12/09 UNIT - 3

Fuzzy logic

→ Fuzzy means uncertain (or) Vaguness (or) Incomplete.

→ Fuzzy logic controller is an excellent tool to deal with uncertainity.

→ The uncertainity arraised in the problem because of the following reasons.

→ Due to incomplete or Vague data.

→ Unreliable data / Uncertain data.

→ Inherent impreossion in the language.

→ Conflict of Information.

→ To deal with the uncertainity in the problem, probability theory is used before introduction of Fuzzy logic.

→ But the main drawback of Probability theory is it will solve only problems which are characterised with the random processes.

→ But the real world problems are characterized with both random & Non-random processes.

→ F.L (Fuzzy logic) is developed in the year 1965 by Lotfi-A-Zadeh to deal with the uncertain problems, whose processor are characterized by Random & Non-Random Nature of.

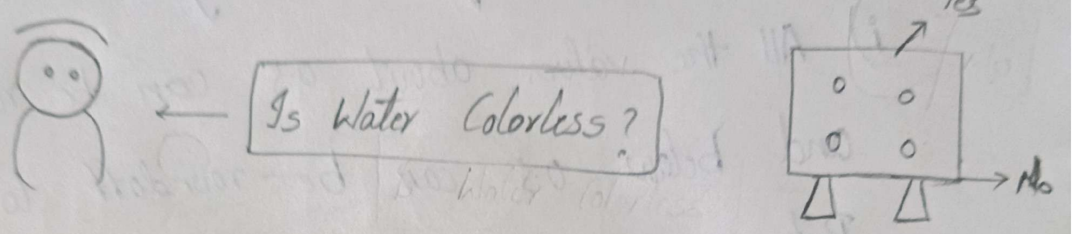
→ The fuzzy logic, is the extension of the Crisp logic.
Classical / linguistic

Crisp Logic Vs Fuzzy logic :-

1. Explain fuzzy versus crisp set operations with an example.

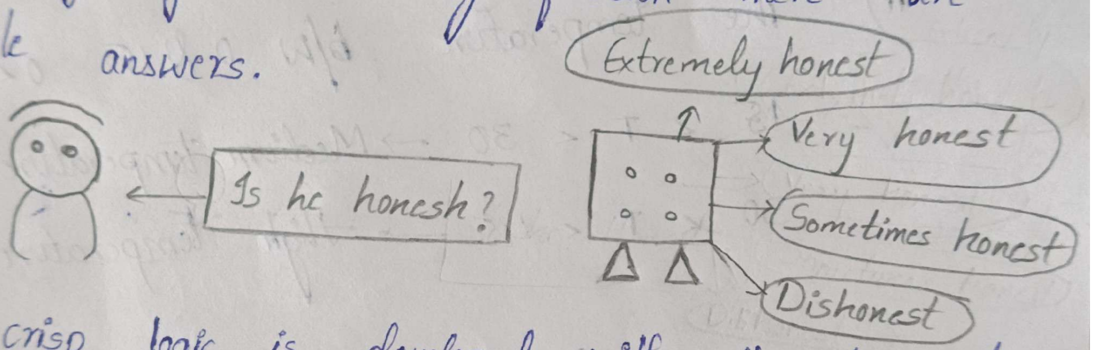
→ In the crisp logic for every question the answer will be either 0/1 or T/F or Yes/No.

Eg:

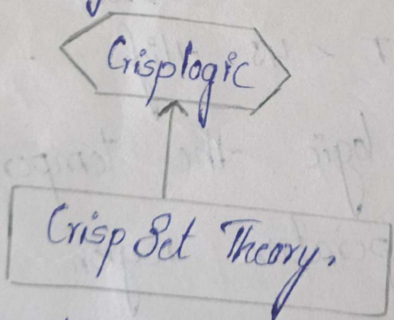


→ In fuzzy logic for every question there are multiple answers.

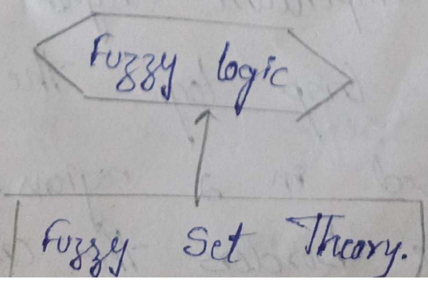
Eg:



→ The crisp logic is developed with the help of crisp set theory.



→ Fuzzy logic developed with the help of Fuzzy set theory.



→ A crisp set is the well defined collection of elements.

→ A fuzzy set theory is also well defined collection of elements but each element is represented with Truth value / Membership value.

→ The crisp set theory is failing for the following.

i) All the values about 0.5 can be round off to 1, and below 0.5 can be roundoff to 0.
But 0.5 can't be roundoff to 1/0.

ii) The temperature b/w 0 & 15 $0 < T < 15$ Low
$15 < T < 30$ → Medium temperature Region
$30 < T < 45$ → High temperature Region.

$0 < T < 15$ Low
$15 < T < 30$ Medium
$30 < T < 45$ High

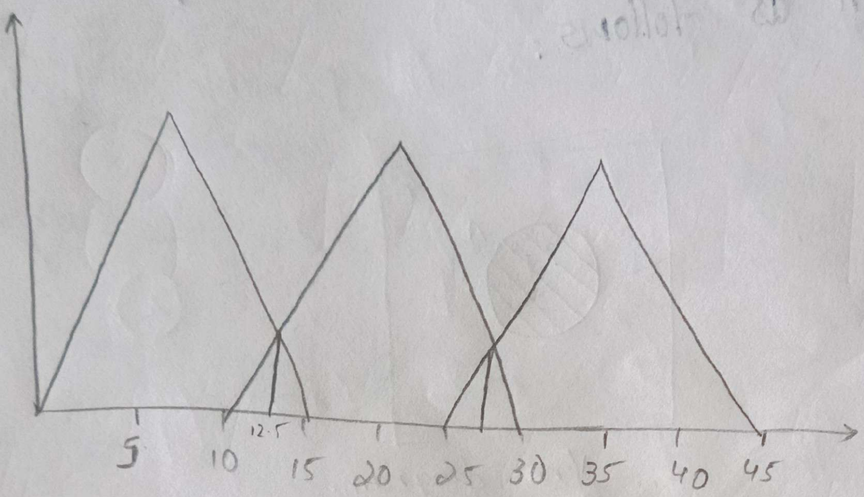
→ As per crisp logic the temperature 14.999 is ≃ 15 is in low temperature region.

→ The temperature 15.001 is ≅ 15 is in the medium temperature region.

→ So as per Crisp logic the same temperature is 15 is discussed in 2 regions.

→ So at the borders the crisp logic is completely failing to provide solution.

→ For such type of Vague problems solution is provided by Fuzzy logic with the help of Membership function as given below.



→ In the above figure 14.99 comes under Medium region & also 15.001 comes under Medium region. Like this fuzzy logic provides solution to the uncertain problems.

Crisp Set theory

What is crisp set? Explain the operations and properties of crisp

i) Universal Set :-

It is defined as collection of All the possible elements withrespect to particular context.

Eg:- All the real Numbers, All students in a university, All freedom fighters, All humans, All birds, All gods.

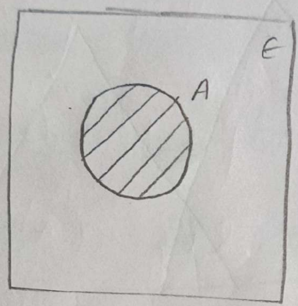
Set :-

Set is the well defined collection of Elements.

A = {1, 2, 3, 4}   B = { Gandhi, Nehru, Bose }

ii) Venn Diagram :

It is Pictorial representation of the set. Let us consider a set A in the universe of the discourse (set) $\mathcal{E}$, the venn diagram will be drawn as follows :



Venn diagram.

Define the following with example

i) Membership
ii) Power set
iii) Super set
iv) Cardinality

iii) Membership :-

An element x is said to be member of set A if set A contains the element x.

Eg:- $A = \{1, 2, 3, 4\}$ $B = \{a, b, c, d\}$

3 is the member of set A is denoted as $3 \in A$.

a is not the member of set A but member of set B. $a \notin A$
$a \in B$.

iv) Cardinality :-

The no: of elements in a set is called as Cardinality.

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Eg:- Set A $\longrightarrow$ Cardinality is denoted as

$n(A)$, $|A|$, $\# A$.

$A = \{1, 2, 3, 4, 5\}$

$|A| = 5$

$B = \{a, b, c\}$

$|B| = 3$

v) Family of Sets :-

The elements of a sets is again the sets the particular set is said to be family of sets.

Eg:- $A = \{\{1,2\}, \{2,3\}, \{3,4\}, \{4,5\}\}$.

$B = \{\{a, b, c\}, \{d, e, f\}\}$

vi) Singleton Set :-

A singleton set is one which contains only one element (or) The set whose cardinality is one is also called as Singleton set.

Eg:- $A = \{1\}$ $B = \{c\}$.

vii) Null Set :-

$\longrightarrow$ A Null set is one which contains zero elements or the set having zero cardinality.

$\longrightarrow$ It is denoted with ϕ or $\{\}$.

Eg:- $A = \{\}$ $B = \phi$

viii) SubSet & Superset :-

Let us consider two sets A & B, if all

the elements of Set B is in Set A, then B is said to be Subset of A, and A is said to be superset of B.

ix) Power Set :-

It is defined as All possible subsets of a given set including null set.

Eg:- ~~All~~ A = {1, 2} ⟶ 2

P(A) = {1, 2, (1, 2), ϕ} ⟶ 4

The no:- of elements in a power set is $2^{\text{cardinality}(A)}$

B = {1, 2, 3} ⟶ 3

P(B) = {1, 2, 3, {1, 2}, {2, 3}, {3, 1}, {1, 2, 3}, ϕ} ⟶ 8

Crisp Set Operations / Operations on Crisp Set

i) Union :-

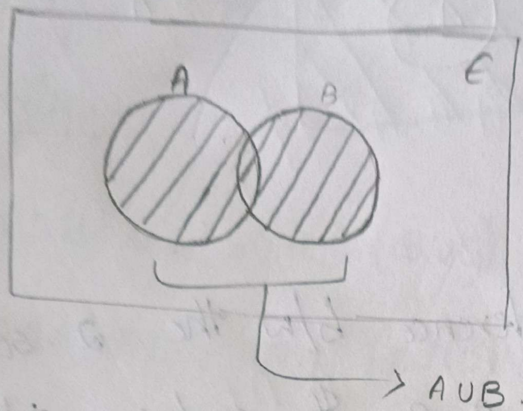
The union of two sets A & B is denoted with A ∪ B; which contains the elements belongs to set A & set B.

It is also defined as the set of elements from A & B without repeatition.

Eg:- $A = \{1,2,3,a,b,c\}$ $B = \{4,5,6,a,b,c\}$

$A \cup B = \{1,2,3,a,b,c,4,5,6\}$.

Venn diagram



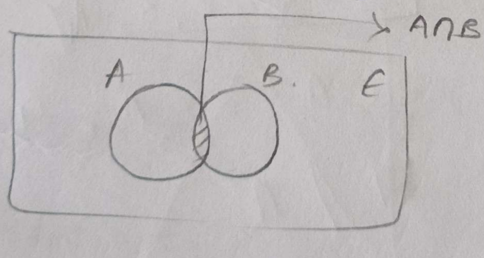
ii) Intersections :-

The intersection of 2 sets A & B contains all the elements that are common in A & B.

It is also defined as set of elements common in both the sets A & B.

$A = \{1,2,3,a,b,c\}$ $B = \{4,5,6,a,b,c\}$

$A \cap B = \{a,b,c\}$.

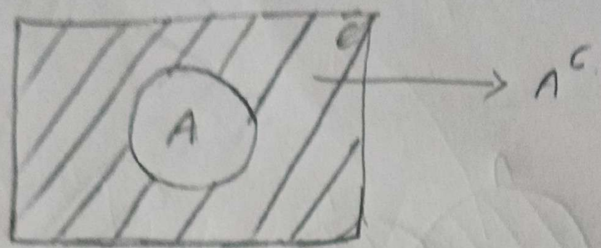


iii) Complement :-

The complement of a given set A is denoted as A^C. It is a set of all the elements in the universal set except the elements in set A.

Eg:- $\xi = \{1, 2, 3 \ldots n\}$

$A = \{2, 3\}$ $A^c = \{1, 4, 5, 6 \ldots n\}$
Except 2, 3.

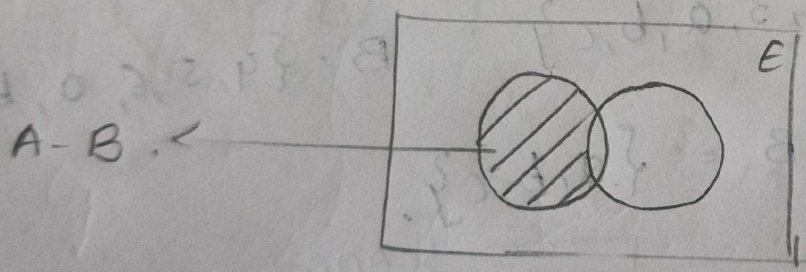


iv) Difference :-

The difference b/w the 2 sets A & B, is denoted as A - B. It contains all the elements of A except the elements of B.

Eg:- $A = \{1, 2, 3, a, b, c\}$ $B = \{a, b, c\}$

$A - B = \{1, 2, 3\}$



Properties of Crisp Set

Commutativity :-

$$A \cup B = B \cup A$$
$$A \cap B = B \cap A$$

Associativity :-

$$(A \cup B) \cup C = A \cup (B \cup C)$$
$$(A \cap B) \cap C = A \cap (B \cap C)$$

Distributivity

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$
$$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$$

Idempodent :-

$$A \cup A = A$$
$$A \cap A = A$$

Identity :-

$$A \cup \phi = A$$
$$A \cap E = E.$$

Law of Absorption :-

$$A \cup (A \cap B) = A$$
$$A \cap (A \cup B) = A$$

Transitivity :- If $A \le B$, $B \le C$ then $A \le C$

Involution :-

$$(A^c)^c = A$$

Law of Contradiction :-

$$A \cap A^c = \phi$$

De-Morgan's Law :-

$$(A \cup B)^c = A^c \cap B^c$$

$$(A \cap B)^c = A^c \cup B^c$$

Fuzzy Set Theory :-

→ A crisp set is defined as well-defined collection of elements.

Eg:- Set $A = \{1, 2, 3, 4\}$

$1 \in A$, $2 \in A$, $3 \in A$, $4 \in A$

$7 \notin A$.

→ In a crisp set theory we can able to tell whether the element belongs to set or not belongs to set.

→ In crisp set theory it is not possible to say how much degree of truth the element is belongs to set.

Fuzzy Set :

Define Fuzzy set?

→ Fuzzy set is also well defined collection of elements but each element is represented with degree of truth or membership value.

→ Degree of truth or membership value is denoted with μ and varies from 0 to 1

→ A singlon (term) fuzzy set is defined as follows.

$$\hat{A} = \{(x, \mu_{\hat{A}}(x))\} \qquad \hat{A} = \left\{\frac{x}{\mu_{\hat{A}}(x)}\right\}$$

Element, Truth value.

$$\hat{A} = \left\{\frac{\mu_{\hat{A}}(x)}{x}\right\}$$

Eg: $\hat{A} = \{(x, 0.4)\}$

→ A multi element fuzzy set is as given below:

$$\hat{A} = \{(x_1, \mu_{\hat{A}}(x_1)) (x_2, \mu_{\hat{A}}(x_2)) (x_3, \mu_{\hat{A}}(x_3)) (x_4, \mu_{\hat{A}}(x_4))\}$$

Eg:-

$$\hat{A} = \{(x_1, 0.4) (x_2, 0.6) (x_3, 0.8) (x_4, 1)\}$$

$$\hat{B} = \{(x_1, 0.1) (x_2, 0.2) (x_3, 0.3)\}$$

$$\hat{C} = \{(a, 0.1), (b, 0.5) (c, 0.7)\}$$

Membership functions:-

Explain in brief about Member ship function of Fuzzy set?

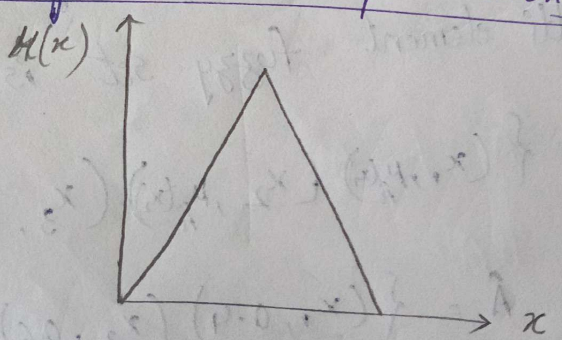
→ A fuzzy logic controller can only understands fuzzy membership functions. Before giving i/p to the fuzzy logic controller crisp logic must be converted into fuzzy membership functions. The Input as well the o/p of fuzzy logic controller is fuzzy membership functions. There are 2 types of functions. They are:-

→ Discrete Membership functions

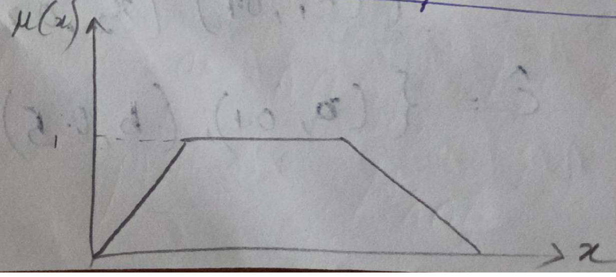
→ Continuous Membership functions.

→ The following are the different continuous membership functions.

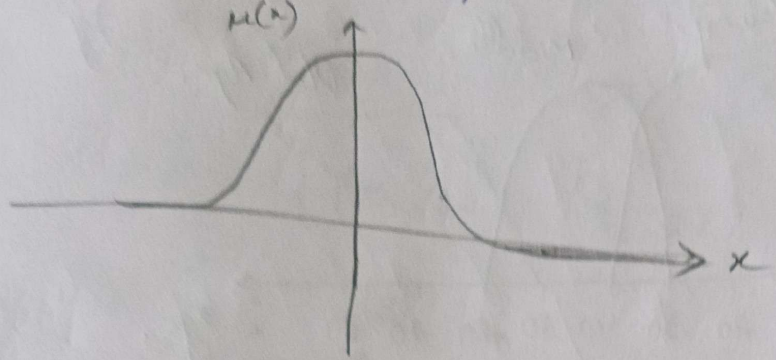
i) Triangular membership function:-



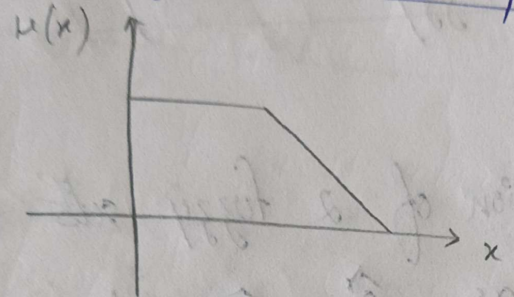
ii) Trapezoidal Membership function:-



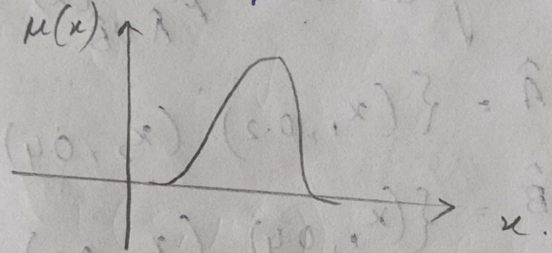
iii) <u>Continuous Membership function</u>



iv) <u>Half Trapezoidal Membership function:-</u>



v) Continuous Membership function in the +ve side only



Let us consider the following Example:-

Let us consider the age b/w 0 to 40 years is considered as Young age. and 40 to 60 years is the middle age. and more than 60 years the age is said to be old age.

$0 < age < 40$ Young

$40 < age < 60$ Middle

$age > 60$ Old.

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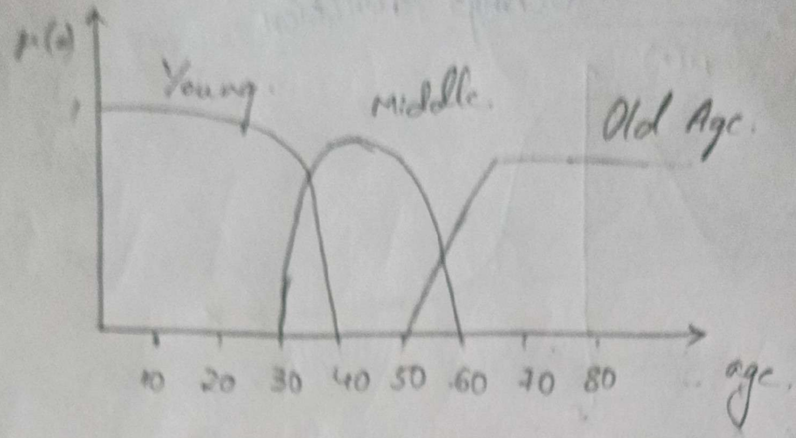


Operators of Fuzzy Set Theory

i) **Union (∪)** :-

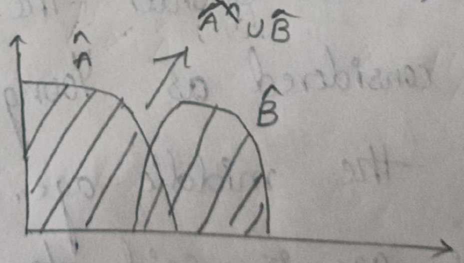
The union of 2 fuzzy sets $\hat{A}$ & $\hat{B}$ is denoted as $\hat{A} \cup \hat{B}$ whose membership value is given as $\mu_{\hat{A} \cup \hat{B}}(x) = \max(\mu_{\hat{A}}(x), \mu_{\hat{B}}(x))$

Eg:- $\hat{A} = \{(x_1, 0.2)\ (x_2, 0.4), (x_3, 0.6)\}$

$\hat{B} = \{(x_1, 0.4), (x_2, 0.2), (x_3, 0.7)\}$

$\hat{A} \cup \hat{B} = \{(x_1, 0.4), (x_2, 0.4), (x_3, 0.7)\}$

~~Intersection (∩)~~ :-



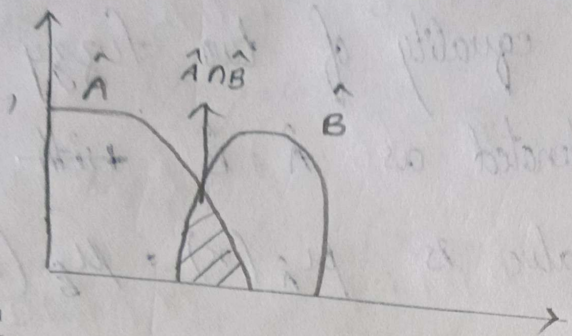
ii) **Intersection (∩)** :-

The intersection of two fuzzy sets $\hat{A}$ & $\hat{B}$ is denoted as $\hat{A} \cap \hat{B}$ whose ~~members~~ membership value is given as $\mu_{\hat{A} \cap \hat{B}}(x) = \min(\mu_{\hat{A}}(x), \mu_{\hat{B}}(x))$

$\hat{A} = \{(x_1, 0.2), (x_2, 0.4) (x_3, 0.6)\}$

$\hat{B} = \{(x_1, 0.4) (x_2, 0.2) (x_3, 0.7)\}$

$\hat{A} \cap \hat{B} = \{(x_1, 0.2) (x_2, 0.2) (x_3, 0.6)\}$

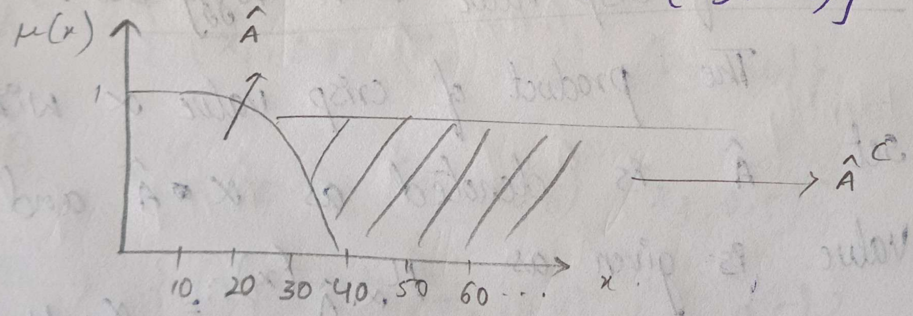


iii) <u>Complement</u> :-

The complement of set $\hat{A}$ is denoted as $\hat{A}^c$ whose membership function is given as $\mu_{\hat{A}^c}(x) = 1 - \mu_{\hat{A}}(x)$.

$\hat{A} = \{(x_1, 0.2) (x_2, 0.4) (x_3, 0.6)\}$

$\hat{A}^c = \{(x_1, 0.8) (x_2, 0.6) (x_3, 0.4)\}$



iv) <u>Product of two fuzzy Sets</u> :-

The product of 2 fuzzy sets $\hat{A}$ & $\hat{B}$ is denoted with $\hat{A} \cdot \hat{B}$ whose membership value is given as $\mu_{\hat{A} \cdot \hat{B}}(x) = \mu_{\hat{A}}(x) * \mu_{\hat{B}}(x)$.

$\hat{A} = \{(x_1, 0.2) (x_2, 0.4) (x_3, 0.6)\}$

$\hat{B} = \{(x_1, 0.1) (x_2, 0.2) (x_3, 0.3)\}$

$\hat{A} \cdot \hat{B} = \{(x_1, 0.02)\ (x_2, 0.08)\ (x_3, 0.18)\}$

v) Equality of Fuzzy Set :

The equality of two fuzzy sets of $\hat{A}$ & $\hat{B}$ is denoted as $\hat{A} = \hat{B}$ ~~with~~ then the membership value is $\mu_{\hat{A}}(x) = \mu_{\hat{B}}(x)$

$$\hat{A} = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.6)\}$$

$$\hat{B} = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.6)\}$$

$$\boxed{\hat{A} = \hat{B}}$$

vi) Product of Crisp value with fuzzy Set :

The product of crisp value α with fuzzy set $\hat{A}$ is denoted as $\alpha \cdot \hat{A}$ and membership value is given as $\mu_{\alpha.\hat{A}}(x) = \alpha \cdot \mu_{\hat{A}}(x)$

$\alpha = 0.2$, $\hat{A} = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.6)\}$

$$\alpha \cdot \hat{A} = \{(x_1, 0.04)\ (x_2, 0.08)\ (x_3, 0.12)\}$$

vii) Difference b/w 2 fuzzy Sets :

The difference b/w the two fuzzy sets $\hat{A}$ & $\hat{B}$ is denoted as $\hat{A} - \hat{B}$ and is calculated

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using formula $\hat{A} \cap \hat{B}^c$

$\hat{A} = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.6)\}$

$\hat{B} = \{(x_1, 0.3)\ (x_2, 0.2)\ (x_3, 0.5)\}$

$\hat{B}^c = \{(x_1, 0.7)\ (x_2, 0.8)\ (x_3, 0.5)\}$

$\hat{A} - \hat{B} = \hat{A} \cap \hat{B}^c = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.5)\}.$

viii) Disjunctive Sum of two fuzzy Sets :

The disjunctive sum of two fuzzy sets can be represented as $\hat{A} \odot \hat{B}$ and is calculated using formula $(\hat{A}^c \cap \hat{B}) \cup (\hat{A} \cap \hat{B}^c)$

$\hat{A} = \{(x_1, 0.2)\ (x_2, 0.4)\ (x_3, 0.6)\}$

$\hat{A}^c = \{(x_1, 0.8)\ (x_2, 0.6)\ (x_3, 0.4)\}$

$\hat{B} = \{(x_1, 0.1)\ (x_2, 0.6)\ (x_3, 0.5)\}$

$\hat{B}^c = \{(x_1, 0.9)\ (x_2, 0.4)\ (x_3, 0.5)\}$

$\hat{A}^c \cap \hat{B} \Rightarrow \{(x_1, 0.2)\ (x_2, 0.6)\ (x_3, 0.4)\}$

$\hat{A} \cap \hat{B}^c \Rightarrow \{(x_1, 0.1)\ (x_2, 0.4)\ (x_3, 0.5)\}$

$(\hat{A}^c \cap \hat{B}) \cup (\hat{A} \cap \hat{B}^c) \Rightarrow \{(x_1, 0.2)\ (x_2, 0.6)\ (x_3, 0.5)\}.$

Properties of Fuzzy Sets :-

Commutativity :- $\hat{A} \cup \hat{B} = \hat{B} \cup \hat{A}$

$\hat{A} \cap \hat{B} = \hat{B} \cap \hat{A}$

Associativity :- $\hat{A} \cup (\hat{B} \cup \hat{C}) = (\hat{A} \cup \hat{B}) \cup \hat{C}$

$\hat{A} \cap (\hat{B} \cap \hat{C}) = (\hat{A} \cap \hat{B}) \cap \hat{C}$

Distributivity :-

$\hat{A} \cup (\hat{B} \cap \hat{C}) = (\hat{A} \cup \hat{B}) \cap (\hat{A} \cup \hat{C})$

$\hat{A} \cap (\hat{B} \cup \hat{C}) = (\hat{A} \cap \hat{B}) \cup (\hat{A} \cap \hat{C})$

Idempodent :-

$\hat{A} \cup \hat{A} = \hat{A}$

$\hat{A} \cap \hat{A} = \hat{A}$

Identity :-

$\hat{A} \cup \phi = \hat{A}$ $\quad$ $\hat{A} \cap \phi = \phi$

$\hat{A} \cup X = \hat{A}$ $\quad$ $\hat{A} \cap X = X$

Transitivity :- If $\hat{A} \leq \hat{B}$, $\hat{B} \leq \hat{C}$ then $\hat{A} \leq \hat{C}$

Involution :- $(\hat{A}^{c})^{c} = \hat{A}$

Demorgan's Law :- $\hat{A} \cap \hat{B} = \hat{A}^{c} \cup \hat{B}^{c}$

$\hat{A} \cup \hat{B} = \hat{A}^{c} \cap \hat{B}^{c}$