18MA002

Hall Ticket Number:

I/IV B.Tech (Regular) DEGREE EXAMINATION

April, 2019
Second Semester

Common to all branches

Numerical Methods & Advanced Calculus

Time: Three Hours
Answer Question No.1 compulsorily.
Answer ONE question from each unit.

Maximum:50 Marks
(10X1 = 10 Marks)
(4X10=40 Marks)

1. Answer all questions (10X1=10 Marks)

a) Develop Newton's iterative formula to find cube root of a natural number. (CLO2) 1M

b) Decompose $A = \begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix}$ as LU. Here L and U are lower and upper triangular matrices. (CLO2) 1M

c) Write Newton's divided difference formula. (CLO1) 1M

d) Give the Simpson's 1/3rd rule of integration. (CLO1) 1M

e) Write the general formula to find y_1 for the initial value problem $\frac{dy}{dx} = f(x,y),\ y(x_0) = y_0$ in Ruge-Kutta method of 4th order. (CLO1) 1M

f) Change $\iint (x^2+y^2)^{n/2}\, dx\, dy$ over the positive quadrant of the circle $x^2+y^2=4$ to polar coordinates. (No need to evaluate) (CLO2) 1M

g) Evaluate $\int_0^1 \int_{y^2}^1 \int_0^{1-x} x\, dx\, dy\, dz$ (CLO1) 1M

h) For $\phi = xy^2 + yz^3$, find its gradient. (CLO2) 1M

i) Find $\nabla \times F$, for $F = (-x^2+yz)I + (4y-z^2x)J + (2xz-4z)K$ (CLO2) 1M

j) State Green's theorem. (CLO1) 1M

UNIT I

2. a) Using Regula-Falsi method find a root of $xe^x = 2$ correct to three decimal places. (CLO2) 5M

b) Apply Gauss-Siedel method to solve the system $2x+17y+4z=35, x+3y+10z=24,\ 28x+4y-z=32$ with (0,0,0) as initial approximation correct to four decimal places. (CLO2) 5M

(OR)

3. a) Find a root of $e^x = 4\sin x$ using bisection method correct to four decimal places. (CLO2) 5M

b) Using Gauss elimination find a solution of $2x_1+4x_2+x_3=3, 3x_1+2x_2-2x_3=-2,\ x_1-x_2+x_3=6$ (CLO2) 5M

UNIT II

4. a) Express the value of θ in terms of x for the data

x	40	50	60	70	80	90
θ	184	204	226	250	276	304

Also find the values of θ at $x = 43$ (CLO2) 5M

b) Find the value of y for $x = 0.1$ using Picard's method for the initial value problem $\frac{dy}{dx} = y,\ y(0) = 1$ (CLO2) 5M

(OR)

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5. a) Apply Lagrange's interpolation formula to find the value of y when x =10 for the data 5M

x	5	6	9	11
y	12	13	14	16

(CLO2)

b) Using Trapezoidal and Simposon's 3/8th rule of integration, find $\int_{4}^{5.2} \ln x\,dx$, for h = 0.2. (CLO2) 5M

UNIT III

6. a) Evaluate $\iint xy(x+y)\,dx\,dy$ over the region bounded by $y=x^2$, $y=x$. (CLO3) 5M

b) Find the volume of the sphere $x^2+y^2+z^2=a^2$. (CLO3) 5M

(OR)

7. a) Find the are between the parabolas $y^2=4x,\ x^2=4y$. (CLO3) 5M

b) Evaluate $\iiint z^2\,dx\,dy\,dz$, taken over the volume bounded by the surfaces $x^2+y^2=a^2$, $x^2+y^2=z$ and z = 0. (CLO3) 5M

UNIT IV

8. a) Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point $(2,-1,2)$. (CLO3) 5M

b) Using Stoke's theorem evaluate $\int_C \left[(x+y)\,dx+(2x-z)\,dy+(y+z)\,dz\right]$, where C is the boundary of the triangle with vertices $(2,0,0)$, $(0,3,0,)$, $(0,0,6)$ (CLO3) 5M

(OR)

9. Verify Gauss Divergence theorem for $F=yI+xJ+z^2K$ over the cylindrical region bounded by $x^2+y^2=9, z=0, z=2$. (CLO3) 10M

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Numerical methods and Advanced Calculus

Scheme of valuation, April, 2019

① ⓐ Let $x = \sqrt[3]{N} \Rightarrow x^3 - N = 0$

Let $f(x) = x^3 - N$ then $f'(x) = 3x^2$

By newton's iterative formula, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$\Rightarrow x_{n+1} = x_n - \frac{x_n^3 - N}{3x_n^2} = \frac{2x_n^3 + N}{3x_n^2}$$

ⓑ Let $A = \begin{bmatrix} 4 & 1 \\ 3 & -5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ a & 1 \end{bmatrix} \begin{bmatrix} b & c \\ 0 & d \end{bmatrix}$

Solving $a = 3/4$, $b = 4$, $c = 1$, $d = -23/4$.

ⓔ By R-K method of furth order

$$y_1 = y_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = h f(x_0, y_0)$$

$$k_2 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2})$$

$$k_3 = h f(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2})$$

$$k_4 = h f(x_0 + h, y_0 + k_3)$$

ⓓ Simpson's $1/3^{rd}$ rule of integration

$$\int_{x_0}^{x_0+nh} (f(x) = y)\, dx = \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + y_5 + \cdots) + 2(y_2 + y_4 + \cdots)]$$

Here the number of intervals is even.

ⓒ Newton's divided difference formula

$$f(x) = f(x_0) + (x - x_0)[x_0, x_1] + (x - x_0)(x - x_1)[x_0, x_1, x_2] + \cdots + (x - x_0)(x - x_1)\cdots(x - x_{n-1})[x_0, x_1, \ldots x_n]$$

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(f) Consider $\iint (x^2+y^2)^{n/2}\, dx\,dy$

By changing to polar form, $x = r\cos\theta,\ y = r\sin\theta$

For positive quadrant $\theta = 0$ to $\pi/2$ $r = 0$ to 2 $dx\,dy = r\,dr\,d\theta$

Then $$\iint (x^2+y^2)^{n/2}\, dx\,dy = \int_{\theta=0}^{\pi/2}\int_{r=0}^{2} r^n \cdot r\, dr\,d\theta = \int_0^{\pi/2}\int_{r=0}^{2} r^{n+1}\, dr\,d\theta$$

(g) Consider $$\int_0^1\int_{y^2}^1\int_0^{1-x} x\, dx\,dy\,dz = \int_{y=0}^1\int_{x=y^2}^1\int_{z=0}^{1-x} x\, dz\,dx\,dy$$

$$= \int_{y=0}^1\int_{x=y^2}^1 x(1-x)\,dx\,dy = \int_{y=0}^1 \left(\frac{x^2}{2}-\frac{x^3}{3}\right)_{x=y^2}^1 dx$$

$$= \int_{y=0}^1 \left(\frac{1}{2}-\frac{1}{3}-\frac{y^4}{2}+\frac{y^6}{3}\right) dy = \left(\frac{1}{6}y-\frac{y^5}{10}+\frac{y^7}{21}\right)_0^1 = \frac{1}{6}-\frac{1}{10}+\frac{1}{21} = \frac{4}{35}$$

(h) Consider $$\nabla\phi = \left(i\frac{\partial}{\partial x}+j\frac{\partial}{\partial y}+k\frac{\partial}{\partial z}\right)(xy^2+yz^3)$$
$$= y^2 i + (2xy+z^3)\,j + 3yz^2 k$$

(i) Consider $$\nabla\times F = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -x^2+yz & 4y-z^2x & 2xz-4z \end{vmatrix}$$
$$= 2xz\, i + (y-2z)\,j + (-z^2-z)\,k.$$

(j) Green's theorem: If $\phi(x,y)$, $\psi(x,y)$, ϕ_y and ψ_x be continuous in a region E of the xy-plane bounded by a closed curve C, then $$\int_C (\phi\, dx + \psi\, dy) = \iint_E \left(\frac{\partial\psi}{\partial x}-\frac{\partial\phi}{\partial y}\right) dx\,dy.$$

②ⓐ Regula falsi method.

Let $f(x) = xe^x - 2$

A root of $f(x)$ is between 0 and 1.

Let $x_0 = 0$, $x_1 = 1$. $x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

Then $x_2 = 0.758$, $x_3 = 0.8395$, $x_4 = 0.8512$, $x_5 = 0.8525$

$x_6 = 0.8526$

② ⓑ Rewriting the given system of equations

$$28x + 4y - z = 32$$
$$2x + 17y + 4z = 35$$
$$x + 3y + 10z = 24$$

Then $x = \frac{1}{28}(32 - 4y + z)$, $y = \frac{1}{17}(35 - 2x - 4z)$, $z = \frac{1}{10}(24 - x - 3y)$

Let $x_0 = y_0 = z_0 = 0$ be an initial approximation

$x_1 = 1.1429$	$y_1 = 1.9244$	$z_1 = 1.7084$
$x_2 = 0.9289$	$y_2 = 1.5476$	$z_2 = 1.8428$
$x_3 = 0.9875$	$y_3 = 1.509$	$z_3 = 1.8485$
$x_4 = 0.9933$	$y_4 = 1.507$	$z_4 = 1.8485$
$x_5 = 0.9935$	$y_5 = 1.507$	$z_5 = 1.8485$
$x_6 = 0.9935$	$y_6 = 1.507$	$z_6 = 1.8485$

③ ⓐ Let $f(x) = 4\sin x - e^x$

$f(0) = -1$, $f(1) = 0.64760$

A root of $f(x)$ is between 0 and 1

using Bisection method $x_2 = 0.5$, $f(x_2) = 0.26898$

$x_3 = 0.375$ $f(x_3) = 0.01009$

$x_4 = 0.3125$ $f(x_4) = -0.13037$

$x_5 = 0.34375$, $f(x_5) =$

$x_6 = 0.35937$, $x_7 = 0.36718$

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③ ⓑ matrix form of the given system is

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 3 & 2 & -2 & -2 \\ 2 & 4 & 1 & 3 \end{array}\right]$$

$$\begin{array}{c} R_2 \to R_2 - 3R_1 \\ \sim \\ R_3 \to R_3 - 2R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 5 & -5 & -20 \\ 0 & 6 & -1 & -9 \end{array}\right]$$

$$\begin{array}{c} R_2 \to R_2/5 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 6 & -1 & -9 \end{array}\right]$$

$$\begin{array}{c} R_3 \to R_3 - 6R_1 \\ \sim \end{array} \left[\begin{array}{ccc|c} 1 & -1 & 1 & 6 \\ 0 & 1 & -1 & -4 \\ 0 & 0 & 5 & 15 \end{array}\right]$$

Then $x_1 - x_2 + x_3 = 6$

$x_2 - x_3 = -4$

$5x_3 = 15$

So, $x_1 = 2$, $x_2 = -1$, $x_3 = 3$

④ⓐ Forward difference table for the given data is

x	θ	$\Delta\theta$	$\Delta^2\theta$	$\Delta^3\theta$
40	184	20	2	0
50	204	22	2	0
60	226	24	2	0
70	250	26	2	
80	276	28		
90	304			

Take $x = x_0 + \theta h$, $x = 43$, $h = 10$, $x_0 = 40$ then $p = 0.3$

By Newton's forward interpolation formula

$$\theta_p = \theta_0 + p\Delta\theta_0 + \frac{p(p-1)}{2!}\Delta^2\theta_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3\theta_0 + \cdots$$

$= 189.79.$

④ⓑ Consider $\frac{dy}{dx} = y, \quad y(0)=1.$

By Picard's method $y_1 = y_0 + \int_0^x y_0\,dx = 1+\int_0^x 1\cdot dx = 1+x$

$$y_2 = y_0 + \int_0^x y_1\,dx = 1+\int_0^x (1+x)\,dx = 1+x+\frac{x^2}{2}$$

$$y_3 = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}$$

$$y_4 = 1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}$$

$$y(0.1) = 1+0.1+\frac{0.1^2}{2!}+\frac{0.1^3}{3!}+\frac{0.1^4}{4!} = 1.10517.$$

⑤ ⓐ Given data

x	5	6	9	11
y	12	13	14	16

using Lagrange's interpolation formula

$$y(x=10) = 12\,\frac{(10-6)(10-9)(10-11)}{(5-6)(5-9)(5-11)} + 13\,\frac{(10-5)(10-9)(10-11)}{(6-5)(6-9)(6-11)} +$$

$$14\,\frac{(10-5)(10-6)(10-11)}{(9-5)(9-6)(9-11)} + 16\,\frac{(10-5)(10-6)(10-9)}{(11-5)(11-6)(11-9)}$$

$$= 2-\frac{13}{3}+\frac{35}{3}+\frac{16}{3} = \frac{42}{3}.$$

5 (b) :

x	4	4.2	4.4	4.6	4.8	5	5.2
$\ln x$	1.3863	1.4351	1.4816	1.5261	1.5686	1.6094	1.6487

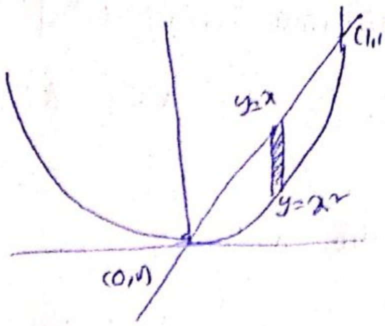
Trapezoidal rule, $\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{h}{2}\left[f(x_0)+f(x_n)+2(f(x_1)+f(x_2)+\cdots+f(x_{n-1})\right]$

$$= 1.8277.$$

Simpson's rule (3/8th), $\int_{x_0}^{x_0+nh} f(x)\,dx = \frac{3h}{8}\left[f(x_0)+f(x_n)+3(f(x_1)+f(x_2)+f(x_4)+f(x_5)+\cdot) + 2(f(x_3)+f(x_6)+\cdots)\right]$

$$= 1.8278$$

⑥ ⓐ Consider $\iint xy(x+y)\,dx\,dy$



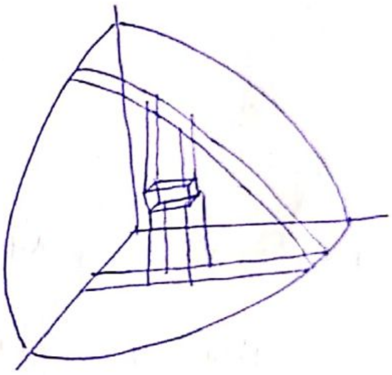
$$= \int_{x=0}^{1}\int_{y=x^2}^{x} (x^2y+xy^2)\,dy\,dx$$

$$= \int_{x=0}^{1} \left(\frac{x^2y^2}{2}+\frac{xy^3}{3}\right)_{y=x^2}^{x} dx$$

$$= \int_{x=0}^{1}\left(\frac{x^4}{2}+\frac{x^4}{3}-\frac{x^6}{2}-\frac{x^7}{3}\right)dx = 3/56$$

⑥ ⓑ Volume of the sphere $x^2+y^2+z^2=a^2$ is

8 × volume of the sphere in the first octant



$$= 8\times \iint_{x^2+y^2\le a^2}\int_{z=0}^{\sqrt{a^2-x^2-y^2}} dz\,dx\,dy$$

$$= 8\iint_{x^2+y^2\le a^2}\sqrt{a^2-x^2-y^2}\,dx\,dy$$

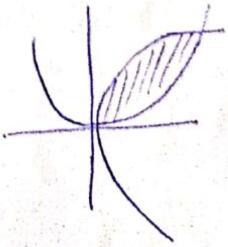
By changing to polar coordinates

$$= 8\int_{\theta=0}^{\pi/2}\int_{r=0}^{a}\sqrt{a^2-r^2}\; r\,dr\,d\theta$$

$$= -\frac{8}{2}\int_{\theta=0}^{\pi/2}\int_{r=0}^{a}(a^2-r^2)^{1/2}(-2r)\,dr\cdot d\theta$$

$$= -4\int_{\theta=0}^{\pi/2}\left(\frac{(a^2-r^2)^{3/2}}{3/2}\right)_{r=0}^{a} d\theta \quad = -\frac{8}{3}\int_{\theta=0}^{\pi/2} -a^3\,d\theta = \boxed{\frac{4}{3}\pi a^3}$$

⑦ ⓐ Area between the parabolas $y^2=4x$ and $x^2=4y$ is



$$\int_{x=0}^{4}\int_{y=x^2/4}^{2\sqrt{x}} dy\,dx = \int_{x=0}^{4}\left(2\sqrt{x}-\frac{x^2}{4}\right)dx$$

$$= \cancel{\frac{8}{4}}\ \frac{16}{3}$$

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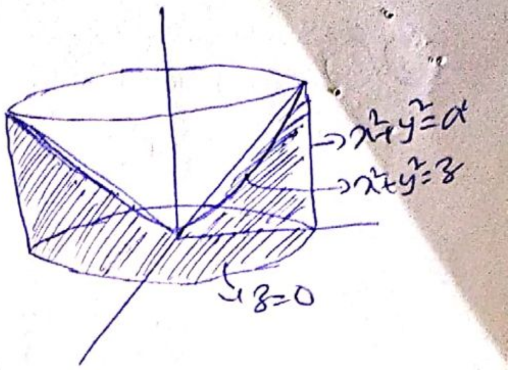
⑦ⓑ Volume bounded by the surfaces $x^2+y^2=a^2$, $x^2+y^2=z$, $z=0$ is shown in the figure.



Consider $\iiint_V z^2\,dx\,dy\,dz = \iint_{x^2+y^2\le a^2} \int_{z=0}^{x^2+y^2} z^2\,dz\,dx\,dy$

$$= \iint_{x^2+y^2\le a^2} \left(\frac{z^3}{3}\right)_{z=0}^{x^2+y^2} dx\,dy$$

$$= \iint_{x^2+y^2\le a^2} \frac{(x^2+y^2)^3}{3}\,dx\,dy$$

$$= \int_{\theta=0}^{2\pi}\int_{r=0}^{a} \frac{r^6}{3}\cdot r\,dr\,d\theta = \frac{1}{3}\times 2\pi\times\frac{a^8}{8} = \frac{\pi a^8}{12}$$

⑧ⓐ Let $f_1 = x^2+y^2+z^2-9$ $\qquad f_2 = x^2+y^2-z-9$

then $\nabla f_1 = 2x\hat{i}+2y\hat{j}+2zk$ $\qquad \nabla f_2 = 2x\hat{i}+2yj-k$

$\nabla f_1)_{(2,-1,2)} = 4\hat{i}-2j+4k$, $\qquad \nabla f_2)_{(2,-1,2)} = 4\hat{i}-2j-k$

Let θ be the angle between the two curves, then

$$\cos\theta = \frac{\nabla f_1\cdot\nabla f_2}{|\nabla f_1||\nabla f_2|} = \frac{(4\hat{i}-2j+4k)\cdot(4\hat{i}-2j-k)}{|4\hat{i}-2j+4k|\,|4\hat{i}-2j-k|}$$

$$= \frac{16+4-4}{6\sqrt{21}} = \frac{8}{3\sqrt{21}}$$

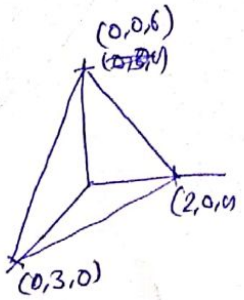
Hence $\theta = \cos^{-1}\left(\frac{8}{3\sqrt{21}}\right)$

(b) By Stoke's theorem, $\int_C F\cdot dR = \iint_S \text{curl}F\cdot N\,ds$.

$F = (x+y)\,i + (2x-z)\,j + (y+z)\,k$

$\text{Curl}F = \nabla\times F = 2\hat{i}+k$

Equation of the plane passing through the points (2,0,0), (0,3,0) and (0,0,6) is $\frac{x}{2}+\frac{y}{3}+\frac{z}{6}=1 \Rightarrow 3x+2y+z-6=0$



$\nabla f = 3i+2j+k$

$N = \frac{3i+2j+k}{\sqrt{14}}$

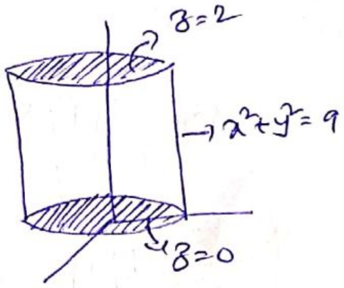
$\int_C F\cdot dR = \iint_S \text{curl}F\cdot N\,ds = \iint_S (2i+k)\cdot \frac{3i+2j+k}{\sqrt{14}}\,ds$

$= \iint_S \frac{7}{\sqrt{14}}\,ds = \frac{7}{\sqrt{14}}$ area of ΔABC.

$= \frac{7}{\sqrt{14}} \times 3\sqrt{14} = \boxed{21}$

⑨ Given $F = yi + xj + z^2k$.

By Gauss Divergence theorem



$\iint_S F\cdot N\,ds = \iiint_V \text{div}F\,dV$

$\text{div}F = 2z$

Then $\iiint_V \text{div}F\,dV = \iint_{x^2+y^2\le 9} \int_{z=0}^{2} 2z\,dz\,dx\,dy$

$= \iint_{x^2+y^2\le 9} 4\,dx\,dy = 4$ (area of the circle $x^2+y^2=9$)

$= \boxed{36\pi}$ —①

(i) For $z=0$, $N=-k$

$F\cdot N = -z^2$

Then $\iint_{S_1} F\cdot N\,ds = 0$ (since $z=0$ on S_1)

(ii) For $z=2$ $n=k$

$F\cdot N = z^2 = 4$

Then $\iint_S F\cdot N\,ds = \iint_{x^2+y^2\le 9} 4\,dx\,dy = 4$ (Area of circle $x^2+y^2=9$)

$= 4(9\pi) = 36\pi$

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(iii) For the cylindrical surface $x^2+y^2=9$

$$\nabla f = 2xi+2yj$$

$$\text{Then } N = \frac{2xi+2yj}{\sqrt{4x^2+4y^2}} = \frac{xi+yj}{3}.$$

$$\iint_S F\cdot N\,ds = \iint_S \cancel{xi+yj}\ \frac{2xy}{3}\cdot\frac{dx\,dz}{|N\cdot j|}$$

$$= \iint_S \frac{2xy}{3}\cdot\frac{dx\,dz}{y/3} = 2\int_{x=-2}^{2} x \int_{z=0}^{2} dz\cdot dx.$$

$$= 2\int_{x=-2}^{2} x\cdot 2\cdot dx$$

$$= 4\left(\frac{x^2}{2}\right)_{-2}^{2} = 0$$

$$\text{Then } \iint_S F\cdot N\,ds = \iint_{S_1} F\cdot N\,ds + \iint_{S_2} F\cdot N\,ds + \iint_{S_3} F\cdot N\,ds$$

$$= 0 + 36\pi + 0$$

$$= 36\pi \quad \text{——} ②$$

From ① and ② $\iiint_V \text{div}F\,dV = \iint_S F\cdot N\,ds$.

Hence ~~s~~ Gauss Divergence theorem verified.

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