

**Unit – 4 (BEE) R19&R20 Regulations – I ECE II Semester**

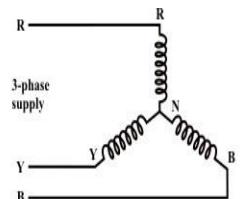
Induction Machine: Principle of operation and construction of three-phase induction motors – slip ring and squirrel cage motors – slip-torque characteristics – efficiency calculation – starting methods Brake test on 3-Phase Induction Motor.

**CONSTRUCTION OF 3-PHASE INDUCTION MOTOR**

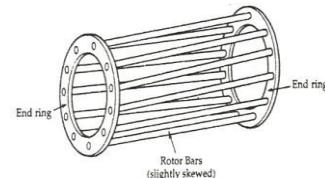
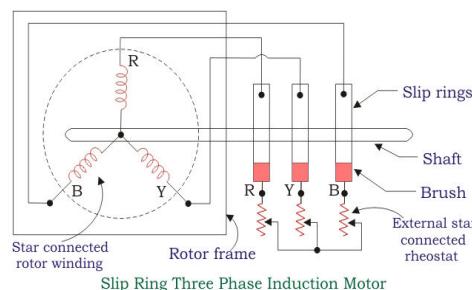
The 3-Phase induction motor consists of mainly two parts namely stator and rotor

**Stator:** The stator consists of

- **Stator frame:** The stator frame is made of cast iron and consists of cooling fins  
It gives the support and protects other parts of the motor
- **Stator core:** The stator core is made of with laminated high grade alloy steel stampings and slotted on the inner periphery and these stampings are insulated.
- **Stator winding:** The stator winding is placed in the stator core, which is connected either in star or delta

**Squirrel cage Rotor:**

1. The rotor core is a cylindrical one built from a high grade alloy steel laminations.
2. It consists of rotor slots in parallel to the shaft axis on the outer periphery.
3. In general the slots are not parallel to the shaft but skewed with some angle to the shat
4. The purpose of the skewing is to prevent interlocking and to reduce the humming noise
5. The rotor copper bars are placed in the rotor slots and the bars are short circuited with end rings.
6. In Cage rotor type there is no chance of adding the external resistance to the rotor to improve the torque developed at starting.

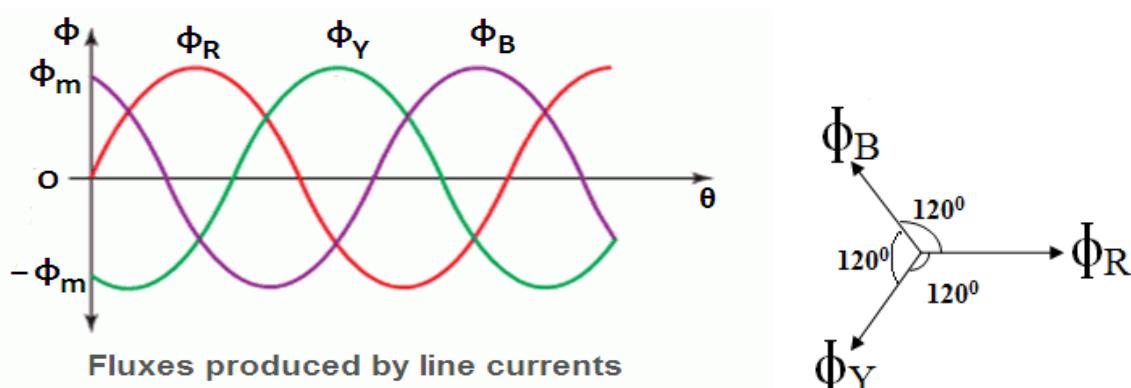
➤ **Slip ring rotor (or) Phase Wound rotor:**

1. The rotor core is a cylindrical one built from a high grade alloy steel laminations.
2. It consists of rotor slots on the outer periphery where the star connected winding is done.
3. The star connected rotor winding is done for the same poles as that of the stator winding
4. The ends of the star connected rotor winding are connected to the three slip rings placed on the shaft.
5. The carbon brushes are mounted on the slip rings, through which an external resistance is added to the rotor.
6. The advantage of the Wound rotor is the starting torque is improved by adding the external resistance to the rotor using slip rings.

### Rotating Magnetic Field

1. The induction motor rotates due to the **rotating magnetic field in 3 phase induction motor**, which is produced by the stator winding in the air gap between the stator and the rotor.
2. The stator has a three phase stationary winding which can be either star connected or delta connected.
3. Whenever the AC supply is connected to the stator windings, line currents  $I_R$ ,  $I_Y$ , and  $I_B$  start flowing.
4. These line currents have phase difference of  $120^\circ$  with respect to each other.
5. Due to each line current, a sinusoidal flux is produced in the air gap.
6. These fluxes have the same frequency as that of the line currents, and they also have the same phase difference of  $120^\circ$  with respect to each other.

Let the flux produced by the line currents  $I_R$ ,  $I_B$ ,  $I_Y$  be  $\phi_R$ ,  $\phi_B$ ,  $\phi_Y$  respectively.



Mathematically, they are represented as follows:

$$\phi_R = \phi_m \sin \omega t = \phi_m \sin \theta$$

$$\phi_Y = \phi_m \sin (\omega t - 120^\circ) = \phi_m \sin (\theta - 120^\circ)$$

$$\phi_B = \phi_m \sin (\omega t - 240^\circ) = \phi_m \sin (\theta - 240^\circ)$$

The effective or total flux ( $\phi_T$ ) in the air gap is equal to the phasor sum of the three components of fluxes  $\phi_R$ ,  $\phi_Y$  and,  $\phi_B$ .

Therefore,  $\phi_T = \phi_R + \phi_Y + \phi_B$

**Step 1:** The values of total flux  $\phi_T$  for different values of  $\theta$  such as 0, 60, 120 , 180 .... 360°. are to be calculated

**Step 2:** For every value of  $\theta$  in step 1, draw the phasor diagram, with the phasor  $\phi_R$  as the reference phasor i.e. all the angles are drawn with respect to this phasor.

**For  $\theta = 0^\circ$**

$$\phi_R = \phi_m \sin \omega t = \phi_m \sin \theta = 0$$

$$\phi_Y = \phi_m \sin (\omega t - 120^\circ) = \phi_m \sin (\theta - 120^\circ) = \phi_m \sin (0 - 120^\circ) = (-)\phi_m \sin 120^\circ = -0.866 \phi_m$$

$$\phi_m$$

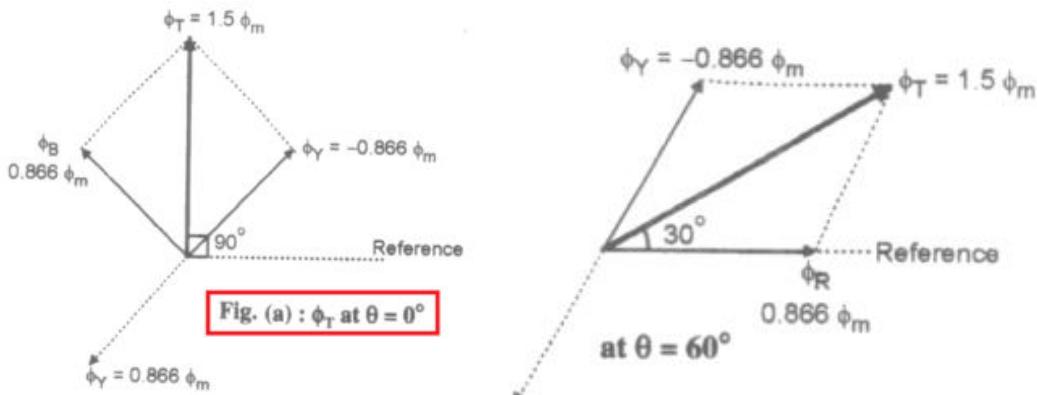
$$\phi_B = \phi_m \sin (\omega t - 240^\circ) = \phi_m \sin (\theta - 240^\circ) = \phi_m \sin (0 - 240^\circ) = (-)\phi_m \sin 240^\circ = 0.866 \phi_m$$

Therefore,  $\phi_T = 0 + \phi_Y + \phi_B = \phi_T = 0 + (-\phi_Y) + \phi_B$

$$\Phi_T = \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B \cos 60^\circ}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \frac{1}{2}}$$

$$\Phi_T = \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \phi_m\right)^2} = \frac{3}{2} \phi_m = 1.5 \phi_m$$



**For  $\theta = 60^\circ$**

$$\phi_R = \phi_m \sin \omega t = \phi_m \sin \theta = \phi_m \sin 60 = 0.866 \phi_m$$

$$\phi_Y = \phi_m \sin (\omega t - 120^\circ) = \phi_m \sin (\theta - 120^\circ) = \phi_m \sin (60 - 120^\circ) = (-)\phi_m \sin 60^\circ = -0.866 \phi_m$$

$$\phi_m$$

$$\phi_B = \phi_m \sin (\omega t - 240^\circ) = \phi_m \sin (\theta - 240^\circ) = \phi_m \sin (60 - 240^\circ) = (-)\phi_m \sin 180^\circ = 0$$

Therefore,  $\phi_T = \phi_R + (-\phi_Y) + 0$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_Y)^2 + 2\Phi_Y\Phi_R \cos 60^\circ}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \frac{1}{2}}$$

$$\Phi_T = \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2} = \frac{3}{2} \varphi_m = 1.5 \varphi_m$$

**For  $\theta = 120^\circ$**

$$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 120^\circ = 0.866 \varphi_m$$

$$\varphi_Y = \varphi_m \sin (\omega t - 120^\circ) = \varphi_m \sin (120^\circ - 120^\circ) = (-)\varphi_m \sin 0^\circ = 0$$

$$\varphi_B = \varphi_m \sin (\omega t - 240^\circ) = \varphi_m \sin (120^\circ - 240^\circ) = (-)\varphi_m \sin 120^\circ = -0.866 \varphi_m$$

Therefore,  $\Phi_T = \Phi_R + 0 + (-\Phi_B)$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_B)^2 + 2\Phi_R\Phi_B \cos 60^\circ}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \frac{1}{2}}$$

$$\Phi_T = \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2} = \frac{3}{2} \varphi_m = 1.5 \varphi_m$$

**For  $\theta = 180^\circ$**

$$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 180^\circ = 0$$

$$\varphi_Y = \varphi_m \sin (\omega t - 120^\circ) = \varphi_m \sin (180^\circ - 120^\circ) = \varphi_m \sin 60^\circ = 0.866 \varphi_m$$

$$\varphi_B = \varphi_m \sin (\omega t - 240^\circ) = \varphi_m \sin (180^\circ - 240^\circ) = (-)\varphi_m \sin 60^\circ = -0.866 \varphi_m$$

Therefore,  $\Phi_T = 0 + \Phi_Y + (-\Phi_B)$

$$\Phi_T = \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B \cos 60^\circ}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \frac{1}{2}}$$

$$\Phi_T = \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2} = \frac{3}{2} \varphi_m = 1.5 \varphi_m$$

**For  $\theta = 240^\circ$**

$$\varphi_R = \varphi_m \sin \omega t = \varphi_m \sin \theta = \varphi_m \sin 240^\circ = -0.866 \varphi_m$$

$$\varphi_Y = \varphi_m \sin (\omega t - 120^\circ) = \varphi_m \sin (240^\circ - 120^\circ) = \varphi_m \sin 120^\circ = 0.866 \varphi_m$$

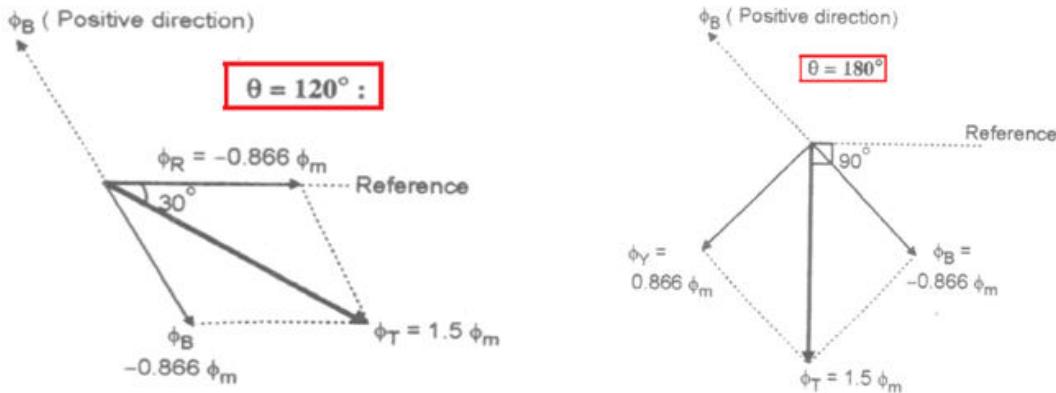
$$\varphi_B = \varphi_m \sin (\omega t - 240^\circ) = \varphi_m \sin (240^\circ - 240^\circ) = \varphi_m \sin 0^\circ = 0$$

Therefore,  $\Phi_T = (-\Phi_R) + \Phi_Y + 0$

$$\Phi_T = \sqrt{(\Phi_R)^2 + (\Phi_Y)^2 + 2\Phi_R\Phi_Y \cos 60^\circ}$$

$$\Phi_T = \sqrt{\left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \left(\frac{\sqrt{3}}{2} \varphi_m\right) \times \frac{1}{2}}$$

$$\Phi_T = \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \varphi_m\right)^2} = \frac{3}{2} \varphi_m = 1.5 \varphi_m$$



**For  $\theta = 300^\circ$**

$$\Phi_R = \phi_m \sin \omega t = \phi_m \sin \theta = \phi_m \sin 300^\circ = -0.866 \phi_m$$

$$\Phi_Y = \phi_m \sin (\omega t - 120^\circ) = \phi_m \sin (300^\circ - 120^\circ) = \phi_m \sin 180^\circ = 0$$

$$\Phi_B = \phi_m \sin (\omega t - 240^\circ) = \phi_m \sin (300^\circ - 240^\circ) = \phi_m \sin 60^\circ = 0.866 \phi_m$$

Therefore,  $\Phi_T = (-\Phi_R) + \Phi_Y + \Phi_B$

$$\begin{aligned} \Phi_T &= \sqrt{(\Phi_R)^2 + (\Phi_B)^2 + 2\Phi_R\Phi_B \cos 60^\circ} \\ \Phi_T &= \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \frac{1}{2}} \\ \Phi_T &= \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \phi_m\right)^2} = \frac{3}{2} \phi_m = 1.5 \phi_m \end{aligned}$$

**For  $\theta = 360^\circ$**

$$\Phi_R = \phi_m \sin \omega t = \phi_m \sin \theta = \phi_m \sin 360^\circ = 0$$

$$\Phi_Y = \phi_m \sin (\omega t - 120^\circ) = \phi_m \sin (360^\circ - 120^\circ) = \phi_m \sin 240^\circ = -0.866 \phi_m$$

$$\Phi_B = \phi_m \sin (\omega t - 240^\circ) = \phi_m \sin (360^\circ - 240^\circ) = \phi_m \sin 60^\circ = 0.866 \phi_m$$

Therefore,  $\Phi_T = 0 + (-\Phi_Y) + \Phi_B$

$$\begin{aligned} \Phi_T &= \sqrt{(\Phi_Y)^2 + (\Phi_B)^2 + 2\Phi_Y\Phi_B \cos 60^\circ} \\ \Phi_T &= \sqrt{\left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + \left(\frac{\sqrt{3}}{2} \phi_m\right)^2 + 2 \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \left(\frac{\sqrt{3}}{2} \phi_m\right) \times \frac{1}{2}} \\ \Phi_T &= \sqrt{3 \times \left(\frac{\sqrt{3}}{2} \phi_m\right)^2} = \frac{3}{2} \phi_m = 1.5 \phi_m \end{aligned}$$

In the similar way as shown in the phasor diagrams the resultant or total flux rotates 60 degrees for every instant and completes one cycle of rotation in the direction of phase sequence of the supply.

Thus when a three phase supply is applied to the three phase winding connected either in star or delta it produces a rotating magnetic field having

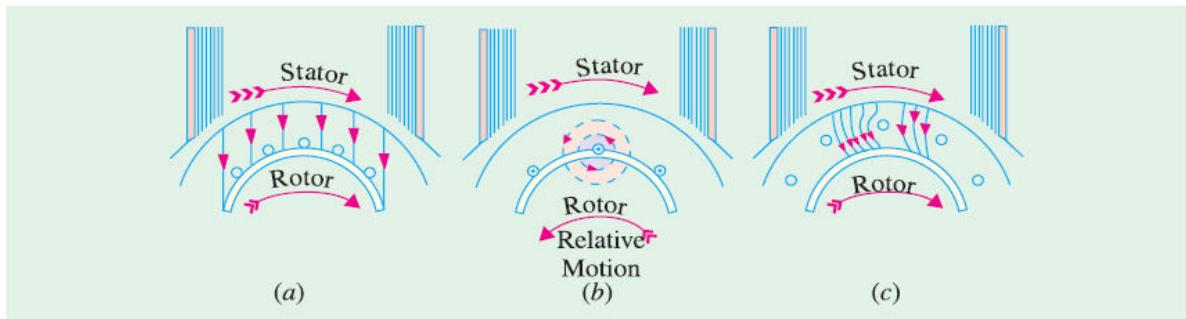
- i. **a constant magnitude of 1.5 times the  $\Phi_m$**
- ii. **a constant speed of synchronous speed  $N_s = 120f/P$**
- iii. **a direction equal to its phase sequence**

## WORKING PRINCIPLE OF 3-PHASE INDUCTION MOTOR

1. The balanced three-phase winding of the stator is supplied with a balanced three-phase voltage.
2. The current in the stator winding produces a rotating magnetic field, with constant magnitude of  $1.5\Phi_m$  and rotates at synchronous speed of  $N_s=120f/P$
3. The magnetic flux lines in the air gap cut both stator and rotor (being stationary, as the motor speed is zero) conductors at the same speed.
4. The emfs in both stator and rotor conductors are induced at the same frequency, i.e. line or supply frequency, with No. of poles for both stator and rotor windings (assuming wound one) being same.
5. As the rotor winding is short-circuited at the slip-rings, current flows in the rotor windings.
6. The electromagnetic torque in the motor is in the same direction as that of the rotating magnetic field, due to the interaction between the rotating flux produced in the air gap by the current in the stator winding, and the current in the rotor winding.
7. This is as per Lenz's law, as the developed torque is in such direction that it will oppose the cause, which results in the current flowing in the rotor winding.
8. As the rotor starts rotating in the same direction, as that of the rotating magnetic field due to production of the torque as stated earlier, the relative velocity decreases, along with lower values of induced emf and current in the rotor.
9. If the rotor speed is equal that of the rotating magnetic field, which is termed as synchronous speed, and also in the same direction, the relative velocity is zero, which causes both the induced emf and current in the rotor to be reduced to zero. Under this condition, torque will not be produced.
10. So, for production of positive (motoring) torque, the rotor speed must always be lower than the synchronous speed. The rotor speed is never equal to the synchronous speed in an IM.

The setting up of the torque for rotating the rotor is explained below :

In Fig (a) is shown the stator field which is assumed to be rotating clockwise. The relative motion of the rotor with respect to the stator is **anticlockwise**. By applying Right-hand rule, the direction of the induced e.m.f. in the rotor is found to be outwards. Hence, the direction of the flux due to rotor current **alone**, is as shown in Fig. (b). Now, by applying the Left-hand rule, or by the effect of combined field [Fig. (c)] it is clear that the rotor conductors experience a force tending to rotate them in clockwise direction. Hence, the rotor is set into rotation in the same direction as that of the stator flux (or field).



### Slip

It is defined as the relative speed or slip speed ( $N_s - N_r$ ) expressed in terms of synchronous speed

$$s = \frac{N_s - N_r}{N_s}$$

- Since the speed at standstill is zero, the slip is 1.
- As the speed of rotor increases the slip decreases.

### **EFFECT OF ROTOR QUANTITIES WITH RESPECT TO THE SLIP IN A 3-PHASE INDUCTION MOTOR**

The rotor quantities that effect with slip are

1. Rotor induced emf ( $E_r$ )
2. rotor emf's frequency ( $f_r$ )
3. induced currents ( $I_r$ )
4. rotor power factor ( $\cos\Phi_r$ )

### Rotor induced emf

Under running conditions the induced emf is directly proportional to the relative speed ( $N_s - N_r$ )

$$\begin{aligned} E_2 \text{ at standstill} &= K N_s, \\ E_r \text{ at running} &= K (N_s - N_r) \end{aligned}$$

$$\frac{E_r}{E_2} = \frac{N_s - N_r}{N_s} = s$$

Therefore  $E_r = sE_2$

### Rotor emf's frequency

Frequency of rotor emf at stand still

Frequency of rotor emf at running

$$f_2 = PN_s/120$$

$$f_r = P(N_s - N_r)/120$$

$$\frac{f_r}{f_2} = \frac{N_s - N_r}{N_s} = s \quad \text{Therefore } f_r = sf_2 = sf$$

### Rotor reactance

Rotor reactance at stand still	X <sub>2</sub> =2ΠfL <sub>2</sub>
Rotor reactance at running	X <sub>r</sub> =2Πf <sub>r</sub> L <sub>2</sub>

$$\frac{X_r}{X_2} = \frac{2\pi f_r L_2}{2\pi f_2 L_2} = \frac{f_r}{f_2} = \frac{sf_2}{f_2} = s \quad \text{Therefore } X_r = sX_2$$

### Rotor impedance

Rotor impedance at stand still	Z <sub>2</sub> =R <sub>2</sub> +jX <sub>2</sub>
Rotor impedance at running	Z <sub>r</sub> = R <sub>2</sub> +jX <sub>r</sub>

$$Z_2 = \sqrt{R_2^2 + X_2^2} \quad \text{and} \quad Z_r = \sqrt{R_2^2 + X_r^2} = \sqrt{R_2^2 + (sX_2)^2}$$

### Rotor induced currents

The rotor current is defined as the ratio of the rotor emf to the rotor impedance.  
Rotor current at stand still is

$$I_2 = \frac{E_2}{Z_2} = \frac{E_2}{\sqrt{r_2^2 + x_2^2}}$$

Rotor current at running is

$$I_r = \frac{E_r}{Z_r} = \frac{sE_2}{\sqrt{r_2^2 + (s^2 x_2^2)}}$$

### Rotor power factor

Power factor is defined as the ratio of the rotor resistance to the rotor impedance  
Rotor power factor at stand still is

$$\cos \phi_2 = \frac{r_2}{Z_2} = \frac{r_2}{\sqrt{r_2^2 + x_2^2}}$$

Rotor power factor at running is

$$\cos \phi_r = \frac{r_2}{Z_r} = \frac{r_2}{\sqrt{r_2^2 + (s^2 x_2^2)}}$$

## TORQUE EQUATION OF 3-Φ INDUCTION MOTOR

### Torque at stand still

The torque in the motor is directly proportional to the product of flux and active component of the rotor current

$$T \propto \phi I_2 \cos \phi_2$$

Here the flux is directly proportional to the rotor induced emf E<sub>2</sub> i.e  $\phi \propto E_2$

The rotor current I<sub>2</sub> and rotor power factor cosΦ<sub>2</sub> are

$$I_2 = \frac{E_2}{Z_2} \text{ and } \cos \phi_2 = \frac{R_2}{Z_2}$$

$$T \propto E_2 \times \frac{E_2}{\sqrt{R_2^2 + X_2^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_2^2}}$$

$$T = k \frac{E_2^2 R_2}{R_2^2 + X_2^2} \text{ where } k = \frac{3}{2\pi n_s} \text{ here } n_s \text{ is synchronous speed in rps}$$

$$\text{Therefore, torque at standstill is } T = \left( \frac{3}{2\pi n_s} \right) \frac{E_2^2 R_2}{R_2^2 + X_2^2}$$

From the above equation torque at stand still depends on rotor resistance ( $R_2$ ), so keeping this  $R_2$  as variable the condition for the maximum torque at standstill is

$$\frac{dT_{st}}{dR_2} = 0$$

Rewriting the stand still torque

$$T = K \frac{R_2}{R_2^2 + X_2^2} \text{ where } K = \frac{3E_2^2}{2\pi n_s} \text{ and } T \propto \frac{R_2}{R_2^2 + X_2^2}$$

$$\frac{d\left(\frac{R_2}{R_2^2 + X_2^2}\right)}{dR_2} = 0 \Rightarrow \frac{(R_2^2 + X_2^2) - R_2(2R_2)}{(R_2^2 + X_2^2)^2} = 0 \Rightarrow (R_2^2 + X_2^2) - R_2(2R_2) = 0$$

$$R_2^2 + X_2^2 = 2R_2^2 \Rightarrow R_2^2 = X_2^2 \Rightarrow R_2 = X_2$$

Therefore on adding the resistance to the rotor such that  $R_2 = X_2$ , the motor will develop maximum torque at stand still.

$$\text{The maximum torque at standstill is } T_{max} = \left( \frac{3}{2\pi n_s} \right) \frac{E_2^2 X_2}{X_2^2 + X_2^2} = \frac{K}{2X_2} \quad T_{max} = \frac{3}{2\pi n_s} \frac{E_2^2}{2X_2}$$

### Torque at running condition

The torque in the motor is directly proportional to the product of flux and active component of the rotor current

$$T \propto \phi I_r \cos \phi_r$$

Here the flux is directly proportional to the rotor induced emf  $E_2$  i.e.  $\phi \propto E_2$

The rotor current  $I_r$  and rotor power factor  $\cos \Phi_r$  are

$$I_r = \frac{E_r}{Z_r} \text{ and } \cos \phi_r = \frac{R_2}{Z_r}$$

$$T \propto E_2 \times \frac{E_r}{\sqrt{R_2^2 + X_r^2}} \times \frac{R_2}{\sqrt{R_2^2 + X_r^2}}$$

$$T \propto E_2 \times \frac{sE_2}{\sqrt{R_2^2 + (sX_2)^2}} \times \frac{R_2}{\sqrt{R_2^2 + (sX_2)^2}}$$

$$T = k \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \text{ where } k = \frac{3}{2\pi n_s} \text{ here } n_s \text{ is synchronous speed in rps}$$

$$\text{Therefore, torque at standstill is } T = \left( \frac{3}{2\pi n_s} \right) \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2}$$

From the above equation torque under running condition depends on slip ( $s$ ), so keeping this ' $s$ ' as variable the condition for the maximum torque at running is

$$\frac{dT}{ds} = 0$$

Rewriting the torque under running

$$T = K \frac{sR_2}{R_2^2 + (sX_2)^2} \text{ where } K = \frac{3E_2^2}{2\pi n_s} \text{ and } T \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

$$\frac{d\left(\frac{sR_2}{R_2^2 + (sX_2)^2}\right)}{ds} = 0 \Rightarrow \frac{(R_2^2 + (sX_2)^2)R_2 - sR_2(2sX_2^2)}{(R_2^2 + (sX_2)^2)^2} = 0 \Rightarrow (R_2^2 + (sX_2)^2)R_2 - sR_2(2sX_2^2) = 0$$

$$(R_2^2 + (sX_2)^2) = s(2sX_2^2) \Rightarrow R_2^2 + (sX_2)^2 = 2s^2 X_2^2 \Rightarrow R_2^2 = (sX_2)^2 \Rightarrow R_2 = (sX_2) \Rightarrow s_m = \frac{R_2}{X_2}$$

Therefore the motor when rotates at a slip  $s_m = \frac{R_2}{X_2}$  then the motor will develop maximum torque at running.

The maximum torque at running is

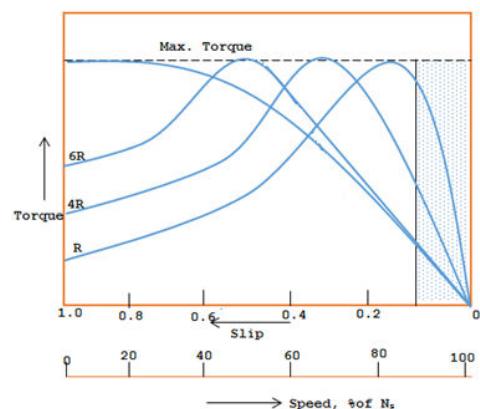
$$T = K \frac{s^2 X_2}{(sX_2)^2 + (sX_2)^2} \text{ where } K = \frac{3E_2^2}{2\pi n_s}$$

$$T = K \frac{s^2 X_2}{2(sX_2)^2} = \frac{K}{2X_2} \quad T_{\max} = \frac{3}{2\pi n_s} \frac{E_2^2}{2X_2}$$

Thus, the magnitude of the maximum torque is same at both standstill and running conditions

### TORQUE - SLIP CHARACTERISTICS

- The torque-slip characteristics in an induction motor shows the variation of the torque developed with respect to changes of slip.
- When the load on the motor is removed gradually the speed increases and the slip



decreases

- Considering, the speed at standstill  $N_r = 0$  the slip  $s = 1$  and as the speed increases from 0 to  $N_s$  the slip  $s$  decreases from 1 to zero, any how the induction motor never rotates at  $N_s$  so the slip never becomes 0
- Let the torque in an induction motor is

$$T = \left( \frac{3}{2\pi n_s} \right) \frac{sE_2^2 R_2}{R_2^2 + (sX_2)^2} \quad T \propto \frac{sR_2}{R_2^2 + (sX_2)^2}$$

- For the smaller values of slips i.e  $0 < s < s_m$ ,  $sX_2 \ll R_2$  so neglecting  $sX_2$ , the torque in this smaller range of slips is

$$T \propto \frac{sR_2}{R_2^2} \quad T \propto \frac{s}{R_2} \quad T \propto s$$

- As the torque is directly proportional to slip  $s$ , Therefore as slip increases the torque increases linearly and attains maximum torque when slip  $s = s_m$
- For the larger values of slips i.e  $s_m < s < 1$ ,  $R_2 \ll sX_2$  so neglecting  $R_2$ , the torque in this larger range of slips is

$$T \propto \frac{R_2}{sX_2^2} \quad T \propto \frac{1}{s}$$

- As the torque is inversely proportional to slip  $s$ , Therefore as slip increases the torque decreases linearly and falls to the value of standstill torque  $T_{st}$  at  $s = 1$

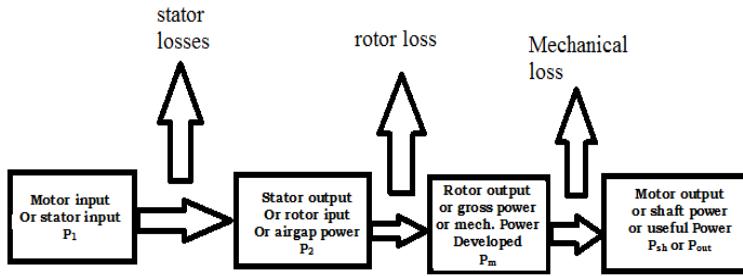
Salient points:

1. The maximum value of the torque is independent to the rotor resistance
2. The slip at which the maximum torque occurs ( $s_m$ ) depends on the rotor resistance
3. The motor develops maximum torque at starting itself by making  $s_m = 1$  which is possible when  $R_2 = X_2$

## POWER STAGES IN A 3-PHASE INDUCTION MOTOR

In a 3-Phase induction motor the power losses occurs in stator and rotor

- Stator losses = Stator core loss + stator Cu loss
- Rotor losses = Rotor Cu loss
- Mechanical loss = Friction and windage loss



Therefore,

- Stator output power ( $P_2$ ) = Stator input power ( $P_1$ ) – stator losses
- Rotor output power ( $P_m$ ) = Rotor input power ( $P_2$ ) – rotor Cu loss
- Motor output power ( $P_{sh}$ ) = Rotor output power ( $P_m$ ) – Mechanical loss

#### Relationship between rotor input, rotor output and slip in a 3-Phase induction motor

The mechanical power in a motor is given by

$$P_m = \frac{2\pi N_r}{60} T \text{ or } P_2 = \frac{2\pi N_s}{60} T$$

Where  $P_m$  is the mechanical or gross output power and  $P_2$  is the air gap power or rotor input.  $T$  = torque in the motor

$$P_m = \omega_r T \quad \dots \dots (1) \quad \text{and} \quad P_2 = \omega_s T \quad \dots \dots (2)$$

$$\text{Eq (2)} - \text{Eq (1)} = P_2 - P_m = \omega_s T - \omega_r T = (\omega_s - \omega_r)T \quad \dots \dots (3)$$

Divide Eq (3) with Eq (2)

$$\frac{P_2 - P_m}{P_2} = \frac{\text{Rotor Cu Loss}}{P_2} = \frac{(\omega_s - \omega_r)}{\omega_s} = s \quad \text{Rotor cu loss (RCL)} = sP_2 \quad \dots \dots (4)$$

Divide Eq (1) with Eq (2)

$$\frac{P_m}{P_2} = \frac{\omega_r}{\omega_s} = \frac{P_2 - RCL}{P_2} = \frac{P_2 - sP_2}{P_2} = \frac{P_2(1-s)}{P_2} = 1-s \quad P_m = (1-s) P_2 \quad \dots \dots (5)$$

Divide Eq (5) with Eq (4)

$$\frac{P_m}{RCL} = \frac{P_2(1-s)}{sP_2} = \frac{(1-s)}{s} \quad P_m = \frac{(1-s)}{s} \times RCL \quad \dots \dots (6)$$

Finally from Eq's (4), (5) & (6)  $P_2 : RCL : P_m = 1 : s : (1-s)$

Thus,

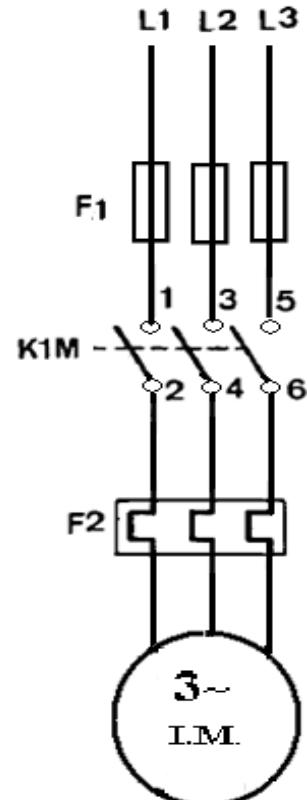
$$\eta = \frac{\text{motor output}}{\text{motor input}} = \frac{\text{motor output}}{\text{motor output} + \text{mech loss} + \text{rotor cu loss} + \text{stator core loss} + \text{stator cu loss}}$$

## STARTING METHOD FOR INDUCTION MOTORS

- A 3-phase induction motor is theoretically self starting. The stator of an induction motor consists of 3-phase windings, which when connected to a 3-phase supply creates a rotating magnetic field. This will link and cut the rotor conductors which in turn will induce a current in the rotor conductors and create a rotor magnetic field. The magnetic field created by the rotor will interact with the rotating magnetic field in the stator and produce rotation.
- Therefore, 3-phase induction motors employ a starting method not to provide a starting torque at the rotor, but because of the following reasons;
  1. Reduce heavy starting currents and prevent motor from overheating.
  2. Provide overload and no-voltage protection.
- There are many methods in use to start 3-phase induction motors. Some of the common methods are;
  - Direct On-Line Starter (DOL)
  - Star-Delta Starter
  - Auto Transformer Starter
  - Rotor resistance Starter

### Direct On-Line Starter (DOL)

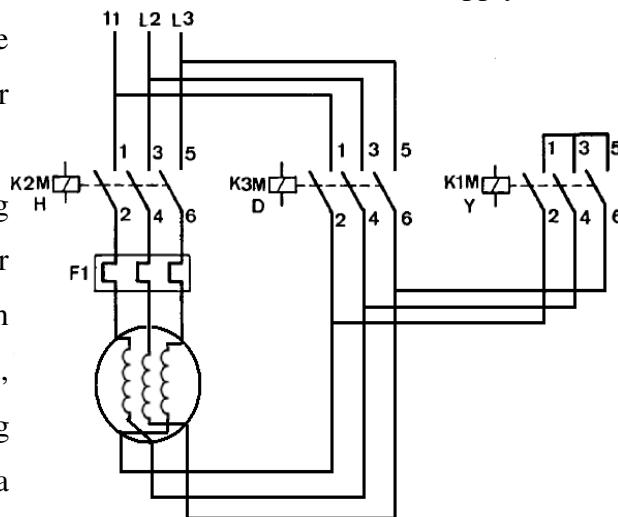
1. The Direct On-Line (DOL) starter is the simplest and the most inexpensive of all starting methods and is usually used for squirrel cage induction motors.
2. It directly connects the contacts of the motor to the full supply voltage. The starting current is very large, normally 6 to 8 times the rated current.
3. The starting torque is likely to be 0.75 to 2 times the full load torque.
4. In order to avoid excessive voltage drops in the supply line due to high starting currents, the DOL starter is used only for motors with a rating of less than 5KW
5. There are safety mechanisms inside the DOL starter which provides protection to the motor as well as the operator of the motor.
6. The DOL starter consists of a coil operated contactor K1M controlled by start and stop push buttons.



7. On pressing the start push button S1, the contactor coil K1M is energized from line L1. The three mains contacts (1-2), (3-4), and (5-6) in fig. are closed. The motor is thus connected to the supply.
8. When the stop push button S2 is pressed, the supply through the contactor K1M is disconnected. Since the K1M is de-energized, the main contacts (1-2), (3-4), and (5-6) are opened. The supply to motor is disconnected and the motor stops.

### **Star-Delta Starter**

1. The star delta starting is a very common type of starter and extensively used, compared to the other types of the starters. This method used reduced supply voltage in starting. Figure shows the connection of a 3phase induction motor with a star – delta starter.



2. The method achieved low starting current by first connecting the stator winding in star configuration, and then after the motor reaches a certain speed, throw switch changes the winding arrangements from star to delta configuration.

3. By connecting the stator windings, first in star and then in delta, the line current drawn by the motor at starting is reduced to one-third as compared to starting current with the windings connected in delta.

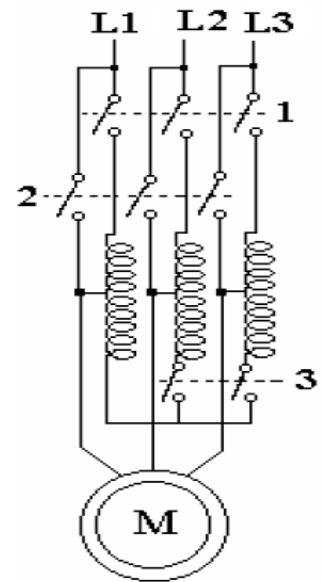
4. At the time of starting when the stator windings are start connected, each stator phase gets voltage  $\frac{V_L}{\sqrt{3}}$  where  $V_L$  is the line voltage.

5. Since the torque developed by an induction motor is proportional to the square of the applied voltage, star- delta starting reduced the starting torque to one – third that obtainable by direct delta starting.

- a. K2M Main Contactor
- b. K3M Delta Contactor
- c. K1M Star Contactor
- d. F1 Thermal Overload Relay

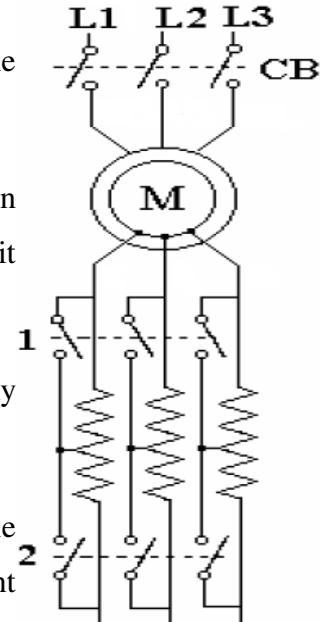
### Auto Transformer Starter

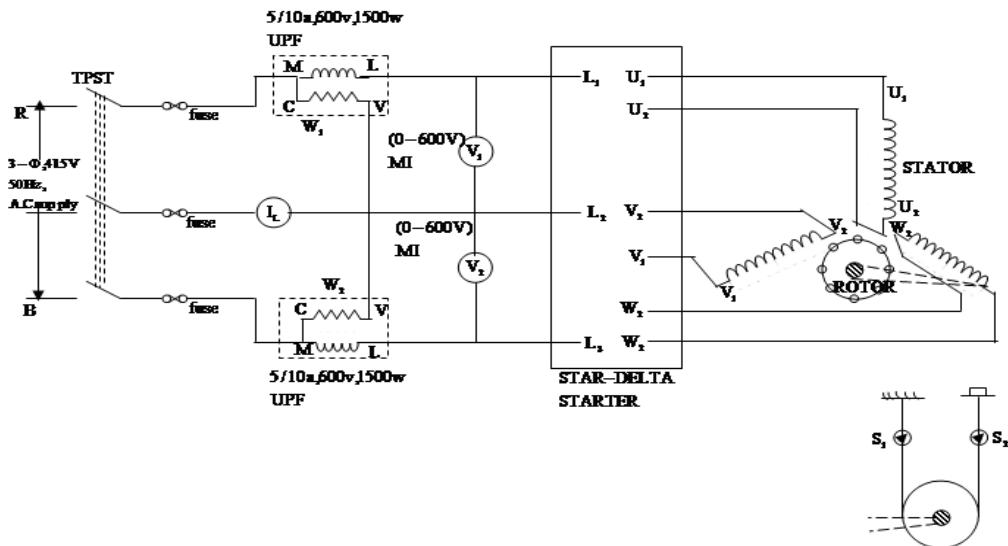
1. The operation principle of auto transformer method is similar to the star delta starter method.
2. The starting current is limited by (using a three phase auto transformer) reduce the initial stator applied voltage.
3. The auto transformer starter is more expensive, more complicated in operation and bulkier in construction when compared with the star – delta starter method.
4. But an auto transformer starter is suitable for both star and delta connected motors, and the starting current and torque can be adjusted to a desired value by taking the correct tapping from the auto transformer.
5. When the star delta method is considered, voltage can be adjusted only by factor of  $\frac{1}{\sqrt{3}}$ .



### Rotor Resistance Starter

1. This method allows external resistance to be connected to the rotor through slip rings and brushes.
2. Initially, the rotor resistance is set to maximum and is then gradually decreased as the motor speed increases, until it becomes zero.
3. The rotor impedance starting mechanism is usually very bulky and expensive when compared with other methods.
4. It also has very high maintenance costs. Also, a considerable amount of heat is generated through the resistors when current runs through them.



**BRAKE TEST ON 3-PHASE INDUCTION MOTOR.**

1. The brake test is a direct method of testing. It consists of applying a brake to a water – cooled pulley mounted on the shaft of the motor.
2. A rope is wound round the pulley and its two ends are attached to two spring balances  $S_1$  and  $S_2$ .
3. The tension of the rope can be adjusted with the help of swivels. Then, the force acting tangentially on the pulley =  $(S_1 - S_2)$  Kgs.
4. If  $r$  is the pulley radius, the torque at the pulley,  $T_{sh} = (S_1 - S_2) r$  kg m.
5. If “ $\omega$ ” is the angular velocity of the motor.  $\omega = 2\pi N/60$ , Where  $N$  is the speed in rpm.
6. Motor output  $P_{out} = 9.81 \times 2\pi N (S_1 - S_2) r$  watts.
7. The motor input can be measured directly from the wattmeter's by summing  $w_1$  and  $w_2$  readings
8. Thus, the efficiency is calculated by taking the ratio of the motor output to the motor input.