Vibrations (AS2080*): Project 01 (50 marks)

The main objective of this project (Home Assignment) is for you to demonstrate how to solve basic vibration problems analytically and numerically. Please submit a report that must include your handwritten solutions, a computation program (using MATLAB/Python/Others), and final plots for each of the problems given below. Kindly make use of the computational method to solve the problems as much as possible, apart from just plotting the results. The program should at least include the following parts: code to obtain roots of the characteristic equations (for SDOF system), code to solve eigenvalue problems (for the TDOF system), and code to plot the results.

Problem 01 (15 marks):

Derive and plot the free-vibration response of a viscously damped single-degree-of-freedom system with m = 4 kg, k = 2500 N/m, $x_0 = 0.15$ m, $\dot{x}_0 = -20$ m/s for the following values of the damping constant:

(i)
$$c = 0 \text{ N-s/m}$$
, (ii) $c = 100 \text{ N-s/m}$, (iii) $c = 200 \text{ N-s/m}$, (iv) $c = 400 \text{ N-s/m}$

Problem 02 (15 marks):

Consider a spring-mass-damper system with k = 4000 N/m, m = 10 kg, and c = 40 N-s/m. Derive and plot the steady-state and total responses of the system under the excitation forces listed below. Initial conditions are $x_0 = 0.1$ m and $\dot{x}_0 = 10$ m/s.

(i)
$$F(t) = 100 \cos 10t$$
, (ii) $F(t) = 100 \cos 20t$

Problem 03 (20 marks):

Derive and plot the free-vibration response of the system given in the figure below. System parameters are as follows: $k_1 = 1000 \text{ N/m}$, $k_2 = 500 \text{ N/m}$, $m_1 = 2 \text{ kg}$, $m_2 = 1 \text{ kg}$. Solve the system for the following two sets of initial conditions and then plot the general solutions.

(i)
$$x_1(0) = 1$$
 m, $x_2(0) = 2$ m, $\dot{x}_1(0) = 1$ m/s, $\dot{x}_2(0) = -2$ m/s

(ii)
$$x_1(0) = 1$$
 m, $x_2(0) = 0$ m, $\dot{x}_1(0) = -1$ m/s, $\dot{x}_2(0) = 0$ m/s

