18MA002

Hall Ticket Number:

I/IV B.Tech. (Supplementary) DEGREE EXAMINATION

December, 2019 — **Common to all branches**

Second Semester — **Numerical Methods and Advanced Calculus**

Time: Three Hours — **Maximum:** 50 Marks

Answer Question No.1 compulsorily. (10X1 = 10 Marks)

Answer ONE question from each unit. (4X10=40 Marks)

1. Answer all questions (10X1=10 Marks)
   - a) Define a transcendental equation. 1M
   - b) Write the iterative formula to compute $\sqrt{N}$ by Newton-Raphson method. 1M
   - c) State Trapezoidal rule of integration 1M
   - d) Write Lagrange's interpolation formula. 1M
   - e) Explain about the Runge-Kutta method of fourth order. 1M
   - f) Transform $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx\,dy$ into polar form. 1M
   - g) Sketch the region of the double integral $\int_0^1 \int_0^{x^2} dx\,dy$. 1M
   - h) Is the vector field $3y^4y^2\, i + 4x^3z^2\, j + 3x^2y^2\, k$ solenoidal? 1M
   - i) Find a vector normal to the surface f(x,y,z) = xyz at the point (1,1,-1) 1M
   - j) State Green's theorem. 1M

UNIT I

2. a) Find a root of the equation $x^3 - 4x - 9 = 0$ using the bisection method 5M

   b) Using Newton's iterative method find the real root of $x\log_e x = 1.2$ correct to the five decimal places. 5M

(OR)

3. a) Solve $10x + y + z = 12; 2x + 10y + z = 13; 2x + 2y + 10z = 14$ by factorisation method. 5M

   b) Using Gauss seidal iteration method to solve the system $27x + 6y - z = 85;\ 6x + 15y + 2z = 72;\ x + y + 54z = 110$. 5M

UNIT II

4. a) The following table gives corresponding values of x and y. Construct the difference table and then express y as a function of x:

x	0	1	2	3	4
y	3	6	11	18	27

5M

   b) Given the values

x	5	7	11	13	17
y	150	392	1452	2366	5202

Evaluate f(9), using Newton's divided difference formula. 5M

(OR)

5. a) Diving the range into 10 equal parts, find the approximate value of $\int \sin x\, dx$ by Simpson's 1/3 rule. 5M

   b) Given $\frac{dy}{dx} = \frac{y-x}{y+x}$ with initial condition y=1 at x=0; find y for x=1 by Euler's method. 5M

UNIT III

6. a) Evaluate $\iint r^3 dr\, d\theta$ over the area bounded between the circle $r = 2\cos\theta$, $r = 4\cos\theta$. 5M

b) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dx\, dy$ and hence evaluate the double integral. 5M

(OR)

7. a) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. 5M

b) Find the volume bounded by the xy-plane, the cylinder $x^2 + y^2 = 1$ and the plane x+y+z=3. 5M

UNIT IV

8. a) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in a direction of the normal to the surface $3xy^2 + y = z$ at $(0,1,1)$. 5M

b) If $\bar{F} = (x^2 + y^2)\bar{i} - 2xy\bar{j}$ evaluate $\oint_C \bar{F}\, d\bar{r}$ where curve C is the rectangle in xy-plane bounded by y=0,y=b, x=0, x=a. 5M

(OR)

9. a) Use Stoke's theorem evaluate $\int_C [(x+y)dx + (2x-z)dy + (y+z)dz]$, where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6). 5M

b) Evaluate $\int_S F\, ds$ where $F = 4xI - 2y^2 J + z^2 K$ and S is the surface boundary the region $x^2 + y^2 = 4, z = 0$ and $z = 3$. 5M

BAPATLA ENGINEERING COLLEGE, BAPATLA

DEPARTMENT OF MATHEMATICS 18MA002

I/IV B.Tech (Supplementary) DEGREE EXAMINATION

Numerical Methods and Advanced calculus December-2019

1 (a). Transcendental equation: An equation involving transcendental coefficients is called a transcendental equation.

Eg: $f(x) = c_1 e^{x} + c_2 e^{-x} = 0$; $f(x) = \sin 2x + \log 5x - \frac{\pi}{2} = 0$.

(b). The iterative formula to compute $\sqrt{N}$ by Newton-Raphson method is

$$x_{n+1} = \frac{1}{2}\left(x_n + \frac{N}{x_n}\right)$$

(c). Trapezoidal rule:

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\cdots+y_{n-1})\right]$$

(d). Lagrange's interpolation formula is

$$f(x) = \frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_2)\cdots(x_0-x_n)}y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)\cdots(x_1-x_n)}y_1 + \cdots + \frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_{n-1}-x_0)(x_{n-1}-x_1)\cdots(x_{n-1}-x_n)}y_n.$$

(e) Runge-kutta method of fourth order is

$$y_{n+1} = y_n + K \;;\quad K = \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

where $K_1 = h\,f(x_n, y_n)$

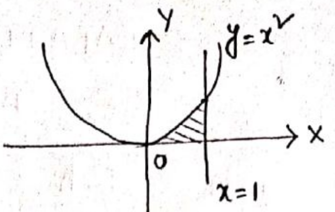
$K_2 = h\,f(x_n + \frac{h}{2}, y_n + \frac{K_1}{2})$

$K_3 = h\,f(x_n + \frac{h}{2}, y_n + \frac{K_2}{2})$

$K_4 = h\,f(x_n + h, y_n + K_3)$.

(f). In polar form $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\,dx\,dy = \int_{\theta=0}^{\pi/2} \int_{r=0}^{\infty} e^{-r^2}\, r\, dr\, d\theta$.

(g). $\int_0^1 \int_0^{x^2} dx\, dy$



(h). Let $F = 3y^6 \vec{i} + 4x^3z^2 \vec{j} + 3x^2y^2 \vec{K}$

$$\text{div } F = \nabla \cdot F = \frac{\partial}{\partial x}(3y^6) + \frac{\partial}{\partial y}(4x^3z^2) + \frac{\partial}{\partial z}(3x^2y^2) = 0.$$

$\Rightarrow F$ is solenoidal.

(i). A vector normal to the surface $f(x,y,z) = xyz$ is

$$\nabla f = yz\,\vec{i} + xz\,\vec{j} + xy\,\vec{k}$$

at the point $(1,1,-1)$, $\nabla f = -\vec{i} - \vec{j} + \vec{k}$

(j). Green's theorem: If $\phi(x,y)$, $\psi(x,y)$, ϕ_y and ψ_x be continuous in a region E of the xy-plane bounded by a closed curve C, then

$$\int_C (\phi\, dx + \psi\, dy) = \iint_E \left(\frac{\partial \psi}{\partial x} - \frac{\partial \phi}{\partial y}\right) dx\, dy.$$

$\boxed{\text{UNIT-I}}$

2(a). Let $f(x) = x^3 - 4x - 9 = 0$

$f(2) = -13$; $f(3) = 6$

Since $f(2)$ is $-$ve and $f(3)$ is $+$ve, a root lies between 2 and 3.

$\therefore x_1 = \frac{1}{2}(2+3) = 2.5$

Then $f(x_1) = x_1^3 - 4x_1 - 9 = -3.375$

$\therefore$ The root lies between x_1 and 3.

Thus $x_2 = \frac{1}{2}(x_1+3) = 2.75$

Then $f(x_2) = x_2^3 - 4x_2 - 9 = 0.7969$

$\therefore$ The root lies between x_1 and x_2.

Thus $x_3 = \frac{1}{2}(x_1 + x_2) = 2.625$

Then $f(x_3) = x_3^3 - 4x_3 - 9 = -1.4121$

$\therefore$ The root lies between x_2 and x_3.

Thus $x_4 = \frac{1}{2}(x_2 + x_3) = 2.6875$

Hence the root is 2.6875 approximately.

2(b). Let $f(x) = x\log_{10} x - 1.2$

$f(1) = -1.2$

$f(2) = -0.59794$

$f(3) = 0.23136$

So a root of $f(x)=0$ lies between 2 and 3. Let $x_0 = 2$

And $f'(x) = \log_{10} x + x \cdot \frac{1}{x} \log_{10} e = \log_{10} x + 0.43429$

$\therefore$ By Newton's iteration formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{0.43429\, x_n + 1.2}{\log_{10} x_n + 0.43429}$$

$$x_1 = \frac{0.43429 \times 2 + 1.2}{\log_{10} 2 + 0.43429} = 2.81$$

$$x_2 = \frac{0.43429 \times 2.81 + 1.2}{\log_{10} 2.81 + 0.43429} = 2.741$$

$$x_3 = \frac{0.43429 \times 2.741 + 1.2}{\log_{10} 2.741 + 0.43429} = 2.74064$$

$$x_4 = \frac{0.43429 \times 2.74064 + 1.2}{\log_{10} 2.74064 + 0.43429} = 2.74065$$

$$x_5 = \frac{0.43429 \times 2.74065 + 1.2}{\log_{10} 2.74065 + 0.43429} = 2.74065$$

clearly $x_4 = x_5$

Hence the required root is 2.74065.

(OR)

3(a). $10x + y + z = 12$; $2x + 10y + z = 13$; $2x + 2y + 10z = 14$.

consider the given system of equations in the matrix form $AX = B$.

$$\text{i.e, } \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$$

Let $A = LU$

$$\Rightarrow \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}.$$

$$\Rightarrow \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ l_{21} u_{11} & l_{21} u_{12} + u_{22} & l_{21} u_{13} + u_{23} \\ l_{31} u_{11} & l_{31} u_{12} + l_{32} u_{22} & l_{31} u_{13} + l_{32} u_{23} + u_{33} \end{bmatrix}$$

$\therefore u_{11} = 10 \quad ; u_{12} = 1 \quad ; u_{13} = 1$

$l_{21} u_{11} = 2 \Rightarrow l_{21} = 0.2$

$l_{21} u_{12} + u_{22} = 10 \Rightarrow u_{22} = 9.8$

$l_{21} u_{13} + u_{23} = 1 \Rightarrow u_{23} = 0.8$

$l_{31} u_{11} = 2 \Rightarrow l_{31} = 0.2$

$l_{31} u_{12} + l_{32} u_{22} = 2 \Rightarrow l_{32} = 0.1837$

$l_{31} u_{13} + l_{32} u_{23} + u_{33} = 10 \Rightarrow u_{33} = 9.6530.$

$$\therefore \begin{bmatrix} 10 & 1 & 1 \\ 2 & 10 & 1 \\ 2 & 2 & 10 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.2 & 0.1837 & 1 \end{bmatrix} \begin{bmatrix} 10 & 1 & 1 \\ 0 & 9.8 & 0.8 \\ 0 & 0 & 9.6530 \end{bmatrix}$$

consider $LY = B \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0.2 & 1 & 0 \\ 0.2 & 0.1837 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 13 \\ 14 \end{bmatrix}$

$\Rightarrow y_1 = 12$

$\Rightarrow 0.2y_1 + y_2 = 13 \Rightarrow y_2 = 10.6$

$\Rightarrow 0.2y_1 + 0.1837y_2 + y_3 = 14 \Rightarrow y_3 = 9.6530.$

Now $UX = Y \Rightarrow \begin{bmatrix} 10 & 1 & 1 \\ 0 & 9.8 & 0.8 \\ 0 & 0 & 9.6530 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 10.6 \\ 9.6530 \end{bmatrix}$

$\Rightarrow 10x + y + z = 12$ —— ①

$9.8y + 0.8z = 10.6$ —— ②

$9.6530 z = 9.6530 \Rightarrow z = 1$

from ②, $y = 1$

from ①, $x = 1.$

$\therefore (x, y, z) = (1, 1, 1).$

3(b). The given system of equations are

$27x + 6y - z = 85$; $6x + 15y + 2z = 72$; $x + y + 54z = 110$.

The given system is diagonally dominant, and we rewrite as

$$x = \frac{1}{27}(85 - 6y + z)$$

$$y = \frac{1}{15}(72 - 6x - 2z)$$

$$z = \frac{1}{54}(110 - x - y).$$

we start iteration by taking $y=0$ and $z=0$.

$x^{(1)} = 3.15$, $y^{(1)} = 3.54$, $z^{(1)} = 1.91$

$x^{(2)} = 2.43$, $y^{(2)} = 3.57$, $z^{(2)} = 1.926$

$x^{(3)} = 2.426$, $y^{(3)} = 3.572$, $z^{(3)} = 1.926$

$x^{(4)} = 2.425$, $y^{(4)} = 3.573$, $z^{(4)} = 1.926$ and

$x^{(5)} = 2.425$, $y^{(5)} = 3.573$, $z^{(5)} = 1.926$.

The solution is $x = 2.425$, $y = 3.573$, $z = 1.926$.

UNIT-II

4(a). The difference table is

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	3			
		3		
1	6		2	
		5		0
2	11		2	
		7		0
3	18		2	
		9		
4	27			

Here $h=1$, $x_0=0$, $p = \frac{x-x_0}{h} = x$.

By Newton's forward interpolation formula,

$$f(x) = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \cdots$$

$$\Rightarrow f(x) = 3 + x(3) + \frac{x(x-1)}{2}(2)$$

$$\Rightarrow f(x) = x^2 + 2x + 3.$$

4(b). The divided difference table is

x	$f(x)$	1st. divided differences	2nd divided differences	3rd divided differences
5	150			
		121		
7	392		24	
		265		1
11	1452		32	
		457		1
13	2366		42	
		709		
17	5202			

Taking $x=9$ in the Newton's divided difference formula, we get

$$f(x) = f(x_0) + (x-x_0)\, f[x_0,x_1] + (x-x_0)(x-x_1)\, f[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)\, f[x_0,x_1,x_2,x_3]$$

$$\Rightarrow f(9) = 150 + (9-5)(121) + (9-5)(9-7)(24) + (9-5)(9-7)(9-11)(1)$$

$$\Rightarrow f(9) = 150 + 484 + 192 - 16 = 810.$$

(OR)

5(a). Here $n=10$ and $h = \frac{\pi - 0}{10} = \frac{\pi}{10}$.

$\therefore$ The table of values is

x	0	$\pi/10$	$2\pi/10$	$3\pi/10$	$4\pi/10$	$5\pi/10$	$6\pi/10$	$7\pi/10$	$8\pi/10$	$9\pi/10$	$10\pi/10$
$y=\sin x$	0	0.3090	0.5878	0.8090	0.9511	1	0.9511	0.8090	0.5878	0.3090	0

By Simpson's rule,

$$\int_{x_0}^{x_n} f(x)\,dx = \frac{h}{3}\left[(y_0+y_n) + 4(y_1+y_3+\cdots+y_{n-1}) + 2(y_2+y_4+\cdots+y_{n-2})\right]$$

$$\Rightarrow \int_0^{\pi} \sin x\,dx = \frac{\pi}{30}\left[(0+0) + 4(0.3090+0.8090+1+0.8090+0.3090) + 2(0.5878+0.9511+0.9511+0.5878)\right]$$

$$= 2.0009.$$

5(b). $$\frac{dy}{dx} = \frac{y-x}{y+x}$$

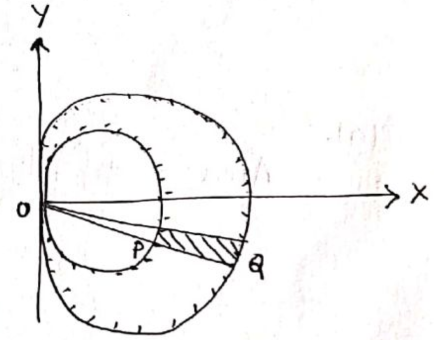
we divide the interval (0,1) into 5 steps. we take $n=5$ and $h=0.2$.

The calculations are

x	y	$dy/dx = (y-x)/(y+x)$	old y + $(0.2)(dy/dx)$ = new y
0	1	1	$1+(0.2)(1) = 1.200$
0.2	1.200	0.7143	$1.200+(0.2)(0.7143) = 1.3429$
0.4	1.3429	0.5410	$1.3429+(0.2)(0.5410) = 1.4511$
0.6	1.4511	0.4149	$1.4511+(0.2)(0.4149) = 1.5341$
0.8	1.5341	0.3145	$1.5341+(0.2)(0.3145) = 1.5970$.
1	1.5970		

Hence the required approximate value of y is 1.5970.

UNIT-III



6(a). $$\iint r^3\, dr\, d\theta = \int_{\theta=-\pi/2}^{\pi/2} \int_{r=2\cos\theta}^{4\cos\theta} r^3\, dr\, d\theta$$

$$= \int_{\theta=-\pi/2}^{\pi/2} \left(\frac{r^4}{4}\right)_{2\cos\theta}^{4\cos\theta} d\theta$$

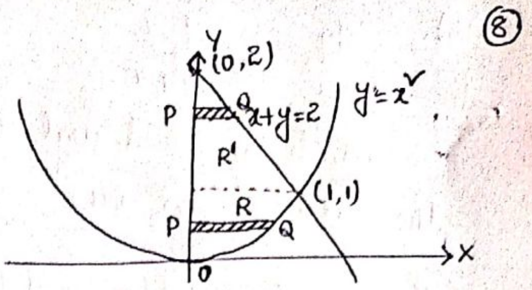
$$= 60\int_{\theta=-\pi/2}^{\pi/2} \cos^4\theta\, d\theta$$

$$= 120\int_{\theta=0}^{\pi/2} \cos^4\theta\, d\theta$$

$$= 120 \times \frac{3\times 1}{4\times 2} \times \frac{\pi}{2}$$

$$= \frac{45\pi}{2}.$$

6(b). $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$



$= \iint_R xy \, dx \, dy + \iint_{R'} xy \, dx \, dy$

$= \int_{y=0}^{1} \int_{x=0}^{\sqrt{y}} xy \, dx \, dy + \int_{y=1}^{2} \int_{x=0}^{2-y} xy \, dx \, dy$

$= \int_{y=0}^{1} \left(\frac{x^2}{2}\right)_0^{\sqrt{y}} y \, dy + \int_{y=1}^{2} \left(\frac{x^2}{2}\right)_0^{2-y} y \, dy$

$= \frac{1}{2}\int_{y=0}^{1} y^2 \, dy + \frac{1}{2}\int_{y=1}^{2} [4y - 4y^2 + y^3] \, dy$

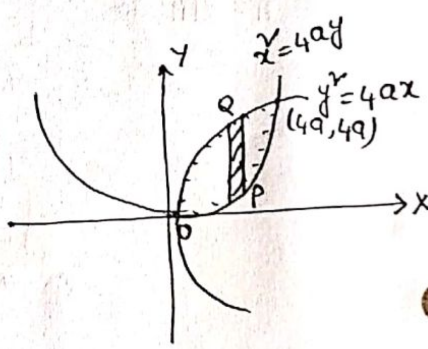
$= \frac{1}{2}\left(\frac{y^3}{3}\right)_0^1 + \frac{1}{2}\left[4\frac{y^2}{2} - 4\frac{y^3}{3} + \frac{y^4}{4}\right]_1^2$

$= \frac{1}{2}\left(\frac{1}{3}\right) + \frac{1}{2}\left[\left(8 - \frac{32}{3} + 4\right) - \left(2 - \frac{4}{3} + \frac{1}{4}\right)\right] = \frac{3}{8}.$

(OR)

7(a).

Area $= \iint_R dy \, dx$



$= \int_{x=0}^{4a} \int_{y=x^2/4a}^{2\sqrt{ax}} dy \, dx$

$= \int_{x=0}^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$

$= \left[2\sqrt{a} \cdot \frac{2}{3} x^{3/2} - \frac{1}{4a} \cdot \frac{x^3}{3}\right]_0^{4a}$

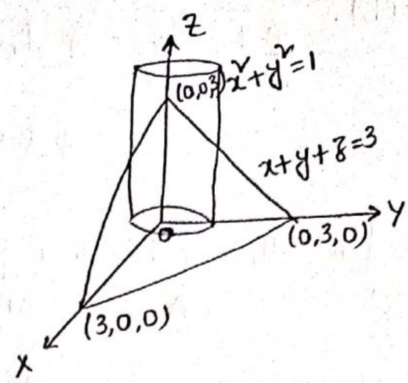
$= \frac{32}{3}a^2 - \frac{16}{3}a^2$

$= \frac{16}{3}a^2.$

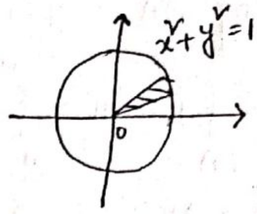
7(b).

$$\text{Volume} = \iiint_V dx\,dy\,dz$$

$$= \int_{x^2+y^2=1}\int \int_{z=0}^{3-x-y} dz\,dy\,dx$$



$$= \iint_{x^2+y^2=1} (3-x-y)\,dy\,dx$$



$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{1} (3 - r\cos\theta - r\sin\theta)\, r\,dr\,d\theta$$

$$= \int_{\theta=0}^{2\pi} \left(\frac{3r^2}{2} - \frac{r^3}{3}\cos\theta - \frac{r^3}{3}\sin\theta\right)_0^1 d\theta$$

$$= \left(\frac{3}{2}\theta - \frac{1}{3}\sin\theta + \frac{1}{3}\cos\theta\right)_0^{2\pi}$$

$$= 3\pi.$$

UNIT-IV

8(a). Let $f(x,y,z) = 3xy^2+y-z=0$.

Vector normal to the surface $f(x,y,z)$ at $(0,1,1)$ is

$$\nabla f = 3y^2\vec{i} + (6xy+1)\vec{j} - \vec{k}$$

$$(\nabla f)_{(0,1,1)} = 3\vec{i} + \vec{j} - \vec{k}$$

Unit vector normal to the surface $f(x,y,z)$ at $(0,1,1)$ is $\dfrac{3\vec{i}+\vec{j}-\vec{k}}{\sqrt{11}}$

Let $g(x,y,z) = xyz^2 + xz$.

Then $\nabla g = (yz^2+z)\vec{i} + xz^2\vec{j} + (2xyz+x)\vec{k}$

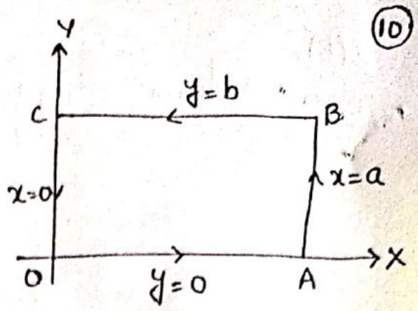
$$(\nabla g)_{(1,1,1)} = 2\vec{i} + \vec{j} + 3\vec{k}.$$

$\therefore$ The directional derivative is $\nabla g \cdot \hat{f} = (2\vec{i}+\vec{j}+3\vec{k})\cdot\left(\dfrac{3\vec{i}+\vec{j}-\vec{k}}{\sqrt{11}}\right)$

$$= \frac{6+1-3}{\sqrt{11}} = \frac{4}{\sqrt{11}}.$$

8(b). $\oint_{C} \bar{F}\cdot \bar{dr} = \oint F_1\,dx + F_2\,dy$

$= \oint (x^2+y^2)\,dx - 2xy\,dy$



$$\int_C F\cdot dr = \int_{OA} F\cdot dr + \int_{AB} F\cdot dr + \int_{BC} F\cdot dr + \int_{CO} F\cdot dr$$

Along OA: $y=0$, $dy=0$ and x varies from 0 to a.

$$\therefore \int_{OA} F\cdot dr = \int_0^a x^2\,dx = \frac{a^3}{3}$$

Along AB: $x=a$, $dx=0$ and y varies from 0 to b.

$$\therefore \int_{AB} F\cdot dr = \int_0^b (-2ay)\,dy = -ab^2$$

Along BC: $y=b$, $dy=0$ and x varies from a to 0.

$$\therefore \int_{BC} F\cdot dr = \int_a^0 (x^2+b^2)\,dx = -\frac{a^3}{3} - ab^2.$$

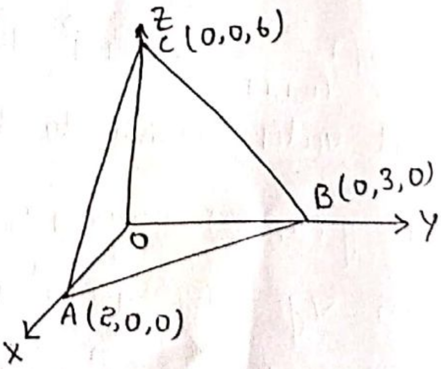
Along CO: $x=0$, $dx=0$ and y varies from b to 0.

$$\therefore \int_{CO} F\cdot dr = \int_b^0 0\,dy = 0.$$

$$\therefore \oint_C F\cdot dr = \frac{a^3}{3} - ab^2 - \frac{a^3}{3} - ab^2 = -2ab^2.$$

(OR)

9(a). Here $F = (x+y)\,I + (2x-z)\,J + (y+z)\,K$



$$\therefore \text{Curl } F = \begin{vmatrix} I & J & K \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x+y & 2x-z & y+z \end{vmatrix} = 2I + K.$$

vector normal to the plane $3x+2y+z=6$ is

$$\nabla(3x+2y+z-6) = 3I + 2J + K.$$

$$\therefore \hat{N} = \frac{3I+2J+K}{\sqrt{3^2+2^2+1^2}} = \frac{1}{\sqrt{14}}(3I+2J+K).$$

Hence $\int_C [(x+y)\,dx + (2x-z)\,dy + (y+z)\,dz]$

$$= \int_C F \cdot dR$$

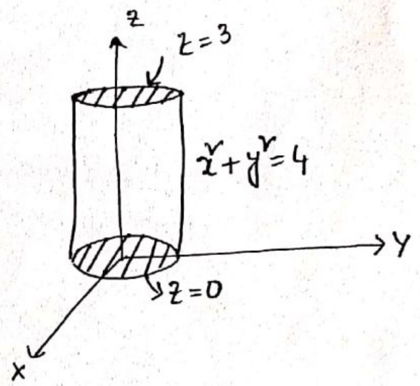
$= \int_S \text{curl } F \cdot \hat{N}\, ds$, where S is the triangle ABC.

$$= \int_S (2I+K) \cdot \left(\frac{3I+2J+K}{\sqrt{14}}\right) ds$$

$$= \int_S \frac{1}{\sqrt{14}} (6+1)\, ds$$

$= \frac{7}{\sqrt{14}}$ (Area of ΔABC) $= \frac{7}{\sqrt{14}} \cdot 3\sqrt{14} = 21$.

9(b). By divergence theorem,



$$\int_S F \cdot ds = \int_V \text{div } F\, dv$$

$$= \int_V \left[\frac{\partial}{\partial x}(4x) + \frac{\partial}{\partial y}(-2y^2) + \frac{\partial}{\partial z}(z^2)\right] dv$$

$$= \iiint_V (4-4y+2z)\, dx\, dy\, dz$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_0^3 (4-4y+2z)\, dz\, dy\, dx$$

$$= \int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} (12-12y+9)\, dy\, dx$$

$$= \int_{-2}^{2} \left(21y - 6y^2\right)_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} dx$$

$$= 42 \int_{-2}^{2} \left(\sqrt{4-x^2}\right) dx$$

$$= 84 \int_0^2 \sqrt{4-x^2}\, dx$$

$$= 84 \left[\frac{x}{2}\sqrt{4-x^2} + \frac{4}{2}\sin^{-1}\left(\frac{x}{2}\right)\right]_0^2 = 84\pi .$$