Hall Ticket Number:

II/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

April, 2017 **Common for all branches**

Fourth Semester **Engineering Mathematics -IV**

Time: Three Hours **Maximum :** 60 Marks

Answer Question No.1 compulsorily. (1X12 = 12 Marks)

Answer ONE question from each unit. (4X12=48 Marks)

1 Answer all questions (1X12=12 Marks)

a) Find the imaginary part of *l*og(-i).

b) State Cauchy-Riemann equations in polar form.

c) Evaluate $\int_C \frac{dz}{z-2}$, where 'C' is the circle $|z-2| = 1$.

d) If z = a is simple pole of f(z), then what is residue of f(z) at z = a?

e) Find the nature of singularity of $f(z) = \frac{z-\sin z}{z^2}$.

f) State Residue theorem.

g) If the distribution function of a random variable is given by $F(x) = \begin{cases} 1-\frac{1}{x^2}; & x>1 \\ 0 \quad ; & x \le 1 \end{cases}$,

Find the probability that this random variable will take on a value less than 3.

h) What is the mean and standard deviation of standard normal distribution?

i) The variance of a population is 2. The size of the sample collected from the population is 169. What is the standard error of mean?

j) What are the errors of sampling?

k) Define point estimation.

l) Define F-distribution.

UNIT I

2 a) Find all roots of the equations $(i)\sqrt[3]{1+i}$ $(ii)\, z^2+z+1=i$. 6M

b) Evaluate $\int_C \frac{z-1}{(z+1)^2(z-2)}dz$, using Cauchy's integral formula, where 'C' is $|z-i| = 2$. 6M

(OR)

3 a) Show that $u = e^{-x}$ (x siny – y cosy) is harmonic and find its conjugate harmonic. 6M

b) If $F(a) = \int_C \frac{4z^2+z+5}{z-a}dz$, where 'C' is the ellipse $\left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1$ then find the value of (i) F(3.5) (ii) F(i), $F^1(-1)$, $F^{11}(-i)$. 6M

UNIT II

4 a) Expand $f(z) = \cos z$ in powers of 'z – π/2'. 6M

b) Using the method of contour integration prove that $\int_{-\infty}^{\infty} \frac{dx}{x^4+1} = \frac{\pi}{\sqrt{2}}$ 6M

(OR)

5 a) Evaluate $\int_C \frac{z\cos z}{\left(z-\frac{\pi}{2}\right)^3}dz$ where 'C' is the circle $|z-1| = 1$, by using Cauchy's Residue theorem. 6M

b) Expand $f(z) = \frac{1}{(z-1)(z-2)}$ in the region (i) $|z| < 1$ (ii) $1 < |z| < 2$ (iii) $|z| > 2$. 6M

UNIT III

6 a) If 20% of the memory chips made in a certain plant are defective, what are the probabilities that in a lot of 100 randomly chosen for inspection using normal approximation to binomial distribution (a) at most 15 will be defective; (b) exactly 15 will be defective? 6M

b) If the mean of breaking strength of copper wire is 575 lbs with a standard deviation 8.3 lbs. How large a sample must be used in order that there will be one chance in 100 that the mean breaking strength of the sample is less than 572 lbs. 6M

(OR)

7 a) If the joint probability density of two random variables is given by

$$f(x_1, x_2) = \begin{cases} 6\, e^{-2x_1 - 3x_2} & \text{for } x_1 > 0, x_2 > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Find the probability that the first random variable will take on a value between '1' and '2' and the second random variable will take on a value between '2' and '3'

(ii) Find the probability that the first random variable will take on a value less than '2' and the second random variable will take on a value greater than '2'. 6M

b) The chi square distribution with 4 degrees of freedom is given by

$f(x) = \begin{cases} \frac{1}{4}\, x\, e^{-x/2}; & \text{for } x > 0 \\ 0\,; & \text{for } x \leq 0 \end{cases}$. Find the probability that the variance of a random sample of size 5 from a normal population with $\boldsymbol{\sigma}$ = 12 will exceed 180. 6M

UNIT IV

8 a) An airline claims that only 6% of all lost luggage is never found. If, in a random sample, 17 of 200 pieces of lost luggage are not found, test the null hypothesis p = 0.06 against the alternative hypothesis p> 0.06 at the 0.05 level of significance. 6M

b) A sample of 400 items is taken from a population whose standard deviation is 10. The mean of the sample is 40. Test whether the sample has come from a population with mean 38. Also calculate 95% confidence interval for the population. 6M

(OR)

9 a) Experience has shown that 20% of a manufactured product is of the top quality. In one day's production of 400 articles only 50 are of top quality. Test the hypothesis at 0.05 level. 6M

b) A random sample of 100 teachers in a large metropolitan area revealed a mean weekly salary of Rs.487 with a standard deviation Rs.48. With what degree of confidence can assert that the average weekly salary of all teachers in the metropolitan area is between Rs.472 to Rs.500 6M