

UNIT-1

Automata theory: It is the study of abstract computing devices or machine.

* It is the model for both h/w & s/w.

Why we need to study automata theory?

Automata is an useful model for many important kinds h/w & s/w.

Some kinds of automata are:-

- i; S/w for designing & checking the behaviour of digital circuits.
- ii; lexical analysis of compiler that breaks input keywords identifying e.g. text into logical units such as valid or not, semantic error \Rightarrow S/O.
- iii; S/w for scanning large bodies of text, collection of webpages to find occurrences of words, phases & other patterns.
- iv; Also S/w for verifying systems of all types that have a finite no. of distinct states.

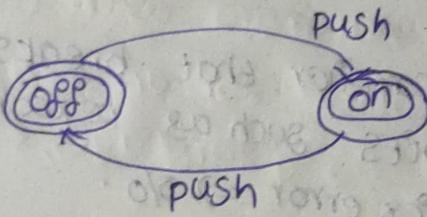
Classification of Automata:

1. Finite Automata (FA)
2. push down Automata (PDA)
3. linear Bounded Automata (LBA)
4. Turing machine (TM)

Finite Automata:-

- * Finite Automata is simplest machine to recognize patterns.
- * Finite Automata / finite state machine is an abstract machine that have 's' elements or tuples.
- * It has a set of states & rules for moving states from one to another, but it depends upon the applied input symbol.

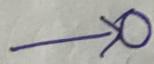
Ex:- on or off switch



* finite automata states are represented by circles labeled by some input symbol.

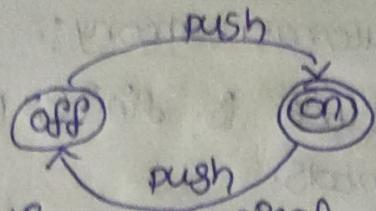
* In the above diagram total no. of states are "2"; input symbol is "push" named as on & off.

* In finite Automata one of the state designated as Start state which is represented by forward arrow followed by circle.



* In the above diagram starting state or initial state is "off".

* It is necessary to indicate one or more states as final or accepting states which is represented by double circle.



In the above diagram final state is "on".

Structural representation of an Automata:

There are 2 important notations plays an important role in study of Automata & their applications.

① Wrommers: Wrommers are useful model that process the data with recursive structure.

Ex:

$$\epsilon \rightarrow a\epsilon/b\epsilon$$

$$\epsilon \rightarrow a/b$$

2. Regular Expression:-

RE also denotes structure of the data especially text string. The unique style of Regular expression is $[A-Z][a-z]^*[*][0-9]^+$

where $[A-Z]$ represents range of characters from A-Z.

$[a-z]^*$ represents range of any no. of lower case characters from a-z.

$[*]$ represents any no. of times

$[+]$ represents atleast one occurrence.

$[0-9]^+$ represents atleast one digit from 0-9.

Note:- Parathesis are used to group the contents of an expression.

Central Concepts of Automata Theory:-

- ① Alphabet (Σ) :- Alphabet is finite, non-empty collection of symbols.
To represent an alphabet symbol " Σ " (sigma) is used.
- Ex:- 1. $\Sigma = \{0, 1\}$ \rightarrow binary alphabet
2. $\Sigma = \{a, b, c, \dots, z\}$ \rightarrow represents set of all lower case alphabet.
3. $\Sigma = \{0, 1, 2, \dots, 9\}$ \rightarrow set of all digits.
4. $\Sigma = \{A, B, \dots, Z\}$ \rightarrow represents set of all upper case alphabet.
5. set of ASCII characters.

- ② String :- It is a finite collection of symbols from alphabet (or)
String is a finite sequence of characters chosen from some alphabet.
- Ex:- i. 0101 is a string chosen from binary alphabet
ii. aba is a string which is chosen from $\Sigma = \{a, b\}$

- ③ empty string (ϵ) :-
It is a string with zero occurrence of symbol denoted by " ϵ " (epsilon)
empty string is a string of length "zero".
- Ex:- 10101 = 4 $|\epsilon| = 0$

4. length of string:
 the no. of positions for symbols in string is called length of the string (or) no. of symbols in string gives length of the string.
 Standard notation for length of the string is $|w|$.

Ex:- $w = a10bc$
 $|w| = 5$

5. Power of an alphabet: - If " Σ " is an alphabet then we define Σ^k to be the set of strings of length "k". Each of whose symbols is in " Σ ".

Ex:- if $\Sigma = \{a, b\}$ then $\Sigma^0 = \{\epsilon\}$
 where " ϵ " is the only string whose length is "zero".
 $\Sigma^1 = \{a, b\}$, $\Sigma^2 = \{aa, ab, ba, bb\}$
 $\Sigma^3 = \{aaa, aab, aba, baa, bba, bab, baa, bbb\}$
 $\Sigma^* \Rightarrow$ set of length of strings $0, 1, 2, 3, \dots$
 $\Sigma^* = \{\epsilon, a, b, aa, ab, ba, bb, aaa, \dots\}$
 $= \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$

Note:- ① set of all strings over the alphabet Σ is denoted by Σ^*
 ② set of non-empty strings from the alphabet Σ is denoted by Σ^+
 $\Sigma^+ = \Sigma^* - \{\epsilon\} = \Sigma^0 \cup \Sigma^1 \cup \Sigma^2 \cup \Sigma^3 \cup \dots$

3. set of all strings over the alphabet Σ is
 $\Sigma^* = \Sigma^+ \cup \{\epsilon\} = \Sigma^0 \Sigma^1 \Sigma^2 \dots$
- ⑥ concatenation of strings:-
 let x & y be two strings then xy denotes the concatenation of x and y where x is a string composed of ' i ' symbol.
 i.e $x = \{a_0, a_1, a_2, a_3, \dots, a_i\}$ and
 y is string composed of ' j ' symbol.
 i.e $y = \{b_0, b_1, b_2, \dots, b_j\}$
 then $xy = a_0 a_1 a_2 \dots a_i b_0 b_1 b_2 \dots b_j$
- Ex:-
 if $x = 0110$ & $y = 100$ then $xy = 0110100$
- Note:-
 for any string w , $\epsilon \cdot w = w$. $\epsilon = w$
 here ϵ is identity for concatenation.
- $\epsilon \text{psilon} = \epsilon$
- $x = a, y = b$
 $xy = ab$
- $x = a, y = \epsilon$
 $xy = a\epsilon = (a\epsilon) = 1$
- ⑦ language:-
 language is a collection of appropriate strings
 (or) language is a set of strings all are which
 is chosen from an alphabet Σ^*
 where Σ is particular alphabet.

language of an automata is denoted by ' L '
if Σ is an alphabet then $L \subseteq \Sigma^*$

Q1:
the language of all strings consists of ~~all~~ n zeroes
followed by n is $\Sigma = \{0, 1\}$ where $n \geq 1$
 $L = \{01, 001, 011, \dots\}$

Ex3:
set of all strings which contains equal no. of 0's
& 1's over the alphabet $\Sigma = \{0, 1\}$
 $L = \{\epsilon, 01, 10, 0011, 1100, 0101, 1010, 1001, 0110, \dots\}$

- Note:
1. ϕ is an empty language i.e it has no strings
 $\phi = \{\}$
 2. $L = \{\epsilon\}$ is not an empty language i.e it contains one string ' ϵ '.
 3. $\phi \neq \{\epsilon\}$ b/c ϕ has no strings where as $\{\epsilon\}$ has one string named epsilon ' ϵ '.

It is common to describe set using formula
i.e. $\{\omega \mid \text{something about } \omega\}$

Ex:-

$L = \{\omega \mid \omega \text{ consists of equal no. of zeroes & ones}\}$

$$\Sigma = \{0, 1\}$$

Ex:-
 $L = \{\omega \mid \omega \text{ is a binary integer whose value is prime}\}$

Ex:-

$L = \{\omega \mid \omega \text{ is a symmetrically correct 'C' pgm.}\}$

Describe the following language over input

Set $A = \{a, b\}$

1. $L_1 = \{a, ab, ab^2, ab^3, \dots\}$

$L_1 = \{\omega \mid \omega \text{ is string which contains only one 'a' followed by any no. of 'b's}\}$

2. $L_2 = \{\epsilon, a, aa, aaa, \dots\}$

$L_2 = \{\omega \mid \omega \text{ is string which contains any no. of 'a's & 0 no. of 'b's}\}$

essay

Deterministic finite automata (DFA):-

DFA consists of :-

1. a finite set of states denoted by Q .
 2. finite set of i/p symbols denoted by Σ .
 3. The transition function (δ) that takes argument as state & i/p symbol and return its state. which can be defined as
- $$\boxed{\delta: Q \times \Sigma \rightarrow Q}$$
4. A start state ' q_0 ' i.e one of the state in Q .
5. set of final/^{accepting} states that are denoted by F .
6. Deterministic finite automata will be referred by it's acronym DFA.
7. DFA in ~~rightable~~ notation

$$A = \{Q, \Sigma, \delta, q_0, F\}$$

Ex:- Design DFA for all the strings which ends with zero over the alphabets $\Sigma = \{0, 1\}$.

Sol:-
DFA with '0'

Sol:-

$$Q = \{q_0, q_f\}$$

$$\Sigma = \{0, 1\}$$

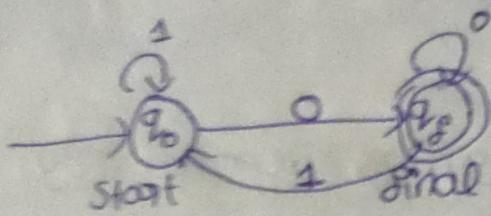
$$\delta(q_0, 0) = q_f$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_f, 0) = q_f$$

$$\delta(q_f, 1) = q_0$$

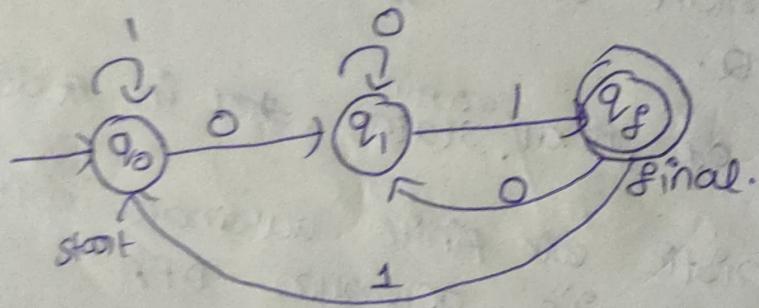
ends with '0'



not above points

Ex2:
design DFA for all the strings which ends with 0,1 and over the alphabets $\Sigma = \{0,1\}$.

Sol:



$$Q = \{q_0, q_1, q_f\}$$

$$\Sigma = \{0,1\}$$

$$\delta(q_0, 0) = q_1$$

$$\delta(q_0, 1) = q_f$$

$$\delta(q_1, 1) = q_f$$

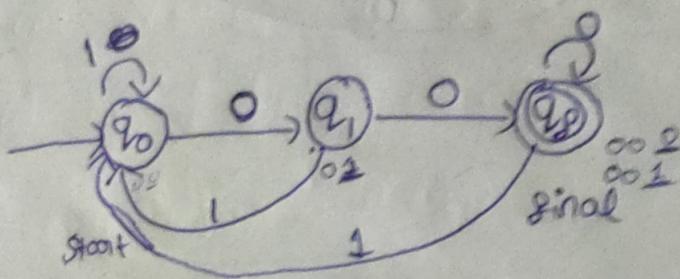
$$\delta(q_1, 0) = q_1$$

$$\delta(q_f, 0) = q_1$$

$$\delta(q_f, 1) = q_0$$

Ex-3: Design DFA for all strings which ends with '00' over the alphabets $\Sigma = \{0, 1\}$

Sol:-



$$Q = \{q_0, q_1, q_f\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_1$$

$$\delta(q_0, 1) = q_0$$

$$\delta(q_1, 0) = q_f$$

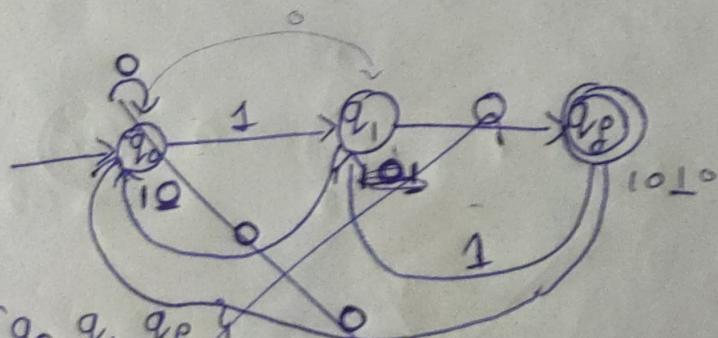
$$\delta(q_1, 1) = q_0$$

$$\delta(q_f, 0) = q_f$$

$$\delta(q_f, 1) = q_0$$

Ex-4: Design DFA for all strings which ends with '10' over the alphabets $\Sigma = \{0, 1\}$

Sol:-



$$Q = \{q_0, q_1, q_f\}$$

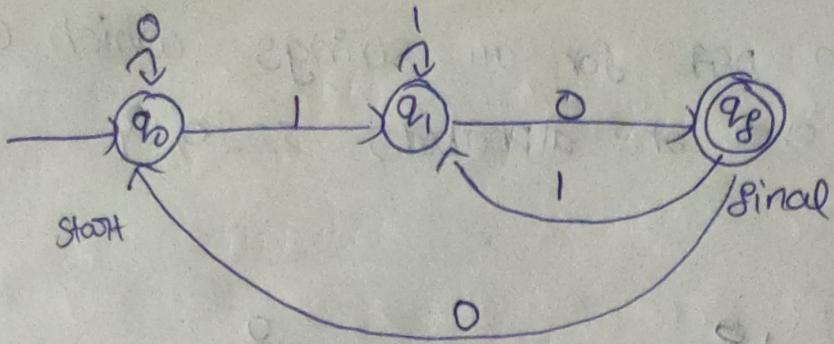
$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_0$$

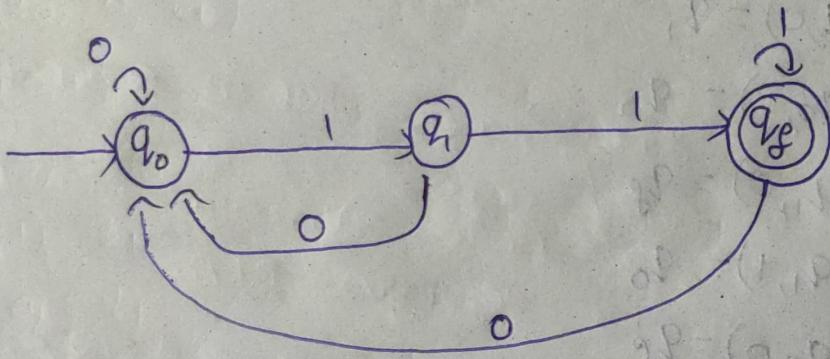
$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_f$$

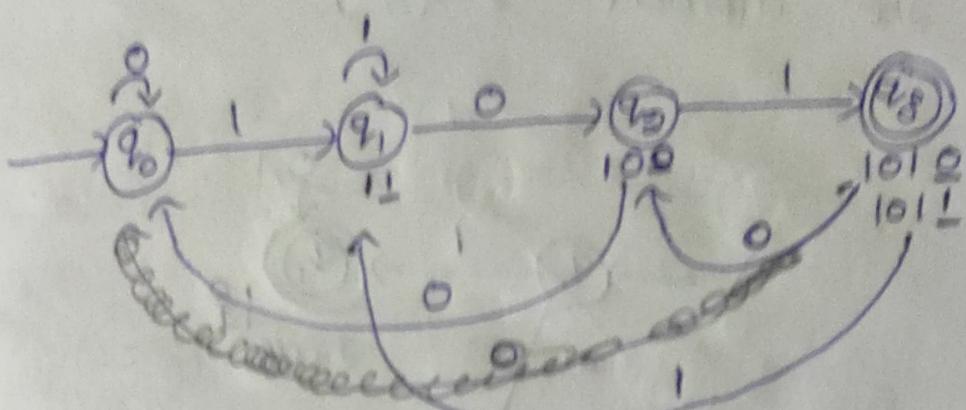
$$\delta(q_1, 1) =$$



Ex-5: design colth DFA for all strings which ends



Q6: Design DFA for all strings which ends with 101 over the alphabets $\Sigma = \{0, 1\}$



$$Q = \{q_0, q_1, q_2, q_f\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_f$$

$$\delta(q_2, 0) = q_0$$

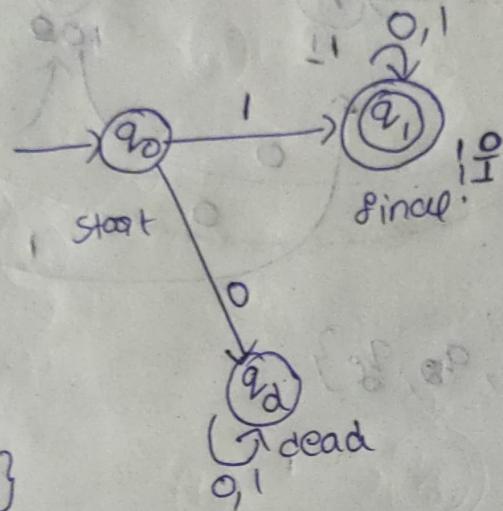
$$\delta(q_2, 1) = q_f$$

$$\delta(q_f, 0) = q_0$$

$$\delta(q_f, 1) = q_1$$

Ex-7:- Design DFA for all strings which accepts for all strings that starts with 1 over the alphabet $\Sigma = \{0, 1\}$.

Sol:-



$$Q = \{q_0, q_1, q_d\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_d$$

$$\delta(q_0, 1) = q_1$$

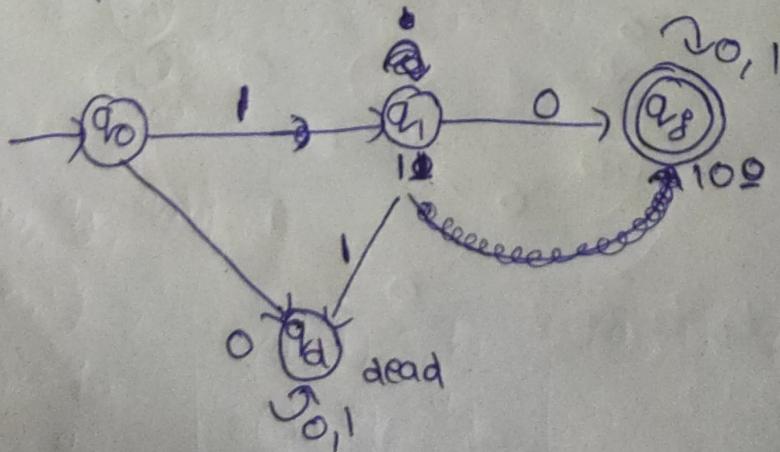
$$\delta(q_1, 0) = q_1$$

$$\delta(q_1, 1) = q_1$$

$$\delta(q_d, 0) = q_d$$

$$\delta(q_d, 1) = q_d$$

Ex-8:- Design DFA which accepts for all strings that starts with 10 over the alphabet $\Sigma = \{0, 1\}$.



$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_d$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_d$$

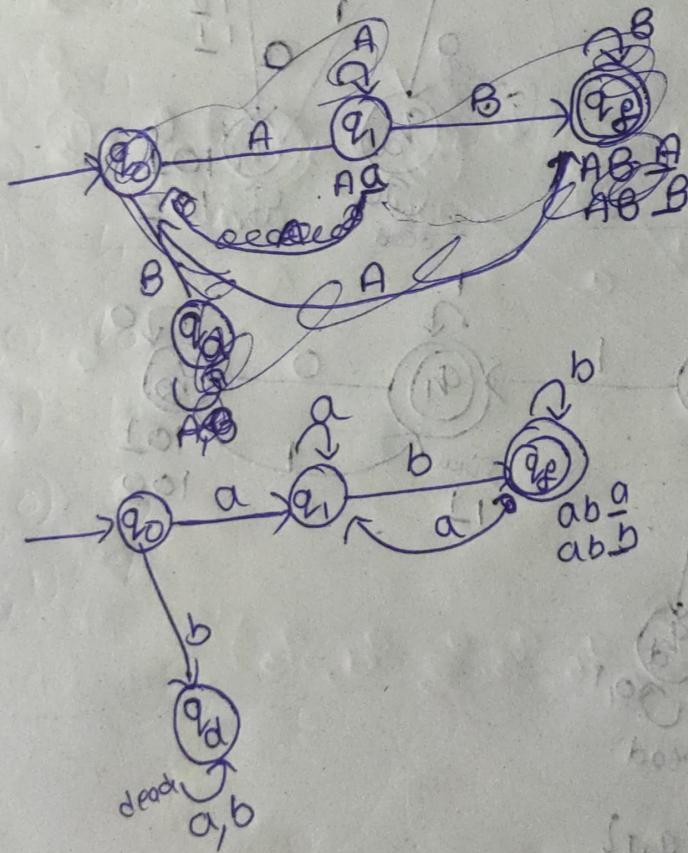
$$\delta(q_2, 0) = q_d$$

$$\delta(q_2, 1) = q_2$$

Ex-8: Design DFA which accepts all strings starting with A & ends with B over the alphabet $\Sigma = \{a, b\}$

$$L = \{ab, aab, abb,$$

Sol:-

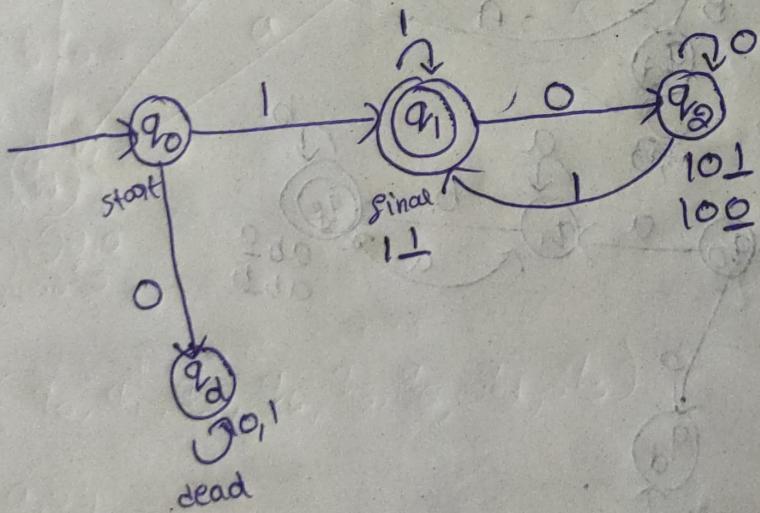
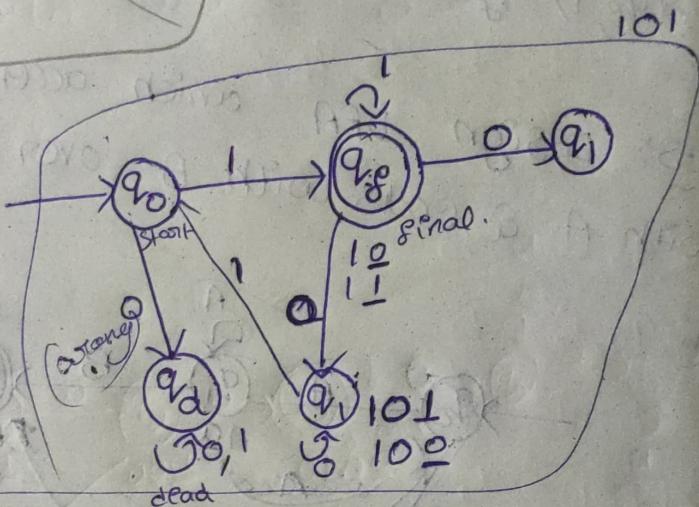
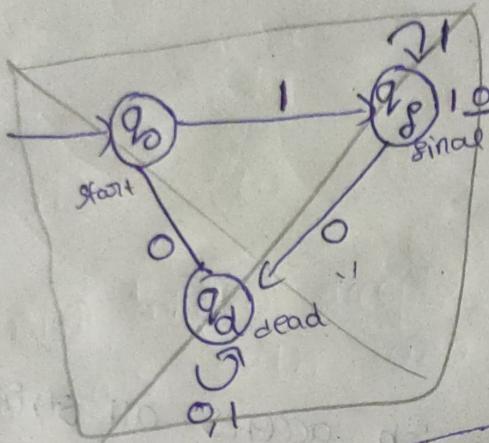


$$Q = \{q_0, q_1, q_2, q_d\}$$

$$\Sigma = \{0, 1\}$$

Ex-9:- design DFA starts $\times \epsilon$ ends width 1 over

Sol:-



$$Q = \{q_0, q_1, q_d\}$$

$$\Sigma = \{0, 1\}$$

$$S(q_0, 0) = q_d$$

$$S(q_0, 1) = q_1$$

$$S(q_1, 0) = q_g$$

$$S(q_1, 1) = q_1$$

$$S(q_d, 0) = q_d$$

$$S(q_d, 1) = q_d$$

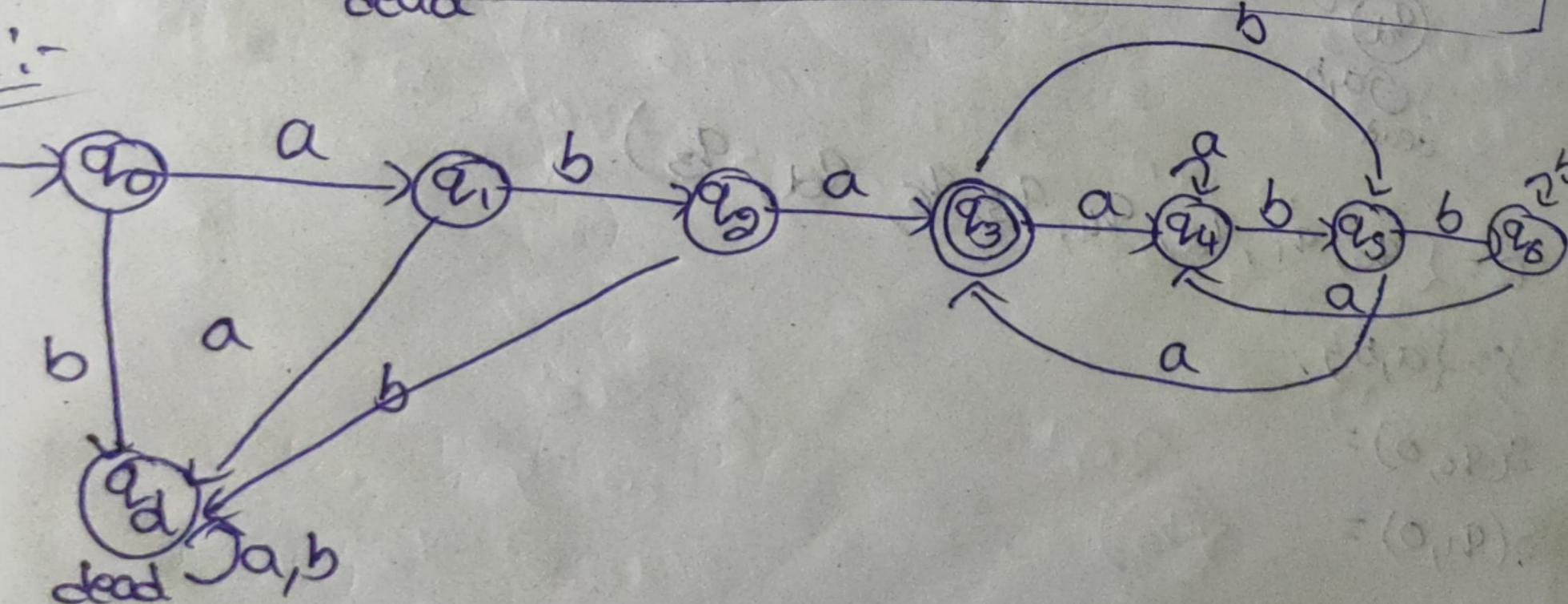
$$S(q_g, 0) = q_g$$

$$S(q_g, 1) = q_1$$

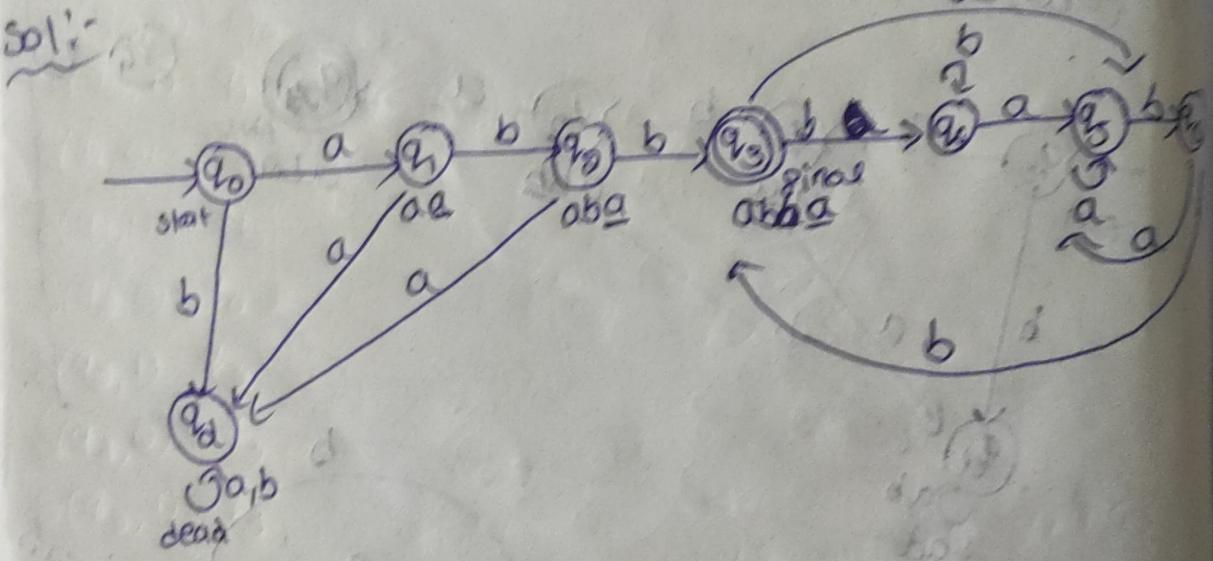
Ex-10: design DFA which accepts all the strings
starts with ABA and ends with ABA

sol:-

dead



Q11:
 Design DFA which accepts all strings that start & ends with abb over the alphabets
 $\Sigma = \{a, b\}$



$$Q = \{q_0, q_1, q_2, q_4, q_5, q_6, q_7, q_3\}$$

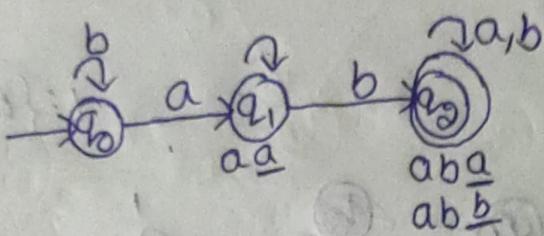
$$\Sigma = \{a, b\}$$

$$\delta(q_0, a) =$$

$$\delta(q_1, a) =$$

Ex-12:- Design DFA which accepts all the strings which contains ab as substring over the alphabets $\Sigma = \{a, b\}$

Sol:-



$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b\}$$

$$\delta(q_0, a) = q_1$$

$$\delta(q_0, b) = q_2$$

$$\delta(q_1, a) = q_2$$

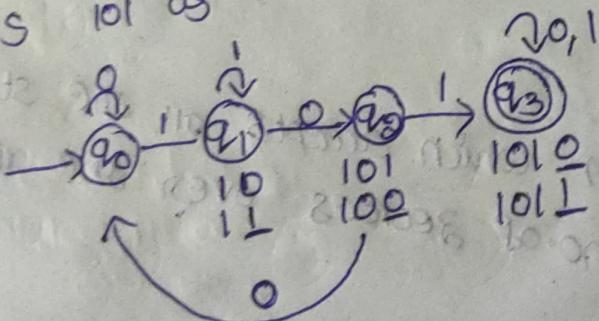
$$\delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_2$$

$$\delta(q_2, b) = q_2$$

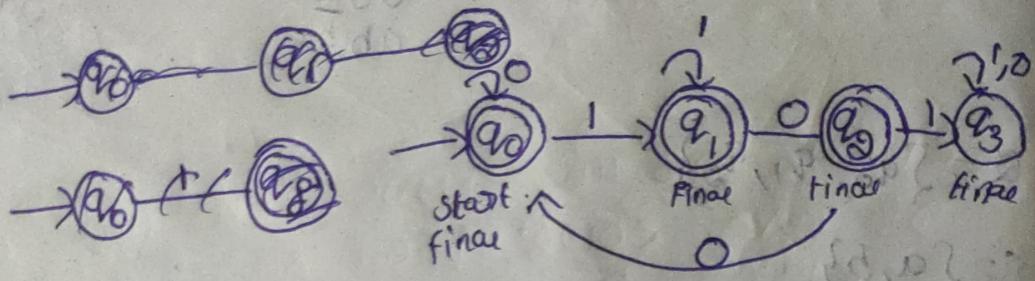
Ex-13:- Design DFA which accepts all the string which contains 101 as substring over the alphabet $\Sigma = \{0, 1\}$

Sol:-



Ex-14: Design DFA which accepts which doesn't contains 101 as substring over the alphabets $\Sigma = \{0, 1\}$

Sol:-



$$Q = \{q_0, q_1, q_2, q_3\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_2$$

$$\delta(q_1, 1) = q_3$$

$$\delta(q_2, 0) = q_0$$

$$\delta(q_2, 1) = q_3$$

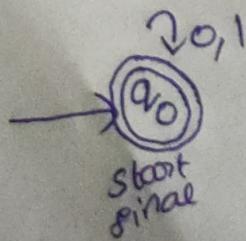
$$\delta(q_3, 0) = q_3$$

$$\delta(q_3, 1) = q_3$$

Ex-15: Design DFA which accepts all the strings contains any no. of zeroes over the alphabet $\Sigma = \{0, 1\}$

$$\Sigma = \{0, 1\}$$

Sol:-



$$Q = \{q_0\}$$

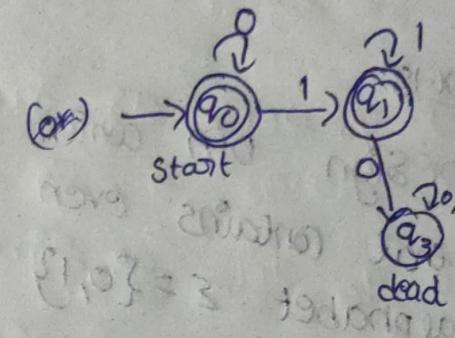
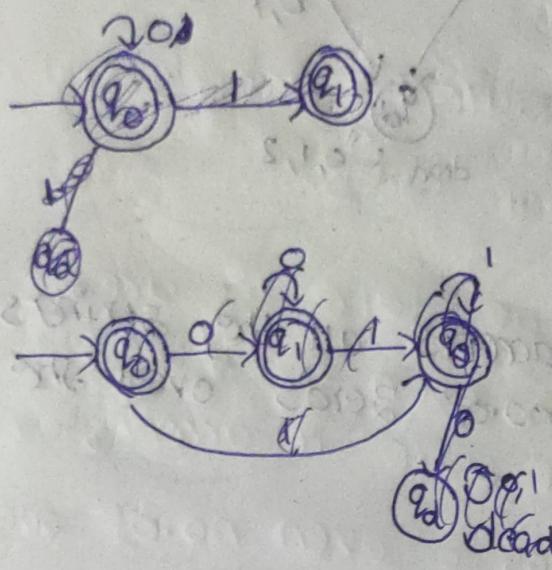
$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_0$$

Ex-16: Design DFA which accepts all the strings in which contains any no. of 0's followed by any no. of 1's over the alphabet $\Sigma = \{0, 1\}$

$$L = \{\epsilon, 0, 1, 00, 11, 01, 000, \\ 111, 001, 011, \\ 100, 110, 101, 010\}$$



$$Q = \{q_0, q_1\}$$

$$\Sigma = \{0, 1\}$$

$$\delta(q_0, 0) = q_0$$

$$\delta(q_0, 1) = q_1$$

$$\delta(q_1, 0) = q_3$$

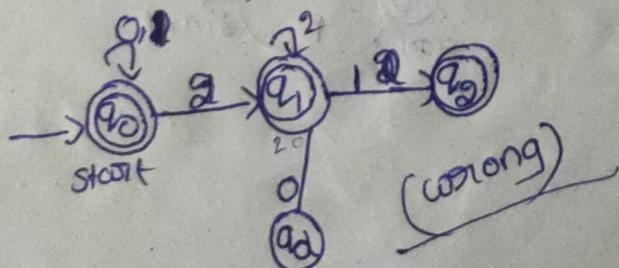
$$\delta(q_1, 1) = q_1$$

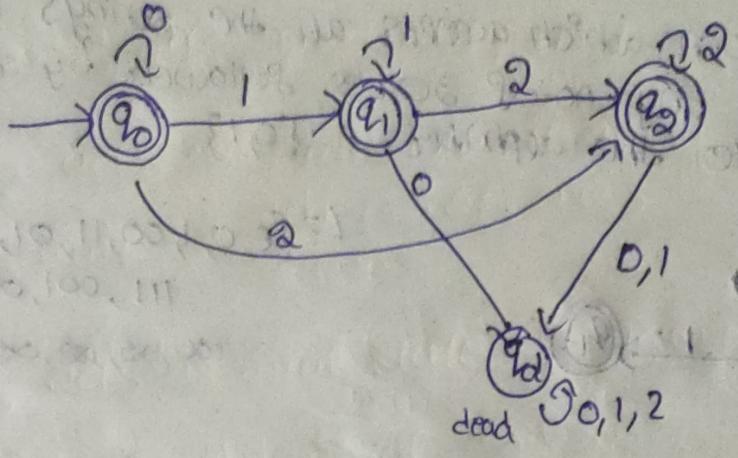
$$\delta(q_3, 0) = q_3$$

$$\delta(q_3, 1) = q_3$$

Ex-17: Design DFA which accepts all strings which contains any no. of 0's followed by any no. of 1's over the alphabet $\Sigma = \{0, 1\}$

$$L = \{\epsilon, 0, 1, 01, \\ 01^2, \dots\}$$



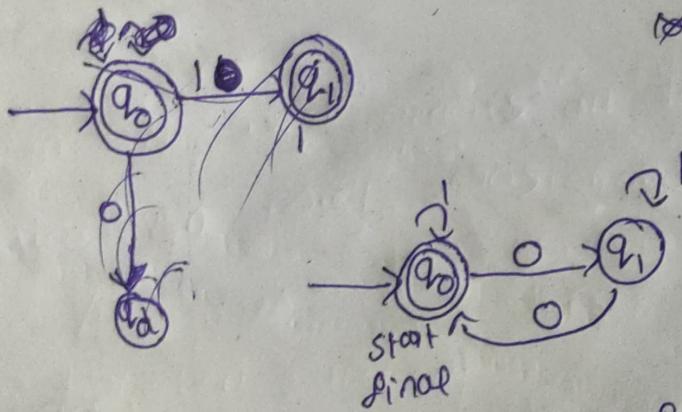


Ex-18:

Design DFA which accepts all the strings that contains even no.of zeros over the alphabet $\Sigma = \{0, 1\}$

Sol:-

$$L = \{\epsilon, 100, 11, 00, 10, \\ 000, 111, 001, 100, \\ 110, 101, 010, \\ 1010, 0000, 1111, 0001, 1000, \\ 1100, 1010, 0100, 1011, 0010, 1101, 0110, 1001, 1110, 0011, 10100, 11000, 00011, 11111, 00001, 10000, \\ 110000, 101000, 010000, 101001, 010010, 101010, 010100, 101100, 011000, 110001, 000110, 110010, 010011, 101011, 011010, 110100, 001100, 111000, 000111, 111100, 011100, 110110, 010110, 101110, 001110, 111110, 011110, 110111, 010111, 101111, 001111, 111111, 011111, 1101110, 0101110, 1011110, 0011110, 1111110, 0111110, 1101111, 0101111, 1011111, 0011111\}$$

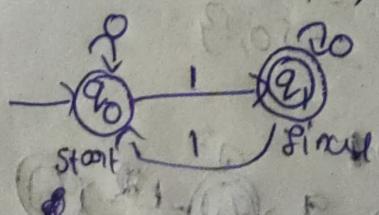
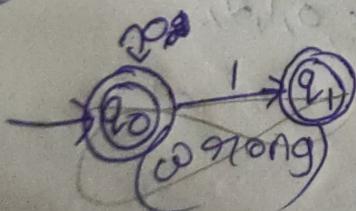


Ex-19:

Design DFA which contain odd no.of one's over the alphabet $\Sigma = \{0, 1\}$

Sol:-

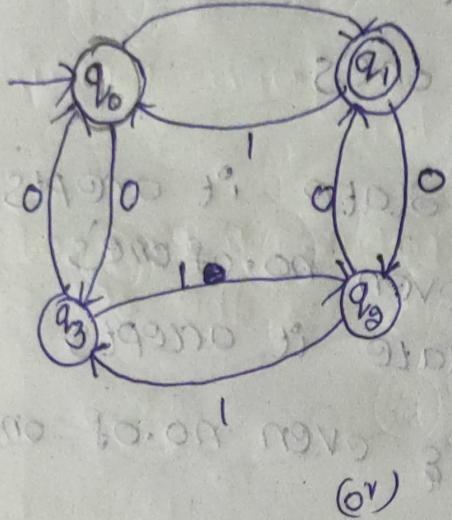
$$L = \{\epsilon, 0, 1, 00, 01, 10, 1, \\ 000, 111, 001, 100, 110, 010, \\ 101, 0000, 1111, 0001, 1000, 1100, 0100, \\ 1010, 0010, 1101, 0110, 1110, 0011, 1011, 0101, 11001, 00110, 11100, 01110, 11011, 01011, 10111, 00111, 11110, 01111, 110111, 010111, 101111, 001111, 111110, 011111, 1101111, 0101111, 1011111, 0011111\}$$



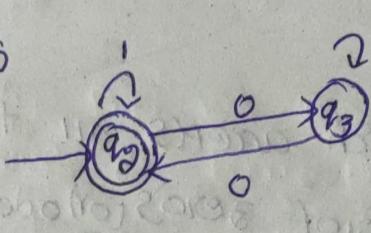
Ex-80: design DFA which accepts all the strings that contains even no. of zero's (0) odd no. of ones over the alphabet, $\Sigma = \{0, 1\}$.

$$L = \{1, 00, 0\}$$

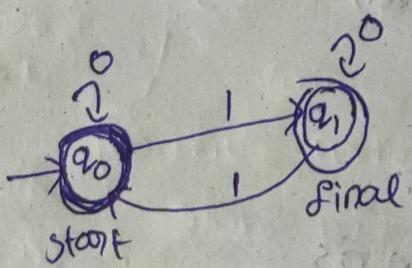
Sol:



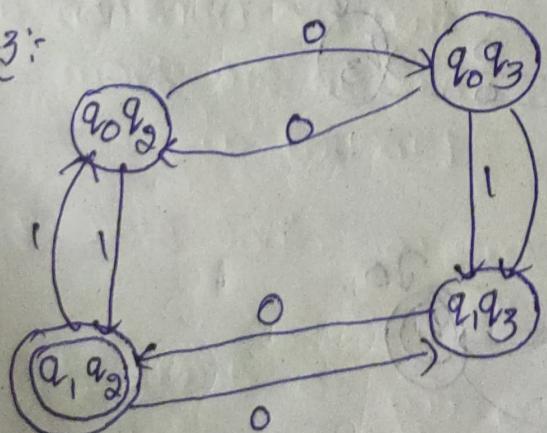
Step-1: even 0's



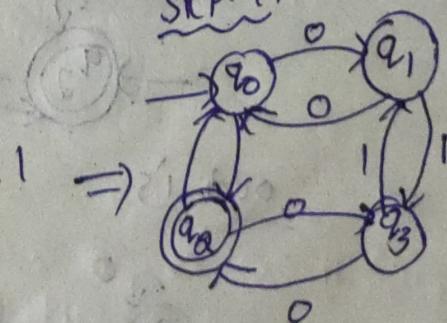
Step-2: odd no. of 1's



Step-3:



Step-4:



consider the diagram they are 4 interpretations.

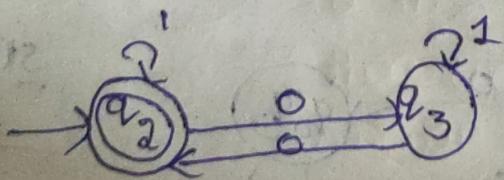
1. q_2 acts as final state it accept even no. of zeros & odd no. of one's
2. q_3 can't accept odd no. of zeros & odd no. of one's.
3. q_1 acts as final state it accepts odd no. of zeros & even no. of one's.
4. q_0 acts as final state it accepts even no. of zeros & even no. of ones.

Ex-2:
Design DFA which accepts all the strings which contain even no. of zeros or odd no. of one's

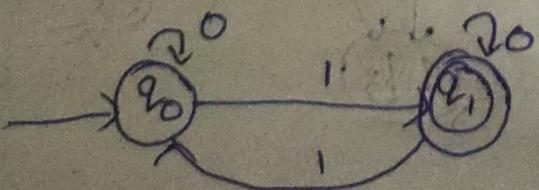
Sol:-

Step-1:-

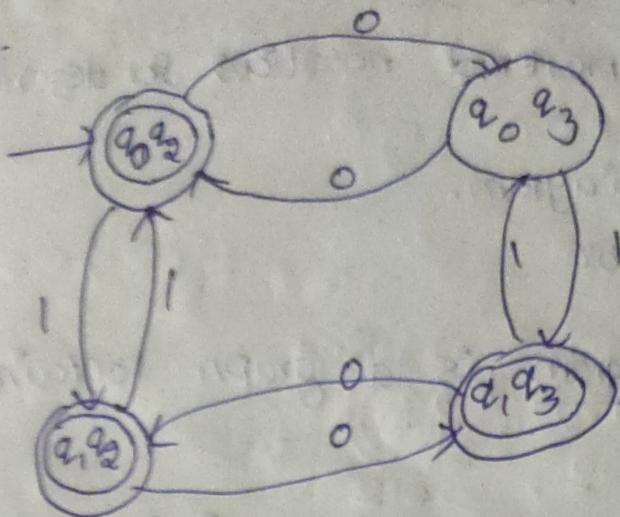
even 0's



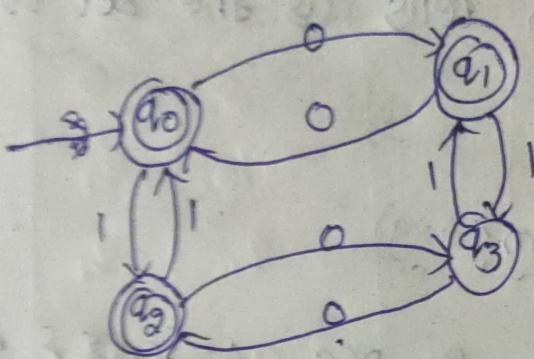
odd 1's



Step 3:



Step 4:



- From the above diagram there are 4 interpretation
- & remaining are final states
1. If q_1 is not final state, it accepts even no. of zeros (or) odd no. of one's.
 2. If q_3 is not final state & remaining are final states then it accepts even no. of zeros (or) even no. of ones.
 3. If q_2 is not final state & remaining are final states then it accepts even no. of odd no. of zeros.
 4. If q_0 is not final state & remaining are final states then it accepts odd no. of 0's & even no. of 1's.

Simple notations for DFA:-

There are 2 preferred notations for describing DFA.

1. Transition diagram.

2. transition table.

* transition diagram is a graph containing state & i/p symbols.

* transition table is tabular listing of delta function (δ) which tells us the set of states & i/p symbols.

1. Transition diagram: (long question)

Transition diagram for DFA $A = \{Q, \Sigma, \delta, q_0, q_F\}$ is a graph defined as follows:-

Q = set of states

Σ = i/p symbol

δ = transition function

q_0 = starting state

q_F = final state

i) for each state q there is a node denoted by circle (O)

ii) for each state q in Q there, for each i/p symbol a in Σ that $\delta(q, a) = p$ then the transition diagram has an arc from q to p labelled 'a'.

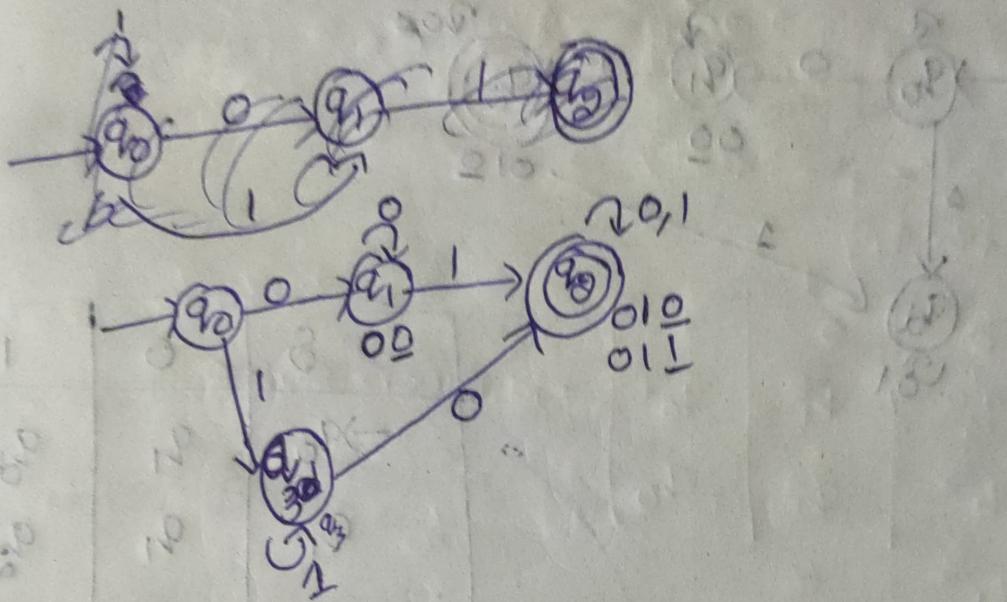
iii) There is an arrow to initial state q_0 labelled start state

iv) the node corresponding to accepting states (F) are marked by double circle (O). states not in F have single circle (O).

Ex: design DFA which accepts all strings that consist atleast one "zero" & one "one" over the alphabet $\Sigma\{0,1\}$

$$L = \{0\}$$

20



2. Transition

transition transition representation take two arguments as its state. table:- table is a conventional, tabular representation of function like $\delta(s)$ that has state & y_p symbol & corresponds to states

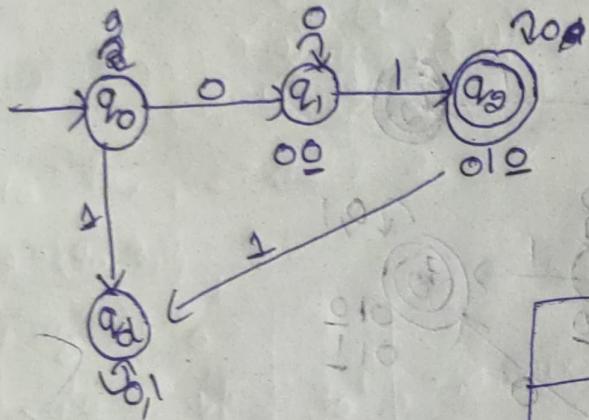
take two arguments
returns it's state.
the rows of the table corresponds to states
columns corresponds to symbols
for above DFA is:

Ex: the transition table for above

s	o	l
$\rightarrow q_0$	q_1	q_3
q_1	q_1	q_2
q_3	q_2	q_3

Ex: design DFA which accepts all the strings that contains atleast one zero followed by atleast one one.

$$\Sigma = \{01, 010,$$



Σ	δ	0	1
$\rightarrow q_0$	q_1	q_d	q_d
q_1	q_1	q_d	q_d
q_d	q_d	q_d	q_d
q_2	q_d	q_d	q_d

Extended Transition Function: - ($\hat{\delta}$)
extended transition function describes what happens when we start with any state & follow any sequence of inputs.

* If ' δ ' is our transition function then the extended transition function constructed from δ will be denoted by $\hat{\delta}$ (delta cap)

proof:-

Basis of induction:-

Let $s(q_0, a) = q_1$ and $s(q_1, b) = q_2$

$$\therefore \hat{s}(q_0, ab) = q_2$$

Induction:-

Suppose $\hat{s}(q_0, \epsilon) = q_0$ & consider any string 'wa'
where w is a string belongs to Σ^* & a is
last alphabet of string 'wa', $a \in \Sigma$.

To prove $\hat{s}(q_0, ab) = q_2$

$$\text{LHS} = \hat{s}(q_0, ab)$$

$$= \hat{s}(s(q_0, a), b)$$

$$= \hat{s}(q_1, b)$$

$$= \hat{s}(s(q_1, b), \epsilon)$$

$$= \hat{s}(q_2, \epsilon) = q_2$$

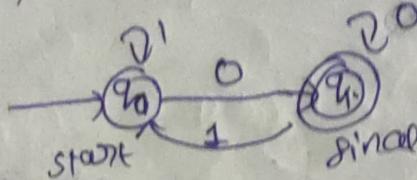
$$= \hat{s}(s(q_2, \epsilon), b) = q_2$$

Hence proved //

$$\boxed{\begin{array}{l} s(q_0, a) = q_1 \\ s(q_1, b) = q_2 \\ \hat{s}(q_2, \epsilon) = q_2 \end{array}}$$

Design DFA which accepts even binary number
 & also check 101010 is accepted by the
 DFA or not.

$$L = \{00, 00, 00, 10, \dots\}$$



$$q_0 = q_0, \Sigma = \{0, 1\}$$

$$Q = \{q_0, q_1\}$$

S	0	1
$\rightarrow q_0$	q_1	q_0
$\star q_1$	q_1	q_0

transition table

$$F = q_1$$

check 101010:-

$$\hat{s}(q_0, 101010) = \hat{s}(\hat{s}(q_0, 1), 01010)$$

$$= \hat{s}(q_0, 01010)$$

$$= \hat{s}(\hat{s}(q_0, 0), 1010)$$

$$= \hat{s}(q_1, 1010)$$

$$= \hat{s}(\hat{s}(q_1, 1), 010)$$

$$= \hat{s}(q_0, 010)$$

$$= \hat{s}(\hat{s}(q_0, 0), 10)$$

$$= \hat{s}(q_1, 10)$$

$$= \hat{s}(\hat{s}(q_1, 1), 0)$$

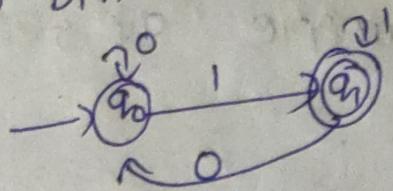
$$= \hat{s}(q_0, 0)$$

$$= \hat{s}(\hat{s}(q_0, 0), \epsilon)$$

$$= \hat{s}(q_1, \epsilon) = q_1$$

Given string is accepted by DFA

design DFA which accepts all odd binary numbers



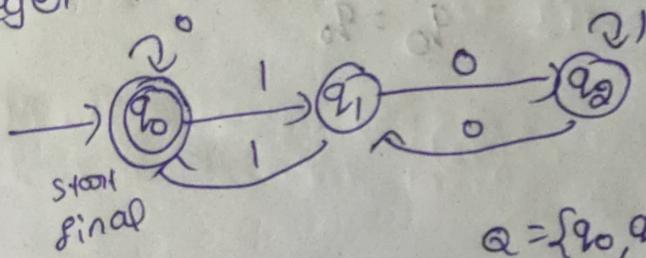
$$Q = \{q_0, q_1\}$$

$$F = q_1$$

δ	0	1
$\rightarrow q_0$	q_0	q_1
$* q_1$	q_0	q_1

$$\begin{aligned} q_0 &= q_0 \\ \Sigma &= \{0, 1\} \end{aligned}$$

design DFA which can be interpreted as binary integer is multiple of 3.



$$Q = \{q_0, q_1, q_2\}$$

$$F = q_0$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$

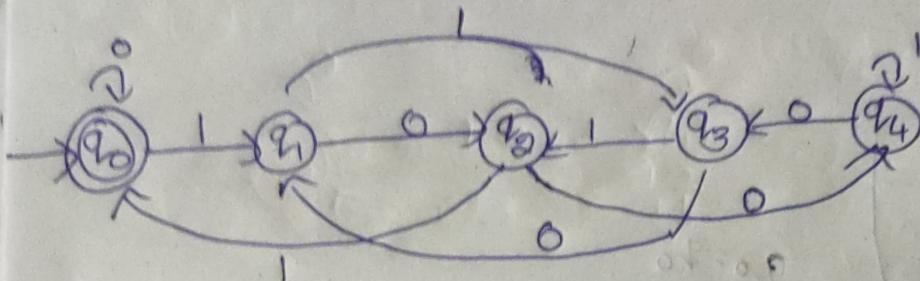
δ	0	1
$\rightarrow q_0$	q_0	q_1
$* q_1$	q_1	

δ	0	1
$* q_1$	q_1	q_2
q_2	q_1	q_2

Design DFA which can be interpreted as binary integer is multiple of 5

$$\{0, 5, 10, 15, 20, 25\}$$

163421
10100



δ	0	1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_0
q_3	q_1	q_2
q_4	q_3	q_4

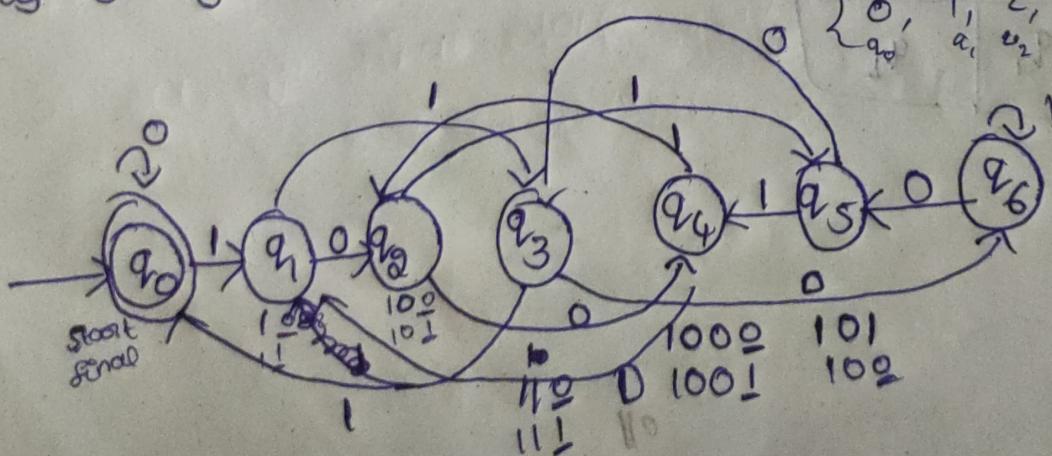
$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$F = q_0$$

$$\Sigma = \{0, 1\}$$

$$q_0 = q_0$$

interpreted
design DFA for all strings which can be
as binary integer divisible by 7.



$$\{0, 1, 2, 3, 4, 5, 6\}$$

$$421$$

$$4_7 = 4$$

$$242 = 7$$

$$4_7 = 7$$

$$6_7 = 6$$

$$7_7 = 1$$

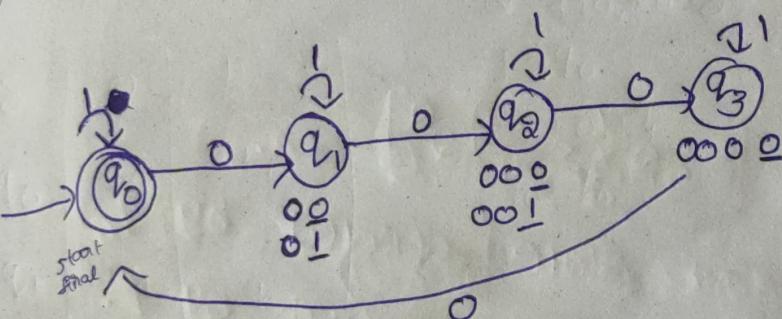
$$8_7 = 1$$

$$7_7 = 1$$

$$0_7 = 0$$

	0	1
*	q_0	q_1
q_0	q_0	q_1
q_1	q_2	q_3
q_2	q_4	q_5
q_3	q_6	q_0
q_4	q_1	q_2
q_5	q_3	q_4
q_6	q_5	q_6

Design DFA for all strings which contains no. of zeros is divisible by 4 over the alphabet $\Sigma = \{0, 1\}$



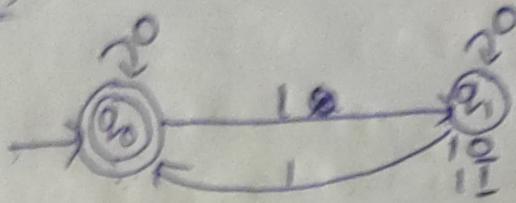
$$\begin{aligned} 2/4 &= 2 \\ 1/4 &= 1 \\ 3/4 &= \\ 4/4 &= 0 \end{aligned}$$

	0	1
*	q_0	q_1
q_0	q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_2
q_3	q_0	q_3

→ Transition table.

Design DFA which accepts all the strings which contains no. of 1's is even & no. of 0's is multiple of 3.

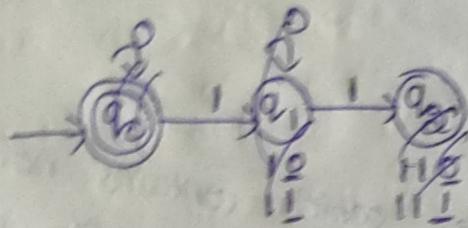
even \Rightarrow



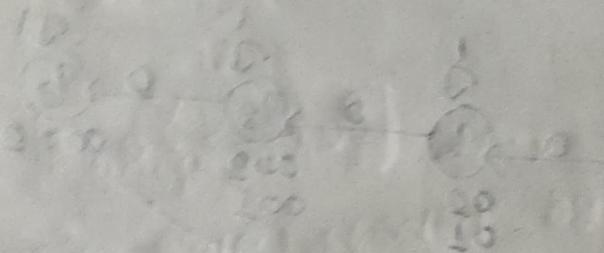
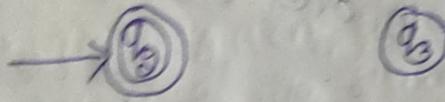
0110
10

0's multiple of 3

0, 3, 6, 9, 12, 15, 18



$3/3 = 0$

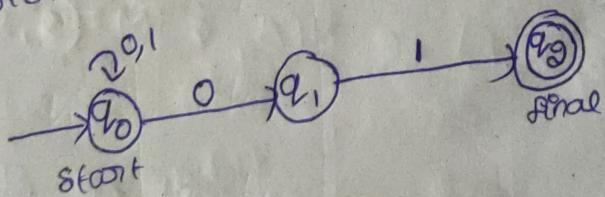


Non-Deterministic Finite Automata (NFA)

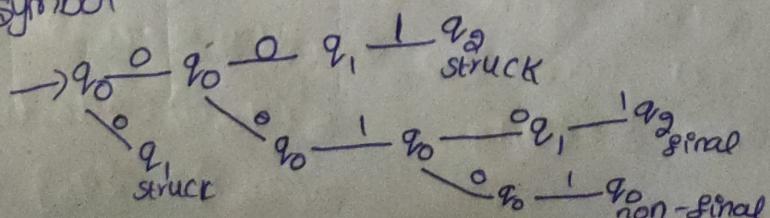
The concept of NFA is exactly the reverse of DFA. The finite automata is called NFA when there exists many paths for specific I/P from current state to next state.

DFA	NFA
<ul style="list-style-type: none"> * DFA is deterministic finite automata * For a given state on a given I/P if reaches to deterministic & unique state * Transition function for DFA is $\delta: Q \times \Sigma \rightarrow Q$ 	<ul style="list-style-type: none"> * NFA is non-deterministic finite automata. * For a given state on a given I/P it may reaches to more than one state. * Transition function for NFA is $\delta: Q \times \Sigma \rightarrow 2^Q$

Design NFA which accepts all the strings that end with 010.



States of NFA designing during a process of I/P symbol



Ex-2:

states of NFA by process of string 11010

→ q_0

NFA:- (definition)

NFA consists of :-

1. Finite set of states denoted by Q .
2. " " " i/p symbols denoted by Σ

3. Transition function as state & i/p states,

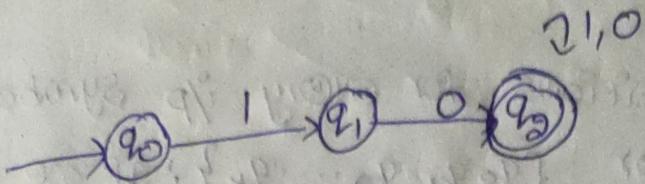
i.e; $S: Q \times \Sigma \rightarrow \mathcal{P}^Q$

data(S) takes arguments symbol then returns set of

4. Start state denoted by q_0
5. Set of final state denoted by F .



Design NFA which accepts all strings starting with 10 over the alphabet $\Sigma = \{0, 1\}$



$$Q = \{$$

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{0, 1\}$$

	0	1
q_0	\emptyset	q_1
q_1	q_2	\emptyset
$*q_2$	q_2	q_2

Conversion of NFA into DFA :-

conversion procedure specifies

following procedure specifies
let $M = (Q, \Sigma, S, q_0, F)$ is an NFA which accepts
the language $L(M)$ there should be equivalent
DFA denoted by $M' = (Q', \Sigma', S', q'_0, F')$ such that

$$L(M) = L(M')$$

Step-1:- Start state of NFA will be the start state
of DFA. Hence add q_0 (NFA) q'_0 of NFA
(start state) to Q'

Step-2:- For each state $(q_0, q_1, q_2, \dots, q_i)$ in Q' the
transition for each input symbol Σ can be obtained
as $\text{① } S'(\{q_0, q_1, q_2, \dots, q_i\}, a) = S(q_0, a) \cup S(q_1, a) \cup \dots \cup S(q_i, a)$
 $= \{q_0, q_1, q_2, \dots, q_k\}^{\text{(some state)}}$

Step-3:-

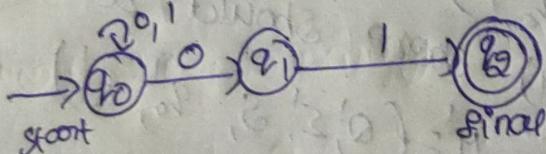
Q Hence add $\{q_0, q_1, q_2, \dots, q_k\}$ to DFA if it is not already present in Q' .

Step-4:- Find transitions for every i/p symbol from Σ for $\{q_0, q_1, q_2, \dots, q_k\}$. If we get some states $\{q_0, q_1, \dots, q_n\}$ which is not in Q' of DFA then add this state to Q' .

Step-5:- If there is no new state generated then rest of the process of finding all transitions can be stopped.

Step-6:- For the state $\{q_0, q_1, \dots, q_n\}$ in Q' of DFA, if any one state q_i is a final state of NFA then $\{q_0, q_1, \dots, q_n\}$ becomes final state. Thus the set of all final state belongs to F' of DFA.

Ex:- Convert the following NFA to DFA.



Sol:- Let $M = (Q, \Sigma, \delta, q_0, F)$ be NFA & $M' = (Q', \Sigma, \delta', q'_0, F')$ be DFA.

Step-1:- Start state of NFA, q_0 , will be start state of DFA then add q_0 to Q'
i.e. $Q' = \{q_0\}$

Step-2:- find transitions for states in Q' of DFA over $\Sigma = \{0, 1\}$ $\therefore \delta'(q_0, 0) = \{q_0, q_1\} \rightarrow$ it is now state.

Hence add $\{q_0, q_1\}$ to q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\})$$

Step-3 :- find transitions for $\{q_0, q_1\}$ over $\Sigma = \{0, 1\}$

$$S(\{q_0, q_1\}, 0) = S(q_0, 0) \cup S(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$$

$$S(\{q_0, q_1\}, 1) = S(q_0, 1) \cup S(q_1, 1) = \{q_0\} \cup \{q_1\} = \{q_0, q_1\}$$

Hence add $\{q_0, q_1\}$ to q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_1\})$$

Step-4 :- find transitions for $\{q_0, q_1\}$ over $\Sigma = \{0, 1\}$

$$S(\{q_0, q_1\}, 0) = S(q_0, 0) \cup S(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$$

$$S(\{q_0, q_1\}, 1) = S(q_0, 1) \cup S(q_1, 1) = \{q_0\} \cup \emptyset = \{q_0\}$$

Hence add $\{q_0\}$ to q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_1\}, \{q_0\})$$

Step-5 :- find transitions for $\{q_0\}$ over $\Sigma = \{0, 1\}$

$$S(q_0, 0) = \emptyset$$

$$S(q_0, 1) = \emptyset$$

\therefore there are no new states generated.

Hence $\{q_0, q_1\}$ becomes final state of DFA b/w
final state of NFA is q_0

Step 2:-

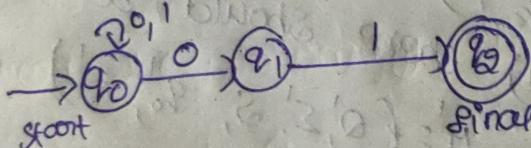
- Q. Hence add $\{q_0, q_1, q_2, \dots, q_k\}$ to DFA if it is not already present in Q' .

Step 3:- Find transitions for every i/p symbol from Σ for $\{q_0, q_1, q_2, \dots, q_k\}$. If we get some states $\{q_0, q_1, \dots, q_n\}$ which is not in Q' of DFA then add this state to Q' .

Step 4:- If there is no new state generated then rest of the process of finding all transitions can be stopped.

Step 3:- For the state $\{q_0, q_1, \dots, q_n\}$ in Q' of DFA, if any one state q_i is a final state of NFA then $\{q_0, q_1, \dots, q_n\}$ becomes final state. Thus the set of all final state belongs to F' of DFA.

Ex:- Convert the following NFA to DFA.



Sol:- Let $M = (Q, \Sigma, \delta, q_0, F)$ be NFA & $M' = (Q', \Sigma, \delta', q'_0, F')$ be DFA.

Step 1:- Start state of NFA, q_0 , will be start state of DFA then add q_0 to Q' i.e. $Q' = \{q_0\}$

Step 2:- Find transitions for states in Q' of DFA over $\Sigma = \{0, 1\}$: $\delta'(q_0, 0) = \{q_0, q_1\} \rightarrow$ it is new state.

$\delta'(q_0, 1) = \{q_2\} \rightarrow$ it is already in Q' .

Hence add $\{q_0, q_3\}$ to Q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_3\})$$

Step-3:- find transitions for $\{q_0, q_1\}$ over $\Sigma = \{0, 1\}$

$$\delta'(\{q_0, q_1\}, 0) = \delta^*(q_0, 0) \cup \delta^*(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$$

it is in state

$$\delta'(\{q_0, q_1\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) = \{q_0\} \cup \{q_2\} = \{q_0, q_2\}$$

it is new state

Hence add $\{q_0, q_2\}$ to Q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\})$$

Step-4:- find transitions for $\{q_0, q_2\}$ over $\Sigma = \{0, 1\}$

$$\delta'(\{q_0, q_2\}, 0) = \delta(q_0, 0) \cup \delta(q_2, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\}$$

$$\delta'(\{q_0, q_2\}, 1) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0\} \cup \emptyset = \{q_0\}$$

$$\delta'(\{q_0, q_2\}, 1) = \delta(q_0, 1) \cup \delta(q_2, 1) = \{q_0\} \cup \emptyset = \{q_0\}$$

it is ~~new state~~
already in Q'

Hence add $\{q_2\}$ to Q' of DFA

$$\therefore Q' = (\{q_0\}, \{q_0, q_1\}, \{q_0, q_2\}, \{q_2\})$$

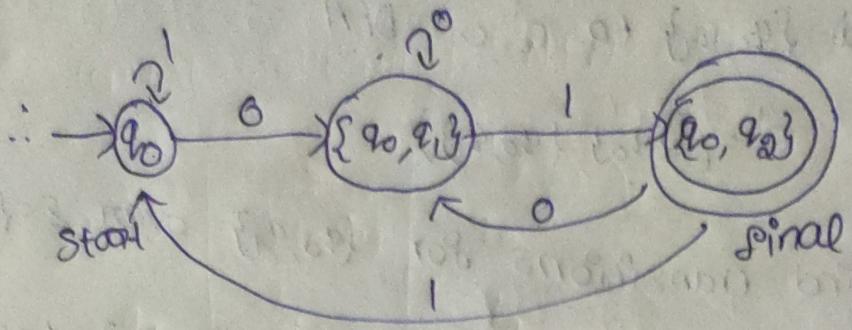
Step-5:- find transitions for $\{q_2\}$ over $\Sigma = \{0, 1\}$

$$\delta'(q_2, 0) = \emptyset$$

$$\delta'(q_2, 1) = \emptyset$$

: there are no new states generated.

Hence $\{q_0, q_2\}$ becomes final state of DFA $\frac{5}{2}$
final state of NFA is q_2

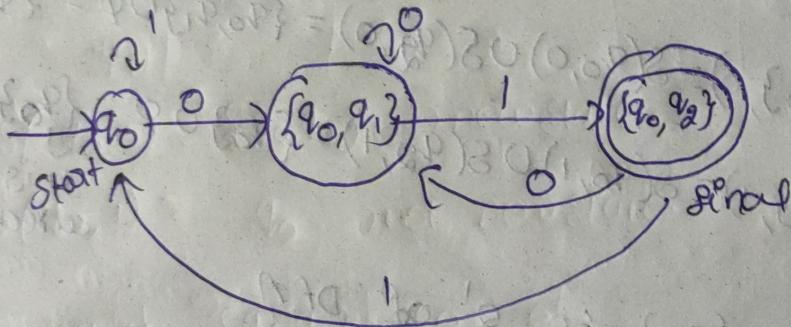


transition table

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
q_1	\emptyset	$\{q_2\}$
$*q_2$	\emptyset	\emptyset

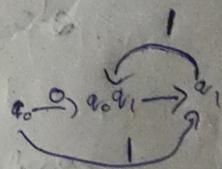
DFA

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_0\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_0\}$	$\{q_0\}$	$\{q_0\}$



Ex-2:- convert the following NFA to DFA

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$
$*q_1$	\emptyset	$\{q_0, q_1\}$



Step-1:- start state of NFA $\rightarrow(q_0)$ will be start state of DFA then add q_0 to Q' i.e $Q' = \{q_0\}$.

Step-2: Find transitions for states in Q' of DFA over $\Sigma = \{0, 1\}$

$$\begin{aligned} s'(\{q_0\}, 0) &= \{\{q_0, q_1\}\} \quad \text{get 1s new states} \\ s'(\{q_0\}, 1) &= \{\{q_1\}\} \end{aligned}$$

Hence add $\{q_0, q_1\}$ & $\{q_1\}$ to Q' of DFA
 $\therefore Q' = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}\}$

Step-3: Find transitions for $\{q_0, q_1\}$ & $\{q_1\}$

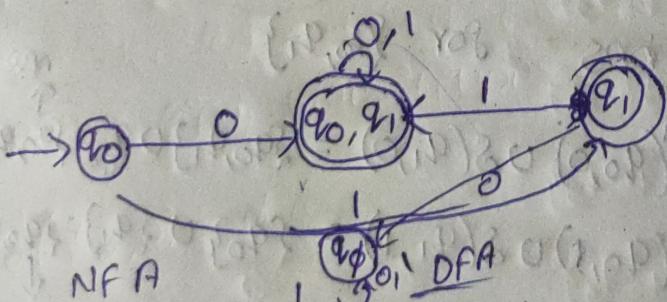
$$\begin{aligned} s'(\{q_0, q_1\}, 0) &= s(q_0, 0) \cup s(q_1, 0) = \{q_0, q_1\} \cup \emptyset = \{q_0, q_1\} \\ s'(\{q_0, q_1\}, 1) &= s(q_0, 1) \cup s(q_1, 1) = \{q_1\} \cup \{q_0, q_1\} = \{q_0, q_1\} \end{aligned}$$

$$\begin{aligned} s'(\{q_1\}, 0) &= \emptyset \rightarrow \text{no new state} \\ s'(\{q_1\}, 1) &= \{q_0, q_1\} \end{aligned}$$

Step-4: add \emptyset to Q'
 $\therefore Q' = \{\{q_0\}, \{q_0, q_1\}, \{q_1\}, \emptyset\}$

$$\begin{aligned} s'(\emptyset, 0) &= \emptyset \\ s'(\emptyset, 1) &= \emptyset \end{aligned}$$

Hence $\{q_0, q_1\}$, $\{q_1\}$ are becomes final states of
DFA b/c final state of NFA is q_1 .



	0	1		0	1	
$\rightarrow q_0$	$\{q_0\}$	$\{q_1\}$		$\{q_0, q_1\}$	$\{q_1\}$	
$* q_1$	\emptyset	$\{q_0, q_1\}$		$\{q_0, q_1\}$	$\{q_0, q_1\}$	

Ex-3:- convert following NFA to DFA

	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
q_1	q_2	q_1
q_2	q_2	q_3
$* q_3$	\emptyset	q_2

Step-1:- start state of NFA (q_0), will be start state of DFA. Then add q_0 to Q' .

Step-2:- find transitions for states in Q' of DFA.

$$S'(q_0, 0) = \{q_0, q_1\} \rightarrow \text{new state}$$

$$S'(q_0, 1) = q_0 \rightarrow \text{not new state.}$$

Hence add $\{q_0, q_1\}$ to Q' .

$$\therefore Q' = \{q_0, \{q_0, q_1\}\}$$

Step-3:- find transitions for $\{q_0, q_1\}$

$$S(\{q_0, q_1\}, 0) = S(q_0, 0) \cup S(q_1, 0) = \{q_0, q_1\} \cup q_2 = \{q_0, q_1, q_2\}$$

$$S(\{q_0, q_1\}, 1) = S(q_0, 1) \cup S(q_1, 1) = \{q_0\} \cup \{q_3\} = \{q_0, q_3\}$$

Hence add $\{q_0, q_1, q_2\}$ to Q' .

$$Q' = \{q_0, \{q_0, q_1\}, \{q_0, q_1, q_2\}\}$$

Step-4: find transitions for $\{q_0, q_1, q_2\}$

$$\delta'(\{q_0, q_1, q_2\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_2, 0) = \{q_0, q_1\} \cup \{q_2\} \cup \{q_3\} = \{q_0, q_1, q_2\}$$

$$\delta'(\{q_0, q_1, q_2\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_2, 1) = \{q_0\} \cup \{q_3\} \cup \{q_3\} = \{q_0, q_1, q_3\}$$

add $\{q_0, q_1, q_3\}$ to Q'



$$\therefore Q' = \{q_0\}, \{q_0, q_1\}, \{q_0, q_1, q_2\}, \{q_0, q_1, q_3\}$$

Step-5: find transitions for $\{q_0, q_1, q_3\}$

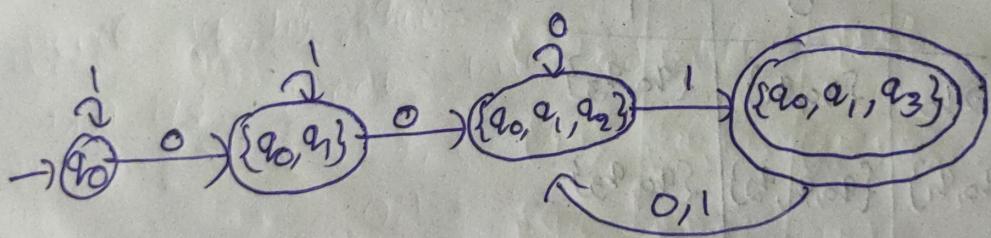
$$\delta'(\{q_0, q_1, q_3\}, 0) = \delta(q_0, 0) \cup \delta(q_1, 0) \cup \delta(q_3, 0) = \{q_0, q_1\} \cup \{q_2\} \cup \{q_3\} = \{q_0, q_1, q_2\}$$

$$\delta'(\{q_0, q_1, q_3\}, 1) = \delta(q_0, 1) \cup \delta(q_1, 1) \cup \delta(q_3, 1) = \{q_0\} \cup \{q_3\} \cup \{q_2\} = \{q_0, q_1, q_2\}$$

\therefore There is no new state.

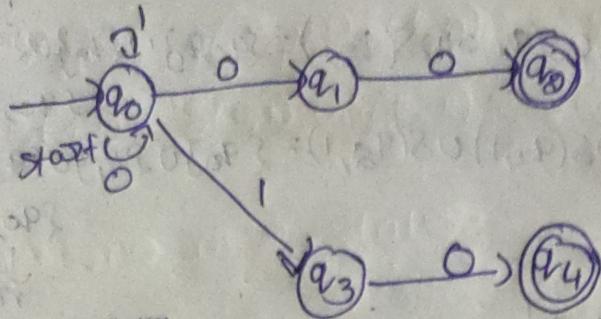
Hence $\{q_0, q_1, q_3\}$ is final state of DFA b/3

final state of NFA is q_3 .



	0	1
$\rightarrow q_0$	$\{q_0, q_1\}$	q_0
$\{q_0, q_1\}$	$\{q_0, q_2\}$	$\{q_0, q_1\}$
$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_3\}$
$* \{q_0, q_1, q_3\}$	$\{q_0, q_2\}$	$\{q_0, q_1, q_2\}$

Expt:- convert NFA to DFA



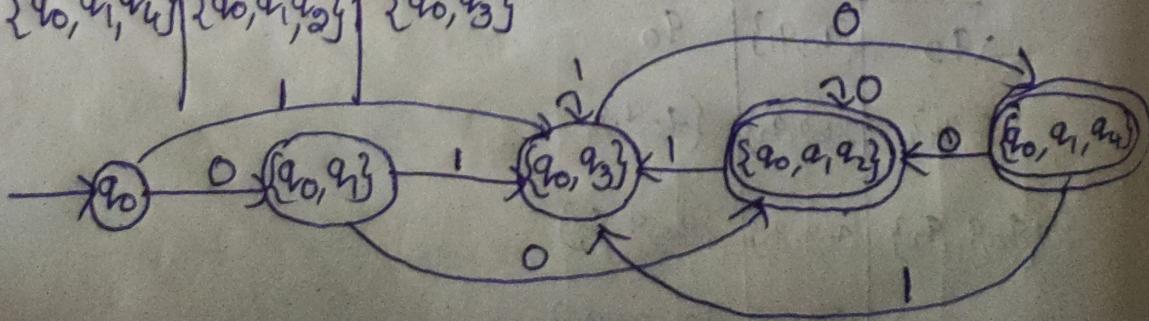
write theory
shortcut

NFA

	0	1
$\rightarrow q_0$	$\{q_0, q_3\}$	$\{q_0, q_3\}$
q_1	$\{q_2\}$	\emptyset
$* q_2$	\emptyset	\emptyset
q_3	$\{q_4\}$	\emptyset
$* q_4$	\emptyset	\emptyset

DFA

	0	1
$\rightarrow q_0$	$\{q_0, q_3\}$	$\{q_0, q_3\}$
$\{q_0, q_1\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$\{q_0, q_3\}$	$\{q_0, q_1, q_4\}$	$\{q_0, q_3\}$
$* \{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$
$* \{q_0, q_1, q_4\}$	$\{q_0, q_1, q_2\}$	$\{q_0, q_3\}$



Ex-5: convert following NFA to DFA

	0	1
$\rightarrow P$	{P, q, s}	{P, t}
q	{q, s}	{t}
*q	{P, q, s}	t
*s	φ	φ
*t	φ	φ

Step-1:- start state of NF (P) will be start state of DFA then add P to Q'

$$Q' = \{P\}$$

Step-2:- find transitions for states in Q' of DFA

$$\delta'(P, 0) = \{P, q\} \rightarrow \text{new state } e$$

$$\delta'(P, 1) = \{P\}$$

\therefore add $\{P, q\}$ to Q'

$$\therefore Q' = \{P, \{P, q\}\}$$

Step-3:- find transitions for $\{P, q\}$

$$\delta'(\{P, q\}, 0) = \delta(P, 0) \cup \delta(q, 0) = \{P\} \cup \{q, s\} = \{P, q, s\}$$

$$\delta'(\{P, q\}, 1) = \delta(P, 1) \cup \delta(q, 1) = \{P\} \cup \{t\} = \{P, t\}$$

\therefore add $\{P, q, s\}, \{P, t\}$ to Q'

$$Q' = \{P, \{P, q\}, \{P, q, s\}, \{P, t\}\}$$

Step-4:- $\delta'(\{P, q, s\}, 0) = \delta^*(P, 0) \cup \delta(q, 0) \cup \delta(s, 0) \cup \delta(P, q, s, 0) = \{P, q, s\} \cup \{q, s\} \cup \{s\} \cup \emptyset = \{P, q, s\}$

$$\delta'(\{P, q, s\}, 1) = \delta(P, 1) \cup \delta(q, 1) \cup \delta(s, 1) \cup \delta(P, q, s, 1) = \{P\} \cup \{t\} \cup \{t\} \cup \{P, t\} = \{P, t\}$$

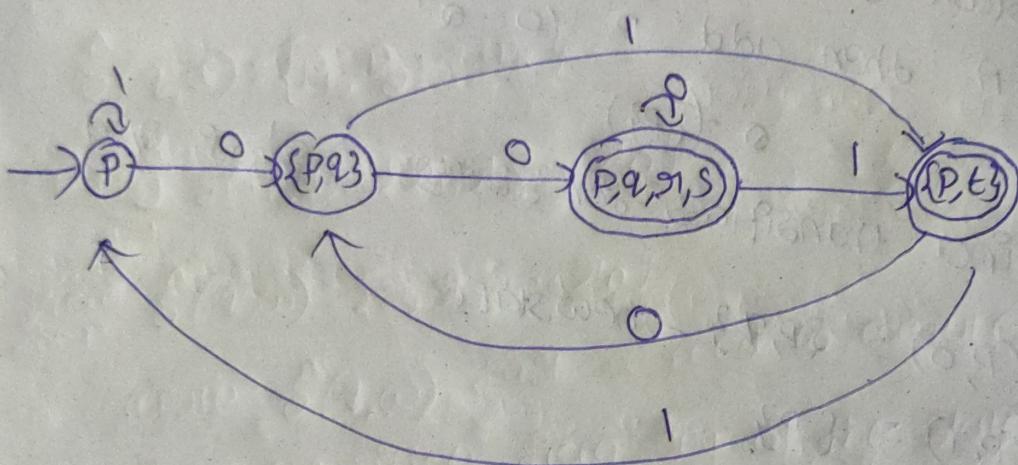
$$\delta'(\{P, \epsilon\}, 0) = \delta(P, 0) \cup \delta(\epsilon, 0) = \{P, Q\} \cup \emptyset = \{P, Q\}$$

$$\delta'(\{P, \epsilon\}, 1) = \delta(P, 1) \cup \delta(\epsilon, 1) = \{P\} \cup \emptyset = \{P\}$$

no new states -

$$Q' = (\{P\}, \{P, Q\}, \{P, Q, S, R\}, \{P, \epsilon\})$$

final state of NFA is $\{P, Q, S, R\}, \{P, \epsilon\}$



DFA

	0	1
$\{P\}$	$\{P, Q\}$	$\{P\}$
$\{P, Q\}$	$\{P, Q, S, R\}$	$\{P, \epsilon\}$
$\{P, Q, S, R\}$	$\{P, Q, S, R\}$	$\{P, \epsilon\}$
$\{P, \epsilon\}$	$\{P, Q\}$	$\{P\}$

Ex-6:

	0	1
$\rightarrow P$	$\{P, q\}$	$\{P\}$
q	g_1	g_1
g_1	S	\emptyset
* S	S	S

	0	1
$\rightarrow P$	$\{P, q\}$	$\{P\}$
$\{P, q\}$	$\{P, q, g_1\}$	$\{P, g_1\}$
$\{P, g_1\}$	$\{P, q, g_1\}$	$\{P, g_1\}$
$\{P, g_1\}$	$\{P, g_1, S\}$	$\{P\}$
* $\{P, q, g_1, S\}$	$\{P, q, g_1, S\}$	$\{P, g_1, S\}$
* $\{P, q, S\}$	$\{P, q, g_1, S\}$	$\{P, g_1, S\}$
* $\{P, g_1, S\}$	$\{P, g_1, S\}$	$\{P, S\}$
* $\{P, S\}$	$\{P, q, S\}$	$\{P, S\}$

Draw diagram

Extended transition function for NFA denoted by δ^*
 extended transition function (δ^*) for NFA takes
 arguments as state & string contains a
 set of i/p symbols denoted by ' w ' & return
 set of states.

Basis of induction:-

$$\hat{S}(q, \epsilon) = \{q\}$$

without treating any i/p symbol we can
only in state begin with q

Induction:-

Suppose 'w' is of the form $w = a\alpha$ where a is the final symbol of w such that $a \in \Sigma$ & $\alpha \in \Sigma^*$

$$\hat{S}(q, a) = \{P_1, P_2, \dots, P_k\}$$

$$\text{let } \bigcup_{i=1}^k S(P_i, a) = \{\pi_1, \pi_2, \dots, \pi_n\}$$

$$\text{from } \hat{S}(q, w) = \{\pi_1, \pi_2, \dots, \pi_n\}$$

then to compute $\hat{S}(q, w)$ first we need to compute $\hat{S}(q, \alpha)$
to compute $\hat{S}(q, \alpha)$ first we need to compute $\hat{S}(q, a)$
and then any transition from any of the states labeled a'

a'

Theorem: ~~if~~ ^{imp}

if $D = (Q_D, \Sigma_D, S_D, \{q_0\}, F_D)$ is constructed from

$N = (Q_N, \Sigma_N, S_N, \{q_0\}, F_N)$ then prove $L(D) = L(N)$

Proof:-

Let us consider string w of length $k+1$ is of the form $w = a\alpha$ where a is final alphabet & α is the rest of the string w .

Basis of induction:-

Let $|w| = 0$ i.e. $w = \epsilon$

$$\text{LHS } \hat{S}_D(q_0, \epsilon) = \{q_0\}$$

$$\text{RHS } \hat{S}_N(q_0, \epsilon) = \{q_0\}$$

$$\therefore \hat{S}_D(q_0, \epsilon) = \hat{S}_N(q_0, \epsilon) = \{q_0\}$$

Inductive hypothesis:

Assume that $\hat{\delta}_D(q_0, \omega) = \hat{\delta}_N(q_0, \omega)$ is true for $|w| = k$

i.e $w = \alpha$

Inductive step:-

To prove $L(D) = L(N)$ for $|w| = k+1$ i.e., $w = \alpha a$

The definition of $\hat{\delta}$ for NFA is $\hat{\delta}_N(q_0, \omega) = \bigcup_{i=1}^N \delta_N(p_i, a)$

The subset construction on the other hand

$$\delta_D(\{p_1, p_2, p_3, \dots, p_k\}, a) = \bigcup_{i=1}^k \delta_N(p_i, a) \rightarrow ②$$

LHS

$$\begin{aligned}\delta_D(\{q_0\}, \omega) &= \delta_D(\{q_0\}, \hat{\delta}(q_0, \alpha), a) \\ &= \delta_D(\{p_1, p_2, \dots, p_k\}, a) \\ &= \bigcup_{i=1}^k \delta(p_i, a) \\ &= \hat{\delta}_N(q_0, \omega)\end{aligned}$$

$$\therefore \hat{\delta}_D(q_0, \omega) = \hat{\delta}_N(q_0, \omega)$$

ϵ -NFA (Finite automata with epsilon (ϵ) transition)

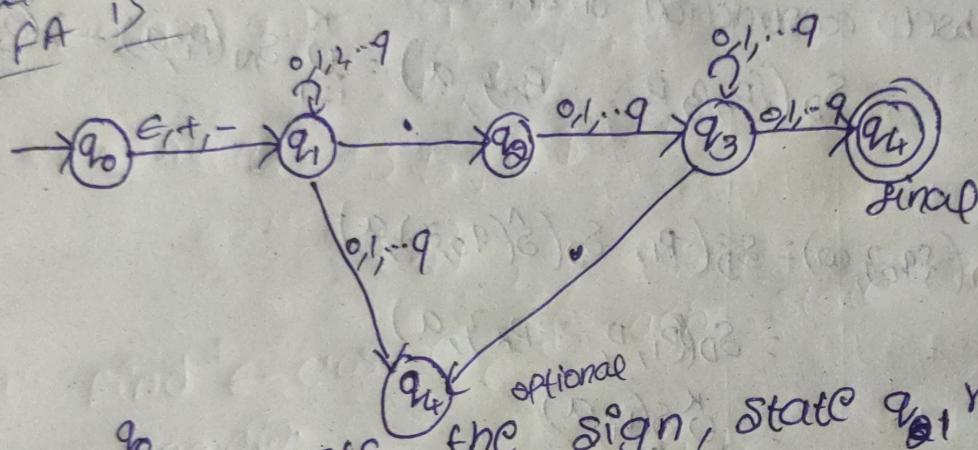
This is another extension of finite automata. The new feature is that the transition of (ϵ) epsilon that is an empty string.

* NFA is allowed to make transition without receiving an i/p symbol such NFA is called as epsilon NFA (ϵ -NFA)

Ex:- E-NFA that accepts decimal numbers consisting of

1. an optional + or - sign.
2. a string of digits 0, 1, 2, ..., 9
3. a decimal point (.)
4. another string of digits 0, 1, 2, ..., 9

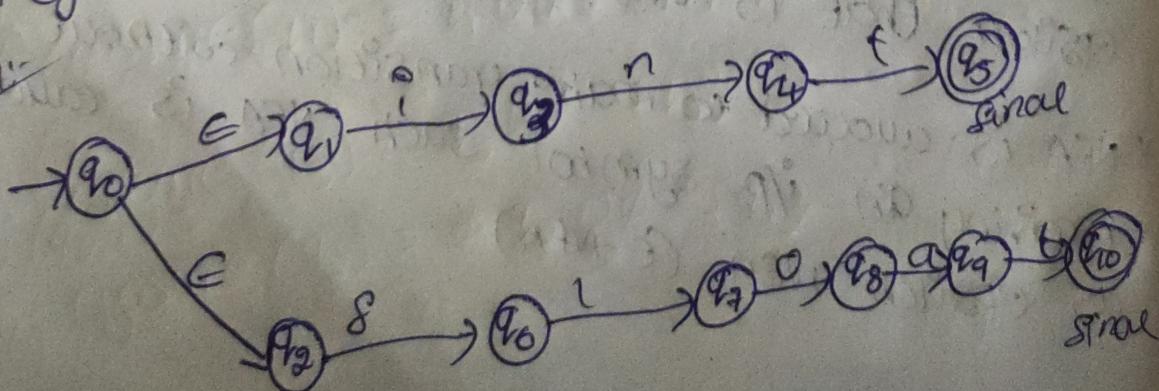
Our NFA is



state q_0 represents the start sign, state q_1 represents the situate decimal point, state q_2 represents at least one digit before the decimal point & after the decimal point, state q_3 represents at least one digit before the decimal point & after the decimal point.

Ex:- construct E-NFA that recognize set of keywords inteflop.

Sol:-



formal definition of ENFA:-

1. E-NFA consists of set of states denoted by Q
2. set of i/p symbols denoted by Σ
3. transition function s takes arguments as state & either i/p symbol or empty state, then that generates set of states
4. start state denoted by q_0 .
5. set of final state denoted by F .

$$s: Q \times \Sigma \cup \{\epsilon\} \rightarrow 2^Q$$

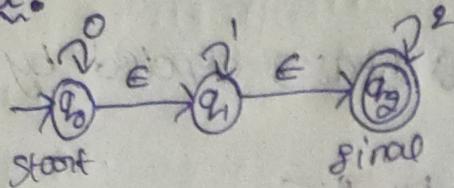
definition of ϵ -closure(P):-

ϵ -closure(P) is set of all states which are reachable from state P on ϵ -transition such that

$$\text{i, } \epsilon\text{-closure}(P) = \{P\} \cup \text{ } \epsilon\text{-transitions from 'P' where } P \in Q$$

$$\text{ii, } \epsilon\text{-closure}(P) = q \text{ & } \epsilon\text{-closure}(q) \text{ } s(q, \epsilon) = q \text{ then } \epsilon\text{-closure}(P) = \{q, q\}$$

Ex:- find ϵ -closure of each state in diagram.



$$\epsilon\text{-closure}(q_0) = \{q_0\} \cup \epsilon\text{-closure}(q_0)$$

$$= \{q_0\} \cup q_1 \cup q_2$$

$$\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\}$$

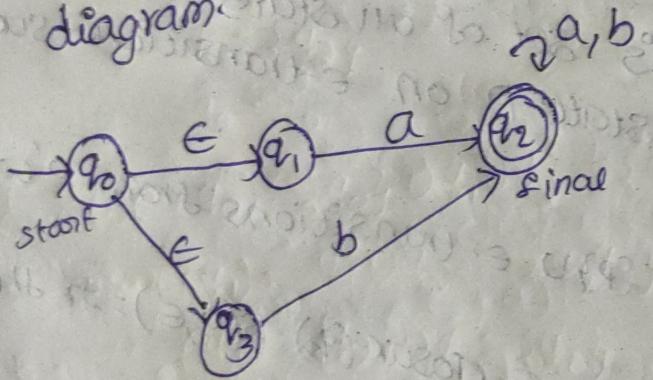
$$\epsilon\text{-closure}(q_1) = q_1 \cup q_2$$

$$= \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = \{q_2\} \cup \emptyset$$

$$= \{q_2\}$$

Ex:- find ϵ -closure of each state in given diagram



$$\epsilon\text{-closure}(q_0) = \{q_0\} \cup \epsilon\text{-closure}(q_1) \cup \epsilon\text{-closure}(q_3)$$

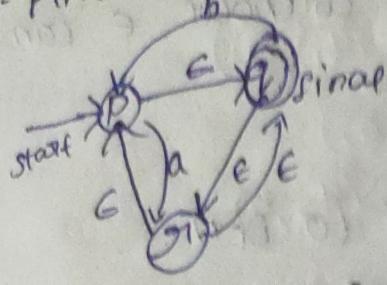
$$= \{q_0, q_1, q_3\}$$

$$\epsilon\text{-closure}(q_1) = q_1 \cup \emptyset = \{q_1\}$$

$$\epsilon\text{-closure}(q_3) = q_3 \cup \emptyset = \{q_3\}$$

$$\epsilon\text{-closure}(q_2) = \emptyset \cup q_2 \cup \emptyset = \{q_2\}$$

Ex: Find ϵ -closure for following.



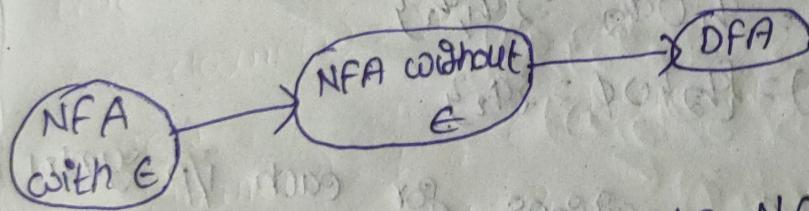
$$\begin{aligned}\epsilon\text{-closure}(p) &= \{p\} \cup \{q\} \cup \{r\} \cup \{q, r\} \\ &= \{p, q, r\}\end{aligned}$$

$$\begin{aligned}\epsilon\text{-closure}(q) &= \{q\} \cup \{p\} \cup \{q\} \cup \{r\} \\ &= \{p, q, r\}\end{aligned}$$

$$\begin{aligned}\epsilon\text{-closure}(r) &= \{r\} \cup \{p\} \cup \{q\} \cup \{p, q\} \\ &= \{p, q, r\}\end{aligned}$$

Conversion from ϵ -NFA to DFA
NFA with ϵ can be converted to NFA without ϵ .
 ϵ can be converted to DFA.

Following diagram represents conversion procedure:-



conversion from ϵ -NFA to NFA :-

Step-1:- Find out all the ϵ transitions from each state in Q that will called as ϵ -closure(q_i) where $q_i \in Q$.

Step-2:- Find s transitions for each i/p symbol s for each state of given ϵ -NFA.

$$\begin{aligned}
 \hat{s}(a_1, b) &= e\text{-closure}(\delta(\hat{s}(a_1, \epsilon), b)) \\
 &= e\text{-closure}(\delta(e\text{-closure}(a_1), b)) \\
 &= e\text{-closure}(\delta(\{a_1, a_2\}, b)) \\
 &= e\text{-closure}(\delta(a_1, b) \cup \delta(a_2, b)) \\
 &= e\text{-clos}(a_1 \cup b) \\
 &= e\text{-closure}(a_1) \\
 &= \{a_1, a_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_1, c) &= e\text{-closure}(\delta(\hat{s}(a_1, \epsilon), c)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_1), c)) \\
 &= e\text{-clos}(\delta(\{a_1, a_2\}, c)) \\
 &= e\text{-clos}(\delta(a_1, c) \cup \delta(a_2, c)) \\
 &= e\text{-clos}(\emptyset \cup a_2) \\
 &= e\text{-clos}(\{a_2\}) \\
 &= \{a_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_2, a) &= e\text{-clos}(\delta(\hat{s}(a_2, \epsilon), a)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_2), a)) \\
 &= e\text{-clos}(\delta(\{a_2\}, a)) \\
 &= e\text{-clos}(\delta(a_2, a)) \\
 &= e\text{-clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_2, b) &= e\text{-clos}(\delta(\hat{s}(a_2, \epsilon), b)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_2), b)) \\
 &= e\text{-clos}(\delta(\{a_2\}, b)) \\
 &= e\text{-clos}(\delta(a_2, b)) \\
 &= e\text{-clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 \delta^*(q_2, c) &= \epsilon\text{-clos}(\delta(\delta^*(q_2, \epsilon), c)) \\
 &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_2), c)) \\
 &= \epsilon\text{-clos}(\delta(q_2), c) \\
 &= \epsilon\text{-clos}(\delta(q_2, c)) \\
 &= \epsilon\text{-clos}(q_2) \\
 &= \{q_2\}
 \end{aligned}$$

Step-3: NFA without ϵ

	a	b	c
* q_0	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
* q_1	\emptyset	$\{q_1, q_2\}$	$\{q_2\}$
* q_2	\emptyset	\emptyset	$\{q_2\}$

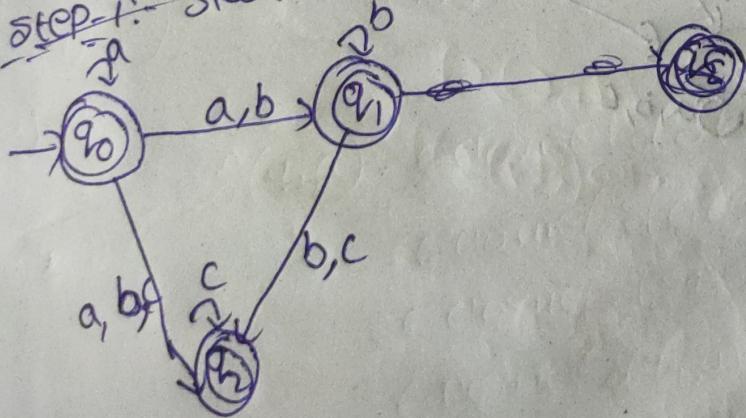
Here q_0, q_1, q_2 acts as final states

ϵ -closure(q_0), ϵ -clos(q_1), & ϵ -clos(q_2)
states of ϵ -NFA i.e q_2 (ϵ -NFA)

of NFA b/z
containing final

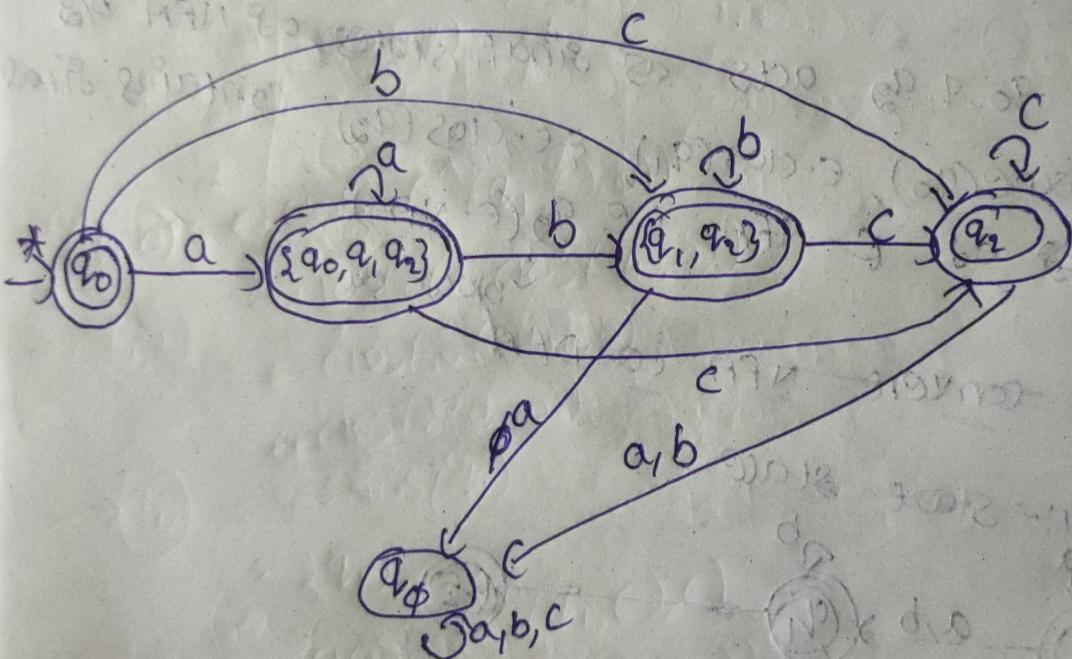
Now convert NFA to DFA

Step-1: Start state

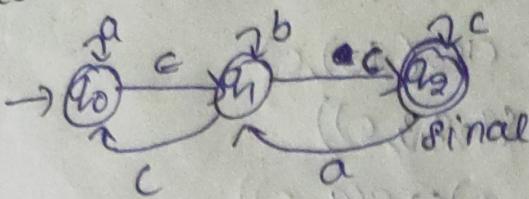


NOC
convert NFA to DFA
shortcut
step-1: state of NFA (q_0) will be start state of DFA

	a	b	c
* $\rightarrow q_0$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_0, q_1, q_2\}$	$\{q_0, q_1, q_2\}$	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_1, q_2\}$	\emptyset	$\{q_1, q_2\}$	$\{q_2\}$
* $\{q_2\}$	\emptyset	\emptyset	$\{q_2\}$
\emptyset	\emptyset	\emptyset	\emptyset



convert following ϵ -NFA to DFA



sol: convert ϵ -NFA to NFA without ϵ given

step-1: Find ϵ -closure for each state in ϵ -NFA

$$Q = \{q_0, q_1, q_2\}$$

$$\Sigma = \{a, b, c\}$$

$$F = \{q_3\}$$

$$q_0 = q_0$$

$$\epsilon\text{-clos}(q_0) = q_0 \cup q_1 = \{q_0, q_1\}$$

$$\epsilon\text{-clos}(q_1) = \emptyset q_1 \cup \emptyset = \{q_1\}$$

$$\epsilon\text{-clos}(q_2) = \emptyset q_2 \cup \emptyset = \{q_2\}$$

ϵ -clos for each i/p symbol

step-2: Find δ transitions for each i/p symbol

$$\begin{aligned}\delta(q_0, a) &= \epsilon\text{-clos}(\delta(\delta(q_0, \epsilon), a)) \\ &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), a)) \\ &= \epsilon\text{-clos}(\delta(\{q_0, q_1\}, a)) \\ &= \epsilon\text{-clos}(\delta(q_0, a) \cup \delta(q_1, a)) \\ &= \epsilon\text{-clos}(\{q_0\} \cup \emptyset) \\ &= \epsilon\text{-clos}(q_0) \\ &= \{q_0, q_1\}\end{aligned}$$

$$\begin{aligned}\delta(q_0, b) &= \epsilon\text{-clos}(\delta(\delta(q_0, \epsilon), b)) \\ &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), b)) \\ &= \epsilon\text{-clos}(\delta(q_0, q_1), b) \\ &= \epsilon\text{-clos}(\delta(q_0, b) \cup \delta(q_1, b)) \\ &= \epsilon\text{-clos}(\emptyset \cup q_1) \\ &= \epsilon\text{-clos}(q_1) \\ &= \{q_1\}\end{aligned}$$

$$\begin{aligned}
 @-\hat{s}(a_0, c) &= e\text{-clos}(\delta(\hat{s}(a_0, e), c)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_0), c)) \\
 &= e\text{-clos}(\delta(\{a_0, a_3, c\})) \\
 &= e\text{-clos}(\delta(a_0, c) \cup \delta(a_1, c)) \\
 &\supseteq e\text{-clos}(\emptyset \cup \{a_2\}) \\
 &= e\text{-clos}(\{a_2\}) \\
 &= \{\{a_2\}\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_1, a) &= e\text{-clos}(\delta(\hat{s}(a_1, e), a)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_1), a)) \\
 &= e\text{-clos}(\delta(\{a_3, a\})) \\
 &= e\text{-clos}(\delta(a_1, a)) \\
 &\supseteq e\text{-clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_1, b) &= e\text{-clos}(\delta(\hat{s}(a_1, e), b)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_1), b)) \\
 &= e\text{-clos}(\delta(\{a_3, b\})) \\
 &= e\text{-clos}(\delta(a_1, b)) \\
 &= e\text{-clos}(a_1) \\
 &= \{\{a_1\}\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(a_1, c) &= e\text{-clos}(\delta(\hat{s}(a_1, e), c)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(a_3), c)) \\
 &= e\text{-clos}(\delta(\{a_3\}), c)) \\
 &= e\text{-clos}(\delta(a_1, c)) \\
 &= e\text{-clos}(\{a_0\}) \\
 &= \{\{a_0\}, \{a_0, a_2\}\}
 \end{aligned}$$

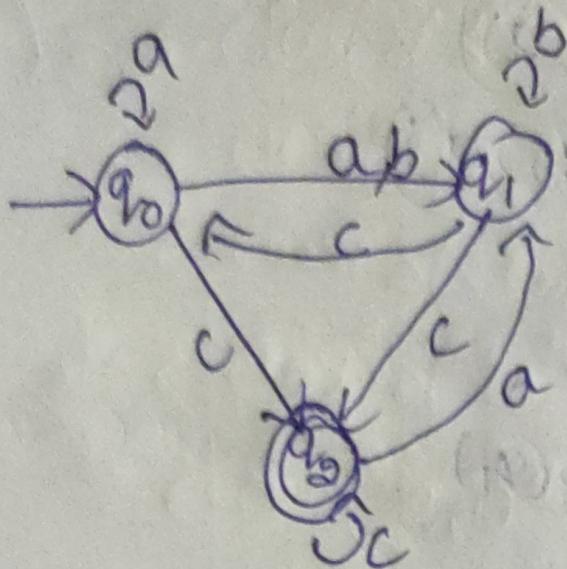
$$\begin{aligned}
 \hat{s}(q_0, a) &= \epsilon\text{-clos}(\delta(\hat{s}(q_0, \epsilon), a)) \\
 &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), a)) \\
 &= \epsilon\text{-clos}(\delta(\{q_0\}, a)) \\
 &= \epsilon\text{-clos}(\delta(q_0, a)) \\
 &= \epsilon\text{-clos}(\emptyset) = \epsilon\text{-clos}(q_1) \\
 &= \emptyset \quad = \{q_1\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(q_0, b) &= \epsilon\text{-clos}(\delta(\hat{s}(q_0, \epsilon), b)) \\
 &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), b)) \\
 &= \epsilon\text{-clos}(\delta(\{q_0\}, b)) \\
 &= \epsilon\text{-clos}(\delta(q_0, b)) \\
 &= \epsilon\text{-clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(q_0, c) &= \epsilon\text{-clos}(\delta(\hat{s}(q_0, \epsilon), c)) \\
 &= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(q_0), c)) \\
 &= \epsilon\text{-clos}(\delta(\{q_0\}, c)) \\
 &= \epsilon\text{-clos}(\delta(q_0, c)) \\
 &= \epsilon\text{-clos}\{q_0\} \\
 &= \{q_0\}
 \end{aligned}$$

Step-3:- NFA without ϵ

		a	b	c
$\rightarrow q_0$		$\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$
q_1		\emptyset	$\{q_1\}$	$\{q_0, q_2\}$
$* q_2$		$\{q_2\}$	\emptyset	$\{q_2\}$

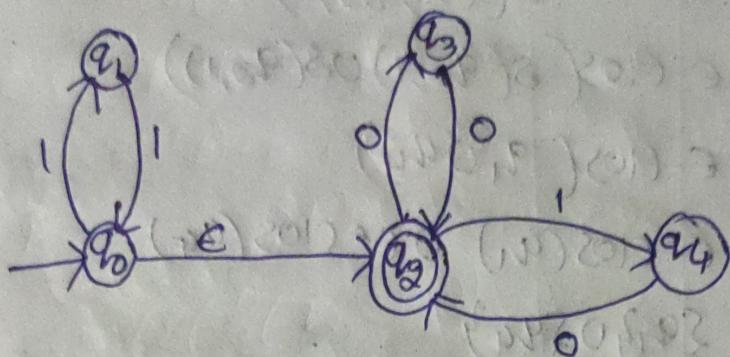


Now convert NFA to DFA

	a	b	c
$\rightarrow q_0$	$\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$
$\{q_0, q_1\}$	$\{q_0, q_1\}$	$\{q_1\}$	$\{q_0, q_2\} \cup \{q_1, q_2\}$
$\{q_1\}$	\emptyset	$\{q_1\}$	$\{q_2\} \cup \{q_0, q_2\}$
$\{q_0, q_2\}$	$\{q_0, q_1\}$	$\{q_1\}$	$\{q_2\}$
$* \{q_2\}$	$\{q_1\}$	\emptyset	$\{q_2\}$
\emptyset	\emptyset	\emptyset	\emptyset

Ex-3 convert following ϵ -NFA to DFA

⑥



Step-1:-

$$Q = \{q_0, q_1, q_2, q_3, q_4\}$$

$$\Sigma = \{0, 1\}$$

$$F = \{q_3\}$$

$$q_0 = q_0$$

$$\epsilon\text{-CLOS}\{q_0\} = q_0 \cup q_2 \quad (= \{q_0, q_2\})$$

$$\epsilon\text{-CLOS}\{q_1\} = \{q_2\}$$

$$\epsilon\text{-CLOS}\{q_2\} = \{q_0, q_2\}$$

$$\epsilon\text{-CLOS}\{q_3\} = \{q_3\}$$

$$\epsilon\text{-CLOS}\{q_4\} = \{q_4\}$$

Step-2:-

$$\hat{\delta}(q_0, 0) = \epsilon\text{-CLOS}(\delta(\hat{\delta}(q_0, \epsilon), 0))$$

$$\hat{\delta}(q_0, 1) = \epsilon\text{-CLOS}(\delta(\epsilon\text{-CLOS}(q_2), 0))$$

$$= \epsilon\text{-CLOS}(\delta(\{q_0, q_2\}), 0)$$

$$= \epsilon\text{-CLOS}(\delta(q_0, 0) \cup \delta(q_2, 0))$$

$$= \epsilon\text{-CLOS}(\emptyset \cup q_3)$$

$$= \epsilon\text{-CLOS}\{q_3\}$$

$$= \{q_3\}$$

$$\begin{aligned}
 \hat{s}(q_0, 1) &= e\text{-clos}(\delta(\hat{s}(q_0, e), 1)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(\{q_0\}), 1)) \\
 &= e\text{-clos}(\delta(\{q_0, q_3\}, 1)) \\
 &= e\text{-clos}(\delta(q_0, 1) \cup \delta(q_3, 1)) \\
 &= e\text{-clos}(q_1 \cup q_4) \\
 &= (e\text{-clos}(q_1) \cup e\text{-clos}(q_4)) \\
 &= \{q_2 \cup q_3\} \\
 &= \{q_1, q_4\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(q_1, 0) &= e\text{-clos}(\delta(\hat{s}(q_1, e), 0)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(\{q_1\}), 0)) \\
 &= e\text{-clos}(\delta(\{q_1, 3\}, 0)) \\
 &= e\text{-clos}(\delta(q_1, 0)) \\
 &= e\text{-clos}(\phi) \\
 &= \phi
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(q_1, 1) &= e\text{-clos}(\delta(\hat{s}(q_1, e), 1)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(\{q_1\}), 1)) \\
 &= e\text{-clos}(\delta(\{q_1\}), \phi) \\
 &= e\text{-clos}(\delta(q_1, 1)) \\
 &= e\text{-clos}(q_0) \\
 &= \{q_0, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 \hat{s}(q_2, 0) &= e\text{-clos}(\delta(\hat{s}(q_2, e), 0)) \\
 &= e\text{-clos}(\delta(e\text{-clos}(\{q_2\}), 0)) \\
 &= e\text{-clos}(\delta(\{q_2\}, 0)) \\
 &= e\text{-clos}(\delta(q_2, 0)) \\
 &= e\text{-clos}(q_3) \\
 &= \{q_3\}
 \end{aligned}$$

$$\hat{s}(q_2, 1) = \epsilon\text{-clos}(\delta(\hat{s}(q_2, \epsilon), 1))$$

$$= \epsilon\text{-clos}(\delta(\hat{s}(q_3, 1)))$$

$$= \epsilon\text{-clos}(\delta(q_3, 1))$$

$$= \epsilon\text{-clos}(\delta(q_2, 1))$$

$$= \epsilon\text{-clos}\{q_4\}$$

$$= \{q_4\}$$

$$\hat{s}(q_3, 0) = \epsilon\text{-clos}(\delta(\hat{s}(q_3, \epsilon), 0))$$

$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_3\}, 0)))$$

$$= \epsilon\text{-clos}(\delta(\{q_3\}, 0))$$

$$= \epsilon\text{-clos}(\delta(q_3, 0))$$

$$= \epsilon\text{-clos}\{q_2\}$$

$$= \{q_2\}$$

$$\hat{s}(q_3, 1) = \epsilon\text{-clos}(\delta(\hat{s}(q_3, \epsilon), 1))$$

$$= \epsilon\text{-clos}(\delta(\epsilon\text{-clos}(\{q_3\}, 1)))$$

$$= \epsilon\text{-clos}(\delta(\{q_3\}, 1))$$

$$= \epsilon\text{-clos}(\delta(q_3, 1))$$

$$= \epsilon\text{-clos}(\emptyset)$$

$$= \emptyset$$

$$\hat{s}(q_4, 0) = \epsilon\text{-clos}(\delta(\hat{s}(q_4, \epsilon), 0))$$

$$= \epsilon\text{-clos}(\delta(q_4, 0))$$

$$= \epsilon\text{-clos}\{q_2\}$$

$$= \{q_2\}$$

$$\hat{s}(q_4, 1) = \epsilon\text{-clos}(\delta(\hat{s}(q_4, \epsilon), 1))$$

$$= \epsilon\text{-clos}(\delta(q_4, 1))$$

$$= \epsilon\text{-clos}(\emptyset)$$

$$= \emptyset$$

NFA without ϵ

	0	1	
$\rightarrow q_0$	q_3	$\{q_1, q_4\}$	$((1, \text{EP})^2, 2) \text{ 2013 } \rightarrow ((1, \text{EP})^2, 2) \text{ 2013 }$
q_1	\emptyset	$\{q_0, q_2\}$	$\{\text{EP}^2, 2013\} \rightarrow$
$* q_2$	$\{q_3\}$	$\{q_4\}$	$\{q_3\} \rightarrow$
q_3	$\{q_2\}$	\emptyset	$((0, (\text{EP})^2, 2) \text{ 2013 } \rightarrow ((0, (\text{EP})^2, 2) \text{ 2013 } \rightarrow$
q_4	$\{q_2\}$	\emptyset	$((0, \text{EP})^2, 2) \text{ 2013 } \rightarrow$

$\xrightarrow{} q_0$

q_3

q_1

q_2

q_4

q_1

$\{q_3\} \text{ 2013 } \rightarrow$

$\{q_3\}$

$((1, (\text{EP})^2, 2) \text{ 2013 } \rightarrow ((1, \text{EP})^2, 2) \text{ 2013 }$

$((1, \text{EP})^2, 2) \text{ 2013 } \rightarrow$

$((1, \text{EP})^2, 2) \text{ 2013 } \rightarrow$

Conversion of ϵ -NFA to DFA (Direct method):-

Consider $M = (Q, \Sigma, \delta, q_0, F)$ be an ϵ -NFA then its equivalent DFA is $M_D = (Q_D, \Sigma_D, \delta_D, q'_0, F_D)$ then

Step-1:- ϵ -closure(q_0) = $\{p_0, p_1, \dots, p_n\}$ be states start state of DFA

Step-2:-

i.e $Q_D = \{\{p_0, p_1, \dots, p_n\}\}$, then find & transition for each state in Q_D on each i/p ϵ .

$\delta_D(\{p_0, p_1, \dots, p_n\}, a) = \epsilon\text{-clos}(\{p_0, p_1, \dots, p_n\}, a)$ where $a \in \Sigma$

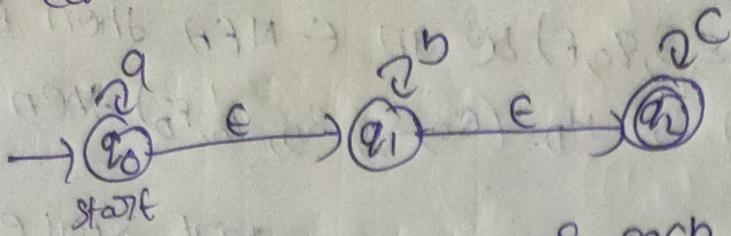
$\delta_D(\{p_0, p_1, \dots, p_n\}, a) = \{q_1, q_2, \dots, q_m\}$ which not in Q_D

then Add(q_1, q_2, \dots, q_m) to Q_D

$\therefore Q_D = \{\{p_0, p_1, \dots, p_n\}, \{q_1, q_2, \dots, q_m\}\}$

Step-3:- repeat step-2 until no new states found.
Step-4:- the states obtained in Q_D , the states containing final state in Q_D acts as final state of DFA.

convert following ϵ -NFA to DFA (direct method)



Step-1:- find ϵ -closure of each state in NFA

$$\epsilon\text{-closure}(q_0) = q_0 \cup q_1 \cup q_2 = \{q_0, q_1, q_2\}$$

$$\epsilon\text{-closure}(q_1) = q_1 \cup q_2 = \{q_1, q_2\}$$

$$\epsilon\text{-closure}(q_2) = q_2 \cup \emptyset = \{q_2\}$$

Step-2:- consider $\epsilon\text{-closure}(q_0) = \{q_0, q_1, q_2\} \rightarrow A$ as start state of DFA

Here A is $Q_D = \{\{q_0, q_1, q_2\}\}$ acts as start state of DFA

Step-3:- Then find S-transition A. on each i/p symbol

$$S_D(A, a) = \epsilon\text{-clos}(S(A, a))$$
$$= \epsilon\text{-clos}(S(\{q_0, q_1, q_2\}, a))$$
$$= \epsilon\text{-clos}(S(q_0, a) \cup S(q_1, a) \cup S(q_2, a))$$
$$= \epsilon\text{-clos}(\{q_0 \cup \emptyset \cup \emptyset\})$$
$$= \epsilon\text{-clos}(\{q_0\})$$
$$= \{q_0, q_1, q_2\} \text{ which is already in } Q_D$$
$$S_D(A, b) = \epsilon\text{-clos}(S(A, b))$$
$$= \epsilon\text{-clos}(S(\{q_0, q_1, q_2\}, b))$$
$$= \epsilon\text{-clos}(S(q_0, b) \cup S(q_1, b) \cup S(q_2, b))$$
$$= \epsilon\text{-clos}(\{\emptyset \cup q_1 \cup \emptyset\})$$
$$= \epsilon\text{-clos}(q_1)$$
$$= \{q_1, q_2\} \rightarrow B$$

$$\therefore Q_D = \left(\frac{\{q_0, q_1, q_2\}}{A}, \frac{\{q_1, q_2\}}{B} \right)$$

$$\begin{aligned}
 s_D(A, C) &= e\text{-clos}(S(A, C)) \\
 &= e\text{-clos}(S(\{q_0, q_1, q_2\}, C)) \\
 &= e\text{-clos}(S(q_0, C) \cup S(q_1, C), S(q_2, C)) \\
 &= e\text{-clos}(\{\phi\} \cup \{q_2\}) \\
 &= e\text{-clos} \{q_2\} \\
 &= \{q_2\} \rightarrow C
 \end{aligned}$$

$$\therefore QD = (\underbrace{\{q_0, q_1, q_2\}}_A, \underbrace{\{q_1, q_2\}}_B, \underbrace{\{q_2\}}_C)$$

$$\begin{aligned}
 s_D(B, A) &= e\text{-clos}(S(B, A)) \\
 &= e\text{-clos}(S(\{q_1, q_2\}, A)) \\
 &= e\text{-clos}(S(q_1, A) \cup S(q_2, A)) \\
 &= e\text{-clos}(\phi \cup \phi) \\
 &= e\text{-clos}(\phi) \\
 &= \phi \rightarrow D
 \end{aligned}$$

$$\therefore QD = (\underbrace{\{q_0, q_1, q_2\}}_A, \underbrace{\{q_1, q_2\}}_B, \underbrace{\{q_2\}}_C, \phi)$$

$$\begin{aligned}
 s_D(B, b) &= e\text{-clos}(S(B, b)) \\
 &= e\text{-clos}(S(\{q_1, q_2\}, b)) \\
 &= e\text{-clos}(S(q_1, b) \cup S(q_2, b)) \\
 &= e\text{-clos}(q_1 \cup \phi) \\
 &= e\text{-clos}(q_1) \\
 &= \{q_1, q_2\}
 \end{aligned}$$

$$\begin{aligned}
 s_D(B, C) &= e\text{-clos}(S(B, C)) \\
 &= e\text{-clos}(S(\{q_1, q_2\}, C)) \\
 &= e\text{-clos}(S(q_1, C) \cup S(q_2, C)) \\
 &= e\text{-clos}(\phi \cup q_2) \\
 &= e\text{-clos}\{q_2\} \\
 &= \{q_2\}
 \end{aligned}$$

$$\begin{aligned}
 s_D(C, a) &= e\text{-clos}(S(C, a)) \\
 &= e\text{-clos}(S(\{q_2\}, a)) \\
 &= e\text{-clos}(S(q_2, a)) \\
 &= e\text{-clos}(\phi) \\
 &= \phi
 \end{aligned}$$

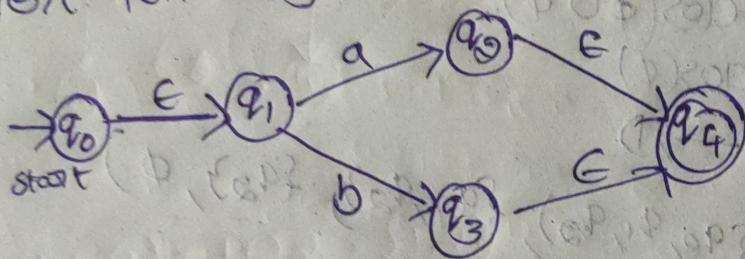
$$\begin{aligned}
 s_D(C, b) &= e\text{-clos}(S(C, b)) \\
 &= e\text{-clos}(S(\{q_2\}, b)) \\
 &= e\text{-clos}(S(q_2, b)) \\
 &= e\text{-clos}(\phi) \\
 &= \phi
 \end{aligned}$$

$$\begin{aligned}
 s_D(C, C) &= e\text{-clos}(S(C, C)) \\
 &= e\text{-clos}(S(\{q_2\}, C)) \\
 &= e\text{-clos}(S(q_2, C)) \\
 &= e\text{-clos}\{q_2\} \\
 &= \{q_2\}
 \end{aligned}$$

	a	b	c	
*A $\{q_0\}$	$\{q_0, q_1, q_3\}$	$\{q_0, q_3\}$	$\{q_2\}$	$\{q_0, q_1, q_3, q_2\}$
*B $\{q_1\}$	\emptyset	$\{q_1, q_3\}$	$\{q_2\}$	$\{q_0, q_1, q_3, q_2\}$
*C $\{q_3\}$	\emptyset	\emptyset	$\{q_2\}$	$\{q_0, q_1, q_3, q_2\}$
*	\emptyset	\emptyset	\emptyset	\emptyset

every state contains final state of
in DFQD every state considered as final state.
ε-NFA so all states considered as final states.

Ex-2:-
convert following ε-NFA to DFA



Step 1:-

$$\epsilon\text{-clos}(q_0) = q_0 \cup q_1 = \{q_0, q_1\}$$

$$\epsilon\text{-clos}(q_1) = q_1 \cup \emptyset = \{q_1\}$$

$$\epsilon\text{-clos}(q_2) = q_2 \cup q_4 = \{q_2, q_4\}$$

$$\epsilon\text{-clos}(q_3) = q_3 \cup q_4 = \{q_3, q_4\}$$

$$\epsilon\text{-clos}(q_4) = q_4 \cup \emptyset = \{q_4\}$$

Step 2:- $\epsilon\text{-clos}(q_0) = \{q_0, q_1\} \rightarrow A$

$$Q_D = \{q_0, q_1\}$$

A is acts as start state of DFA

Step = 2

$$\begin{aligned}
 S_D(A, a) &= e - \text{clos}(S(A, a)) \\
 &= e - \text{clos}(S(\{a_0, a_1, a\}, a)) \\
 &= e - \text{clos}(S(a_0, a) \cup S(a_1, a)) \\
 &= e - \text{clos}(\emptyset \cup a_2) \\
 &= e - \text{clos}(a_2) \\
 &= \{a_2, a_4\} \rightarrow B \text{ (named as)} \\
 Q_D &= \left(\frac{\{a_0, a_1\}}{A}, \frac{\{a_2, a_4\}}{B} \right)
 \end{aligned}$$

$$\begin{aligned}
 S_D(A, b) &= e - \text{clos}(S(A, b)) \\
 &= e - \text{clos}(S(\{a_0, a_1, b\}, b)) \\
 &= e - \text{clos}(S(a_0, b) \cup S(a_1, b)) \\
 &= e - \text{clos}(\emptyset \cup a_3) \\
 &= e - \text{clos}(a_3) \\
 &= \{a_3, a_4\} \rightarrow C
 \end{aligned}$$

$$Q_D = \left(\frac{\{a_0, a_1\}}{A}, \frac{\{a_2, a_4\}}{B}, \frac{\{a_3, a_4\}}{C} \right)$$

$$\begin{aligned}
 S_D(B, a) &= e - \text{clos}(S(B, a)) \\
 &= e - \text{clos}(S(\{a_2, a_4\}, a)) \\
 &= e - \text{clos}(S(a_2, a) \cup S(a_4, a)) \\
 &= e - \text{clos}(\emptyset \cup \emptyset) \\
 &= e - \text{clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 S_D(B, b) &= e - \text{clos}(S(B, b)) \\
 &= e - \text{clos}(S(\{a_2, a_4\}, b)) \\
 &= e - \text{clos}(S(a_2, b) \cup S(a_4, b)) \\
 &= e - \text{clos}(\emptyset \cup \emptyset) \\
 &\Rightarrow \emptyset
 \end{aligned}$$

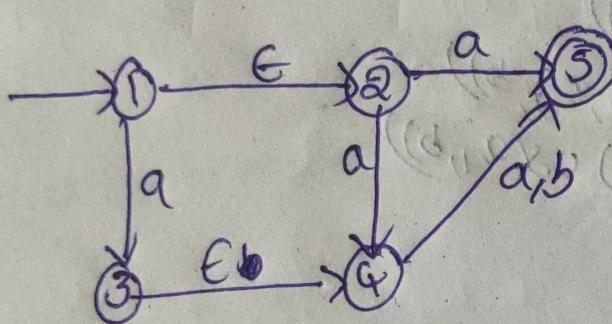
$$\begin{aligned}
 S_D(C, a) &= e - \text{clos}(S(C, a)) \\
 &= e - \text{clos}(S(\{a_3, a_4\}, a)) \\
 &= e - \text{clos}(S(a_3, a) \cup S(a_4, a)) \\
 &= e - \text{clos}(\emptyset \cup \emptyset) \\
 &= \emptyset
 \end{aligned}$$

$$\begin{aligned}
 S_D(C, b) &= e - \text{clos}(S(C, b)) \\
 &= e - \text{clos}(S(\{a_3, a_4\}, b)) \\
 &= e - \text{clos}(\emptyset \cup \emptyset) \\
 &= e - \text{clos}(\emptyset) \\
 &= \emptyset
 \end{aligned}$$

	a	b	
→ A * {q ₀ , q ₁ }	{q ₂ , q ₄ }	{q ₃ , q ₄ }	(q ₀ , q ₁)
* B * {q ₂ , q ₄ }	∅	∅	(q ₀ , q ₂)
* C * {q ₃ , q ₄ }	∅	∅	(q ₃ , q ₄)
∅	∅	∅	(q ₀ , q ₃)

Ex-3:-

ε-NFA \leftrightarrow DFA



Step:-

$$\text{ε-CLOS}(1) = 1 \cup 2 = \{1, 2\}$$

$$\text{ε-CLOS}(2) = 2 \cup \emptyset = \{2\}$$

$$\text{ε-CLOS}(3) = 3 \cup \emptyset = \{3\}$$

$$\text{ε-CLOS}(4) = 4 \cup \emptyset = \{4\}$$

$$\text{ε-CLOS}(5) = 5 \cup \emptyset = \emptyset$$

$$\text{ε-CLOS}(1) = \{1, 2\} \rightarrow A$$

$$Q_0 = \{1, 2\}$$

A acts as start

state of DFA

Step 2:

$$\begin{aligned}
 SD(A, a) &= e^{-\text{clos}(\delta(A, a))} \\
 &= e^{-\text{clos}(\delta(\{1, 2\}, a))} \\
 &= e^{-\text{clos}(\delta(1, a) \cup \delta(2, a))} \\
 &= e^{-\text{clos}(\{3, 4, 5\})} \\
 &= e^{-\text{clos}(\{3, 5, 4\})} \\
 &= e^{-\text{clos}(\{3\})} \circ e^{-\text{clos}(\{5\})} \circ e^{-\text{clos}(\{4\})} \\
 &= \{3\} \cup \{5\} \cup \{4\} \\
 &= \{3, 4, 5\} \rightarrow B
 \end{aligned}$$

$$\begin{aligned}
 SD(A, b) &= e^{-\text{clos}(\delta(A, b))} \\
 &= e^{-\text{clos}(\delta(1, b) \cup \delta(2, b))} \\
 &= e^{-\text{clos}(\phi \cup \phi)} \\
 &= \phi
 \end{aligned}$$

$$SD(B, QD = \left\{ \frac{\{1, 2\}}{A}, \frac{\{3, 4, 5\}}{B} \right\})$$

$$\begin{aligned}
 SD(B, a) &= e^{-\text{clos}(\delta(B, a))} \\
 &= e^{-\text{clos}(\delta(\{3, 4, 5\}, a))} \\
 &= e^{-\text{clos}(\delta(3, a) \cup \delta(4, a) \cup \delta(5, a))} \\
 &= e^{-\text{clos}(\phi \cup \{5\} \cup \phi)} \\
 &= e^{-\text{clos}(\{5\})} \\
 &= \{5\} \rightarrow C
 \end{aligned}$$

$$\begin{aligned}
 SD(B, b) &= e^{-\text{clos}(\delta(B, b))} \\
 &= e^{-\text{clos}(\delta(\{3, 5\}) \cup \delta(\{4\}))} \\
 &= e^{-\text{clos}(\phi \cup \{5\} \cup \phi)} \\
 &= e^{-\text{clos}(\{5\})} \\
 &= e^{-\text{clos}(\{4\})} \circ e^{-\text{clos}(\{5\})} \\
 &= \{4\} \rightarrow D \\
 &= \{5\} \rightarrow D
 \end{aligned}$$

$$QD = \left\{ \frac{\{1, 2\}}{A}, \frac{\{3, 4, 5\}}{B}, \frac{\{5\}}{C}, \frac{\{4, 5\}}{D} \right\}$$

$$\begin{aligned}
 SD(C, a) &= e^{-\text{clos}(\delta(C, a))} \\
 &= e^{-\text{clos}(\delta(\{5\}, a))} \\
 &= e^{-\text{clos}(\delta(5, a))} \\
 &= \phi
 \end{aligned}$$

$$\begin{aligned}
 SD(C, b) &= e^{-\text{clos}(\delta(C, b))} \\
 &= \phi
 \end{aligned}$$

$$\begin{aligned}
 SD(D, a) &= e^{-\text{clos}(\delta(D, a))} \\
 &= e^{-\text{clos}(\delta(\{4, 5\}, a))} \\
 &= e^{-\text{clos}(\delta(4, a) \cup \delta(5, a))} \\
 &= e^{-\text{clos}(\{5\})} \\
 &= \{5\}
 \end{aligned}$$

$$\begin{aligned}
 SD(D, b) &= e^{-\text{clos}(\delta(D, b))} \\
 &= e^{-\text{clos}(\delta(\{5\}, b))} \\
 &= e^{-\text{clos}(\{5\})} \\
 &= \{5\}
 \end{aligned}$$

	α	β
$\{1,2\}A$	$\{3,4,5\}$	\emptyset $(1,2)(3,4)(5)$ $(1,3)(2,4)(5)$ $(1,4)(2,3)(5)$ $(1,5)(2,3)(4)$ $(2,3)(1,4,5)$ $(2,4)(1,3,5)$ $(2,5)(1,3,4)$ $(3,4)(1,2,5)$ $(3,5)(1,2,4)$ $(4,5)(1,2,3)$
$\{3,4,5\}B$	$\{5\}$	$\{4,5\}$ $(1,2,3)(4,5)$ $(1,3,2)(4,5)$ $(1,4,2)(3,5)$ $(1,5,2)(3,4)$ $(2,3,1)(4,5)$ $(2,4,1)(3,5)$ $(2,5,1)(3,4)$ $(3,4,1)(2,5)$ $(3,5,1)(2,4)$ $(4,5,1)(2,3)$
$C\{5\}$	\emptyset	\emptyset $(1,2,3,4)$ $(1,3,2,4)$ $(1,4,2,3)$ $(1,5,2,3,4)$ $(2,3,4,5)$ $(2,4,3,5)$ $(2,5,3,4)$ $(3,4,5,2)$ $(3,5,4,2)$ $(4,5,2,3)$
$\{4,5\}D$	$\{5\}$	$\{5\}$ $(1,2,3,4)$ $(1,3,2,4)$ $(1,4,2,3)$ $(1,5,2,3,4)$ $(2,3,4,5)$ $(2,4,3,5)$ $(2,5,3,4)$ $(3,4,5,2)$ $(3,5,4,2)$ $(4,5,2,3)$
\emptyset	\emptyset	\emptyset $((1,2)(3,4)(5))$ $((1,3)(2,4)(5))$ $((1,4)(2,3)(5))$ $((1,5)(2,3)(4))$ $((2,3)(1,4,5))$ $((2,4)(1,3,5))$ $((2,5)(1,3,4))$ $((3,4)(1,2,5))$ $((3,5)(1,2,4))$ $((4,5)(1,2,3))$

$((1,2)(3,4)(5))$
 $((1,3)(2,4)(5))$
 $((1,4)(2,3)(5))$

$((1,5)(2,3)(4))$
 $((2,3,1)(4,5))$
 $((2,4,1)(3,5))$

$((2,5,1)(3,4))$
 $((3,4,1)(2,5))$
 $((3,5,1)(2,4))$

$((4,5,1)(2,3))$
 $((1,2,3,4)(5))$
 $((1,3,2,4)(5))$

$((1,4,2,3)(5))$
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\emptyset
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