# Unit - 4

Noise in analog communication
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- Noise in DSB-SC receivers
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**Noise:-** It is an unwanted electrical (or) electromagnetic energy that interferes with the transmitted message and degrade the quantity of message signal

(or)

Any unwanted signal interfering with the reception of unwanted unwanted signal


**Classification of Noise :-**

Noise
- Internal Noise
  - Thermal Noise
  - shot Noise
  - Transit time Noise
- External Noise
  - Atmospheric Noise
  - Extra terrestrial Noise
  - Manmade Noise

* Internal Noises are generated within the receiver or communication system.

* If the noise gets added to the signal, then it is known as "Additive Noise".

$$x(t)+n(t) \Rightarrow \text{Additive Noise}$$

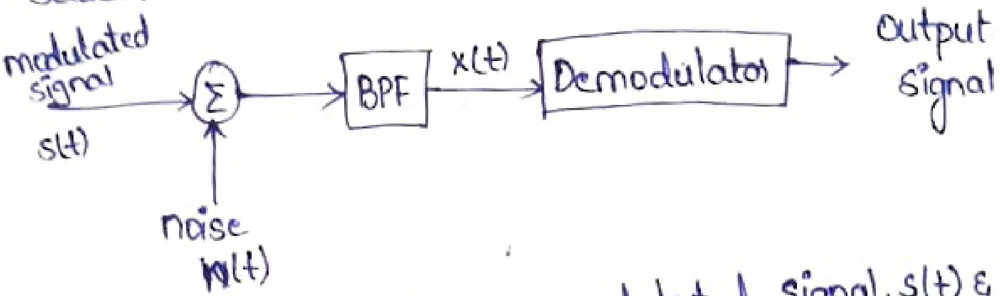
* If the noise gets multiplied to the signal, then it is known as "Fading".

$$x(t) * n(t) \Rightarrow \text{Fading}$$

## Receiver Model →

**Receiver:-** A receiver is a collection of electronic circuits designed to convert the signal back to the original information.

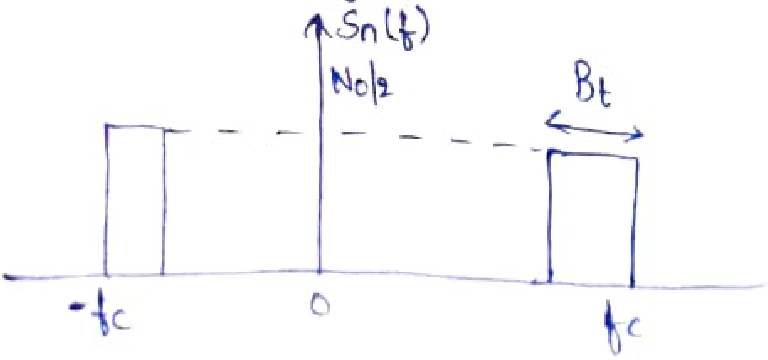
* It consists of amplifier, detector, mixer, oscillator & tran-sducer etc.,




* The model consists of modulated signal, s(t) & noise Signal, n(t). The receiver i/p is the sum of s(t) & n(t).

* BPF is used for filtering action of tuned amplifier for the purpose of signal amplification prior to demodulation.

* The Bandwidth of a BPF is kept just wide enough to pass the modulated signal, s(t) without distortion.



$\frac{No}{2}$ → power spectral density of noise, n(t) for both +ve & -ve frequencies

No → Avg noise power

$B_T$ → Bandwidth of BPF (or) s(t)

* Bandwidth of BPF is equal to the transmission bandwidth of the modulated signal s(t) & it is denoted as '$B_t$' (or) 'W'.
* Midband frequency is equal to the carrier frequency & is denoted as '$f_c$'.
* The carrier frequency $f_c >> B_t$, & therefore we may consider the filtered noise n(t) as narrowband noise & is defined as,

$$n(t) = \underbrace{n_I(t)}_{\substack{\text{in-phase} \\ \text{noise component}}} \cos(2\pi f_c t) - \underbrace{n_Q(t)}_{\substack{\text{quadrature} \\ \text{noise component}}} \sin(2\pi f_c t) \rightarrow ①$$

* The filtered signal x(t) available for demodulation is given by,

$$x(t) = s(t) + n(t) \rightarrow ②$$

* The Average noise power $= \left[\frac{\text{Avg noise power}}{\text{Unit B.W}}\right] \times \text{Bandwidth}$

$$= 2 \times \frac{No}{2} \times Bt$$

$$= No \cdot Bt \text{ (or) } No \cdot W.$$

* Input signal to Noise ratio

$$(S/N)_i = \frac{\text{Avg. power of modulated signal}}{\text{Avg. power of filtered noise}}$$

* Output signal to Noise ratio

$$(S/N)_o = \frac{\text{Avg power of demodulated message signal}}{\text{Avg. power of filtered noise}}$$

* Figure of merit is the ratio of signal to noise ratio at o/p to the signal to noise ratio at i/p i.e.,

$$\text{FOM} = \frac{(S/N)_o}{(S/N)_I} \rightarrow (3)$$

**Noise Figure:-**

$$\text{Noise figure} = \frac{(S/N)_I}{(S/N)_o} \rightarrow (4)$$

* Higher the value of figure of merit, better the performance of the receiver.
* The value of the figure of merit also depends upon the type of modulation used.

**Noise in DSB-SC Receiver →**

$$s(t) = A_c \cos 2\pi f_c t \; m(t)$$

$$s(t) = A_c \cdot A_m \cos 2\pi f_c t \cos 2\pi f_m t$$

$$S_i = \left(\frac{A_c}{\sqrt{2}}\right)^2 \left(\frac{A_m}{\sqrt{2}}\right)^2 \text{ (or) } \frac{A_c^2}{2} \cdot m^2(t)$$

$$S_i = \frac{A_c^2 \cdot A_m^2}{4}$$

$$\boxed{\frac{A_m^2}{2} = P}$$

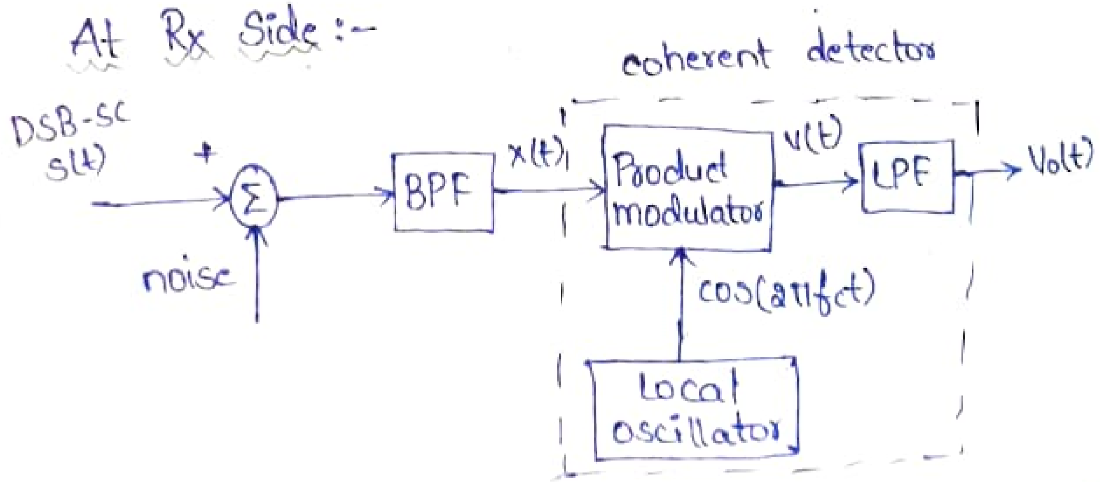
$$S_i = \frac{A_c^2}{2} \cdot P$$

where $P \rightarrow$ power of message $= \frac{A_m^2}{2} = m(t)$,

$$\frac{S_i}{N_i} = \frac{A_c^2 P}{2 N_o W}$$

$$\therefore N_i = N_o \cdot W$$

**At Rx Side :-**



* Input to BPF is signal with noise having amplitude 1/2
so the o/p of BPF is noise & m(t) is selected
* Input to product modulator is (DSB-SC + Noise)

$x(t) = [A_c m(t) \cos(2\pi f_c t) + n_i(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)]$

o/p of product modulator is multiplied with $\cos(2\pi f_c t)$

$$x(t) = [A_c \cdot m(t) \cos(2\pi f_c t) + n_i(t) \cos(2\pi f_c t) - n_q(t) \sin(2\pi f_c t)] * \cos(2\pi f_c t)$$

$$A_c m(t) \cos^2(2\pi f_c t) + n_i(t) \cos^2(2\pi f_c t) - n_q(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

where $\cos^2(\theta) = \frac{1+\cos 2\theta}{2}$

$$\frac{\sin(A+B) + \sin(A-B)}{2} = \sin A \cos B$$

$$s(t) = A_c m(t) \left[\frac{1 + \cos(4\pi f_c t)}{2}\right] + n_i(t) \left[\frac{1+\cos 4\pi f_c t}{2}\right] - \frac{n_q(t) \sin(4\pi f_c t)}{2}$$

o/p of LPF is,

$$\frac{A_c \cdot m(t)}{2} + \frac{n_c(t)}{2}$$

$\frac{A_c}{2} \rightarrow$ scaling factor

$$\boxed{s(t) = \frac{A_c^2 \cdot P}{4}} \qquad \therefore m^2(t) = P$$

$n_c(t) = 2 N_0 \cdot W$

but scaling factor is 1/2

So, $N_o = 2 \cdot N_0 \cdot W \times \frac{1}{4}$

$$\boxed{N_o = \frac{N_0 W}{2}}$$

$$(S/N)_o = \frac{\frac{A_c^2 P}{4}}{\frac{N_o W}{2}}$$

$$= \frac{A_c^2 P}{4} \times \frac{2}{N_o W}$$

$$\boxed{(S/N)_o = \frac{A_c^2 P}{2 N_o W}}$$

$$\text{FOM of DSB-SC} = \frac{(S/N)_o}{(S/N)_I}$$

$$= \frac{\frac{A_c^2 P}{2N_o W}}{\frac{A_c^2 P}{2N_o W}}$$

$$= \frac{A_c^2 P}{2N_o W} \times \frac{2N_o W}{A_c^2 P}$$

$$\boxed{\text{FOM of DSB-SC} = 1}$$

Noise in AM Receiver →

$$s(t) = A_c[1 + k_a m(t)] \cos 2\pi f_c t$$

$$s(t) = A_c \cos 2\pi f_c t + A_c k_a m(t) \cos 2\pi f_c t$$

$$S_i = \left(\frac{A_c}{\sqrt{2}}\right)^2 + \left(\frac{A_c k_a m(t)}{\sqrt{2}}\right)^2$$

$$S_i = \frac{A_c^2}{2} + \frac{A_c^2 k_a^2 m^2(t)}{2}$$

$$\therefore \frac{A_c^2}{2} = P_c \quad \& \quad m^2(t) = P$$

$$\boxed{S_i = \frac{A_c^2}{2}[1 + k_a^2 P]}$$

$$n_i = N_o W$$

$(S/N)_I$ : $\dfrac{A_c^2[1+k_a^2 P]}{2N_0 W}$ ... (1)

Noise + AM $\rightarrow$ [BPF] $\rightarrow$ [Envelope detector] $\rightarrow$

i/p to envelope detector = AM + noise

$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos 2\pi f_c t + n_c(t) \cos 2\pi f_c t$
$+ n_q(t) \sin 2\pi f_c t$

$= [A_c + A_c k_a m(t) + n_c(t)] \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t$

$= \sqrt{\underbrace{[A_c + A_c k_a m(t) + n_c(t)]^2}_{} + n_q^2(t)}$

term is very high compared to $n_q(t)$.

$\therefore$ So $n_q(t)$ is neglected

$s(t) = A_c + A_c k_a m(t) + n_c(t)$

$m^2(t) = P$

$S_0 = A_c^2 \cdot k_a^2 \cdot P$

$N_0 = 2N_0 \cdot W$

$$\left(\frac{S}{N}\right)_0 = \frac{A_c^2 k_a^2 P}{2N_0 W} \rightarrow (2)$$

$$FOM = \frac{(S/N)_0}{(S/N)_I} = \frac{A_c^2 k_a^2 P / 2N_0 W}{A_c^2[1+k_a^2 P] / 2N_0 W}$$

$$= \frac{A_c^2 k_a^2 P}{2N_0 W} \times \frac{2N_0 W}{A_c^2[1+k_a^2 P]}$$

$$FOM = \frac{k_a^2 P}{1+k_a^2 P}$$

$P = m^2(t) = A_m^2/2$

$k_a \cdot A_m = \mu$

$$FoM = \frac{k_a^2 A_m^2/2}{1 + k_a^2 A_m^2/2}$$

$$= \frac{\mu^2/2}{1 + \frac{\mu^2}{2}}$$

$$= \frac{\mu^2/2}{2+\mu^2/2}$$

$$FOM = \frac{\mu^2}{2} \times \frac{2}{2+\mu^2}$$

$$\boxed{FOM = \frac{\mu^2}{2+\mu^2}}$$

* When envelope detector is used, max value of $\mu$ is '1'

$$\boxed{FOM = \frac{1}{2+1} = \frac{1}{3}}$$

**Noise in FM Receiver →**

* WKT, the modulated signal of an FM wave is given as,

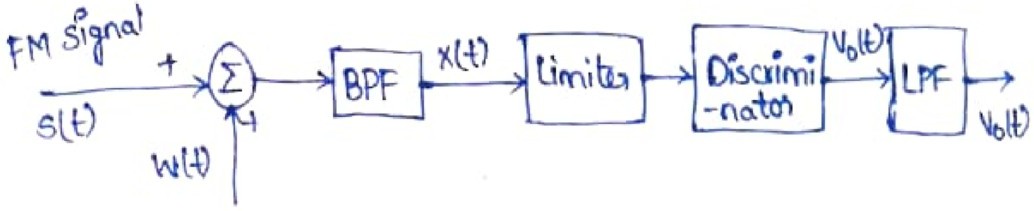
$$s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int_0^t m(t)\,dt\right]$$

$$S_i = \frac{A_c^2}{2}$$

$$n_i = N_0 \cdot W$$

$$\boxed{(S/N)_i = \frac{A_c^2}{2 N_0 \cdot W}}$$

**At rx side:-**




**SNR at o/p:-**

* The o/p of BPF is distorted FM signal

* It is passed through a limiter, which is a type of clipper circuit.
* It clips the undesired amplitude levels & produces a clipped FM wave.
* The o/p of a limiter is passed through a discriminator which performs two operations as a differentiator & then as a envelope detector.
* Finally, the o/p of the discriminator is passed through a LPF to recover the original modulating signal.

we have, $s(t) = A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right]$

$n(t) \rightarrow$ narrow Band noise

$\therefore\ n(t) = n_c(t) \cos 2\pi f_c t + n_Q(t) \sin 2\pi f_c t \rightarrow$ rectangular form

* The discriminator consists of an envelope detector, we convert the narrow Band noise which is in rectangular form to polar form.

polar representation of $n(t) \Rightarrow$

$$n(t) = \underbrace{r(t)}_{\text{envelope}} \cos(2\pi f_c t + \underbrace{\phi_n(t)}_{\text{phase}})$$

where $r(t) = \sqrt{n_I^2(t) + n_Q^2(t)}$

$$\phi_n(t) = \tan^{-1}\left(\frac{n_Q(t)}{n_I(t)}\right)$$

* the i/p to FM demodulator = FM signal + noise

$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int m(t)\,dt\right] + n_I(t) \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t$

* Assume that the carrier is not modulated $\Rightarrow m(t) = 0$

$= A_c \cos 2\pi f_c t + n_I(t) \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t$

$= \left[A_c + n_c(t)\right] \cos 2\pi f_c t + n_q(t) \sin 2\pi f_c t$

$r(t) = \sqrt{n_I^2(t) + n_Q^2(t)} \Rightarrow \sqrt{(A_c + n_c(t))^2 + n_q^2(t)} \cos(2\pi f_c t + \phi) \rightarrow (1)$

$$\phi = \tan^{-1}\left[\frac{n_q(t)}{A_c + n_c(t)}\right]$$

$$\therefore s(t) = A_c \cos \underset{\substack{\downarrow \\ 2\pi f_c t + \phi}}{\theta(t)}$$

$$\phi \approx \tan^{-1}\left[\frac{n_q(t)}{A_c}\right]$$

+ neglecting $n_c(t)$ when compared to $A_c$, $A_c$ is very high compared to $n_q(t)$

So, $$\phi = -\frac{n_q(t)}{A_c}$$

So, $$\tan^{-1}\left(\frac{n_q(t)}{A_c}\right) < 1$$

Sub $\phi$ in 1

$$= \sqrt{[A_c + n_c(t)]^2 + n_q^2(t)} \cdot \cos\underbrace{\left[2\pi f_c t - \frac{n_q(t)}{A_c}\right]}_{\theta(t)}$$

$$f_i = \frac{1}{2\pi}\frac{d\theta(t)}{dt}$$

$$= \frac{1}{2\pi}\left[\frac{d}{dt}\left(2\pi f_c t - \frac{n_q(t)}{A_c}\right)\right]$$

$$= \frac{1}{2\pi}\left[2\pi f_c - \frac{1}{A_c}\frac{d}{dt} n_q(t)\right]$$

$$f_i = f_c - \frac{1}{2\pi A_c}\frac{d}{dt} n_q(t)$$

$$f_i = \underbrace{f_c + k_f m(t)}_{\text{due to FM signal}} - \underbrace{\frac{1}{2\pi A_c}\frac{d}{dt} n_q(t)}_{\text{due to noise}}$$

o/p of demodulator, $V_0 = K \cdot f_i$

$$V_0 = K \cdot f_c + K \cdot k_f\, m(t) - \frac{K}{2\pi A_c} \frac{d}{dt} n_q(t)$$

$$\Downarrow$$

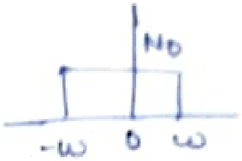
$$0 \;[\text{when } f_i = f_c]$$

$$V_0 = K \cdot k_f\, m(t) - \frac{K}{2\pi A_c} \frac{d}{dt} n_q(t)$$

$$K \cdot k_f\, m(t) \longrightarrow K^2 k_f^2 \cdot P$$

$$S_0 = K^2 k_f^2 P$$

$$n_q(t) \xrightarrow[\text{power spectrum density}]{\text{PSD}} N_0 \qquad -W < f_c < W$$




$$\frac{K}{2\pi A_c} \frac{d}{dt}[n_q(t)] \longrightarrow \frac{K}{2\pi A_c} j\omega N_0$$

$$= \frac{K^2}{4\pi^2 A_c^2} (2\pi f)^2 N_0$$

$$= \frac{K^2 f^2 N_0}{A_c^2}$$

$$PSD = \frac{K^2 N_0}{A_c^2} \cdot f^2$$

$$\text{Noise power} = \int_{-W}^{W} \frac{K^2 N_0}{A_c^2} f^2 \cdot df$$

$$= \frac{K^2 N_0}{A_c^2} \int_{-W}^{W} f^2 \cdot df$$

$$= \frac{K^2 N_0}{A_c^2} \left[\frac{f^3}{3}\right]_{-W}^{W}$$

$$= \frac{K^2 N_0}{3A_c^2} \left[W^3 - (-W^3)\right]$$

$$n_0 = \frac{K^2 N_0}{3A_c^2} \left[2W^3\right]$$

$$(S/N)_o = \frac{k^2 k_f^2 \cdot P \cdot 3A_c^2}{k^2 N_0 \, 2\omega^3}$$

$$= \frac{3}{2} \frac{k_f^2 P A_c^2}{N_0 \cdot \omega^3}$$

$$FOM = \frac{\frac{3}{2} \cdot k_f^2 P A_c^2}{N_0 \cdot \omega^3} \times \frac{2N_0\omega}{A_c^2}$$

$$= \frac{3}{\cancel{2}} \times \frac{k_f^2 \cdot P}{\omega^3} \times \cancel{2}\omega$$

For single tone modulation of FM $= \dfrac{3k_f^2 A_m^2}{2f_m^2}$

$$k_f \cdot A_m = \Delta f$$

$$\frac{\Delta f}{f_m} = \beta$$

$$\boxed{\therefore FOM = \frac{3}{2}\beta^2}$$

Threshold Effect in AM Receiver →

* When the value of the i/p signal to noise ratio falls below a particular value, a rapid fall of the o/p signal to noise ratio is observed by the receiver.
* Threshold effect occurs due to the presence of large noise in the modulated signal.
* Due to the threshold effect, the detection of the message signal becomes difficult.
* threshold effect can be improved by applying feedback in demodulator circuits such as phase locked loops & frequency locked loops.
* the o/p of SNR value does not fall rapidly.

* The relation between i/p SNR & o/p SNR is

$$(SNR)_{o/p} = 3\beta^2 (SNR)_{i/p}$$

where $\beta$ = modulation index

Capture effect →

* In FM signal can be affected by another frequency modulated signal whose frequency content is close to "$f_c$" of desired FM wave.
* Receiver may lock such an interference signal & suppress the desired '$f_m$', when interference signal is stronger than the desired signal.
* When strength of desired signal & interference signal are nearly equal, the receiver fluctuates back & forth between them i.e., receives lock interference signal for some time & desired signal for some time & this goes on randomly. This phenomenon is called "capture effect".

FM Threshold Effect →

* The $(S/N)_o$ of an FM signal is

$$(S/N)_o = \frac{3A_c^2 K_f^2 P}{2N_0 W^3}$$

is valid only if (S/N) measured at discriminator i/p. is high compared with unity. It is observed that as the (S/N) ratio decreases & receiver i/p noise power increases the (S/N) ratio ... breaks.

* Initially individual clicks are heard in receiver o/p & as (S/N) decreases still further, clicks rapidly merge into a signal.

Cracking or sputtering sound :-

* At breaking point $(S/N)_o$ begins to fall by predicting values of o/p (S/N) larger than the actual ones. This

phenomenon is known as " threshold effect".

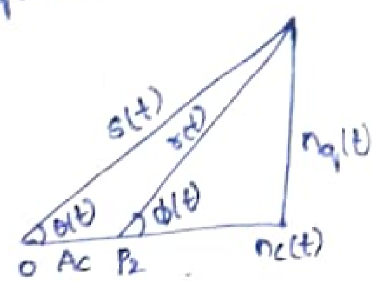
* Threshold effect is defined as the minimum carrier to noise ratio (S/N) that gives the $(S/N)_o$ not less than the value predicted by usual (S/N) formula assuming a small noise power.

* The composite signal at frequency discriminator i/p is given by,

$$s(t) = [A_c + n_c(t)]\cos(2\pi f_c t) + n_q(t)\sin(2\pi f_c t)$$

* It is realized that when (S/N) is high amplitudes of $n_c(t)$ & $n_q(t)$ are small compared to $A_c$ & $\theta(t)$ increases

* We can derive the condition required to clicks to occur




Conditions for '+ve' clicks:-

$$r(t) > A_c$$

$$\varphi(t) < \pi < \varphi(t) + d\varphi(t)$$

$$\frac{d\varphi(t)}{dt} > 0$$

Conditions for '-ve' clicks:-

$$r(t) > A_c$$

$$\varphi(t) > -\pi > \varphi(t) + d\varphi(t)$$

$$\frac{d\varphi(t)}{dt} < 0$$

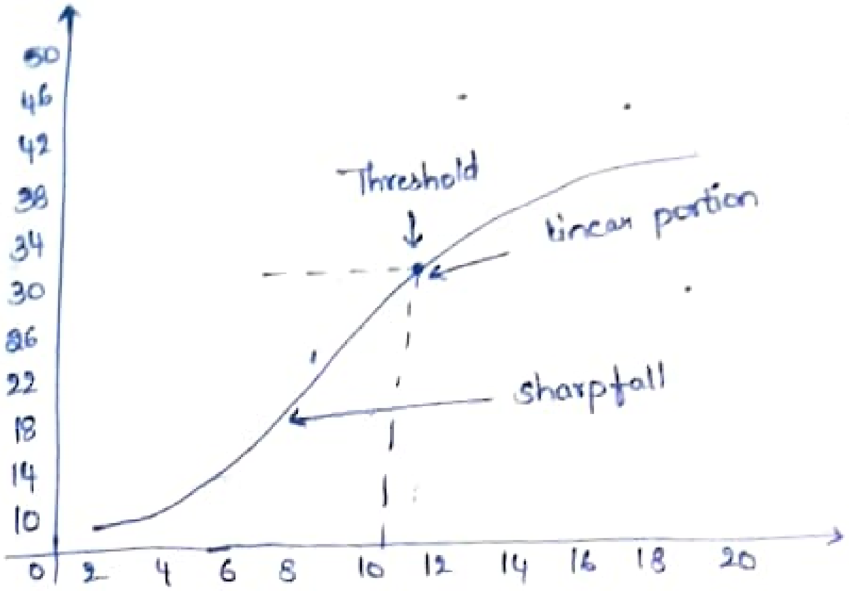
* The conditions for '+ve' clicks ensure that $\theta(t)$ changes by $2\pi$ radiants during time increment dt & conditions '-ve' clicks ensures that $\theta(t)$ changes by $-2\pi$ radiants during the time increment dt.

$$(S/N)_i = \frac{A_c^2}{2N_0 W}$$

* Average number of clicks per unit time are inversely proportional to $(S/N)_i$

* It is observed the $(S/N)_o$ is a linear function of (S/N) b

when $\left(\frac{S}{N}\right)_i > 10db$ However, it falls sharply for lower values of $(S/N)_i$ than 10db




$$(S/N)_i = 10\log_{10}\left(\frac{S}{N}\right)_0$$

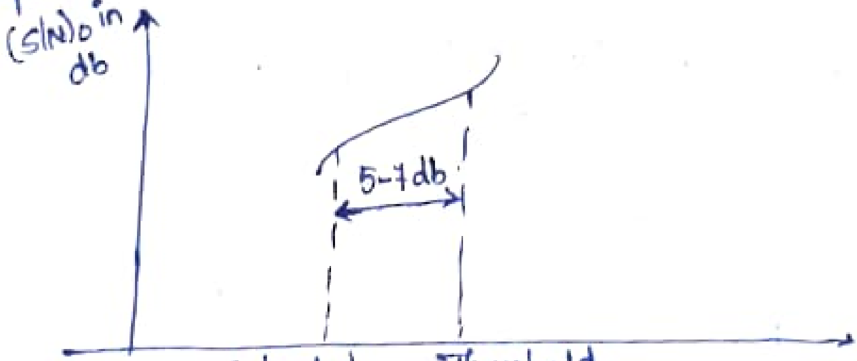
* Threshold can be avoided by keeping $\left(\frac{S}{N}\right)_i > 20$ i.e., 13 db.

$$\frac{A_c^2}{2WN_0} \geq 20$$

$$\frac{A_c^2}{2} \geq 20N_0W$$

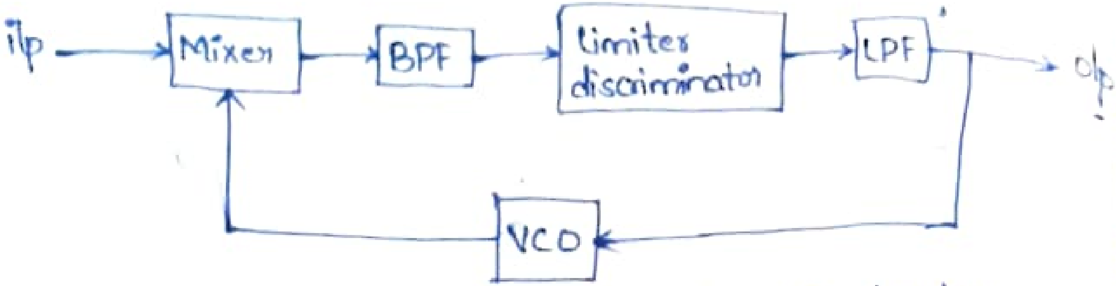
FM Threshold Reduction :-

* In specific applications, such as space communications we require lesser threshold in a FM receiver.



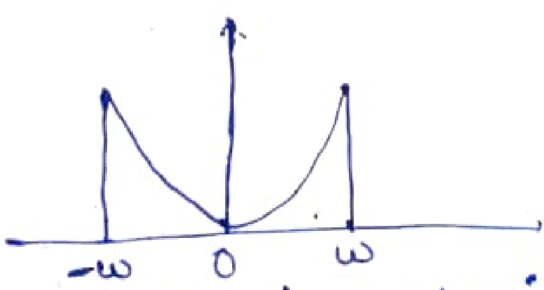

* For such applications FM threshold can be reduced by using FM demodulator with '-ve' feed back (or) using

a PLL demodulator. Such devices are referred to as "extended threshold demodulators".

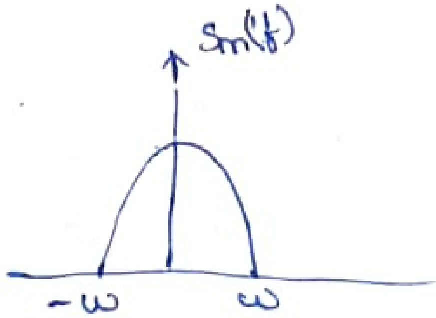



* FM demodulator with '-ve' feedback is also known as "FMFB demodulator".
* In this demodulator the local oscillator is replaced by VCO.
* The instantaneous o/p frequency of such VCO is controlled by demodulated signal.
* The Bandwidth of noise to which FMFB Receiver responds is precisely the band of noise that the VCO tracks.
* Thus, the FMFB receiver acts as a tracking filter, that can track only the slowly varying frequency of a wide band FM signal & therefore it responds only to a narrow Band of noise centred about the instantaneous carrier frequency. As a result, FMFB receiver allow a threshold extension.
* Like, the FMFB demodulator, PLL is also a tracking filter & hence it also provides threshold extension.

Pre-emphasis & De-emphasis in FM →

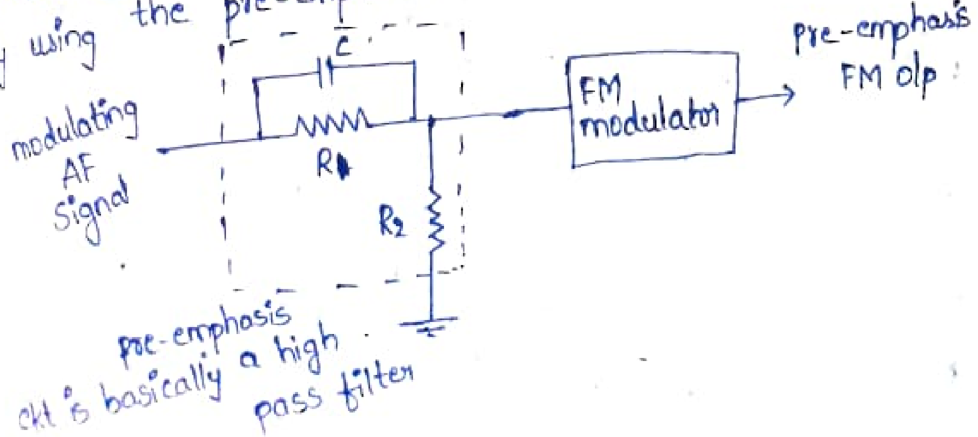



Power spectral density of noise at FM receiver o/p

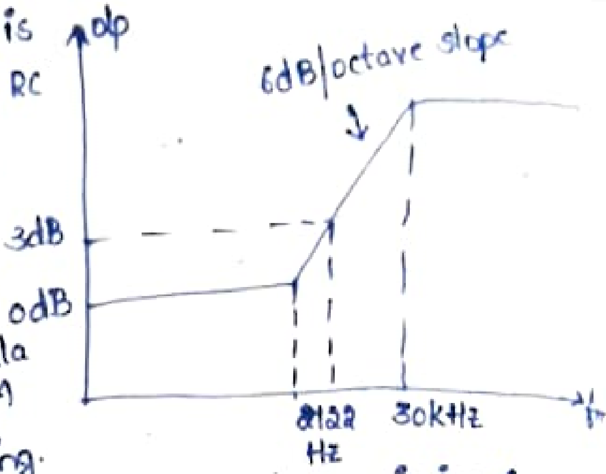



Power spectral density of a typical msg source.

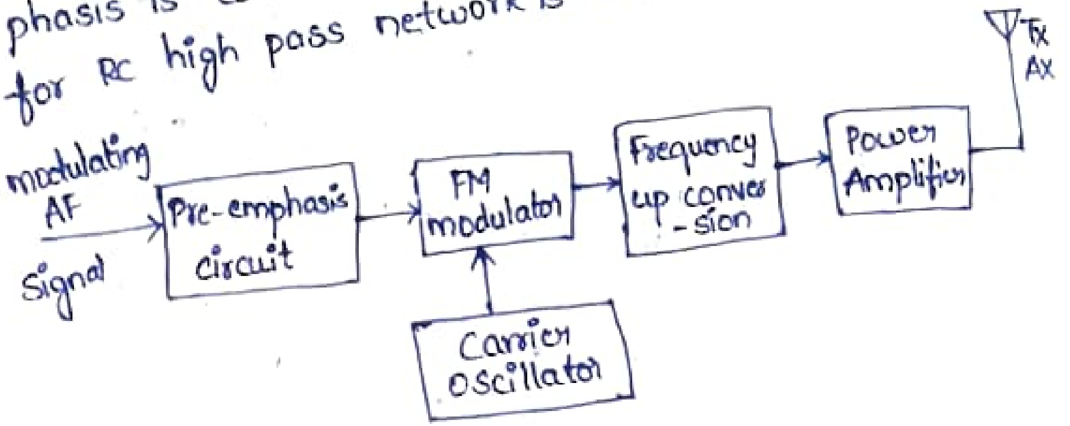
* The power spectral density of message usually falls off at higher frequencies. On the other hand, the power spectral density of o/p noise increases rapidly with frequency.
* Thus at $f = fw$, the relative spectral density of message is quite low, whereas that o/p noise is quite high in comparison.
* It is clear that the message is not utilizing the frequency band allowed to it, in an effective manner.
* One way of improving noise performance is to slightly reduce the bandwidth of post detection LPF. So, as to reject a large amount of noise power.
* A more satisfactory approach to efficient utilization of allowed frequency band is based on the use of pre-emphasis in transmitter & de-emphasis in receiver.
* In FM, the noise has a greater effect on the higher modulating frequencies. This effect can be reduced by increasing the value of modulation index ($m_f$) for high modulating frequencies ($f_m$).
* This can be done by increasing the deviation ($\Delta f$) & $\Delta f$ can be increased by increasing the amplitude of modulating signal at higher modulating frequencies.
* If we boost the amplitude of higher frequency modulating signals artificially, then it will be possible to improve the noise immunity at higher modulating frequencies.
* "The artificial boosting of higher modulating frequencies is called as pre-emphasis".
* Boosting of higher frequency modulating signal is achieved by using the pre-emphasis circuit.



* The modulating AF signal is passed through a high pass RC filter, before applying it to the FM modulator.




* As '$f_m$' increases, reactance of 'c' decreases & modulating voltage applied to FM modulator goes on increasing.
* The amount of pre-emphasis in FM transmission & sound transmission in TV has been standardized at 75μsec.
* The pre-emphasis circuit is basically a HPF. The pre-emphasis is carried out at the transmitter. The frequency for RC high pass network is 2122Hz.

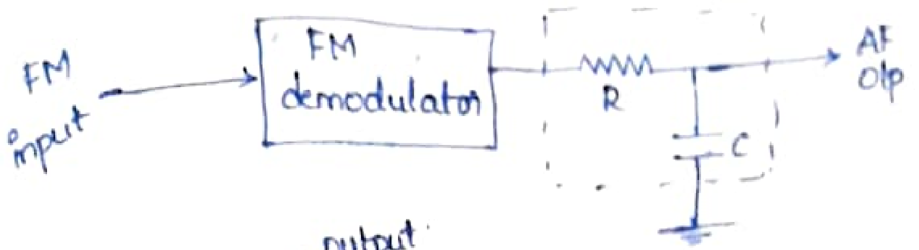


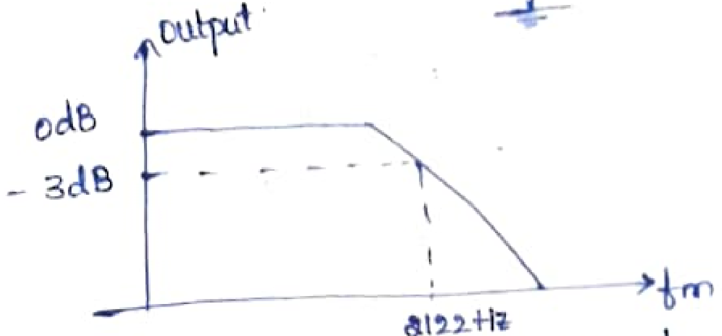

De-emphasis →

* The process that is used at the receiver end to nullify (or) compensate the artificial boosting given to the higher modulating frequencies in the process of pre-emphasis is called "De-emphasis".
* That means the artificially boosted high frequency signals are brought to their original amplitude using de-emphasis circuit.
* The 75μsec de-emphasis circuit is standard.
* It shows that it is a low pass filter. 75μsec de-emphasis corresponds to a frequency response curve

that is 3dB down at a frequency whose RC time constant is 75µsec i.e.,

$$f = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 75 \times 10^{-6}} = 2122\ Hz$$




* The demodulated FM is applied to the de-emphasis circuit with increase in fm, the reactance of 'c' goes on decreasing & the o/p of de-emphasis circuit will also reduce.