**(c) Inventory Carrying/Holding Costs:** Holding cost is associated with storing an item in inventory. It is proportional to the amount of inventory and the time over which it is held. It includes the following:

**(i) The cost of capital tied up in the inventory.**

**(ii) Cost of warehouses, racks, salary of warehouse employees.**

**(iii) Insurance Cost.**

**(iv) Pilferage cost.**

**(v) Obsolescence cost.**

**(vi) Handling cost associated with the movement of stocks such as cost of labour, and other machinery used for this purpose.**

**(vii) Storage operation cost including cost of maintaining inventory records and security expenses.**

**Carrying Cost = (Cost of carrying one unit of an item in the inventory for a given length of time) * (Average number of units of an item carried in the inventory for a given length of time).**

**(d) <u>Shortage or Stock out Cost</u>:** Stock out cost is the cost of sales lost due to shortage of stock or loss of goodwill due to delay in supply of finished goods.

***Total Inventory Cost = Purchase Cost + Ordering Cost + Carrying Cost + Shortage Cost.***

**(2) <u>Demand</u>:** The demand pattern of an item may be either:

**(i) <u>Deterministic demand</u>:** In this, the quantities needed over subsequent periods of time are known with certainty and expressed over equal periods of times.

**(ii) Probabilistic demand:** This occurs when quantities needed over a certain period of time are not known with certainty, but their pattern can be expressed by a known probability distribution. It can be stationary or non-stationary.

**(3) Ordering Cycle:** It refers to the time period between two successive placement of orders, which can be determined in following ways:

**(i) Continuous Review:** In this system, a record of the inventory level is updated until a certain lower limit (called re-order point) is reached at which point a new order of fixed amount is placed. This is also known as two-bin system.

**(ii) Periodic Review:** In this case, the orders are placed at equal intervals of time through periodic review. Quantity of orders is always decided on the basis of consumption between two review periods.

**(4) Time Horizon:** It is known as planning period over which inventory is to be controlled and is generally done on annual basis.

**(5) Delivery Lag or Lead Time:** It is the time between the requisition for an item and its receipt. It can be deterministic or probabilistic. It includes:

**(i)** Administrative lead time.

**(ii)** Supplier's lead time.

**(iii)** Transportation lead time.

**(iv)** Inspection lead time.

**(6) Safety Stock:** It is the minimum additional inventory which serve as a safety margin or cushion to meet an unanticipated increase in demand.

**Safety stock = (Max. lead time – Avg. lead time) * Demand rate.**

**(7) Re-order Level:** It is the amount of stock that is on hand at the time of the placement of the replenishment order. This level is enough to serve the customers during the lead time.

**Re-order level = Safety stock + Lead time demand.**

**Steps involved in Inventory Model Building:**

(a)Collect the data regarding the pattern of demand, the replenishment policy, planning horizon and relevant inventory costs.

(b)Develop the total annual inventory relevant cost.

(c)Transform the total annual cost from formula to a mathematical formula.

(d)Find the optimum quantity to be ordered and when to re-order and the total relevant cost.

(e)Estimate lead time, safety stock and reorder level.

(f)Develop the inventory model.

(g)Review the position and make suitable changes depending upon the current constraints.

## The Economic Order Quantity (EOQ):

Economic order quantity is the size of the order representing standard quality of material and it is the one for which the aggregate of the costs of procuring the inventory and costs of holding the inventory is minimum.

**Single-item deterministic inventory control models without shortages:**

(a)The total number of units required for 1 year is known exactly.

(b)The demand is known & constant & is resupplied instantaneously.

(c)Orders are received instantaneously.

(d)Ordering costs are the same regardless of order size.

(e)The purchase price does not fluctuate during the period considered.

(f)There is sufficient space, and money to allow the procurement of any quantity desired.

**Model 1: Basic Economic Order Quantity Model with Infinite supply:**

**Basic Assumptions:**

(a) Annual demand, carrying cost and ordering cost is known with certainty and is constant over time.

(b) Avg. inventory level for a material is order quantity divided by two. It means that materials are entirely used up when the next order arrives.

(c) Volume discounts do not exist.

(d) There are not stockout costs, i.e. inventory is replenished immediately as the stock level is almost zero.

(e) Lead time is known with certainty.

(f) Infinite planning horizon is assumed.

**Definitions:**

(a)D = annual demand for a material (units per year)

(b)Q = Quantity of material ordered at each order point.

(c)$C_h$= Cost of carrying one unit in inventory for one year.

(d)$C_o$= Average cost of completing an order for material.

**Formulas:**

(a) Annual carrying cost = Avg. inventory level * carrying cost per unit per year.

$= (Q/2)* C_h$

(b) Annual Ordering cost = No. of orders placed per year * ordering cost per order.

= (annual demand/no. of units in each order) * ordering cost per order.

(c) Total annual cost = Annual carrying cost + Annual ordering cost.

(d) Average Inventory level = (Maximum inventory + Minimum inventory)/2

(e) Optimal time between two orders = EOQ/Annual demand

**Q1:** The XYZ manufacturing company has determined from an analysis of its accounting and production data for 'part number alpha', that its cost to purchase is Rs. 36 per order and Rs.2 per part. Its inventory carrying charge is 9% of the average inventory. The demand of this part is 10,000 units per annum. Determine:

(a) What should be the economic order quantity?

(b) What is the optimal number of orders?

**Q2:** A company, one of the A-class items, place 6 orders each of size 200 in a year. Given ordering cost = Rs. 600, holding cost = 40%, cost per unit = Rs. 40. Find out the loss to the company in not operating scientific inventory policy? What are your recommendations for the future?

**Q3:** The production department of a company works 50 weeks a year and has demand for an item which is constant at 100 units a week. The cost of each unit is Rs. 200 and the company aims for a return of 20% on capital invested. Annual warehouse costs are estimated to be 5% of the value of goods stored. The purchasing department of the company costs Rs. 4,50,000 a year and send out an average of 2,000 orders. Determine.

(a) Optimal order quantity for the item and optimal number of orders per year.

(b) Optimal time between two consecutive orders, and

(c) the minimum annual cost of carrying the item.

**Model 2: Economic Production Quantity Model with Finite Replenishment (Supply) Rate:** This model is similar to the previous one except that here supply is gradual rather than instantaneous.

**There can be three possible relationships between demand rate & supply rate (or production rate):**

**(a)Demand rate > Production rate:** In this case shortage will occur which is ignored in this model. It is assumed that shortage is not permitted.

**(b)Demand rate = Production rate:** In this case there will be no need of holding inventory in stock as both demand & production rates are equal.

**(c)Demand rate < Production rate:** In this case if production process remains continuous, inventory stock will go on increasing which is the main concern of this model.

- The problem involved in this model is to determine the optimal number of units to be produced in one production run as production continuity is not possible.
- This is useful where products must be ordered from a production department within the organisation.
- Production occurs at a specific rate greater than demand and finished goods are transferred gradually from production to finished goods inventory as they are produced.

**Assumptions:**

(a)Annual demand, carrying cost & ordering cost is known with certainty.

(b)No safety stock is utilized, materials are supplied and used at a uniform rate and materials are entirely used up when the next orders begins to arrive.

(c)Volume discounts do not exist.

(d)There are not stockout costs.

(e)Supply rate is greater than demand rate.

(f)Production begins immediately after production set-up.

**Definitions:**

(a) d = rate at which units are used out of inventory (D/number of working days)

(b) p = rate at which units are supplied to inventory.

**Formulae:**

**(a) Inventory Accumulation rate =** (p – d)

**(b) Maximum Inventory Level =** Inventory accumulation rate + delivery period

$$= \frac{(p-d)Q}{p}$$

**(c) Minimum inventory Level** = 0

**(d) Average Inventory Level =** $\frac{(p-d)Q}{2p}$

**(f) Annual Carrying Cost** = $\frac{(p-d)Q}{2p} * C_h$

**(g) Annual Ordering Cost** = $(D/Q) * C_o$

**(h) Total annual cost** = Annual carrying cost + Annual ordering cost.

**(i) EOQ** = $\sqrt{\frac{2\,DC_o\,(p)}{C_h\,(p-d)}}$

**Q4:** XYZ co. uses 10,000 units of a particular valve per year. Each valve costs Rs. 32. The production engineering department estimates set-up cost as Rs. 55 and the accounting department estimates the holding cost as 12.5% of the value of inventory. Replenishment rate is uniform 120 valves per day. Assuming 250 working days, calculate,

(a) Optimal order quantity

(b) Total inventory cost on the basis of optimal policy, and

(c) Optimal number of set-ups.

**Q5:** (i) At present a company is purchasing an item 'X' from outside suppliers. The consumption is 10,000 units per year. The cost of the item is Rs. 50 per unit and the ordering cost is estimated to be Rs. 1,000 per order. The cost of carrying inventory is 25%. If the consumption rate is uniform, determine the economic purchasing quantity.

(ii) In the above problem assume that company is going to manufacture the above item with the equipment that is estimated to produce 100 units per day. The cost of the unit thus produced is Rs. 35 per unit. The set-up cost is Rs. 1500 per set-up and the inventory carrying charge is 25%. How has your answer changed? Assume 250 working days in a year.

**Model 3: Inventory Control Model with Planned Shortages:**

•Earlier models were based on the assumption that shortage cost was not allowed. But in some situations economic advantage may be gained by allowing shortages to occur.

•One advantage is allowing shortages is to increase the cycle time so that the ordering costs is spread over a longer period.

•Another advantage is when the unit value of the inventory as well as carrying cost is high.

**Assumptions:**

(a)This model is also known as back order or planned shortages inventory model as Stockouts and back ordering is allowed.

(b)Back order is the situation in which a customer places an order, finds that the material is out of stock, and waits for the next shipment to arrive.

(c)This model assumes that the customer's sale will not be lost due to stockout.

**Notations:**

(a)$C_b$ = Stockout or backorder cost per unit back-ordered per period.

(b)S = remaining units after the back-order is satisfied.

(c)Q – S = Number of shortages per order (back-order quantity)

(d)$t_1$ = Time during which inventory is on hand.

(e)$T_2$= Time during which a shortage exists.

(f)T = Time between receipt of orders.

**Formulae:**

$$\text{total shortage costs} = C_s \frac{S^2}{2Q}$$

$$\text{total carrying cost} = C_c \frac{(Q-S)^2}{2Q}$$

$$\text{total ordering cost} = C_o \frac{D}{Q}$$

$$TC = C_s \frac{S^2}{2Q} + C_c \frac{(Q-S)^2}{2Q} + C_o \frac{D}{Q}$$

$$Q_{opt} = \sqrt{\frac{2C_o D}{C_c}\left(\frac{C_s + C_c}{C_s}\right)}$$

$$S_{opt} = Q_{opt}\left(\frac{C_c}{C_c + C_s}\right)$$

**Maximum number of back orders** = $Q\ (C_h / C_h + C_s)$

**Number of orders per year** = $D/Q$

**Q6:** The demand for an item is deterministic and constant over the time and it is equal to 600 units per year. The per unit cost of the item is Rs. 50 while the cost of placing an order is Rs. 5. The inventory carrying cost is 20% of the cost of inventory per annum and the cost of shortage is Re. 1 per unit per month. Find the optimal ordering quantity when stockouts are permitted. If the stockouts are not permitted, what would be the loss to the company?