Hall Ticket Number:

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I/IV B.Tech (Regular / Supplementary) DEGREE EXAMINATION

November, 2020 — Common to All Branches

Second Semester — Numerical Methods And Advanced Calculus

Time: Three Hours — Maximum: 50 Marks

*Answer ALL Questions from PART-A.* (1X10 = 10 Marks)

*Answer ANY FOUR questions from PART-B.* (4X10=40 Marks)

## PART-A

| | | | |
|---|---|---|---|
| 1. | a) | What is the order of convergence of Bisection method? | CO1 |
| | b) | State diagonal dominance property. | CO1 |
| | c) | Write Newton's backward interpolation formula. | CO1 |
| | d) | State Trapezoidal rule of integration. | CO2 |
| | e) | Write the Euler's iterative formula for $y' = f(x, y)$, $y(x_0) = y_0$. | CO2 |
| | f) | Evaluate the double integral $\int_0^1\int_1^2 xy\,dy\,dx$ | CO3 |
| | g) | What is formula to find the area enclosed by the plane curves? | CO3 |
| | h) | Find the value of grad f for $f(x,y,z) = xyz$. | CO4 |
| | i) | Is the vector function $\overline{F} = 2x\,I + 3y\,J + 4z\,K$ is irrotational. | CO4 |
| | j) | State stoke's theorem. | CO4 |

## PART-B

| | | | | |
|---|---|---|---|---|
| 2. | a) | Using Newton – Raphson method find a root of the equation $x^3 - 2x - 5 = 0$. | CO1 | 5M |
| | b) | Solve the system of equation $x + 4y - z = -5$; $x + y - 6z = -12$; $3x - y - z = 4$ using Gauss Elimination method. | CO1 | 5M |
| 3. | a) | Find a root of the equation $xe^x - 2 = 0$ using the method of false position. | CO1 | 5M |
| | b) | Solve the system of equations $5x + 2y + z = 12$; $x + 4y + 2z = 15$; $x + 2y + 5z = 20$ using Gauss- Seidel iteration method. Do five iterations. | CO1 | 5M |
| 4. | a) | Find the cubic polynomial which takes the following values (0,1),(1,2), (2,1) and (3,10) using Newton's forward interpolation formula | CO2 | 5M |
| | b) | Estimate the value of f(9) using Lagrange's interpolation formula from the following data: | CO2 | 5M |

| | | | | |
|---|---|---|---|---|
| x | 5 | 7 | 11 | 13 |
| f(x) | 15 | 39 | 14 | 23 |

| | | | | |
|---|---|---|---|---|
| 5. | a) | Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using Simpson's one third rule of integration. Take n = 6. | CO2 | 5M |
| | b) | Apply Runge – Kutta method of 4th order find an approximate value of y for x = 0.2 if $dy/dx = x + y^2$, $y(0) = 1$. Take h = ·1. | CO2 | 5M |

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| | | | | |
|---|---|---|---|---|
| 6. | a) | Evaluate by changing the order of integration $\int_0^{\infty}\int_x^{\infty}\frac{e^{-y}}{y}\,dydx$ | CO3 | 5M |
| | b) | Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $16a^2/3$. | CO3 | 5M |
| 7. | a) | Evaluate the triple integral $\int_{-1}^{1}\int_{0}^{z}\int_{x-z}^{x+z}(x+y+z)\,dy\,dx\,dz$ | CO3 | 5M |
| | b) | Find the volume of the solid bounded by the planes $x = 0$, $y = 0$, $x + y + z = 1$ and $z = 0$. | CO3 | 5M |
| 8. | a) | Find the directional derivative of $f(x,y,z) = xy^2 + yz^3$ at the point (2,-1,1) in the direction of the vector $I + 2J + 2K$. In what directional the directional derivative is maximum? | CO4 | 5M |
| | b) | If $\overline{F}=3xy\,I-y^2J$ evaluate $\int_C \overline{F}.d\overline{R}$, where C is the curve in the xy-plane $y = 2x^2$ from (0,0) to (1,2). | CO4 | 5M |
| 9. | a) | Find the area of a circle of radius a using Green's theorem. | CO4 | 5M |
| | b) | Evaluate $\iint(xdydz + ydzdx + zdxdy)$ over the surface of a sphere of radius a. | CO4 | 5M |

Bapatla Engg. College, Bapatla

I/IV B.Tech (Regular) Degree Examination

November, 2020, II Semester

18MA002, NMAC Scheme of Evaluation

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One mark questions

1)

PART-A

(a) linear (or) I order

(b) In each equation the absolute value of the largest coefficient is greater than the sum of the absolute values of the remaining coefficients.

(c) $y_p = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \cdots$

(d) $\int_a^b f(x)\,dx = \frac{h}{2}\left[(y_0+y_n)+2(y_1+y_2+\cdots+y_{n-1})\right]$

(e) $y_{n+1} = y_n + h f(x_{n-1}, y_{n-1})$ ; $n = 0, 1, 2, \ldots$

(f) 3/4

(g) Area $= \int_{x_1}^{x_2}\int_{f_1(x)}^{f_2(x)} dy\,dx = \int_{y_1}^{y_2}\int_{g_1(y)}^{g_2(y)} dx\,dy$

(h) grad $f = yz\,\bar{i} + xz\,\bar{j} + xy\,\bar{k}$

(i) Yes, curl $\bar{F} = \bar{0}$

(j) If S is an open surface bounded by a closed curve C and $\bar{F} = f_1\bar{i} + f_2\bar{j} + f_3\bar{k}$ be any continuously differentiable vector point function, then

$$\int_C \bar{F}\cdot d\bar{R} = \iint_S (\nabla\times\bar{F})\cdot\bar{N}\,ds$$

where $\bar{N}$ is the unit external normal vector to the surface S.

## Part-B

2 (a) Let $f(x) = x^3 - 2x - 5$, $f'(x) = 3x^2 - 2$

$f(2) = -1$, $-ve$

$f(3) = 16$, $+ve$

A root of $f(x) = 0$ lies between 2 and 3.

choose $x_0 = 2$

Formula, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$ ; $n = 0, 1, 2, 3, \dots$

$n=0$, $x_1 = 2.10000$

$n=1$, $x_2 = 2.09457$

$n=2$, $x_3 = 2.09455$

$n=3$, $x_4 = \underline{2.09455}$

(b) $[A|B] = \left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 1 & 1 & -6 & -12 \\ 3 & -1 & -1 & 4 \end{array}\right]$ $R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 3R_1$

$\sim \left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & -13 & 2 & 19 \end{array}\right]$ $R_3 \to R_3 - \frac{13}{3}R_2$

$\sim \left[\begin{array}{ccc|c} 1 & 4 & -1 & -5 \\ 0 & -3 & -5 & -7 \\ 0 & 0 & \frac{71}{3} & \frac{148}{3} \end{array}\right]$

$\Rightarrow$ $x + 4y - z = -5 \to (1)$

$-3y - 5z = -7 \to (2)$

$\frac{71}{3} z = \frac{148}{3}$

By back substitution, $z = \frac{148}{71}$, $y = -\frac{81}{71}$, $x = \frac{117}{71}$.

(3) (a) Let $f(x) = xe^x - 2$, $f(0) = -2$, -ve, $f(1) = 0.7183$, +ve

~~$f(2) = 12.7781$~~ $\therefore$ root lies b/w 0 and 1

Take $x_0 = 0$ and $x_1 = 1$.

$$x_2 = x_0 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_0) = 0.7358, \quad f(x_2) = -0.4644$$

$x_3 = 0.8395$, $f(x_3) = -0.0563$

$x_4 = 0.8512$, $f(x_4) = -0.0062$

$x_5 = 0.8525$, $f(x_5) = -0.007$

$x_6 = \underline{0.8526}$,

(b) $x_{n+1} = \frac{1}{5}\left[12 - 2y_n - z_n\right]$

$y_{n+1} = \frac{1}{4}\left[15 - x_{n+1} - 2z_n\right]$ $\quad n = 0, 1, 2, 3, --$

$z_{n+1} = \frac{1}{5}\left[20 - x_{n+1} - 2y_{n+1}\right]$ $\quad x_0 = y_0 = z_0 = 0$

$n = 0$, $x_1 = 2.4$, $y_1 = 3.15$, $z_1 = 2.26$

$n = 1$, $x_2 = 0.688$, $y_2 = 2.448$, $z_2 = 2.8832$

$n = 2$, $x_3 = 0.8442$, $y_3 = 2.0974$, $z_3 = 2.9922$

$n = 3$, $x_4 = 0.9626$, $y_4 = 2.0132$, $z_4 = 3.0022$

$n = 4$, $x_5 = 0.9943$, $y_5 = 2.0003$, $z_5 = 3.001$

$\therefore$ $x = 1$, $y = 2$ and $z = 3$ is the solution.

4 (a) Difference Table

| $x$ | $f(x)$ | $\Delta f(x)$ | $\Delta^2 f(x)$ | $\Delta^3 f(x)$ |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | 1 | | |
| 1 | 2 | | -2 | |
| | | -1 | | 12 |
| 2 | 1 | | 10 | |
| | | 9 | | |
| 3 | 10 | | | |

Take $x_0 = 0$, $p = \frac{x - x_0}{h} = x$, $h = 1$

$$f(x) = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + x \cdot 1 + \frac{x(x-1)}{2}(-2) + \frac{x(x-1)(x-2)}{6}(12)$$

$$= 2x^3 - 7x^2 + 6x + 1.$$

(b) $$f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

$$f(9) = \frac{16}{-96} \times 15 + \frac{32}{48} \times 39 + \frac{-32}{-48} \times 14 + \frac{-16}{96} \times 23$$

$$= 29$$

(5) (a) $a = 0$, $b = 6$, $f(x) = \frac{1}{1+x^2}$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x$: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $f(x)$: | 1 | 0.5 | 0.2 | 0.1 | 0.0588 | 0.0385 | 0.027 |

By ~~FA~~ simpson's $\frac{1}{3}$rd rule, we have

$$\int_0^6 \frac{dx}{1+x^2} = \frac{h}{3}\left[(y_0 + y_6) + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4)\right]$$

$$= 1.3662$$

5 (b) $f(x,y) = x + y^2$, $x_0 = 0$, $y_0 = -1$, $h = .1$

$k_1 = h f(x_0, y_0) = 0.1000$

$k_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}) = 0.1152$

$k_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}) = 0.1168$

$k_4 = h\, f(x_0+h, y_0+k_3) = 0.1347$

$k = \frac{1}{6}(k_1+2k_2+2k_3+k_4) = 0.1165$

$y(0.1) = y_0 + k = 1.1165$

$x_1 = x_0 + h,\ y_1 = 1.1165,\ h = 0.1$

$k_1 = h f(x_1, y_1) = 0.1347$

$k_2 = h\, f(x_1+\frac{h}{2}, y_1+\frac{k_1}{2}) = 0.1551$

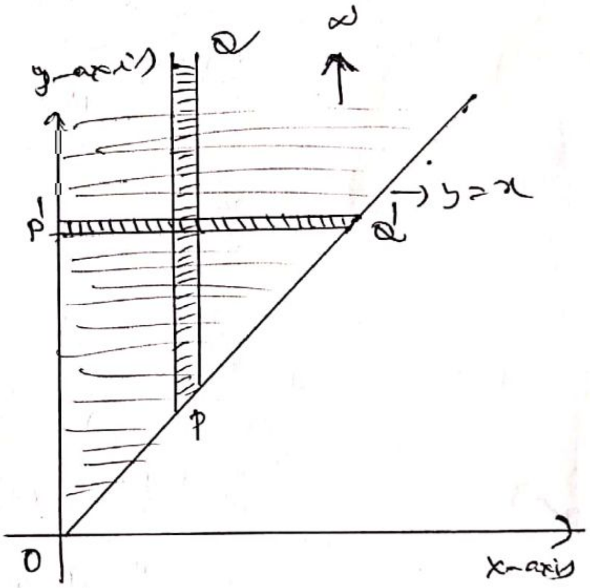
$k_3 = h\, f(x_1+\frac{h}{2}, y_1+\frac{k_2}{2}) = 0.1576$

$k_4 = h\, f(x_1+h, y_1+k_3) = 0.1823$

$k = 0.1571$

$y(0.2) = y_1 + k = 1.2736$

6 (a) Given $I = \int_0^\infty \int_x^\infty \frac{e^{-y}}{y}\, dy\, dx$




After changing the order of integration

$$I = \int_0^\infty \int_0^y \frac{e^{-y}}{y}\, dx\, dy$$

$$= \int_0^\infty \frac{e^{-y}}{y} (x)_0^y\, dy$$

$$= \int_0^\infty \frac{e^{-y}}{\cancel{y}} (\cancel{y-0})\, dy$$

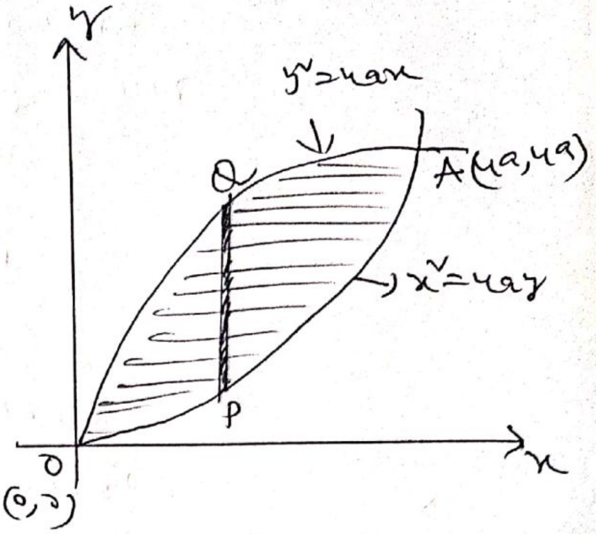
$$= (-e^{-y})_0^\infty$$

$$= -e^{-\infty} + e^0$$

$$= 1.$$

(b) Required Area

$$= \int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dy\, dx$$




$$= \int_0^{4a} \left(2\sqrt{ax} - \frac{x^2}{4a}\right) dx$$

$$= \left(2\sqrt{a}\, \frac{x^{3/2}}{3/2} - \frac{x^3}{12a}\right)_0^{4a}$$

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2 \text{ sq. units}$$

7(a) Let $I = \int_{-1}^{1} \int_0^{z} \int_{x-z}^{x+z} (x+y+z)\, dx\, dy\, dx$ with respect to y first

$$= \int_{-1}^{1} \int_0^{z} \left(xy + \frac{y^2}{2} + yz\right)_{x-z}^{x+z} dx\, dz$$

$$= \int_{-1}^{1} \int_0^{z} \left[(x+z)(2z) + \frac{4}{2}xz\right] dx\, dz$$

$$= 2\int_{-1}^{1} \left(\frac{x^2 z}{2} + xz^2 + \frac{x^2}{2} z\right)_0^{z} dz$$

$$= 2\int_{-1}^{1} \left(\frac{z^3}{2} + z^3 + \frac{z^3}{2}\right) dz$$

$$= 4\left(\frac{z^4}{4}\right)_{-1}^{1}$$

$$= 0$$

(b) Required volume $= \int_0^1 \int_0^{1-x} \int_0^{1-x-y} dz\, dy\, dx$

$$= \int_0^1 \int_0^{1-x} (1-x-y)\, dy\, dx$$

$$= \int_0^1 \left(y - xy - \frac{y^2}{2}\right)_0^{1-x} dx = \int_0^1 \left[(1-x)y - \frac{y^2}{2}\right]_0^{1-x} dx$$

$$= \frac{1}{2} \int_0^1 (1-x)^2 dx = 1/6 \text{ cubic units}$$

8 (a) $\nabla f = y^2 i + (2xy + z^3) j + 3yz^2 k$

$\nabla f$ at $P(2,-1,1) = i - 3j - 3k$

directional derivative $= (i - 3j - 3k) \cdot \dfrac{i + 2j + 2k}{3}$

$$= \frac{1-6-6}{3} = -\frac{11}{3}$$

(b) $\int_C \bar{F} \cdot d\bar{R} = \int_C (3xy\, dx - y^2 dy)$

$$= \int_0^1 3x(2x^2)\, dx - (2x^2)^2 d(2x^2)$$

$$= \int_0^1 (6x^3 - 16x^5)\, dx = -7/6$$

9 (a) By Green's Theorem, Area $= \frac{1}{2} \oint_C (x\,dy - y\,dx)$

For a circle of radius a, $x = a\cos t$, $y = a\sin t$

and $0 \le t \le 2\pi$

$$\therefore \text{Area} = \frac{1}{2} \int_0^{2\pi} [a\cos t(a\cos t)\,dt - a\sin t(-a\sin t)\,dt]$$

$$= \frac{a^2}{2} \int_0^{2\pi} (\cos^2 t + \sin^2 t)\, dt = \frac{a^2}{2} 2\pi = \pi a^2 \text{ sq. units}$$

(b) By Gauss-Divergence Theorem

$$\iint_S (x\,dy\,dz + y\,dz\,dx + z\,dx\,dy) = \iiint_V \left(\frac{\partial}{\partial x} x + \frac{\partial}{\partial y} y + \frac{\partial}{\partial z} z\right) dx\,dy\,dz$$

$$= \iiint_V (1+1+1)\, dx\,dy\,dz$$

$$= 3 \iiint_V dx\,dy\,dz \qquad \left(\text{volume of } x^2+y^2+z^2=a^2 \text{ is } \frac{4}{3}\pi a^3\right)$$

$$= 3 \cdot \frac{4}{3}\pi a^3$$

$$= 4\pi a^3.$$

P.V. Saradhi
(Dr. P. Vijaya Saradhi)
HOD, Maths.

Prepared by
T. Srinivasa Rao
(DR T. SRINIVASA RAO)
Associate professor
Dept of Mathematics
BEC, Bapatla