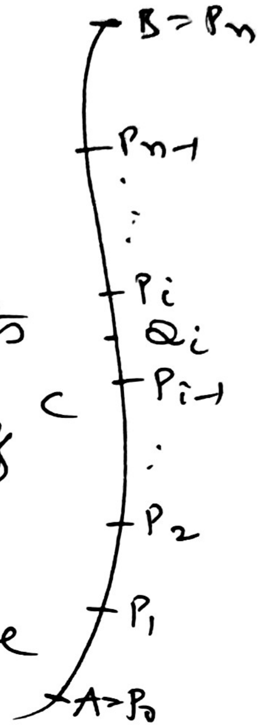
Line integrals

Any integral which is to be evaluated along a curve is called a line integral.

Let $\bar{F}(P)$ be a continuous vector point function defined at every point of a curve C in space. Divide the curve C into n parts by the points $A = P_0, P_1, P_2, \dots, P_n = B$ & let $\bar{R}_0, \bar{R}_1, \dots, \bar{R}_n$ be the position vectors of these points. Let Q_i be any point on the arc $P_{i-1}P_i$.



consider the sum $\sum_{i=1}^{n} \bar{F}(Q_i)\cdot\delta\bar{R}_i$ where $\delta\bar{R}_i = \bar{R}_i - \bar{R}_{i-1}$. The limit of the above sum as $n \to \infty$ and every $|\delta\bar{R}_i| \to 0$, if it exists, is called a line integral of $\bar{F}$ along a curve C and is denoted by $\int_C \bar{F}\cdot d\bar{R}$ (or) $\int_C \bar{F}\cdot\frac{d\bar{R}}{dt}\,dt$. If C is a closed curve, then the line integral is denoted by $\oint_C \bar{F}\cdot d\bar{R}$, which is a scalar.

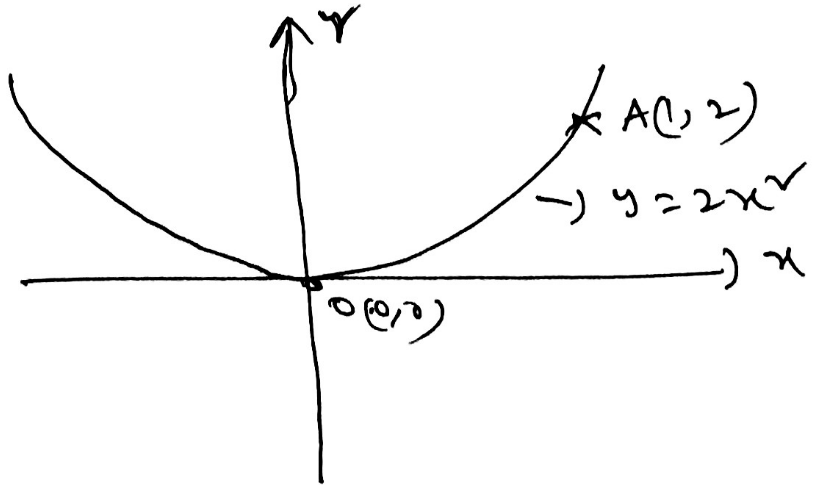
Remark (i): If $\bar{F}$ represents the velocity of a fluid particle then the line integral $\int_C \bar{F} \cdot d\bar{R}$ is called the circulation of $\bar{F}$ around the curve C. When the circulation of $\bar{F}$ around every closed curve in a region E vanishes, $\bar{F}$ is said to be irrotational in E.

(ii) The total work done by $\bar{F}$ during the displacement from A to B is given by the line integral $\int_A^B \bar{F} \cdot d\bar{R}$.

Two other types of line integrals are $\int_C \bar{F} \times d\bar{R}$ and $\int_C f \, d\bar{R}$ which are both vectors.

Problem (1): If $\bar{F} = 3xy\,I - y^2 J$, evaluate $\int_C \bar{F} \cdot d\bar{R}$, where C is the curve in the xy-plane $y = 2x^2$ from (0,0) to (1,2).

Solution:



As C is a curve in the xy-plane, $z=0$.

So, $\bar{R} = xI + yJ + 0K$

$= xI + yJ$

$\bar{F}\cdot d\bar{R} = (3xy\,I - y^2 J)\cdot(dx\,I + dy\,J)$

$= 3xy\,dx - y^2\,dy$

Also along the curve C, $y = 2x^2$ and $dy = 4x\,dx$

when $y=0$, $2x^2 = 0$ and hence $x=0$

when $y=2$, $2x^2=2$ and hence $x=1$

So, along the curve C x varies from 0 to 1

$$\therefore \int_C \bar{F}\cdot d\bar{R} = \int_0^1 [3x(2x^2)\,dx - 4x^4\,4x\,dx]$$

$$= \int_0^1 (6x^3 - 16x^5)\,dx$$

$$= \left[6\frac{x^4}{4} - 16\frac{x^6}{6}\right]_0^1$$

$$= \frac{6}{4} - \frac{16}{6}$$

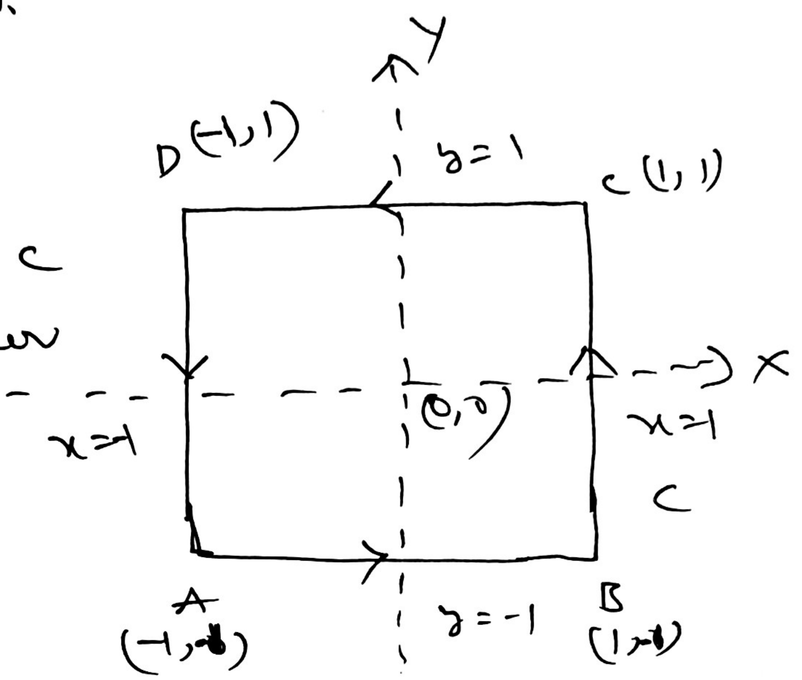
$$= \frac{3}{2} - \frac{8}{3}$$

$$= -\frac{7}{6}.$$

Problem (2): Evaluate $\int_C [(x^2+xy)\,dx + (x^2+y^2)\,dy]$ where C is the square formed by the lines $x = \pm$ and $y = \pm 1$.

Solution:

Here the curve C is composed of four lines $x=1$, $x=-1$, $y=1$ and $y=-1$.



Clearly, $\bar{F} = (x^2+xy)\,I + (x^2+y^2)\,J$ and $\bar{R} = xI + yJ$ so that $d\bar{R} = dx\,I + dy\,J$.

$$\oint_C \bar{F}\cdot d\bar{R} = \int_{AB} \bar{F}\cdot d\bar{R} + \int_{BC} \bar{F}\cdot d\bar{R} + \int_{CD} \bar{F}\cdot d\bar{R} + \int_{DA} \bar{F}\cdot d\bar{R}$$

(i) Along AB, $y=-1$ and $dy=0$. x varies from -1 to 1.

(ii) Along BC, $x=1$ and $dx=0$. y varies from -1 to 1.

(iii) Along CD, $y=1$ and $dy=0$. x varies from 1 to -1.

(iv) Along DA, $x = -1$ and $dx = 0$. y varies from 1 to -1.

$* \int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

$$\therefore \oint_C \left[(x^2+xy)\,dx + (x^2+y^2)\,dy\right] = \int_{-1}^{1}(x^2-x)\,dx + \int_{-1}^{1}(1+y^2)\,dy + \int_{1}^{-1}(x^2+x)\,dx + \int_{1}^{-1}(1+y^2)\,dy$$

$$\Rightarrow \int_C \bar{F}\cdot d\bar{R} = \cancel{\int_{-1}^{1} x^2\,dx} - \int_{-1}^{1} x\,dx - \cancel{\int_{1}^{-1}(1+y^2)\,dy}\ *$$
$$* - \cancel{\int_{-1}^{1} x^2\,dx} - \overset{*}{\int_{-1}^{1}} x\,dx + \cancel{\int_{1}^{-1}(1+y^2)\,dy}$$

$$= -2\int_{-1}^{1} x\,dx \qquad \left(\int_{-l}^{l} f(x)\,dx = 0 \text{ if } f(x) \text{ is odd}\right).$$

$$= 0$$

$$\therefore \int_C \left[(x^2+xy)\,dx + (x^2+y^2)\,dy\right] = 0.$$

Problem(3): If $\bar{A} = (3x^2+6y)I - 14yzJ + 20xz^2K$

evaluate $\int \bar{A} \cdot d\bar{R}$ from $(0,0,0)$ to $(1,1,1)$ along the path $x = t,\ y = t^2,\ z = t^3$.

solution: Given $\bar{A} = (3x^2 + 6y)I - 14yzJ + 20xz^2K$

let $\bar{R} = xI + yJ + zK$. Then $d\bar{R} = dxI + dyJ + dzK$

Then $\bar{A} \cdot d\bar{R} = (3x^2 + 6y)\,dx - 14yz\,dy + 20xz^2\,dz$

Let $A = (0,0,0)$ and $B = (1,1,1)$. The parametric equations of the curve are $x = t,\ y = t^2,\ z = t^3$

when $x = 0,\ t = 0$ and when $x = 1,\ t = 1$.

consider $\int_A^B \bar{A} \cdot d\bar{R} = \int_{(0,0,0)}^{(1,1,1)} [(3x^2 + 6y)dx - 14yz\,dy + 20xz^2 dz]$

$$= \int_0^1 [(3t^2 + 6t^2)dt - 14t^5(2t\,dt) + 20t^7(3t^2 dt)]$$

$$= \int_0^1 (9t^2 - 28t^6 + 60t^9)\,dt$$

$$= \left(9\frac{t^3}{3} - 28\frac{t^7}{7} + 60\frac{t^{10}}{10}\right)_0^1$$

$$= 3 - 4 + 6$$

$$= 5$$

Problem (4): Find the total work done by the force $\bar{F} = 3xy\,I - y\,J + 2zx\,K$ in moving a particle around the circle $x^2+y^2=4$.

Solution: Given $\bar{F} = 3xy\,I - y\,J + 2zx\,K$

Let $\bar{R} = x\,I + y\,J + z\,K$ & $d\bar{R} = dx\,I + dy\,J + dz\,K$

The parametric equations of the circle are $x = 2\cos t$ and $y = 2\sin t$ where $0 \le t \le 2\pi$

$$\therefore \oint_C \bar{F}\cdot d\bar{R} = \oint_C \left[3xy\,dx - y\,dy + 2zx\,dz\right]$$

But on the xy-plane $z=0$ & $dz=0$

$$\therefore \oint_C \bar{F}\cdot d\bar{R} = \oint_C \left[3xy\,dx - y\,dy\right]$$

$$= \int_0^{2\pi} \left[3(2\cos t\, 2\sin t)(-2\sin t\,dt) - 2\sin t\,(2\cos t\,dt)\right]$$

$$= \int_0^{2\pi} \left[24\cos t\sin^2 t - 4\sin t\cos t\right] dt$$

$$= \left[-24\frac{\sin^3 t}{3} - 4\frac{\sin^2 t}{2}\right]_0^{2\pi}$$

$$= (-8(0) - 2(0)) - (-8(0) - 2(0))$$

$$= 0.$$

Thus, the work done by the force $\bar{F}$ in moving a particle around the circle is 0.

Problem (5): If $\bar{F} = 2yI - zJ + xK$, evaluate $\oint_C \bar{F} \times d\bar{R}$ along the curve $x = \cos t$, $y = \sin t$, $z = 2\cos t$ from $t = 0$ to $t = \frac{\pi}{2}$.

Solution: Given $\bar{F} = 2yI - zJ + xK$

let $\bar{R} = xI + yJ + zK$ & $d\bar{R} = dxI + dyJ + dzK$

$$\text{Now } \bar{F} \times d\bar{R} = \begin{vmatrix} I & J & K \\ 2y & -z & x \\ dx & dy & dz \end{vmatrix}$$

$$= (-z\,dz - x\,dy)I + (x\,dx - 2y\,dz)J + (2y\,dy + z\,dx)K$$

When $x = \cos t$, $y = \sin t$ and $z = 2\cos t$

$$\bar{F} \times d\bar{R} = [-2\cos t(-2\sin t\,dt) - \cos t(\cos t\,dt)]I$$
$$+ [\cos t(-\sin t\,dt) - 2\sin t(-2\sin t\,dt)]J$$
$$+ [2\sin t(\cos t\,dt) + 2\cos t(-\sin t\,dt)]K$$

$$= (4\cos t\sin t - \cos^2 t)I + (4\sin^2 t - \sin t\cos t)J$$
$$+ 0K$$

$$\therefore \oint_C \bar{F} \times d\bar{R} = \int_0^{\pi/2} \left[\left(4\cos t \sin t - \cos^2 t\right) I + 4\sin^2 t - \cos t \sin t \, J\right] dt$$

$$= \left(4\left(\frac{\sin^2 t}{2}\right)_0^{\pi/2} - \frac{1}{2}\cdot\frac{\pi}{2}\right) I + \left(4 \cdot \frac{1}{2}\cdot\frac{\pi}{2} - \left(\frac{\sin^2 t}{2}\right)_0^{\pi/2}\right) J$$

$$= \left(2 - \frac{\pi}{4}\right) I + \left(\pi - \frac{1}{2}\right) J.$$

Problem (6): Find the work done in moving a particle in the force field $\bar{F} = 3x^2 I + (2xz - y) J + z K$ along the curve defined by $x^2 = 4y$, $3x^3 = 8z$ from $x = 0$ to $x = 2$.

Solution: Given $\bar{F} = 3x^2 I + (2xz - y) J + z K$

Let $\bar{R} = xI + yJ + zK$ & $d\bar{R} = dx\, I + dy\, J + dz\, K$

Now $\bar{F} \cdot d\bar{R} = 3x^2 dx + (2xz - y)\, dy + z\, dz$

The parametric equations of the given curve are $x = t$, $y = \frac{t^2}{4}$, $z = \frac{3}{8}t^3$.

Clearly when $x = 0$, $t = 0$ and when $x = 2$, $t = 2$.

So t varies from 0 to 2.

$$\therefore \text{work done} = \int_C \bar{F}\cdot d\bar{R}$$

$$= \int_0^2 \left[3t^2 dt + \left(2t \cdot \frac{3}{8}t^3 - \frac{t^2}{4}\right)\frac{2t}{4} dt + \frac{3}{8}t^3 \, \frac{9t^2}{8} dt\right]$$

$$= \int_0^2 \left(3t^2 + \frac{3}{8}t^5 - \frac{1}{8}t^3 + \frac{27}{64}t^5\right) dt$$

$$= \int_0^2 \left(3t^2 - \frac{1}{8}t^3 + \frac{51}{64}t^5\right) dt$$

$$= \left[t^3 - \frac{t^4}{32} + \frac{51}{64\times 6}t^6\right]_0^2$$

$$= 8 - \frac{16}{32} + \frac{51}{64\times 6} 64$$

$$= 8 - \frac{1}{2} + \frac{51}{6}$$

$$= \frac{48-3+51}{6}$$

$$= \frac{96}{6} = 16.$$

Practice problems

1) If $\bar{F} = (5xy - 6x^2) I + (2y - 4x) J$, evaluate $\int_C \bar{F} \cdot d\bar{R}$ along the curve c in the xy-plane, $y = x^3$ from $(1, 1)$ to $(2, 8)$.

2) Compute the line integral $\int_C (y^2 dx - x^2 dy)$ about the triangle whole vertices are $(1, 0)$, $(0, 1)$ and $(-1, 0)$.

3) Find the total work done in moving a particle in a force field given by $\bar{F} = 3xy I - 5z J + 10x K$ along the curve $x = t^2 + 1$, $y = 2t^2$, $z = t^3$ from $t = 1$ to $t = 2$.

Surface integral: Any integral which is to be evaluated over a surface is called a surface integral.

Let $\bar{F}(P)$ be a continuous vector point function and S is a two sided surface. Divide the surface S into a finite number of sub-surfaces $\delta S_1, \delta S_2, \ldots, \delta S_K$. Let P_i be any point in δS_i and $\bar{n}_i$ be the unit vector at P in the direction of the outward normal to the surface at P_i. Then the limit of the sum

$$\sum_{i=1}^{K} \bar{F}(P_i) \cdot \bar{n}_i \, \delta S_i, \text{ as } K \to \infty \text{ and each}$$

$\delta S_i \to 0$ is called surface integral of $\bar{F}(P)$ over S and is denoted by $\iint_S \bar{F} \cdot \bar{n} \, ds$ (or)

$$\iint_S \bar{F} \cdot d\bar{S}.$$

Remark: If $\bar{F}$ represents the velocity of a fluid particle then the total outward flux of $\bar{F}$ across a closed surface S is the surface integral $\int_S \bar{F} \cdot d\bar{S}$ (or) $\iint_S \bar{F} \cdot d\bar{S}$.

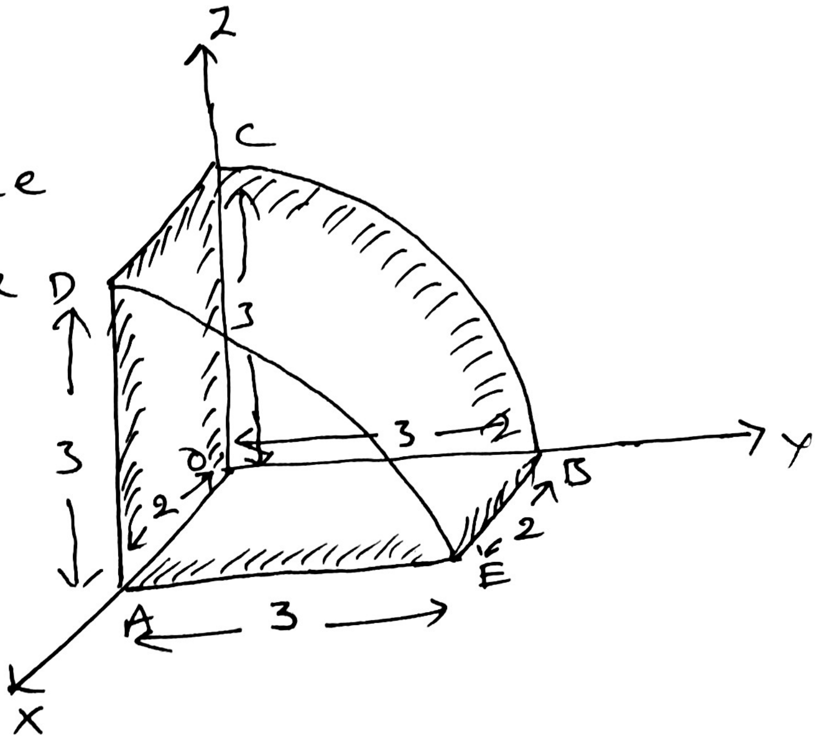
when the flux of $\bar{F}$ across every closed surface S in a region E vanishes then $\bar{F}$ is said to be a solenoidal vector point function in E.

<u>Note</u>: Flux means the volume emerging from a surface S per unit time.

Problem (1): Evaluate $\int_S \bar{F}.\bar{N}\,dS$ where $\bar{F} = 2x^2y\,I - y^2 J + 4xz^2 K$ and S is the closed surface of the region in the first octant bounded by the cylinder $y^2+z^2=9$ and the planes $x=0$, $x=2$, $y=0$ and $z=0$.

Solution:

Here closed surface S is composed of five sub-surfaces namely S_1, S_2, S_3, S_4 and S_5



where S_1: the rectangular face OAEB in the xy-plane

S_2: the rectangular face OADC in the xz-plane.

S_3: OBC in the yz-plane

S_4: AED ~~..~~ and

S_5: the curved surface BCDE.

$$\therefore \int_S \bar{F}\cdot\bar{N}\,ds = \int_{S_1} \bar{F}\cdot\bar{N}\,ds + \int_{S_2} \bar{F}\cdot\bar{N}\,ds + \int_{S_3} \bar{F}\cdot\bar{N}\,ds$$

$$+ \int_{S_4} \bar{F}\cdot\bar{N}\,ds + \int_{S_5} \bar{F}\cdot\bar{N}\,ds \rightarrow (1)$$

consider $\int_{S_1} \bar{F}\cdot\bar{N}\,ds = \int_{S_1} (2x^2y\,I - y^2 J + 4xz^2 K)\cdot(-K)\,ds$

$$= \int_{S_1} -4xz^2\,ds$$

$$= -4\int_{S_1} x\,0^2\,ds \quad \text{(on the xy-plane } z=0)$$

$$= 0$$

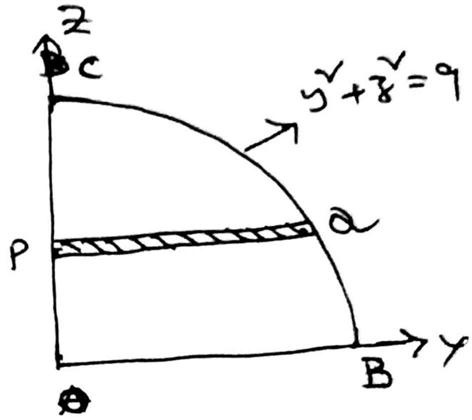
consider $\int_{S_2} \bar{F}\cdot\bar{N}\,ds = \int_{S_2} (2x^2y\,I - y^2 J + 4xz^2 K)\cdot(-J)\,ds$

$$= \int_{S_2} -y^2\,ds = 0 \quad \text{(on the xz-plane } y=0).$$

Similarly, $\int_{S_3} \bar{F}\cdot\bar{N}\,ds = 0$

consider $\int_{S_4} \bar{F}\cdot\bar{N}\,ds = \int_{S_4} (2x^2 y I - y^2 J + 4xz^2 K)\cdot I\,ds$

Here equation of the plane AED is $x = 2$.



* $ds = \dfrac{dy\,dz}{|\bar{N}\cdot I|}$

$= \dfrac{dy\,dz}{|I\cdot I|}$

$= dy\,dz$

$\therefore \int_{S_4} \bar{F}\cdot\bar{N}\,ds = \int_0^3 \int_0^{\sqrt{9-z^2}} 2(2)^2 y\,dy\,dz$

$= 8\int_0^3 \left(\frac{y^2}{2}\right)_0^{\sqrt{9-z^2}} dz$

$= 4\int_0^3 (9-z^2)\,dz$

$= 4\left(9z - \frac{z^3}{3}\right)_0^3$

$= 4\left(27 - \frac{27}{3}\right)$

$= \frac{4}{3}(54) = 72$

* when we projecting the surface on the yz plane then $ds = \dfrac{dy\,dz}{|\bar{N}\cdot I|}$

Finally, S_5 is a curved surface. To find unit external normal vector we use gradient.

Equation of the surface BCDE is $y^2+z^2=9$.

So, $\nabla(y^2+z^2) = 2y J + 2z K$

$$\Rightarrow \bar{N} = \frac{2y J + 2z K}{\sqrt{(2y)^2+(2z)^2}}$$

$$= \frac{2y J + 2z K}{\sqrt{4(y^2+z^2)}}$$

$$= \frac{2(y J + z K)}{2\sqrt{9}}$$

$$= \frac{y J + z K}{3}$$

The projection of BCDE lies on the xy-plane. So,

$$ds = \frac{dx\,dy}{|\bar{N}\cdot K|}$$

$$= \frac{dx\,dy}{\left|\frac{y J + z K}{3}\cdot K\right|}$$

$$= \frac{dx\,dy}{\left(\frac{z}{3}\right)}$$

$$\bar{F}\cdot\bar{N} = (2x^2y\, I - y^2 J + 4xz^2 K)\cdot\left(\frac{y J + z K}{3}\right) = \frac{-y^3+4xz^3}{3}$$

Thus $\int_{S_5} \bar{F}\cdot\bar{N}\,dS = \int_0^2\int_0^3 \frac{-y^3+4xz^3}{\cancel{z}}\,\frac{dy\,dx}{z/\cancel{3}}$

$$= \int_0^2\int_0^3 \left(\frac{-y^3}{z} + 4xz^2\right) dy\,dx$$

put $y = 3\sin\theta$

$z = 3\cos\theta$

$dy = 3\cos\theta\,d\theta$

$\theta: 0 \to \pi/2$

$$= \int_0^2\int_0^{\pi/2}\left[\frac{-27\sin^3\theta}{3\cos\theta} + 4x(9\cos^2\theta)\right] \times 3\cos\theta\,d\theta\,dx$$

$$= \int_0^2\int_0^{\pi/2}\left[-27\sin^3\theta + 108x\cos^3\theta\right] d\theta\,dx$$

$$= \int_0^2 \left(-27\cdot\frac{2}{3} + 108x\cdot\frac{2}{3}\right) dx$$

$$= \int_0^2 (-18 + 72x)\,dx$$

$$= \left(-18x + 72\frac{x^2}{2}\right)_0^2$$

$$= -18\times 2 + 36\times 4$$

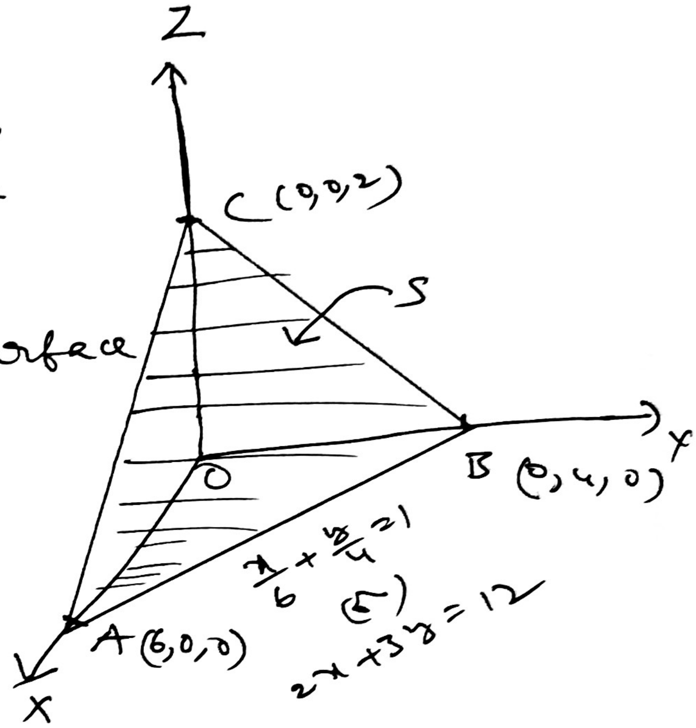
$$= 144 - 36$$

$$= 108$$

$\therefore \int_S \bar{F}\cdot\bar{N}\,dS = 0+0+0+72+108 = 180.$

Problem (2): Evaluate $\int_S \bar{F}\cdot\bar{N}\,ds$ where $\bar{F} = 18z\,I - 12J + 3y\,K$ and S is the portion of the plane $2x+3y+6z=12$ in the first octant.

Solution:



Given $\bar{F} = 18z\,I - 12J + 3y\,K$

and given surface is $f = 2x+3y+6z=12$

normal vector to the surface is $\nabla(2x+3y+6z)$

$\Rightarrow \nabla f = 2I + 3J + 6K$

unit normal vector to the surface S is $\frac{\nabla f}{|\nabla f|} = \bar{N}$

i.e., $$\bar{N} = \frac{2I+3J+6K}{\sqrt{4+9+36}} = \frac{2I+3J+6K}{\sqrt{49}} = \frac{2I+3J+6K}{7}$$

$$ds = \frac{dx\,dy}{|\bar{N}\cdot K|} = \frac{dx\,dy}{\left|\frac{2I+3J+6K}{7}\cdot K\right|} = \frac{dx\,dy}{\frac{6}{7}}$$

$$\bar{F}\cdot\bar{N} = (18z\,I - 12J + 3y)\cdot\frac{2I+3J+6K}{7}$$

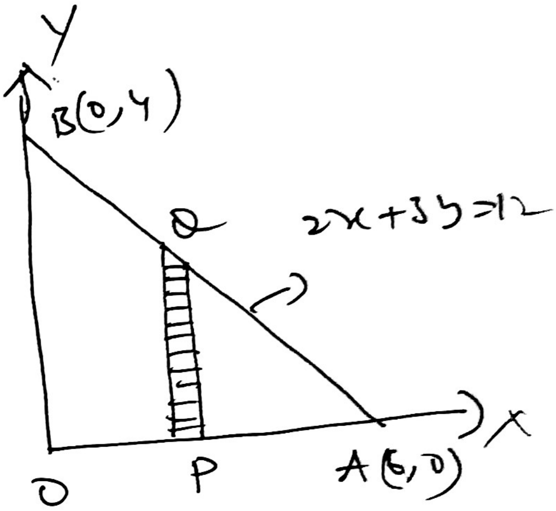
$$= \frac{36z}{7} - \frac{36}{7} + \frac{18}{7}y = \frac{36}{7}z + \frac{18}{7}y - \frac{36}{7}$$

$$\text{Now } \int_S \bar{F}\cdot\bar{n}\,dS = \iint_E \left[36\left(\frac{12-2x-3y}{6}\right) + 18y - 36\right]\frac{dx\,dy}{6}$$

(on the plane $z = \frac{12-2x-3y}{6}$)

$$= \iint_E (6-2x)\,dx\,dy$$

where E is the region bounded by $x=0$, $y=0$ and $2x+3y=12$



$$\therefore \int_S \bar{F}\cdot\bar{n}\,dS = \int_0^6 \int_0^{\frac{12-2x}{3}} 6-2x\,dy\,dx$$

$$= \int_0^6 (6-2x)\,(y)_0^{\frac{12-2x}{3}}\,dx$$

$$= \int_0^6 (6-2x)\left(\frac{12-2x}{3}\right)dx$$

$$= \frac{1}{3}\int_0^6 (72-36x+4x^2)\,dx$$

$$= \frac{1}{3}\left(72x-18x^2+\frac{4}{3}x^3\right)_0^6$$

$$= \frac{1}{3}\left[72\times6 - 18\times36 + \frac{4}{3}6^3\right]$$

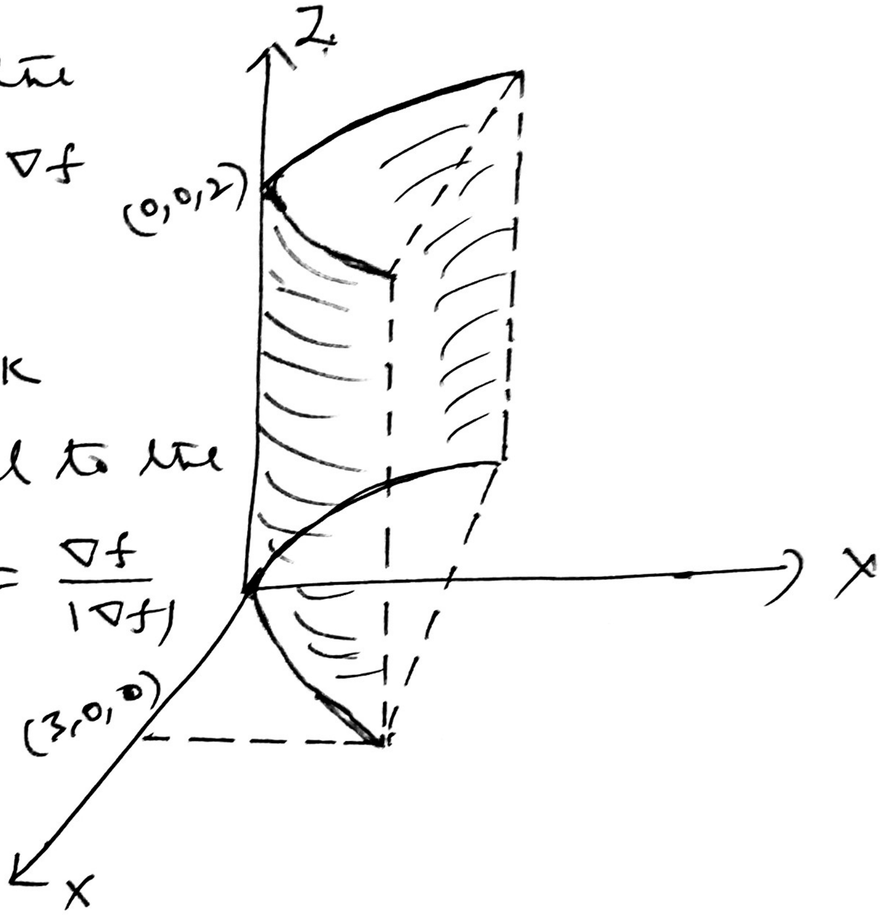
$$= \frac{1}{3}\left[432-648+288\right]$$

$$= \frac{1}{3}(720-648)$$

$$= 24.$$

Problem (3): If the velocity vector is $\bar{F} = yI + 2J + xzK$ m/sec., show that the flux of water through the parabolic cylinder $y = x^2$, $0 \le x \le 3$, $0 \le z \le 2$ is 69 m^3/sec.

Solution: Given $\bar{F} = yI + 2J + xzK$ and given surface is $x^2 - y = 0$, $0 \le x \le 3$, $0 \le z \le 2$



normal vector to the given surface is ∇f where $f = x^2 - y$

$\therefore \nabla f = 2xI - J + 0K$

unit vector normal to the given surface is $\bar{n} = \dfrac{\nabla f}{|\nabla f|}$

ie, $\bar{n} = \dfrac{2xI - J}{\sqrt{4x^2+1}}$

Now $\bar{F} \cdot \bar{n} = (yI + 2J + xzK) \cdot \dfrac{2xI - J}{\sqrt{4x^2+1}}$

$$= \frac{2xy - 2}{\sqrt{4x^2+1}}$$

$$= \frac{2x(x^2) - 2}{\sqrt{4x^2+1}} \quad \text{(on the surface, } y = x^2\text{)}$$

$$\Rightarrow \bar{F}\cdot\bar{N} = \frac{2x^3-2}{\sqrt{4x^2+1}}$$

$$dS = \frac{dx\,dz}{|\bar{N}\cdot J|} = \frac{dx\,dz}{\left|\frac{2xI - J}{\sqrt{4x^2+1}}\cdot J\right|}$$

$$= \frac{dx\,dz}{\frac{1}{\sqrt{4x^2+1}}}$$

Thus $$\int_S \bar{F}\cdot\bar{N}\,dS = \int_0^2\int_0^3 \frac{2x^3-2}{\sqrt{4x^2+1}}\;\frac{dx\,dz}{\frac{1}{\sqrt{4x^2+1}}}$$

$$= \int_0^2\int_0^3 2(x^3-1)\,dx\,dz$$

$$= 2\int_0^2 \left(\frac{x^4}{4}-x\right)_0^3 dz$$

$$= 2\int_0^2\left(\frac{81}{4}-3\right)dz$$

$$= 2\times\frac{69}{4}\,(z)_0^2$$

$$= 2\times 2\times\frac{69}{4}$$

$$= 69\ m^3/\text{sec.}$$

Practice problems:

1) Evaluate $\int_S \bar{F} \cdot d\bar{S}$ where $\bar{F} = xI + (z^2 - zx)J - xyK$ and S is the triangular surface with vertices (2, 0, 0), (0, 2, 0) and (0, 0, 4).

2) If $\bar{F} = 2yI - 3J + x^2K$ and S is the surface of the parabolic cylinder $y^2 = 8x$ in the first octant bounded by the planes $y = 4$, $z = 6$, ~~and~~ show that $\int_S \bar{F} \cdot \bar{n}\, dS = 132$.