4. MEASUREMENT OF PARAMETERS

T.G.S.Akhil Kumar

Introduction :-

The measurement of resistance is as important as the measurement of any other electrical parameter. From the point of view of measurement, the basic knowledge of resistance measurement is necessary to understand the working of other instruments used for the measurement of other electrical quantities.

Classification of Resistances :-

The resistances are classified as follows:

1) Low resistances :- All the resistances of order of 1Ω or less are classified as low resistances.

* Availability of the stable d.c. standard supply.
* Due to contact and lead resistances are eliminated.
* Low accuracy.

2) Medium resistances :- From 1Ω onwards upto 0.1 MΩ resistances are classified as medium resistances.

3) High resistances :- Resistances of the order of 0.1MΩ and higher are classified as high resistances.

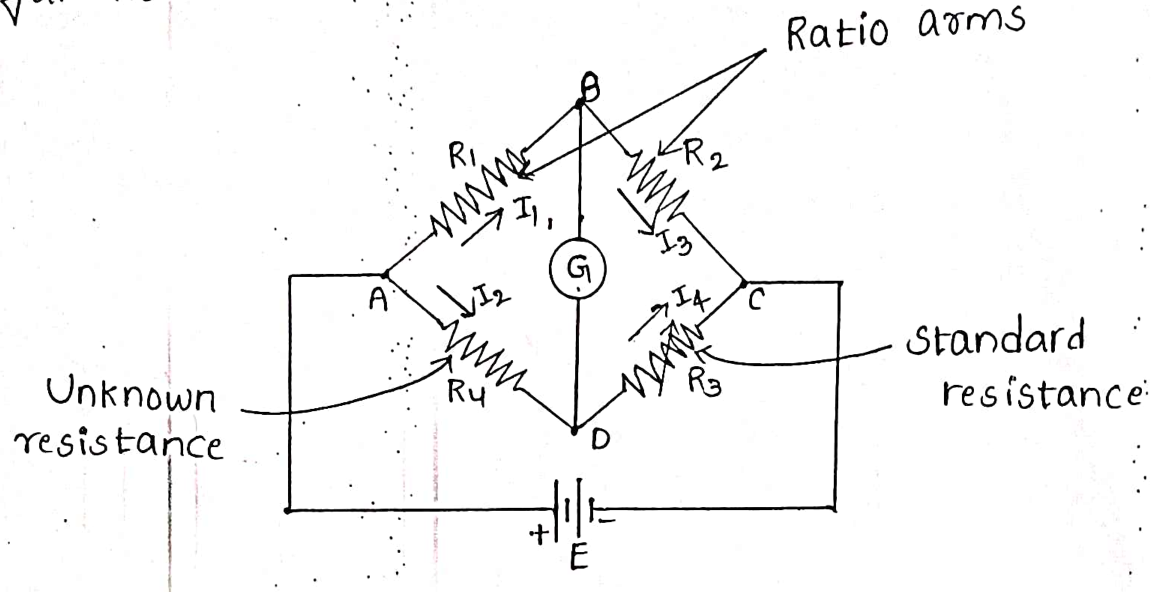
→ The methods used for the measurement of high resistances are as follows :-

i) Loss of charge method   ii) Megger

→ Instead of using the ohmmeter or ammeter voltmeter method, the bridges are used in practice which give high degree of accuracy.

*Wheat stone Bridge :-

The bridge consists of four resistive arms together with a source of e.m.f. and a null detector. The galvanometer is used as a null detector.



→ The arms consisting the resistances R_1 & R_2 are called ratio arms. The arm consisting the standard known resistance 'R_3' is called standard arm. The resistance 'R_4' is the unknown resistance to be measured. The battery is connected between A and C while galvanometer is connected between B and D.

Balance condition :-

→ When the bridge is balanced, the galvanometer carries zero current and it does not show any deflection.

Thus bridge works on the principle of null deflection or null indication.

→ To have zero current through galvanometer, the points B and D must be at the same potential. Thus potential across arm AB must be same as the

Potential across arm AD.

Thus $I_1 R_1 = I_2 R_4$ $\longrightarrow$ (1)

→ As galvanometer current is zero,

$$I_1 = I_3 \text{ and } I_2 = I_4$$

Considering the battery path under balanced condition,

$$I_1 = I_3 = \frac{E}{R_1 + R_2} \longrightarrow (2)$$

and

$$I_2 = I_4 = \frac{E}{R_3 + R_4} \longrightarrow (3)$$

→ Using equations (2) & (3) in equation (1),

$$\frac{E}{R_1 + R_2} \times R_1 = \frac{E}{R_3 + R_4} \times R_4$$

$$R_1(R_3 + R_4) = R_4(R_1 + R_2)$$

$$R_1 R_3 + R_1 R_4 = R_1 R_4 + R_2 R_4$$

$$\boxed{R_4 = R_3 \frac{R_1}{R_2}} \longrightarrow (4)$$

This is required balance condition of wheat stone bridge.

*The following points can be observed,

1. It depends on the ratio of R_1 and R_2 hence these arms are called ratio arms.
2. As it works on null indication, the results are not independent on the callibration and characteristics of galvanometer.

3. The standard resistance 'R_3' can be varied to obtain the required balance.

* Sensitivity of Wheatstone Bridge :-

When the bridge is balanced, the current through galvanometer is zero. But when bridge is not balanced current flows through the galvanometer causing the deflection. The amount of deflection depends on the sensitivity of the galvanometer. Thus the sensitivity can be expressed as amount of deflection per unit current.

$$\boxed{\text{Sensitivity } S = \frac{\text{Deflection } D}{\text{Current } I}}$$

* As the current is in micro ampere and deflection can be measured in mm, radians or degrees, the sensitivity is expressed as mm/uA, radians/uA or degrees/uA

⟶ The amount of deflection per unit voltage across the galvanometer. This is called "voltage sensitivity" of the galvanometer. Mathematically it is denoted as,

$$\boxed{S_v = \frac{\theta}{e}}$$

where

e = Voltage across galvanometer

θ = Deflection of galvanometer

* It is used to measured in degrees per volts or radians per volts.

* While the "bridge sensitivity" is defined as the deflection of the galvanometer per unit fractional change in the unknown resistance. It is denoted as "S_B".

$$\boxed{S_B = \frac{\theta}{\Delta R / R}}$$

Where $\Delta R/R$ = Unit fractional change in unknown resistance

*Wheatstone Bridge Under Small Unbalance :-

→ The bridge sensitivity can be calculated by solving the bridge for small unbalance.

→ At balance condition, $R_4 = R_3 \frac{R_1}{R_2}$

i.e, $$\frac{R_4}{R_3} = \frac{R_1}{R_2}$$

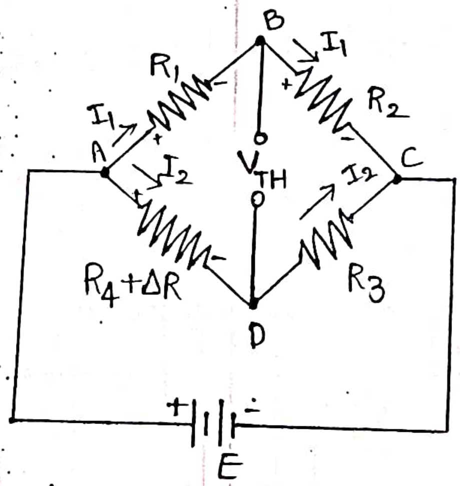


Fig:- Bridge under Unbalance

Let the resistance "R_4" is changed by ΔR creating the unbalance. Due to this, the e.m.f. appears across the galvanometer. To obtain this e.m.f., let us use Thevenin's method. Remove the branch of galvanometer and obtain the voltage across the open circuit teminals.

$$E_{AB} = I_1 R_1 \longrightarrow (1)$$

$$I_1 = \frac{E}{R_1+R_2} \longrightarrow (2)$$

$$E_{AD} = I_2 (R_4+\Delta R) \longrightarrow (3)$$

$$I_2 = \frac{E}{R_3 + R_4 + \Delta R} \longrightarrow (4)$$

$$V_{BD} = V_{TH} = E_{AD} - E_{AB} \longrightarrow (5)$$

$$\longrightarrow V_{TH} = \frac{E(R_4 + \Delta R)}{R_3 + R_4 + \Delta R} - \frac{E}{R_1 + R_2} R_1$$

$$V_{TH} = E\left[\frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_1}{R_1 + R_2}\right] \longrightarrow (6)$$

As, $\frac{R_4}{R_3} = \frac{R_1}{R_2}$ then, $\frac{R_1}{R_1 + R_2} = \frac{R_4}{R_4 + R_3}$

$\longrightarrow$ Using above relation in equation (6),

$$V_{TH} = E\left[\frac{R_4 + \Delta R}{R_3 + R_4 + \Delta R} - \frac{R_4}{R_3 + R_4}\right]$$

$$= E\left[\frac{R_3R_4 + R_3\Delta R + R_4^2 + R_4\Delta R - R_3R_4 - R_4^2 - R_4\Delta R}{(R_3 + R_4)(R_3 + R_4 + \Delta R)}\right]$$

$$= \frac{ER_3\Delta R}{(R_3 + R_4)^2 + (R_3 + R_4)\Delta R}$$

But as ΔR is very small, $(R_3 + R_4)\Delta R << (R_3 + R_4)^2$

$$\longrightarrow \boxed{V_{TH} = V_g = \frac{ER_3\Delta R}{(R_3 + R_4)^2}} \longrightarrow (7)$$

and $\Delta R/R = \Delta R/R_4$ as there is change in 'R_4'.

→ From the galvanometer sensitivity S_V,

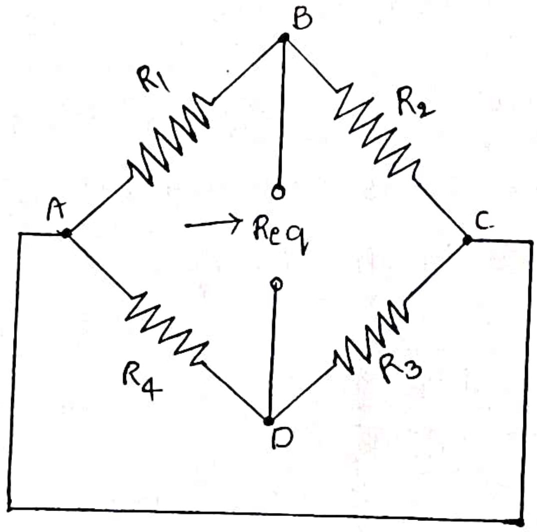
$\theta = S_V \times e$ where e = voltage across galvanometer = V_g

Using 'θ' in the expression of S_B,

$$\therefore S_B = \frac{S_V V_g}{\Delta R / R_4} = \frac{S_V E R_3 \Delta R R_4}{(R_3+R_4)^2} = \frac{S_V E R_3 R_4}{R_3^2 + 2R_3R_4 + R_4^2}$$

$$\boxed{\therefore S_B = \frac{S_V E}{\frac{R_3}{R_4} + 2 + \frac{R_4}{R_3}}}$$

Thevenin's Equivalent and Galvanometer Current :-



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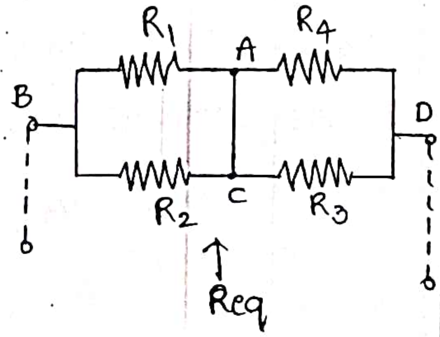
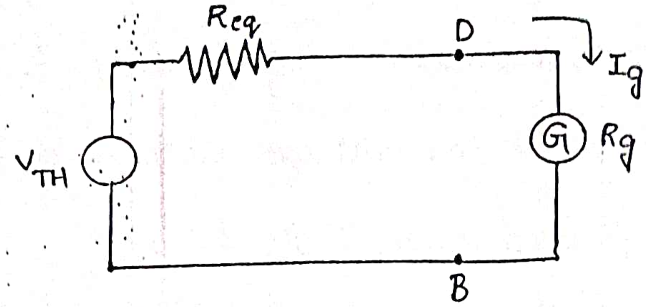


→ $R_{eq} = (R_1 \| R_2) + (R_3 \| R_4)$

$V_{TH} = E_{AD} - E_{AB}$ with R_4 not changed by ΔR.

$$= I_2 R_4 - I_1 R_1 = \frac{E}{R_3+R_4} R_4 - \frac{E}{R_1+R_2} R_1$$

$$\boxed{V_{TH} = E\left[\frac{R_4}{R_3+R_4} - \frac{R_1}{R_1+R_2}\right]}$$



→ Thus Thevenin's equivalent is as shown in above figure

Let R_g = Galvanometer resistance

I_g = Galvanometer current

$$\boxed{I_g = \frac{V_{TH}}{R_{eq}+R_g}}$$

where,

V_{TH} = Thevenin's voltage

Galvanometer Current Under Unbalanced Condition :-

Let the resistance R_4 is changed by ΔR which has caused the unbalance in the bridge.

As derived earlier,

$$V_{TH} = \frac{E R_3 \Delta R}{(R_3+R_4)^2}$$

→ $R_{eq} = (R_1 \| R_2) + (R_3 \| R_4 + \Delta R)$

$$= \frac{R_3(R_4+\Delta R)}{R_3+R_4+\Delta R} + \frac{R_1 R_2}{R_1+R_2}$$

→ Neglecting ΔR compared to R_3 and R_4,

$$R_{eq} = \frac{R_3 R_4}{R_3+R_4} + \frac{R_1 R_2}{R_1+R_2}$$

$$\rightarrow \quad I_g = \frac{V_{TH}}{R_{eq} + R_g}$$

For bridge with equal arms $R_1 = R_2 = R_3 = R_4 = R$ then

$$V_{TH} = \frac{ER\Delta R}{4R^2} = \frac{E\Delta R}{4R}$$

· and $$R_{eq} = \frac{R^2}{2R} + \frac{R^2}{2R} = R$$

$$\boxed{\therefore I_g = \frac{\frac{E\Delta R}{4R}}{R + R_g} = \frac{E(\Delta R/4R)}{R + R_g}}$$

S_B interms of Current Sensitivity of Galvanometer :-

The deflection of galvanometer for a small change in unknown resistance R_4 is,

$$\boxed{\theta = S_v e = S_v V_g = \frac{S_v E R_3 \Delta R}{(R_3 + R_4)^2}}$$

while, $$S_v = \frac{S_i}{R_{eq} + R_g}$$

where, S_i = Current sensitivity of galvanometer

$$\theta = \frac{S_i E R_3 \Delta R}{(R_{eq} + R_g)(R_3 + R_4)^2}$$

and $$\boxed{S_B = \frac{\theta}{\Delta R/R_4} = \frac{S_i E R_3 R_4}{(R_{eq} + R_g)(R_3 + R_4)^2}}$$

where $R_{eq} = (R_1 || R_2) + (R_3 || R_4 + \Delta R)$

* This is the bridge sensitivity interms of current sensi-tivity of the galvanometer.

* Advantages of Wheatstone Bridge :-

The various advantages of wheatstone bridge are,

1. The results are not dependent on the calibration and characteristics of galvanometer as it works on null deflection.
2. The source emf & inaccuracies due to the source fluctu-ations do not affect the balance of the bridge. Hence the corresponding errors are completely avoided.
3. Due to null deflection method used, the accuracy and sensitivity is higher than direct deflection meters.

* Limitations of Wheatstone Bridge :-

→ The effect of lead resistance and contact resistance is very much significant while measuring low resistances.

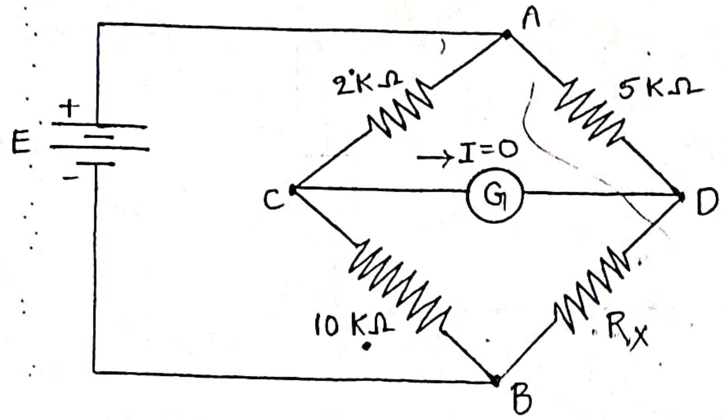
→ The bridge cannot be used for high resistance measurement.

* Applications of Wheatstone Bridge :-

* The wheatstone bridge is basically a d.c. bridge and used to measure the resistances in the range 1Ω to low megaohm.
* It is used to measure the resistance of motor winding, relay coils etc.
* It is used by the telephone companies to locate the cable faults.

Problems :-

(1) The Wheatstone bridge is shown in the below figure. calculate the value of unknown resistance, assuming the bridge to be in balanced condition.



Sol:-

Given data :-

$R_1 = 10 K\Omega$

$R_2 = 2 K\Omega$

$R_3 = 5 K\Omega$ and

$R_4 = R_X = ?$

Calculation :-

→ Under balanced condition,

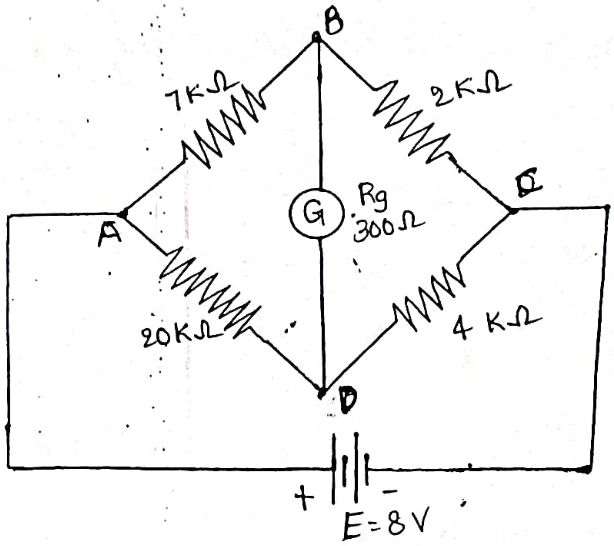
$$\boxed{R_4 R_2 = R_1 R_3}$$

$$R_X = R_4 = \frac{R_1}{R_2} R_3 = \frac{10}{2} \times 5$$

$$R_X = 5 \times 5 = 25 K\Omega \qquad \boxed{R_X = 25 K\Omega}$$

* Thus unknown resistance is 25 KΩ

(2) Calculate the current through the galvanometer for the bridge shown in below figure.



Sol:-

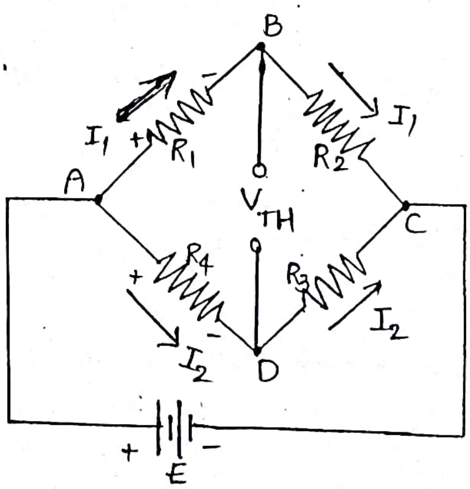
Given data:-

$R_1 = 7K\Omega$, $R_2 = 2K\Omega$

$R_3 = 4K\Omega$, $R_4 = 20K\Omega$, $E = 8V$

calculation:-

→ Use Thevenin's equivalent for I_g



$$V_{TH} = V_{BD} = V_{AD} - V_{AD}$$

$$= I_2 R_4 - I_1 R_1$$

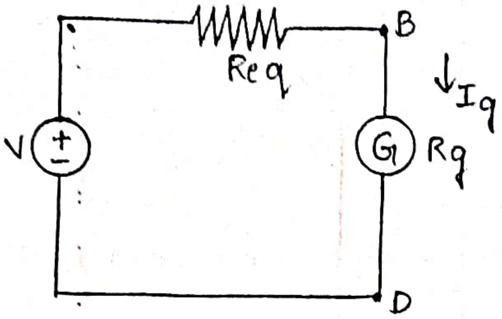
$$= \frac{E}{R_3 + R_4} R_4 - \frac{E}{R_1 + R_2} R_1$$

$$= 8 \left\{ \frac{20}{20+4} - \frac{7}{7+2} \right\}$$

$$= 0.44V$$

Thus 'B' is positive with respect to 'D'

Now $R_{eq} = [R_1 \| R_2] + [R_3 \| R_4]$ ..... with "E" shorted.



$$= \frac{R_1 R_2}{R_1+R_2} + \frac{R_3 R_4}{R_3+R_4}$$

$$= 4.888\ K\Omega$$

$$I_g = \frac{V_{TH}}{R_{eq}+R_g} = \frac{0.444}{4.888\times10^3+300}$$

$$\boxed{I_g = 85.62\ \mu A}$$

This is the current through galvanometer

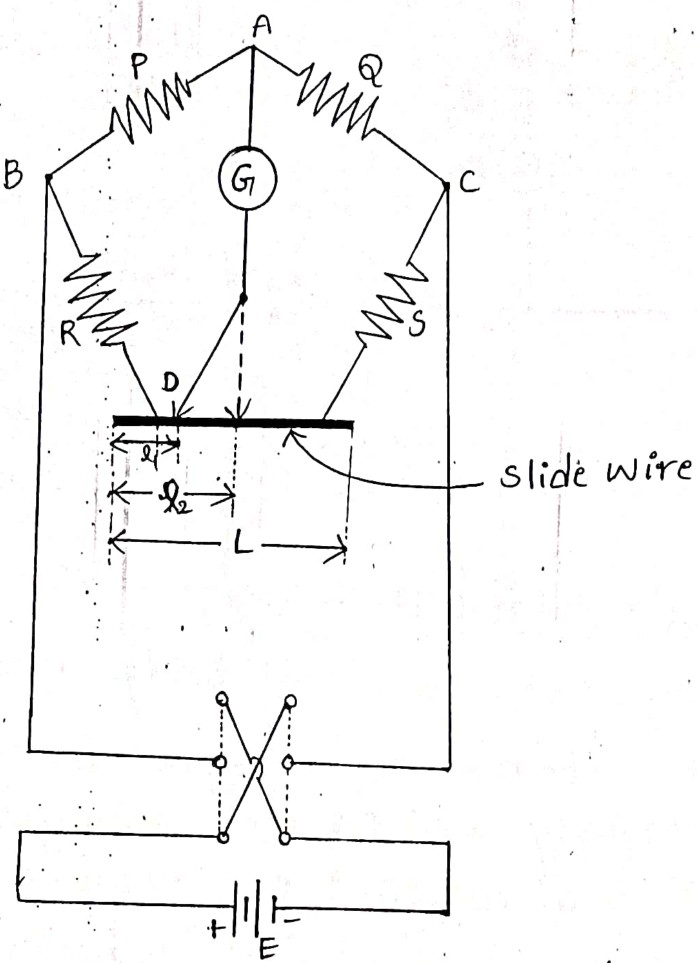
*Carey-Foster Slide Wire Bridge :-

A carey-Foster slide wire bridge is the elaborated from of the wheatstone bridge. This type of the bridge is most extensively used for the comparison of the two nearly equal resistances. The circuit arrangement for the Carey-Foster bridge is as shown in the below figure.

→ The arms consisting resistances 'P' and 'Q' are nominal equal ratio arms. The resistance 'R' is the resistance under test; while 'S' is the standard resistance. A slide wire of length 'L' is introduced between the resistances 'R' and 'S'.

→ Initially a balanced condition is obtained by adjusting the sliding contact on the slide wire at a distance "l_1" from the left hand side of the slide wire. After this the positions of resistances 'R' and 'S' are interchanged and a new balance point is obtained.

→ Let the distance from the left hand of the slide wire be "l_2".



→ The first balance condition is given by,

$$\boxed{\frac{P}{Q} = \frac{R+l_1 r}{S+(l-l_1)r}} \longrightarrow (1)$$

Where r = resistance of slide wire per unit length of the slide wire.

→ Similarly the second balance condition is given by,

$$\boxed{\frac{P}{Q} = \frac{S+l_2 r}{R+(l-l_2)r}} \longrightarrow (2)$$

Comparing eq'ns (1) & (2), we can write,

$$\frac{R+l_1 r}{S+(l-l_1)r} = \frac{S+l_2 r}{R+(l-l_2)r}$$

Adding "1" on both sides of above equations, We get,

$$\frac{R+l_1 r}{S+(l-l_1)r}+1 = \frac{S+l_2 r}{R+(l-l_2)r}+1$$

$$\frac{R+\cancel{l_1 r}+S+lr-\cancel{l_1 r}}{S+(l-l_1)r} = \frac{S+\cancel{l_2 r}+R+lr-\cancel{l_2 r}}{R+(l-l_2)r}$$

$$\frac{R+S+lr}{S+(l-l_1)r} = \frac{S+R+lr}{R+(l-l_2)r}$$

$$R+(l-l_2)r = S+(l-l_1)r$$

$$\boxed{(S-R) = (l_1-l_2)r} \longrightarrow (3)$$

→ The slide wire of resistance 'r' per unit length of the slide wire can be calibrated by shunting resistances 'R' or 'S' with a known high value resistance. Thus effective value of the standard resistance 'S' reduces to 'S''. Now the complete procedure is repeated and two new balance points are obtained at lengths l_1' and l_2' from the left hand side of the slide wire.

→ Then eq'n(3) can be modified as,

$$(S'-R) = (l_1'-l_2')r \longrightarrow (4)$$

Simplyfying eq'ns (3) & (4) by dividing eq(3) by eq(4), we get,

$$\frac{S-R}{S'-R} = \frac{l_1-l_2}{l_1'-l_2'}$$

$$(S-R)(l_1'-l_2') = (S'-R)(l_1-l_2)$$

$$S(l_1' - l_2') - R(l_1' - l_2') = S'(l_1 - l_2) - R(l_1 - l_2)$$

$$S(l_1' - l_2') - S'(l_1 - l_2) = R(l_1' - l_2') - R(l_1 - l_2)$$

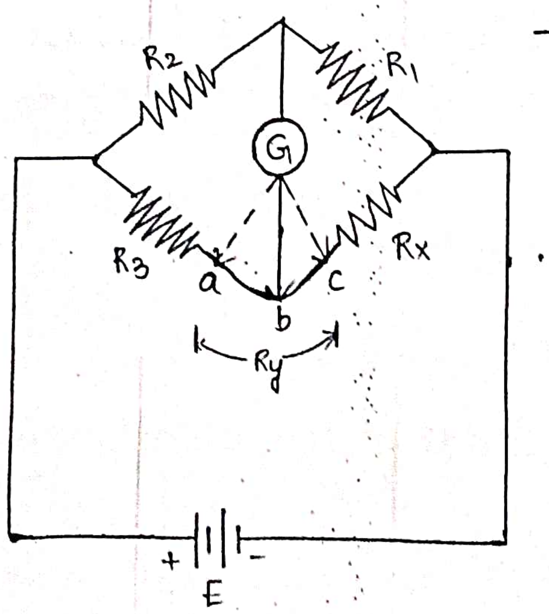
$$\boxed{R = \frac{S(l_1' - l_2') - S'(l_1 - l_2)}{l_1' - l_2' - l_1 + l_2}} \rightarrow (5).$$

From eq'n (5), it is clear that this method compares resistances 'S' and 'R' directly interms of lengths only and the resistances P, Q and the lead resistances are being eliminated.

*Kelvin Bridge - Measurement of Low Resistance :-

→ Measuring the values of resistance below 1Ω, the modified form of wheatstone bridge is used, known as Kelvin bridge.

→ The consideration of the effect of contact and lead resistances is the basic aim of the Kelvin bridge.



→ The resistance 'R_y' represents the resistance of the connecting leads from 'R_3' to 'R_x'.

The resistance "R_x" is the unknown resistance to be measured.

→ The galvanometer can be connected to either terminal a, b or terminal 'c'. When it is connected to 'a', the lead resistance 'R_y' gets added to 'R_x' hence the value measured by the bridge, indicates much higher value of 'R_x'.

→ If the galvanometer is connected to terminal 'c', then 'R_y' gets added to R_3.

→ The point 'b' is in between the points 'a' and 'c', in such a way that the ratio of the resistance from 'c' to 'b' and that from 'a' to 'b' is equal to the ratio of R_1 & R_2.

$$\therefore \frac{R_{bc}}{R_{ab}} = \frac{R_1}{R_2} \longrightarrow (1)$$

→ Now the bridge balance equation in its standard form is,

$$R_1 R_3 = R_2 R_x \longrightarrow (2)$$

But 'R_3' & 'R_x' now are changed to $R_3 + R_{ab}$ and $R_x + R_{bc}$ respectively due to lead resistance.

$$\boxed{\therefore R_1 (R_3 + R_{ab}) = R_2 (R_x + R_{bc})} \longrightarrow (3)$$

$$(R_x + R_{bc}) = \frac{R_1}{R_2}(R_3 + R_{ab}) \longrightarrow (4)$$

→ Now we have, $\frac{R_{bc}}{R_{ab}} = \frac{R_1}{R_2}$

$$\therefore \frac{R_{bc}}{R_{ab}} + 1 = \frac{R_1}{R_2} + 1$$

$$\frac{R_{bc} + R_{ab}}{R_{ab}} = \frac{R_1 + R_2}{R_2} \longrightarrow (5)$$

→ But, $R_{bc} + R_{ab} = R_y$ → (6)

Sub eq(6) in eq(5), we get,

$$\frac{R_y}{R_{ab}} = \frac{R_1 + R_2}{R_2}$$

$$R_{ab} = \frac{R_2 R_y}{R_1 + R_2} \longrightarrow (7)$$

→ Substitute eq(7) in eq(6), we get

$$R_{bc} = R_y - R_{ab} = R_y - \left[\frac{R_2 R_y}{R_1 + R_2}\right]$$

$$R_{bc} = R_y\left[1 - \frac{R_2}{R_1 + R_2}\right] = \frac{R_1 R_y}{R_1 + R_2} \longrightarrow (8)$$

→ Substituting these values of R_{bc} and R_{ab} in the eq(4) we get,

$$R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1}{R_2}\left(R_3 + \frac{R_2 R_y}{R_1 + R_2}\right)$$

$$\therefore \quad R_x + \frac{R_1 R_y}{R_1 + R_2} = \frac{R_1 R_3}{R_2} + \frac{R_1 R_y}{R_1 + R_2}$$

$$\boxed{R_x = \frac{R_1 R_3}{R_2}}$$

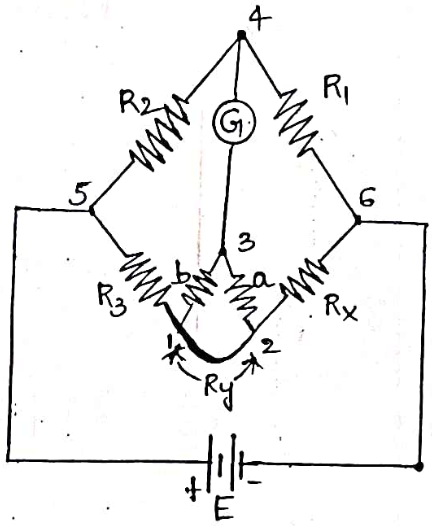
→ Thus the effect of the connecting lead resistance is completely eliminated by connecting the galvanometer to an intermediate position 'b'.

→ This principle forms the basis of the construction of Kelvin Double Bridge which is popularly called Kelvin Bridge.

Kelvin's Double Bridge Method for Low Resistance Measurement :-

This bridge consists of another set of ratio arms hence called double bridge.

The second set of ratio arms is the resistances 'a' and 'b'. With the help of these resistances the galvanometer is connected to point '3'. The galvanometer gives null indication when the potential of the terminal '3' is same as the potential of the terminal '4'.



Thus $E_{45} = E_{513} \longrightarrow (1)$

Here E_{45} = Potential across 'R_2'.

E_{513} = Potential across 'R_3' & 'b'.

The ratio of the resistances 'a' & 'b' is the same as the ratio of 'R_1' & 'R_2'.

$$\therefore \frac{a}{b} = \frac{R_1}{R_2} \longrightarrow (2)$$

Now, $$E_{45} = R_2 \frac{E}{R_1+R_2} \longrightarrow (3)$$

Consider the path from 51-26 back to '5' through the battery 'E'. The resistance between the terminals 1-2 is the parallel combination of R_y and $(a+b)$.

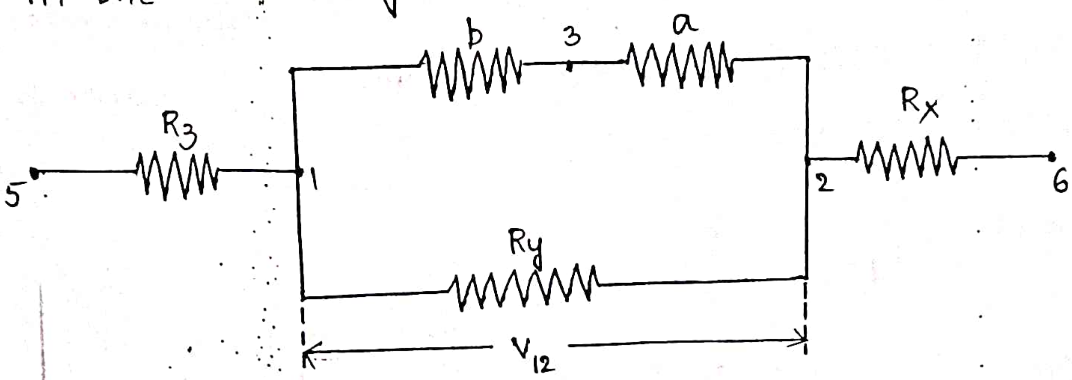
$$E = I \times [R_3 + R_y \| (a+b) + R_x]$$

$$\therefore \quad E = I\left[R_3 + R_x + \frac{R_y(a+b)}{R_y+a+b}\right] \longrightarrow (4)$$

Substitute in (3),

$$E_{45} = \frac{R_2}{R_1+R_2} \times I\left[R_3 + R_x + \frac{R_y(a+b)}{R_y+a+b}\right] \longrightarrow (5)$$

For E_{513}, consider the path from the terminal 5 to 2 as shown in the below figure.



We can write,

$$V_{12} = I \times \left(\frac{R_y(a+b)}{R_y+a+b}\right)$$

and $\quad V_{13} = \dfrac{b}{a+b} \cdot V_{12}$

$$V_{13} = \frac{b}{a+b} \cdot I\left(\frac{R_y(a+b)}{R_y+a+b}\right)$$

$$\therefore E_{513} = IR_3 + V_{13}$$

$$E_{513} = IR_3 + I\frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]$$

$$E_{513} = I\left[R_3 + \frac{b}{a+b}\left[\frac{R_y(a+b)}{R_y+a+b}\right]\right] \longrightarrow (7)$$

$E_{45} = E_{513}$ for balancing

Now

$$\therefore \frac{IR_2}{R_1+R_2}\left(R_3+R_x+\frac{(a+b)R_y}{a+b+R_y}\right) = I\left[R_3+\frac{b}{a+b}\left\{\frac{R_y(a+b)}{R_y+a+b}\right\}\right]$$

$$R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = \frac{R_1+R_2}{R_2}\left[R_3+\frac{b}{(a+b)}\left\{\frac{R_y(a+b)}{a+b+R_y}\right\}\right]$$

$$\therefore R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = \left[1+\frac{R_1}{R_2}\right]\left[R_3+\frac{bR_y}{a+b+R_y}\right]$$

$$R_3+R_x+\frac{(a+b)R_y}{a+b+R_y} = R_3+\frac{R_1R_3}{R_2}+\frac{bR_y}{a+b+R_y}+\frac{R_1bR_y}{R_2(R_y+a+b)}$$

$$R_x = \frac{R_1R_3}{R_2}+\frac{bR_y}{a+b+R_y}+\frac{R_1bR_y}{R_2(R_y+a+b)}-\frac{(a+b)R_y}{R_y+a+b}$$

$$R_x = \frac{R_1R_3}{R_2}+\frac{bR_1R_y}{R_2(R_y+a+b)}-\frac{aR_y}{R_y+a+b}$$

$$\boxed{R_x = \frac{R_1R_3}{R_2}+\frac{bR_y}{R_y+a+b}\left(\frac{R_1}{R_2}-\frac{a}{b}\right)}$$

Now, $\frac{a}{b} = \frac{R_1}{R_2}$ thus $\frac{R_1}{R_2}-\frac{a}{b} = 0$

$$\boxed{R_x = \frac{R_1R_3}{R}}$$

Thus the effect of lead and contact resistances is completely eliminated.

Problem :-

In a Kelvin's double bridge, there is error due to mismatch between the ratios of outer and inner arm resistances. The bridge uses,

Standard resistance = $100.03\,\mu\Omega$

Inner ratio arms = $100.31\,\Omega$ and $200\,\Omega$

Outer ratio arms = $100.24\,\Omega$ and $200\,\Omega$

The resistance of the connecting leads from standard to unknown resistance is $700\,\mu\Omega$. calculate the unknown resistance under this condition.

Sol :-

Given data :-

$R_3 = 100.03\,\mu\Omega$, $R_2 = 100.24\,\Omega$, $R_1 = 200\,\Omega$

$b = 100.31\,\Omega$, $a = 200\,\Omega$, $R_y = 700\,\mu\Omega$

→ Thus unknown resistance is,

$$R_X = \frac{R_1 R_3}{R_2} + \frac{bR_y}{(R_y + a + b)}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

$$= \frac{200 \times 100.03 \times 10^{-6}}{100.24} + \frac{100.31 \times 700 \times 10^{-6}}{(700 \times 10^{-6} + 200 + 100.31)}\left\{\frac{200}{100.24} - \frac{200}{100.31}\right\}$$

$$= 1.9958 \times 10^{-4} + (2.3381 \times 10^{-4})(1.3923 \times 10^{-3})$$

$$R_X = 1.999 \times 10^{-4}\,\Omega = 199.905\,\mu\Omega$$

Loss of Charge Method :-

This is very typical method of measuring the insulation resistance of very high value.

→ The resistance to be measured is shunted by a known value capacitor. The voltage across parallel combi--nation is measured using electrostatic voltmeter. The circuit is driven by a d.c. voltage source of value 'v'. This voltage is applied to the (crit) circuit through a switch.

→ Initially switch is kept open. When the switch is closed at certain instant, the capacitor 'C' starts charging. The voltage across 'C' is given by.

$$\rightarrow V_C = V\left(1 - e^{-\frac{1}{RC}}\right] \longrightarrow (1)$$

Then at certain instant say $t = t_1$, switch is opened. Then capacitor 'C' starts discharging through 'R'. Then at instant voltage is given by,

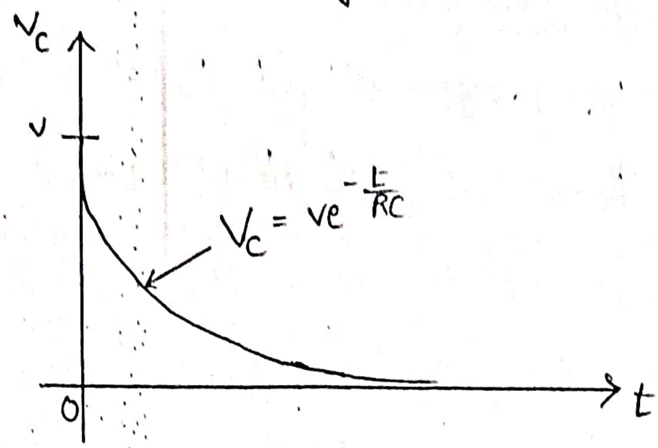
$$V_c = Ve^{-t/RC}$$

$$\therefore \frac{V_c}{V} = e^{-\frac{t}{RC}}$$

Simplifying,

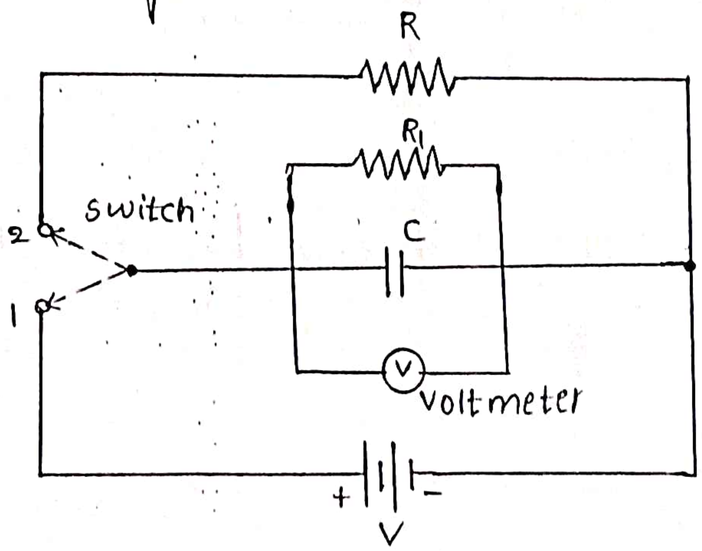
$$\therefore R = \frac{t}{C \ln \frac{V}{V_c}} = \frac{0.4343\,t}{C \log_{10} \frac{V}{V_c}}$$

→ The varation of voltage across 'c' is given by,



If the value of Resistance 'R' is very large, then capacitor 'C' requires more time for discharging. In such cases, the process becomes time consuming.

→ This method can be used effectively for measurement of high resistances, but it needs a capacitor with a high leakage resistance. The typical circuit arrangement is as below figure.



→ The circuit consists high insulation resistance 'R' to be measured along with capacitor of known value 'c' shunted with electrostatic voltmeter and leakage resistance 'R_1'.

→ Initially, capacitor 'C' is charged to suitable voltage say "V_1" by moving switch to position '1'. Then switch is moved to position '2'. The capacitor starts discharging through parallel combination of 'R' & 'R_1'. At certain instant 't' voltage across capacitor 'C' i.e. 'V_2' is measured using electrostatic voltmeter. Thus in time 't', voltage across capacitor drops down from V_1 to V_2. Let the equivalent resistance through which 'C' discharges be denoted by "R'" where $R \| R_2$.

→ The expression for current any instant 't' is given by,

$$i = \frac{-dq}{dt} = -C\frac{dv}{dt} \longrightarrow (2)$$

But $$i = \frac{\text{Potential drop across } R'}{R'} = \frac{V}{R'} \longrightarrow (3)$$

→ Comparing eq'ns (2) & (3), we can write,

$$\frac{V}{R'} = -C\frac{dv}{dt} \quad \text{or} \quad \frac{dV}{V} = \frac{-dt}{RC}.$$

→ Integrating both sides,

$$[\ln V]_{V_1}^{V_2} = \left[\frac{-t}{R'C}\right]_0^t.$$

i.e, $$\ln\frac{V_2}{V_1} = \frac{-t}{R'C}$$

$$\boxed{V_2 = V_1 e^{\frac{-t}{R_1C}}} \longrightarrow (4)$$

→ Thus if time 't' is known, the resistance "R'" can be obtaining by measuring voltages V_1 & V_2. The same test is repeated with unknown resistance 'R' removed. Then 'C' discharges through only "R_1". Then expression is given by,

$$\boxed{V_2 = V_1 e^{\frac{-t}{R_1 C}}} \longrightarrow (5)$$

Thus the value of the leakage resistance of the capacitor can also be found out using this method.

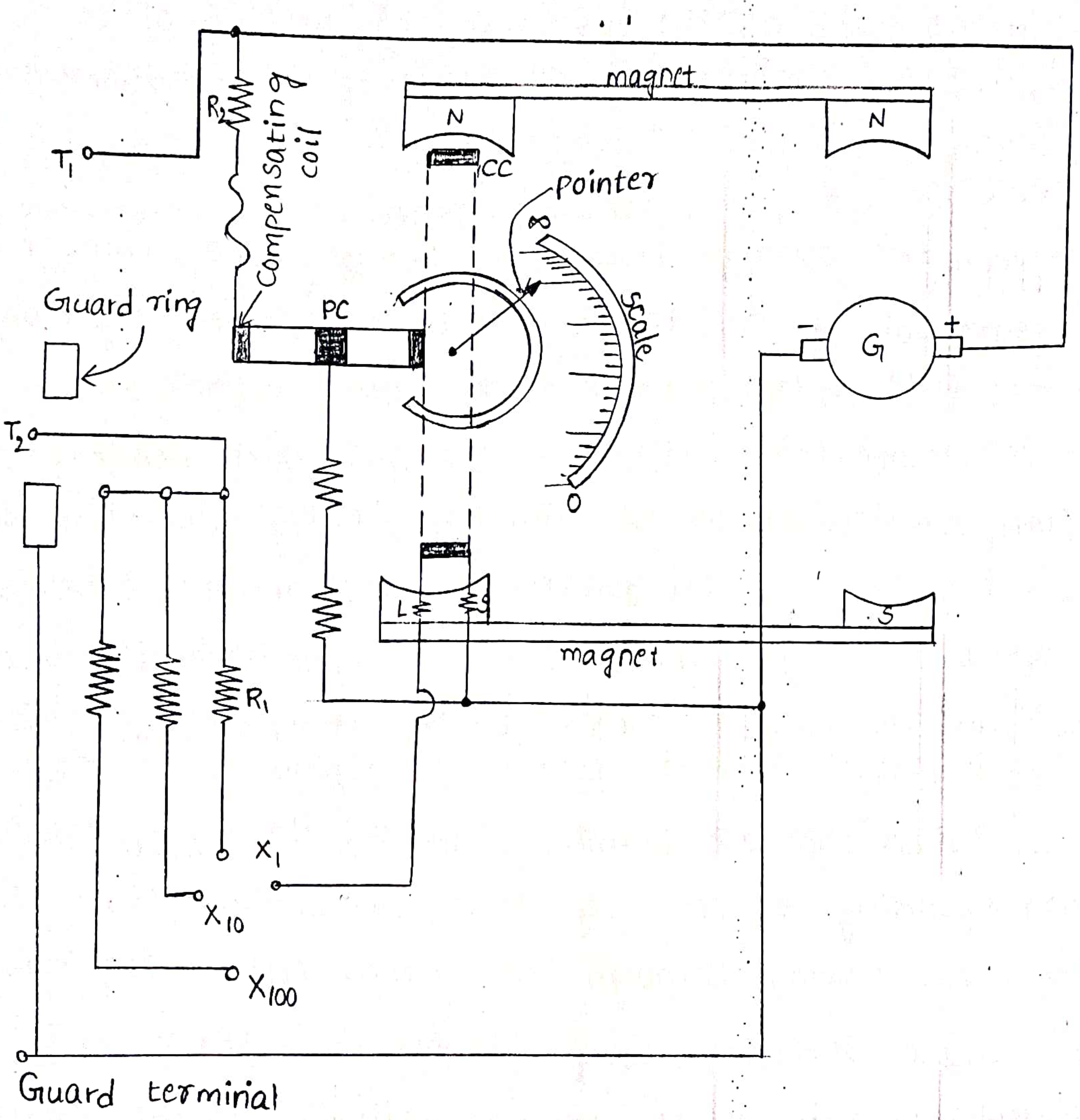
* Megger :-

→ Resistances of the order of 0.1 MΩ and upwards are classified as high resistances. These high resistances are measured by portable, instrument known as megger. It is also used for testing the insulation resistance of cables.

Principle of Operation :-

→ It is based on the principle of electromagnetic induction.

→ When a current carrying conductor is placed in a uniform magnetic field it experiences a mechanical force whose magnetude depends upon the strength of current and magnetic field. While its direction depends on the direction of current and magnetic field.



Construction :-

It consists of a permanent magnet which provides the field for both the generator 'G' and ohmmeter. The moving element of the ohmmeter consist of three coil viz. current or deflection coil, pressure or control coil and compensating coil. These coils are mounted on a central shaft which are free to rotate over a stationary C-shaped iron core.

The current coil is connected in series with resistance "R_1" between one generator terminal and the test terminal 'T_2'. The series resistance "R_1" protects the current coil in the event of the test terminals getting short circuited and also controls the range of the instrument.

Working :-

→ When the current flows from the generator, through the pressure coil, the coil tends to set itself at the right angles to the field of the permanent magnet.

→ When the test terminals are open, corresponding to infinite resistance, no current flows through deflection coil. Thus the pressure coil governs the motion of the moving element making it move to its extreme anticlockwise position. The pointer comes to rest at the infinity end of the scale.

→ When the test terminals are short circuited i.e., corresponding to zero resistance, the current from the generator flowing through the current coil is large enough to produce sufficient torque to overcome the counter-clockwise torque of the pressure coil. Due to this, pointer moves over a scale showing zero resistance.

→ When the high resistance to be tested is connected between terminals T_1 and T_2 the opposing torques of the coils balance each other so that pointer attains a stationary position at some intermediate point on scale. The scale is calibrated in megaohms so that the resistance is directly indicated by pointer.

→ The guard ring is provided to eliminate the error due to leakage current. The supply to the meter is usually given by a hand-driven permanent magnet d.c. generator sometimes motor-driven generator may also be used.

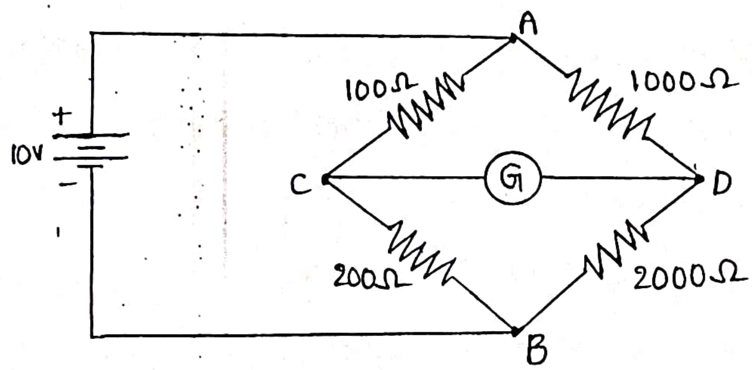
Applications :-

* The megger can be used to determine whether there is sufficiently high resistance between the conducting part of a circuit and the ground. This resistance is called insulation resistance.

* The megger can also be used to test continuity between any two points. When connected to the two positions of points, if pointer shows full deflection then there is an electrical continuity between them.

Problems :-

(1) The wheatstone bridge is shown in the below figure. The galvanometer has a current sensitivity of 12 mm/μA. The internal resistance of galvanometer is 200Ω. Calculate the deflection of the galvanometer caused due to 5Ω unbalance in the arm BD.



Sol:- Given data :-

From the given bridge,

$R_1 = 100\,\Omega$

$R_2 = 1000\,\Omega$

$R_3 = 200\,\Omega$

$R_4 = 2000\,\Omega$

→ Now, $R_1 R_4 = 100 \times 2000 = 200000$

$R_2 R_3 = 200 \times 1000 = 200000$

→ For $R_4 = 2000\Omega$, there is unbalance of 5Ω in the resistance of arm BD i.e.,

$R_4 = 2000 + 5 = 2005\,\Omega$

Due to this imbalance current will flow through the galvanometer.

→ By Thevenin's equivalent,

$$V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 10\left[\frac{200}{100+200} - \frac{2005}{1000+2005}\right]$$

$$= 10[0.6667 - 0.6672]$$

$$= -5.213\ mV$$

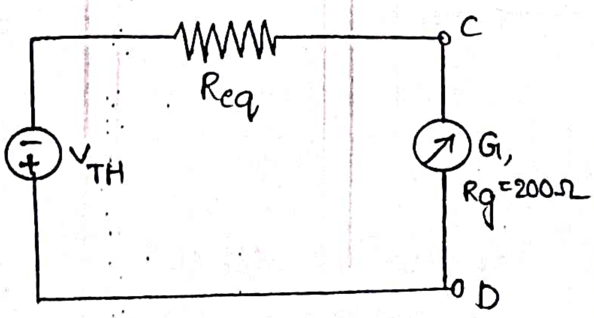
The negative sign indicates that "D" is more positive than 'C'.

$$\rightarrow R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

$$= \frac{100 \times 200}{100+200} + \frac{1000 \times 2005}{1000+2005}$$

$$R_{eq} = 733.888\,\Omega$$

Hence Thevenin's equivalent is,



$$\rightarrow \therefore I_g = \frac{V_{TH}}{R_{eq}+R_g}$$

$$= \frac{5.213 \times 10^{-3}}{733.888+200}$$

$$= 5.582\ \mu A$$

→Now deflection of galvanometer is proportional to its sensitivity.

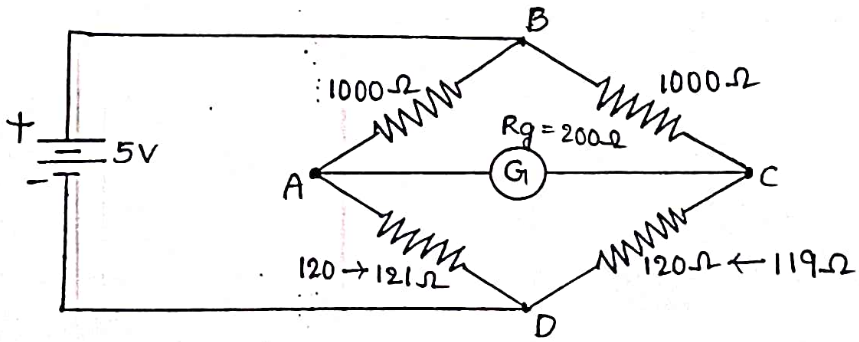
$$\boxed{S = \frac{D}{I}}$$

$$D = S \times I = 12\ mm/\mu A \times 5.582\ \mu A$$

$$\boxed{D = 66.98\ mm}$$

② The four arms of the wheatstone bridge have the following resistances, AB = 1000 Ω, BC = 1000 Ω, CD = 120 Ω, DA = 120 Ω. The bridge is used for strain measurement and supplied from 5V ideal battery. The galvanometer has sensitivity of 1 mm/μA with internal resistance of 200 Ω. Determine the deflection of the galvanometer if arm DA increases to 121 Ω and arm CD decreases to 119 Ω.

Sol: The bridge is given by,



→Now, $R_1 = 1000\Omega$ $R_2 = 1000\Omega$

$R_3 = 121\Omega$ $R_4 = 119\Omega$

Let us calculate Thevenin's equivalent due to change in R_3' and R_4'.

$$\rightarrow V_{TH} = E\left[\frac{R_3}{R_1+R_3} - \frac{R_4}{R_2+R_4}\right]$$

$$= 5\left[\frac{121}{1000+121} - \frac{119}{1000+119}\right]$$

$$= 5\left[0.1079 - 0.1063\right]$$

$$V_{TH} = 7.975\ mV$$

$$\rightarrow R_{eq} = \frac{R_1 R_3}{R_1+R_3} + \frac{R_2 R_4}{R_2+R_4}$$

$$= \frac{121 \times 1000}{121+1000} + \frac{119 \times 1000}{119+1000}$$

$$= 107.9393 + 106.3449$$

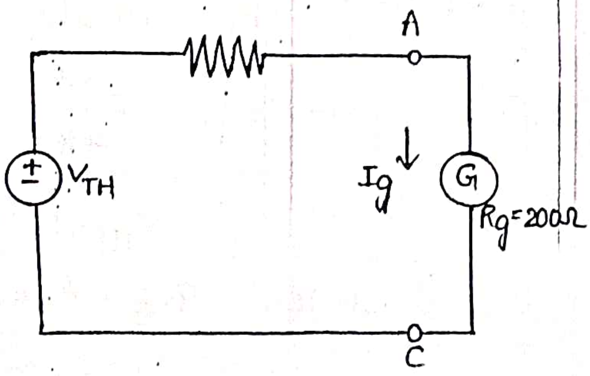
$$R_{eq} = 214.2842\Omega$$

→Thevenin's equivalent circuit is,

$$I_g = \frac{V_{TH}}{R_{eq}+R_g}$$

$$= \frac{7.975 \times 10^{-3}}{214.2842+200}$$

$$\boxed{I_g = 19.24\ \mu A}$$



Now the deflection of the galvanometer is proportional to its sensitivity.

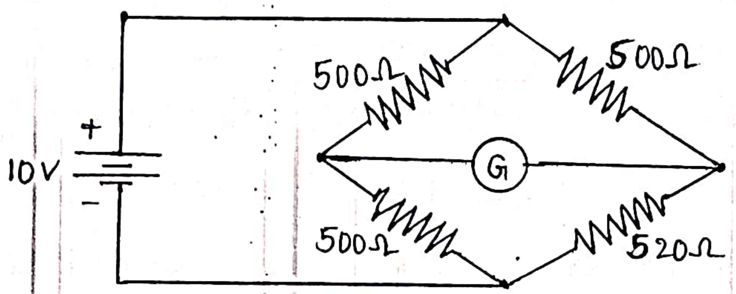
$$S = \frac{D}{I}$$

$$\therefore \quad D = S \times I = 1\,mm/\mu A \times 19.24\,\mu A$$

$$\boxed{D = 19.24\,mm}$$

This is the deflection of the galvanometer.

(3) Using the approximation of slightly unbalanced bridge, calculate the current through the galvanometer having internal resistance of 125Ω, for the bridge as shown in the below figure.



Sol:-

Given data :-

$R = 500\Omega$ and $\Delta r = 20\Omega$

→ Using approximate result,

$$\rightarrow V_{TH} = \frac{E\Delta r}{4R} = \frac{10 \times 20}{4 \times 500}$$

$$V_{TH} = 0.1\,V$$

→ while, $R_{eq} = R = 500\,\Omega$

$$R_g = 125\,\Omega$$

$$\rightarrow I_g = \frac{V_{TH}}{R_{eq} + R_g} = \frac{0.1}{500 + 125} = 160\,\mu A$$

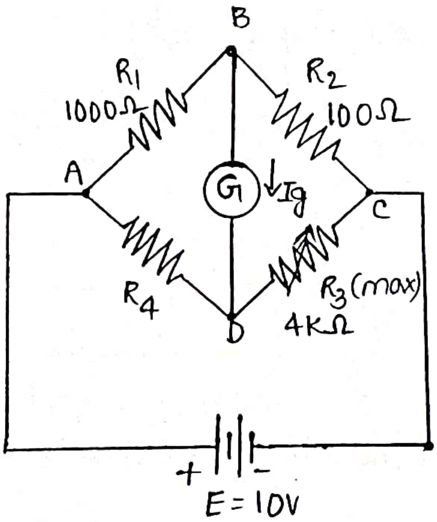
④ A wheatstone's bridge circuit consists of ratio arms as 1000Ω and 100Ω. The adjustable arm is adjusted to its maximum of $4K\Omega$. The supply voltage of 10V is used for the bridge.

i) Draw the circuit diagram.

ii) Determine maximum unknown resistance which can be mea-sured.

iii) If galvanometer internal resistance is 80Ω and its sensitivity is $70mm/\mu A$, find the unbalance in bridge required to cause the deflection of 3mm if the unknown resistance equal to its maximum value is used in the circuit. Neglect internal resistance of the battery.

Sol: i) Circuit diagram is shown as below figure,



ii) At bridge balance :

$$\longrightarrow R_1 R_3 = R_2 R_4$$

$$R_4 = \frac{R_1 R_3}{R_2} = \frac{1000 \times 4 \times 10^3}{100}$$

$$= 40 K\Omega$$

iii) $R_{TH} = [R_1 \| R_2] + [R_3 \| R_4]$

$$= \left[\frac{1000 \times 100}{1000 + 100}\right] + \left[\frac{40\times10^3 \times 4\times10^3}{40\times10^3 + 4\times10^3}\right]$$

$$= 90.9090 + 3.6363\times10^3 = 3.7272\ K\Omega$$

$\rightarrow S_i$ = current sensitivity $= 70\ mm/\mu A = 70\times10^{+6}\ mm/A$

Now, $\theta = \dfrac{S_i E R_3 \Delta R}{(R_{TH}+R_g)(R_3+R_4)^2}$

Assuming $R_{eq} = R_{TH}$

$$3 = \frac{[70\times10^6]\times10\times4\times10^3\times\Delta R}{(3.7272\times10^3+80)(4\times10^3+40\times10^3)^2}$$

$$\Delta R = \frac{3\times3807.2\times1.936\times10^9}{70\times10^6\times10\times4\times10^3}$$

$$\therefore \quad \boxed{\Delta R = 7.8972\,\Omega}$$

This much unbalance is necessary to cause the deflection of 3mm.

(5) A Kelvin double Bridge is balanced with the following constants :

Outer ratio arm = 100Ω and 1000Ω

Inner ratio arms = 99.92Ω and 1000.6Ω,

Resistance of link = 0.1Ω,

Standard resistance = 0.00377 Ω,
calculate the value of unknown resistance.

(4) Given data :-

$R_1 = 100\,\Omega$

$R_2 = 1000\,\Omega$

R_3 = standard resistance = $0.00377\,\Omega$

$a = 99.92\,\Omega$

$b = 1000.6\,\Omega$

R_y = resistance of link = $0.1\,\Omega$

→ Hence by formula, the unknown resistance 'R_x' is given by,

$$R_x = \frac{R_1 R_3}{R_2} + \frac{bR_y}{(R_y+a+b)}\left\{\frac{R_1}{R_2} - \frac{a}{b}\right\}$$

$$= \frac{(100)(0.00377)}{1000} + \frac{(1000.6)(0.1)}{[0.1+99.92+1000.6]} - \left\{\frac{100}{1000} - \frac{99.92}{1000.6}\right\}$$

$$\boxed{R_x = 0.3827\ m\Omega}$$

(5) A highly sensitive galvanometer can detect a current as low as 0.1 nA. This galvanometer is used in wheatstone bridge as a detector. The resistance of galvanometer is negligible. Each arm of the bridge has a resistance of 1kΩ. The input voltage applied to the bridge is 20V. Calculate the smallest change in the resistance, which can be detected.

Sol:- Given data:-

For the bridge,

$R = 1000\Omega$, $E = 20V$

→ The current which can be detected by the galvanometer is 0·1nA.

→ $I_g = 0.1\,nA = 0.1 \times 10^{-9}\,A$

For small change in the resistance Δr, the Thevenin's approximate voltage is,

$$\rightarrow \quad V_{TH} = \frac{E\Delta r}{4R}$$

while, $R_{eq} = R$

$$I_g = \frac{V_{TH}}{R_{eq}} \quad \text{as } R_g = 0\Omega$$

$$\therefore 0.1 \times 10^{-9} = \frac{E\,\Delta r}{4R \times R}$$

$$0.1 \times 10^{-9} = \frac{20 \times \Delta r}{4 \times 1000 \times 1000}$$

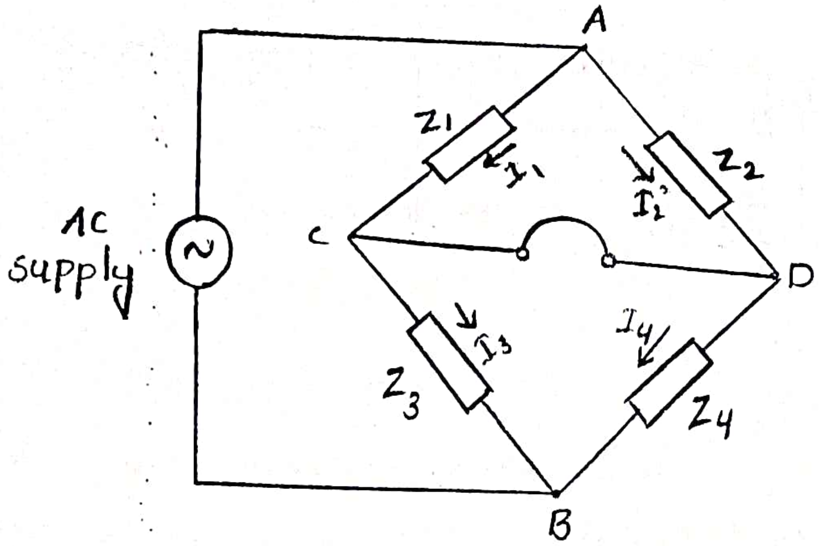
$$\Delta r = \frac{4 \times 10^{6} \times 0.1 \times 10^{-9}}{20}$$

$$\boxed{\Delta r = 20\mu\Omega}$$

Thus the smallest change in the resistance which can be detected is $200\mu\Omega$.

AC BRIDGES

Basic Bridge:-



Bridge Balance Equation :-

For bridge balance, the potential of point C must be same as the potential of point D. these potential must be equal interms of amplitude ase well as phase.

Thus the drop from A to C must be equal to drop across A to D, in both magnitude and phase for the bridge balance.

$$\bar{E}_{AC} = \bar{E}_{AD} \longrightarrow (1)$$

The vector notation indicates, both amplitude and phase to be considered.

$$\overline{I_1 Z_1} = \overline{I_2 Z_2} \longrightarrow (2)$$

When the bridge is balanced, no current flows through the headphones $\therefore \bar{I}_3 = \bar{I}_1$ and $\bar{I}_4 = \bar{I}_2$

Now $I_1 = \dfrac{\bar{E}_1}{\bar{Z}_1 + \bar{Z}_3} \rightarrow (3)$

and $I_2 = \dfrac{\bar{E}}{\bar{Z}_2 + \bar{Z}_4} \rightarrow (4)$

Substituting equation (3) and (4) in eq (2)

$$\frac{\bar{E} \cdot Z_1}{\bar{Z}_1 + \bar{Z}_3} = \frac{\bar{E} \cdot \bar{Z}_2}{\bar{Z}_2 + \bar{Z}_4}$$

$$\overline{Z_1 Z_2} + \overline{Z_1 Z_4} = \overline{Z_1 Z_2} + \overline{Z_2 Z_3}$$

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3} \rightarrow (5)$$

Eq (5) is the balancing equation in the impedence form.

In the admittance form the condition can be expressed as,

$$\overline{Y_1 Y_4} = \overline{Y_2 Y_3}$$

The admittance is the reciprocal of the impedence.

Now in the polar form the impendence are expressed as

$$\bar{Z}_1 = Z_1 \angle \theta_1$$

$$\bar{Z}_2 = Z_2 \angle \theta_2$$

$$\bar{Z}_3 = Z_3 \angle \theta_3$$

$$\bar{Z}_4 = Z_4 \angle \theta_4$$

where Z_1, Z_2, Z_3, Z_4 are the magnitudes and $\theta_1, \theta_2, \theta_3$ and θ_4 are the phase angles.

Equating magnitude of both sides we get the magnitude condition as ;

$$\boxed{Z_1 Z_4 = Z_2 Z_3}$$

Equating phase angles we get

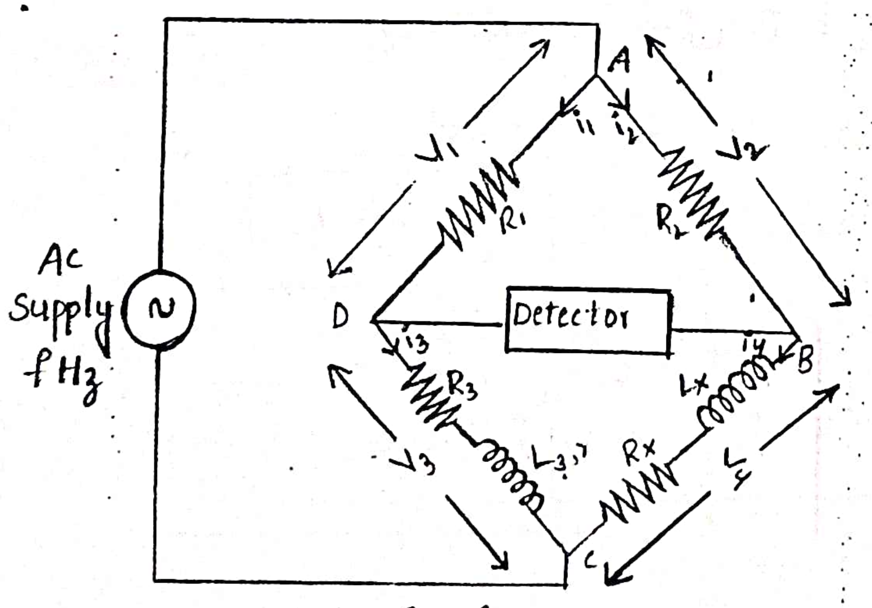
$$\boxed{\theta_1 + \theta_4 = \theta_2 + \theta_3}$$

Maxwell's Bridge :-

Maxwell's bridge can be used to measure inductance by comparison either with a variable standard self inductance (or) with a standard variable capacitance. These two measurements can be done by using the Maxwell's bridge in two different form.

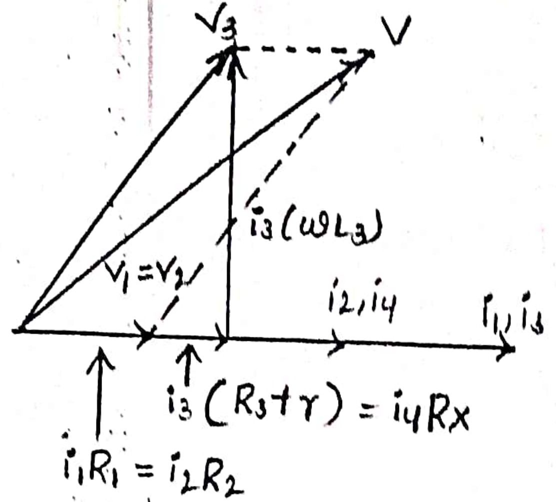
Maxwell's Inductance Bridge :-

Using this bridge, we can measure inductance by comparing it with a standard variable self inductance arranged in bridge circuit as shown in below fig.



(a) circuit diagram

Phasor diagram:-



Consider Maxwell's inductance bridge as shown in the fig: Two branches consist of non-inductive resistances R_1 and R_2. One of the arms consist variable inductance with series resistance r. The remaining arm consists unknown L_x.

At balance, we get condition as

$$\frac{R_1}{(R_3+r)+j\omega L_3} = \frac{R_2}{R_x+j\omega L_x}$$

$$\therefore \quad R_1[R_x + j\omega L_x] = R_2[(R_3+r) + j\omega L_3]$$

$$\therefore \quad R_1R_x + j\omega R_1L_x = R_2(R_3+r) + j\omega R_2L_3$$

Equating imaginary terms, we get

$$R_1L_x = R_2L_3 \quad \Rightarrow \quad \boxed{L_x = \frac{R_2}{R_1}L_3}$$

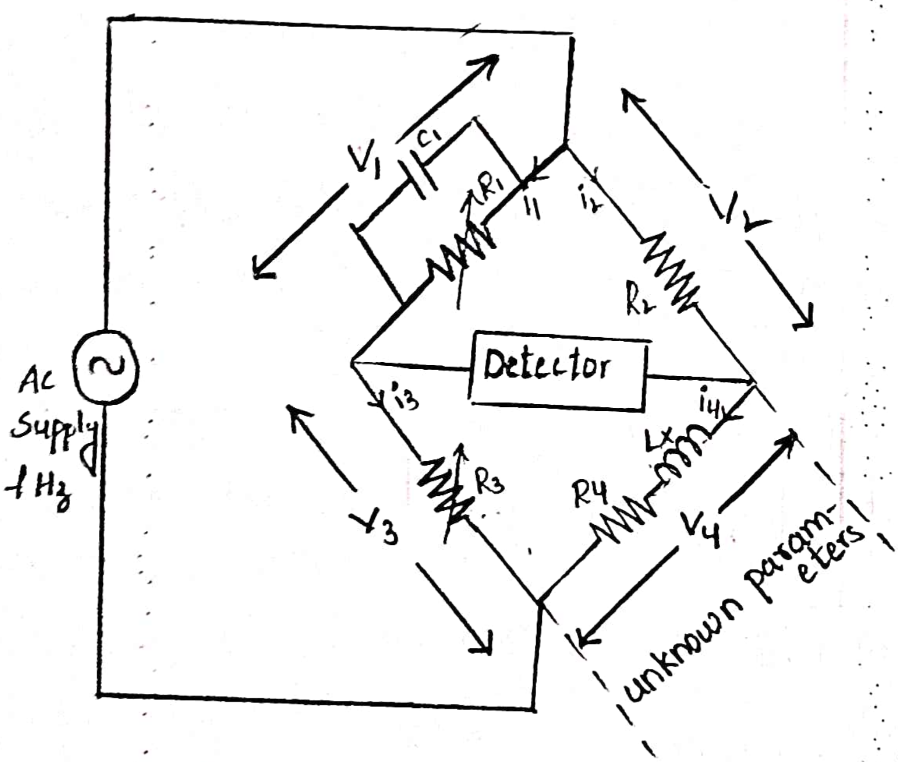
Equating real terms, we get

$$R_1R_x = R_2(R_3+r) \quad \Rightarrow \quad \boxed{R_x = \frac{R_2}{R_1}(R_3+r)}$$

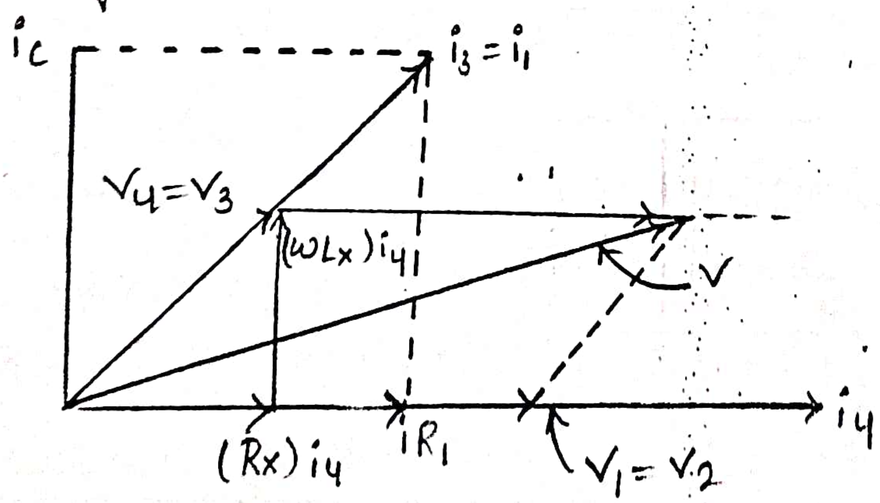
Maxwell's Inductance Capacitance Bridge:-

Using this bridge, we can measure inductance by comparing with a variable standard capacitor. The bridge circuit diagram is as shown in the below fig.

Circuit diagram:-



Phasor diagram:-



One of the ratio arms consists of resistance and capacitance in parallel. Hence it is simple to write the bridge equations in the admittance form.

The general bridge balance equation is

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

$$\overline{Z_x} = \frac{\overline{Z_2 Z_3}}{\overline{Z_1}} = \overline{Z_2 Z_3}\,\overline{Y_1} \longrightarrow (1)$$

where $\overline{Y_1} = \frac{1}{Z_1}$ i.e R_1 in parallel with C_1

$\overline{Z_2} = R_2$

$\overline{Z_3} = R_3$

$\overline{Z_x} = R_x + j\omega L_x$, as L_x in series with R_x

Now $\overline{Y_1} = \frac{1}{R_1} + j\omega C_1 \longrightarrow (2)$

as $\overline{Z_1} = R_1 \| j\left[\frac{1}{\omega C_1}\right]$ as $\frac{1}{j} = -j$

Substituting all the values in eq (1) we get

$$R_x + j\omega L_x = R_2 R_3\left[\frac{1}{R_1} + j\omega C_1\right]$$

$$R_x + j\omega L_x = \frac{R_2 R_3}{R_1} + j R_2 R_3 \omega C_1 \longrightarrow (3)$$

Equating real parts,

$$\boxed{R_x = \frac{R_2 R_3}{R_1}}$$

Equating imaginary parts

$$\omega L_x = R_2 R_3 \omega C_1$$

$$\boxed{L_x = R_2 R_3 C_1}$$

The resistances are expressed in ohms, the inductances in henries and capacitance in farads.

The quality factor of the coil is given by

$$Q = \frac{\omega L_x}{R_x} = \frac{\omega R_2 R_3 C_1}{\left(\frac{R_2 R_3}{R_1}\right)}$$

$$\boxed{Q = \omega R_1 C_1}$$

The advantages of using standard known capacitor for measurement are,

1, The capacitors are less expensive than stable and accurate standard inductors.

2, The capacitors are almost lossless.

3, External fields have less effect on a capacitor. The standard inductor requires well shielding in order to eliminate the effect of stray magnetic fields.

4, The standard inductor will not present its rated value of inductance unless current flow through it precisely adjusted.

5. The capacitor are smaller in size.

This bridge is also called Maxwell Wien Bridge.

Advantages of Maxwell Bridge:-

The advantages of the Maxwell bridge are,

1, The balance equation is independent of losses associated with inductance.

2, The balance equation is independent of frequency of measurement.

3, The scale of the resistance can be calibrated to read the inductance directly.

4, The scale of R_1 can be calibrated to read the Q value directly.

Disadvantages of Maxwell Bridge:-

The disadvantages of Maxwell Bridge are,

1, It cannot be used for the measurement of high Q values. It use is limited to the measurement of low Q values from 1 to 10. This can be proved from phase angle balance condition which says that, sum of the angles of one pair of opposite arms must be equal.

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

2, There is an interaction between the resistance and reactance balances. Getting the balance adjustment is little difficult.

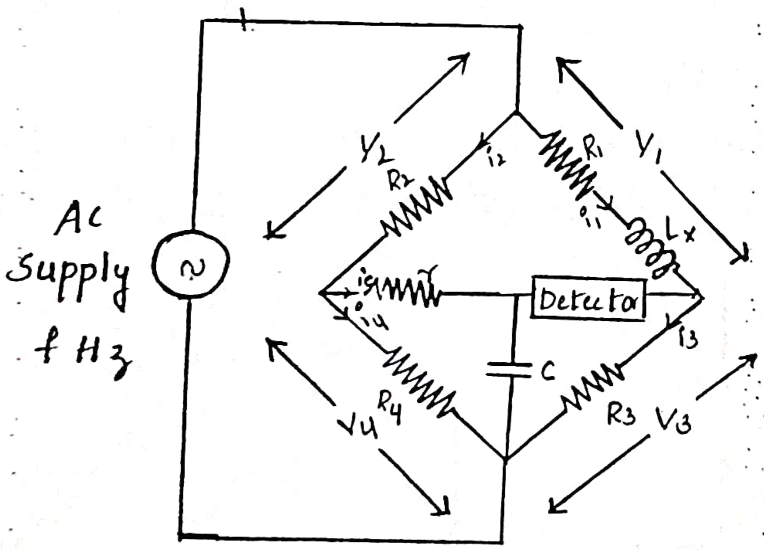
3, It is unsuited for the coils with low Q values, less than one, because of balance convergence problems.

Anderson Bridge:-

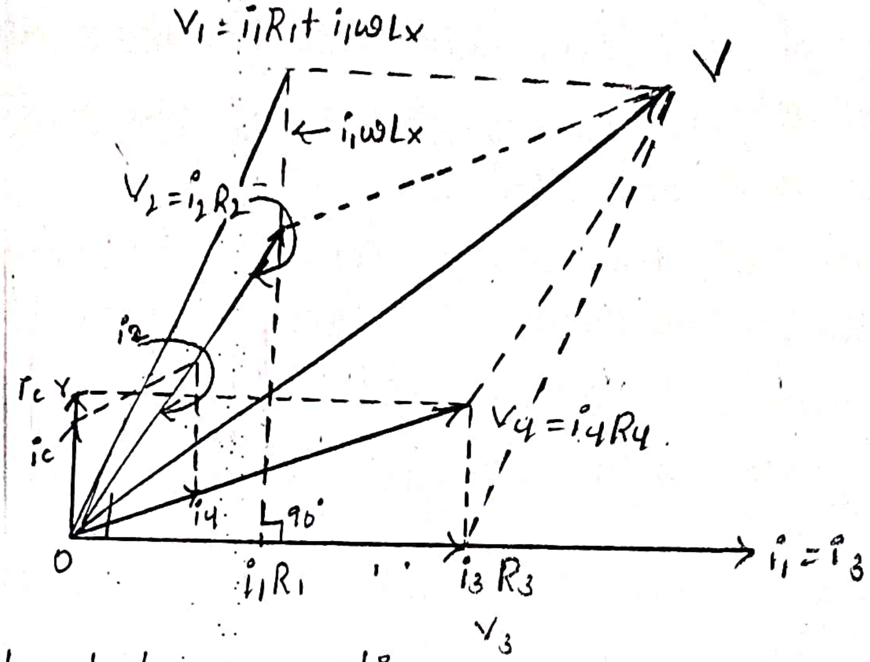
It is another important a.c bridge used for the measurement of self inductance interms of a standard capacitor. Actually this bridge is nothing but modified Maxwell's standard capacitor.

One arm of the bridge consists of unknown inductor L_x with known resistance in series with L_x. This resistance R_1 includes resistance of the inductor. C is the standard capacitor with r, R_2, R_3 and R_4 are non-inductive known resistances.

circuit diagram:-



Phasor diagram:-



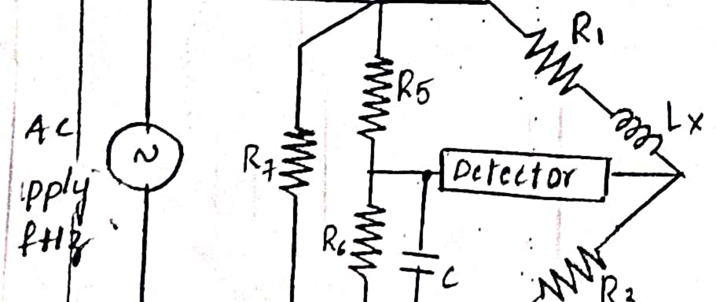
The bridge balance equations are

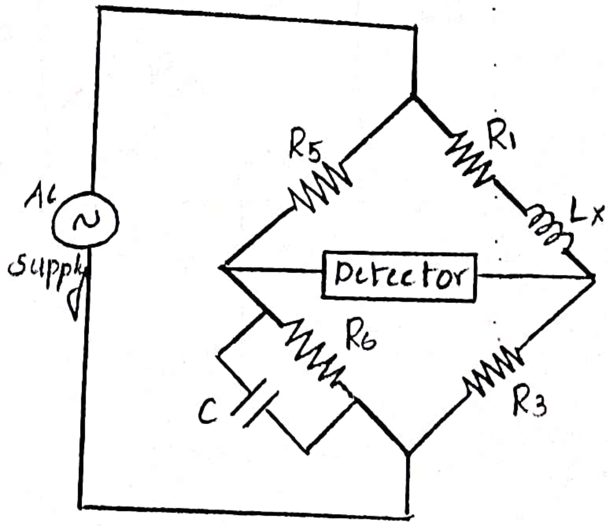
$i_1 = i_3$, $i_2 = i_4 + i_c$, $V_2 = i_2R_2$, $V_3 = i_3R_3$

$V_1 = V_2 + i_c r$ and $V_4 = V_3 + i_c r$, $V_1 = i_1R_1 + i_1\omega L_1$, $V_4 = i_4R_4$

$V = \bar{V_2} + \bar{V_4} = \bar{V_1} + \bar{V_3}$

To find balance equations transforming a star formed by R_2, R_4 and r into its equivalent delta as shown in the below fig.





The elements in equivalent delta are given by

$$R_5 = \frac{R_2 r + R_4 r + R_2 R_4}{R_4}$$

$$R_6 = \frac{R_2 r + R_4 r + R_2 R_4}{R_2}$$

$$R_7 = \frac{R_2 r + R_4 r + R_2 R_4}{r}$$

Now R_7 shunts the source, hence it doesnot effect the balance condition. Thus by neglecting R_7 and rearranging a network a shown in the fig, we get a Maxwell inductance bridge.

Thus, balance equations are given by

$$L_x = C R_3 R_5 \text{ and}$$

$$R_1 = R_3 \frac{R_5}{R_6}$$

Subotituing values of R_5 and R_6, we can write,

$$L_x = \frac{CR_3}{R_4}\left[R_2 r + R_4 r + R_2 R_4\right] \text{ and}$$

$$R_1 = \frac{R_2 R_3}{R_4}$$

<u>advantages of Anderson Bridge.</u>

The advantages of Anderson's bridge are,

1. Can be used for accurate measurement of capacitance interms of inductance.
2. Other bridges require variable capacitor but a fixed

capacitor can be used for Anderson's bridge.

3. The bridge is easy to balance from convergence point of view compared to Maxwell's bridge in case of low values of Q.

Disadvantages of Anderson Bridge:-

The disadvantages of Anderson's bridge are

1. It is more complicated than other bridges.
2. Uses more number of components.
3. Balance equations are also complicated to derive.
4. Bridge cannot be easily shielded due to additional junction point, to avoid the effects of stray capacitances.

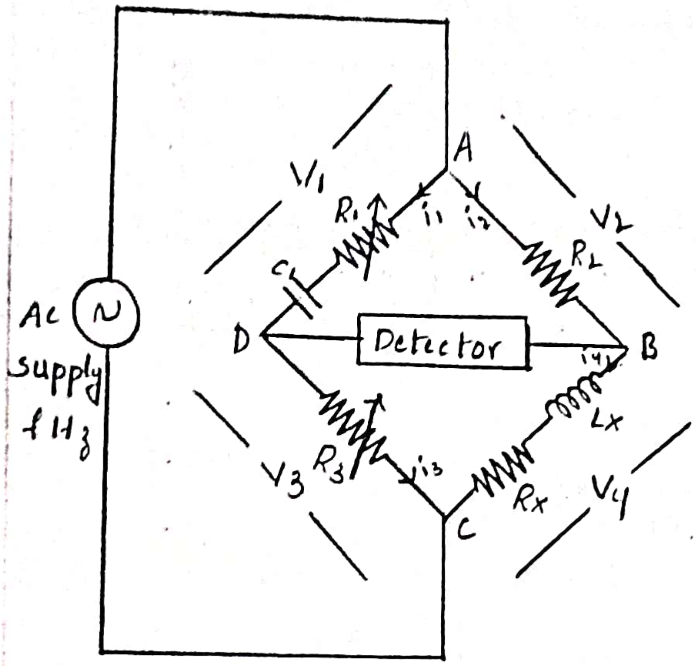
Hay's Bridge :-

The limination of Maxwell's bridge is that it can't be used for high Q values. The Hay's bridge is suitable for the coils having high Q values.

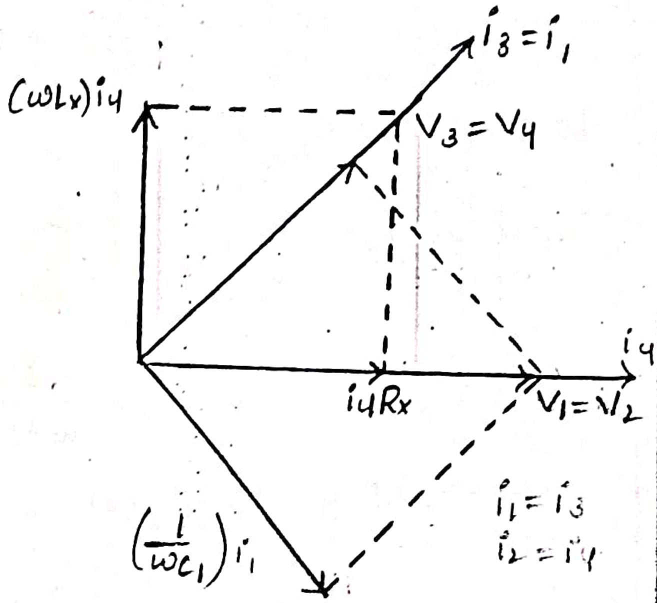
The differences in Maxwell's bridge and Hay's bridge is that the Hay's bridge consists of resistance R_1 in series with the standard capacitor C_1 in one of the ratio arms. Hence for larger phase angles R_1 needed is very low, which is praticable. Hence bridge can be used for the coils with high Q values. the Hay's bridge is shown in the fig.

Under balanced condition, the phasor diagram is as shown in the fig.

circuit diagram:-



Phasor diagram



The various constants of the bridge are

$$Z_1 = R_1 - jX_{C_1} = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2 \text{ and } Z_3 = R_3$$

$$Z_4 = Z_x = R_x + j(\omega L_x)$$

At the balance condition,

$$\overline{Z_1 Z_x} = \overline{Z_2 Z_3}$$

$$\left[R_1 - j\left[\frac{1}{\omega C_1}\right]\right]\left[R_x + j(\omega L_x)\right] = R_2 R_3$$

$$R_1 R_x - j\left[\frac{R_x}{\omega C_1}\right] + j\omega R_1 L_x + \frac{L_x}{C_1} = R_2 R_3$$

$$\left[R_x R_1 + \frac{L_x}{C_1}\right] + j\left[\omega R_1 L_x - \frac{R_x}{\omega C_1}\right] = R_2 R_3 \rightarrow (1)$$

Equating the real parts of both sides,

$$R_x R_1 + \frac{L_x}{C_1} = R_2 R_3 \rightarrow (2)$$

Equating the imaginary parts of both sides of eq (1)

$$\omega R_1 L_x - \frac{R_x}{\omega C_1} = 0 \rightarrow (3)$$

To obtain R_x and L_x, solve eq (2) and eq (3) simultaneously

From eq (3) $\omega R_1 L_x = \frac{R_x}{\omega C_1}$

$$L_x = \frac{R_x}{\omega^2 R_1 C_1} \rightarrow (4)$$

Substituting in eq (2)

$$R_1 R_x + \frac{R_x}{\omega^2 R_1 C_1^2} = R_2 R_3$$

$$R_x \left[R_1 + \frac{1}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

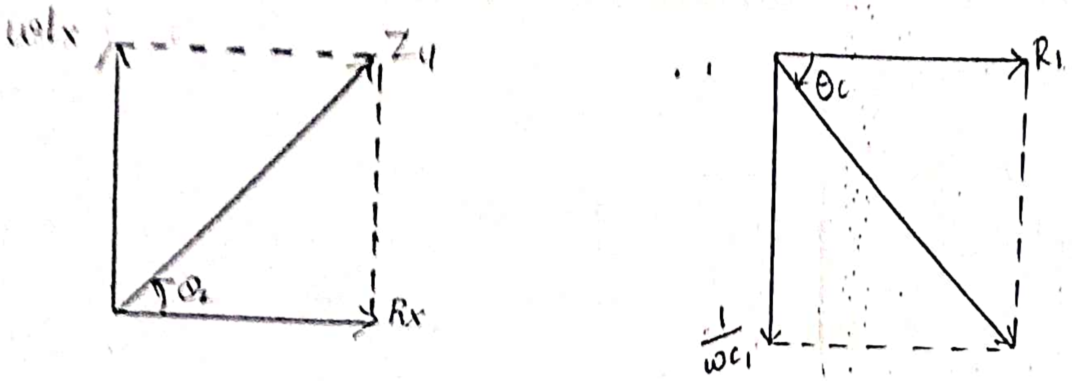
$$R_x \left[\frac{1+\omega^2 R_1^2 C_1^2}{\omega^2 R_1 C_1^2} \right] = R_2 R_3$$

$$\boxed{R_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{1+\omega^2 R_1^2 C_1^2}} \rightarrow (5)$$

substituting eq (5) in eq (4) we get,

$$L_x = \frac{\omega^2 R_1 C_1^2 R_2 R_3}{(1+\omega^2 R_1^2 C_1^2)\,\omega^2 R_1 C_1}, \quad \boxed{L_x = \frac{R_2 R_3 C_1}{1+\omega^2 R_1^2 C_1^2}}$$

The inductive and capacitive phase angles can be determined from the impedence triangles shown in the fig.



$$\tan\theta_L = \frac{X_{Lx}}{R_x} = \frac{\omega L_x}{R_x}$$

$$\tan\theta_L = Q \longrightarrow (7)$$

while $\tan\theta_c = \dfrac{X_{C1}}{R_1} = \dfrac{1}{\omega C_1 R_1} \longrightarrow (8)$

The two angles must be equal at bridge balance

$$\boxed{Q = \frac{1}{\omega C_1 R_1}}$$

Substituting in the equation (6) we get

$$\boxed{L_x = \frac{R_2 R_2 C_1}{1+\left[\frac{1}{Q}\right]^2}}$$

For a large value of Q, $1/Q^2$ becomes small and can be neglected.

$$\boxed{L_x = R_2 R_3 C_1}$$

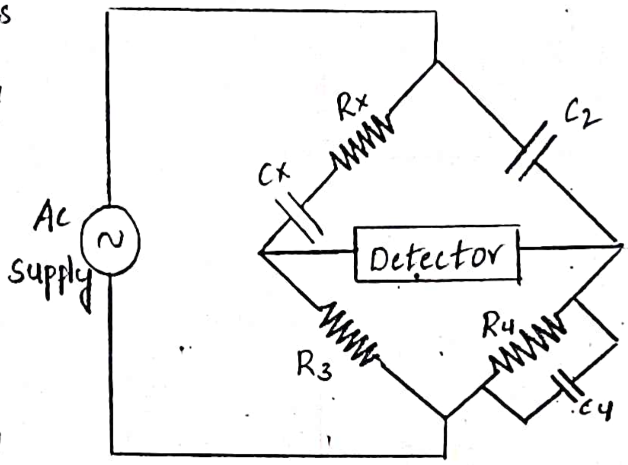
This is same as Maxwell's bridge equation.

The commerical Hay's bridge measure the inductances in the range 1uH - 100H with ± 2% error.

Schering Bridge :-

It is one of the most widely used a.c bridges for the measurement of unknown capacitors, dielectric loss and power factor.

The circuit diagram shows the connections of Schering bridge. It can be used for low voltages. The C_x is perfect capacitor to be measured. R_x is series resistance. C_2 is standard air capacitor having very stable value. R_3 and R_4 are non-inductive resistances while C_4 is variable capacitor. From the general balance equation,



$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

Now $$Z_1 = R_x - j\frac{1}{\omega C_x}$$

$$Z_2 = -\frac{1}{\omega C_2}$$

$$Z_3 = R_3$$

$$Z_4 = R_4 \parallel \frac{-j}{\omega C_4} = \frac{R_4\left[-\frac{j}{\omega C_4}\right]}{\left[R_4 - j\frac{1}{\omega C_4}\right]}$$

$$Z_4 = \frac{-jR_4}{\omega R_4 C_4 - j} = \frac{-jR_4(\omega R_4 C_4 + j)}{(\omega R_4 C_4 - j)(\omega R_4 C_4 + j)} = \frac{R_4 - j\omega R_4^2 C_4}{\omega^2 R_4^2 C_4^2 + 1}$$

$$Z_1 = \frac{Z_2 Z_3}{Z_3} = \frac{\left(\frac{-j}{\omega C_2}\right) R_3}{\left[\frac{R_4 - j\omega R_4^2 C_4}{1+\omega^2 R_4^2 C_4^2}\right]} = \frac{(1+\omega^2 R_4^2 C_4^2) R_3 \left(-\frac{j}{\omega C_2}\right)}{(R_4 - j\omega R_4^2 C_4)}$$

Rationalising $Z_1 = R_3 (1+\omega^2 R_4^2 C_4^2)\left[\frac{\frac{-j}{\omega C_2}(R_4 + j\omega R_4^2 C_4)}{R_4^2 + \omega^2 R_4^2 C_4^2}\right]$

$$= R_x - j\frac{1}{\omega C_x} = \frac{R_3 (1+\omega^2 R_4^2 C_4^2)}{R_4^2 (1+\omega^2 R_4^2 C_4^2)}\left[\frac{R_4^2 C_4}{C_2} - \frac{j R_4}{\omega C_2}\right]$$

Equating real and imaginary parts

$$\boxed{R_x = \frac{R_3}{R_4^2} \times \frac{R_4^2 C_4}{C_2} = \frac{R_3 C_4}{C_2}}$$

$$-j\frac{1}{\omega C_x} = -j\frac{R_3}{R_4^2} \times \frac{R_4}{\omega C_2} = -j\left[\frac{1}{\frac{R_4}{R_3}\omega C_2}\right]$$

$$\omega C_x = \frac{R_4}{R_3}\omega C_2$$

$$\boxed{C_x = \frac{R_4}{R_3} C_2}$$

Power factor (p.f) :- The power factor of the series RC combination is defined as the cosine of the phase angle of the circuit. Thus,

$$p.f = \cos\phi_x = \frac{R_x}{Z_x}$$

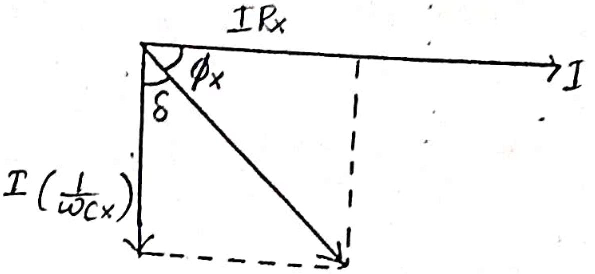
For phase angles very close to 90°, the reactance is almost equal to the impedence

$$p.f = \frac{R_x}{X_x} = \frac{R_x}{\left(\frac{1}{\omega C_x}\right)}$$

$$\boxed{p.f = \omega R_x C_x}$$

Loss angle:- For a series combination of R_x and C_x, the angle between the voltage across the series combination and voltage across the capacitor C_x is called loss angles. This is shown in the fig.

Now $\tan\delta = \dfrac{IR_x}{I\left(\frac{1}{\omega C_x}\right)} = \omega R_x C_x$



$$\tan\delta = \omega\left(\frac{R_3 C_4}{C_2}\right)\left(\frac{R_4}{R_3} C_2\right)$$
$$= \omega R_4 C_4$$

Thus loss angle can be measured, knowing the values of ω, R_4 and C_4.

Dissipation factor (D):-

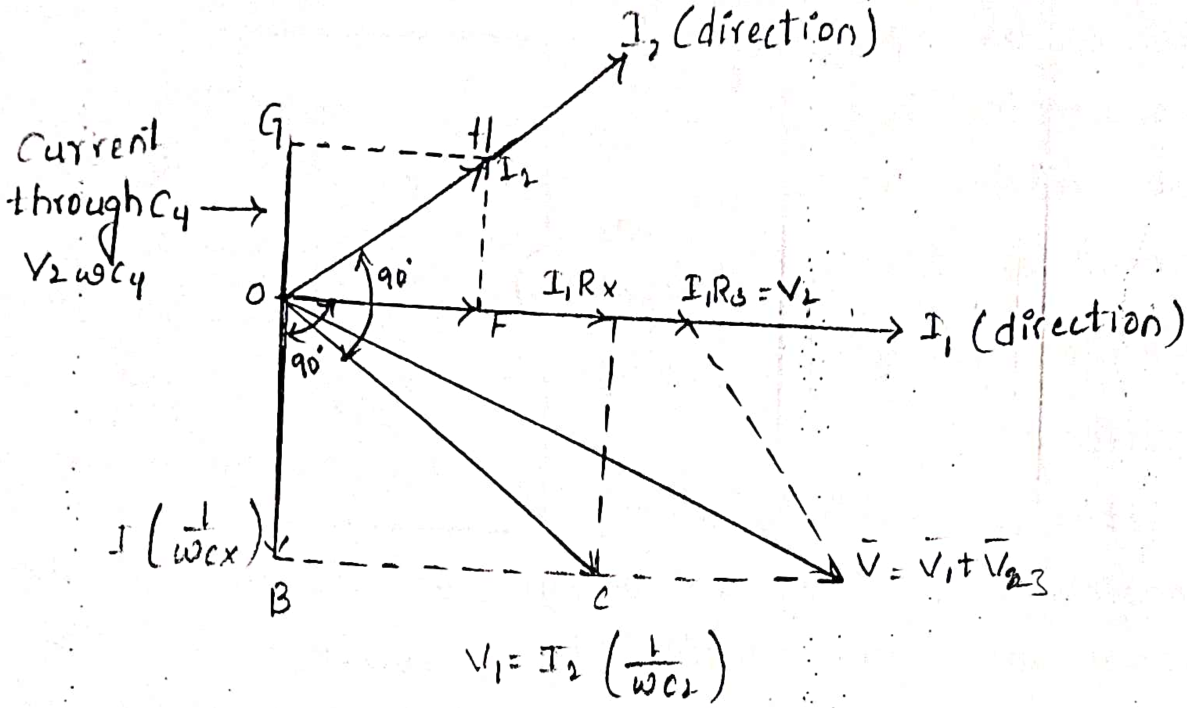
For $R_x - C_x$ series circuit, it is cotangent of the phase angle ϕ_x.

$$D = \cot\phi_x = \frac{1}{\tan\phi_x} = \frac{1}{\left[\dfrac{I\left(\frac{1}{\omega C_x}\right)}{IR_x}\right]} = \omega R_x C_x = \omega R_4 C_4.$$

Key point:-

The bridge is widely used for testing small capacitors at low voltages with very high precision.

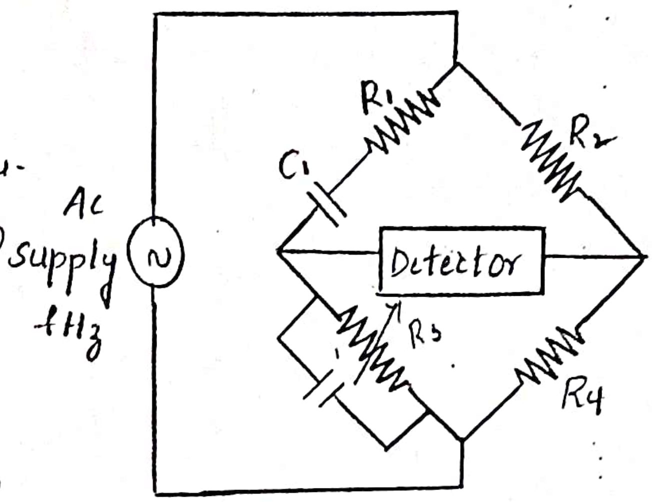
The phasor diagram is shown in fig at the balance condition.



I_1 is chosen reference. Now V_1 is drop across R_x and C_x. Thus $OA = I_1 R_x$ and $OB = I_1\left(\frac{1}{\omega C_x}\right)$ and I_1 leads capacitor drop by 90°. Thus $OC = V_1$ is $\overline{OA} + \overline{OB}$. Now $V_1 = I_2\left(\frac{1}{\omega C_2}\right)$ i.e drop across C_2. So current I_2 leads V_2 by 90°. Then $V_2 = I_1 R_3$ which is OD in phase with I_1. And $\bar{V}_1 + \bar{V}_2 = V$ is supply voltage OE. V_2 is also drop across R_4 and C_4. Let OF is current through R_4 thus $OF = V_2/R_4$ in phase with V_2 while OG is current through C_4 which is $V_2 \omega C_4$ and leading V_2 by 90°. The $OH = I_2$ which is vector sum of OF and OG.

Wein Bridge:-

Basically the bridge is used for the frequency measurement but it is also used for the measurement of the unknown capacitor with great accuracy.



It's one ratio arm consists of a series RC circuit i.e R_1 and C_1. The second ratio arm consists of a resistance R_2. The third arm consists of the parallel combination of resistance and capacitor i.e R_3 and C_3. The circuit of the Wein bridge is as shown in the above fig.

From the fig we can write

$$Z_1 = R_1 - j\left(\frac{1}{\omega C_1}\right)$$

$$Z_2 = R_2$$

$$Z_3 = R_3 \| C_3$$

$$Y_3 = \frac{1}{R_3} + j\omega C_3$$

and $$Z_4 = R_4$$

The balance condition is,

$$\overline{Z_1 Z_4} = \overline{Z_2 Z_3}$$

$$\bar{Z}_2 = \frac{\bar{Z}_1 \bar{Z}_4}{\bar{Z}_3} = Z_1 \bar{Z}_4 Y_3$$

$$R_2 = \left[R_1 - j\left(\frac{1}{\omega C_1}\right)\right] R_4 \left[\frac{1}{R_3} + j\omega C_3\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + j\omega R_1 C_3 - j\frac{1}{\omega C_1 R_3} + \frac{C_3}{C_1}\right]$$

$$R_2 = R_4 \left[\frac{R_1}{R_3} + \frac{C_3}{C_1}\right] + jR_4\left[\omega R_1 C_3 - \frac{1}{\omega C_1 R_3}\right]$$

Equating real parts of both sides,

$$R_2 = \frac{R_4 R_1}{R_3} + \frac{C_3 R_4}{C_1}$$

$$\boxed{\frac{R_2}{R_4} = \frac{R_1}{R_3} + \frac{C_3}{C_1}} \longrightarrow (1)$$

Equating imaginary parts of both sides,

$$\omega R_1 C_3 - \frac{1}{\omega C_1 R_3} = 0$$

$$\omega^2 = \frac{1}{R_1 R_3 C_1 C_3}$$

$$\omega = \frac{1}{\sqrt{R_1 C_1 R_3 C_3}} \longrightarrow (2)$$

$$\boxed{f = \frac{1}{2\pi\sqrt{R_1 C_1 R_3 C_3}}} \longrightarrow (3)$$

The eq (1) gives the resistance ratio while the eq (3) gives the frequency of applied voltage.

Generally, in wein bridge, the selection of the compents in such that

$$R_1 = R_3 = R$$

and $C_1 = C_3 = C$

$$\frac{R_2}{R_4} = 2 \longrightarrow (4)$$

and $$f = \frac{1}{2\pi RC_1} \longrightarrow (5)$$

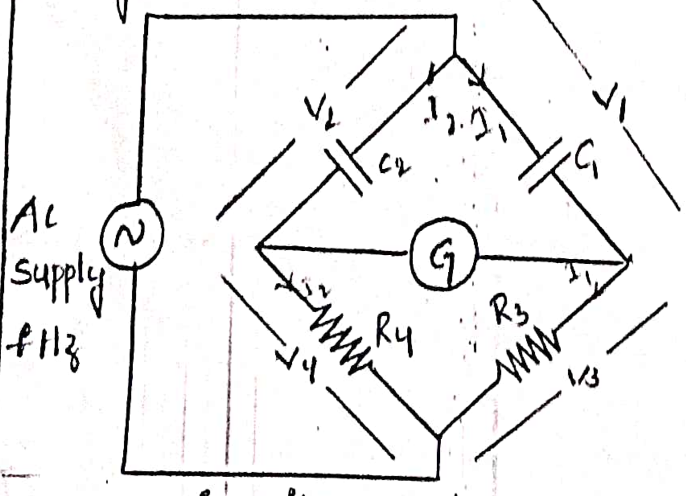
The eq (5) is the general eq for the frequency of the bridge circuit.

Applications:-

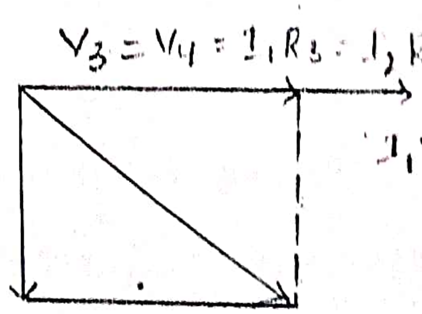
The bridge is used to measure the frequency in audio range. The audio range is [20 - 200 - 20K - 20K] Hz. The resistances are used for the range changing while the capacitors are used for fine frequency control.

Desauty Bridge:-

The basic applications of the De sauty bridge is to compare two capacitance. The circuit diagram of the De Sauty bridge is as shown in the fig.



circuit diagram



$$V_1 = V_2 = \frac{I_1}{\omega C_1} = \frac{I_2}{\omega C_2}$$

phasor diagram.

The De Sauty Bridge consists a capacitor C_1 which is capacitance under test in branch AB. The branch AD consist a known, standard capacitor. The remaining branches BC and CD consists non-inductive resistances R_3 and R_4 respectively.

At balance, we get condition as

$$\frac{-jX_{C_1}}{R_3} = \frac{-jX_{C_2}}{R_4}$$

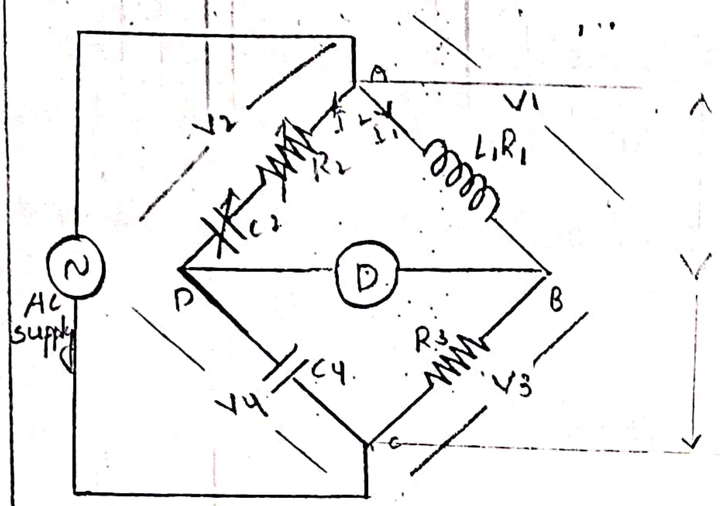
$$\left[\frac{-j}{\omega C_1}\right] R_4 = \left[\frac{-j}{\omega C_2}\right] R_3$$

$$C_1 = \frac{C_2 \cdot R_4}{R_3}$$

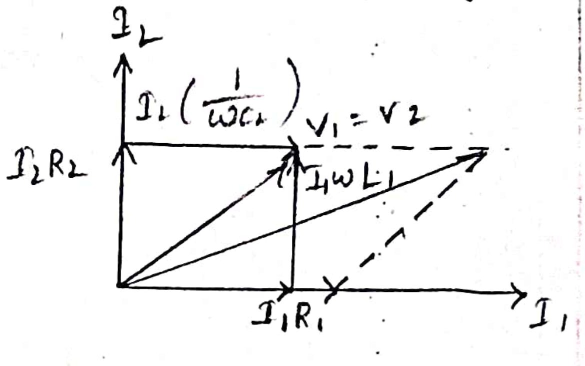
The balance in the bridge can be achieved by varying either R_3 or R_4. Under the balanced condition the vector diagram for the De sauty bridge is as shown in the fig.

Owen's Bridge:-

The bridge can be used to measure the inductance interms of capacitance. the basic arrangement of the Owen's bridge is as shown in the Fig.



a) Circuit diagram.



b) phasor diagram

In branch AB unknown self inductance with resistance R_1 is connected. In branch BC non-inductive resistance R_3 is used. The branch CD consists the fixed value standard capacitor C_4 while the branch DA consists variable non-inductive resistance R_2 in series with variable capacitor C_2,

At balance we get condition as,

$$(R_1 + j\omega L_1)\left[\frac{1}{j\omega C_4}\right] = R_3\left[R_2 + \frac{1}{j\omega C_2}\right]$$

$$\left[\frac{R_1 + j\omega L_1}{j\omega C_4}\right] = \frac{R_2 R_3 (j\omega C_2) + R_3}{j\omega C_2}$$

$$R_1 + j\omega L_1 = \left[\frac{C_4}{C_2}\right] R_3 \left[1 + j\omega R_2 C_2\right]$$

$$R_1 + j\omega L_1 = R_3 \frac{C_4}{C_2} + j\omega R_2 R_3 C_4$$

comparing real terms, we get

$$R_1 = R_3 \frac{C_4}{C_2}$$

Similarly comparing imaginary terms, we get

$$L_1 = R_2 R_3 C_4$$

Advantages of Owen's Bridge:-

The advantages of the Owen's bridge are,

1. From the equation of the balance condition, it is clear that both conditions are independent of each other even if R_2 and C_2 are variables. Moreover these two elements are connected in the same arm in series. Hence it is very easy to achieve balance condition properly.
2. The balance equations are of very simple form
3. The balance equations are independent of the frequency term.
4. It is possible to use this bridge over a width over a wide range of inductances values.

Disadvantages of Owen's Bridge:-

The disadvantages of Owen's bridge are

1. With the inductances of large Q-factor values of C_2 becomes very large.

2. For measurement of wide range of inductances interms of capacitance, the capacitor C_2 is required to be variable. This increases the cost of the bridge.

1. Explain with neat diagram Schering bridge & derive power factor, loss angle (δ), Quality factor (Q) & Dissipation factor.
2. Explain Kelvin's double bridge.