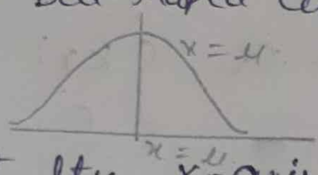
1. write any two properties of Normal distribution.

A:- (i) The Graph of normal distribution is a bell-shaped curve



(ii) It is Symmetrical about $x = \mu$

(iii) The Area under the normal curve about the x-axis is equal to the unity.

(iii) The mean, mode and median all are equal.

2. Define the kth moment of a continuous random variable X about its mean.

A:- The kth moment of a random variable X about its mean is defined as $\mu_k = E(X^k)$, Thus, the mean is the first moment $\mu = \mu_1$ and the variance can be found from first and Second moments, $\sigma^2 = \mu_2 - \mu_1^2$

3.) Find the Value of $z_{0.05}$

A:-

$P(z > z_{0.05}) = 0.05$

$F(z_{0.05}) = 1 - 0.05 = 0.9500$

$z_{0.05} = 1.645$

4.) The mean and variance of gamma distribution are respectively 8 and 32. Find α, β.

A:- The mean of gamma distribution is $\alpha\beta = 8 \Rightarrow \alpha = 8/\beta$

The Variance of gamma distribution is $\alpha\beta^2 = 32$

$\Rightarrow 8/\beta \times \beta^2 = 32 \Rightarrow \boxed{\beta = 4}$

$\alpha = 8/4 = 2 \Rightarrow \boxed{\alpha = 2}$

5.) what are the mean and variance of Gamma distribution.

A:- The mean of gamma distribution is $\alpha\beta$

The variance of gamma distribution is $\alpha\beta^2$

6.) For $\alpha = 0.2$ and $\beta = 0.5$. find the mean of weibull distribution.

A:- The mean of weibull distribution is $\alpha^{-1/\beta} \gamma(1/\beta + 1)$

$\alpha = 0.2$ & $\beta = 0.5$.

$\Rightarrow (0.2)^{-1/0.5} \gamma\left(\frac{1}{0.5}+1\right) \Rightarrow (0.2)^{-2} \gamma(2+1) \Rightarrow 25 \times 2! = \underline{\underline{50}}$.

7.) If $f(x) = \begin{cases} kx^3 & \text{for } 0<x<1 \\ 0 & \text{else where} \end{cases}$ is a probability density function of a random variable X, find the value of k.

A:- Given $f(x) - \int_{-\infty}^{\infty} f(x)dx \Rightarrow \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx$.

$= 0 + \int_0^1 kx^3 dx + 0 = 1$

$= k\left[\frac{x^4}{4}\right]_0^1 = 1 \Rightarrow k\left[\frac{1}{4} - 0\right] = 1 \Rightarrow \boxed{k=4}$

8) when the random variables x and y are said to be independent

A:- The random variable X & Y are said to be independent if it satisfies the condition $f(x,y) = f_1(x) \cdot f_2(y)$ for all x, y.

9) Define the marginal densities of the continuous random Variable X & Y.

A:- Let $f(x,y)$ be a joint probability density of x, y then the marginal probability density of X is defined as

$f_1(x) = \int_{y=-\infty}^{\infty} f(x,y)dy$ and y is $f_2(y) = \int_{x=-\infty}^{\infty} f(x,y)dx$.

10) Define the conditional probability distribution of X given Y=y.

A:- The conditional probability distribution of X given Y=y is denoted by $f_1(x/y) = \frac{f(x,y)}{f_2(y)} \; \forall x$.

1. If the probability density of a random variable is given by

$$f(x) = \begin{cases} 3e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$

Find the probabilities that the random variable will take on a value

(i) between 1 and 3; (ii) greater than 0.5.

Sol:-

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx$$

given $f(x) = \begin{cases} 3.e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

Clearly, $f(x) \geq 0$ and consider $\int_{-\infty}^{\infty} f(x)\,dx$

$$\int_{-\infty}^{\infty} f(x)dx = \int_{0}^{\infty} 3.e^{-3x}dx$$

$$= 3\left[\frac{e^{-3x}}{-3}\right]_0^{\infty} = -1[0-1] = 1$$ $f(x)$ is a probability density function.

(i) between 1 and 3.

$$P(1 < X < 3) = \int_{1}^{3} f(x)dx = \int_{1}^{3} 3.e^{-3x}dx \Rightarrow 3\left[\frac{e^{-3x}}{-3}\right]_1^3 \Rightarrow -1[e^{-9} - e^{-3}] =$$

$$\Rightarrow e^{-3} - e^{-9} = 0.0496$$

(ii) greater than 0.5.

$$P(X > 0.5) = \int_{0.5}^{\infty} f(x)dx$$

$$= \int_{0.5}^{\infty} 3.e^{-3x}.dx$$

$$= 3.\left[\frac{e^{-3x}}{-3}\right]_{0.5}^{\infty}$$

$$= [e^{-3(0.5)}]$$

$$= e^{-1.5} = \underline{\underline{0.2231}}$$

2. A random variable has a normal distribution with $\sigma = 10$. If the probability is 0.8212 that it will take on a value less than 82.5, what is the probability that it will take on a value greater than 58.3.

Sol:- given $\sigma = 10$

Let X be a random variable of a normal distribution with mean μ. and variance $\sigma = 10$

moreover probability of $X < 82.5 = 0.8212$

$P(X < 82.5) = 0.8212$

$\Rightarrow P\left(\frac{X-\mu}{\sigma} < \frac{82.5-\mu}{\sigma}\right) = 0.8212$

$\Rightarrow P\left(z < \frac{82.5-\mu}{10}\right) = 0.8212$

$\Rightarrow F\left(\frac{825-\mu}{10}\right) = 0.8212$

$\Rightarrow F\left(\frac{82.5-\mu}{10}\right) = F(0.92)$

$\Rightarrow \frac{82.5-\mu}{10} = 0.92$

$\Rightarrow 82.5-\mu = 9.2$

$\Rightarrow \boxed{\mu = 73.3}$

$P(X > 58.3)$

$\Rightarrow 1 - P(X \leq 58.3)$

$\Rightarrow 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{58.3-73.3}{10}\right)$

$\Rightarrow 1 - P(z \leq -1.5)$

$\Rightarrow 1 - P(z \leq -1.5)$

$\Rightarrow 1 - F(1.5)$

$\Rightarrow 1 - 0.9332$

$= \underline{\underline{0.070}}$

3. If 20% of the memory clips made in a certain plant are defective, what are the probabilities that in a lot of 100 randomly chosen for inspection.

(i) At most 16 will be defective.

(ii) Exacly 16 will be defective.

Sol:- Sample size $n = 100$

given that 20% of memory clips are defective

$\therefore p = N$

$p = 20\% = 0.2$

$q = 1 - p = 0.8$

mean $np = 20 = (\mu)$

Variance $\sigma^2 = \sqrt{npq} = \sqrt{16} = \underline{\underline{4}}$.

(i) probability of at most 16 will be defective is

$P(X \leq 16)$

$\Rightarrow P(X \leq 16.5)$ $\quad \{\because P(X \leq a) = P(X \leq a + 0.5)\}$

$\Rightarrow P\left(\frac{X-np}{\sqrt{npq}} \leq \frac{16.5-np}{\sqrt{npq}}\right)$

$\Rightarrow P\left(Z \le \frac{16.5-(100)(0.2)}{\sqrt{100 \times 0.2 \times 0.8}}\right)$

$\Rightarrow P\left(Z \le \frac{16.5-20}{4}\right)$

$\Rightarrow P\left(Z \le -\frac{3.5}{4}\right)$

$\Rightarrow P(Z \le -0.87)$

$\Rightarrow F(-0.87)$

$= \underline{0.1922}$

(ii) Exactly 16 will be defective.

$P(X=16) \Rightarrow P(16-0.5 \le X \le 16+0.5)$

$\Rightarrow P(15.5 \le X \le 16.5)$

$\Rightarrow P\left(\frac{15.5-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{16.5-\mu}{\sigma}\right)$

$\Rightarrow P\left(\frac{15.5-20}{4} \le Z \le \frac{16.5-20}{4}\right)$

$\Rightarrow P\left(-\frac{4.5}{4} \le Z \le -\frac{3.5}{4}\right)$

$\Rightarrow P(-1.125 \le Z \le -0.875)$

$\Rightarrow (F(0.1314) - F(0.1922))$

$\Rightarrow F(-0.87) - F(-1.12)$

$\Rightarrow 0.1922 - 0.1314 = \underline{0.0608}$

4) In a certain city the daily consumption of electric power (in millions of kilowatt-hours) can be treated as a random variable having a gamma distribution with $\alpha=2$ and $\beta=3$. If the city's power plant has a daily capacity of 10MKWH, what is the probability that this power supply will be inadequate on any given day?

Sol:- Given that the daily capacity of the power plant is 10MKWH, gamma distribution with $\alpha=2$ and $\beta=3$

$$f(x) = \begin{cases} \frac{1}{\beta^{\alpha}\gamma(\alpha)} x^{\alpha-1} \cdot e^{-x/\beta} \end{cases} \quad \begin{matrix} x>0; \\ \beta>0; \\ \alpha>0; \end{matrix}$$

$$f(x) = \frac{1}{\cancel{2^3\gamma(3)}} \cdot \frac{1}{3^2\gamma(2)} \cdot x^{2-1} \cdot e^{-x/3} \cdot dx.$$

$\{\because \gamma(\alpha+1) = \alpha\}$

$$= \frac{1}{9} \cdot x \cdot e^{-x/3} \cdot dx.$$

the probability that the power supply will adequate is probability of $P(X>10)$

$$P(X>10) = \int_{10}^{\infty} f(x)\,dx.$$

$$P(X>12) = \int_{10}^{\alpha} f(x)\,dx$$

$$= \int_{10}^{\alpha} \frac{1}{9}\, x \cdot e^{-x/3}\, dx$$

$$= \frac{1}{9}\left[x \cdot \frac{e^{-x/3}}{-1/3} - 1 \cdot \frac{e^{-x/3}}{\left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)} \right]_{10}^{\alpha}$$

$$= \frac{1}{9}\left[-3x \cdot e^{-x/3} - 9\, e^{-x/3} \right]_{10}^{\alpha}$$

$$= \frac{-3}{9}\left[x \cdot e^{-x/3} + 3\, e^{-x/3} \right]_{10}^{\alpha}$$

$$= -\frac{1}{9}\left[0 - \left[10 \cdot e^{-10/3} + 3 \cdot e^{-10/3} \right] \right]$$

$$= \frac{1}{9}\left[13 e^{-10/3} \right] = \underline{\underline{0.1516}}$$

5.) In a certain country, the probability of highway sections requiring repairs in any given year is a random variable having the beta distribution with $\alpha=3$ & $\beta=2$. find

(i) on the average, what percentage of highway sections require repairs in any given year?

(ii) find the probability that at most half of highway sections will require repairs in any given year.

Sol:- The random variable having the β-distribution with $\alpha=3$ & $\beta=2$

(i) mean $(\mu) = \frac{\alpha}{\alpha+\beta}$

The average percentage of the highway sections require repairs

$= \frac{3}{3+2} = \frac{3}{5} = 0.6 = 60\%$.

(ii) $P(X \leq 1/2) = \int_0^{1/2} f(x)\,dx$.

$$= \int_0^{1/2} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha).\Gamma(\beta)} x^{\alpha-1} \cdot (1-x)^{\beta-1} dx$$

$$= \int_0^{1/2} \frac{\Gamma(5)}{\Gamma(3).\Gamma(2)} \cdot x^2(1-x)\,dx$$

$$\Rightarrow \frac{4!}{2!1!} \int_0^{1/2} x^2(1-x)\,dx = \frac{24}{2}\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{1/2} = 12\left[\frac{1/8}{3} - \frac{1/16}{4}\right]$$

$$\Rightarrow 12\left[\frac{1}{24} - \frac{1}{64}\right] = \underline{0.3125}$$

6.) Suppose that the lifetime of a certain kind of an emergency backup battery (in hours) is a random variable X having weibull distribution with $\alpha=0.1$ and $\beta=0.5$. find

(i) The mean of life of these batteries.

(ii) The probability that such a battery will last more than 300 hours.

Sol:- Given that x is a random variable of weibull distribution with $\alpha = 0.1$ and $\beta = 0.5$

a) mean $\mu = \alpha^{-1/\beta}\gamma(1+\frac{1}{\beta}) \Rightarrow 0.1^{-1/0.5}\gamma(1+\frac{1}{0.5})$ $\quad \gamma 3 = 2!$

$\Rightarrow 0.1^{-1/0.5}\gamma(3) \Rightarrow 0.1^{-2}\gamma(3) \Rightarrow 0.1^{-2} \cdot 2!$

$= 100.(2) = 200$ hours.

b) probability that a battery is $P(X > 300) = \int_{300}^{\infty} \alpha\beta\, x^{\beta-1} \cdot e^{-\alpha x^{\beta}} dx$

$= \int_{300}^{\infty} (0.1)(0.5)\, x^{0.5-1} \cdot e^{-0.1 \cdot x^{0.5}} dx.$

$= \int_{300}^{\infty} (0.1)(0.5)\, x^{-0.5} \cdot e^{-0.1 \cdot x^{0.5}} dx.$

$\boxed{x^{0.5} = t \Rightarrow 0.5 \cdot x^{-0.5} dx = dt}$

$\Rightarrow \int_{300}^{\infty} (0.1) dt \cdot e^{-0.1t} dx$

$\Rightarrow \int_{300}^{\infty} (0.1) e^{-0.1t} dt \Rightarrow 0.1 \int_{\sqrt{300}}^{\infty} e^{-0.1t} dt$ $\quad \left[\begin{array}{l} \because \text{if } x = 300 \\ (0.1)(300)^{0.5} = t \\ \frac{0.1}{\sqrt{300}} = t \end{array}\right]$

$\Rightarrow 0.1 \int_{\sqrt{300}}^{\infty} e^{-0.1t} dt$

$\Rightarrow \cancel{0}.1 \left[\frac{e^{-0.1t}}{-\cancel{0}.1}\right]_{\sqrt{300}}^{\infty}$

$\Rightarrow \left[e^{-0.1t}\right]_{\sqrt{300}}^{\infty}$

$\Rightarrow \left[e^{-0.1\sqrt{300}}\right]$

$\Rightarrow 0.177$

7.) Two scanners are needed for an expeiment of the five available, two have electronic defects, another one has a defect in the memory and two are in good working order. Two units are selected at random.

a.) Find the joint probability distribution of X_1 = the number with electronic defects, and

X_2 = the number with a defect in memory.

b) find the probability of 0 or 1 total defects among the 1000 selected.

c) find the marginal probability distribution of X_1

Sol:- Given, no. of available scanners = 5

no. of scanners have electronic defects. = 2

no. of scanners have memory defect = 1

Good Conditioned scanners = 2

let X_1 be the random variable corresponds to the number with electronic defects and X_2 with memory defects

E.D	M.D	
X_1	X_2	G.C
0	0	2
0	1	1
1	0	1
1	1	0
2	0	0

$x_1 = 0, 1, 2$

$x_2 = 0, 1$

$G.C = 2 - (x_1 + x_2)$

$\left\{ \because {}^nC_r = \frac{n!}{(n-r)!} \right\}$

(i) $f(x_1, x_2) = \dfrac{2C_{x_1} \cdot 1C_{x_2} \cdot 2C_{2-(x_1+x_2)}}{5C_2}$

$f(0,0) = \dfrac{2C_0 \cdot 1C_0 \cdot 2C_2}{5C_2} = \dfrac{2!}{(2-0)!} \cdot \dfrac{1}{1} \cdot \dfrac{2!}{(2-2)} = 0.1$

$f(0,1) = \dfrac{2C_0 \cdot 1C_1 \cdot 2C_1}{5C_2} = 0.2$

$f(1,0) = 0.4, \; f(1,1) = 0.4.$

$f(2,0) = 0.1, \; f(2,1) = 0$

$f(x_1, x_2)$	0	1	2	
0	0.1	0.4	0.1	0.6
1	0.2	0.2	0	0.4
	0.3	0.6	0.1	

The joint distribution table.

(ii) The probability of 0 (or) 1 total defects among the two select is $f(0,0) + f(0,1) + f(1,0)$

$= 0.1 + 0.2 + 0.4$

$= \underline{\underline{0.7}}$.

(iii) Marginal probability distribution of X_1 is $f(x_1) = \sum_{x_2} f(x_1, x_2)$

then, $f_1(0) = \sum_{x_2=0,1} f(0, x_2) = f(0,0) + f(0,1) = 0.1 + 0.2 = 0.3$

$f_1(1) = \sum_{x_2=0,1} f(1, x_2) = f(1,0) + f(1,1) \Rightarrow 0.4 + 0.2 \Rightarrow 0.6$

$f_2(1) = \sum_{x_2=0,1} f(2, x_2) = f(2,0) + f(2,1) \Rightarrow 0.1 + 0 = 0.1$

8.) If two random variable have the joint density :

$$f(x_1,x_2) = \begin{cases} x_1x_2 & \text{for } 0<x_1<1, 0<x_2<2 \\ 0 & \text{elsewhere.} \end{cases}$$

find the probability that (i) Both random variables will take on variables will be less than 1.

(ii) The sum of the values taken on by the two random variables will be less than 1.

(iii) Find the marginal densities of the two random variables

Sol:- Given joint distribution $f(x_1,x_2) = \begin{cases} x_1x_2 & \text{for } 0<x_1<1 \\ & 0<x_2<2 \\ 0 & \text{elsewhere} \end{cases}$

(i) $P(x_1>1, x_2>1)$

$$= \int_0^1\int_0^1 x_1x_2\,dx_1dx_2 \Rightarrow \int_0^1 x_1\left[\frac{x_2^2}{2}\right]_0^1 dx_1 \Rightarrow \frac{1}{2}\left(\frac{x^2}{2}\right]_0^1 \Rightarrow \frac{1}{2}\times\frac{1}{2} = \frac{1}{4}$$

(ii) $$\int_{x_1=0}^{1}\int_{x_2=0}^{1-x_1} x_1x_2\,dx_1dx_2$$

$$= \int_{x_1=0}^{1} x_1\left(\frac{x_2^2}{2}\right]_0^{1-x_1} dx_1$$

$$= \frac{1}{2}\int_{x_1=0}^{1} x_1(1-x_1)^2\,dx_1 \Rightarrow \frac{1}{2}\int_0^1 (x_1+x_1^3-2x_1^2)\,dx_1$$

$$= \frac{1}{2}\left[\frac{x_1^2}{2}+\frac{x_1^4}{4}-\frac{2x_1^3}{3}\right]_0^1 \Rightarrow \frac{1}{2}\left[\frac{1}{2}+\frac{1}{4}-\frac{2}{3}\right] = \frac{1}{24}$$

(iii) The marginal probability densities are-

$$f_1(x_1) = \int_{-\infty}^{\infty} f(x_1,x_2)\,dx_2$$

$$= x_1\int_0^1 x_2\,dx_2 = \frac{x_1(1-0)}{2} = \frac{x_1}{2}.$$

$$f_2(x_2) = \int_{-\infty}^{\infty} f(x_1,x_2)\,dx_1$$

$$\Rightarrow x_2\int_0^2 x_1\,dx_1$$

$$\Rightarrow x_2\left[\frac{x_1^2}{2}\right]_0^2$$

$$\Rightarrow x_2\left[\frac{4}{2}-\frac{0}{2}\right]$$

$$\Rightarrow 2x_2$$