, & then applied to the i/p of the vco. This voltage then changes the frequency of the vco & keeps the loop in locked condition



* The frequency of vco can be controlled by the external voltage. In a vco, the oscillation frequency varies linearly with its input voltage
* The o/p of the multiplier is passed through a LPF (or) loop filter, & then applied to the i/p of the vco. This voltage then changes the frequency of the vco & keeps the loop in locked condition.



* Initially, the control voltage to vco is zero, then vco is adjusted so that,

1. The frequency of the vco is exactly made equal to the unmodulated carrier frequency "f_c".
2. The vco o/p has a phase shift of 90° w.r.t the unmodulated carrier wave

The i/p to the PLL is given by

$$s(t) = A_c \sin[2\pi f_c t + \phi_1(t)] \rightarrow ①$$

with a modulating signal m(t), we have

$$\phi_1(t) = 2\pi k_f \int_0^t m(t).dt \rightarrow ②$$

If r(t) denotes the o/p of vco, then

$$r(t) = A_v \cos[2\pi f_c t + \phi_2(t)] \rightarrow ③$$

If v(t) is the control voltage applied as i/p voltage to vco, then

$$\phi_2(t) = \underbrace{2\pi k_v}_{\text{freq sensitivity of vco}} \int_0^t v(t)\, dt \rightarrow ④$$

* The incoming FM wave, s(t) & the vco o/p r(t) are the two

ilp's to multiplier. The olp $e(t)$ is given by

$$e(t) = \underbrace{k_m}_{\text{multiplier gain}} [s(t)][r(t)] \rightarrow ⑤$$

$$e(t) = k_m [A_c \sin(2\pi f_c t + \phi_1(t))][A_v \cos(2\pi f_c t + \phi_2(t))] \rightarrow ⑥$$

W.K.T $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$

$$e(t) = \frac{k_m}{2}\left[A_c \cdot A_v \{\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_1(t) - \phi_2(t))\}\right] \rightarrow ⑦$$

$$e(t) = \frac{1}{2} k_m A_c A_v \left[\{\sin(4\pi f_c t + \phi_1(t) + \phi_2(t)) + \sin(\phi_1(t) - \phi_2(t))\}\right] \rightarrow ⑧$$

The LPF olp is given by,

$$e(t) = \frac{1}{2} k_m A_c A_v \sin[\phi_1(t) - \phi_2(t)] \rightarrow ⑨$$

$$e(t) = \frac{1}{2} k_m A_c A_v \sin[\phi_e(t)]$$

Where, $\phi_e(t)$ is the phase error

$$\phi_e(t) = \phi_1(t) - \phi_2(t)$$

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t v(t)\, dt \rightarrow ⑩$$

The loop filter operates on its ilp, $e(t)$ to produce the olp,

$$v(t) = \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, d\tau \rightarrow ⑪$$

where $h(t)$ is the impulse response of filter.

Sub 11 in 10, then

$$\phi_e(t) = \phi_1(t) - 2\pi k_v \int_0^t \int_{-\infty}^{\infty} e(\tau)\, h(t-\tau)\, dt \rightarrow ⑫$$

differentiating eqn 12 w.r.t time we get,

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_m A_c A_v k_v \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau \rightarrow ⑬$$

$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_0 \int_{-\infty}^{\infty} \sin[\phi_e(\tau)]\, h(t-\tau)\, d\tau \rightarrow ⑭$$

Where k_0 is loop parameter given by

$$k_0 = k_m k_v A_c A_v \rightarrow ⑮$$

Non-linear model of PLL →

* Eqn 14 produces non-linear model. This model includes the relationship between $v(t)$ & $e(t)$ as represented by eqn 9 & 12.