Marella. Pujitha
Reg.No:- Y20AIT467.

1. Write the test statics for two means large sample case.

Null Hypothesis, $H_0: \mu_1 - \mu_2 = \delta_0$

Alternate Hypothesis, $H_1: \mu_1 - \mu_2 < \delta_0$

or $H_1: \mu_1 - \mu_2 > \delta_0$

or $H_1: \mu_1 - \mu_2 \neq \delta_0$

sample sizes: n_1 and n_2 (both are greater than or equal to 30)

sample means: $\bar{x}$ and $\bar{y}$

sample variances: s_1^2 and s_2^2

Level of significance, α (1% or 5%)

Test statics, $$Z = \frac{(\bar{x}-\bar{y}) - \delta_0}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

2. What are the confidence limits for difference of two means in large sample case.

A confidence interval of the difference of two mean dynamic modulus is given by

$$(\bar{x}-\bar{y}) + Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{x}-\bar{y}) - Z_{\alpha/2}\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

3. What is the test static for two means in sample sample case

Null Hypothesis, $H_0: \mu_1 - \mu_2 = \delta$

Alternate Hypothesis, $H_1: \mu_1 - \mu_2 < \delta$

or $H_1: \mu_1 - \mu_2 > \delta$

or $H_1: \mu_1 - \mu_2 \neq \delta$

sample sizes: n_1 and n_2 (n_1, n_2 or both less than 30)

sample means: $\bar{x}$ and $\bar{y}$

sample variances: s_1^2 and s_2^2

level of significance, α (1% or 5%)

Test Statistic, $t = \dfrac{(\bar{x}-\bar{y})-\delta}{\sqrt{(n_1-1)s_1^2+(n_2-1)s_2^2}} \sqrt{\dfrac{n_1 n_2 (n_1+n_2-2)}{n_1+n_2}}$

4. What is the test static for one variance.

Null hypothesis $\sigma^2 = \sigma_0^2$

Alternative Hypothesis $H_1: \sigma^2 < \sigma_0^2$

or $H_1: \sigma^2 > \sigma_0^2$

or $H_1: \sigma^2 \neq \sigma_0^2$

Test statistic $\chi^2 = \dfrac{(n-1)s^2}{\sigma_0^2}$

5. Write the critical region for testing null hypothesis $\sigma_1^2 = \sigma_2^2$

Test Statistic, $F = \dfrac{s_1^2}{s_2^2}$

critical regions for testing $H_0: \sigma_1^2 = \sigma_2^2$

Alternate Hypothesis	Test Stastic	Reject null hypothesis if
$\sigma_1^2 < \sigma_2^2$	$F = \dfrac{s_2^2}{s_1^2}$	$F > F_\alpha(n_2-1, n_1-1)$
$\sigma_1^2 > \sigma_2^2$	$F = \dfrac{s_1^2}{s_2^2}$	$F > F_\alpha(n_1-1, n_2-1)$
$\sigma_1^2 \neq \sigma_2^2$	$F = \dfrac{s_M^2}{s_m^2}$	$F > F_{\alpha/2}(n_M-1, n_m-1)$

6. What is the test statistic for one proportion and critical region for testing it.

Null hypothesis $H_0 : P = P_0$

Alternative hypothesis $H_1 : P < P_0$

or $H_1 : P > P_0$

or $H_2 : P \neq P_0$

Test statistic $z = \dfrac{x - np_0}{\sqrt{np_0(1-P_0)}}$.

critical region for testing the null hypothesis is $H_0 : P_0 = P_0$

Alternative hypothesis	Reject null hypothesis
$P < P_0$	$Z < -z_\alpha$
$P > P_0$	$Z > z_\alpha$
$P \neq P_0$	$Z < -z_{\alpha/2}$ or $Z > z_{\alpha/2}$

7. What is the test statistic for two proportions and state its critical region.

Null hypothesis $H_0 : P_1 = P_2$

Alternative hypothesis $H_1 : P_1 < P_2$

$H_1 : P_1 > P_2$

$H_1 : P_1 \neq P_2$

Test statistic $z = \dfrac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{P}(1-\hat{P})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$\hat{P} = \dfrac{x_1 + x_2}{n_1 + n_2}$

Alternative hypothesic	Reject null hypothesis
$P_1 < P_2$	$Z < -Z_\alpha$
$P_1 > P_2$	$Z > Z_\alpha$
$P_1 \neq P_2$	$Z < -Z_{\alpha/2}$ or $Z > Z_{\alpha/2}$

1. Two types of new cars produced in U.S.A are tested for petrol mileage, one sample is consisting of 42 cars averaged 15 kmpl while the other sample consisting of 80 cars averaged 11.5 kmpl with population variances as $\sigma_1^2 = 2.0$ and $\sigma_2^2 = 1.5$ respectively. Test whether there is any significance difference in the petrol consumption of these two types of cars (use $\alpha = 0.01$)

Step-1:- Null hypothesis is $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis is $H_1: \mu_1 - \mu_2 \neq 0$

Step-2:- Level of significance $\alpha = 0.05$

Step-3:- Criteria:- The Null hypothesis is rejected if $Z > Z_{\alpha/2}$ or $Z < Z_{\alpha/2}$

Here, the test statistics $Z = \dfrac{(\bar{x} - \bar{y}) - \delta}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$

$Z_{\alpha/2} = Z_{\frac{0.01}{2}} = Z_{0.005} = 2.58$

Step-4:- Calculations:-

Given $n_1 = 42$ $\quad n_2 = 80$

$\bar{x} = 15$ $\quad \bar{y} = 11.5$

$\sigma_1^2 = 2.0$ $\quad \sigma_2^2 = 1.5$

$\sigma_1 = 1.41421$ $\quad \sigma_2 = 1.2247$

$$Z = \frac{(15-11.5)-0}{\sqrt{\frac{2.0}{42} + \frac{1.5}{80}}} = \frac{3.5}{\cancel{0.2592}\ 0.2576} = \cancel{9.9943}\ 13.5869$$

step-5:- $Z = \cancel{9.99445}$ 13.5869 is greater than $Z_{\alpha/2} = 2.58$ then the null hypothesis is rejected means we accept the alternative hypothesis

2. A simple sample of the height of 6400 Englishmen has a mean of 67.85 inches and a S.D. if 2.56 inches while a simple sample of heights of 1600 Australians has a mean of 68.55 inches and S.D of 2.52 inches. Do the data indicate the Australians are on the average taller than the Englishmen? use 0.05 level of significance.

step-1:- Null hypothesis $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1: \mu_1 < \mu_2$

step-2:- Level of significance $\alpha = 0.05$

Step-3:- criteria:- the null hypothesis is rejected if $Z < -Z_\alpha$

Here the test stastic $Z = \frac{(\bar{x}-\bar{y})-\delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n}}}$

$Z_\alpha = Z_{0.05} = -1.645$

Step-4:- calculations:-

$n_1 = 6400$ $\qquad$ $n_2 = 1600$

$\bar{x} = 67.85$ $\qquad$ $\bar{y} = 68.55$

$\sigma_1 = 2.56$ $\qquad$ $\sigma_2 = 2.52$

$$Z = \frac{(67.85 - 68.55)}{\sqrt{\frac{2.56^2}{6400} + \frac{2.52^2}{1600}}} = \frac{-0.7}{0.0707} = \cancel{9.6181}\ -9.9009$$

step 5: conclusion: Since $Z = -9.9009$ is less than $Z_\alpha = -1.645$ then the null hypothesis is rejected means we accept the alternative hypothesis.

3. Two independent sample of 8 and 7 items respectively have the following values

Sample I	11	11	13	11	15	9	12	14
sample II	9	11	10	13	9	8	19	-

Is the difference between the means of means of sample significance. use $\alpha = 0.05$

step-1: Null hypothesis $H_0: \mu_1 - \mu_2 = 0$

Alternative hypothesis $H_1: \mu_1 - \mu_2 \neq 0$

step-2: Level of significance $\alpha = 0.05$

step-3: criteria: The null hypothesis is rejected if $t > t_{\alpha/2}$ or $t < t_{\alpha/2}$

Here test statistics $t = \dfrac{(\bar{x}_1 - \bar{x}_2) - \delta}{\sqrt{(n_1-1)s_1^2 + (n_2-1)s_2^2}} \sqrt{\dfrac{n_1 n_2 (n_1+n_2-2)}{n_1+n_2}}$

$t_{\alpha/2} = t_{\frac{0.05}{2}} = t_{0.025} = -2.160$

step-4: calculations:

$n_1 = 8 \quad n_2 = 7$

$\bar{x}_1 = 12 \quad \bar{x}_2 = 11.2857$

$s_1^2 = 3.7143 \quad s_2^2 = 14.2381$

$$t = \frac{(12 - 11.2857) - 0}{\sqrt{(7)(3.7143) + (6)(14.2381)}} \sqrt{\frac{56(13)}{15}} = \frac{0.7143}{10.5559} (6.9666)$$

$= 0.4714$

step-5: conclusion: since $t = 0.474$ is ~~less~~ greater than $t_{\alpha/2} = 2.160$ then we ~~reject~~ accept the null hypothesis means we ~~accept~~ reject the alternative hypothesis

4. Playing 10 rounds of gold on his home course, a gold professional averaged 71.3. Test the null hypothesis that the consistency of his game on his home course is actually measured by $\sigma = 1.20$, against the alternative hypothesis that he is less consistent. use the level of significance $\alpha = 0.05$

sol:- step-1:- Null hypothesis H_0: $\sigma = 1.20$

Alternative hypothesis H_1: $\sigma > 1.20$

step-2:- level of significance $\alpha = 0.05$

~~step-3:- criten~~

step-3: criteria: The null hypothesis is rejected if $\chi^2 > \chi^2_\alpha$

Test stastic $\chi^2 = \frac{(n-1)s^2}{\sigma_0^2} = \frac{(10-1)(1.32)^2}{(1.20)^2}$, $\chi^2_\alpha = 16.919$

$= 10.89$

condusion: $\chi^2 = 10.89$ does not exceeds 16.919, the value of $\chi^2_{0.05}$ for 9 degrees, the null hyp can not accept.

5. In one sample of 10 observations, the sum of the squares of the deviations of the sample values from sample mean was 120 and in the other sample of 12 observations, it was 314. Test whether the difference in variances is significance at 5% in variance level.

step-1: Null hypothesis $H_0: \sigma_1^2 = \sigma_2^2$

Alternative hypothesis $H_1: \sigma_1^2 \neq \sigma_2^2$

step-2: level of significance $\alpha = 0.05$

step-3: criteria:- Null hypothesis is rejected if $F > F_{\alpha/2}(n_M - 1, n_m - 1)$

Here the ~~test~~ stastics $F = \frac{S_M^2}{S_m^2}$

$F_\alpha = F_{0.05}$ $\nu_1 = 10, \nu_2 = 9$

Step 4:- calculations

$F = \frac{S_2^2}{S_1^2}$. Given $n_1 = 10$ $\Sigma(x_1 - x_2)^2 = 120$

$n_2 = 12$ $\Sigma(x_2 - x_1)^2 = 314$

$$= \frac{\frac{\Sigma(x_2-x_1)^2}{n_2-1}}{\frac{\Sigma(x_1-x_2)^2}{n_1-1}} = \frac{\frac{314}{11}}{\frac{120}{9}} = \frac{28.545}{13.323}$$

$F = 2.1409$

The samples may have been drawn from two popolations having the same variances. The difference is not sigginificant at 5%. level of significance

6. Among 100 fish caught in large lake, 18 were inedible due to the pollution of the environment. With what confidente tha can we assert that the error of this estimate is at most 0.065?

Given that sample size $n = 100$

max error of estimate, $e = 0.065$

P = sample proportion of inedible fish $= \frac{18}{100} = 0.18$

$\therefore$ ~~q>1~~ $q = 1 - P = 1 - 0.18 = 0.82$

maximum error of estimate ~~from~~ for true proportion

$$E = Z_{\alpha/2} \sqrt{\frac{pq}{n}}$$

$$0.065 = Z_{\alpha/2} \sqrt{\frac{(0.82)(0.18)}{1000}}$$

$$0.065 = Z_{\alpha/2} (0.0384)$$

$$= \frac{0.065}{0.0384}$$

$$Z_{\alpha/2} = 1.6927.$$

7. In a random sample of 125 cool drinkers, 68 said they prefer thumsup to pepsi. Test the null hypothesis $P=0.5$ against the alternative hypothesis $P>0.5$.

Sol step-1: Null hypothesis $P=0.5$

Alternative hypothesis $H_1: P>0.5$

step-2: Level of significance $\alpha = 0.05$.

step-3: Criteria: The null hypothesis is rejected if $Z > Z_\alpha$

here the test statistics $Z = \dfrac{x - nP_0}{\sqrt{nP_0(1-P_0)}}$

$Z_\alpha = Z_{0.05} = 1.645$

step4: calculations:-

Given $n = 125$, $x = 68$

$$Z = \frac{68 - 125(0.5)}{\sqrt{125(0.5)(1-0.5)}} = \frac{5.5}{5.5902} = 0.9839$$

$Z_\alpha = 1.645$

step-5: since $Z_\alpha = 1.645$ is greater than the $Z = 0.9839$ then we accept the null hypothesis mean we reject the alternative hypothesis.

8. A manufacturer of electronic equipment subjects samples of two completing brands of transistors to an accelerated performance test. If 45 of 180 transistors of the first kind and 34 of 120 transistors of the second kind fail the test, what can he conclude at the level of significance $\alpha = 0.05$ about the difference between the corresponding sample proportions.

sol step-1: Null hypothesis $H_0: P_1 = P_2$

Alternative hypothesis $H_1: P_1 \neq P_2$

step-2: Level of significance $\alpha = 0.05$.

step-3: criteria: the null hypothesis is rejected if $Z > Z_{\alpha/2}$ or $Z < -Z_{\alpha/2}$

Here the test statistic $Z = \dfrac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{P}(1-\hat{P})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$

$Z_{\alpha/2} = Z_{0.025} = 1.96$

step-4 : calculations :-

Given $n_1 = 180$ $\qquad$ $n_2 = 120$

$x_1 = 45$ $\qquad$ $x_2 = 34$

$$\hat{p} = \frac{x_1 + x_2}{n_1 + n_2} = \cancel{0.2}\frac{45+34}{180+120} = 0.2633$$

$$Z = \frac{\frac{x_1}{n_1} - \frac{x_2}{n_2}}{\sqrt{\hat{p}(1-\hat{p})}\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = \frac{\frac{45}{180} - \frac{34}{120}}{\sqrt{0.2633(1-0.2633)}\sqrt{\frac{1}{180} + \frac{1}{120}}}$$

$$= \frac{-0.0333}{0.4404(0.1179)} = \frac{-0.0333}{0.0519} = -0.6416$$

$Z_{\alpha/2} = 1.96$

step-5 : since $Z = -0.6416$ is less than $Z_{\alpha/2} = 1.96$ then we ~~re~~ accept the null hypothesis means we reject the alternative hypothesis.