Maths - Home Assaignment - 1

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SUB :- MATHS (M3)

1. Write any two properties of Normal distribution?

* The graph of Normal distribution is a bell shaped curve.

* It is symmetrical about $x = \mu$.

* The area under the normal curve about the x-axis is equal to unity.

2. Define the k^{th} moment of a continuous random variable X about its mean.

The k^{th} moment of a random variable X about the mean is defined by $\mu_k = \int_{-\infty}^{\infty} (x-\mu)^k f(x)\,dx$.

3. Find the value of $Z_{0.05}$

WKT $F(Z_\alpha) = 1-\alpha$

$F(Z_{0.05}) = 1-0.05$

$= 0.95$

In the standard normal distribution tables 0.95 is closest to 0.9495 and 0.9505 corresponding to $z=1.64$ and $z=1.65$

i.e., $F(Z_{0.05}) = 0.95 = F\left(\frac{1.64+1.65}{2}\right) = F(1.645)$

$Z_{0.05} = 1.645$

4. The mean and variance of gamma distribution are respective 8 and 32. Find α, β.

Given

mean(μ) = $\alpha\beta = 8 \Rightarrow \beta = \frac{8}{\alpha} \rightarrow (1)$

variance$(\sigma^2) = \alpha\beta^2 = 32 \rightarrow (2)$

sub (1) in (2)

$\alpha\left(\frac{8}{\alpha}\right)^2 = 32$

$\cancel{\alpha}\left(\frac{64}{\alpha^{\cancel{2}}}\right) = 32$

$32\alpha = 64$

$\alpha = \frac{\cancel{64}^{2}}{\cancel{32}}$

$\boxed{\alpha = 2}$ in (1)

$\beta = \frac{\cancel{8}^{4}}{\cancel{2}}$

$\therefore \boxed{\beta = 4}$

$\therefore \alpha = 2$ and $\beta = 4$

5. What are the mean and variance of gamma distribution.

The mean & variance of gamma distribution are:-

mean $(\mu) = \alpha\beta$

variance $(\sigma^2) = \alpha\beta^2$

8/20

6. For $\alpha=0.2$ and $\beta=0.5$ find the mean of the weibull distribution.

Sol: The mean $\mu = \alpha^{-1/\beta}\Gamma\left(1+\frac{1}{\beta}\right)$

given $\alpha=0.2, \beta=0.5$

$\Rightarrow \mu = (0.2)^{-\frac{1}{0.5}}\Gamma\left(1+\frac{1}{0.5}\right)$

$= 25\cdot\Gamma(3)$

$= 25\cdot 2!$

$= 25\times 2$

$\boxed{\mu = 50}$

7. If $f(x)=\begin{cases} kx^3 & \text{for } 0<x<1 \\ 0 & \text{else where}\end{cases}$ is the probability density function of a random variable. X find the value of K.

given $f(x)$ is a probability density function

$\Rightarrow \int_{-\infty}^{\infty} f(x)\,dx = 1$

$\Rightarrow \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{0} + \int_{0}^{1} f(x)\,dx + \underbrace{\int_{1}^{\infty} f(x)\,dx}_{0} = 1$

$\Rightarrow \int_{0}^{1} f(x)\,dx = 1$

$\Rightarrow \int_{0}^{1} kx^3\,dx = 1$

$k\cdot\left[\frac{x^4}{4}\right]_0^1 = 1 \Rightarrow k\left[\frac{1}{4}\right] = 1$

$\boxed{k=4}$

8. When the random variables x and y are said to be independent.

If x and y are two random variables and the distribution of x is not influenced by the values taken by y, and vice versa the two random variables are said to be independent.

9. Define the marginal densities of the continuous random variables x and y.

Sol:-

1. If the probability density of a random variable is given by
$$f(x) = \begin{cases} 3e^{-3x} & \text{for } x \geq 0 \\ 0 & \text{otherwise.} \end{cases}$$
Find the probabilities that the random variable will take on a value (i) b/w 1 and 3 (ii) greater than 0.5

Sol: when $x=0$, & $x>0$

$$f(x) = 3e^{-3x}$$
$$= 3 \cdot \frac{1}{e^{3x}} \geq 0$$

consider

$$\int_{-\infty}^{\infty} f(x)dx + \int_{-\infty}^{0} f(x)dx + \int_{0}^{\infty} f(x)dx$$

$$\overline{0} \qquad \overline{0}$$

$$= \int_0^\infty 3 \cdot e^{-3x} dx$$

$$= \cancel{3}\left[\frac{e^{-3x}}{-\cancel{3}}\right]_0^\infty$$

$$= -[0-1]$$

$$= 1$$

(i) $P(1 \leq X \leq 3)$

$$\int_1^3 f(x)dx = \int_1^3 3 \cdot e^{-3x} dx$$

$$= \cancel{3}\left(\frac{e^{-3x}}{-\cancel{3}}\right)_1^3$$

$$= -\left[e^{-9} - e^{-3}\right]$$

$$= e^{-3} - e^{-9}$$

$$= 0.0498 - 0.0001$$

$$= 0.0497$$

ii) $P(x > 0.5)$

$$\int_{0.5}^{\infty} f(x)\,dx = \int_{0.5}^{\infty} 3 \cdot e^{-3x}\,dx$$

$$= \cancel{3} \cdot \left[\frac{e^{-3x}}{-\cancel{3}}\right]_{0.5}^{\infty}$$

$$= -\left[\cancel{e} - \cancel{e}\right]$$

$$= -\left[0 - e^{-1.5}\right]$$

$$= e^{-1.5}$$

$$= 0.2231$$

2.

3. If 20% of the memory chips made in a certain plant are defective, what are the probabilities that in a lot of 100 randomly chosen for inspection

i, At most 16 will be defective

ii, Exactly 16 will be defective.

Sol Give $n=100$, $p=20\% = 0.2$

$q = 1-p = 1-0.2$

$\boxed{q=0.8}$

Mean $\mu = np = 20$

standard deviation, $\sigma = \sqrt{npq} = \sqrt{100(0.2)(0.8)} = \sqrt{16} = 4$

i, $P(x \leq 16) = P(x \leq 16+0.5)$

$= P(x \leq 16.5)$

$= P\left(\frac{x-\mu}{\sigma} \leq \frac{16.5-\mu}{\sigma}\right) = P\left(z \leq \frac{16.5-20}{4}\right)$

$= P(z \leq -0.88)$

$= F(-0.88)$

$= \cancel{0.4681} = 0.1894$

ii, $P(x=16) = P(15.5 \leq x \leq 16.5)$

$= \cancel{P\left(\frac{x-\mu}{}\right.}$

$= P\left(\frac{15.5-\mu}{\sigma} \leq \frac{x-\mu}{\sigma} \leq \frac{16.5-\mu}{\sigma}\right)$

$= P\left(\frac{15.5-20}{4} \leq z \leq \frac{16.5-20}{4}\right)$

$\Rightarrow P(-1.13 \leq Z \leq -0.88)$

$= F(-1.13) - F(-0.88)$

$= 0.1292 - 0.1894$

$= -0.06$

4. In a certain city, the daily consumption of electric power can be treated as a random variable having a gamma distribution with $\alpha=2$ and $\beta=3$. If the city's power plant has a daily capacity of 10MKWH. What is the probability that this power supply will be inadequate on any given day?

The daily capacity of the power plant is 10MKWH

The daily capacity of electric power is the Given the daily consumption of electric power with $\alpha=2$ random variable having a gamma distribution & $\beta=3$ let it be X.

The probability that 10MKWH power supply will be inadequate is

$$P(X>10) = \int_{10}^{\infty} f(x)\,dx$$

$$= \int_{10}^{\infty} \frac{1}{\beta^{\alpha}\cdot\Gamma(\alpha)} x^{\alpha-1}\cdot e^{-x/\beta}\,dx$$

$$= \int_{10}^{\infty} \frac{1}{3^{2}\cdot\Gamma(2)} x^{1}\cdot e^{-x/3}\,dx \qquad \Gamma(2)=1!$$

$$= \frac{1}{9\cdot 1}\int_{10}^{\infty} x\, e^{-x/3}\,dx$$

$$= \frac{1}{9}\cdot\left[x\cdot\frac{e^{-x/3}}{-1/3} - 1\cdot\frac{e^{-x/3}}{1/9} + 0\right]_{10}^{\infty}$$

$$= \frac{1}{9}\left[-3e^{-x/3} - 9e^{-x/3}\right]_{10}^{\infty}$$

$$= \frac{1}{9}\left[-12\cdot e^{-x/3}\right]_{10}^{\infty}$$

$$= \frac{1}{9}\left[0 - (-12)(e)^{-10/3}\right]$$

$$= \frac{1}{\cancel{9}_{3}}\left(\cancel{12}^{4}\cdot e^{-3.3333}\right)$$

$$= \frac{1}{3}\left(4e^{-3.333}\right)$$

$$= \frac{1}{3}\left(4\times 0.0357\right)$$

$$= \frac{0.1428}{3}$$

$$= 0.0476$$

~~8. In a certain city, the daily consumption~~

5\. In a certain country, the proportion of highway sections ~~require~~ requiring repairs in any given year is a random variable having the beta distribution with $\alpha=3$ and $\beta=2$ find

a, on the avg, what percentage of highway sections require repairs in any given year.

b, Find the probability that atmost half of the highway sections will require repairs in any given year.

Sol- let x be a random variable having beta distribution with $\alpha=3$ & $\beta=2$

a, Mean $(\mu) = \frac{\alpha}{\alpha+\beta} = \frac{3}{5} = 0.6$

$= 60\%$.

b, $P(X \leq 1/2) = \int_{-\infty}^{1/2} f(x)\,dx$

$= \underbrace{\int_{-\infty}^{0} f(x)\,dx}_{0} + \int_{0}^{1/2} f(x)\,dx$

$= \int_{0}^{1/2} \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\cdot\Gamma(\beta)} x^{\alpha-1} \cdot (1-x)^{\beta-1}$

$= \int_{0}^{1/2} \frac{\Gamma(5)}{\Gamma(3)\Gamma(2)} x^{2}(1-x)\,dx$

$= \frac{4!}{2!\,1!} \int_{0}^{1/2} x^{2}(1-x)\,dx$

$= 12\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^{1/2}$

$= 12\left[\frac{1}{24} - \frac{1}{64}\right]$

$= \frac{5}{16}$

$= 0.3125$

6. suppose that the lifetime of a certain kind of an emergency backup battery (in hours) is a random variable x having weibull distribution with $\alpha=0.1$ & $\beta=0.5$ Find

i, The mean life of these batteries

ii, The probability that such a battery will last more than 300 hours.

Sol Given x is a random variable having weibal distribution with $\alpha=0.1, \beta=0.5$

i, WKT $$\text{mean}(\mu) = \alpha^{-1/\beta}\,\Gamma\left(1+\frac{1}{\beta}\right)$$

$$=(0.1)^{-1/0.5}\,\Gamma\left(1+\frac{1}{0.5}\right)$$

$$=(0.1)^{-2}\,\Gamma(3)$$

$$=100\cdot 2!$$

$$=200 \text{ hours}$$

ii, $$P(X>300) = \int_{300}^{\infty} f(x)\,dx$$

$$=\int_{300}^{\infty} \alpha\beta \cdot x^{\beta-1} \cdot e^{-\alpha x^{\beta}}\,dx$$

$$=\alpha\beta\int_{300}^{\infty} x^{\beta-1} \cdot e^{-\alpha x^{\beta}}\,dx$$

$$=(0.1)(0.5)\int_{30}^{\infty} (x)^{0.5-1} \cdot e^{(-0.1)x^{0.5}}\,dx$$

$$=0.05\int_{300}^{\infty} x^{-0.5} \cdot e^{(-0.1)x^{0.5}}\,dx$$

$$= 0.05 \int_{\sqrt{300}}^{\infty} \frac{1}{0.5} \cdot e^{(-0.1)t} \, dt$$

let $x^{0.5} = t$

$0.5 x^{-0.5} dx = dt$

when $x = 300$

$t = (300)^{0.5}$

$= \sqrt{300}$

when $x = \infty$

$t = \infty$

$$= \frac{0.05}{0.5} \left[\frac{e^{-0.1t}}{-0.1} \right]_{\sqrt{300}}^{\infty}$$

$$= \frac{0.05}{0.5} \left[\frac{e^{-0.1(\infty)}}{-0.1} - \frac{e^{-0.1(\sqrt{300})}}{-0.1} \right]$$

$$= 0.1 \left(0 + \frac{e^{-0.1(\sqrt{300})}}{0.1} \right)$$

$= 0.1 \times 1.7692$

$= 0.177$

7. Two scanners are needed for an experiment of the five available, two have electronic defects another one has a defect in the memory & two are in good working order. Two units are selected random

a, Find the joint probability distribution of X_1 = the number with electronic defects, and X_2 = the number with a defect in memory.

b, Find the probability of 0 or 1 total defects among the two selected.

c, Find the marginal probability distribution of X_1.

Sol: given total no.of scanners = 5

No.of electronic defect scanners = 2

No.of good conditioned scanners = 2

No.of defects in the memory scanners = 1.

Let x_1 be the random variable corresponds to the number with electronic defects the $x_1 = \{0,1,2\}$ and

x_2 be the random variable corresponds to the no.of scanners with defect in the memory, then $x_2 = \{0,1\}$

a,

$$P(x_1 = x_1, x_2 = x_2) = f(x_1, x_2) = \frac{{}^2C_{x_1} \, {}^1C_{x_2} \, {}^2C_{2-(x_1+x_2)}}{{}^5C_2}$$

$x_1 = \{0,1,2\}$

$x_2 = \{0,1\}$

$$f(0,0) = \frac{{}^2C_0 \, {}^1C_0 \, {}^2C_2}{{}^5C_2} = \frac{1}{10} = 0.1$$

$$f(0,1) = \frac{{}^2C_0 \, {}^1C_1 \, {}^2C_1}{{}^5C_2} = \frac{2}{10} = 0.2$$

$$f(1,0) = \frac{{}^2C_1 \cdot {}^1C_0 \cdot {}^2C_1}{{}^5C_2} = \frac{4}{10} = 0.4$$

$$f(1,1) = \frac{{}^2C_1 \cdot {}^1C_1 \cdot {}^2C_0}{{}^5C_2} = \frac{1}{5} = 0.2$$

$$f(2,0) = \frac{{}^2C_2 \, {}^1C_0 \, {}^2C_0}{{}^5C_2} = \frac{1}{10} = 0.1$$

$$f(2,1) = 0$$

b, The probability of 0 or 1 total defects among the two selected is

$$f(0,0)+f(0,1)+f(1,0) = \frac{{}^2C_0\,{}^1C_0\,{}^2C_2}{{}^5C_2} + \frac{{}^2C_1\,{}^1C_0\,{}^2C_1}{{}^5C_2} + \frac{{}^2C_0\,{}^1C_1\,{}^2C_1}{{}^5C_2}$$

$$= \frac{1}{10} + \frac{4}{10} + \frac{2}{10}$$

$$= 0.7$$

Joint probability distribution table

		x_1		
		0	1	2
x_2	0	0.1	0.4	0.1
	1	0.2	0.2	0

c, the marginal distribution of x_1 is $f_1(x_1) = \sum_{x_2} f(x_1, x_2)$

$$f_1(0) = f(0,0) + f(0,1)$$
$$= 0.1 + 0.2$$
$$= 0.3$$

$$f_1(1) = f(1,0) + f(1,1)$$
$$= 0.4 + 0.2$$
$$= 0.6$$

$$f_1(2) = f(2,0) + f(2,1)$$
$$= 0.1 + 0 = 0.1$$

		X_1			
		0	1	2	
X_2	0	0.1	0.4	0.1	$f_2(0)=0.6$
	1	0.2	0.2	0	$f_2(1)=0.4$
		$f_1(0)=0.3$	$f_1(1)=0.6$	$f_1(2)=0.1$	1 (grand total)

8. If two random variables have the joint density

$$f(x_1, x_2) = \begin{cases} x_1 x_2 & \text{for } 0<x_1<1, 0<x_2<2 \\ 0 & \text{else where} \end{cases}$$

Find the probabilities that

i, Both random variables will take on value less than 1

ii, The sum of the values taken on by the two random variables will be less than 1.

iii, Find the marginal densities of the two random variables.