

Recurrence relation (R.R)

generating functions :-

* The generating function of a sequence $a_0, a_1, a_2, a_3, \dots, a_n$ of real numbers is written as the series, the given below.

$$G(z) = a_0 + a_1 z + a_2 z^2 + a_3 z^3 + \dots + a_n z^n$$

$$G(z) = \sum_{n=0}^{\infty} a_n z^n$$

find the generating function for the sequence $1, 3, 3^2, 3^3, \dots$ (or) find the generating function for the sequence

of a_n with $a_n = 3^n$.

Sol :- given series $1, 3, 3^2, 3^3, \dots$

$$a_n = 3^n$$

The generating function of given series

$$\text{is } G(z) = \sum_{n=0}^{\infty} 3^n z^n$$

Find the generating function for the sequence $1, 2, 3, 4$

Sol :- given series, $1, 2, 3, 4$

$$a_n = n + 1$$

The generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} (n+1) z^n.$$

find the generating function of the following sequences

(i) 0, 1, -2, 3, -4, - - -

(ii) 0, 2, 6, 12, 20, 30, 42, - - -

Sol :- (i) given series, 0, 1, -2, 3, -4, - - -

$$a_n = (-1)^{n+1} \cdot n$$

The generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} (-1)^{n+1} \cdot n z^n$$

$$(-1)^{n+1} \cdot n = 0 \quad n=0$$

$$(-1)^{1+1} \cdot 1 = 1 \quad n=1$$

$$(-1)^{2+1} \cdot 2 = -2 \quad n=2$$

(ii) given series, 0, 2, 6, 12, 20, 30, 42, $\frac{(-1)^{3+1}}{2} \Rightarrow n=3$

$$a_n = \frac{2n(n+1)}{2}$$

$$(-1)^{4+1} \cdot 4 \Rightarrow n=-4$$

The generating function for the given series is

$$G(z) = \sum_{n=0}^{\infty} \frac{2n(n+1)}{2} z^n$$

$$n=3 \Rightarrow \frac{2 \cdot 3 \cdot 4}{2} = 12$$

$$n=0 \Rightarrow 0$$

$$n=1 \Rightarrow \frac{2 \cdot 2}{2} = 2$$

$$n=4 \Rightarrow \frac{2 \cdot 4 \cdot 5}{2} = 20$$

$$n=2 \Rightarrow \frac{2(2)(3)}{2} = 6$$

$$n=5 \Rightarrow \frac{2 \cdot 5 \cdot 6}{2} = 30$$

$$n=6 \Rightarrow \frac{2 \cdot 6 \cdot 7}{2} = 42$$

sequence (a_n)

generating function
 $G(z)$

| | | |
|---|-------------------------|---|
| ① | a^n | $\frac{1}{1-az}$ |
| ② | ka^n | $\frac{k}{1-az}$ |
| ③ | bna^n | $\frac{ba z}{(1-az)^2}$ |
| ④ | 1 | $\frac{1}{1-z}$ |
| ⑤ | $n+1$ | $\frac{1}{(1-z)^2}$ |
| ⑥ | $\frac{1}{n!}$ | e^z |
| ⑦ | $\frac{(-1)^{n+1}}{n!}$ | $\log(1+z) = z - \frac{z^2}{2} + \frac{z^3}{3} - \frac{z^4}{4} + \dots$ |
| ⑧ | nC_k | $(1+x)^n$ |
| ⑨ | $nC_k a^n$ | $(1+ax)^n$ |
| ⑩ | $n-k+1 C_k$ | $\frac{1}{(1-x)^n}$ |
| ⑪ | $(-1)^k n+k-1 C_k$ | $\frac{1}{(1-x)^n}$ |

Problems :-

using generating function to solve the sequence relation using generating function $a_n = 3a_{n-1} + 2$, $n \geq 1$ with $a_0 = 1$

Sol: given $a_n = 3a_{n-1} + 2$ $n \geq 1$ with $a_0 = 1$

Taking both sides

$$\sum_{n=0}^{\infty} a_n z^n = 3 \sum_{n=0}^{\infty} a_{n-1} z^n + 2 \sum_{n=0}^{\infty} z^n$$

$$\sum_{n=0}^{\infty} a_n z^n = 3z \sum_{n=1}^{\infty} a_{n-1} z^{n-1} + 2 \sum_{n=1}^{\infty} z^n$$

$$(G(z) - a_0) = 3z G(z) + 2 \frac{1}{(1-z)}$$

$$G(z) - 1 - 3z G(z) = \frac{2z}{1-z}$$

$$G(z) [1-3z] = \frac{2z}{1-z} + 1$$

$$G(z) = \frac{2z+1-2}{(1-z)(1-3z)}$$

$$G(z) = \frac{z+1}{(1-z)(1-3z)}$$

$$G(z) = \frac{z+1}{(1-z)(1-3z)} = \frac{A}{(1-z)} + \frac{B}{(1-3z)} \rightarrow (1)$$

$$\frac{z+1}{(1-z)(1-3z)} = \frac{A(1-3z) + B(1-z)}{(1-z)(1-3z)}$$

$$z+1 = A(1-3z) + B(1-z) \rightarrow (2)$$

put $z=1$ in eq (2) we get

$$G(z) = a_0 + a_1 z + \dots$$

$$G(z) - a_0 = a_1 z + z^2$$

$$G(z) - a_0 z = a_1 z^2$$

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$$z = A(z) + 0$$

$$\boxed{A = -1}$$

put $z = \frac{1}{3}$ in eq(2) we get

$$Y_3 + 1 = 0 + B(1 - Y_3)$$

$$\frac{1}{Y_3} = B \frac{2}{3}$$

$$\boxed{B = 2}$$

$$G(z) = \frac{z+1}{(1-z)(1-3z)} = \frac{-1}{1-z} + \frac{2}{1-3z}$$

$$G(z) = \frac{-1}{1-z} + \frac{2}{1-3z}$$

$$G(z) = -1\left(\frac{1}{1-z}\right) + 2\left(\frac{1}{1-3z}\right)$$

$$a_n = -1(1) + 2(3^n)$$

$$\boxed{a_n = -1 + 2(3^n)}$$

- ② Using the method of generating function to solve recurrence relation

$$a_n - 2a_{n-1} - 3a_{n-2} = 0, n \geq 2, \text{ with } a_0 = 3, a_1 = 1$$

Sol :- given,

$$a_n - 2a_{n-1} - 3a_{n-2} = 0, n \geq 2,$$

$$\sum_{n=2}^{\infty} a_n z^n - 2 \sum_{n=2}^{\infty} a_{n-1} z^n - 3 \sum_{n=2}^{\infty} a_{n-2} z^n = 0$$

$$(G(z) - a_0 - a_1 z) = 2z \sum_{n=2}^{\infty} a_{n-1} z^{n-1} - 3z^2 \sum_{n=2}^{\infty} a_{n-2} z^{n-2} = 0$$

$$(G(z) - a_0 - a_1 z) - 2z (G(z) - a_0) - 3z^2 G(z) = 0$$

$$(G(z) - 3 - z) - 2z (G(z) - 3) - 3z^2 G(z) = 0$$

$$G(z) [-3z^2 - 2z + 1] - 3 - z + 6z = 0$$

$$G(z) [-3z^2 - 2z + 1] - 3 + 5z = 0$$

$$G(z) = \frac{3 - 5z}{(-3z^2 - 2z + 1)}$$

$$G(z) = \frac{3 - 5z}{(1+z)(1-3z)}$$

$$G(z) = \frac{3 - 5z}{(1+z)(1-3z)} = \frac{A}{(1+z)} + \frac{B}{(1-3z)} \rightarrow (1)$$

$$\frac{3 - 5z}{(1+z)(1-3z)} = \frac{A(1-3z) + B(1+z)}{(1+z)(1-3z)}$$

$$3 - 5z = A(1 - 3z) + B(1 + z) \rightarrow (2)$$

put $z = -1$ in eq ②, we get

$$3 - 5(-1) = A(1 - 3(-1)) + B(1 - 1)$$

$$3 + 5 = A(1 + 3) + B(0)$$

$$8 = A(4)$$

$$\boxed{A = 2}$$

Put $z = \frac{1}{3}$ in eq ①, we get

$$3 - 5\left(\frac{1}{3}\right) = A\left(1 - 3\left(\frac{1}{3}\right)\right) + B\left(1 + \frac{1}{3}\right)$$

$$3 - \frac{5}{3} = A(1 - \frac{3}{5}) + B(-)$$

$$3 - \frac{5}{3} = A(1 - 1) + B(\frac{4}{3})$$

$$\frac{9-5}{3} = B(\frac{4}{3})$$

$$\frac{4}{3} = B(\frac{4}{3})$$

$$\boxed{B=1}$$

$$G(z) = \frac{2}{1+z} + \frac{1}{1-3z}$$

$$G(z) = 2\left(\frac{1}{1-(z)}\right) + \frac{1}{1-3z}$$

$$a_n = 2(-1)^n + (3^n)$$

$$a_n = -2 + 3^n$$

*Recurrence relation :-

An equation that express a_n in terms of one or more of the previous terms of the sequence $\{a_0, a_1, a_2, \dots, a_n\}$ is called a recurrence relation for the sequence $\{a_n\}$.

- 1) find the first five terms of the sequence define by each of the following recurrence relation and initial conditions

$$(i) a_n = a_{n-1}^2 - 1, a_1 = 2$$

$$(ii) a_n = n a_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1$$

$$(iii) a_n = a_{n-1} + a_{n-3}, a_0 = 1, a_1 = 2, a_2 = 0$$

(i) given R.R is $a_n = a_{n-1}^2$

Sol:

$$\text{put } a = 2$$

$$a_2 = a_{2-1}^2$$

$$a_2 = a_1^2$$

$$a_2 = 4$$

$$a_3 = a_2^2 = 16$$

$$a_4 = a_3^2 = (16)^2 = 256,$$

$$a_5 = a_4^2 = (256)^2 = 65536$$

$$a_6 = a_5^2 = (65536)^2 = 4294967296.$$

$$(ii) a_n = n a_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1$$

given R.R is $a_n = n a_{n-1} + n^2 a_{n-2}$

$$\text{put } n = 2$$

$$a_2 = 2 a_{2-1} + 2^2 a_{2-2}$$

$$a_2 = 2a_1 + 4a_0$$

$$a_2 = 2(1) + 4(1)$$

$$a_2 = 2 + 4$$

$$a_2 = 6$$

$$\Rightarrow a_3 = 3 a_{3-1} + 3^2 a_{3-2}$$

$$= 3 a_2 + 9 a_1$$

$$= 3(6) + 9(1)$$

$$= 18 + 9 \Rightarrow 27$$

$$\begin{aligned}
 a_4 &= 4a_{4-1} + (4)^2 a_{4-2} \\
 &= 4a_3 + 16a_2 \\
 &= 4(27) + 16(6) \\
 &= 108 + 96 \\
 &= 204
 \end{aligned}$$

$$\begin{aligned}
 a_5 &= 5a_{5-1} + (5)^2 a_{5-2} \\
 &= 5a_4 + 25a_3 \\
 &= 5(204) + 25(27)
 \end{aligned}$$

$$\begin{aligned}
 &= 1020 + 675 \\
 a_5 &= 1695 \\
 a_6 &= 6(a_5) + 36a_4 \\
 &= 6(1695) + 36(204) \\
 &= 17514
 \end{aligned}$$

(iii) given RRIS, $a_n = a_{n-1} + a_{n-3}$, $a_0 = 1, a_1 = 2, a_2 = 0$

put $n = 3$

$$\rightarrow a_3 = a_{3-1} + a_{3-3}$$

$$a_3 = a_2 + a_0$$

$$a_3 = 0 + 1$$

$$a_3 = 1$$

$$\rightarrow a_4 = a_{4-1} + a_{4-3}$$

$$a_4 = a_3 + a_1$$

$$a_4 = 1 + 2$$

$$a_4 = 3$$

$$\rightarrow a_5 = a_{5-1} + a_{5-3}$$

$$= a_4 + a_2 \Rightarrow 3 + 0 \Rightarrow 3$$

$$\rightarrow a_6 = a_{6-1} + a_{6-3}$$

$$= a_5 + a_3$$

$$= 3 + 1$$

$$a_7 = 4$$

$$\rightarrow a_7 = a_{7-1} + a_{7-3}$$

$$= a_6 + a_4$$

$$= 4 + 3$$

$$a_7 = 7$$

By using an iterative approach find the solutions to each of these sequences relation with the given initial conditions

$$(i) a_n = a_{n-1} + 2, a_0 = 3$$

$$(ii) a_n = a_{n-1} + n, a_0 = 1$$

$$(iii) a_n = a_{n-1} + 2n + 3, a_0 = 4$$

$$(iv) a_n = 3a_{n-1} + 1, a_0 = 1$$

(i) given R.R is $a_n = a_{n-1} + 2$

put $n=1$

$$a_1 = a_0 + 2$$

$$a_1 = 3 + 2$$

$$a_1 = 5$$

put $n=2$

$$a_2 = a_1 + 2$$

$$a_2 = 5 + 2$$

$$= 7$$

put $n=3$

$$a_3 = a_2 + 2$$

$$a_3 = 9$$

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put $n=4$

$$\begin{aligned}a_4 &= a_{4-1} + 4 \\&= a_3 + 4 \\&= 7 + 4 \\&= 11\end{aligned}$$

$$1 + \frac{n+1}{2} \cdot n$$

$$\begin{aligned}\Rightarrow \text{put } n=1 &\Rightarrow 1 + \frac{1+1}{2} \cdot 1 = 2 \\&\Rightarrow \text{put } n=2 \Rightarrow 1 + \frac{2+1}{2} \cdot 2 = 4 \\&\Rightarrow \text{put } n=3 \Rightarrow 1 + \frac{3+1}{2} \cdot 3 = 7 \\&\Rightarrow \text{put } n=4 \Rightarrow 1 + \frac{4+1}{2} \cdot 4 = 11\end{aligned}$$

$$a_n = 1 + \left(\frac{n+1}{2}\right) \cdot n$$

(ii)

given R.R is $a_n = a_{n-1} + 2n + 3$, $a_0 = 4$

put $n=1$

$$\begin{aligned}a_1 &= a_{1-1} + 2(1) + 3 \\&= a_0 + 2 + 3 \\&= a_0 + 5 \\&= 4 + 5 \\&= 9\end{aligned}$$

put $n=2$

$$\begin{aligned}a_2 &= a_{2-1} + 2(2) + 3 \\&= a_1 + 4 + 3 \\&= a_1 + 7 \\&= 9 + 7 \\&= 16\end{aligned}$$

put $n=3$

$$\begin{aligned}a_3 &= a_{3-1} + 2(3) + 3 \\&= a_2 + 6 + 3 \\&= a_2 + 9 = 16 + 9 \\&= 25\end{aligned}$$

put $n=4$

$$\begin{aligned}a_4 &= a_{4-1} + 2(4) + 3 \\&= a_3 + 8 + 3 \\&= a_3 + 11 \\&= 95 + 11 \\&= 36\end{aligned}$$

$$\begin{aligned}\Rightarrow a_0 &= n^2 + 0 \times 4 + 4 \Rightarrow n=0 \\&\Rightarrow a_1 = n^2 + 1 \times 4 + 4 \Rightarrow n=1 \\&\Rightarrow a_2 = n^2 + 2 \times 4 + 4 \Rightarrow n=2 \\&\Rightarrow a_3 = n^2 + 3 \times 4 + 4 \Rightarrow n=3\end{aligned}$$

1
1
1
1

$$a_n = n^2 + n \cdot 4 + 4$$

$$a_n = n^2 + 4n + 4$$

(iv) given R.R is $a_n = 3a_{n-1} + 1$, $a_0 = 1$

put $n=1$

$$a_1 = 3a_{1-1} + 1$$

$$= 3a_0 + 1$$

$$= 3(1) + 1$$

$$\therefore a_1 = 3 + 1$$

$$\therefore a_1 = 4$$

put $n=2$

$$a_2 = 3a_{2-1} + 1$$

$$= 3a_1 + 1$$

$$= 3(4) + 1$$

$$= 13$$

put $n=3$

$$a_3 = 3a_{3-1} + 1$$

$$\begin{aligned}
 a_3 &= 3a_2 + 1 \\
 &= 3(13) + 1 \\
 &= 39 + 1 \\
 &= 40 \\
 \text{put } n = 4 & \Rightarrow a_n = \frac{3^{n+1} - 1}{2} \\
 a_4 &= 3a_3 + 1 \\
 &= 3(40) + 1 \\
 &= 121 \\
 \text{put } n = 5 & \Rightarrow a_n = \frac{3^{n+1} - 1}{2} \\
 a_5 &= 3a_4 + 1 \\
 &= 3(121) + 1 \\
 &= 364 + 1 \\
 &= 365 \\
 \text{put } n = 6 & \Rightarrow a_n = \frac{3^{n+1} - 1}{2} \\
 a_6 &= 3a_5 + 1 \\
 &= 3(365) + 1 \\
 &= 1095 + 1 \\
 &= 1096 \\
 \text{put } n = 7 & \Rightarrow a_n = \frac{3^{n+1} - 1}{2} \\
 a_7 &= 3a_6 + 1 \\
 &= 3(1096) + 1 \\
 &= 3288 + 1 \\
 &= 3289
 \end{aligned}$$

Characteristic roots : consider the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k}$, where $c_1, c_2, c_3, \dots, c_k$ are real numbers.

→ The characteristic equation of Recurrence relation,

$$\lambda^k - c_1 \lambda^{k-1} - c_2 \lambda^{k-2} - \dots - c_k = 0$$

→ The solutions of characteristic equations are three types

(1) if roots are real & different then the solution is,

$$a_n = c_1 \lambda_1^n + c_2 \lambda_2^n$$

(2) if roots are real & equal then solution is,

$$a_n = (c_1 + c_2 n) \lambda^n$$

(3) if roots are complex roots, then solution is,

$$a_n = \lambda^n [c_1 \cos \theta + c_2 \sin \theta]$$

Problems :-

1. solve the Recurrence relation, $a_n = 5a_{n-1} + 6a_{n-2}$ for $n \geq 2$, $a_0 = 1$, $a_1 = 0$.

Sol :- given R.R is,

$$a_n = 5a_{n-1} + 6a_{n-2}$$

By simplifying,

$a_n - 5a_{n-1} + 6a_{n-2} = 0$
characteristic eq. of R.R is $\lambda^2 - 5\lambda + 6 = 0$
roots, $\lambda = 2, 3$

∴ the given roots are real and different, then the solution is

$$a_n = c_1 \lambda_1^n + c_2 \lambda_2^n$$

$$a_n = c_1(2)^n + c_2(3)^n$$

Now, put $n=0$

$$a_0 = c_1 2^0 + c_2 3^0$$

$$1 = c_1 + c_2 \rightarrow (1)$$

Now, put $n=1$

$$a_1 = c_1 2^1 + c_2 3^1$$

$$0 = 2c_1 + 3c_2 \rightarrow (2)$$

Now (1) & (2) becomes,

$$c_1 + c_2 = 1 \times (2)$$

$$2c_1 + 3c_2 = 0 \times (1)$$

$$\underline{2c_1 + 2c_2 = 2}$$

$$\underline{2c_1 + 3c_2 = 0}$$

$$\underline{-c_2 = 2}$$

$$c_2 = -2$$

$$\text{From, } -2 + c_1 = 1$$

$$c_1 = 1 + 2$$

$$c_1 = 3$$

$$\therefore a_n = 3 \cdot 2^n + 2 \cdot 3^n$$

2) solve the sequence relation of

$$a_n - 6a_{n-1} + 9a_{n-2} = 0 \quad n \geq 2 \quad a_0 = 5, a_1 = 12, \\ n > i = 2$$

Sol :- Given $a_0 = 5$
 $a_1 = 12$

R.R is $a_n - 6a_{n-1} + 9a_{n-2} = 0$

characteristic equation

$$\lambda^2 - 6\lambda + 9 = 0$$

$$\lambda^2 - 3\lambda - 3\lambda + 9 = 0$$

$$\lambda(\lambda - 3) - 3(\lambda - 3) = 0$$

$$(\lambda - 3)(\lambda - 3) = 0$$

$$\text{roots} = 3, 3$$

the given roots are real and equal

the solution will be

$$a_n = (c_1 + c_2 n) 3^n$$

put $n=0$, $a_0 = (c_1 + c_2(0)) 3^0$

$$5 = c_1 \rightarrow (1)$$

put $n=1$, $a_1 = (c_1 + c_2(1)) 3^1$

$$12 = (c_1 + c_2) 3 \rightarrow (2)$$

$$c_1 \neq 15$$

$$5 = c_1$$

$$12 = (c_1 + c_2) 3$$

$$15 + 3c_2 = 12$$

$$3c_2 = 12 - 15$$

$$3c_2 = -3$$

$$c_2 = -1$$

$$a_n = (5 - 1) 3^n$$

3) solve the recurrence relation

$$a_n = 8a_{n-1} - 16a_{n-2} \text{ for } n \geq 2, a_0 = 16,$$

$$a_1 = 80.$$

sol:

given R.R is

$$a_n = 8a_{n-1} - 16a_{n-2}$$

By simplifying,

$$a_n - 8a_{n-1} + 16a_{n-2} = 0$$

characteristic eq of R.R is $\lambda^2 - 8\lambda + 16 = 0$

$$\lambda^2 - 8\lambda + 16 = 0$$

$$\lambda^2 - 4\lambda - 4\lambda + 16 = 0$$

$$\lambda(\lambda - 4) - 4(\lambda - 4) = 0$$

$$(\lambda - 4)(\lambda - 4) = 0$$

$$\lambda = 4, 4$$

The given roots are real and equal
the solution will be,

$$a_n = (c_1 + c_2 n) 4^n$$

$$\text{put } n=0$$

$$a_0 = (c_1 + c_2(0)) 4^0$$

$$16 = c_1$$

$$\text{put } n=1$$

$$a_1 = (c_1 + c_2(1)) 4^1$$

$$80 = (16 + c_2) 4$$

$$64 + 4c_2 = 80$$

$$4c_2 = 80 - 64$$

$$c_2 = 4$$

$$a_n = (16+4) \cdot 4^n$$

④ solve the sequence relation

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3} \text{ for } n=3,4,5\dots$$

$$\text{with } a_0 = 3, a_1 = 6, a_2 = 0$$

Sol:- given sequence relation is

$$a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$$

By simplifying,

$$a_n - 2a_{n-1} + a_{n-2} + 2a_{n-3} = 0$$

characteristic of sequence relation is

$$\lambda^3 - 2\lambda^2 - \lambda + 2 = 0$$

$$\text{roots} = 1, 2, -1$$

The roots are real & different

The solution will be

$$\begin{array}{r|rrr} 1 & 1 & -2 & 1 & 2 \\ & 0 & 1 & -1 & -2 \\ \hline & 1 & -1 & -2 & 0 \end{array}$$

$$\lambda(\lambda-2) + 1(\lambda-2) = 0$$

$$a_n = c_1 \lambda_1^n + c_2 \lambda_2^n + c_3 \lambda_3^n \quad (\lambda-2)(\lambda+1) = 0$$

$$a_n = c_1(1)^n + c_2(2)^n + c_3(-1)^n \quad \lambda = 2, -1$$

$$a_n = c_1(1)^n + c_2(-1)^n + c_3(2)^n$$

$$\text{put } n=0$$

$$a_0 = c_1(1)^0 + c_2(-1)^0 + c_3(2)^0$$

$$a_0 = c_1 1^0 + c_2 (-1)^0 + c_3 (2)^0$$

$$3 = c_1 + c_2 + c_3 \rightarrow (1)$$

$$\text{put } n=1$$

$$a_1 = c_1(1)^1 + c_2(-1)^1 + c_3(2)^1$$

$c_1 =$

$$6 = c_1 - c_2 + 2c_3 \rightarrow (2)$$

put $n=2$

$$a_2 = c_1(1)^2 + c_2(-1)^2 + c_3(2)^2 \rightarrow (3)$$

$$0 = c_1 + c_2 + 4c_3 \rightarrow (4)$$

solve (1) & (2)

$$c_1 + c_2 + c_3 = 3$$

$$c_1 - c_2 + 2c_3 = 6$$

$$\underline{2c_1 + 3c_3 = 9} \rightarrow (5)$$

solve (2) & (3)

$$c_1 - c_2 + 2c_3 = 6$$

$$c_1 + c_2 + 4c_3 = 0$$

$$\underline{2c_1 + 6c_3 = 6} \rightarrow (6)$$

solve (4) & (5)

$$2c_1 + 3c_3 = 9$$

$$\underline{2c_1 + 6c_3 = 6}$$

$$-3c_3 = 3$$

$$c_3 = -1$$

sub c_3 value in (5)

$$2c_1 + 6(-1) = 6$$

$$2c_1 = 12$$

$$\boxed{c_1 = 6}$$

sub c_1 & c_3 in (2)

$$6 - c_2 - 2 = 6$$

$$-c_2 = 2$$

$$\boxed{c_2 = -2}$$

$$\boxed{c_3 = -1}$$

Solutions of inhomogeneous sequence relation

→ A linear inhomogeneous or non-homogeneous sequence relation with constant coefficients of degree k is a sequence relation of the form $a_n = c_1 a_{n-1} + c_2 a_{n-2} + \dots + c_k a_{n-k} + G(n)$, where c_1, c_2, \dots, c_k are real numbers and $G(n)$ is a function not identically zero depending only on (n) .

Particular solution for $G(n)$:-

| $G(n)$ | P.I |
|--|--|
| ① constant c | constant d |
| ② linear function $(c_0 + c_1 n)$ | $d_0 + d_1 k$ |
| ③ m^{th} degree polynomial $c_0 + c_1 n + c_2 n^2 + \dots + c_m n^m$ | m^{th} degree polynomial $d_0 + d_1 k + d_2 k^2 + \dots + d_m k^m$ |
| ④ π^n $\pi \in \mathbb{R}$ | $d \pi^n$ |

1) solve the sequence relation

$$a_n = 3a_{n-1} + 2^n, a_0 = 1, n \geq 1$$

Sol :-

Given $a_n = 3a_{n-1} + 2^n$

it is a non-homogeneous linear equation,

$$a_n - 3a_{n-1} = 2^n$$

general solution

$$a_n - 3a_{n-1} = 0$$

The characteristic equation of given eqn,
 $r - 3 = 0$

roots 1

$$r = 3$$

The roots are real solution will be
 $a_n = C_1(3)^n$

Put $n=0$

$$a_0 = C_1(3)^0$$

$$a_0 = C_1$$

$$\boxed{C_1 = 1}$$

$$a_n = (3)^n$$

Now, we can PI,

$$PI = 2^n$$

$$d2^n - 3d2^{n-1} = 2^n$$

$$2^{\cancel{n}} \left(d - 3\cancel{2^{n-1}} \right) = 2^n$$

$$2d - 3d = 2$$

$$-d = 2$$

$$\boxed{d = -2}$$

this is of the form dq^n

$$dq^n = (-2)2^n \Rightarrow PI$$

$$\text{Now, } a_n = G + PI$$

$$a_n = (3)^n + (-2)2^n$$