D.Sasikala

1) Write any two properties of Normal distribution:

- The graph of Normal distribution is a bell shaped curve
- It is symmetrical about $\Rightarrow \boxed{x = \mu}$.
- The area under the normal curve above the x-axis is equal to unity.

2) Define the k^{th} moment of a continuous random variable X about its Mean.

The k^{th} moment of the random variable X about the mean is defined by

$$\mu_k = \int_{-\infty}^{\infty} (x-\mu)^k f(x)\, dx$$

3) Find the value of $Z_{0.05}$

we know that $F(Z_\alpha) = 1-\alpha$

$F(Z_{0.05}) = 1-0.05 = 0.95$

In the standard normal distribution table-3, 0.95 is closest to 0.9495 and 0.9505 corressponding to $z = 1.64$ and $z = 1.65$

$$F(Z_{0.05}) = 0.95 = F\left(\frac{1.64+1.65}{2}\right) = F(1.645)$$

$$Z_{0.05} = \underline{1.645}$$

4) The mean and variance of gamma distribution are respectively 8 and 32. Find α, β.

Given

Mean $= \alpha\beta = 8$

Variance $\sigma^2 = \alpha\beta^2 = 32$

$\mu = \alpha\beta = 8$ $\qquad \sigma^2 \Rightarrow \alpha\beta^2 = 32$

$\boxed{\alpha = \frac{8}{\beta}} \Rightarrow$ substitute α

$$\Rightarrow \left(\frac{8}{\cancel{\beta}}\right)\beta^{\cancel{2}} = 32$$

$$\beta = \frac{\cancel{32}\,4}{\cancel{8}\,1} \Rightarrow \boxed{\beta = 4}$$

$$\mu \Rightarrow \alpha\beta = 8$$

$$= \alpha(4) = 8$$

$$\alpha = \frac{\cancel{8}\,2}{\cancel{4}}$$

$\boxed{\alpha = 2}$ $\qquad \boxed{\alpha, \beta = 2, 4}$

5) what are the mean and variance of the gamma distribution.

Mean $\mu = \alpha\beta$ variance $\sigma^2 = \alpha\beta^2$ are the gamma distribution.

6. For $\alpha = 0.2$ and $\beta = 0.5$ find the mean of weibull distribution.

Given that $\alpha = 0.2$ $\beta = 0.5$

we know that

$$\mu = \alpha^{-1/\beta}\, \Gamma\left(1 + \frac{1}{\beta}\right)$$

$$\mu = (0.2)^{-1/0.5}\, \Gamma\left(1 + \frac{1}{0.5}\right)$$

$$= (0.2)^{-2}\, \Gamma(3)$$

$$=$$

7) If $f(x) = \begin{cases} kx^3 & \text{for } 0 < x < 1 \\ 0 & \text{elsewher} \end{cases}$ is a probability desity of the random variable X, Find the value of k.

Given that $f(x)$ is a probability density function

So, $\int_{-\infty}^{\infty} f(x)\, dx = 1$

$\Rightarrow \int_{-\infty}^{0} f(x)dx + \int_{0}^{1} f(x)dx + \int_{1}^{\infty} f(x)dx = 1$

$\Rightarrow \int_{0}^{1} f(x)dx = 1$

$\Rightarrow \int_{0}^{1} kx^2 dx = 1$

$k\left[\frac{x^3}{3}\right]_0^1 = 1$

$k\left[\frac{1}{3}\right] = 1 \quad \boxed{k=3}$

5) When the random variable X and Y are said to be Independent.

In random variable X and Y are said to be independent $p(X=x, Y=y) = P(X=x)P(Y=y_i)$ for all x, y.

when the ran

10) Define the conditional [illegible] of

X given $Y=y$.

7MARKS

1) If the probability density of a random variable is given by $f(x) = \begin{cases} 3e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$. Find the probabilities of the random variable will take a value

(i) between 1 and 3

(ii) greater than 5.

Given that $f(x) = \begin{cases} 3e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise} \end{cases}$

(i) $P(a \leq X \leq b) = \int_a^b f(x)\, dx$

$P(1 \leq X \leq 3) = \int_1^3 3e^{-3x}\, dx$

$= \cancel{3}\left[\frac{e^{-3x}}{-\cancel{3}}\right]_1^3 \Rightarrow e^{-3(1)} - e^{-3(3)}$

$= e^{-3} - e^{-9}$

$= 0.0497$ //

(ii) $P(X>0.5) = \int_{0.5}^{\infty} 3e^{-3x}\,dx$

$$= \cancel{3}\left[\frac{e^{-3x}}{-\cancel{3}}\right]_{0.5}^{\infty}$$

$$= -\left[e^{-3(\infty)} - e^{-3(0.5)}\right]$$

$$= \left[e^{-1.5} - e^{-3(\infty)}\right]$$

$$= 0 + e^{-1.5} = 0.223$$

2) A random variable has a normal distribution with $\sigma = 10$. If the probability is 0.8212 that it will take on a value less than 82.5, what is the probability that it will take on a value greater than 58.3.

$\sigma = 10$ the probability will take on a value less than 82.5 $P(x < 82.5) = 0.8212$

$$P\left(\frac{x-\mu}{\sigma} < \frac{82.5-\mu}{\sigma}\right) = 0.8212$$

$$P\left(z < \frac{82.5-\mu}{10}\right) = 0.8212$$

$$\cancel{F}\left(\frac{82.5-\mu}{10}\right) = \cancel{F}(0.92)$$

$$82.5 - \mu = 0.92 \times 10$$

$$82.5 - \mu = 9.2$$

$$\mu = 82.5 - 9.2$$

$$\boxed{\mu = 73.3}$$

$$P(X>58.3) = P\left(\frac{x-\mu}{\sigma} > \frac{58.3-\mu}{10}\right)$$

$$= P\left(z > \frac{58.3-73.3}{10}\right)$$

$$= P\left(z > \frac{-15}{10}\right)$$

$$= P(z > -1.5)$$

$$= 1 - F(-1.5)$$

$$= 1 - F(-1.5)$$

$$= 1 - 0.0668$$

$$P(x>58.3) = \underline{\underline{0.9332}}$$

3) If 20% of the memory chips made in a certain plant are defective, what are the probabilities that in a lot of 100 randomly chosen for inspection

(i) At most 16 will be defective

(ii) Exactly 16 will be defective.

Given $n = 100$

$p = 20\% = 0.20 \quad q = 0.80$

$\mu = np = 100(0.20) = 20 \qquad \sigma = \sqrt{npq} = \sqrt{16} = 4$

$P(X \leq 16)$

$= P(X \leq 16+0.5)$

$= P(X \leq 16.5)$

$= P\left(\frac{X-\mu}{\sigma} \leq \frac{16.5-\mu}{\sigma}\right)$

$= P\left(Z \leq \frac{16.5-20}{4}\right) = P(Z \leq -0.875)$

$= F(-0.875)$

$= 0.1922$

(ii) $P(X=a) = P(X=16)$

$= P(15.5 \leq X \leq 16.5)$

$= P\left(\frac{15.5-\mu}{\sigma} \leq \frac{X-\mu}{\sigma} \leq \frac{16.5-\mu}{\sigma}\right)$

$= P\left(\frac{15.5-20}{4} \leq Z \leq \frac{16.5-20}{4}\right)$

$= P(-1.125 \leq Z \leq -0.875)$

$= F(-0.875) - F(-1.125)$

$= 0.1922 - 0.1314 \Rightarrow \boxed{0.0608}$

4) In a certain city, the daily consumption of electric power (in millions) can be treated as a random variable having a gamma distribution with $\alpha=3$ and $\beta=2$. If the city's power plant has a daily capacity of 12MKWH, what is the probability that this power supply will be inadequant on any given day?

The daily capacity of the power plant is 12MKWH

Given the daily consumption of electric power is the random variable having a gamma distribution with $\alpha=3$ and $\beta=2$

$$P(X>12) = \int_{12}^{\infty} f(x)\,dx$$

$$= \int_{12}^{\infty} \frac{1}{\beta^{\alpha}\Gamma(\alpha)} x^{\alpha-1} e^{-x/\beta}\,dx$$

$$= \int_{12}^{\infty} \frac{1}{2^{3}\Gamma(3)} x^{3-1} e^{-x/2}\,dx$$

$$= \frac{1}{16}\int_{12}^{\infty} x^{2} e^{-x/2}\,dx$$

$$= \frac{1}{16}\left[-2x^{2}e^{-x/2} - 8xe^{-x/2} - 16e^{-x/2}\right]_{12}^{\infty}$$

$$= \frac{1}{16}\left[0-\left[-2(24)e^{-6} - 8(12)e^{-6} - 16e^{-6}\right]\right]$$

$$= \frac{1}{16}\left[\overset{18}{\cancel{288}}e^{-6} + \overset{6}{\cancel{96}}e^{-6} + \overset{1}{\cancel{16}}e^{-6}\right]$$

$$= 18e^{-6} + 6e^{-6} + e^{-6} = 25e^{-6} = 0.6197$$

5. In a certain country, the proportion of highway sections required repairs in any given year is a random variable having the beta distribution with $\alpha = 3$ and $\beta = 2$. Find.

(a) On the average, what percentage of the highway sections require repairs in any given year?

(b) Find the probability that at most half of the highway sections will require repairs in any given year.

Let X be a random variable having beta distribution with $\alpha = 3$ and $\beta = 2$

(a) Average Mean $\mu = \dfrac{\alpha}{\alpha+\beta} = \dfrac{3}{5} = 0.6$

$\therefore$ On Average 60% of the highway sections require repairs in any given year.

(b) The probability most half of the highway sections will require repairs in any given year is

$$P(X \le \tfrac{1}{2}) = \int_{-\infty}^{1/2} f(x)\,dx$$

$$\therefore \int_0^{1/2} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1}dx$$

$$= \int_0^{1/2} f(x)\,dx$$

$$= \int_0^{1/2} \frac{\Gamma(5)}{\Gamma(3)\Gamma(2)} x^2 (1-x)\,dx$$

$$= \frac{4!}{2!1!} \int_0^{1/2} x^2(1-x)\,dx$$

$$= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{1/2}$$

$$= 12\left[\frac{1}{24} - \frac{1}{64}\right]$$

$$= \frac{5}{16} = 0.3125$$

6) Supper that the lifeline of a certain kind of an emergency back up battery (in hours) is a random variable x having weibull distribution will $\alpha = 0.1$ and $\beta = 0.5$ find.

(i) The mean life if these batteries

(ii) The probability that such a battery will last more than 300 hours.

Sol Given is a random variable having weibull distribution with $\alpha = 0.1, \beta = 0.5$

(i) W.K.T. mean $(\mu) = \alpha^{-1/\beta}\, \Gamma\left(1+\frac{1}{\beta}\right) = (0.1)^{-1/0.5}\, \Gamma\left(1+\frac{1}{0.3}\right)$

$$= (0.1)^{-2}\, \Gamma(3)$$

$$= 100 \cdot 2! = 200 \text{ hours.}$$

(ii) $p(x>300) = \int_{300}^{\infty} f(x)\,dx$

$$= \int_{300}^{\infty} \alpha\beta \cdot x^{\beta-1} \cdot e^{-\alpha x^\beta} \cdot dx = \alpha\beta \int_{300}^{\infty} x^{\beta-1} e^{-\alpha x^\beta}\,dx$$

$$=(0.1)(0.5)\int_{30}^{\infty}(x)^{0.5-1}\cdot e^{(-0.1)x^{0.5}}\,dx$$

$$=0.105\int_{300}^{\infty}x^{-0.5}\,e^{(-0.1)x^{0.5}}\,dx$$

$$=0.05\int_{300}^{\infty}\frac{1}{0.5}\,e^{(-0.1)t}\,dt$$

let $x^{0.5}=t$

$0.5x^{-0.5}\,dx=dt$

when $x=300$

$t=(300)^{0.5}$

$=\sqrt{300}$

when $x=\infty$; $t=\infty$

$$=\frac{0.05}{0.5}\left[\frac{e^{-0.1t}}{-0.1}\right]_{\sqrt{300}}^{\infty}$$

$$=\frac{0.05}{0.5}\left[\frac{e^{-0.1(\infty)}}{-0.1}-\frac{e^{-0.1(\sqrt{300})}}{-0.1}\right]$$

$$=0.1\left[\frac{e^{-0.1(\sqrt{300})}}{0.1}\right]=0.1\times1.7692=0.177$$

Two scanners are needed for an experiment of the fine available, two have electronic defects another on has a defect in the mamory & two are in good working order. Two units with are selected random probability.

(a) Find the joint, distribution of x_1 = the number with electronic defects and x_2 = the number with a defect in memory.

(b) Find the probability of 0 or 1 total defects among the two selected

(c) find the marginal probability distribution of x_1.

Given tatal number of scanners = 5

No. of electronic defect scanners = 2

No. of Good conditioned scanners = 2

no. of defects in the memory scanners =1.

let X_1 be the random variable corresponding to the number with electronic defects the $X_1 = \{0, 1, 2\}$ and X_2 be the random variable corresponds to the no. of scanners with defect in the memory, then $x_2 = \{0, 1\}$

(a) $P(X_1 = x_1, X_2 = x_2) = f(x_1, x_2) = \dfrac{{}^2C_{x_1}\,{}^1C_{x_1}\,{}^2C_{2-(x_1+x_2)}}{{}^5C_2}$

$x_1 = \{0, 1, 2\}$ $x_2 = \{0, 1\}$

$f(0,0) = \dfrac{{}^2C_0\,{}^1C_0\,{}^2C_2}{{}^5C_2} = \dfrac{1}{10} = 0.1$

$f(0,1) = \dfrac{{}^2C_0\,{}^1C_1\,{}^2C_1}{{}^5C_2} = \dfrac{2}{10} = 0.2$

$f(1,0) = \dfrac{{}^2C_1\,{}^1C_0\,{}^2C_1}{{}^5C_2} = \dfrac{4}{10} = 0.4$

$f(1,1) = \dfrac{{}^2C_1 \cdot {}^1C_1 \cdot {}^2C_0}{{}^5C_2} = \dfrac{1}{10} = 0.1$

$f(2,1) = 0.$

(b) The probability of 0 or 1 total defects among the two selected is

$f(0,0) + f(0,1) + f(1,0)$

$= \dfrac{{}^2C_0\,{}^1C_0\,{}^2C_2}{{}^5C_2} + \dfrac{{}^2C_1\,{}^1C_0\,{}^2C_1}{{}^5C_2} + \dfrac{{}^2C_0\,{}^1C_1\,{}^2C_1}{{}^5C_2}$

$= \dfrac{1}{10} + \dfrac{4}{10} + \dfrac{2}{10} = 0.7$

joint probability of distribution table

		X_1		
		0	1	2
X_2	0	0.1	0.4	0.1
	1	0.2	0.2	0

(c) the marginal distribution of X_1 is $f_1(x_1) = \sum_{x_2} f(x_1, x_2)$

$f_1(0) = f(0,0) + f(0,1) = 0.1 + 0.2 = 0.3$

$f_1(1) = f(1,0) + f(1,1) = 0.4 + 0.2 = 0.6$

$f_1(2) = f(2,0) + f(2,1) = 0.1 + 0 = 0.1$

		X_1			
		0	1	2	
X_2	0	0.1	0.4	0.1	$f_2(0) = 0.6$
	1	0.2	0.2	0	$f_2(1) = 0.4$
		$f_1(0) = 0.3$	$f_1(1) = 0.6$	$f_1(2) = 0.1$	1 Grand total

If two random variables have the joint density $f(x_1, x_2) = \begin{cases} x_1 x_2 & \text{for } 0 < x_1 < 1,\ 0 < x_2 < 2 \\ 0 & \text{elsewhere} \end{cases}$

Find the probabilityes that

(i) Both random variables will take on value less than 1.

(ii) The sum of the values taken on by the two random variables with be less than 1.

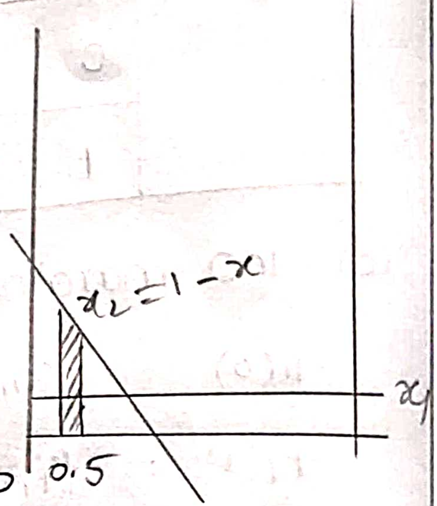
(iii) Find the marginal density of the two random variables.

Given $P(x_1<1, x_2<1) = \int_{-\infty}^{1}\int_{-\infty}^{1} x_1 x_2 \, dx_1 \, dx_2$

$= \int_0^1\int_0^1 x_1 x_2 \, dx_1 \, dx_2 = \int_0^1 x_1 \left(\frac{x_2^2}{2}\right]_0^1 dx_1$

$= \int_0^1 x_1 \left[\frac{1}{2}\right] dx_1$

$= \frac{1}{2}\left[\frac{x_1^2}{2}\right]_0^1 = \frac{1}{2}\left[\frac{1}{2}\right] = \frac{1}{4}$



(ii) $x_1 + x_2 < 1 \quad x_2 = 1 - x_1$

$P(x_1+x_2<1) = \int_{x_1=0}^{1}\int_{x_2=0}^{1-x} x_1 x_2 \, dx_2 \, dx_1$

$= \int_0^1 x_1 \left(\frac{x^2}{2}\right)_0^{1-x} dx_1$

$= \int_0^1 x_1\left(\frac{(1-x)^2}{2}\right) dx_1 = \int_0^1 x_1\left(\frac{x^2+1-2x}{2}\right) dx$

$= \frac{1}{2}\int_0^1 (x_1^3 + x_1 - 2x_1^2) dx$

$= \frac{1}{2}\left[\frac{x_1^4}{4} + \frac{x_1^2}{2} - 2\frac{x^3}{3}\right]_0^1$

$= \frac{1}{2}\left[\frac{1}{4}+\frac{1}{2}-\frac{2}{3}\right]$

$= \frac{1}{74}$

(iii) $f_1(x_1) = \int_{x_2=-\infty}^{\infty} f(x_1, x_2) \, dx_2$

$= \int_0^1 x_1 x_2 \, dx_2$

$= x_1 \left.\frac{x_2^2}{2}\right]_0^1 \Rightarrow \frac{x_1}{2}, \quad x_1 > 0.$

$$f_2(x_2) = \int_{x_1=-\infty}^{\infty} f(x_1, x_2)\, dx_1$$

$$= \int_0^2 x_1 x_2 \, dx_1$$

$$= x_2 \left[\frac{x_1^2}{2}\right]_0^2$$

$$= x_2 \frac{\cancel{4}^{2}}{\cancel{2}} - 0$$

$$= 2x_2 \, , \quad x_2 > 0$$