Time Complexity Analysis

Asymptotic notation

Asymptotic Analysis

- In mathematical analysis, **asymptotic analysis**, also known as **asymptotics**, is a method of describing limiting behavior.
- As an illustration, suppose that we are interested in the limiting behavior or growth properties of a function $f(n)$ as n becomes very large.
- If $T(n)=f(n) = n^2 + 3n$, then as n becomes very large, the term $3n$ becomes insignificant compared to n^2.

Asymptotic Analysis

n	n^2	$3n$
2	4	6
50	2500	150
150	22,500	450
500	2,50,000	1500
1000	10,00,000	3000

Asymptotic Analysis

- The function $f(n)$ is said to be "*asymptotically equivalent* to n^2, as $n \to \infty$".
- This is often written symbolically as $f(n) \sim n^2$, which is read as "$f(n)$ is asymptotic to n^2".
- The idea is we compare the growth rate of the given function $f(n)$ with the known behaviors of standard functions represented by $g(n)$.

Asymptotic Analysis

- $T(n) = f(n) = \; n^3 + 3n^2 + 2n + 1$
- What is $f(n)$ *asymptotically equivalent* to as n→ ∞?
- What is $g(n)$?

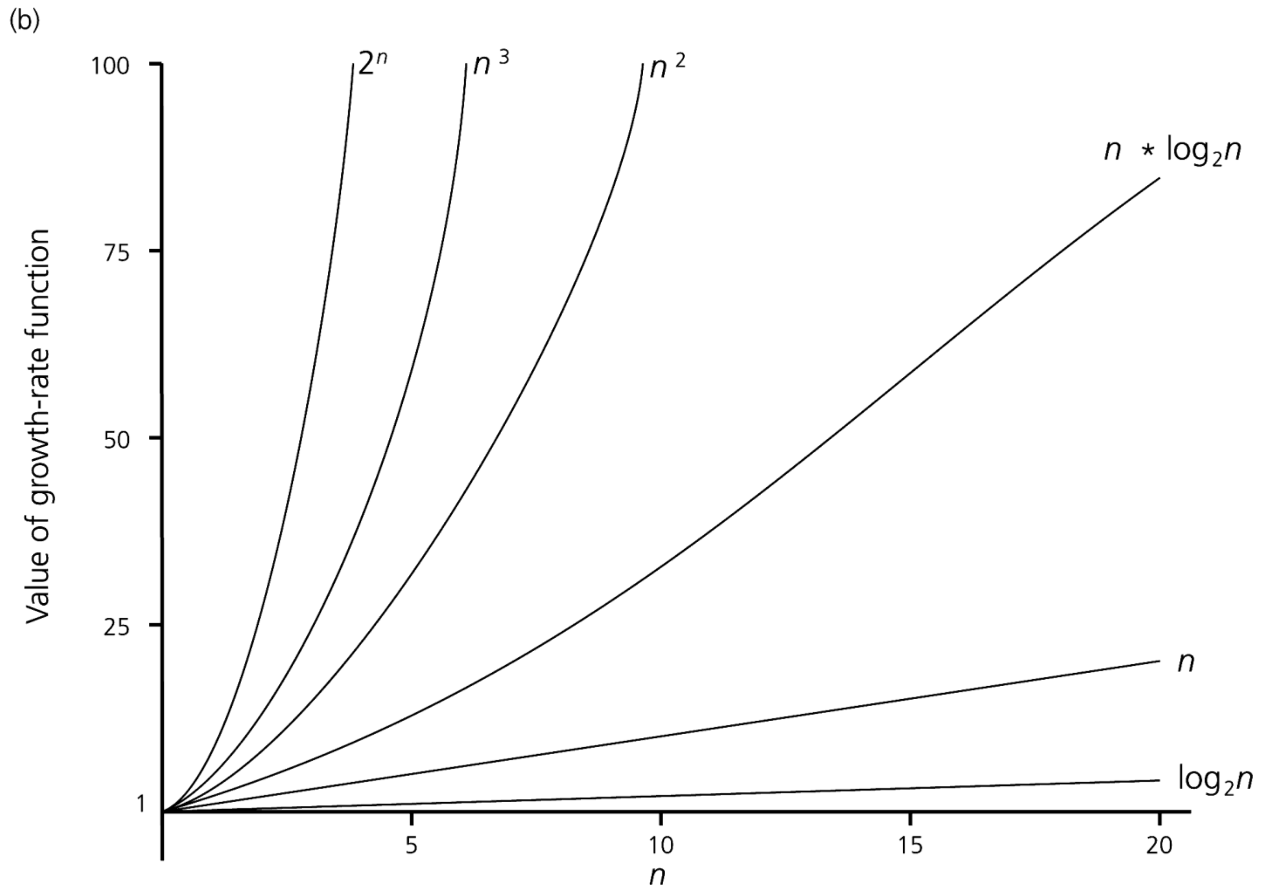
Asymptotic Analysis

- The family of $g(n)$ includes $1,\ n,\ n^2,\ n^k,\ \log n,\ nlog\ n,\ 2^n, n!$
- The same idea with a specfic notation is adopted in the Algorithm Analysis which is called as asymptotic notation/analysis.
- In the case of Algorithm Analysis, the function $T(n)$- the step count takes the place of $f(n)$, and we estimate the growth rate of $T(n)$ in terms of $g(n)$.

Asymptotic Analysis

$g(n) = 1$	Constant function
$g(n) = \log n$	Logarithmic function
$g(n) = n$	Linear function
$g(n) = n^2$	Quadratic function
$g(n) = n^3$	Cubic function
$g(n) = 2^n$	Exponential function

Asymptotic Analysis



Asymptotic Analysis

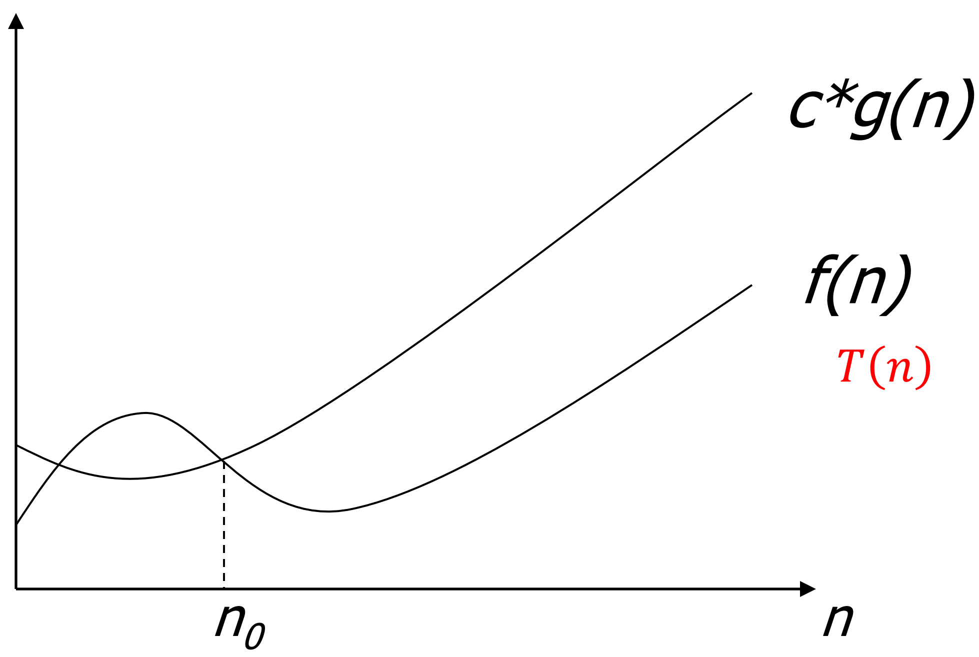
- The analysis is more meaningful because we wound be interested in the behavior of the algorithm as the input size n increases.
- The asymptotic analysis defines identifies three kinds of $g(n)$-

Asymptotic Analysis

- $g(n)$ whose growth rate is greater or equal to the growth rate of $T(n)$.
- $g(n)$ whose growth rate is same as $T(n)$
- $g(n)$ whose growth rate is smaller or equal to the growth rate of $T(n)$.

Asymptotic Analysis

- $T(n)$ is said to be $O(g(n))$ iff there exists constants $c \; and \; n_0$ such that $T(n) \leq c.g(n) \; for \; n \geq n_0$



Asymptotic Analysis

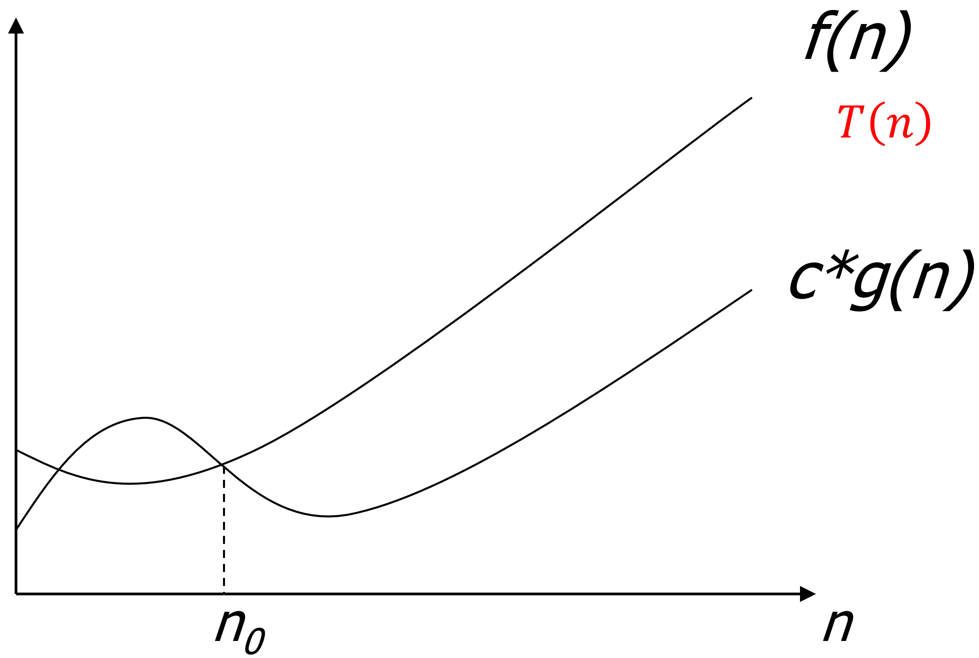
- $T(n)$ will have growth rate which is smaller than $g(n)$. In other words $g(n)$ is the upper bound on $T(n)$. $g(n)$ should be as small as possible.
- $T(n) = 3n + 2 = O(n)$
- $3n + 2 \ \leq 4n \ for \ n \geq 2, \mathrm{c} = 4 \text{ and } \mathrm{n}_0 = 2$

Asymptotic Analysis

- $T(n) = 10n^2 + 4n + 2$
- Find c , n_0, g(n)
- $10n^2 + 4n + 2 \leq 11\, n^2\ for\ n \geq 5$
- $T(n) = O(n^2)$
- $T(n) = n^2 + 6 * 2^n = O(2^n)$
- Find c , n_0, g(n)
- $n^2 + 6 * 2^n \leq 7 * \ 2^n\ for\ n \geq 4$
- $T(n) = O(2^n)$

Asymptotic Analysis

- $T(n)$ is said to be $\Omega(g(n))$ iff there exists constants $c\ and\ n_0$ such that $T(n) \geq c.g(n) for\ n \geq n_0$. $g(n)$ should be as large as possible.

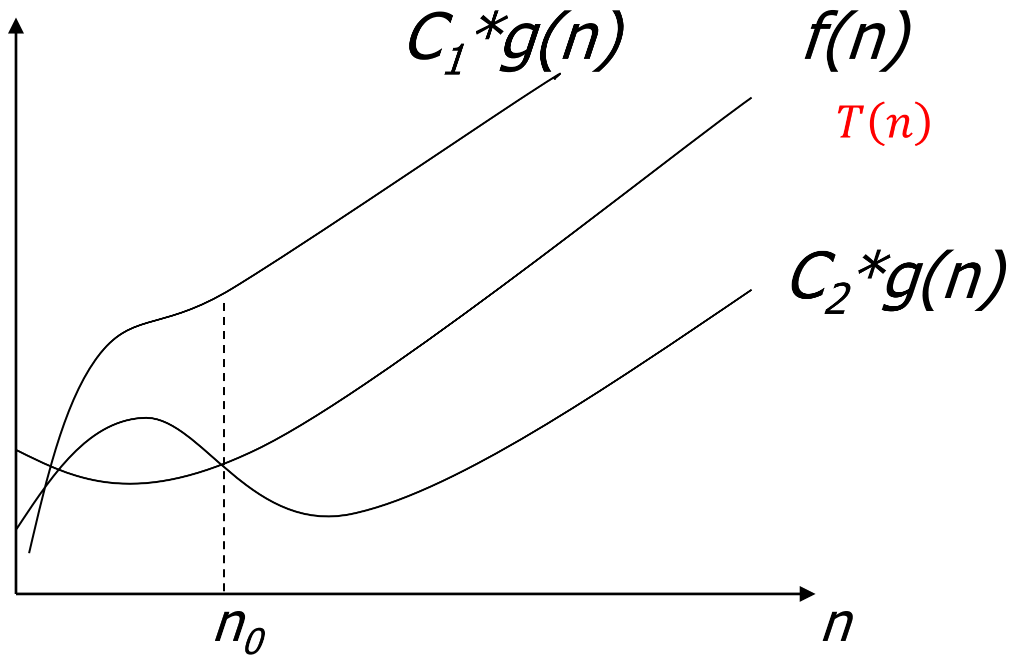


Asymptotic Analysis

- *T*(*n*) will have growth rate which is larger than *g*(*n*). In other words *g*(*n*) is the lower bound on *T*(*n*).
- $T(n) = 3n + 2 = \Omega(n)$
- $3n + 2 \geq 3n\ for\ n \geq 1, \mathrm{c} = 3 \text{ and } \mathrm{n}_0 = 1$
- $T(n) = 10n^2 + 4n + 2 = \Omega(n^2)$
- $10n^2 + 4n + 2 \geq\ n^2\ for\ n \geq 1$

Asymptotic Analysis

- $T(n)$ is said to be $\theta(g(n))$ iff there exists constants $c_1,\ c_2\ and\ n_0$ such that $T(n) \leq c_1 g(n)\ and\ T(n) \geq c_2 g(n)\ for\ n \geq n_0$



Asymptotic Analysis

- *T*(*n*) will have growth rate which is same as *g*(*n*).
- If $T(n)$ is a polynomial of degree n, then
- $T(n) = 3n^2 + 2n + 3 = \theta(n^2)$?
- $T(n) = 3n^2 + 2n + 3 \leq 4n^2$ for $n \geq 3$
- $T(n) = 3n^2 + 2n + 3 \geq n^2$ for $n \geq 1$

Asymptotic Analysis

- $T(n)$ is said to be $\mathrm{o}(g(n))$ iff for any positive constant c, $T(n) < c.g(n)\ for\ n \geq n_0$
- Suppose $T(n) = 2n + 3$
- $T(n) = 2n + 3 < 6n\ for\ n \geq 1$
- but
- $T(n) = 2n + 3 < cn\ for\ n \geq 1$ for $c = 1,2,3,4,5$

Asymptotic Analysis

- $T(n) = 2n + 3 < cn^2 \; for \; n \geq 1$
- Hence
- $T(n) = o(n^2)$