BITS

1. Write the test statistic for two mean large sample case.

Test statistic, $z = \dfrac{(\bar{x}-\bar{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

2. What are the confidence limits for difference of two means in large sample case.

The $(1-\alpha)100\%$ large sample confidence interval for the difference means is given by

$$(\bar{x}-\bar{y}) \pm z_{\alpha/2} \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

3. What is the test statistic for two means in simple sample case.

Test statistic, $t = \dfrac{(\bar{x}-\bar{y}) - \delta}{s_p\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

Where $s_p^2 = \dfrac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$

$\therefore n_1+n_2-2 =$ degrees of freedom.

(4). What is the test statistic for one variance.

Test statistic, $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

(5). What is the critical region for testing null hypothesis $\sigma_1^2 = \sigma_2^2$

Alternate Hypothesis	Test statistic	Reject null hypothesis if
$\sigma_1^2 < \sigma_2^2$	$F = \dfrac{s_2^2}{s_1^2}$	$F > F_\alpha (n_2-1, n_1-1)$
$\sigma_1^2 > \sigma_2^2$	$F = \dfrac{s_1^2}{s_2^2}$	$F > F_\alpha (n_1-1, n_2-1)$
$\sigma_1^2 \neq \sigma_2^2$	$F = \dfrac{s_M^2}{s_m^2}$	$F > F_{\alpha/2} (n_M-1, n_m-1)$

(6). What is the test statistic for one proportion and critical region for testing it.

Test statistic $z = \dfrac{x - np_0}{\sqrt{np_0(1-p_0)}}$

critical region for Testing $p = p_0$

Alternate Hypothesis	Reject null hypothesis if
$p < p_0$	$z < -z_\alpha$
$p > p_0$	$z > z_\alpha$
$p \neq p_0$	$z < -z_{\alpha/2}$ (or) $z > z_{\alpha/2}$

(7) What is the test statistic for two proportions and state its critical region.

Test statistic, $Z = \dfrac{\dfrac{x_1}{n_1} - \dfrac{x_2}{n_2}}{\sqrt{\bar{p}(1-\bar{p})}\sqrt{\dfrac{1}{n_1}+\dfrac{1}{n_2}}}$ with $\bar{p} = \dfrac{x_1+x_2}{n_1+n_2}$

Critical Region for testing :-

Alternate Hypothesis	Reject Null Hypothesis if
$P_1 < P_2$	$Z < -Z_\alpha$
$P_1 > P_2$	$Z > Z_\alpha$
$P_1 \neq P_2$	$Z < -Z_{\alpha/2}$ (or) $Z > Z_{\alpha/2}$

Main Questions :-

1. Two types of new cars produced in U.S.A. are tested for petrol mileage, one sample is consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variences as $\sigma_1^2 = 2.0$ and $\sigma_2^2 = 1.5$ respectively. Test whether there is any significance in the petrol consumption of these two types of cars. (use $\alpha = 0.01$).

Sol:- Let the two types of new cars named as A and B.

No. of cars of type A is $n_1 = 42$

No. of car of type B is $n_2 = 80$

Average mileage of car A is $\bar{x} = 15$

and varience of car A is $\sigma_1^2 = 2.0$

Average mileage of car B is $\bar{y} = 11.5$

and varience of car B is $\sigma_2^2 = 1.5$.

Null Hypothesis $H_0 : \mu_1 - \mu_2 = 0$

Alternate hypothesis $H_1 : \mu_1 - \mu_2 \neq 0$.

$\therefore s_1^2 = 2.0$

$s_2^2 = 1.5$

Test Statistic $Z = \dfrac{(\bar{x}-\bar{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{(15 - 11.5) - 0}{\sqrt{\dfrac{2}{42} + \dfrac{1.5}{80}}}$

$= \dfrac{3.5}{\sqrt{0.0476 + 0.0188}}$

$= \dfrac{3.5}{\sqrt{0.0664}} = \dfrac{3.5}{0.2576}$

$\therefore Z = 13.5870.$

Level of significance $\alpha = 0.01$

$Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005}$ $\quad$ WKt $Z_\alpha = 1 - \alpha$

$= 1 - 0.005$

$= 0.9950$

$\therefore$ $Z_\alpha = 2.5750$ from table '3'

0.9949
0.4951

$\therefore$ $Z = 13.5870 > Z_\alpha = 2.5750$.

since, we reject Null Hypothesis H_0 at 0.01 level of significance and conclude that there is a significant difference in petrol consumption.

(2). A simple sample of the height of 6400 Englishmen has a mean of 67.85 inches and a S.D of 2.56 inches while a simple sample of heights of 1600 Australians has a mean of 68.55 inches and S.D of 2.52 inches. Do the data indicate that the Australians are on the avg taller than the englishmen? Use 0.05 level of significance.

Sol:- given that.

size of first sample $n_1 = 6400$

size of second sample $n_2 = 1600$

mean of first sample $\bar{x} = 67.85$

mean of second sample $\bar{y} = 68.55$

~~var~~ standard deviation of first sample $S_1 = 2.56$

standard deviation of second sample $S_2 = 2.52$

Null Hypothesis H_0 : $\mu_1 - \mu_2 = 0$

Alternate hypothesis H_1 : $\mu_1 - \mu_2 < 0$

level of significance $\alpha = 0.05$

The test statistic $Z = \dfrac{(\bar{x}-\bar{y}) - \delta_0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$.

$$Z = \frac{(67.85 - 68.55)-0}{\sqrt{\dfrac{(2.56)^2}{6400} + \dfrac{(2.52)^2}{1600}}} = \frac{-0.7000}{\sqrt{0.0010 + 0.0040}} = \frac{-0.7000}{\sqrt{0.0050}}$$

$$= \frac{-0.7000}{0.0707}$$

$$= -9.9010.$$

$\therefore Z = -9.9010.$

$Z_\alpha = Z_{0.05} = 1 - \alpha = 1 - 0.05$
$= 0.9500$

$Z < -Z_\alpha$

$\therefore Z_\alpha = 1.6450$

$\therefore Z = -9.9010 < -Z_\alpha = 1.6450.$

Hence, we reject the null hypothesis at 0.05 level of significance and conclude the Australians are taller than englishmen.

3. Two independent sample of 8 and 7 items respectively have the following values:

Sample I	11	11	13	11	15	9	12	14
Sample II	9	11	10	13	9	8	19	—

Is the difference b/w the means of sample significant.
use $\alpha = 0.05$.

Sol:- given that

Null Hypothesis $H_0 : \mu_1 - \mu_2 = 0$

Alternate Hypothesis $H_1 : \mu_1 - \mu_2 \neq 0$.

size of sample1 is $n_1 = 8$

size of sample2 is $n_2 = 7$

mean of sample1 is $\bar{x} = \frac{11+11+13+11+15+9+12+14}{8} = 12$

mean of sample2 is $\bar{y} = \frac{9+11+10+13+9+8+19}{7} = 11.2857$

Varience of sample1 is

$$s_1^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n_1 - 1)}$$

$$\therefore = \frac{(11-12)^2 + (11-12)^2 + (13-12)^2 + (11-12)^2 + (15-12)^2 + (9-12)^2 + (12-12)^2 + (14-12)^2}{7}$$

$$= 1+1+1+1+9+9+0+4/7 = 3.7143$$

$$\therefore s_1^2 = 3.7143$$

$$\therefore s_1 = 1.9272$$

Varience of sample2 is

$$s_2^2 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{(n_2 - 1)}$$

$$= \frac{(9-11.2857)^2 + (11-11.2857)^2 + (10-11.2857)^2 + (9-11.2857)^2 + (8-11.2857)^2 + (19-11.2857)^2}{6}$$

$$= 5.2244 + 0.0816 + 1.6530 + 5.2244 + 10.7958 + 59.5104/6$$

$s_2^2 = 13.7483$. $\therefore s_2 = 3.7079$

Test statistic $t = \dfrac{(\bar{x}-\bar{y}) - \delta}{sp\sqrt{\dfrac{1}{n_1} + \dfrac{1}{n_2}}}$

$$sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}.$$

$$\therefore \quad t = \frac{(12-11.2857)-0}{sp\sqrt{\frac{1}{8}+\frac{1}{7}}} = \frac{0.7143}{sp\,(0.5175)}$$

$$sp^2 = \frac{(7)(3.7143) + (6)(13.7483)}{13} = 8.3454$$

$$\therefore sp = \sqrt{8.3454} = 2.8888.$$

$$t = \frac{0.7143}{(2.8888)(0.5175)} = \frac{0.7143}{1.4950} = 0.4778.$$

$$\therefore \quad t = 0.4778.$$

$$t_\alpha = t_{0.05} = t_{\frac{0.05}{2}} = t_{0.025}$$

$$\therefore t_\alpha = 1-\alpha$$
$$\Rightarrow 1 - 0.025 = 0.9750.$$

$$\therefore t_{0.025} = 1.9600$$

?

Test statistic $z = \dfrac{(\bar{x}-\bar{y}) - \delta}{sp\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$$sp^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}.$$

$$\therefore \quad t = \frac{(12 - 11.2857) - 0}{sp\sqrt{\frac{1}{8} + \frac{1}{7}}} = \frac{0.7143}{sp\,(0.5175)}$$

$$sp^2 = \frac{(7)(3.7143) + (6)(13.7483)}{13} = 8.3454$$

$$\therefore sp = \sqrt{8.3454} = 2.8888.$$

$$t = \frac{0.7143}{(2.8888)(0.5175)} = \frac{0.7143}{1.4950} = 0.4778.$$

$$\therefore \quad t = 0.4778.$$

$$t_\alpha = t_{0.05} = t_{\frac{0.05}{2}} = t_{0.025}$$

$$\therefore t_\alpha = 1-\alpha$$

$$\Rightarrow 1 - 0.025 = 0.9750.$$

$$\therefore t_{0.025} = 1.9600$$

?

4. Playing 10 rounds of gold on his home course, a gold profesional averaged 71.3. Test the null hypothesis that the consistency of his game on his home course is actually measured by $\sigma = 1.20$, against the alternative hypothesis that he is less consistent. Use the level of significance $\alpha = 0.05$.

Sol:-

Null Hypothesis : $\sigma = 1.20$

Alternate hypothesis : $\sigma > 1.20$

level of significance : $\alpha = 0.05$

$n = 10$

$\sigma = 1.20$

$S = 1.32$

Test statistic, $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = \frac{9 \times (1.32)^2}{(1.20)^2} = \frac{15.6816}{1.4400}$

$\therefore \chi^2 = 10.8900$

The critical region for testing $\sigma^2 = \sigma_0^2$

Alternate Hypothesis	Reject the null hypothesis if
$\sigma^2 > \sigma_0^2$	$\chi^2 > \chi_\alpha^2$

Since $\chi^2 = \frac{(n-1)S^2}{\sigma_0^2} = 10.8900$ doesn't exceed 16.919, the value of $\chi^2_{0.05}$ for 9 degrees of freedom, the null hypothesis cannot be rejected, so, accept the null hypothesis.

5. In an sample of 10 observations, the sum of squares of the deviation of the sample values from sample mean was 120 and in other sample of 12 observations, it was 314. Test whether the difference in variences is significant at 5% in the varience level.

Sol:- given that

$n_1 = 10$, $\Sigma(x_1 - \bar{x}_1)^2 = 120$

$n_2 = 12$, $\Sigma(x_2 - \bar{x}_2)^2 = 314$

Null hypothesis H_0: $\sigma_1^2 = \sigma_2^2$

Alternate hypothesis : $\sigma_1^2 \neq \sigma_2^2$

Applying 'F' test

$$F = \frac{S_2^2}{S_1^2} = \frac{\frac{\Sigma(x_2 - \bar{x}_2)^2}{n_2 - 1}}{\frac{\Sigma(x_1 - \bar{x}_1)^2}{(n_1 - 1)}} = \frac{\frac{314}{11}}{\frac{120}{9}} = \frac{28.545}{13.333}$$

$$\therefore F = 2.1409$$

$\alpha = 0.05$ The value of F at 5% level for

$v_1 = 11$

$v_2 = 9$

since, $F < F_{0.05}$ we accept H_0.

The samples may have been drawn from two populations having the same variences. The difference is not significant at 5% level of significance.

(6). Among 100 fish caught in a large lake, 18 were inedible due to the pollution of the environment. With what confidence can we assert that the error of this estimate is at most 0.065?

Sol. given that

sample size $n = 100$

max. error of estimate, $E = 0.065$

p = sample proportion of inedible fish $= \frac{18}{100} = 0.18$

$\therefore q = 1 - p = 1 - 0.18 = 0.82$

maximum error of estimate for true proportion $= E = Z_{\alpha/2} \sqrt{\frac{pq}{n}}$

$$\Rightarrow 0.065 = Z_{\alpha/2} \sqrt{\frac{(0.82)(0.18)}{100}}$$

$$0.065 = Z_{\alpha/2} (0.0384)$$

$$= \frac{0.065}{0.0384}$$

$$Z_{\alpha/2} = 1.6927$$

$$\alpha = ?$$

(7). In a random sample of 125 cool drinkers, 68 said they prefer Thumsup to pepsi. Test the null hypothesis $p = 0.5$ against the alternate hypothesis $p > 0.5$.

Sol:- given that

$n = 125$

$x = 68$

$\cancel{\Sigma (x_1 - \bar{x}_1)^2 = 120}$ $\quad p = \frac{x}{n} = \frac{68}{125} = 0.5440$

$\cancel{\Sigma (x_2 - \bar{x}_2)^2 = 314}$

Null Hypothesis H_0: $P_0 = 0.5$

Alternate hypothesis H_1: $P > 0.5$

level of significance $\alpha = 0.05$

$$Z_\alpha = 1.645$$

$$Z = \frac{p - P_0}{\sqrt{\frac{PQ}{n}}}$$

$$= \frac{0.544 - 0.5}{\sqrt{\frac{0.5 \times 0.5}{125}}} = \frac{0.0440}{0.0447}$$

$$= 0.9843.$$

(8) A manufacturer of electronic equipment subjects samples of two competing brands of transistors to an accelerated performance test. If 45 of 180 transistors of the first kind and 34 of 120 transistors of the second kind fail the test, what can he conclude that the level of significance $\alpha = 0.05$ about the difference b/w the corresponding sample proportions.

Sol:- given that

$n_1 = 180$, $x_1 = 45$, $\alpha = 0.05$

$n_2 = 120$, $x_2 = 34$

Null Hypothesis $H_0: P_1 = P_2$

Alternate hypothesis $H_1: P_1 \neq P_2$

$$Z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\bar{p}(1-\bar{p})} \sqrt{\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}}$$

$$\bar{p} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{45+34}{180+120} = \frac{79}{300} = 0.2633$$

$$\therefore \bar{p} = 0.2633$$

$$\Rightarrow Z = \frac{\frac{45}{180} - \frac{34}{120}}{\sqrt{(0.2633)(0.7367)} \sqrt{(0.0056 + 0.0083)}}$$

$$Z = \frac{0.25 - 0.2833}{(0.4404)(0.1179)} = \frac{-0.0333}{0.0519} = -0.6416$$

$\therefore Z = -0.6416.$

$Z_{0.05/2} = Z_{0.025}$

$\therefore Z_\alpha = 1 - \alpha$

$= 1 - 0.025$

$= 0.9750.$

from table '3' $Z_{0.025} = 1.96.$