CS7015 (Deep Learning) : Lecture 3

Sigmoid Neurons, Gradient Descent, Feedforward Neural Networks, Representation Power of Feedforward Neural Networks

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Acknowledgements

- For Module 3.4, I have borrowed ideas from the videos by Ryan Harris on "visualize backpropagation" (available on youtube)
- For Module 3.5, I have borrowed ideas from this excellent book [a] which is available online
- I am sure I would have been influenced and borrowed ideas from other sources and I apologize if I have failed to acknowledge them

[a] http://neuralnetworksanddeeplearning.com/chap4.html

Module 3.1: Sigmoid Neuron

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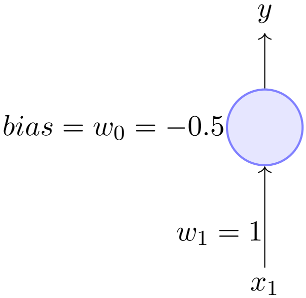
- Can we have a network which can (approximately) represent such functions ?
- Before answering the above question we will have to first graduate from ***perceptrons*** to ***sigmoidal neurons*** ...

Recall

- A perceptron will fire if the weighted sum of its inputs is greater than the threshold (-w_0)



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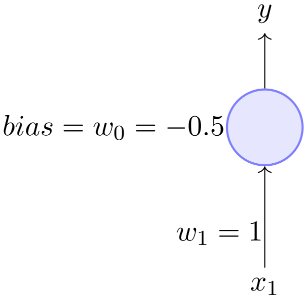

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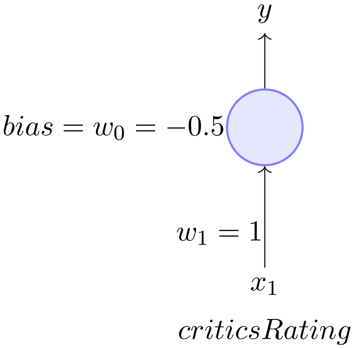

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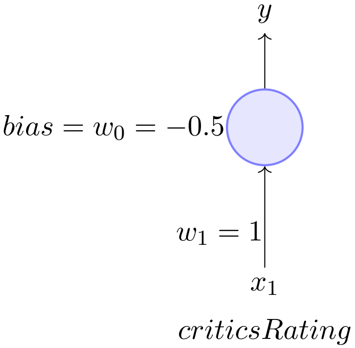

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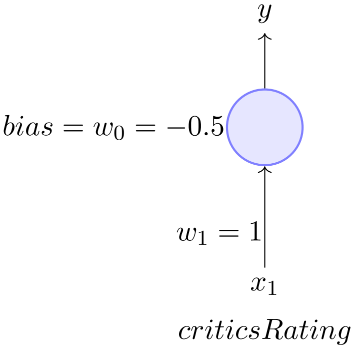
- If the threshold is 0.5 ($w_0 = -0.5$) and $w_1 = 1$ then what would be the decision for a movie with $criticsRating = 0.51$?



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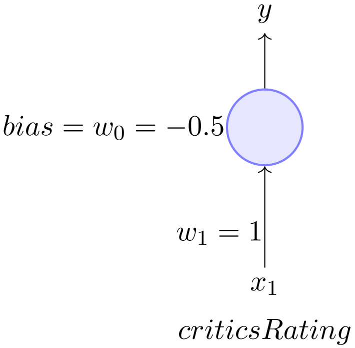

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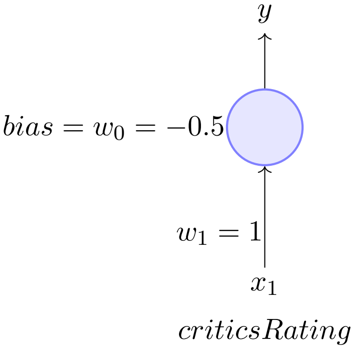

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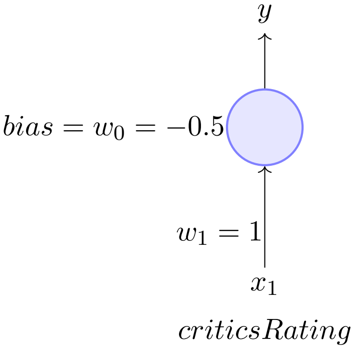

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- What about a movie with $criticsRating = 0.49$? (dislike)
- It seems harsh that we would like a movie with rating 0.51 but not one with a rating of 0.49

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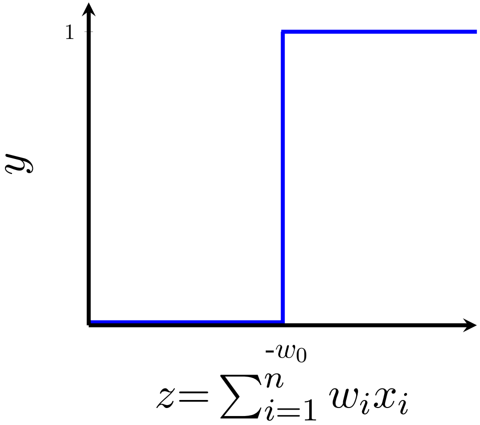

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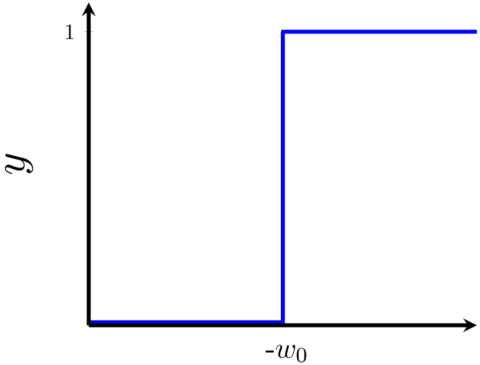

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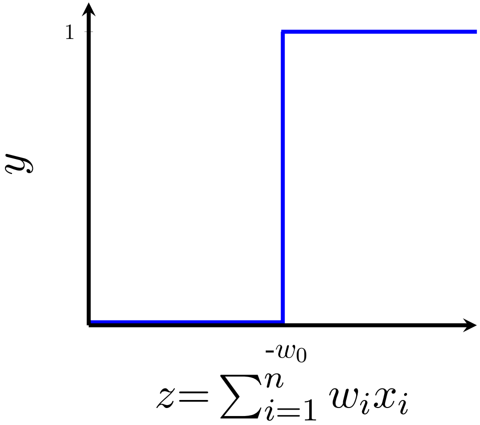

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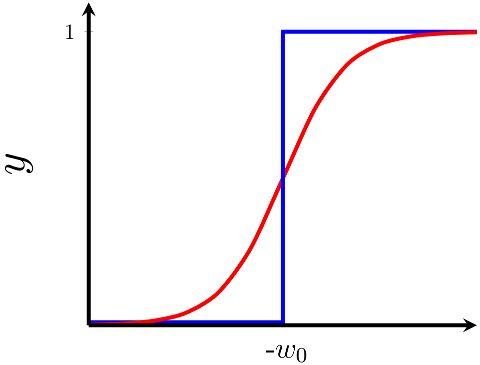
- There will always be this sudden change in the decision (from 0 to 1) when $\sum_{i=1}^{n} w_i x_i$ crosses the threshold ($-w_0$)
- For most real world applications we would expect a smoother decision function which gradually changes from 0 to 1



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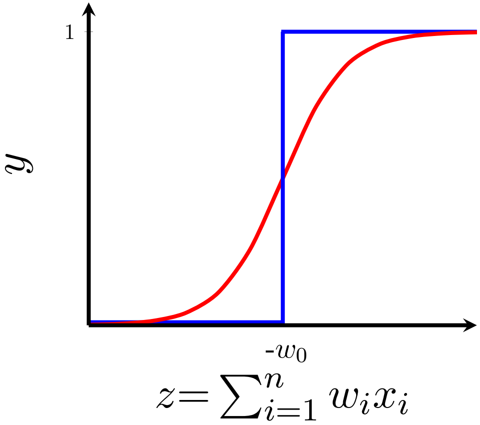

- Introducing sigmoid neurons where the output function is much smoother than the step function
- Here is one form of the sigmoid function called the logistic function

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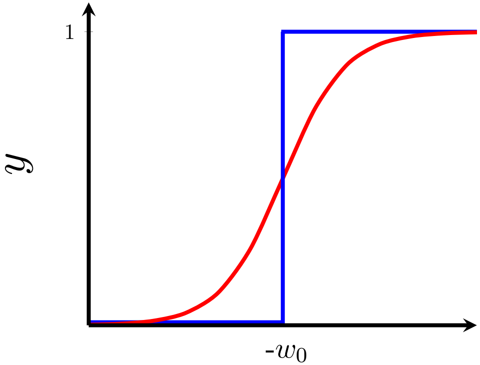

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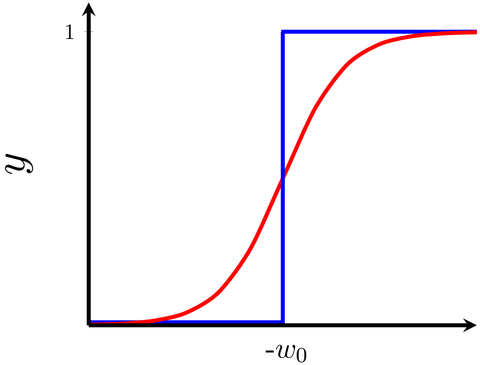

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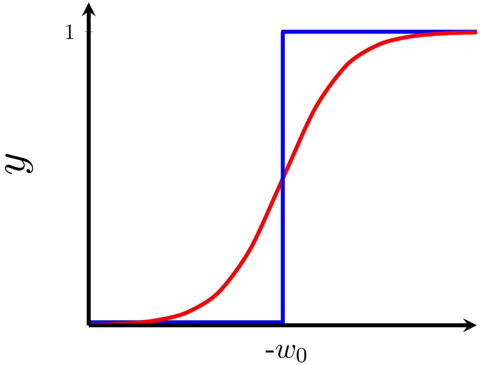
- Also the output y is no longer binary but a real value between 0 and 1 which can be interpreted as a probability



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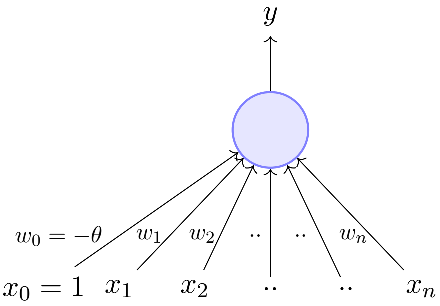
- Instead of a like/dislike decision we get the probability of liking the movie

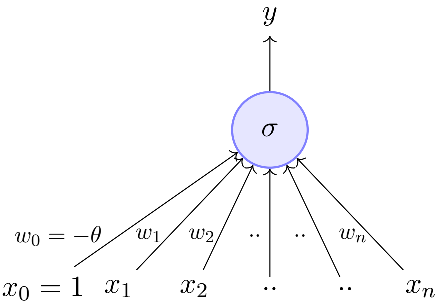
Perceptron



$$y = 1 \quad if \sum_{i=0}^{n} w_i * x_i \geq 0$$

$$= 0 \quad if \sum_{i=0}^{n} w_i * x_i < 0$$

Sigmoid (logistic) Neuron



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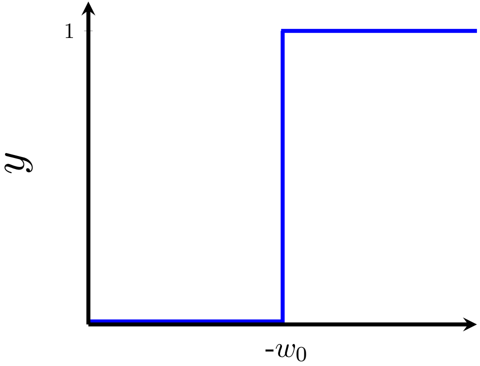

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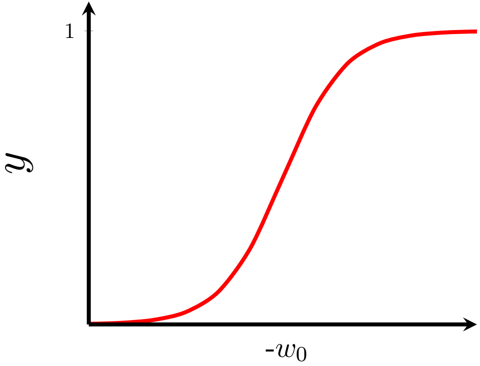

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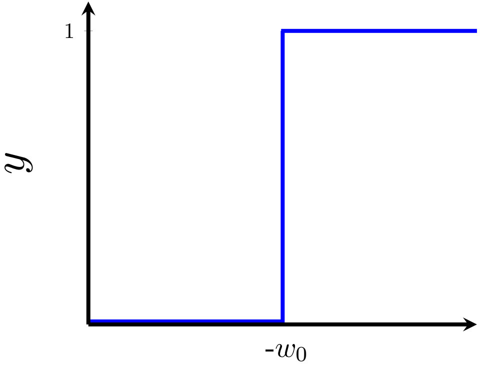

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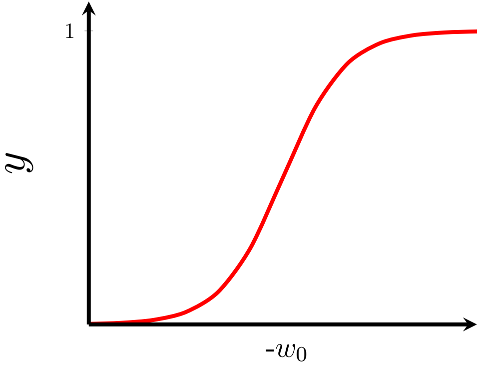

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Module 3.2: A typical Supervised Machine Learning Setup

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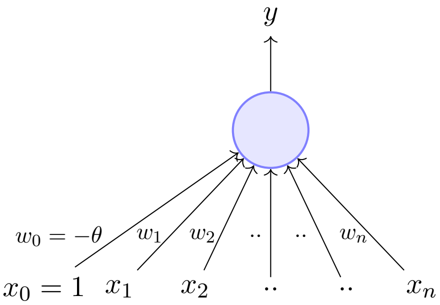
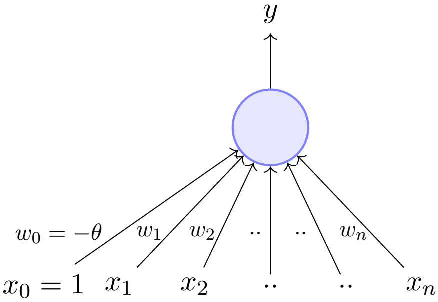


- What next ?

Sigmoid (logistic) Neuron



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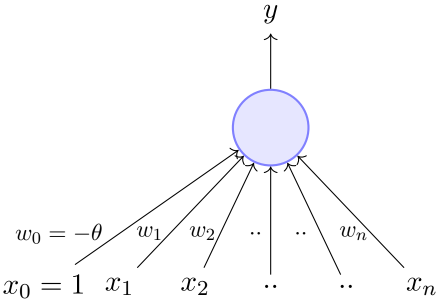

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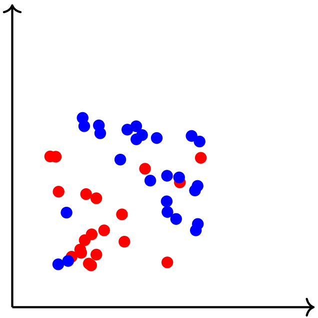
- Before we see such an algorithm we will revisit the concept of **error**



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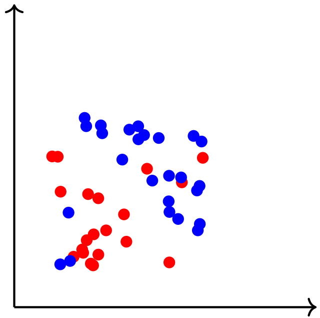

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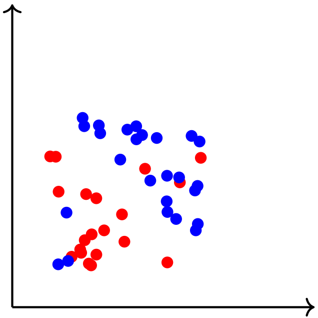

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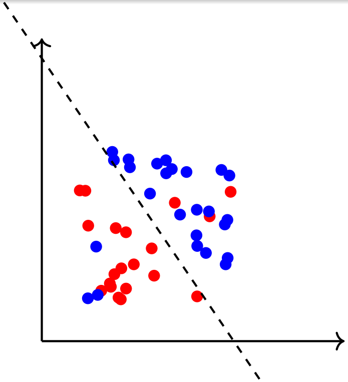

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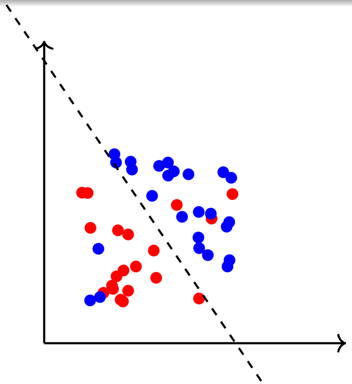

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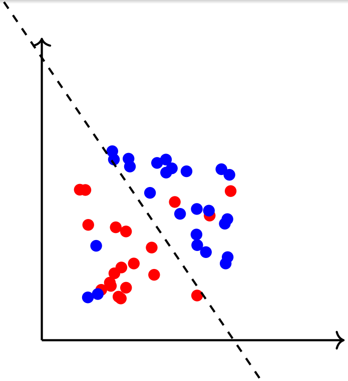

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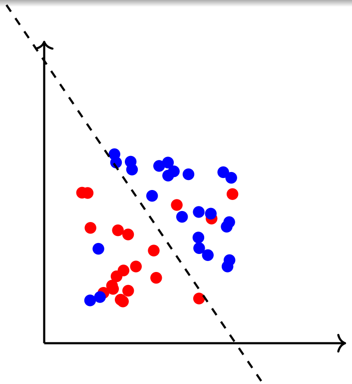

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- From now on, we will accept that it is hard to drive the error to 0 in most cases and will instead aim to reach the minimum possible error

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- **Parameter:** $\mathbf{w}$

As an illustration, consider our movie example

- **Data:** $\{x_i = movie, y_i = like/dislike\}_{i=1}^{n}$
- **Model:** Our approximation of the relation between $\mathbf{x}$ and y (the probability of liking a movie).

$$\hat{y} = \frac{1}{1 + e^{-(\mathbf{w^T x})}}$$

- **Parameter:** $\mathbf{w}$
- **Learning algorithm:** Gradient Descent [we will see soon]

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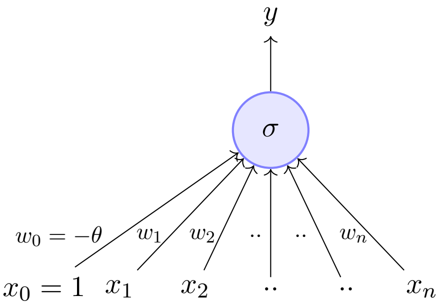
$$\mathscr{L}(\mathbf{w}) = \sum_{i=1}^{n} (\hat{y}_i - y_i)^2$$

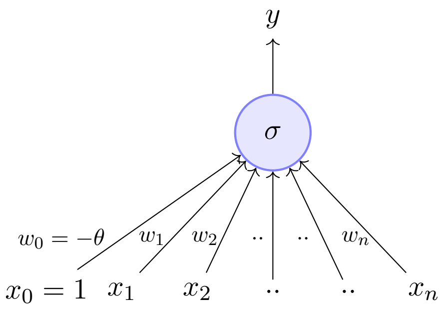
The learning algorithm should aim to find a w which minimizes the above function (squared error between y and $\hat{y}$)

Module 3.3: Learning Parameters: (Infeasible) guess work



$$f(x) = \frac{1}{1+e^{-(w \cdot x+b)}}$$

- Keeping this supervised ML setup in mind, we will now focus on this **model** and discuss an **algorithm** for learning the **parameters** of this model from some given **data** using an appropriate **objective function**



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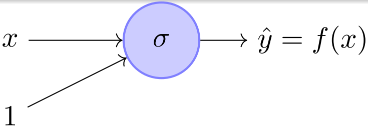

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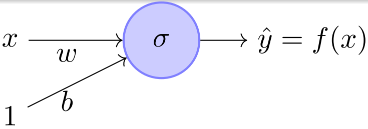

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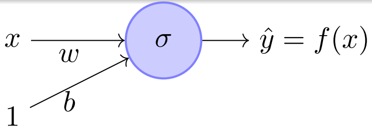
- Further to be consistent with the literature, from now on, we will refer to w_0 as b (bias)



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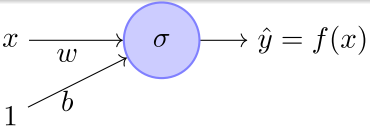
- For ease of explanation, we will consider a very simplified version of the model having just 1 input
- Further to be consistent with the literature, from now on, we will refer to w_0 as b (bias)
- Lastly, instead of considering the problem of predicting like/dislike, we will assume that we want to predict $criticsRating(y)$ given $imdbRating(x)$ (for no particular reason)



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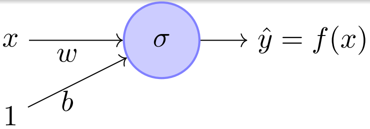
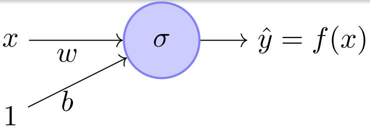


$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

Input for training

$\{x_i, y_i\}_{i=1}^{N} \rightarrow N$ pairs of (x, y)



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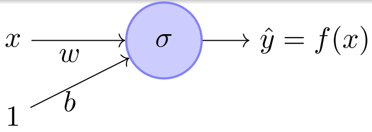
Find w and b such that:

$$\underset{w,b}{\text{minimize}}\, \mathscr{L}(w,b) = \sum_{i=1}^{N} (y_i - f(x_i))^2$$



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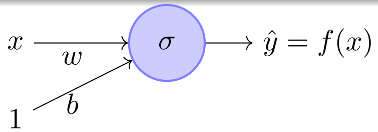

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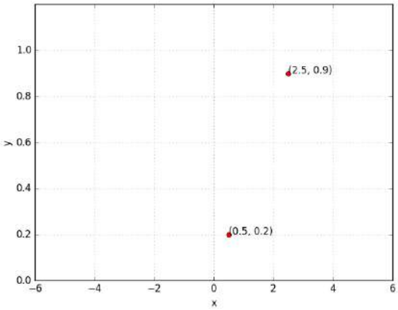
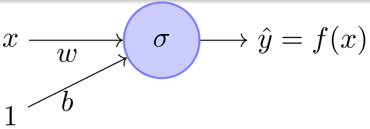


What does it mean to train the network?

- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$



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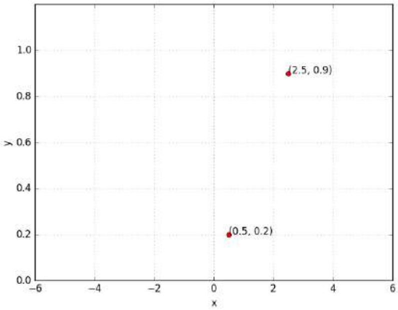

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- Suppose we train the network with $(x, y) = (0.5, 0.2)$ and $(2.5, 0.9)$
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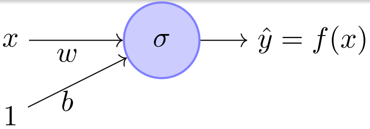

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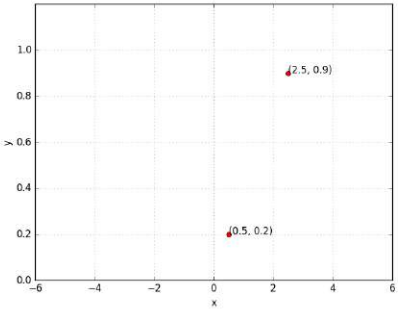

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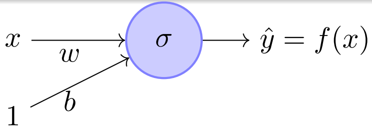

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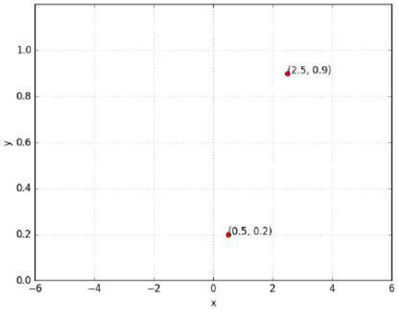

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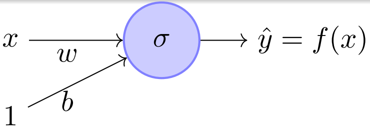
In other words...

- We hope to find a sigmoid function such that $(0.5, 0.2)$ and $(2.5, 0.9)$ lie on this sigmoid



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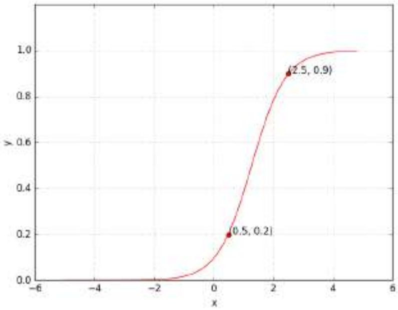

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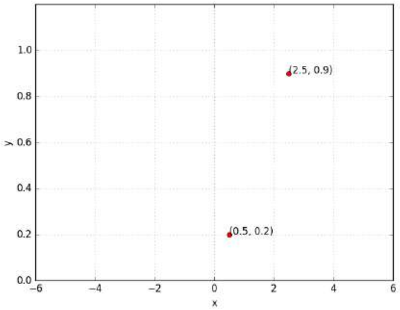
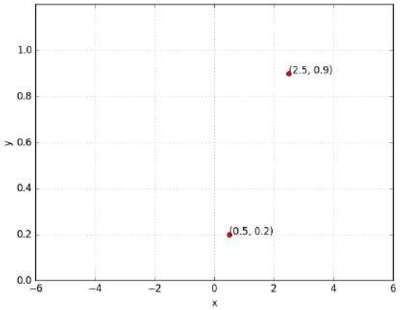
Let us see this in more detail....



$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

- Can we try to find such a w^*, b^* manually



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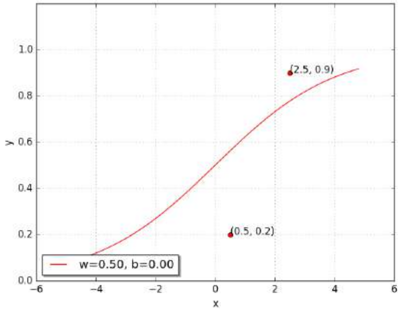
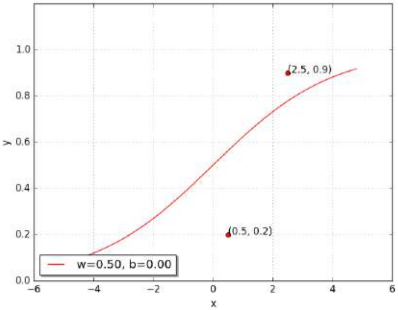


$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)



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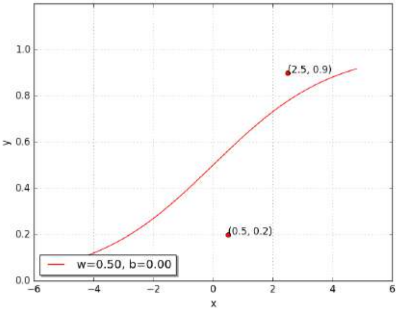
- Clearly not good, but how bad is it ?



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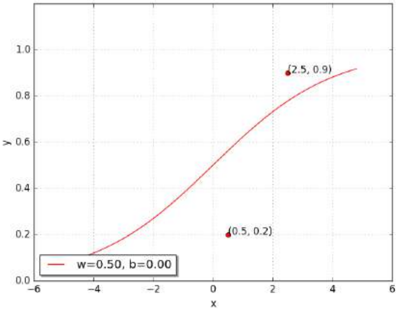
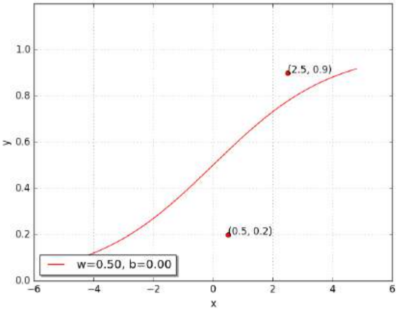
- Can we try to find such a w^*, b^* manually
- Let us try a random guess.. (say, $w = 0.5, b = 0$)
- Clearly not good, but how bad is it ?
- Let us revisit $\mathscr{L}(w, b)$ to see how bad it is ...



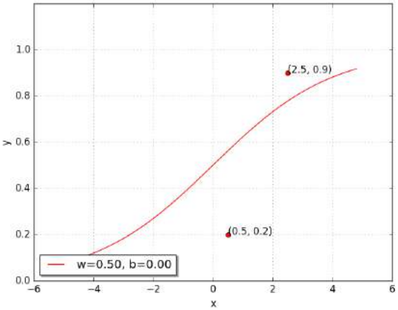
$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$



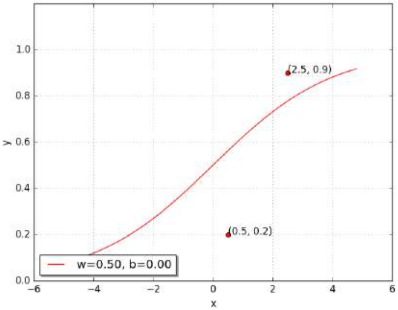
$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$
$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$



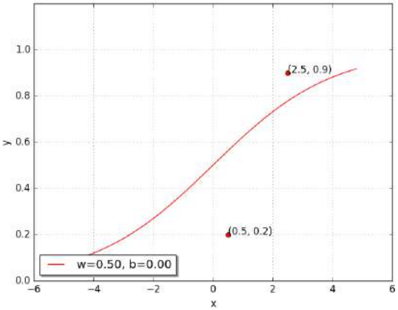
$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$
$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$
$$= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2$$

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$



$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

$$\begin{aligned}
\mathscr{L}(w,b) &= \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2 \\
&= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2 \\
&= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2 \\
&= 0.073
\end{aligned}$$

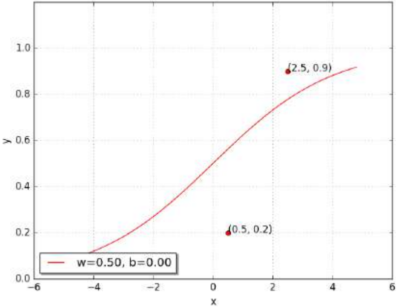


$$\mathscr{L}(w,b) = \frac{1}{2} * \sum_{i=1}^{N} (y_i - f(x_i))^2$$
$$= \frac{1}{2} * (y_1 - f(x_1))^2 + (y_2 - f(x_2))^2$$
$$= \frac{1}{2} * (0.9 - f(2.5))^2 + (0.2 - f(0.5))^2$$
$$= 0.073$$

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

We want $\mathscr{L}(w,b)$ to be as close to 0 as possible

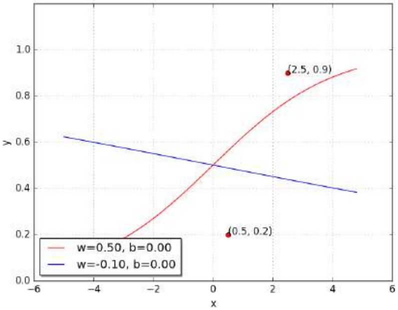
Let us try some other values of w, b



w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Let us try some other values of w, b

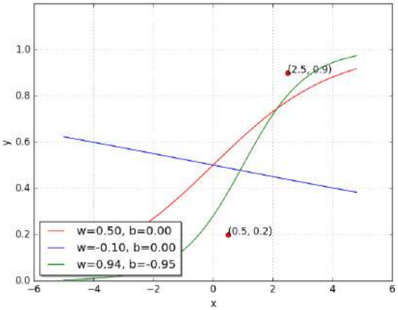


w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Oops!! this made things even worse...

Let us try some other values of w, b

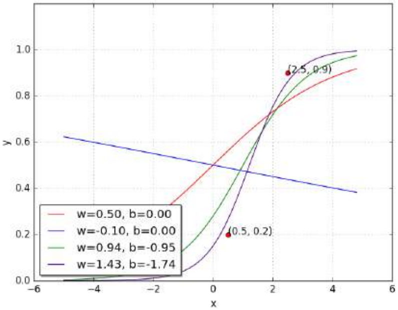


w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214

$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$$

Perhaps it would help to push w and b in the other direction...

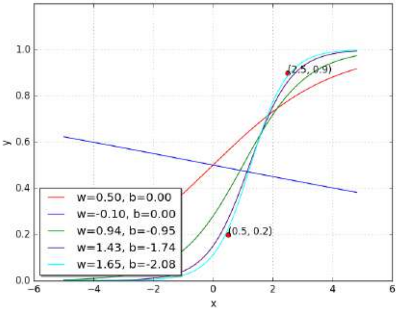
Let us try some other values of w, b



w	b	$\mathscr{L}(w,b)$
0.50	0.00	0.0730
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Let us keep going in this direction, *i.e.*, increase w and decrease b



Let us try some other values of w, b

w	b	$\mathscr{L}(w,b)$
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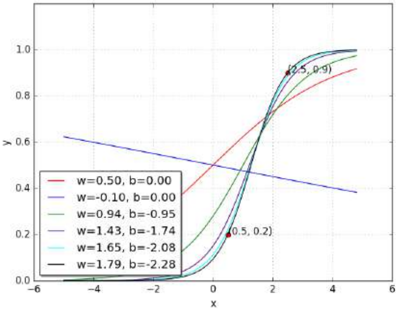
-0.10	0.00	0.1481
0.94	-0.94	0.0214
1.42	-1.73	0.0028
1.65	-2.08	0.0003

$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

Let us keep going in this direction, *i.e.*, increase w and decrease b

Let us try some other values of w, b



w	b	$\mathscr{L}(w,b)$
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-0.10	0.00	0.1481
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1.42	-1.73	0.0028
1.65	-2.08	0.0003
1.78	-2.27	0.0000

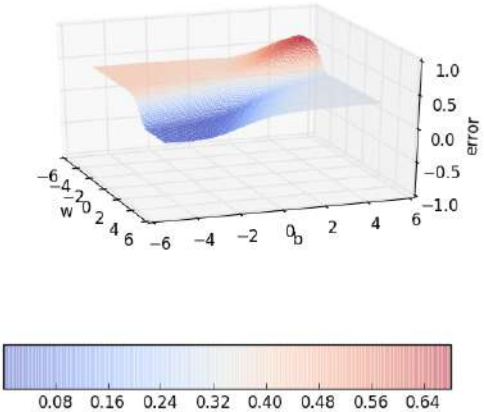
$$\sigma(x) = \frac{1}{1 + e^{-(wx+b)}}$$

With some guess work and intuition we were able to find the right values for w and b

Let us look at something better than our "guess work" algorithm....

- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w,b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w,b)$ is minimum

Random search on error surface



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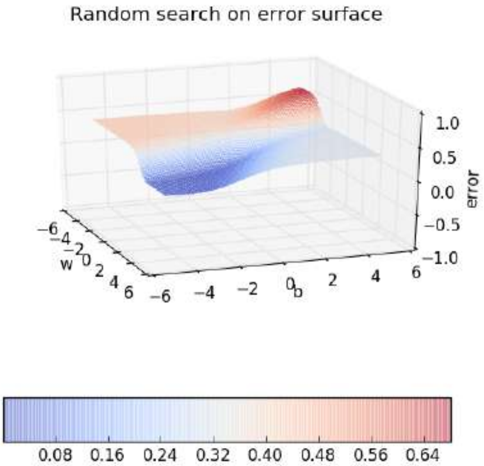
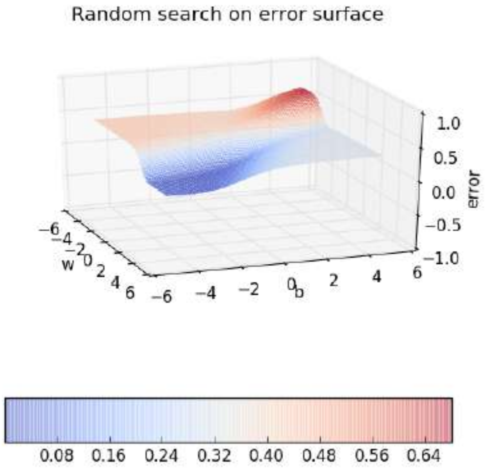
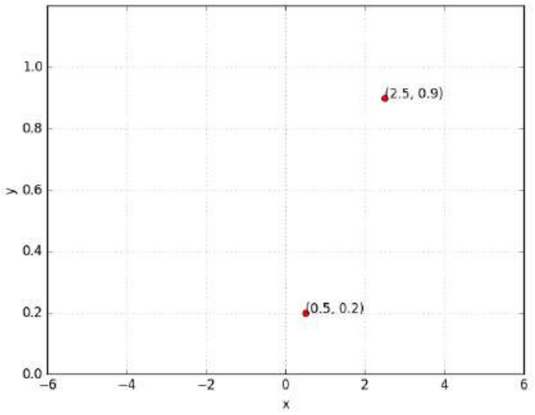


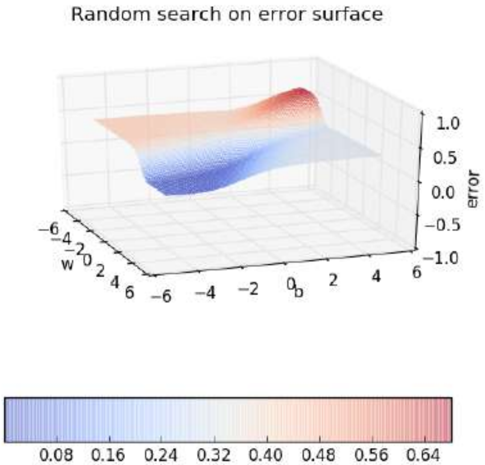
- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!

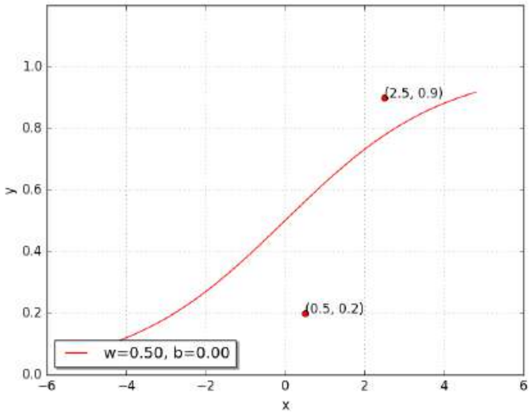


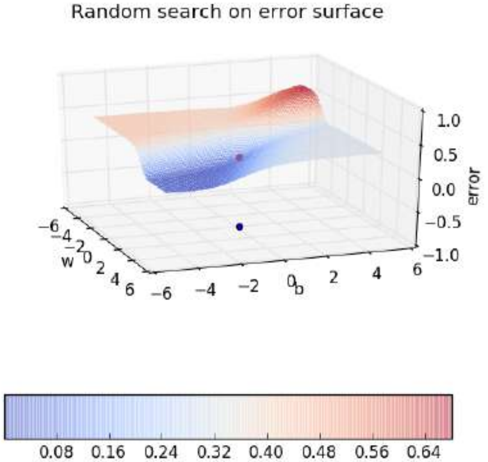
- Since we have only 2 points and 2 parameters $(w,\ b)$ we can easily plot $\mathscr{L}(w, b)$ for different values of $(w,\ b)$ and pick the one where $\mathscr{L}(w, b)$ is minimum
- But of course this becomes intractable once you have many more data points and many more parameters !!
- Further, even here we have plotted the error surface only for a small range of $(w,\ b)$ [from $(-6, 6)$ and not from $(-\inf, \inf)$]

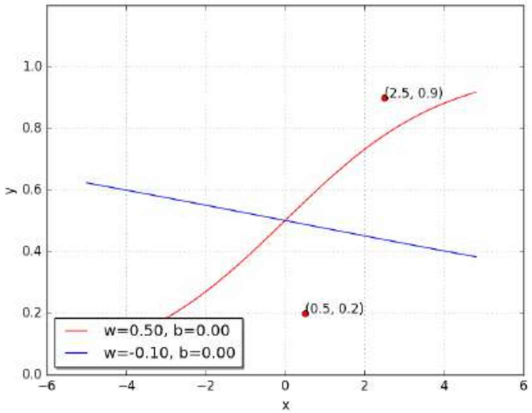
Let us look at the geometric interpretation of our "guess work" algorithm in terms of this error surface

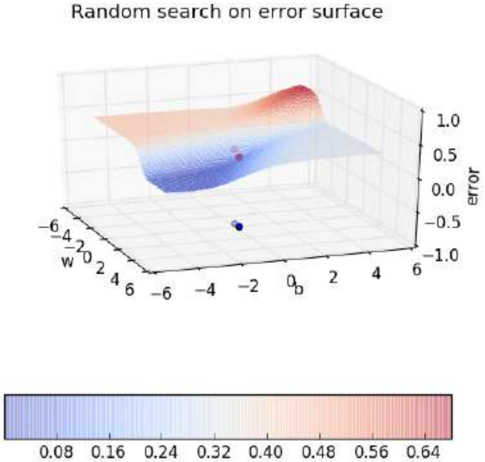

(2.5, 0.9)
(0.5, 0.2)
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

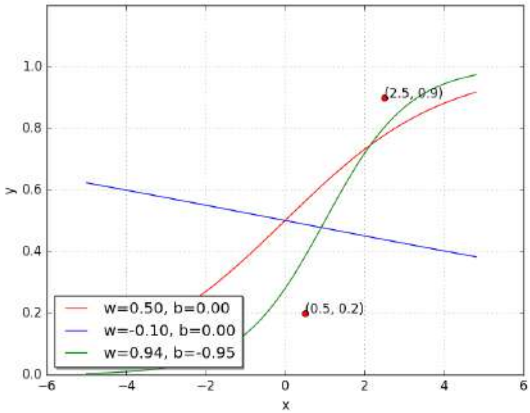

Random search on error surface
error
w
b
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
y
x

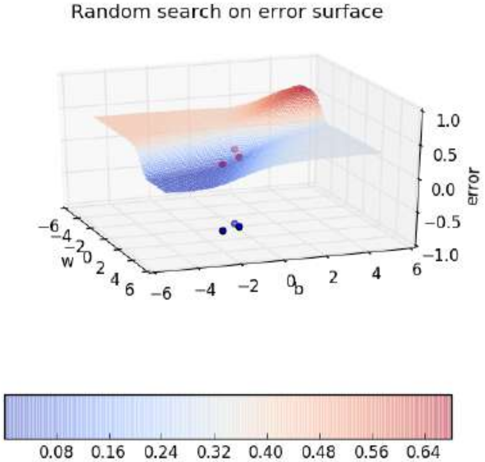

Random search on error surface
w
b
error


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
y
x
1.0
0.8
0.6
0.4
0.2
0.0
−6
−4
−2
0
2
4
6

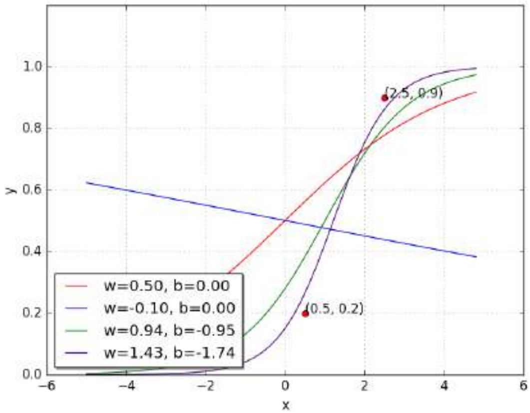

Random search on error surface
error
w
b
1.0
0.5
0.0
−0.5
−1.0
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64

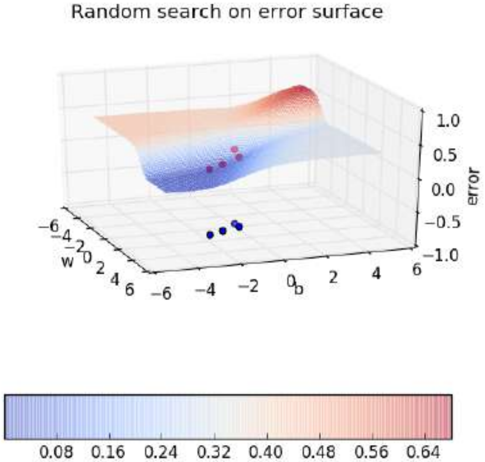


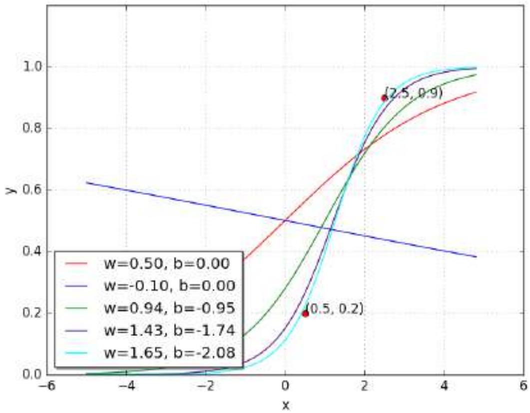
(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
x
y



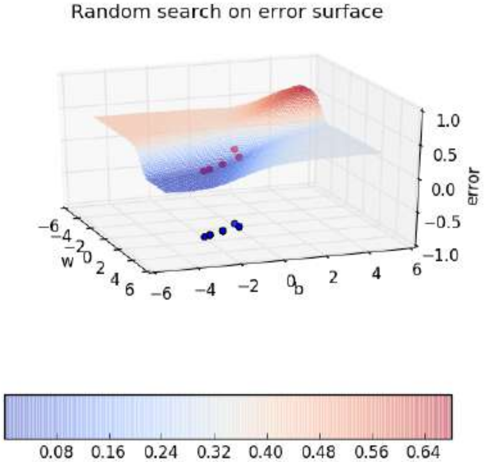
Random search on error surface
error
w
b


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
y
x

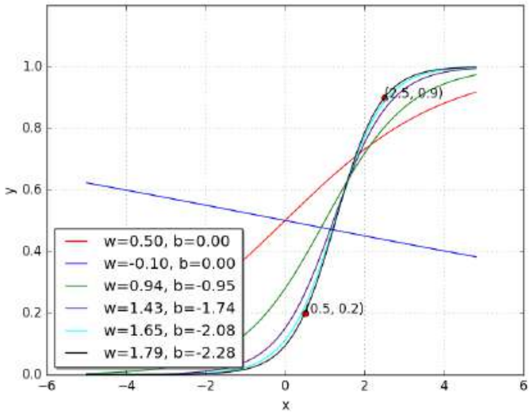

Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

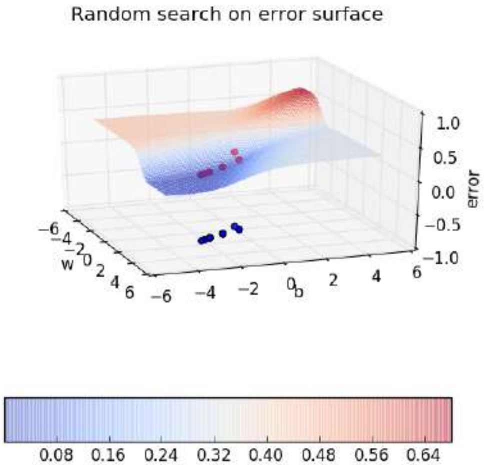


(2.5, 0.9)
(0.5, 0.2)
w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
w=1.65, b=-2.08
y
x



Random search on error surface
error
w
b
0.08
0.16
0.24
0.32
0.40
0.48
0.56
0.64


w=0.50, b=0.00
w=-0.10, b=0.00
w=0.94, b=-0.95
w=1.43, b=-1.74
w=1.65, b=-2.08
w=1.79, b=-2.28
(2.5, 0.9)
(0.5, 0.2)
x
y


Random search on error surface
error
w
b
0.08 0.16 0.24 0.32 0.40 0.48 0.56 0.64

Module 3.4: Learning Parameters : Gradient Descent

Now let us see if there is a more efficient and principled way of doing this

Goal

Find a better way of traversing the error surface so that we can reach the minimum value quickly without resorting to brute force search!

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b

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$$\theta = [w, b]$$

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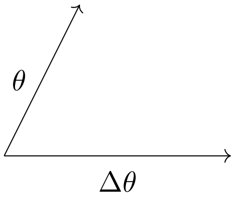
change in the values of w, b



vector of parameters, say, randomly initialized

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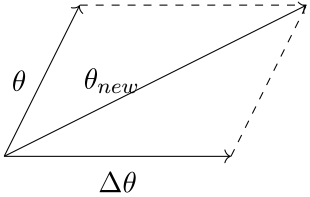
change in the values of w, b

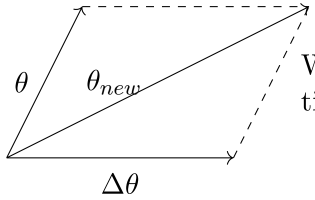


vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b



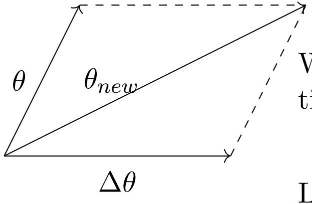
We moved in the direction of $\Delta\theta$

vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b



We moved in the direction of $\Delta\theta$

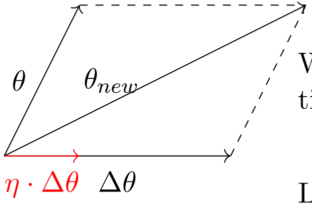
Let us be a bit conservative: move only by a small amount η

vector of parameters, say, randomly initialized

$\theta = [w, b]$

$\Delta\theta = [\Delta w, \Delta b]$

change in the values of w, b



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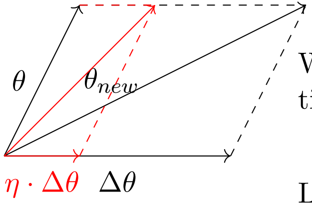


We moved in the direction of $\Delta\theta$

Let us be a bit conservative: move only by a small amount η

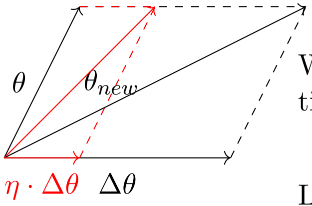
vector of parameters, say, randomly initialized

$$\theta = [w, b]$$

$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b

$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$



We moved in the direction of $\Delta\theta$

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vector of parameters, say, randomly initialized

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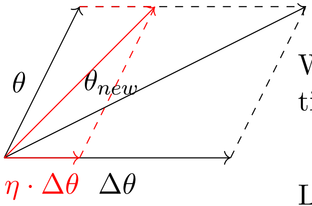

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$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

Question: What is the right $\Delta\theta$ to use ?

vector of parameters, say, randomly initialized

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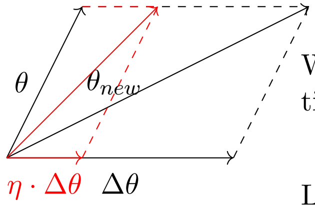
$$\Delta\theta = [\Delta w, \Delta b]$$

change in the values of w, b



We moved in the direction of $\Delta\theta$

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$$\theta_{new} = \theta + \eta \cdot \Delta\theta$$

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The answer comes from Taylor series

For ease of notation, let $\Delta\theta = u$, then from Taylor series, we have,

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\mathscr{L}(\theta + \eta u) &= \mathscr{L}(\theta) + \eta * u^T \nabla_\theta \mathscr{L}(\theta) + \frac{\eta^2}{2!} * u^T \nabla^2 \mathscr{L}(\theta) u + \frac{\eta^3}{3!} * \ldots + \frac{\eta^4}{4!} * \ldots \\
&= \mathscr{L}(\theta) + \eta * u^T \nabla_\theta \mathscr{L}(\theta) \ [\eta \textit{ is typically small, so } \eta^2, \eta^3, .. \rightarrow 0]
\end{aligned}$$

Note that the move (ηu) would be favorable only if,

$$\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) < 0 \ [\textit{i.e., if the new loss is less than the previous loss}]$$

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Note that the move (ηu) would be favorable only if,

$$\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) < 0 \ [\textit{i.e., if the new loss is less than the previous loss}]$$

This implies,

$$u^T \nabla_\theta \mathscr{L}(\theta) < 0$$

Okay, so we have,

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But, what is the range of $u^T \nabla_\theta \mathscr{L}(\theta)$?

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Let β be the angle between u and $\nabla_\theta \mathscr{L}(\theta)$, then we know that,

$$-1 \leq cos(\beta) = \frac{u^T \nabla_\theta \mathscr{L}(\theta)}{||u|| * ||\nabla_\theta \mathscr{L}(\theta)||} \leq 1$$

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$$-1 \leq cos(\beta) = \frac{u^T \nabla_\theta \mathscr{L}(\theta)}{||u|| * ||\nabla_\theta \mathscr{L}(\theta)||} \leq 1$$

multiply throughout by $k = ||u|| * ||\nabla_\theta \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla_\theta \mathscr{L}(\theta) \leq k$$

Okay, so we have,

$$u^T \nabla_\theta \mathscr{L}(\theta) < 0$$

But, what is the range of $u^T \nabla_\theta \mathscr{L}(\theta)$? Let us see....
Let β be the angle between u and $\nabla_\theta \mathscr{L}(\theta)$, then we know that,

$$-1 \leq cos(\beta) = \frac{u^T \nabla_\theta \mathscr{L}(\theta)}{||u|| * ||\nabla_\theta \mathscr{L}(\theta)||} \leq 1$$

multiply throughout by $k = ||u|| * ||\nabla_\theta \mathscr{L}(\theta)||$

$$-k \leq k * cos(\beta) = u^T \nabla_\theta \mathscr{L}(\theta) \leq k$$

Thus, $\mathscr{L}(\theta + \eta u) - \mathscr{L}(\theta) = u^T \nabla_\theta \mathscr{L}(\theta) = k * cos(\beta)$ will be most negative when $cos(\beta) = -1$ *i.e.*, when β is 180°

Gradient Descent Rule

- The direction u that we intend to move in should be at $180°$ w.r.t. the gradient

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- The direction u that we intend to move in should be at $180°$ w.r.t. the gradient
- In other words, move in a direction opposite to the gradient

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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$

$$b_{t+1} = b_t - \eta \nabla b_t$$

$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t,\, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t,\, b = b_t}$$

Gradient Descent Rule

- The direction u that we intend to move in should be at $180°$ w.r.t. the gradient
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Parameter Update Equations

$$w_{t+1} = w_t - \eta \nabla w_t$$
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$$where, \nabla w_t = \frac{\partial \mathscr{L}(w,b)}{\partial w}_{at\ w = w_t, b = b_t}, \nabla b = \frac{\partial \mathscr{L}(w,b)}{\partial b}_{at\ w = w_t, b = b_t}$$

So we now have a more principled way of moving in the w-b plane than our "guess work" algorithm

- Let us create an algorithm from this rule ...

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Algorithm: gradient_descent()

$t \leftarrow 0$;
$max\_iterations \leftarrow 1000$;
while $t < max\_iterations$ **do**
  $w_{t+1} \leftarrow w_t - \eta \nabla w_t$;
  $b_{t+1} \leftarrow b_t - \eta \nabla b_t$;
  $t \leftarrow t + 1$;
end

- Let us create an algorithm from this rule ...

Algorithm: gradient_descent()

$t \leftarrow 0;$

$max\_iterations \leftarrow 1000;$

while $t < max\_iterations$ **do**

$\quad w_{t+1} \leftarrow w_t - \eta \nabla w_t;$

$\quad b_{t+1} \leftarrow b_t - \eta \nabla b_t;$

$\quad t \leftarrow t + 1;$

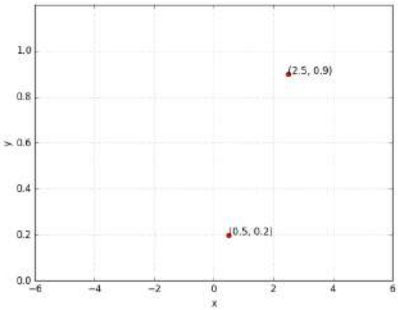
end

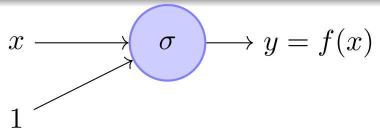
- To see this algorithm in practice let us first derive ∇w and ∇b for our toy neural network

$$x \longrightarrow \sigma \longrightarrow y = f(x)$$

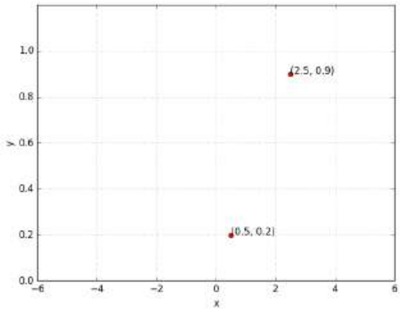
$$1 \nearrow$$

$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$

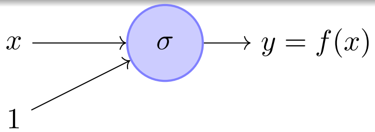




$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$



Let's assume there is only 1 point to fit (x, y)



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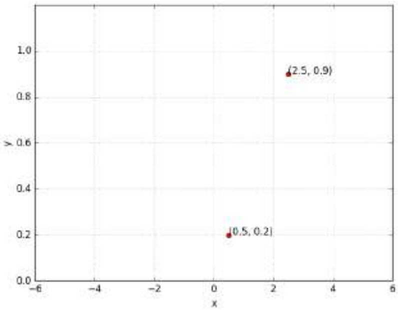
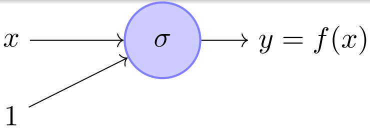
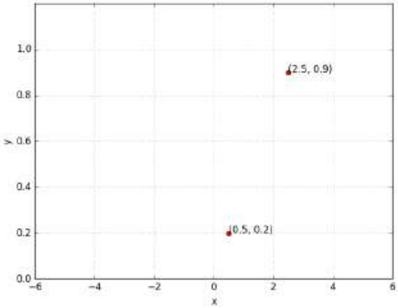


Let's assume there is only 1 point to fit (x, y)

$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$



$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$



Let's assume there is only 1 point to fit (x, y)

$$\mathscr{L}(w, b) = \frac{1}{2} * (f(x) - y)^2$$

$$\nabla w = \frac{\partial \mathscr{L}(w, b)}{\partial w} = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$\nabla w = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$

$$\nabla w = \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2]$$
$$= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)]$$

$$
\begin{aligned}
\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)] \\
&= (f(x) - y) * \frac{\partial}{\partial w}(f(x))
\end{aligned}
$$

$$
\begin{aligned}
\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)] \\
&= (f(x) - y) * \frac{\partial}{\partial w}(f(x)) \\
&= (f(x) - y) * \frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right)
\end{aligned}
$$

$$
\begin{aligned}
\nabla w &= \frac{\partial}{\partial w}[\frac{1}{2} * (f(x) - y)^2] \\
&= \frac{1}{2} * [2 * (f(x) - y) * \frac{\partial}{\partial w}(f(x) - y)] \\
&= (f(x) - y) * \frac{\partial}{\partial w}(f(x)) \\
&= (f(x) - y) * \frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right)
\end{aligned}
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$$\frac{\partial}{\partial w}\left(\frac{1}{1 + e^{-(wx+b)}}\right)$$
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&= {\color{red}(f(x) - y) * f(x) * (1 - f(x)) * x}
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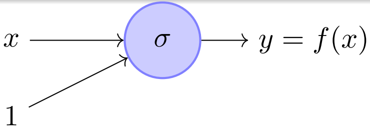
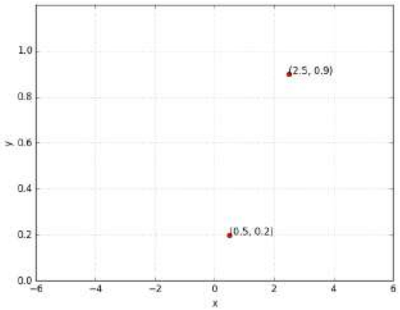
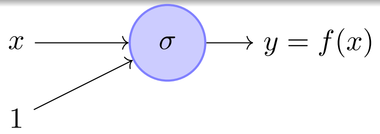


$$f(x) = \frac{1}{1+e^{-(w \cdot x+b)}}$$



So if there is only 1 point (x, y), we have,



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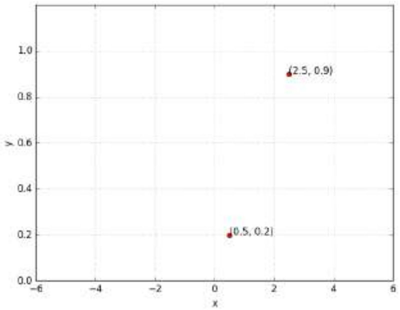

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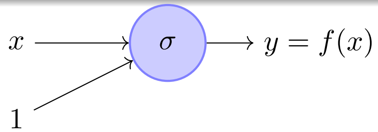
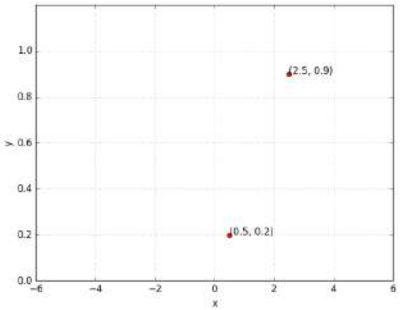
$$\nabla w = (f(x) - y) * f(x) * (1 - f(x)) * x$$



$$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$$



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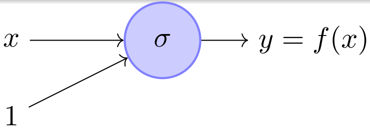
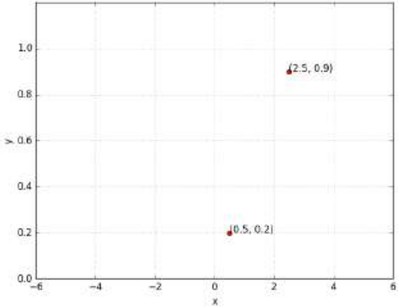
For two points,



$f(x) = \frac{1}{1+e^{-(w \cdot x + b)}}$



So if there is only 1 point (x, y), we have,

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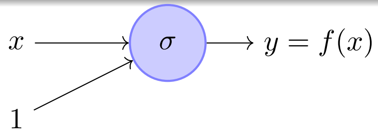
$$\nabla w = \sum_{i=1}^{2} (f(x_i) - y_i) * f(x_i) * (1 - f(x_i)) * x_i$$



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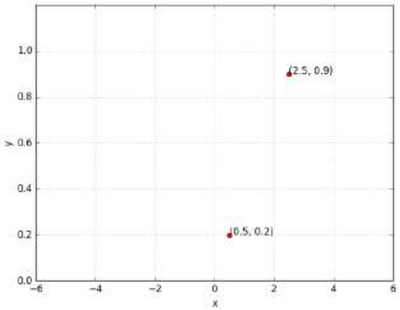

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$$\nabla b = \sum_{i=1}^{2} (f(x_i) - y_i) * f(x_i) * (1 - f(x_i))$$

```
X = [0.5, 2.5]
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```
X = [0.5, 2.5]
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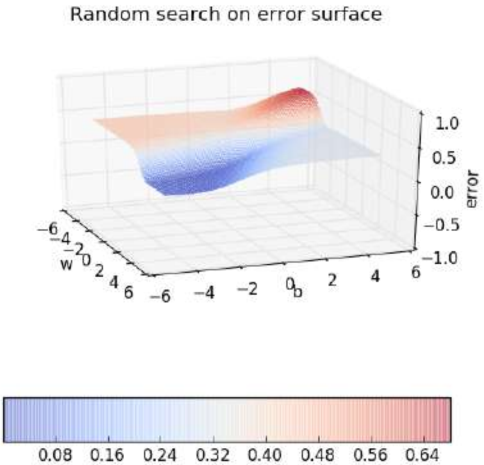

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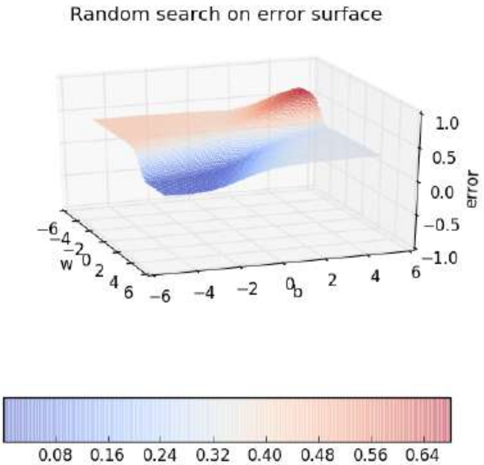

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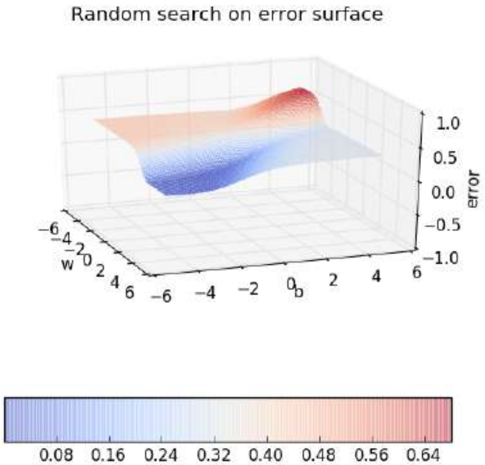

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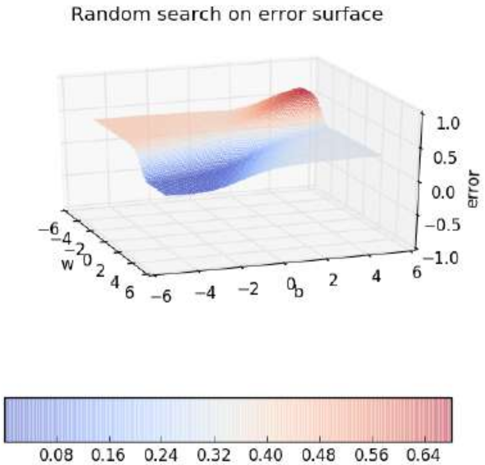

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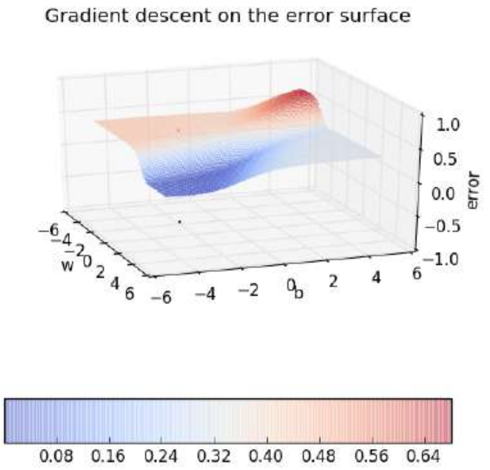

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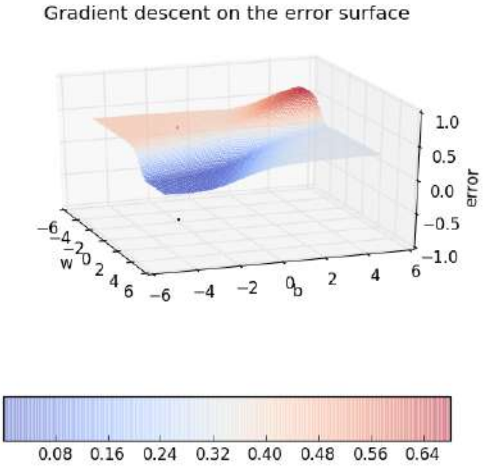

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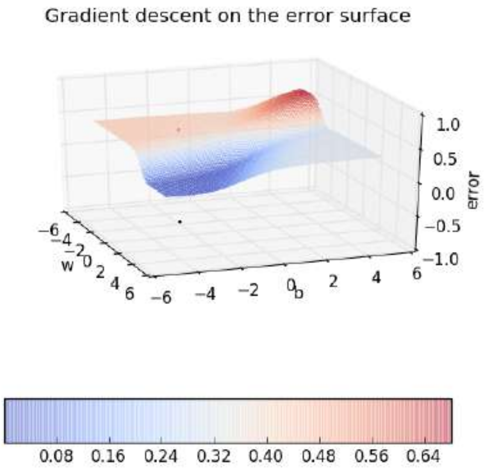

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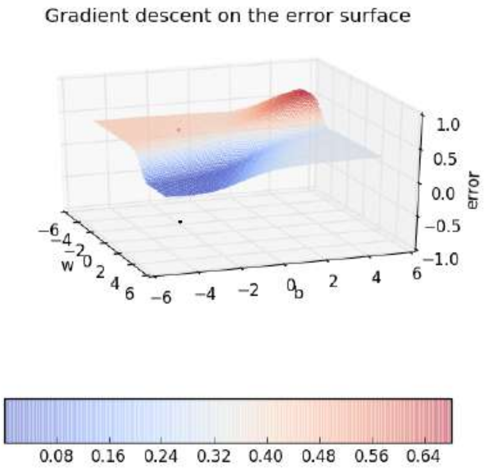

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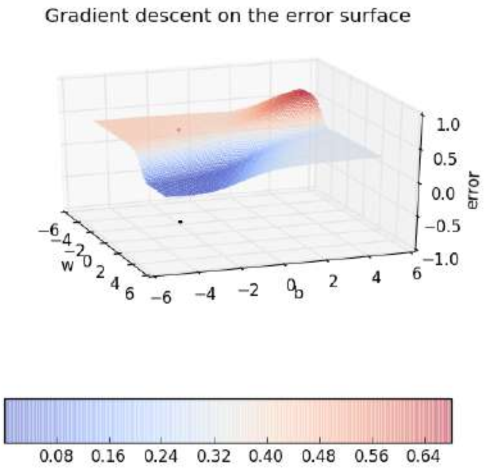

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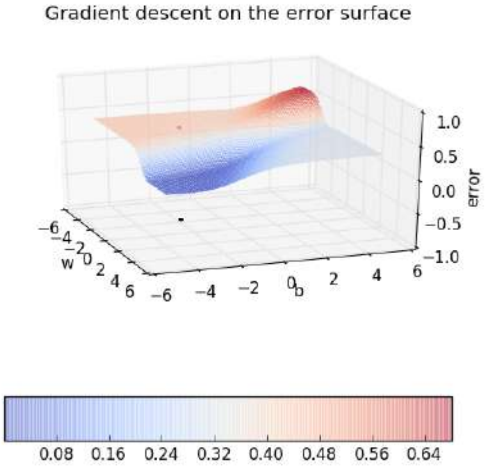

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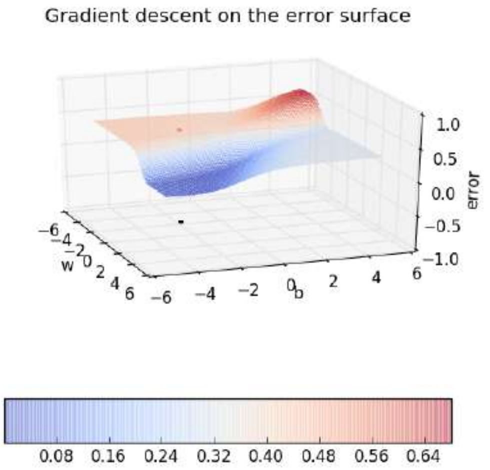

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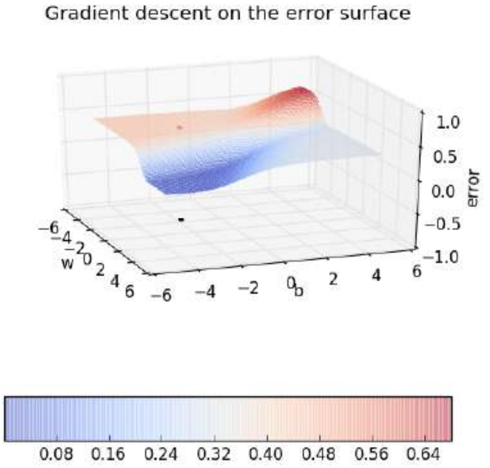

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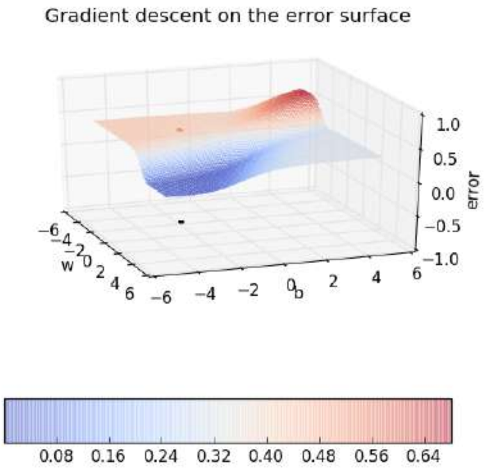

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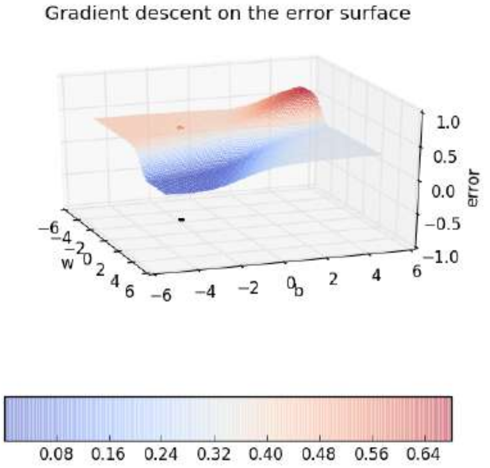

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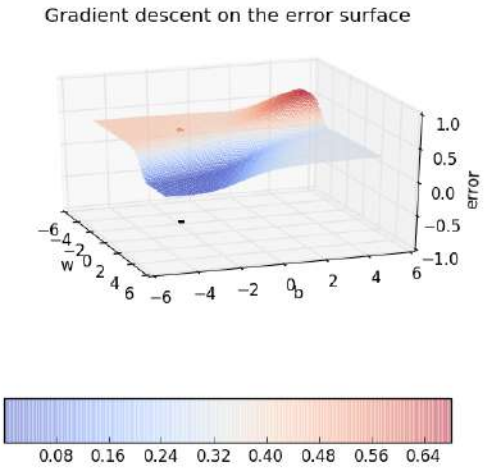

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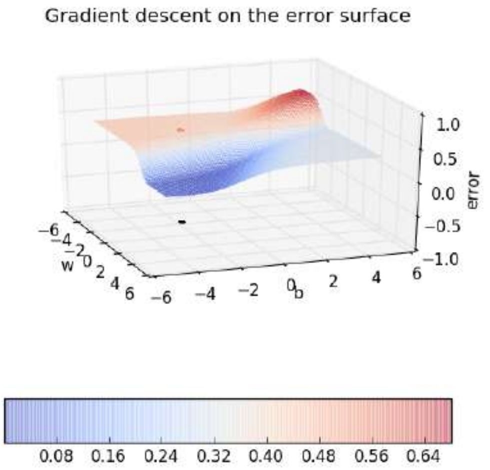

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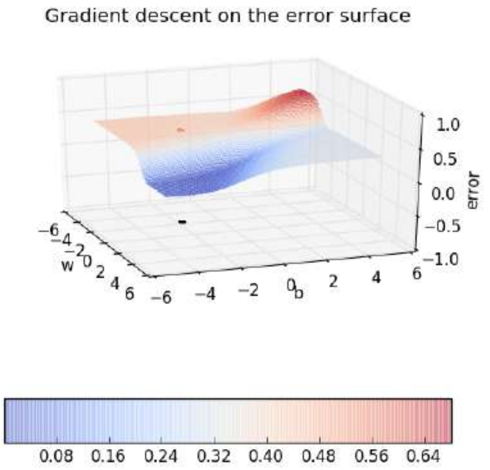

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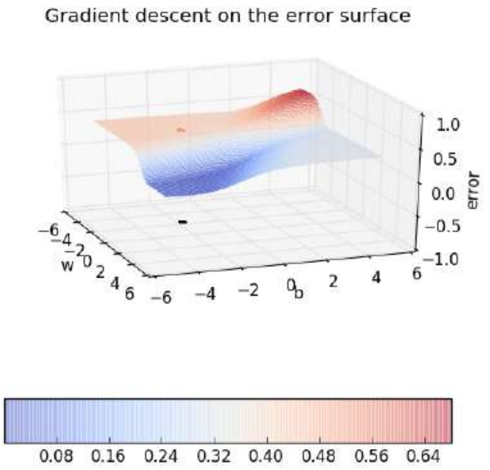

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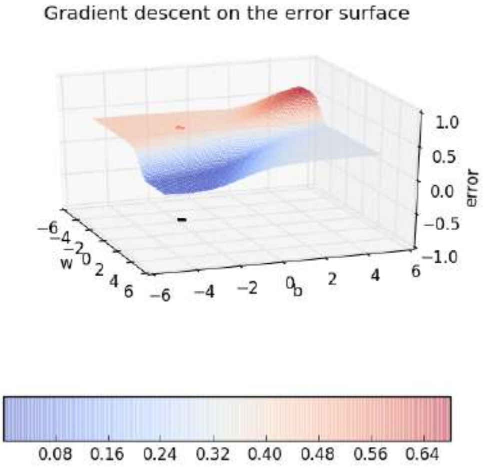

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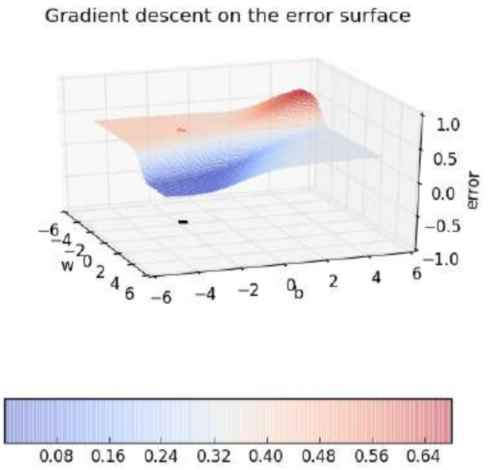

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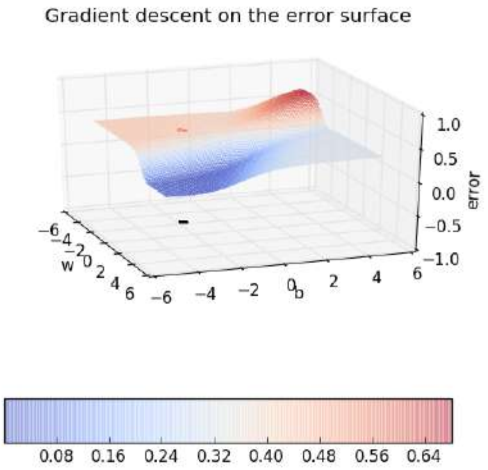

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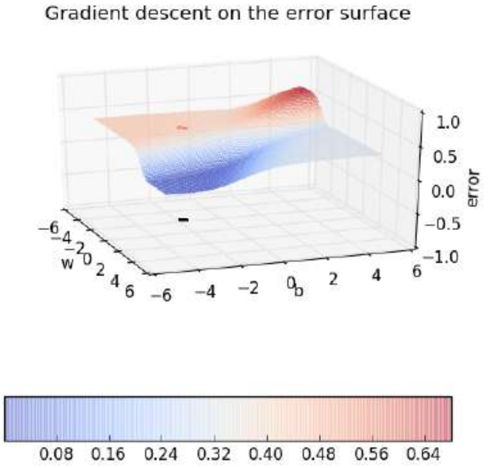

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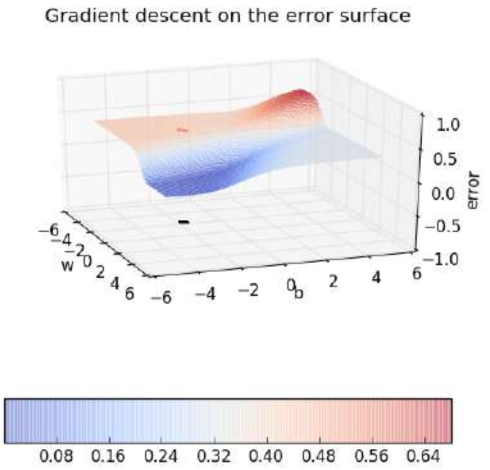

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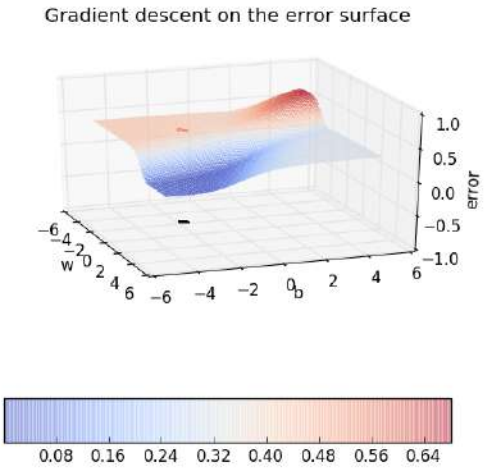

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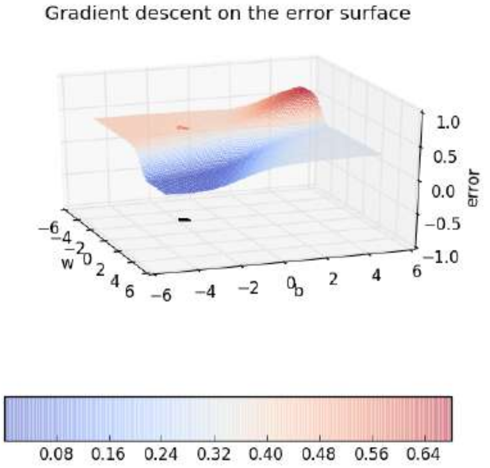

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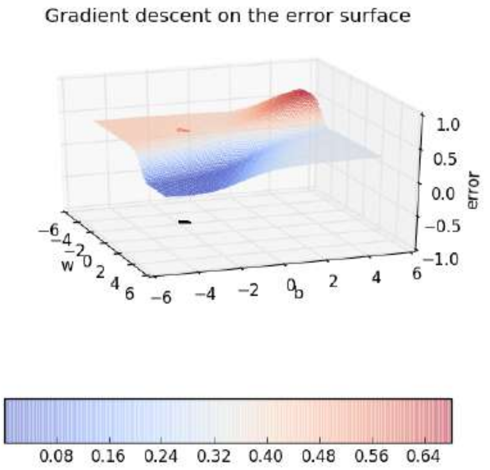

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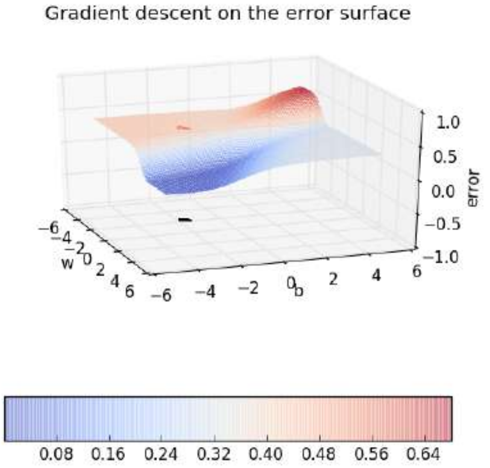

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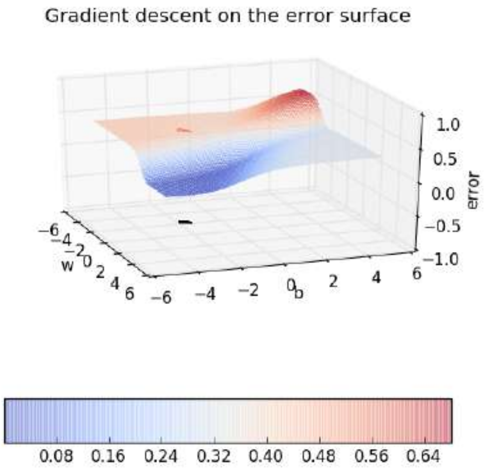

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def f(w,b,x) : #sigmoid with parameters w,b
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def error (w, b) :
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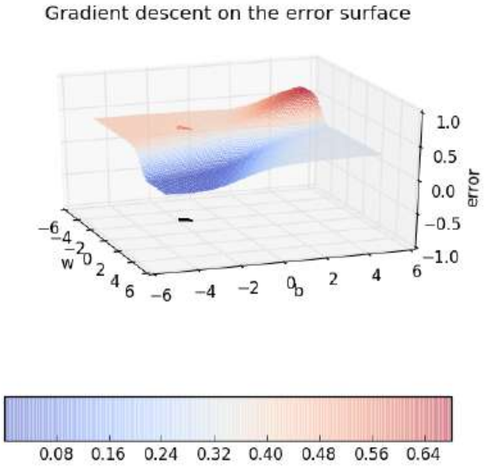

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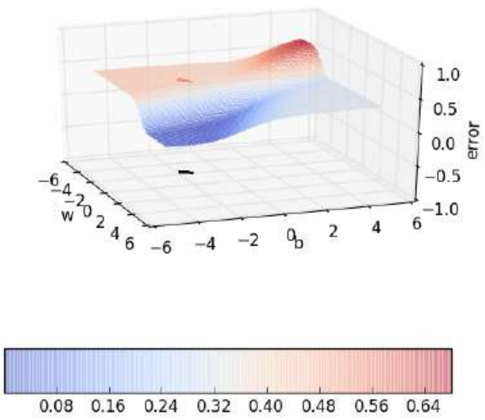
Gradient descent on the error surface



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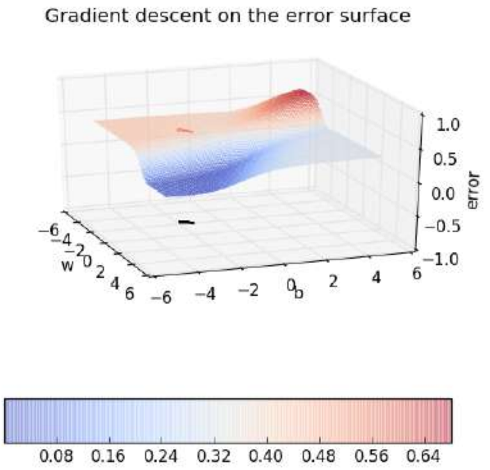

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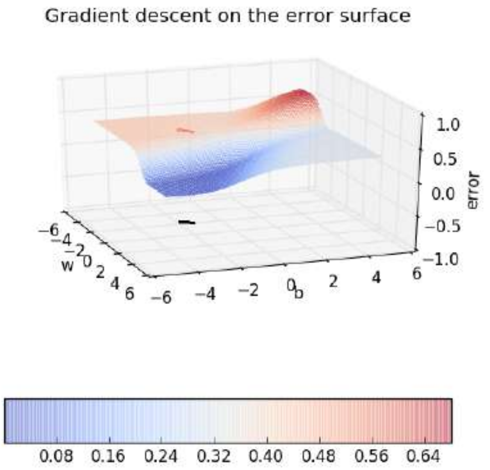

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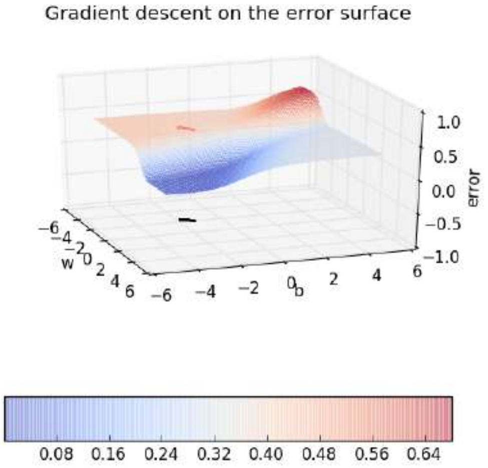

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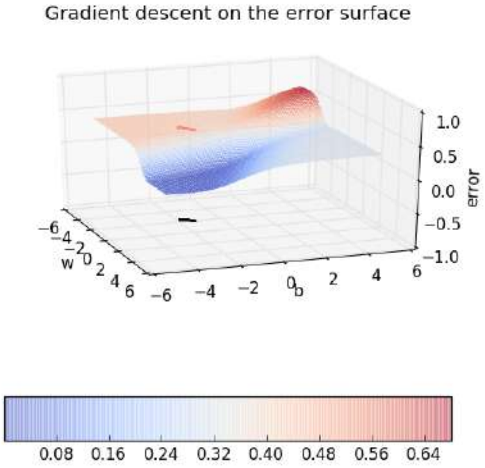

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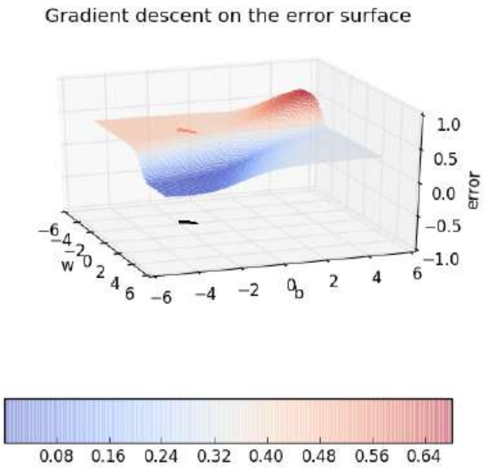

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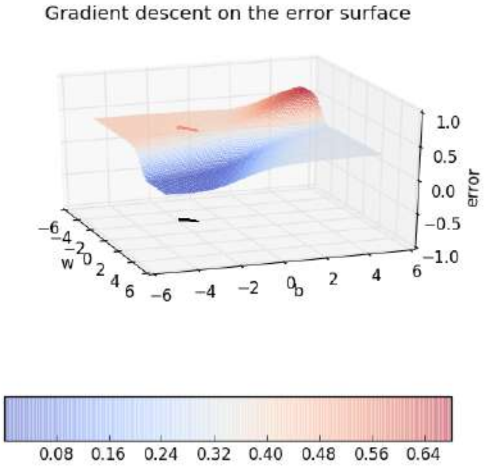

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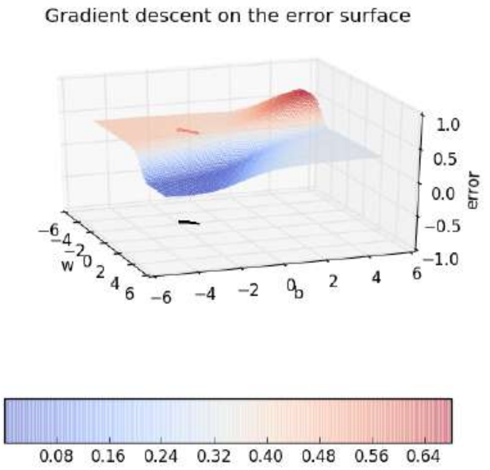

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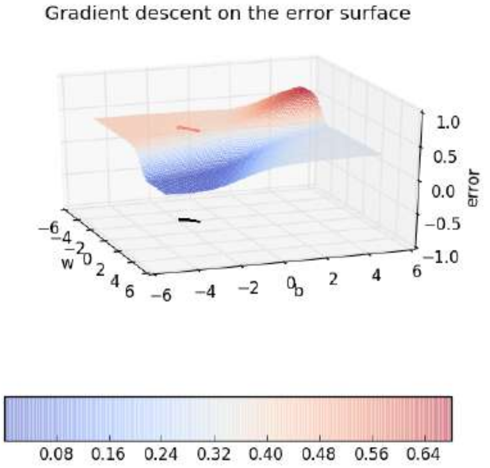

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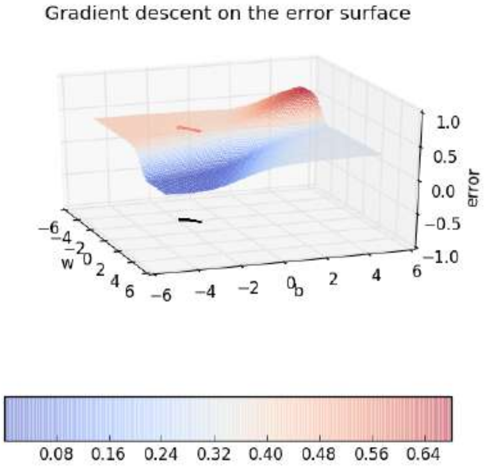

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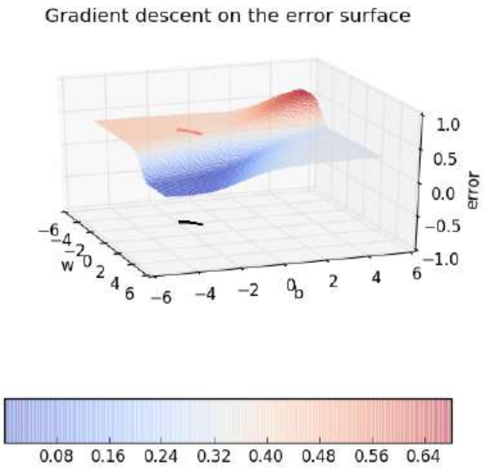

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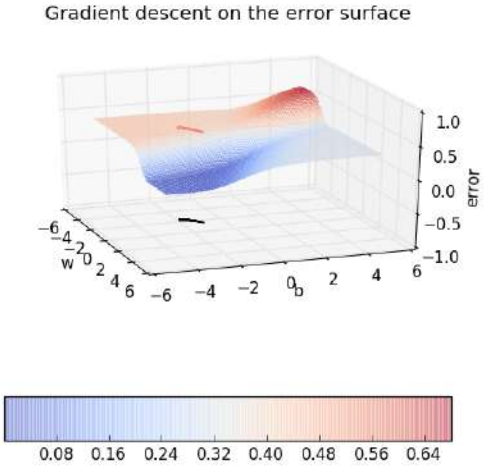

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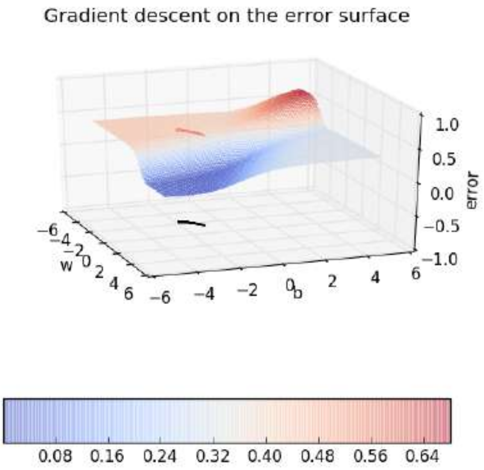

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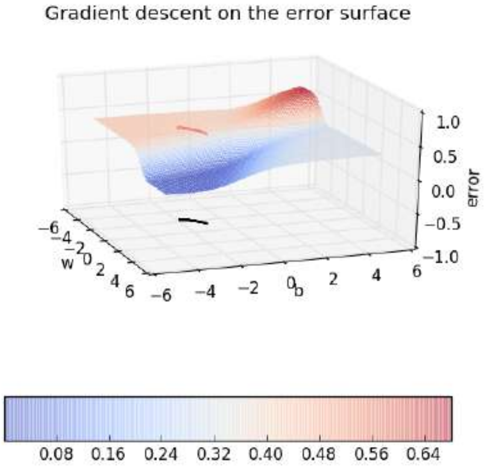

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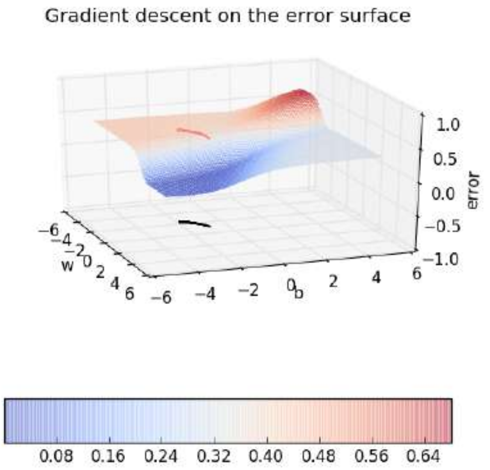

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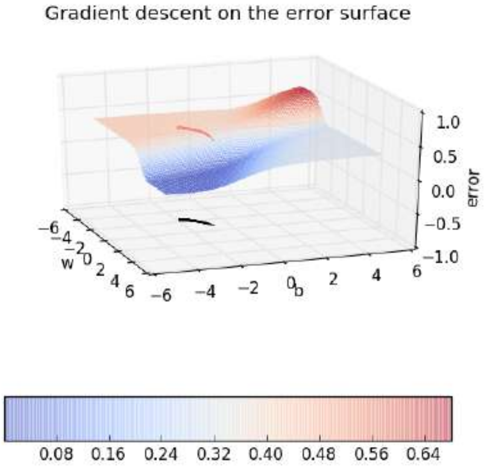

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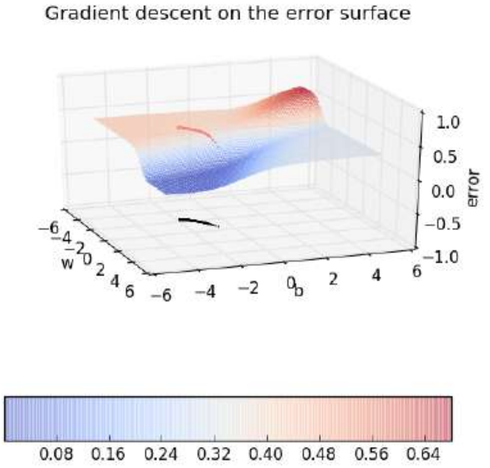

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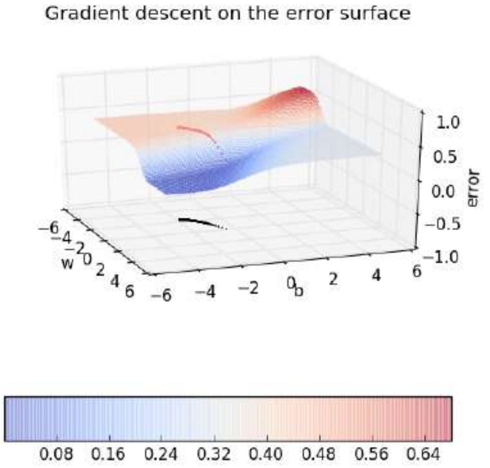

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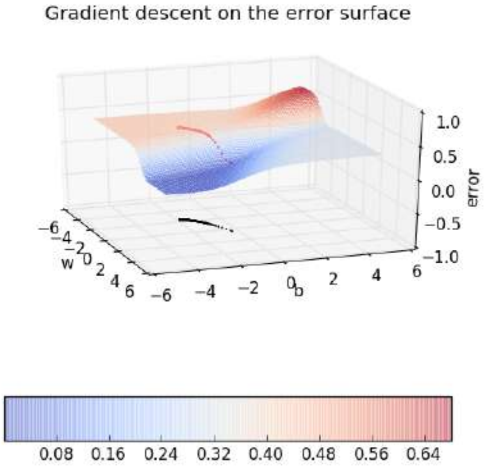

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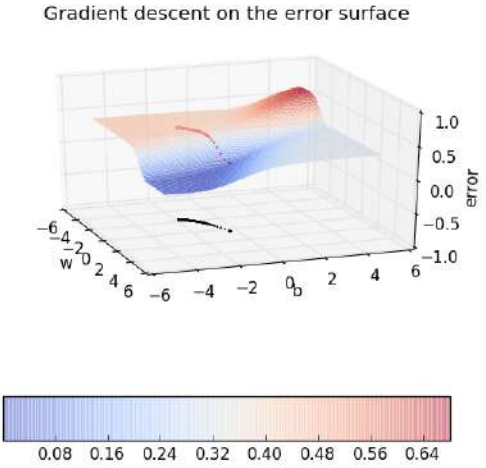

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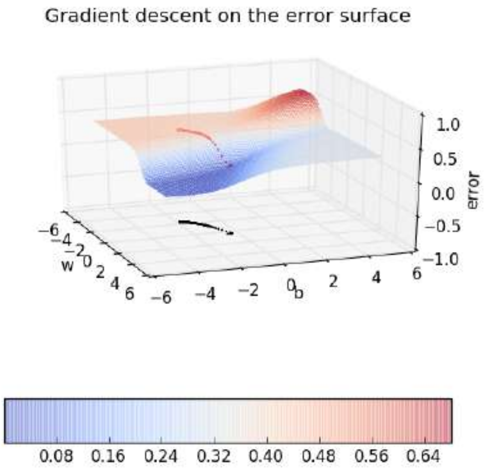

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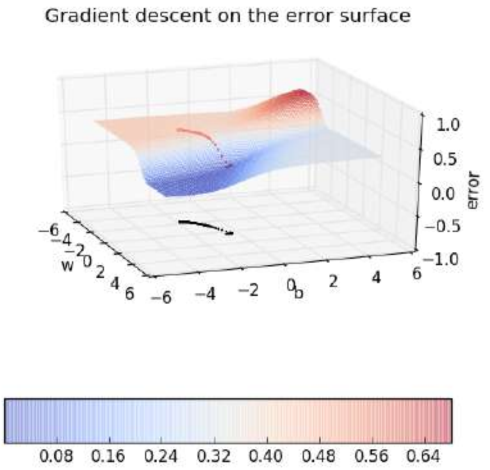

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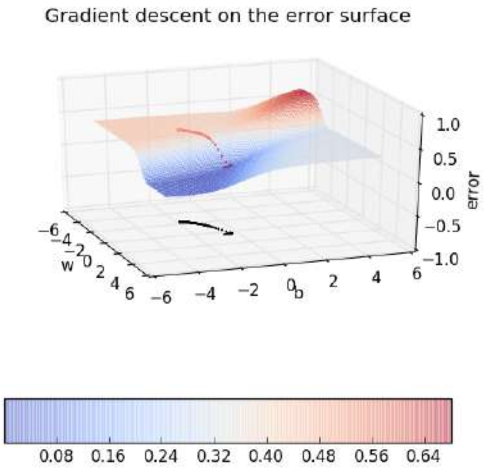

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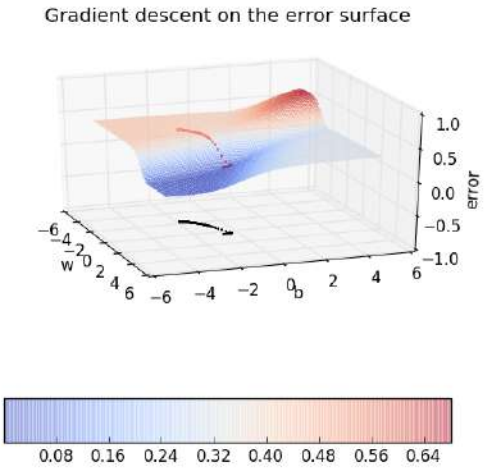

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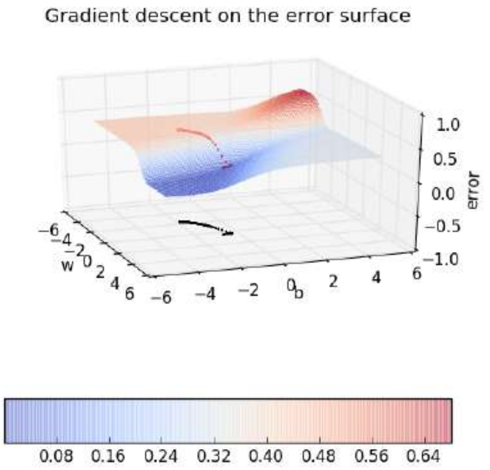

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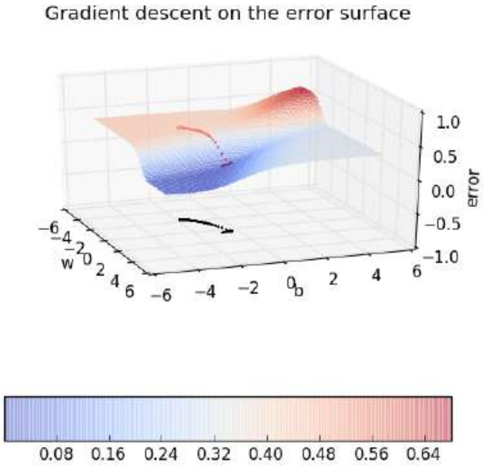

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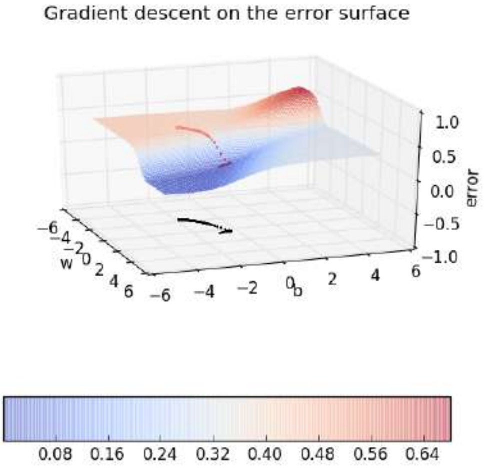

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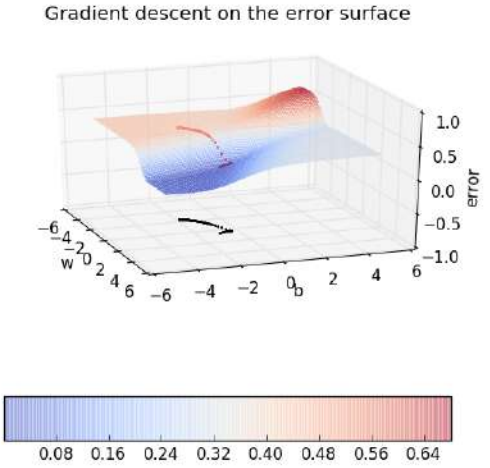

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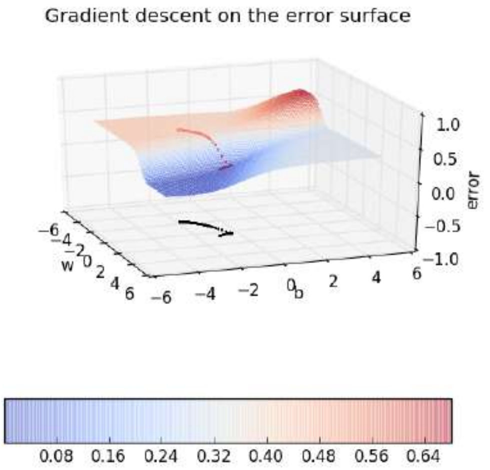

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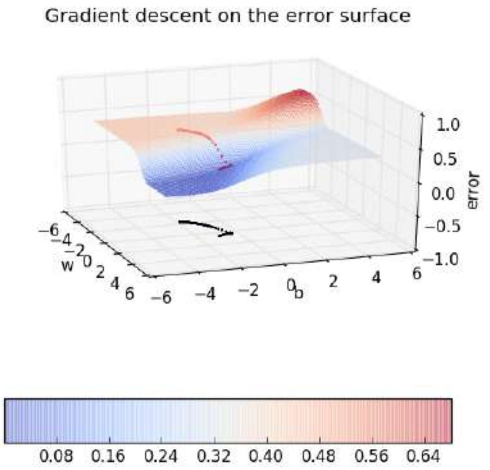

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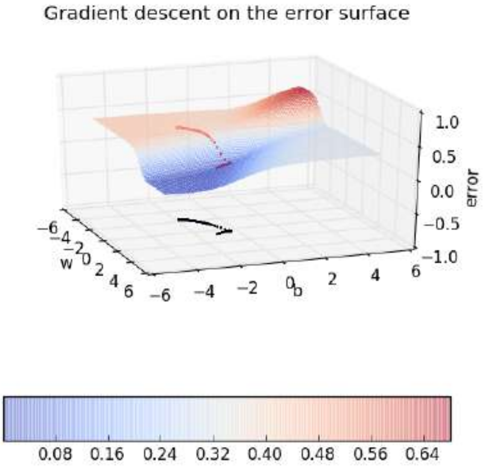

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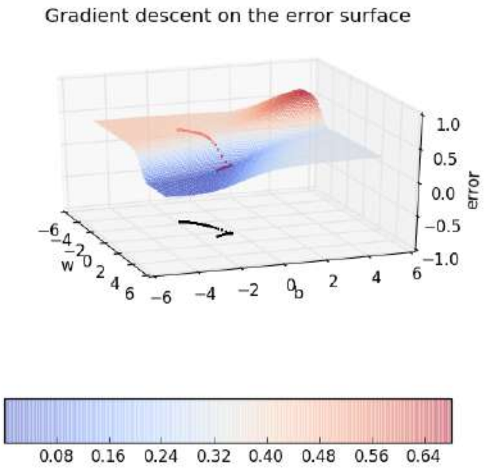

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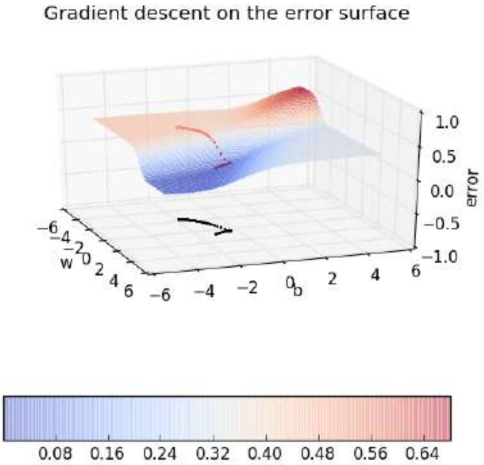

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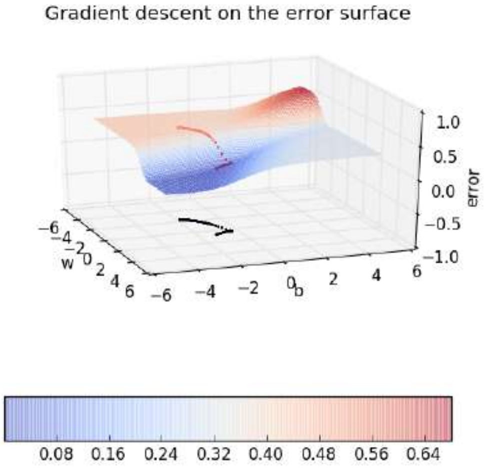

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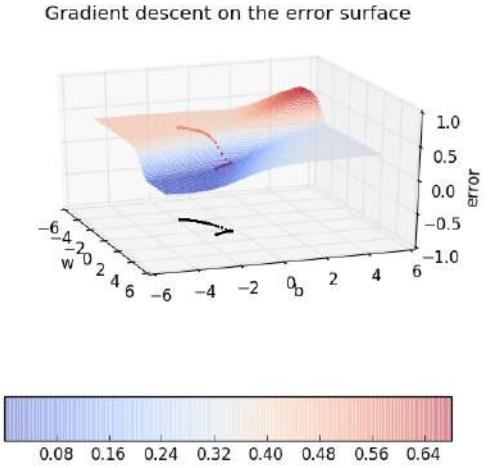

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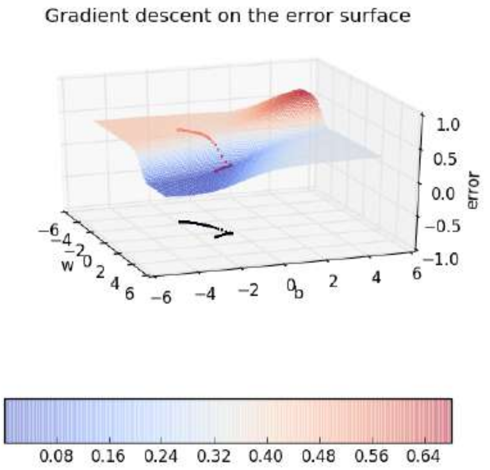

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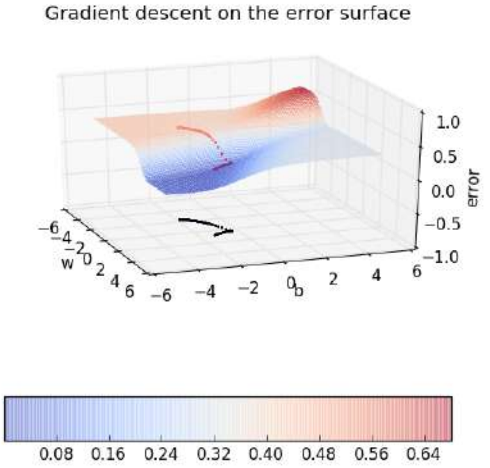

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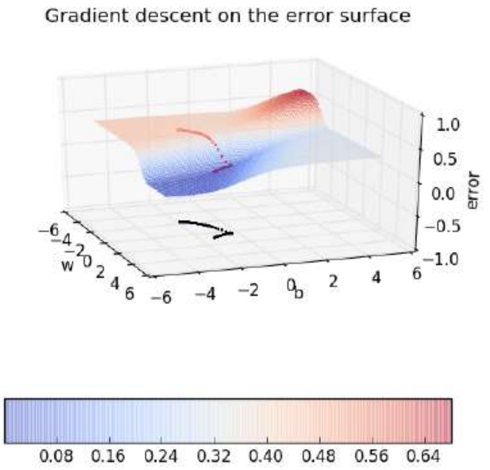

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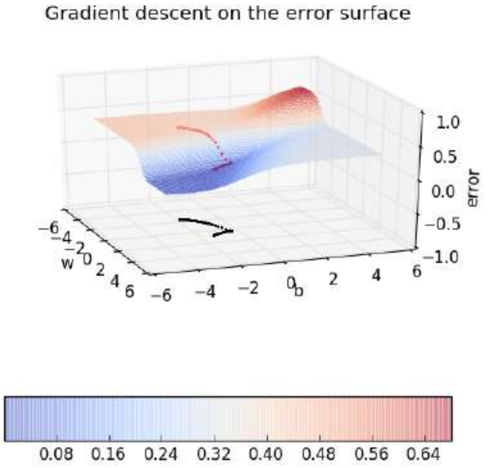

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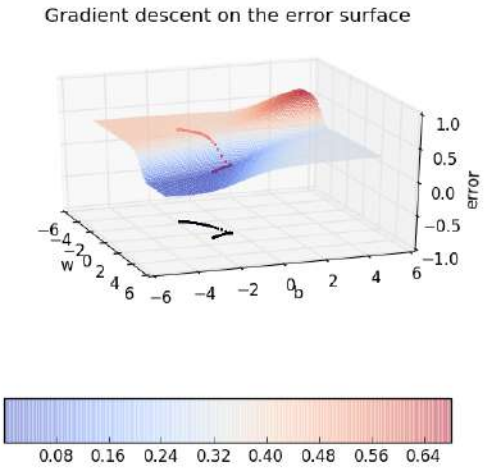

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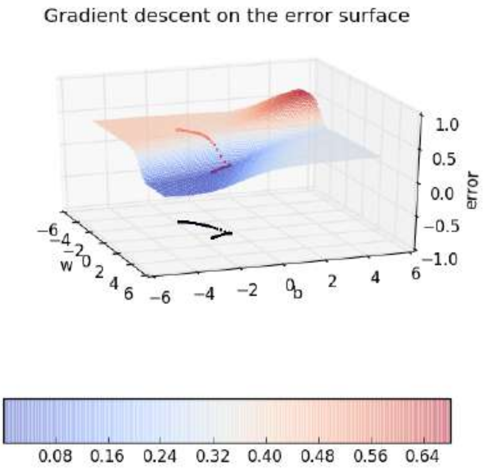

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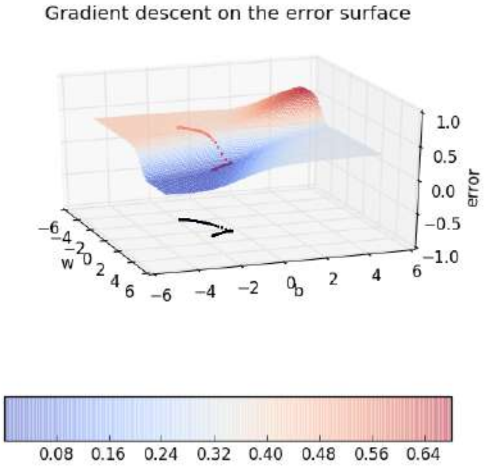

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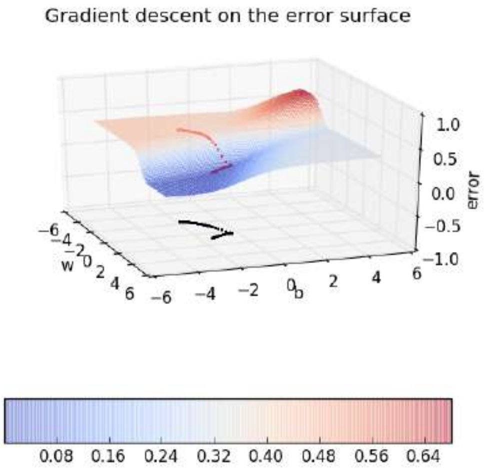

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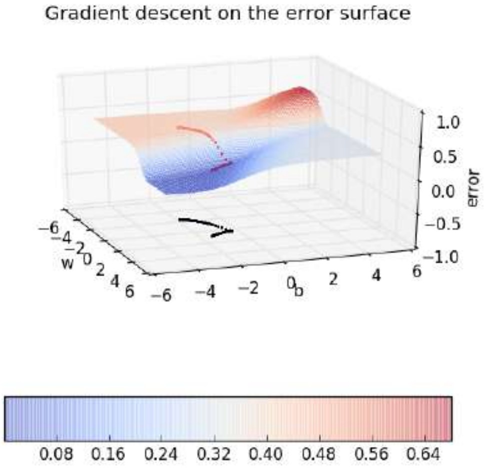

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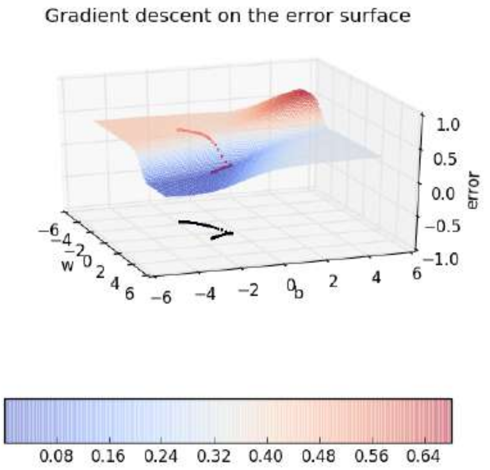

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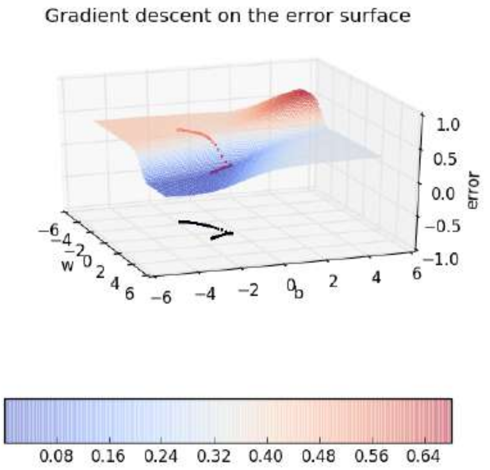

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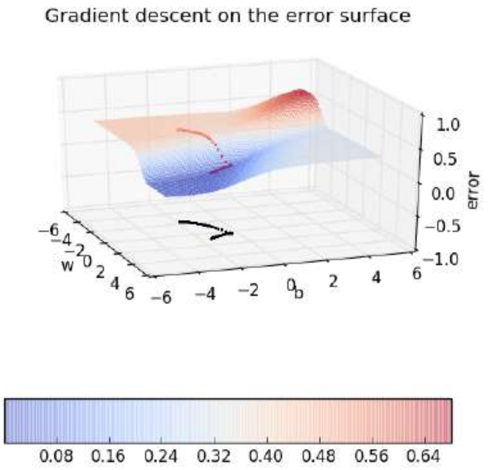

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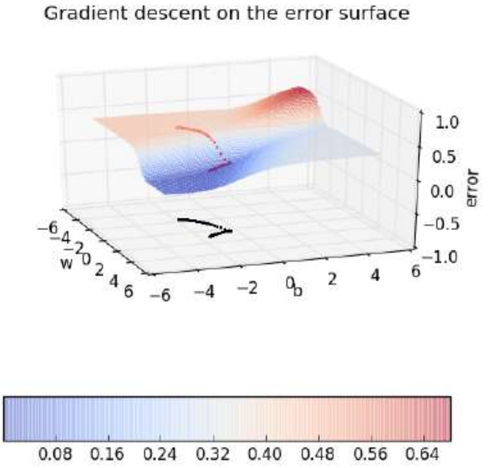

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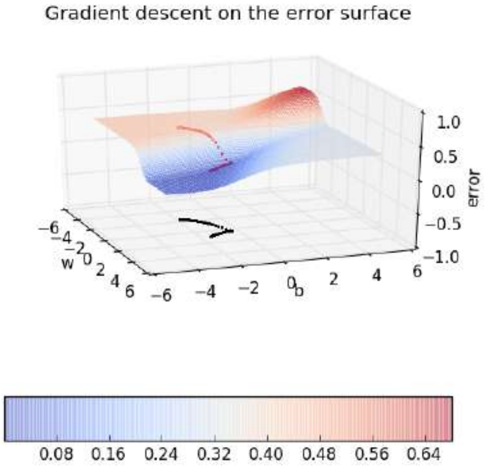

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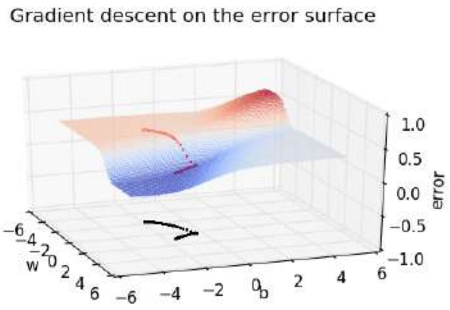

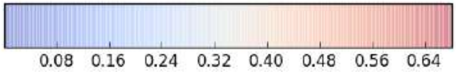

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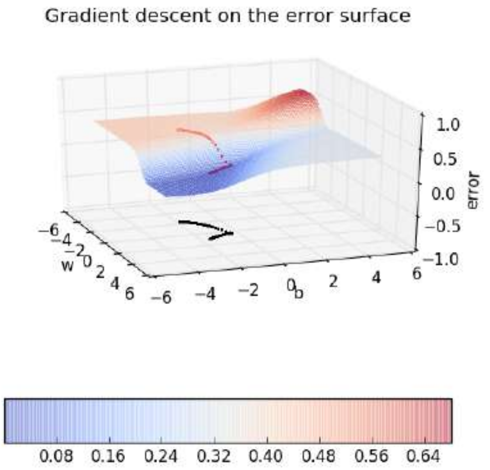

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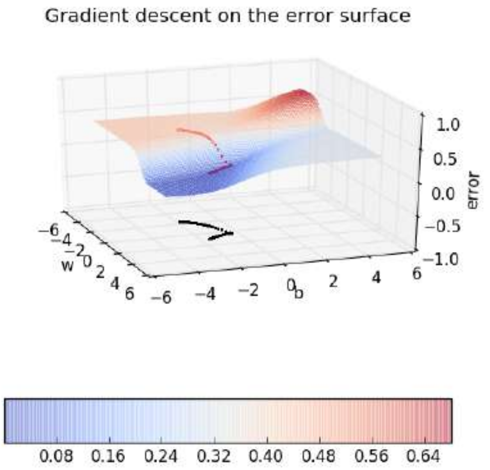

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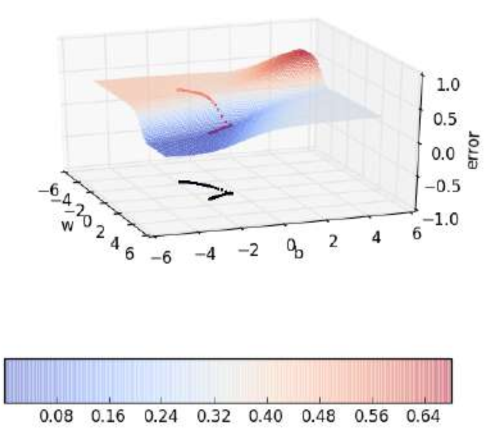
Gradient descent on the error surface



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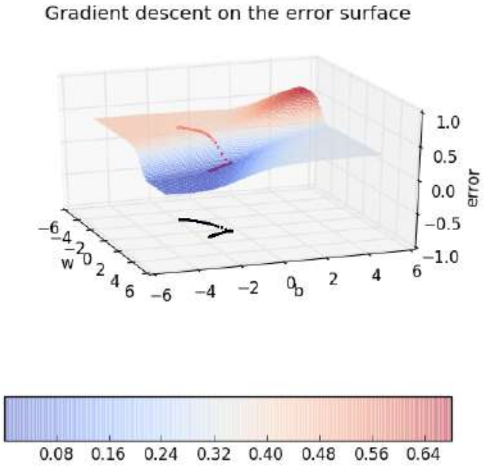

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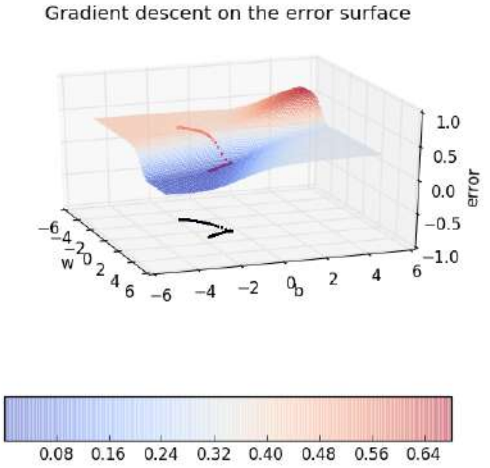

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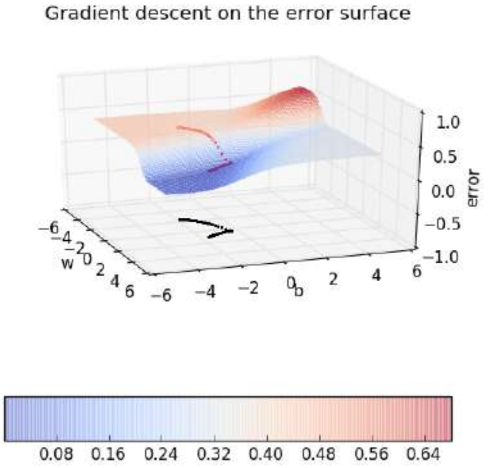

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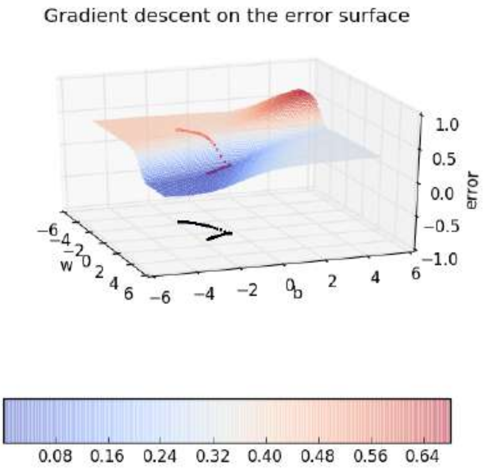

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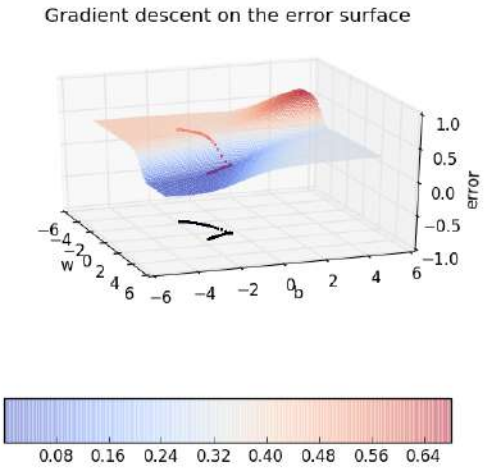

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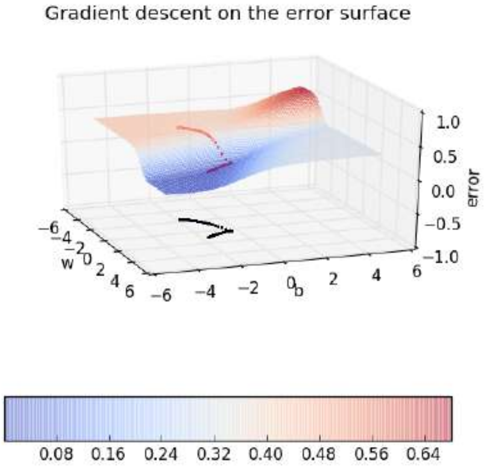

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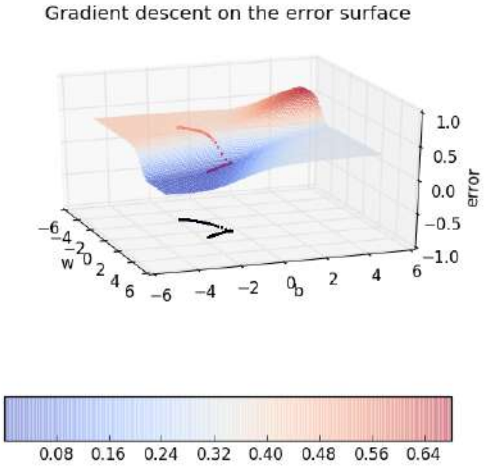

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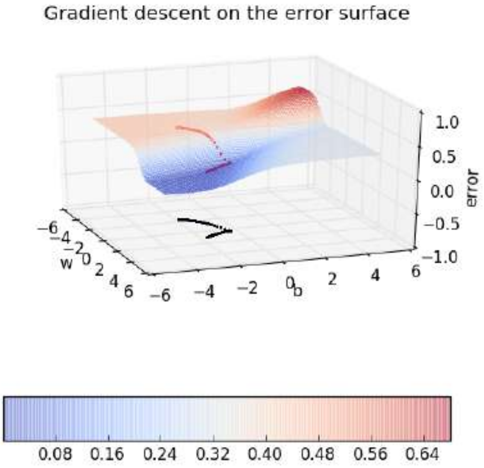

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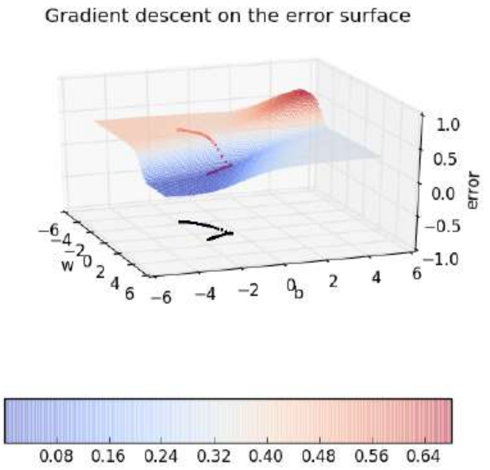

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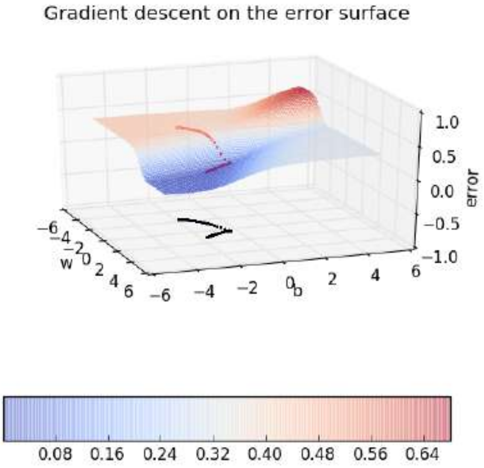

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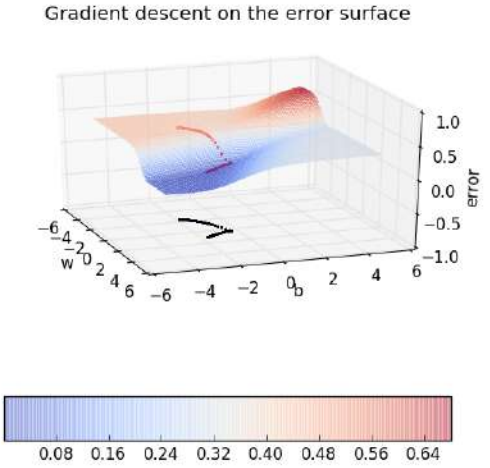

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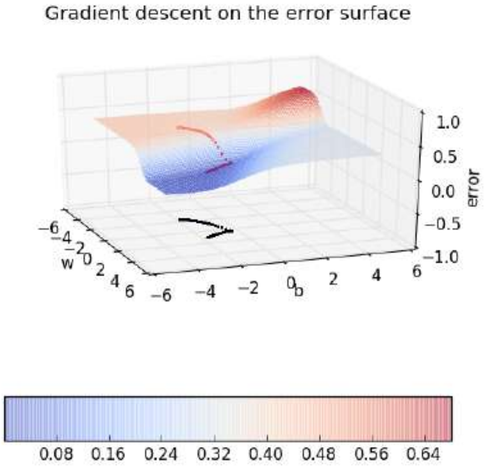

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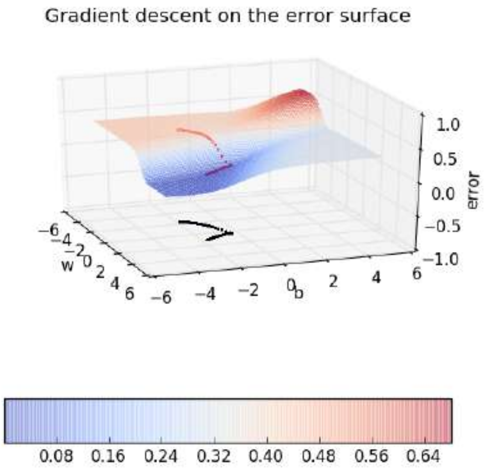

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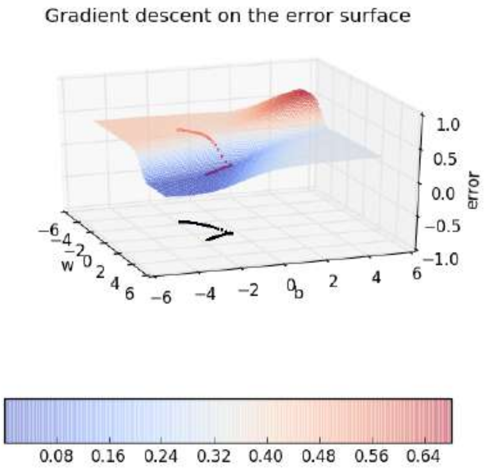

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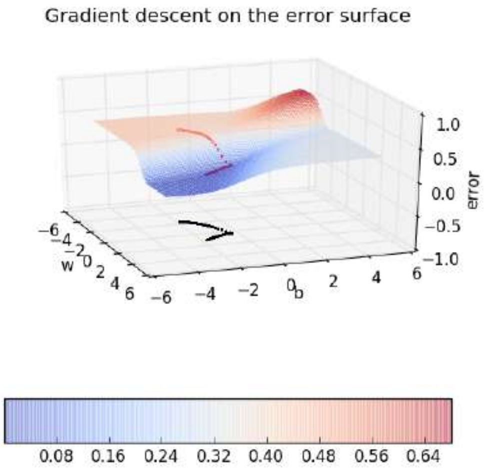

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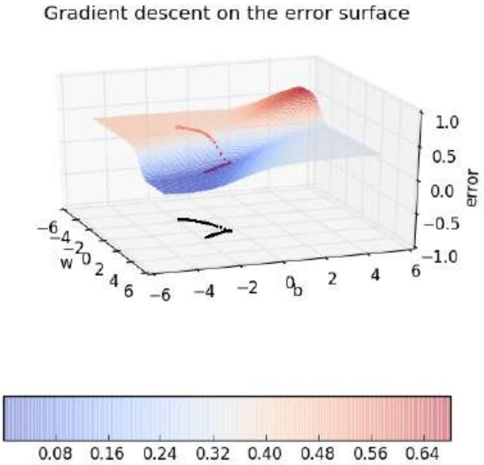

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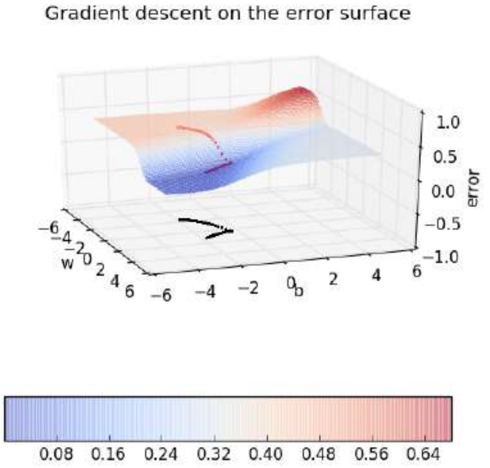

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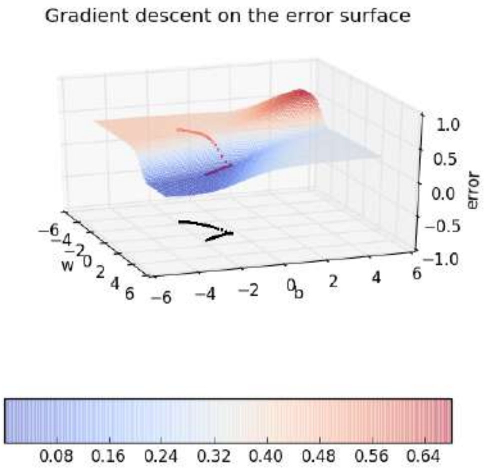

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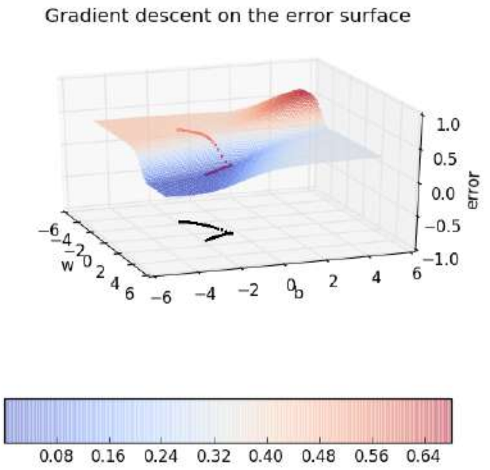

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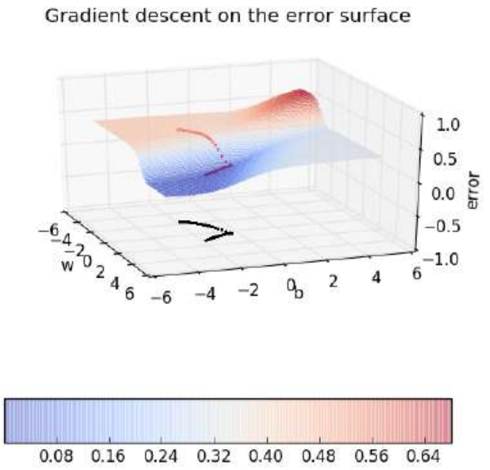

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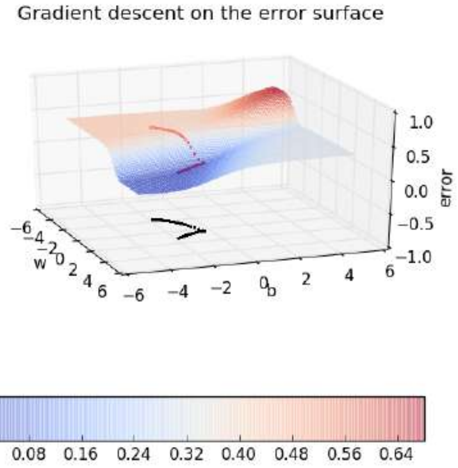

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- So where do we head from here ?

Module 3.5: Representation Power of a Multilayer Network of Sigmoid Neurons

Representation power of a multilayer network of perceptrons

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

Representation power of a multilayer network of perceptrons

A multilayer network of perceptrons with a single hidden layer can be used to represent any boolean function precisely (no errors)

Representation power of a multilayer network of sigmoid neurons

A multilayer network of neurons with a single hidden layer can be used to approximate any continuous function to any desired precision

In other words, there is a guarantee that for any function $f(x) : \mathbb{R}^n \rightarrow \mathbb{R}^m$, we can always find a neural network (with 1 hidden layer containing enough neurons) whose output g(x) satisfies $|g(x) - f(x)| < \epsilon$!!

Representation power of a multilayer network of perceptrons

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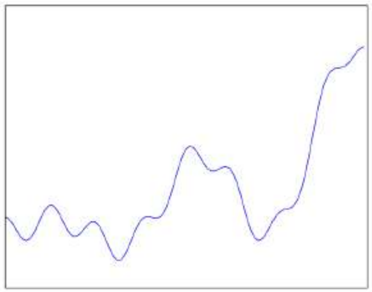
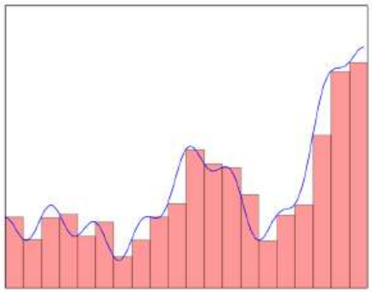
Proof: We will see an illustrative proof of this... [Cybenko, 1989], [Hornik, 1991]

- See this link* for an excellent illustration of this proof
- The discussion in the next few slides is based on the ideas presented at the above link

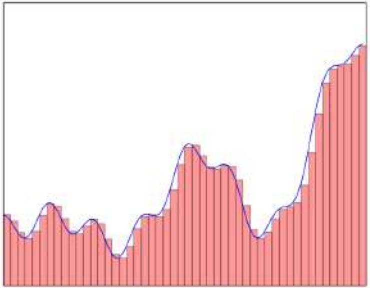
*http://neuralnetworksanddeeplearning.com/chap4.html



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)



- We are interested in knowing whether a network of neurons can be used to represent an arbitrary function (like the one shown in the figure)
- We observe that such an arbitrary function can be approximated by several "tower" functions



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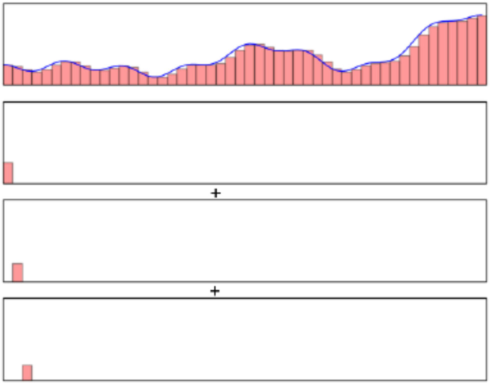

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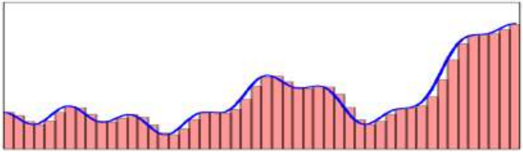
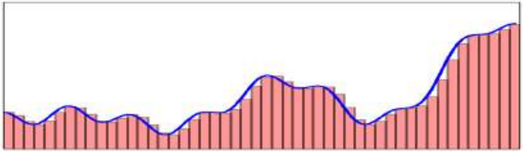
- We observe that such an arbitrary function can be approximated by several "tower" functions
- More the number of such "tower" functions, better the approximation
- To be more precise, we can approximate any arbitrary function by a sum of such "tower" functions



- We make a few observations



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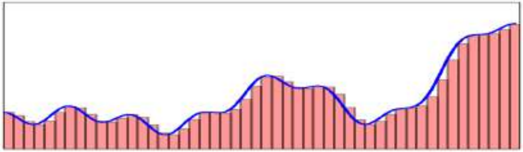
- All these “tower” functions are similar and only differ in their heights and positions on the x-axis



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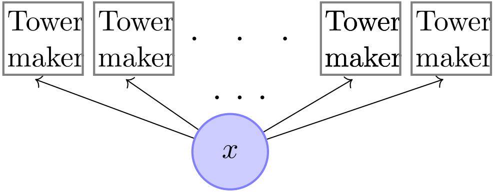
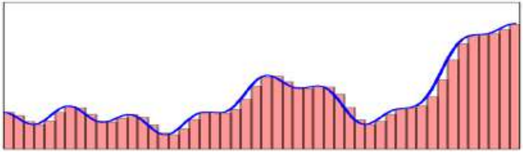
- Suppose there is a black box which takes the original input (x) and constructs these tower functions





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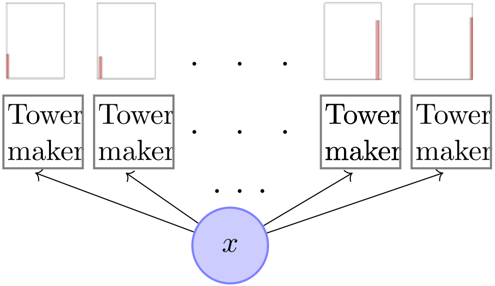

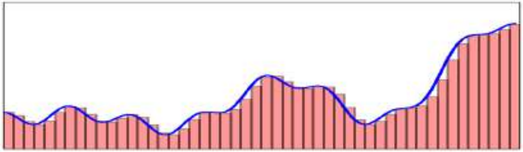

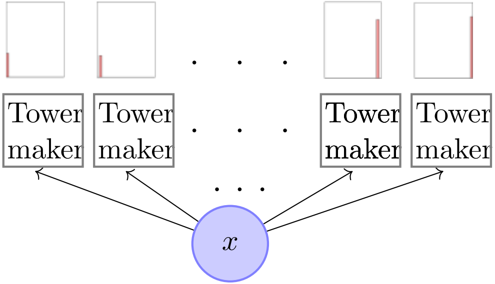

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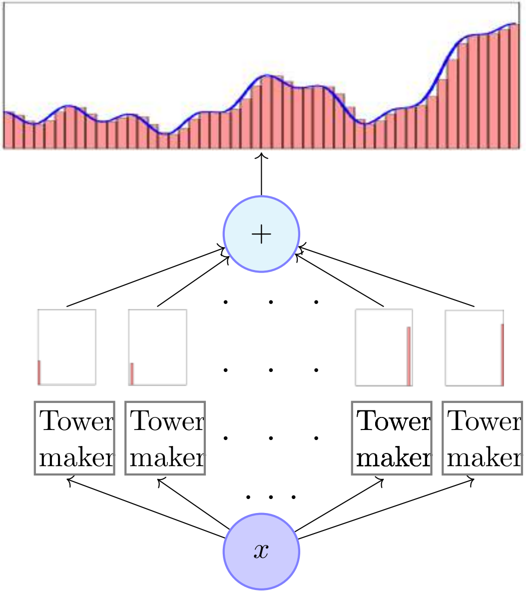

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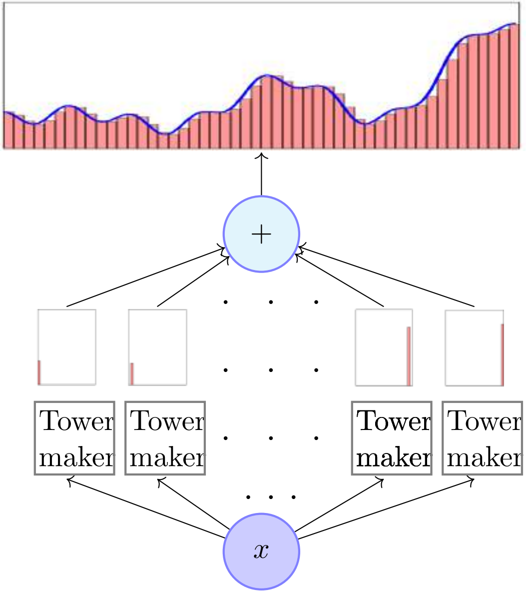

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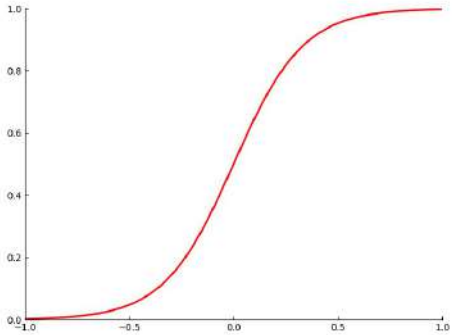
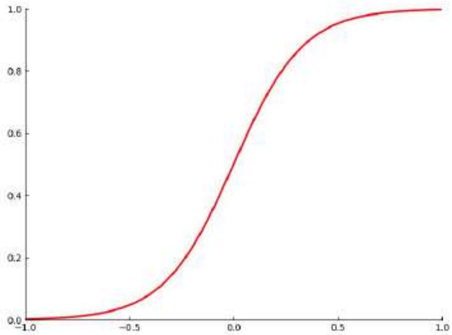
- Suppose there is a black box which takes the original input (x) and constructs these tower functions
- We can then have a simple network which can just add them up to approximate the function
- Our job now is to figure out what is inside this blackbox

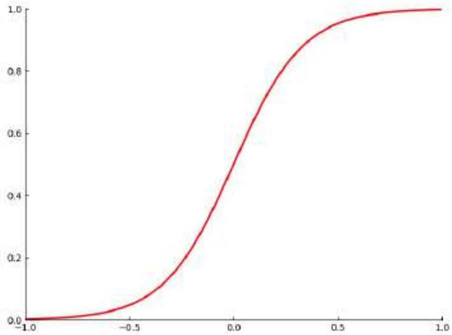
We will figure this out over the next few slides ...



- If we take the logistic function and set w to a very high value we will recover the step function



- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 0, b = 0$

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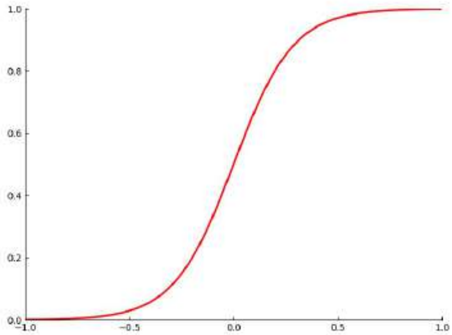

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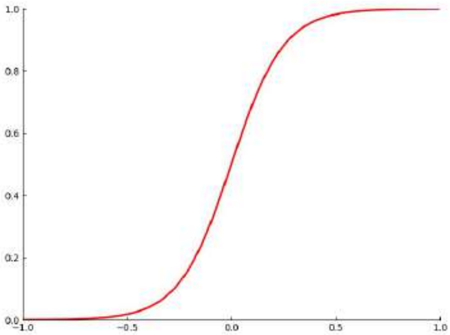

$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 2, b = 0$$

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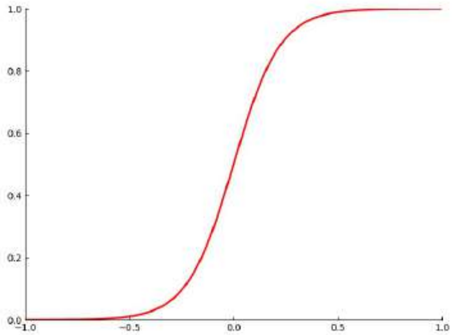

$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 3, b = 0$$

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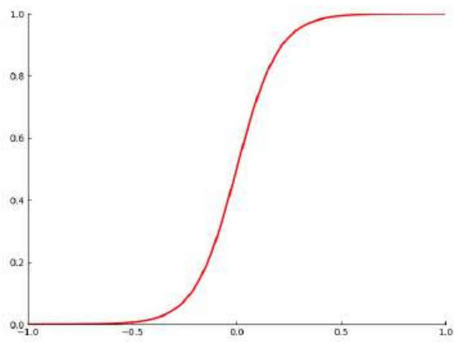

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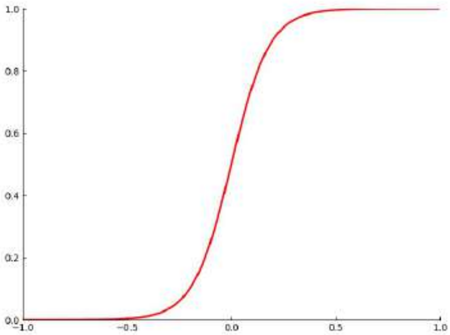

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- If we take the logistic function and set w to a very high value we will recover the step function
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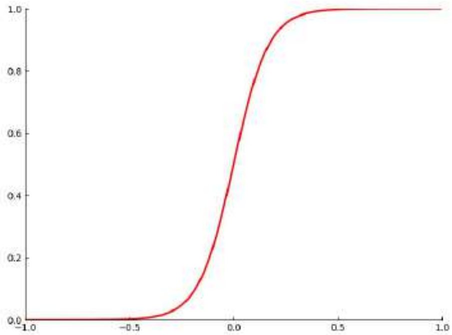
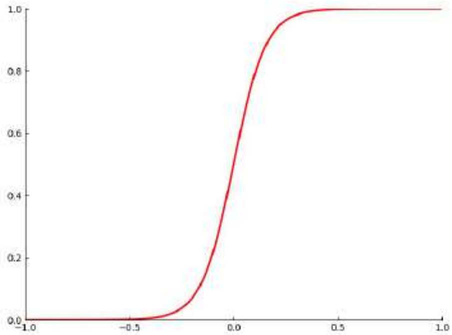


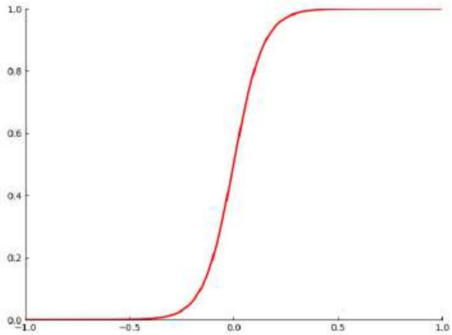
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 6, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



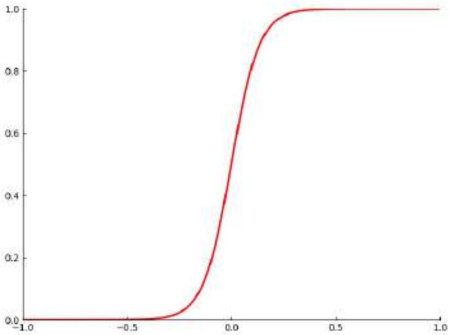
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 7, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



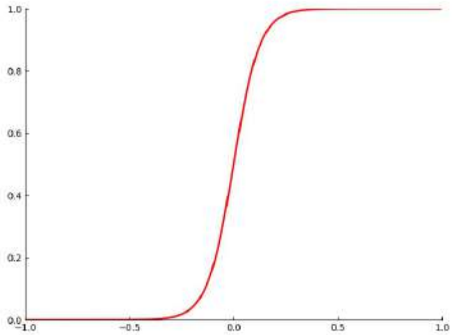
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 8, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



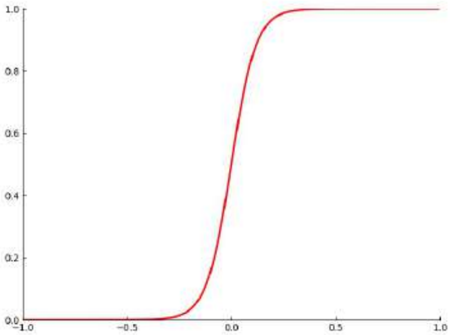
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 9, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



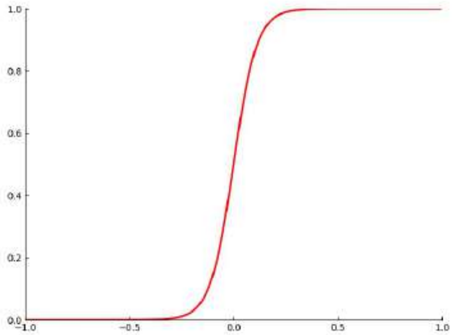
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 10, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



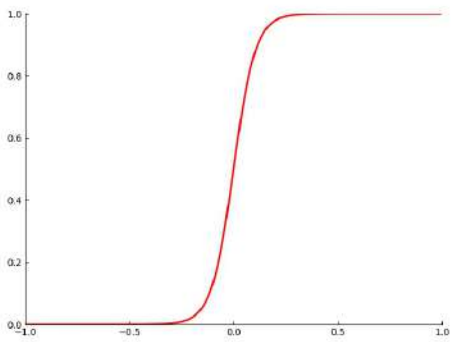
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 11, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



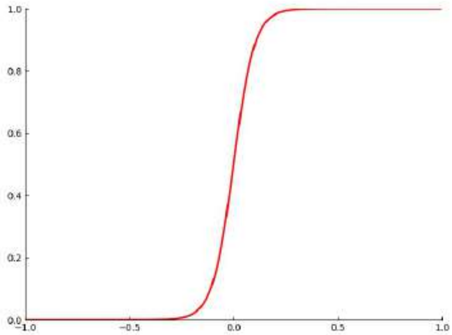
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 12, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



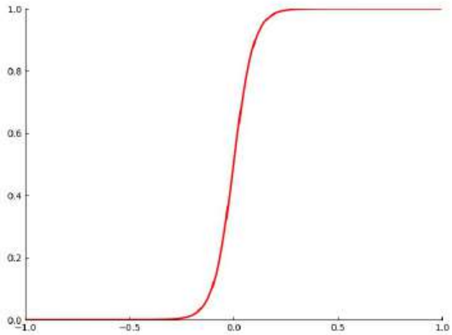
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 13, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



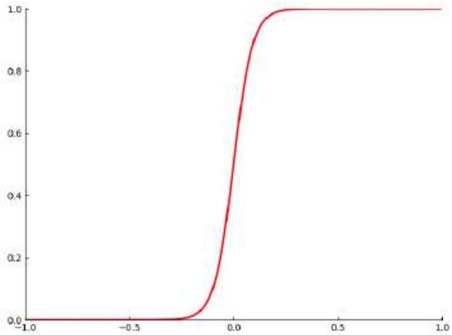
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 14, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



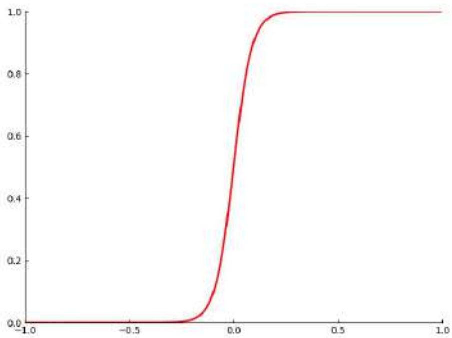
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 15, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



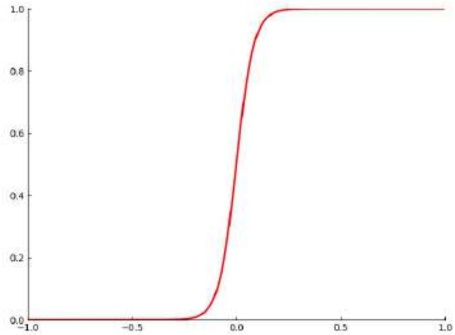
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 16, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



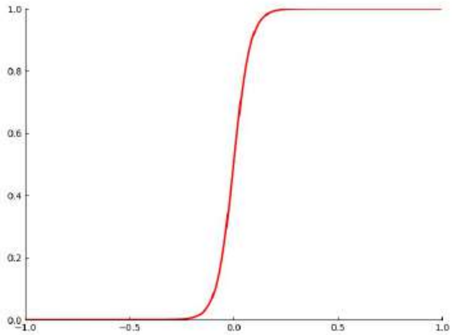
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 17, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



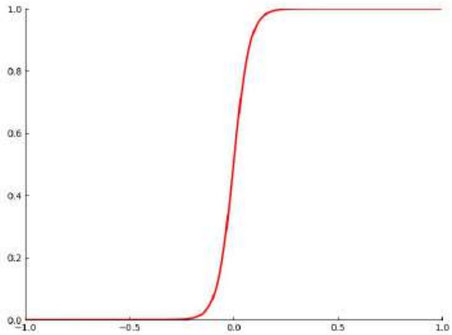
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 18, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



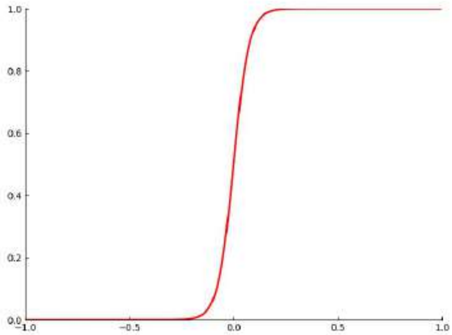
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 19, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



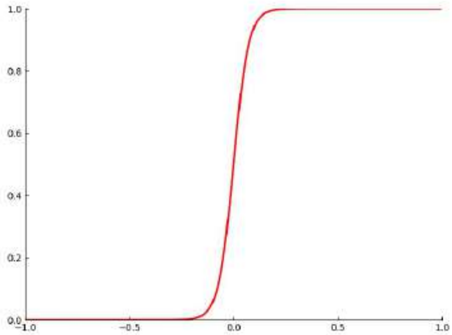
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 20, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



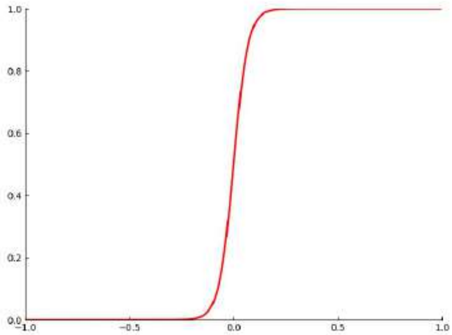
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 21, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



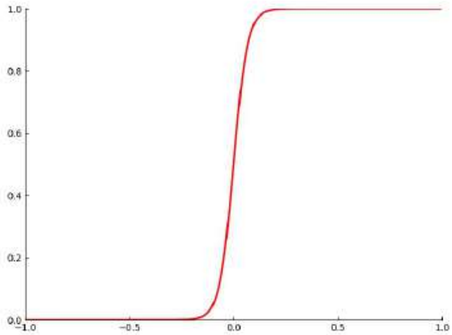
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 22, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



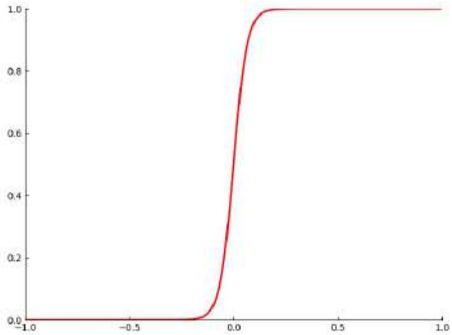
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 23, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



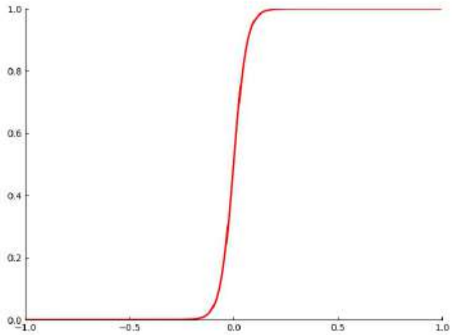
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 24, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



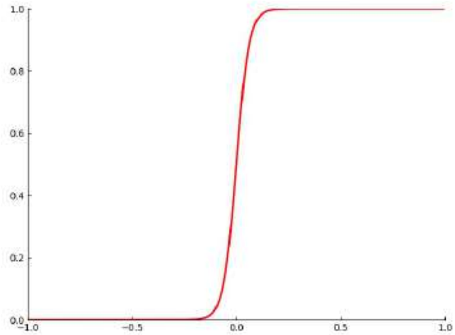
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 25, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



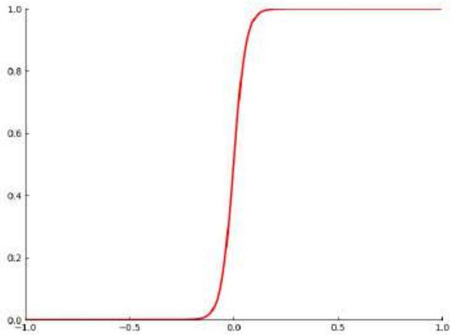
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 26, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



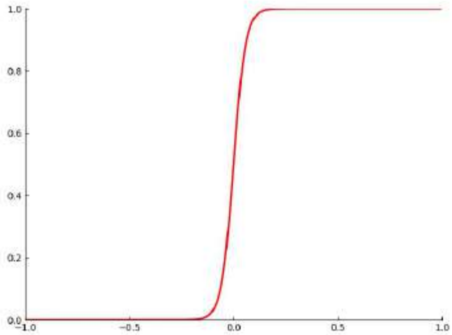
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 27, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



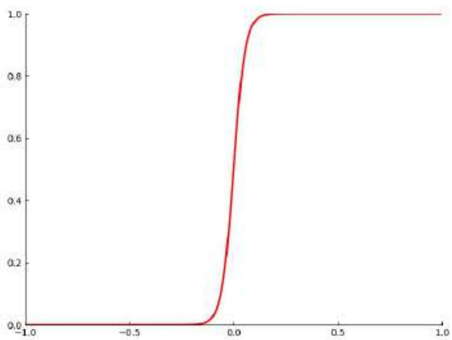
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 28, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



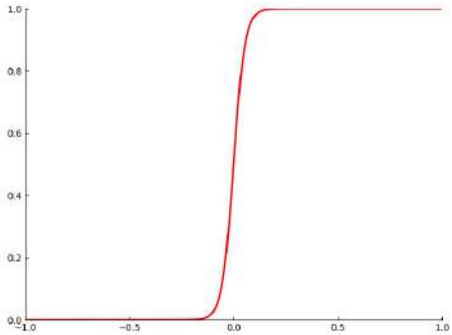
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 29, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



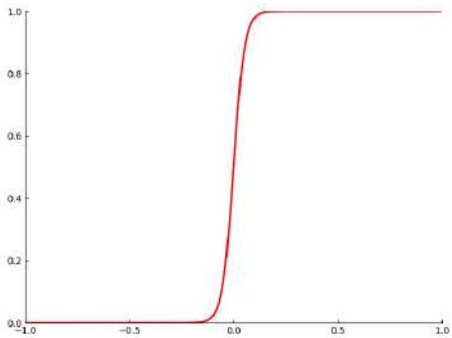
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 30, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



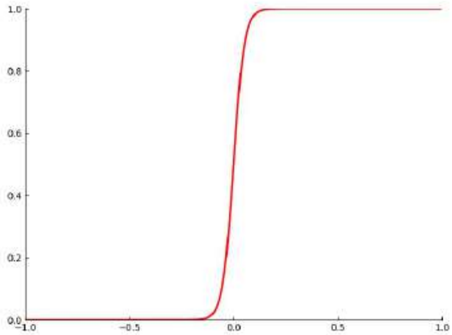
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 31, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



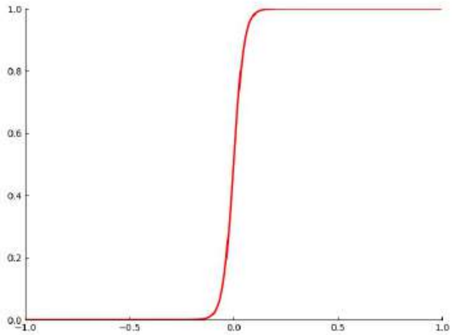
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 32, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



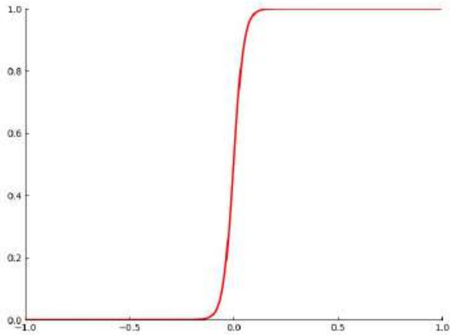
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 33, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



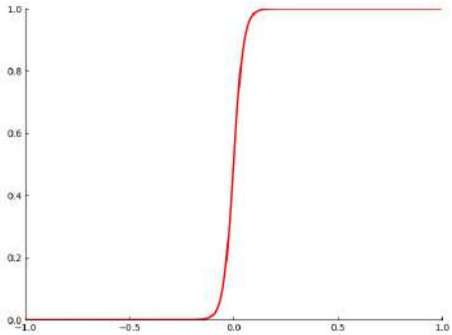
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 34, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



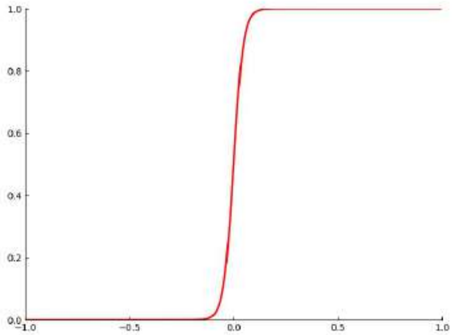
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 35, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



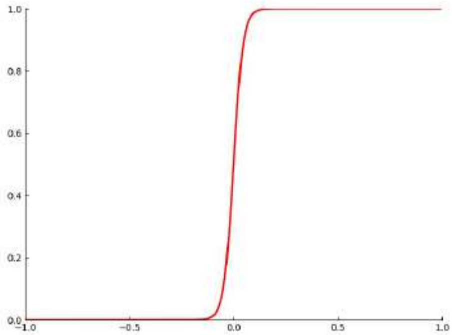
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 36, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



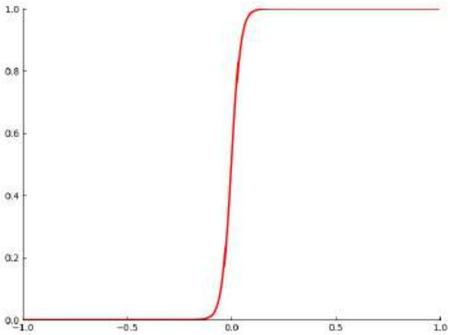
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 37, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



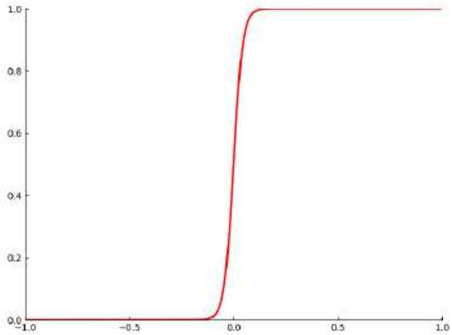
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 38, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



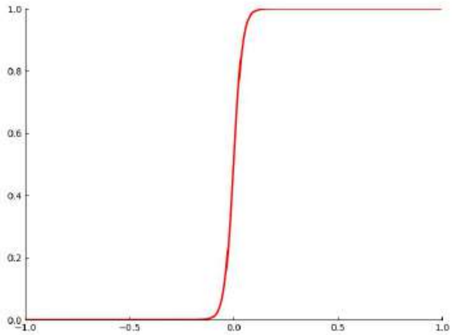
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 39, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



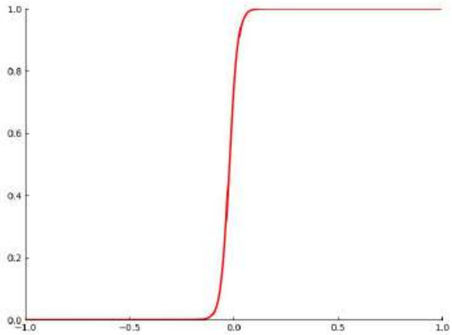
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 40, b = 0$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



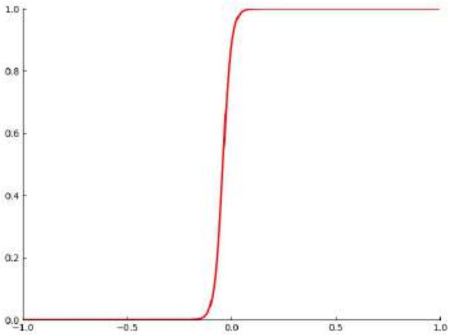
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 41, b = 0$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w



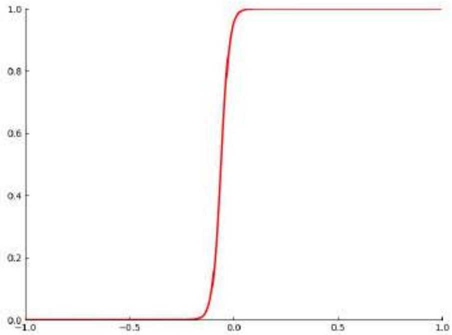
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 1$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



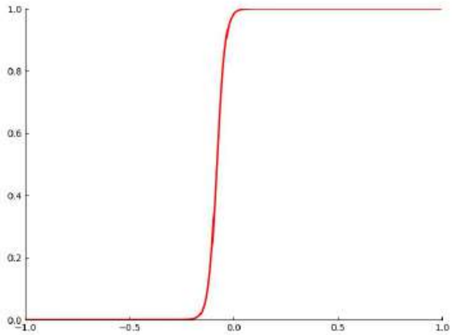
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 2$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



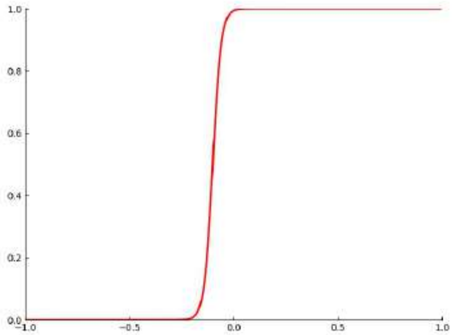
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 3$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



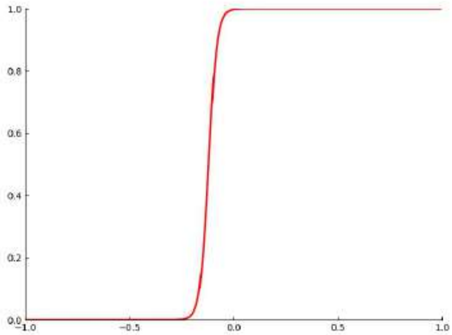
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 4$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



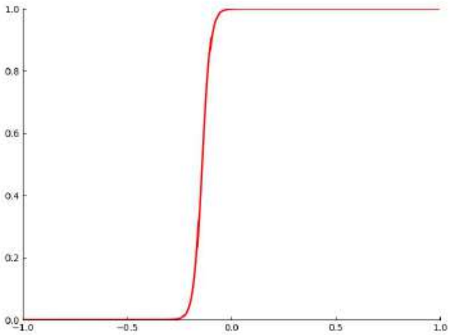
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 5$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



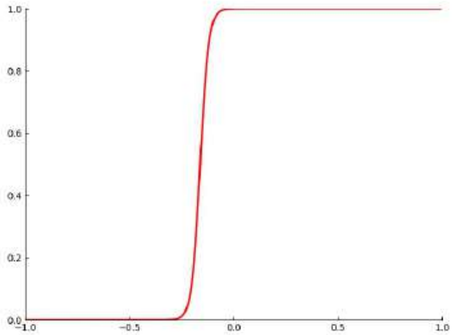
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 6$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



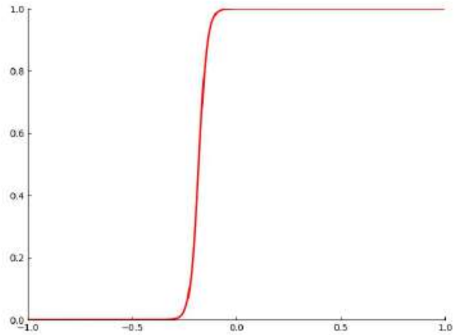
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 7$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



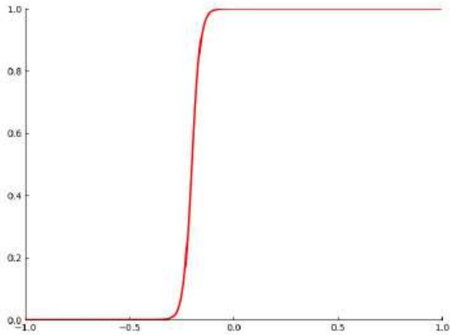
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 8$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



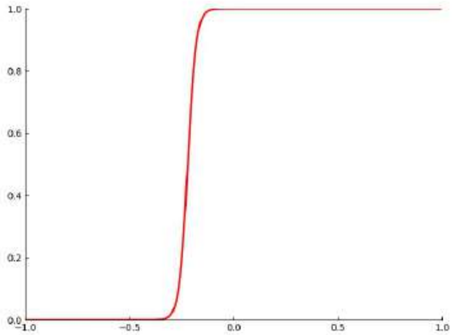
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 9$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



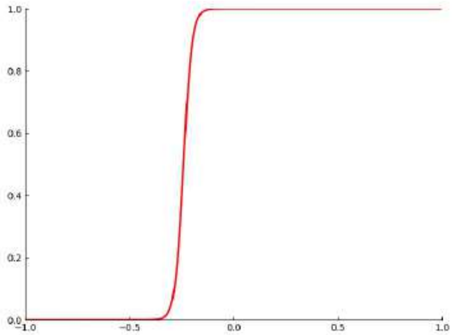
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 10$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



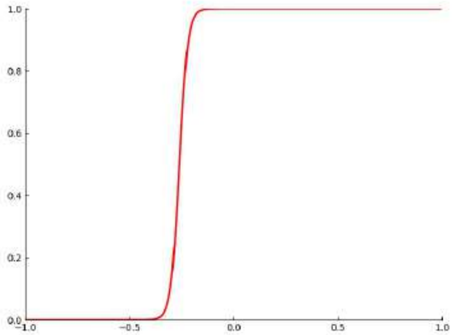
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 11$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



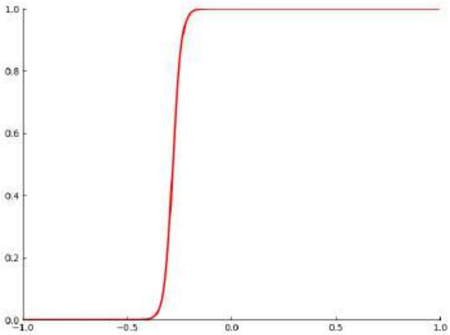
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 12$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



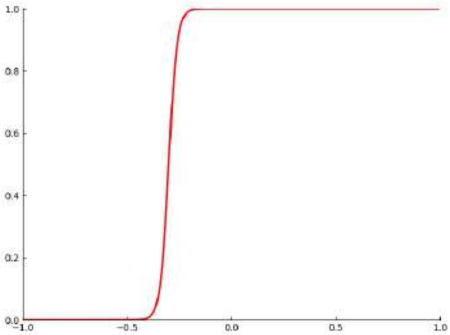
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 13$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



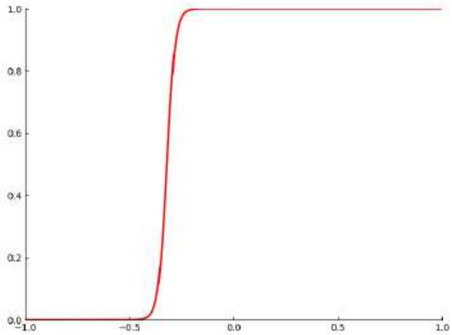
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 14$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



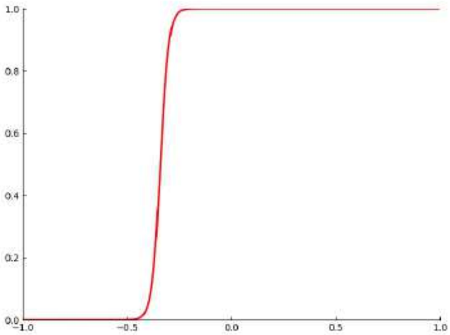
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 15$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



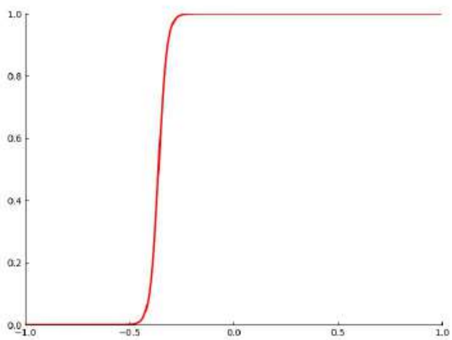
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 16$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



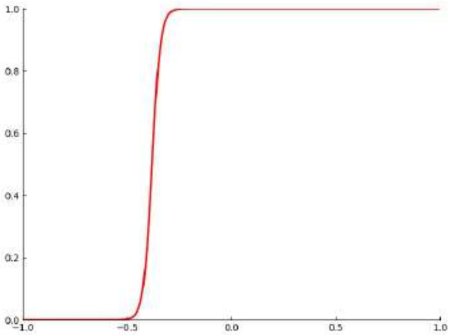
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 17$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



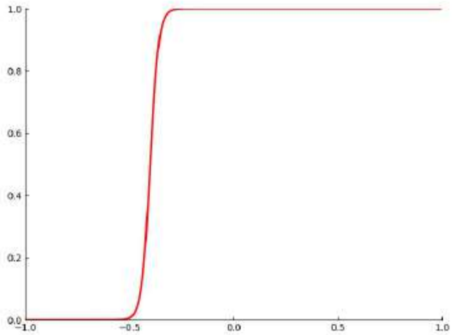
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 18$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



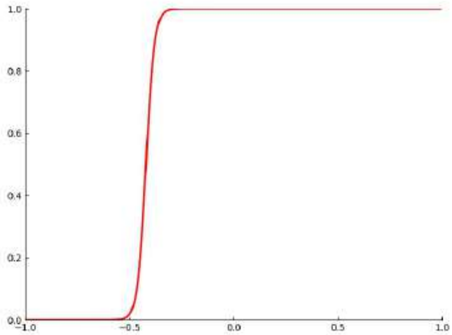
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 19$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



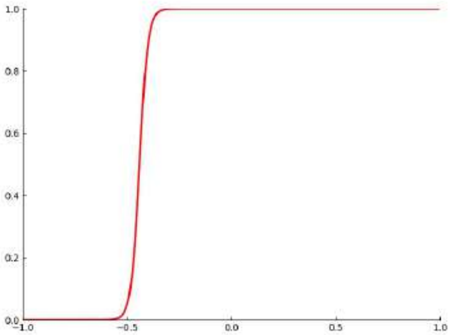
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 20$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



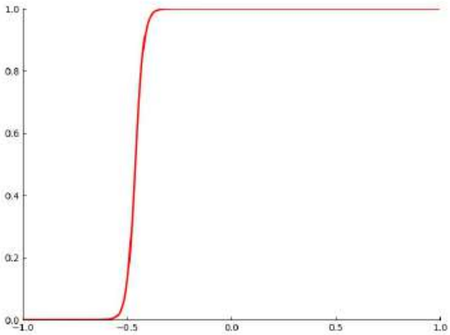
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 21$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



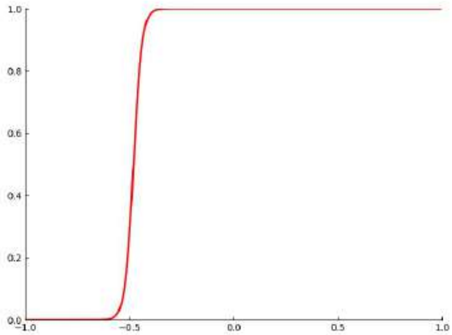
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 22$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



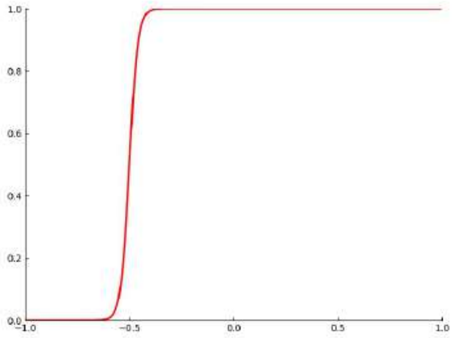
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 23$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



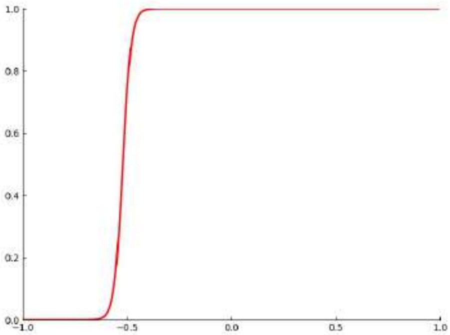
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 24$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



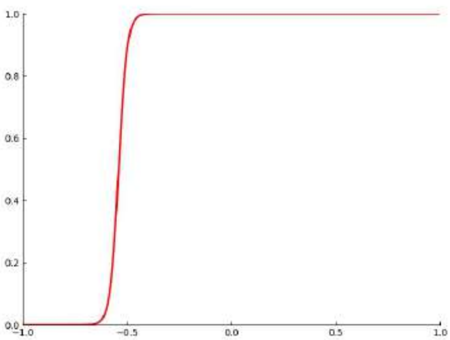
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 25$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



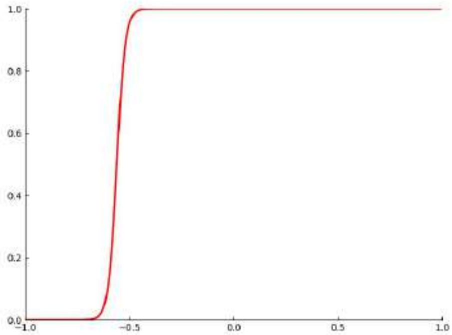
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 26$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



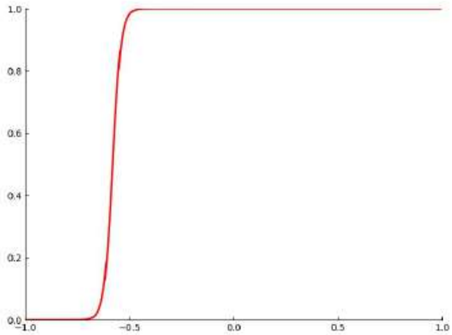
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 27$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



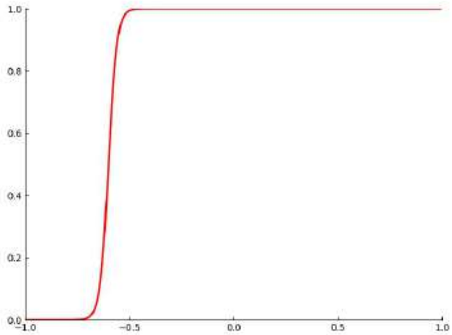
- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1

$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 28$$



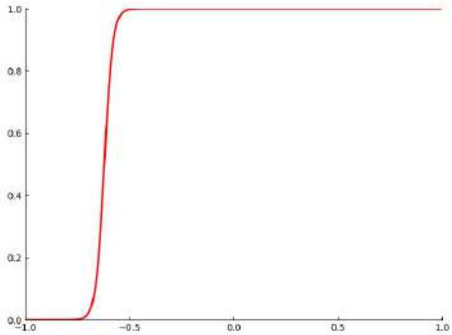
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 29$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



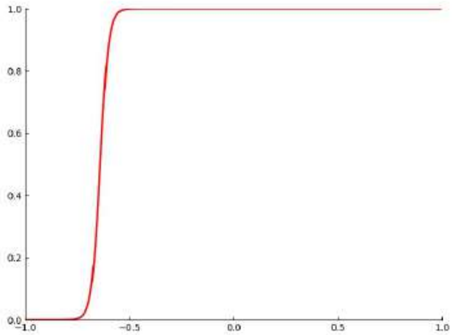
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 30$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



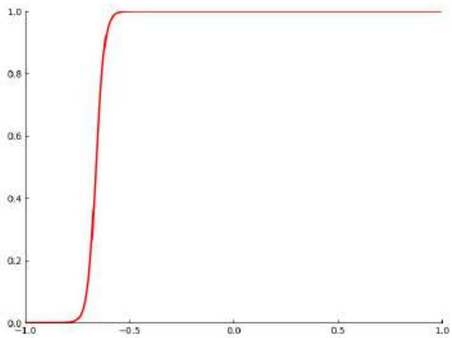
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 31$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



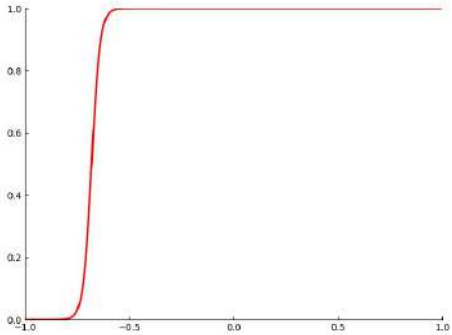
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 32$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



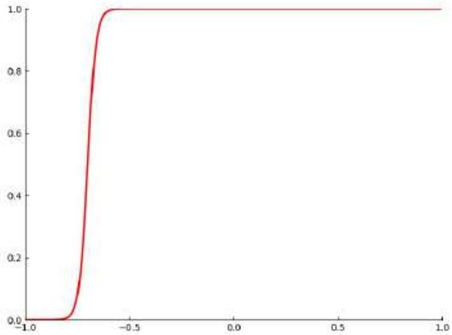
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 33$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



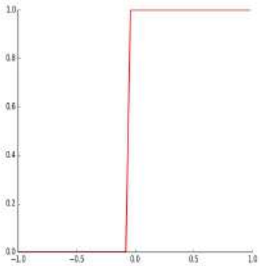
$\sigma(x) = \frac{1}{1+e^{-(wx+b)}}$ $w = 50, b = 34$

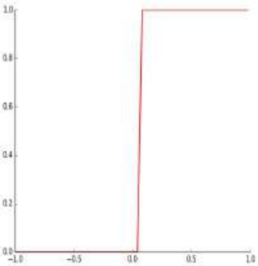
- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1



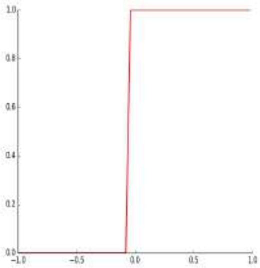
$$\sigma(x) = \frac{1}{1+e^{-(wx+b)}} \quad w = 50, b = 35$$

- If we take the logistic function and set w to a very high value we will recover the step function
- Let us see what happens as we change the value of w
- Further we can adjust the value of b to control the position on the x-axis at which the function transitions from 0 to 1





- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other



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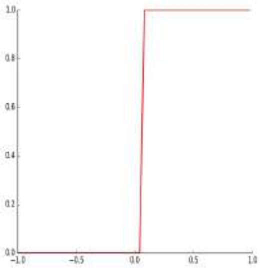
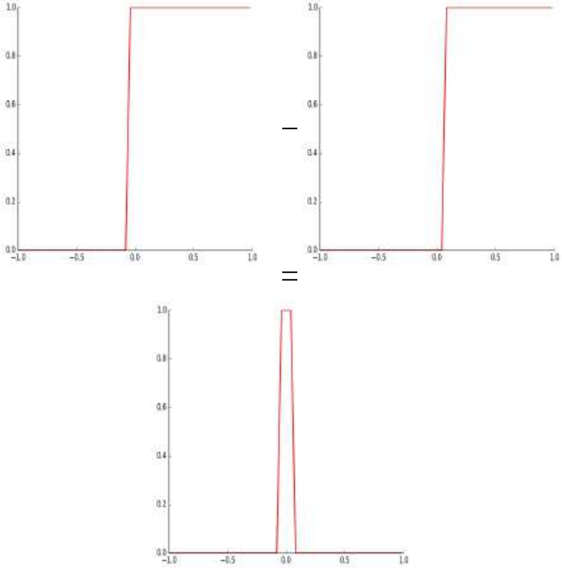


- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other



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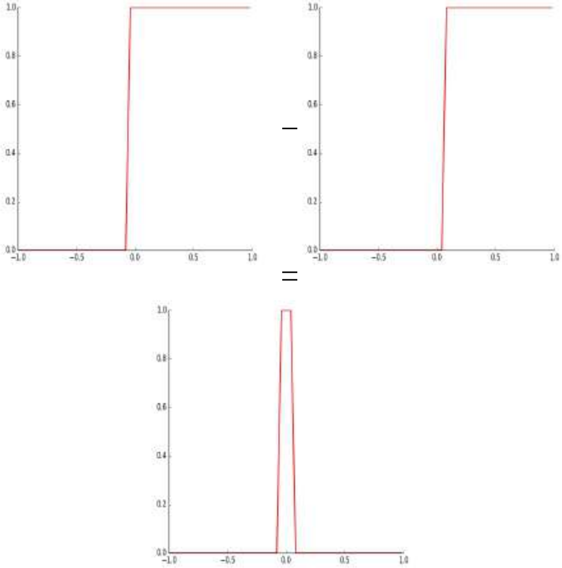
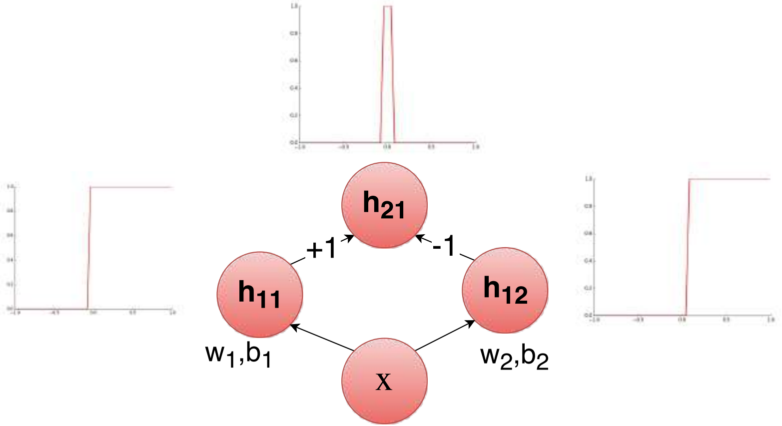


- Now let us see what we get by taking two such sigmoid functions (with different b's) and subtracting one from the other
- Voila! We have our tower function !!

- Can we come up with a neural network to represent this operation of subtracting one sigmoid function from another ?


h21
+1
-1
h11
h12
w1,b1
w2,b2
x

- What if we have more than one input?

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- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)

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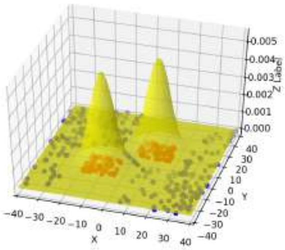
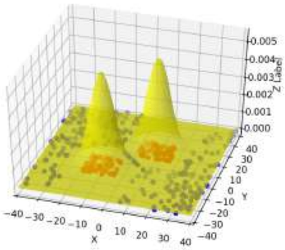
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)



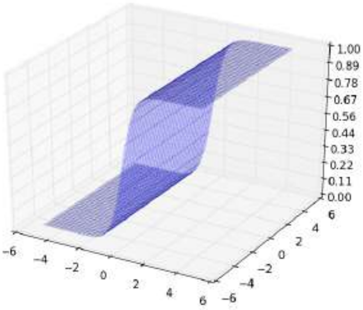
- What if we have more than one input?
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)
- We are given some data and it seems that $y(\text{oil}|\text{no-oil})$ is a complex function of x_1 and x_2



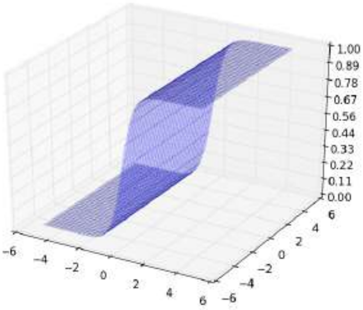
- What if we have more than one input?
- Suppose we are trying to take a decision about whether we will find oil at a particular location on the ocean bed(Yes/No)
- Further, suppose we base our decision on two factors: Salinity (x_1) and Pressure (x_2)
- We are given some data and it seems that y(oil|no-oil) is a complex function of x_1 and x_2
- We want a neural network to approximate this function

$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$

- This is what a 2-dimensional sigmoid looks like

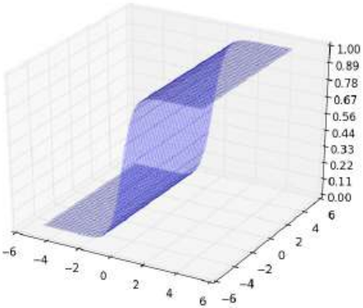


$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case

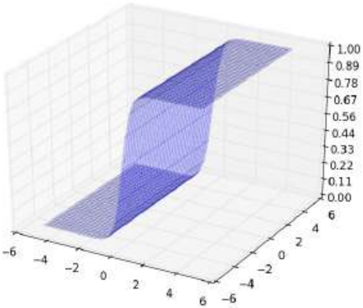
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 2, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

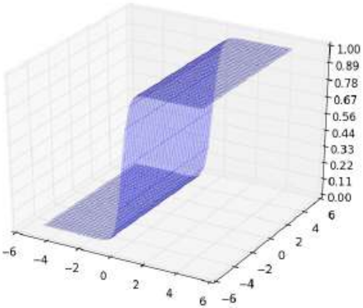
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 3, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

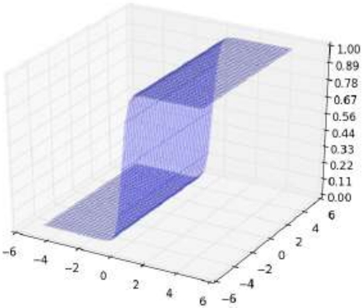
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 4, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

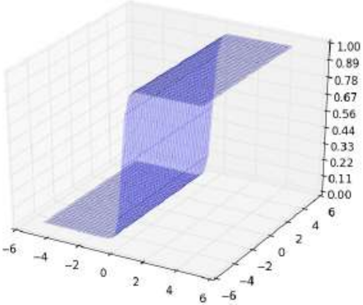
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 5, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

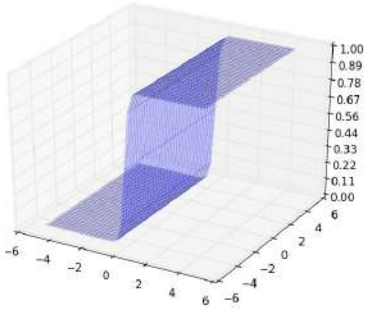
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 6, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

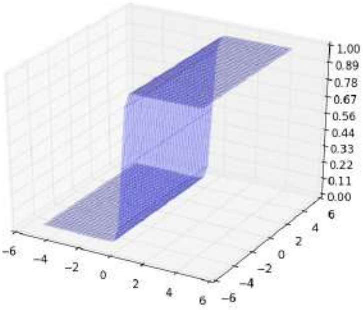
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 7, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

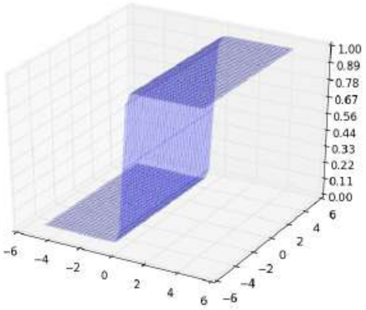
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 8, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

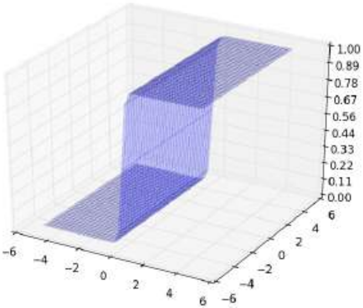
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 9, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

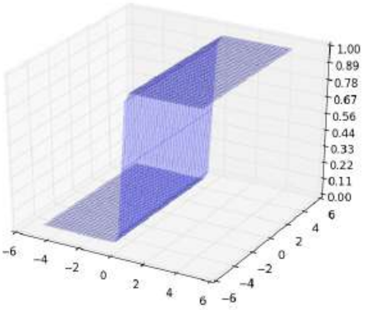
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 10, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

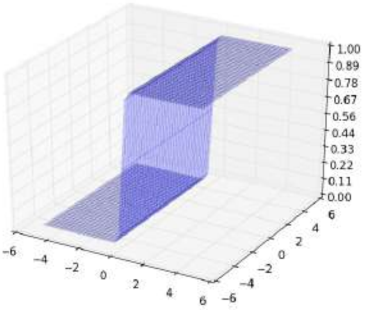
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 11, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

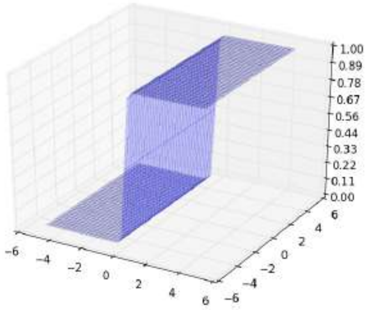
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 12, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

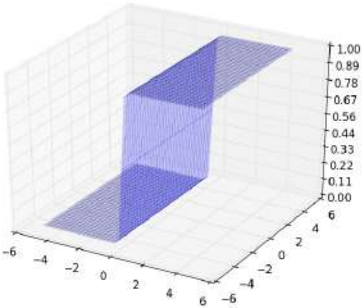
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 13, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

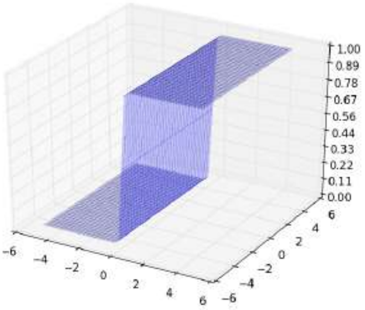
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 14, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

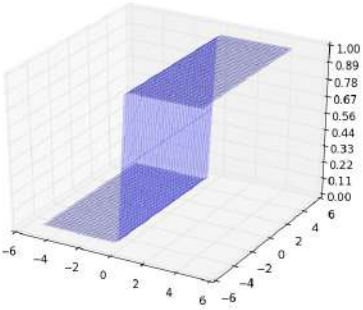
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 15, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

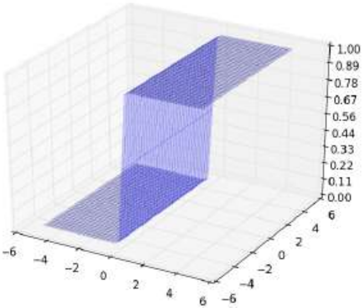
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 16, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

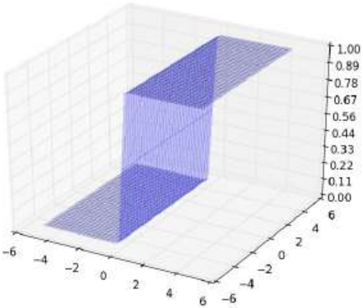
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 17, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

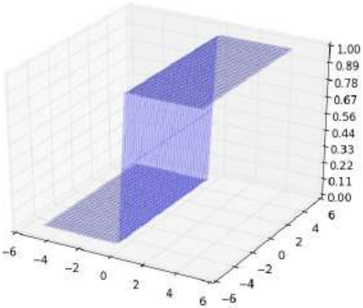
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 18, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

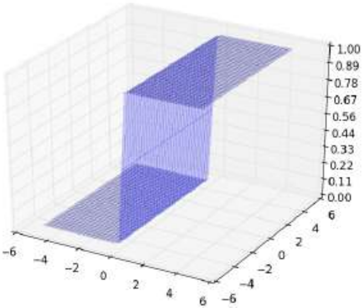
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 19, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

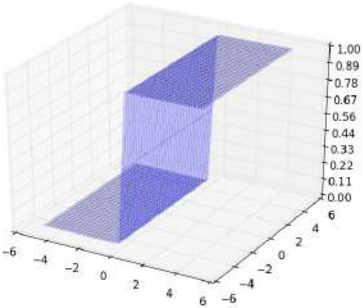
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 20, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

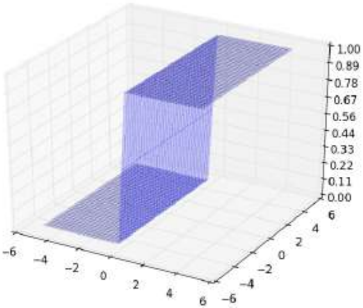
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 21, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

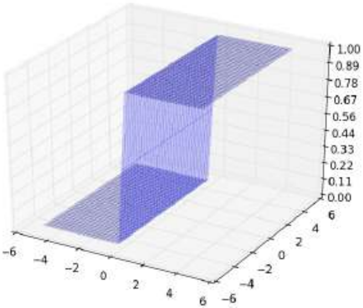
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 22, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

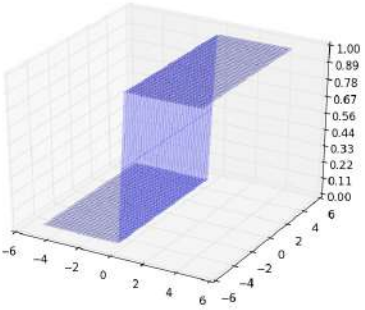
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 23, w_2 = 0, b = 0$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

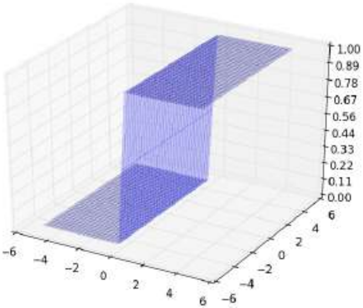
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 24, w_2 = 0, b = 0$

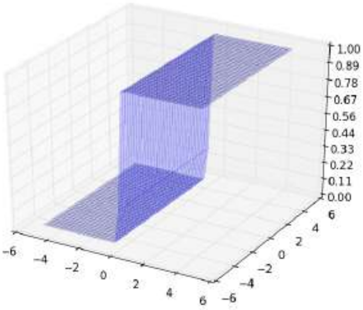
- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

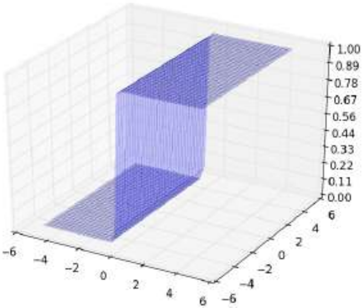
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 5$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

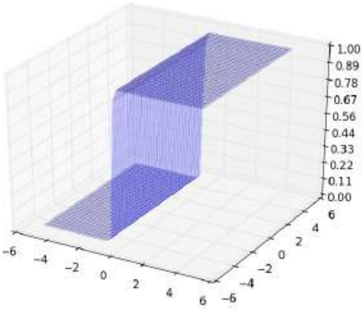
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 10$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

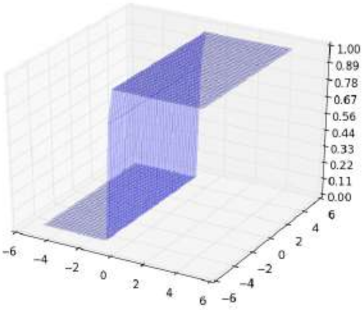
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 15$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

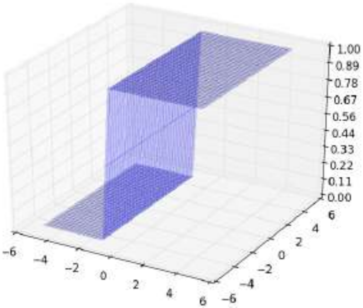
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 20$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

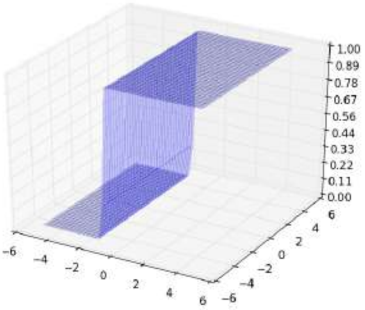
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$$w_1 = 25, w_2 = 0, b = 25$$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

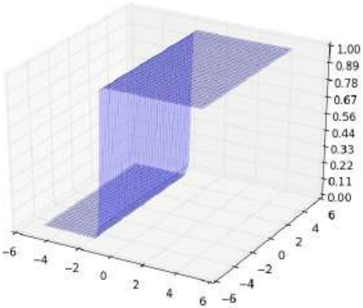
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 30$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

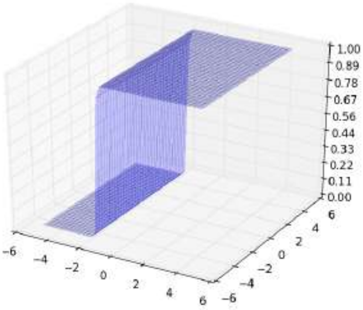
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 35$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

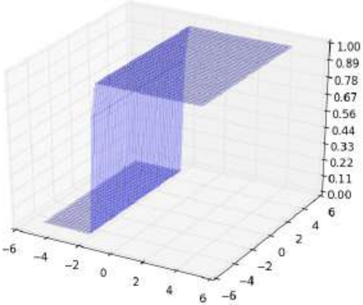
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 25, w_2 = 0, b = 40$

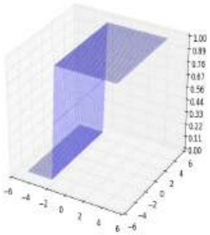
- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

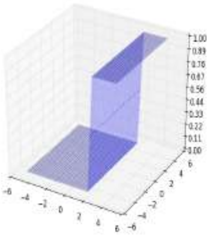
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



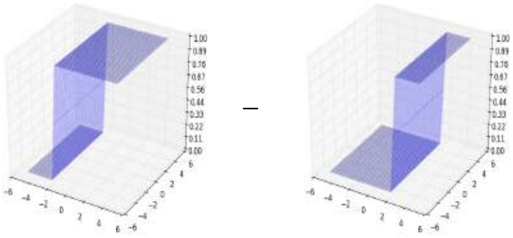
$w_1 = 25, w_2 = 0, b = 45$

- This is what a 2-dimensional sigmoid looks like
- We need to figure out how to get a tower in this case
- First, let us set w_2 to 0 and see if we can get a two dimensional step function
- What would happen if we change b ?

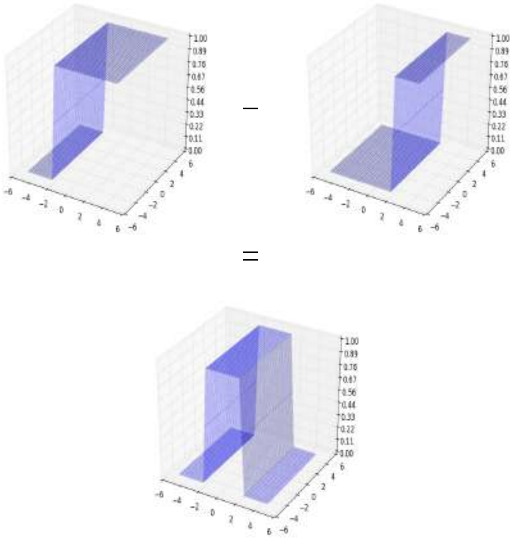




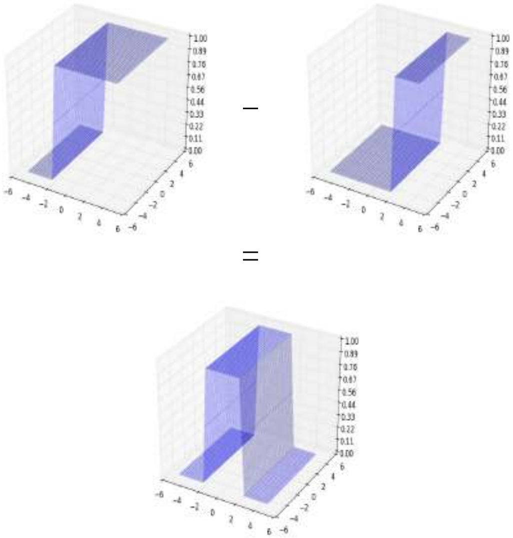
- What if we take two such step functions (with different b values) and subtract one from the other



- What if we take two such step functions (with different b values) and subtract one from the other

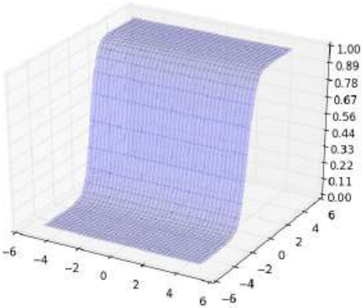


- What if we take two such step functions (with different b values) and subtract one from the other



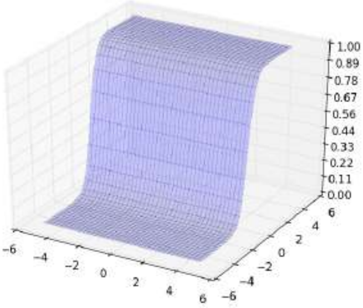
- What if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

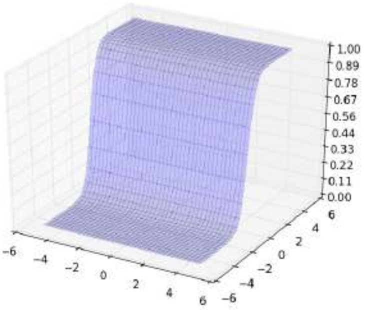
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

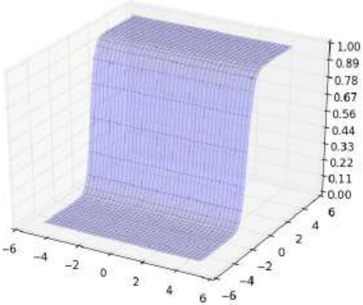
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 2, b = 0$

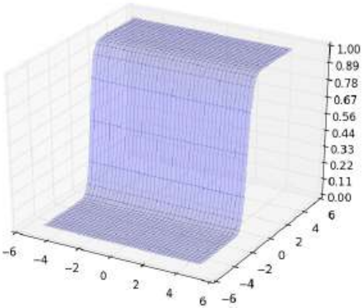
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 3, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

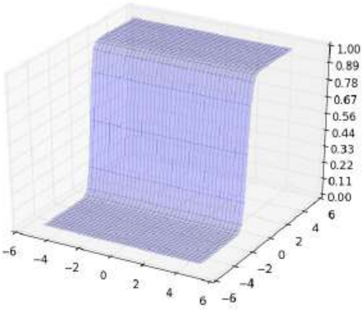
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 4, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

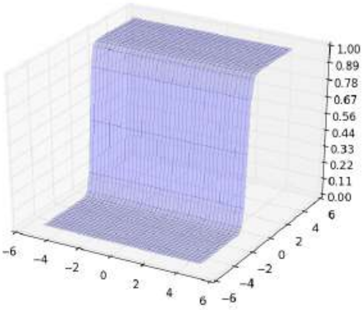
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 5, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

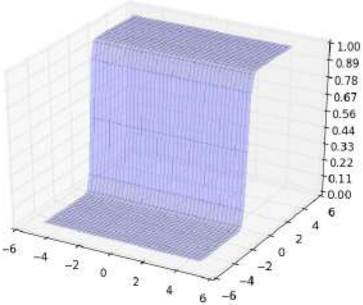
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 6, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

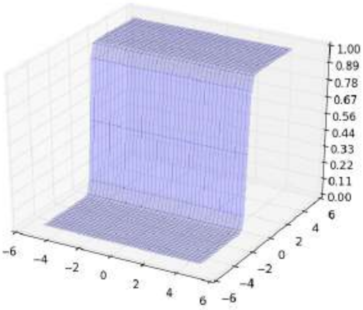


$w_1 = 0, w_2 = 7, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

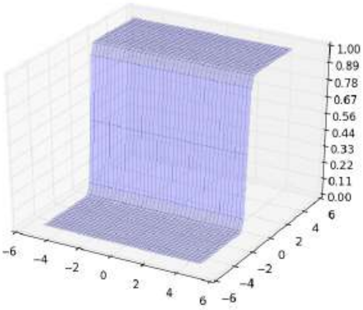
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 8, b = 0$

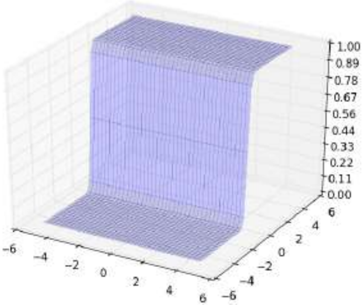
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 9, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

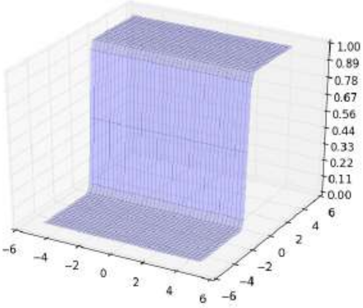
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 10, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

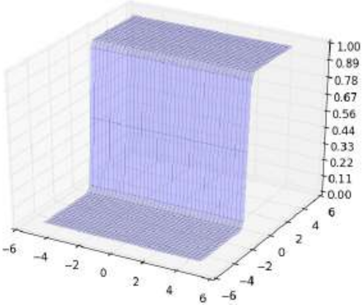
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 11, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

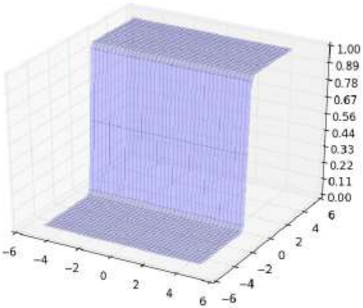
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 0, w_2 = 12, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

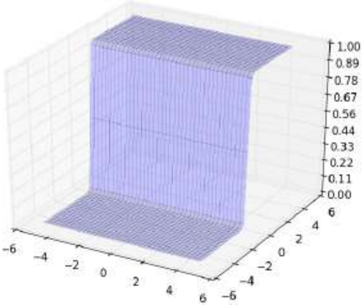


$w_1 = 0, w_2 = 13, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

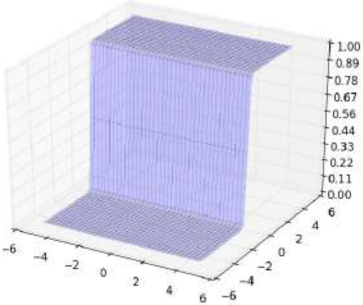
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 14, b = 0$

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

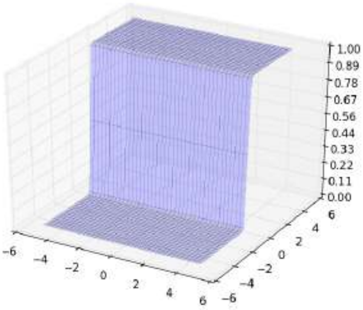


$w_1 = 0, w_2 = 15, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

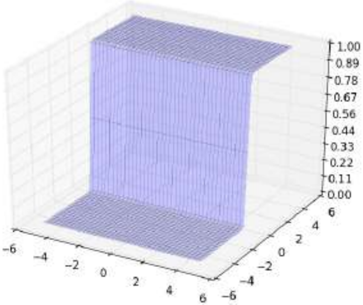
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 16, b = 0$

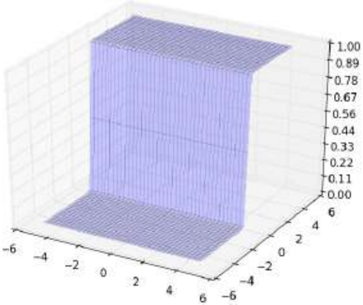
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 17, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

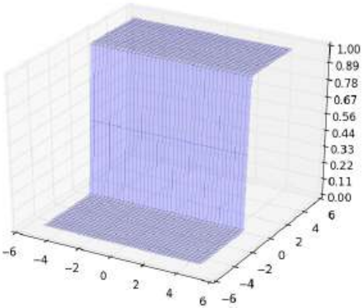
$$y = \frac{1}{1 + e^{-(w_1x_1 + w_2x_2 + b)}}$$



$w_1 = 0, w_2 = 18, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

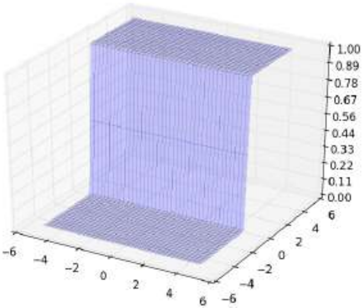
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 19, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

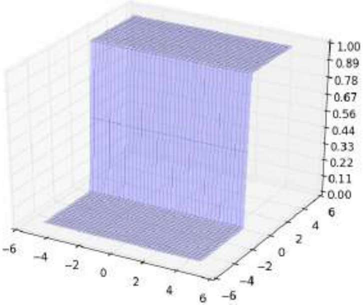
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 20, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

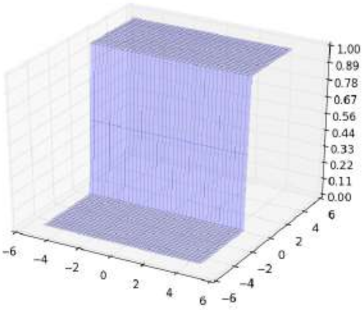


$w_1 = 0, w_2 = 21, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

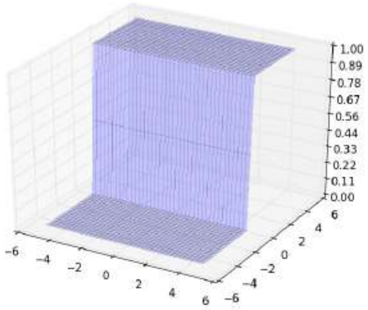
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation



$w_1 = 0, w_2 = 22, b = 0$

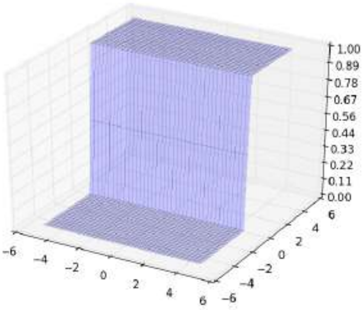
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 23, b = 0$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

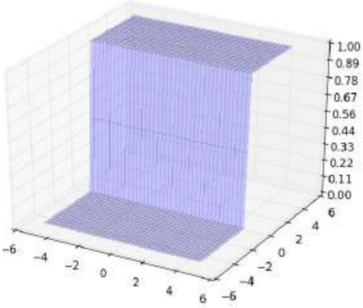
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 24, b = 0$

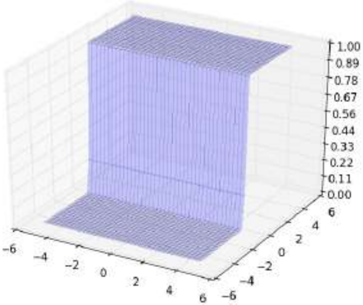
- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation

$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

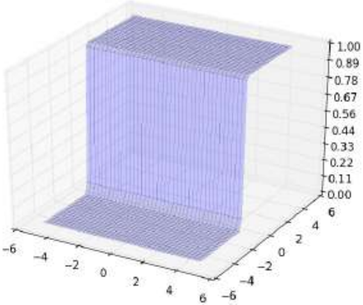
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 5$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

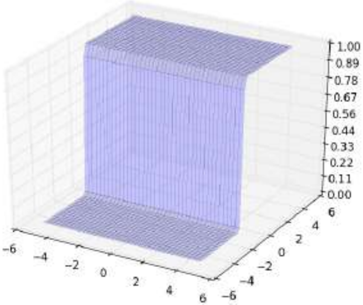
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 10$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

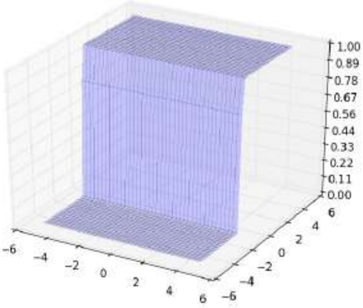
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 15$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

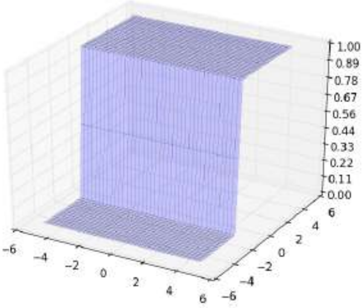
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 20$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

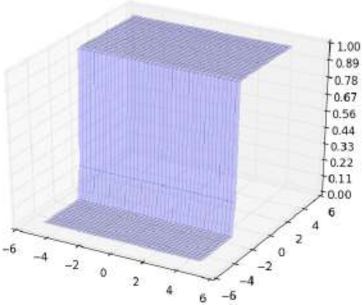
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 25$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

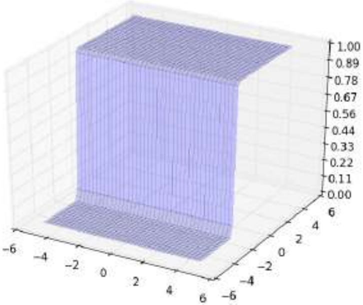
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 30$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

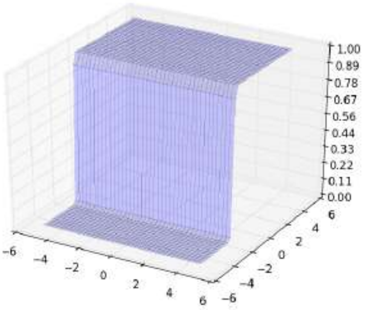
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 35$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

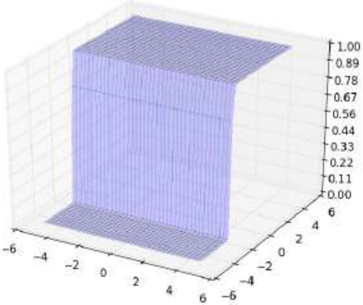
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



$w_1 = 0, w_2 = 25, b = 40$

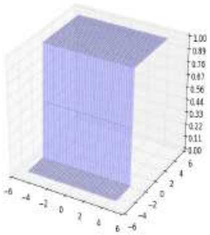
- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b

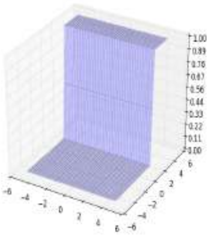
$$y = \frac{1}{1 + e^{-(w_1 x_1 + w_2 x_2 + b)}}$$



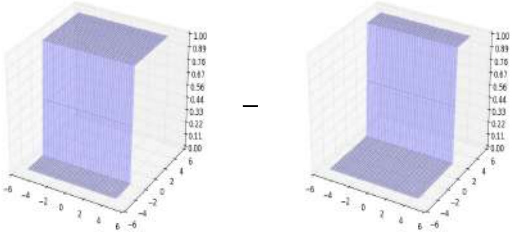
$w_1 = 0, w_2 = 25, b = 45$

- Now let us set w_1 to 0 and adjust w_2 to get a 2-dimensional step function with a different orientation
- And now we change b



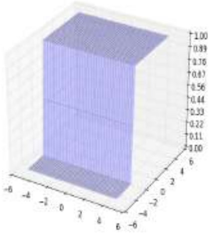


- Again, what if we take two such step functions (with different b values) and subtract one from the other



- Again, what if we take two such step functions (with different b values) and subtract one from the other

- Again, what if we take two such step functions (with different b values) and subtract one from the other



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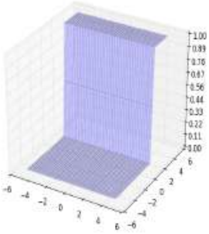
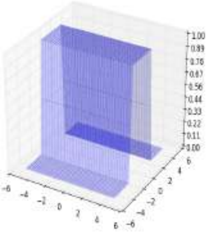
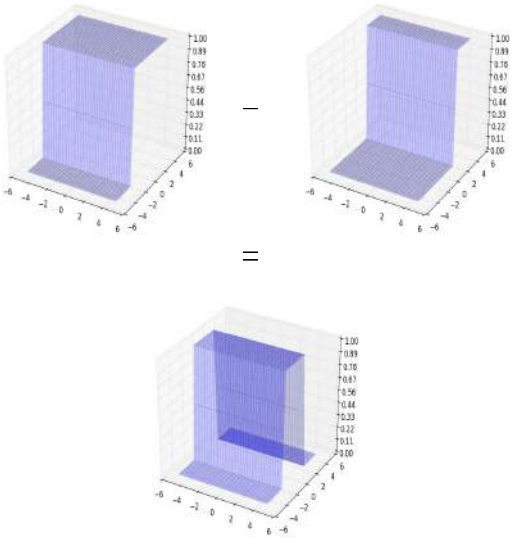
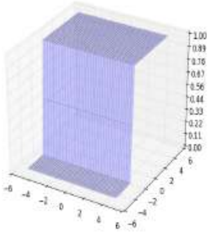


$=$





- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)



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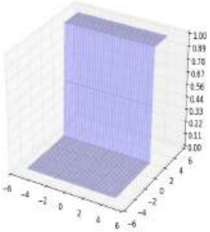
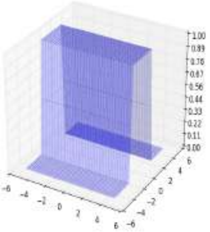
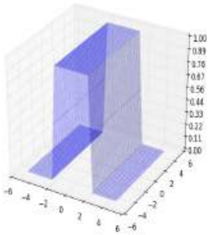
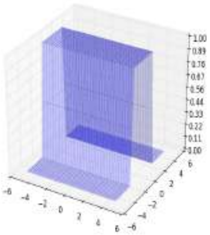


$=$

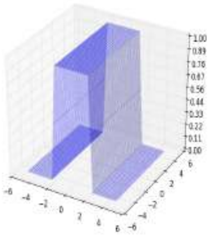


- Again, what if we take two such step functions (with different b values) and subtract one from the other
- We still don't get a tower (or we get a tower which is open from two sides)
- Notice that this open tower has a different orientation from the previous one





- Now what will we get by adding two such open towers ?



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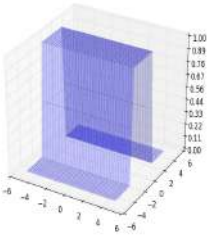
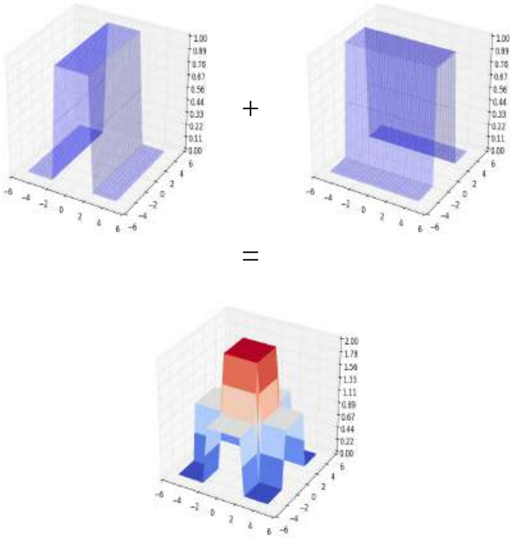
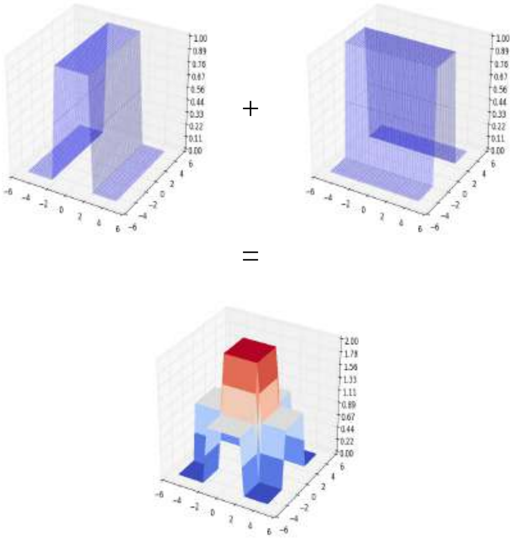


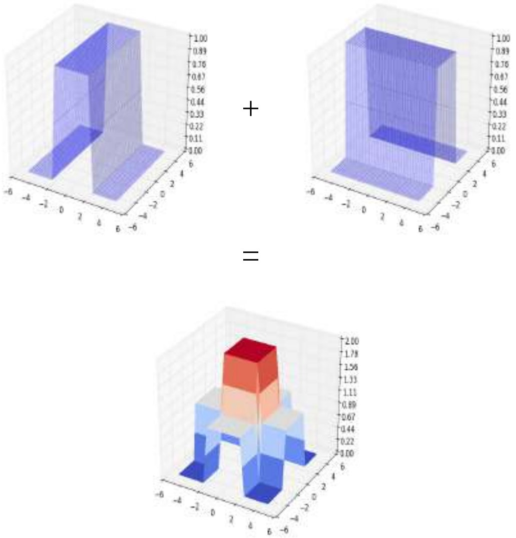
- Now what will we get by adding two such open towers ?



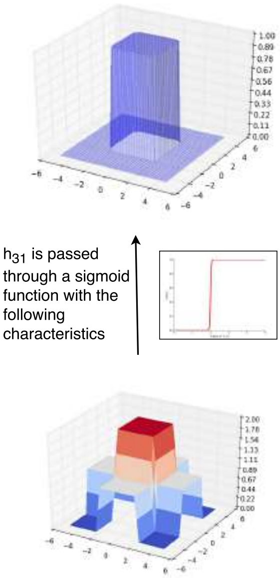
- Now what will we get by adding two such open towers ?



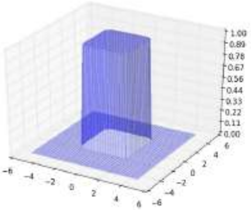
- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base



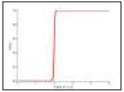
- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !

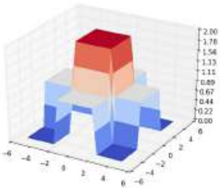


- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !

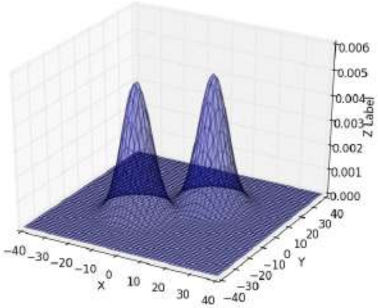


h_{31} is passed through a sigmoid function with the following characteristics



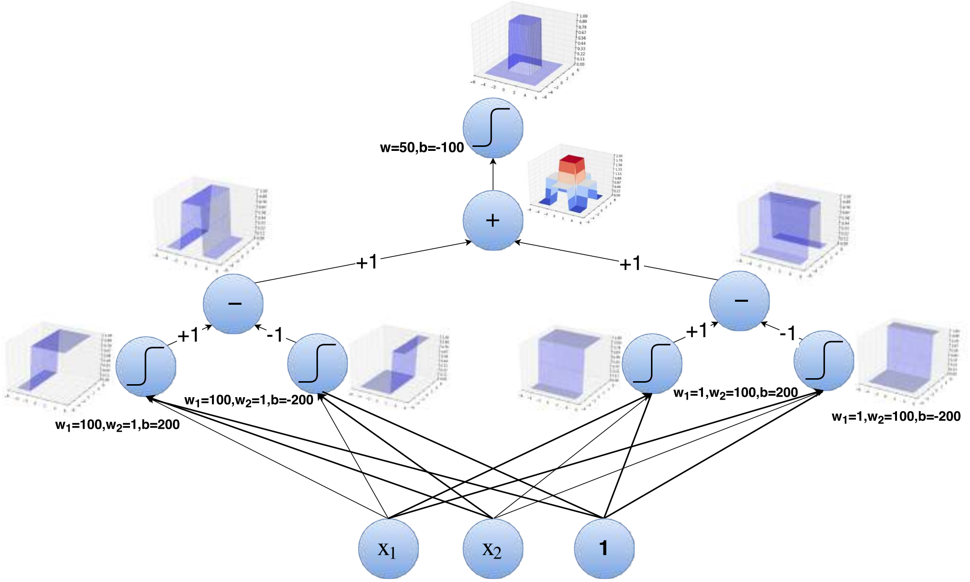


- Now what will we get by adding two such open towers ?
- We get a tower standing on an elevated base
- We can now pass this output through another sigmoid neuron to get the desired tower !
- We can now approximate any function by summing up many such towers

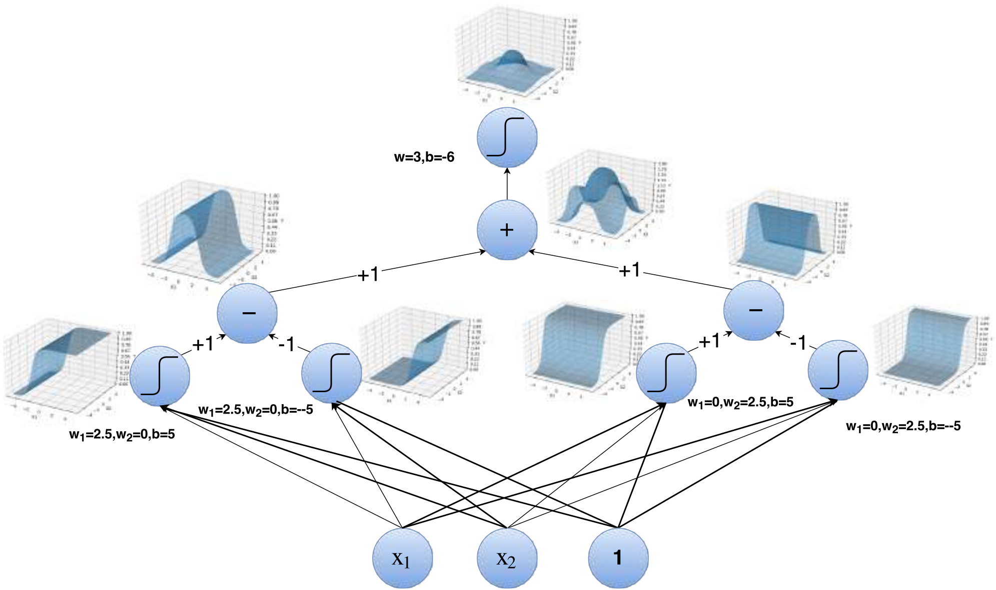


- For example, we could approximate the following function using a sum of several towers

- Can we come up with a neural network to represent this entire procedure of constructing a 3 dimensional tower ?



w=50,b=-100
+
+1
+1
–
–
+1
-1
+1
-1
w1=100,w2=1,b=200
w1=100,w2=1,b=-200
w1=1,w2=100,b=200
w1=1,w2=100,b=-200
x1
x2
1

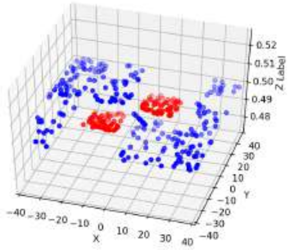

w=3,b=-6
+
+1
+1
−
−
+1
-1
+1
-1
w1=2.5,w2=0,b=5
w1=2.5,w2=0,b=--5
w1=0,w2=2.5,b=5
w1=0,w2=2.5,b=--5
x1
x2
1

Think

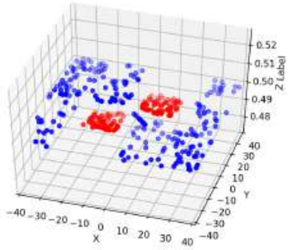
- For 1 dimensional input we needed 2 neurons to construct a tower
- For 2 dimensional input we needed 4 neurons to construct a tower
- How many neurons will you need to construct a tower in n dimensions ?

Time to retrospect

- Why do we care about approximating any arbitrary function ?
- Can we tie all this back to the classification problem that we have been dealing with ?



- We are interested in separating the blue points from the red points



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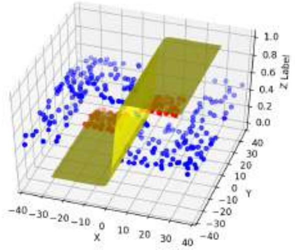
- Suppose we use a single sigmoidal neuron to approximate the relation between $x = [x_1, x_2]$ and y



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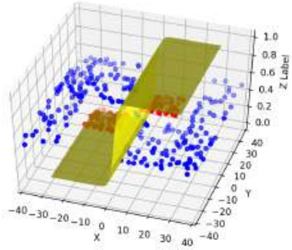

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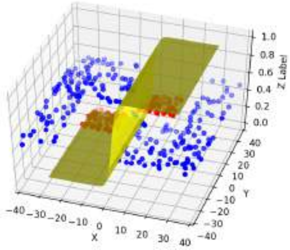

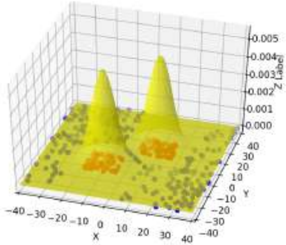

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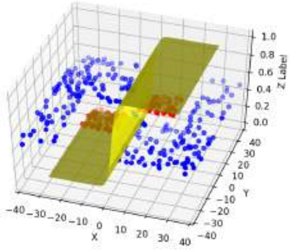

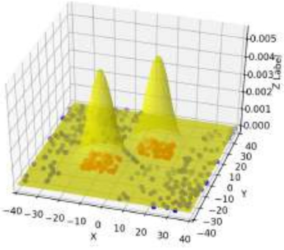

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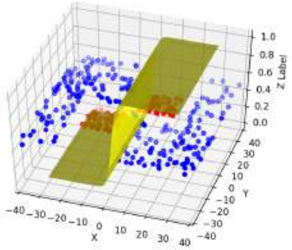
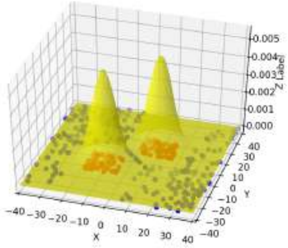
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers





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- This is what we actually want
- The illustrative proof that we just saw tells us that we can have a neural network with two hidden layers which can approximate the above function by a sum of towers
- Which means we can have a neural network which can exactly separate the blue points from the red points !!