

31/10/22  
Monday

## UNIT-2

### Regular Language and Regular Expression :-

Language is said to be a regular if there exist a finite automaton for it.

Every regular language is described by a DFA and NFA.

The language accepted by a finite automata can be easily described by simple expressions called regular expression.

Regular expression is most efficient way to represent any language.

Language accepted by regular expression is known as regular language.

### Regular Expression :-

One of the way to describe the regular language is regular expression i.e. an algebraic description of notation describes exactly same language as finite automata.

Regular Expression serves as input language for many systems that crosses the string.

### Applications of Regular Expressions :-

search commands such as

1. Unix, Grep

2. Lexical analysis generators such as lex & flex.

## Formal definition of Regular Expression :-

Regular Expression defines any Regular language which is defined by finite automata.

Regular Expression involves combination of strings of symbols from some alphabets  $\Sigma$ ,  $($ ,  $)$ , and the operators  $+$  and  $*$  and  $\cdot$ .

Where  $+$  is used to denote union operation.

$\cdot$  is used to denote concatenation operation.

$*$  is used to denote Kleene closure

### NOTE

1.  $\emptyset$  is a regular expression denoting the empty set.

ie  $\{\}$

2.  $\epsilon/\lambda$  is regular expression denoting  $\{\epsilon\}$

(epsilon or lambda)

3.  $\forall a \in \Sigma$ ,  $a$  is regular expression denoting language  $\{a\}$

### Operations on Regular Languages :-

There are '3' basic languages operations on Regular languages.

1. Union

2. Concatenation

3. Kleene closure



(1). Union:- Union of two languages  $L$  and  $M$  is  $L \cup M$ , which is the set of strings that are either in  $L$  or  $M$  or both.

If  $L = \{0, 01, 110\}$  and

$M = \{1, 11, 01\}$ .

$L \cup M = \{0, 1, 01, 11, 110\}$

(2). Concatenation:- Concatenation of two languages  $L$  and  $M$  is set of strings that can be formed by taking any string in  $L$  and concatenating it with any string in  $M$ .

To denote the concatenation of languages  $L$  and  $M$  either with operator  $\cdot$  or no operator at all.

i.e.  $L.M$  or  $LM$

Example:-

If  $L = \{01, 11, 110\}$  and

$M = \{\epsilon, 1, 101\}$

then,  $L.M = \{01, 11, 110, 011, 111, 1101, 01101, 11101, 110101\}$

(3). Kleene Closure:- Kleene closure of Language is denoted by  $L^*$ . And it represents a set of strings that can be formed by taking any no. of strings from  $L$ .

$$L^* = \bigcup_{i \geq 0} L^i = L^0 \cup L^1 \cup L^2 \cup \dots$$

$$L^* = L^0 \cup L^1 \cup L^2 \cup \dots$$

Example:- If  $L = \{0, 1\}$  then

$L^0 = \{\epsilon\}$ ,  $L^1 = \{0, 1\}$ ,  $L^2 = \{00, 11\}$   
 $L^3 = \{000, 011, 110, 111\}$

$$L = \{0, 1\}$$

$$L^0 = \{\epsilon\}$$

$$L^1 = \{0, 1\}$$

$$L^2 = L \cdot L = \{0, 1\} \cdot \{0, 1\} = \{00, 01, 10, 11\}$$

$$L^3 = L^2 \cdot L = \{00, 01, 10, 11\} \cdot \{0, 1\}$$

$$= \{000, 001, 010, 011, 100, 101, 110, 111\}$$

NOTE :-

Clouse of language  $\phi$  is denoted by ~~any language~~

$$\phi^* = \{\epsilon\}$$

clouse of null string & empty string is denoted by

$$\epsilon^* = \epsilon$$

Identity Rules :-

Let  $Q, P, R$  be a regular expressions then the

identity rules are as follows:

$$1. \epsilon \cdot R = R \cdot \epsilon = R$$

$$2. \epsilon^* = \epsilon$$

$$3. \phi^* = \epsilon$$

$$4. \phi \cdot R = R \cdot \phi = \phi$$

$$5. \phi + R = R + \phi = R = R + R$$

$$\boxed{\epsilon + R = R + \epsilon = R + \epsilon}$$

$$6. R \cdot R^* = R^+ \cdot R = R^+$$



$$7. (R^+)^+ = E + R \cdot R^+ \quad \text{--- } R \cdot R^+ \cdot R^+ \text{ ---}$$

$$= \cancel{E + R^+} (E + R^+)$$

$$= R^+$$

$$8. (E+R)^+ = R^+$$

$$9. R^+ (E+R) = (E+R) R^+ = R^+$$

$$10. (PQ)^+ \cdot P = P(QP)^+$$

$$11. (P^+ Q^+)^+ = (P+Q)^+$$

$$12. R + R R^+ = R^+ R$$

$$(E+R)^+ = R^+$$

(i) Prove the expression  $E+1 + (E+1)^+ (E+1)^+ (E+1)^+ = 1^+$

Sol. Take L.H.S

$$= E+1 + (E+1) (E+1)^+ (E+1)$$

$$= (E+1) [E + (E+1)^+ (E+1)]$$

$$= (E+1) (E+1)^+$$

$$= (E+1)^+$$

$$= 1^+ //$$

$$E+R \cdot R^+ = R^+$$

$$R \cdot R^+ = R^+$$

$$\therefore R \cdot R^+ = R^+$$

$$\therefore (E+R)^+ = R^+$$

$$\therefore [E+R^+ R = R^+]$$

$$R \cdot R \cdot R^+ = R \cdot R$$

$$(P^+ Q)^+ = (PQ)^+$$

$$(P^+ Q)^+ = R^+$$

$$(P^+ Q)^+ = P^+$$

$$R^+ (E+1) =$$

$$(E+1) R^+ =$$

$$E \cdot R = R \cdot E = E$$

$$R \cdot R^+ = R^+$$

$$E^+ \cdot R = R \cdot E^+ = R$$

$$R \cdot E^+ = R$$

$$R \cdot R^+ = R^+$$

$$R^+ \cdot R = R^+$$

$$(PQ)^+ \cdot P =$$

$$P(QP)^+$$

2/11/22  
Wednesday

Q. prove that  $(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*$

$$(0 + 10^*1) =$$

$$0^*1(0 + 10^*1)^*$$

take L.H.S:-

$$(1 + 00^*1) + (1 + 00^*1)(0 + 10^*1)^*(0 + 10^*1)$$

$$(1 + 00^*1)[\epsilon + (0 + 10^*1)^*(0 + 10^*1)]$$

$$00^* = 0^+$$

$$(1 + 0^+1)[\epsilon + (0 + 10^*1)^*(0 + 10^*1)]$$

$$(1 + 00^*1)[\epsilon + (0 + 10^*1)^*(0 + 10^*1)]$$

$$\therefore \epsilon + R.R^* = R^+$$

$$(1 + 00^*1)(0 + 10^*1)^*$$

$$0^+(0 + 10^*1)^*$$

$$0^+1(0 + 10^*1)^*$$

$$0^*1(0 + 10^*1)^*$$

Construct a regular expression from given step.

1. Write a regular expression for language accepting

any combination of a's except null string over

$$\Sigma = a.$$

2. write a regular expression for language accepting



all combination of a except null string  
over  $\Sigma = \{a\}$

Ans:-  $R.E = a^+$

3. Write a regular expression for language accepting any combination of a and b over

Ans:-

Examples:- 1. Construct R.E for language contains all strings having any no. of a's and b's except null string.

Ans:-  $R.E = (a+b)^+$

2. Construct R.E for language contains all strings any no. of a's followed by any no. of b's

Ans:-  $R.E = a^*b^*$

3. Construct R.E language accepts all strings ending with 00 over  $\Sigma = \{0,1\}$

Ans:-  $(0+1)^*00$

4. Construct R.E language accepts all strings that contains 00 as substring, over  $\Sigma = \{0,1\}$

Ans:-  $(0+1)^*00(0+1)^*$

5. Construct R.E language accepts all strings starts with a and ends with b over the alphabet

Ans:-  $a(a+b)^*b$

6- construct R.E language accept all strings

with atleast 2 b's over  $\Sigma = \{a, b\}$

Ans:-  $(a+b)^* b (a+b)^* b (a+b)^*$

7- construct R.E language accept all strings with exact

2 b's over  $\Sigma = \{a, b\}$

Ans:-  $a^* b a^* b a^*$

8- Construct R.E language accept all strings,

which contains 2 consecutive b's

Ans:-  $(a+b)^* b b (a+b)^*$

9- construct R.E language which accept atleast

All the strings which contains one a followed by atleast one b followed by atleast one c.

Ans:-  ~~$a^* b^* c^*$~~   $a^+ b^+ c^+$

10- construct R.E language which accept all strings

in which <sup>3rd</sup> symbol from right ~~end~~ is b over  $\Sigma = \{a, b\}$ .

Ans:-  $(a+b)^* b (a+b)(a+b)^*$



11. write R.E language, which accept all the strings and which begin and end with either 00 or 11 over  $\Sigma = \{0, 1\}$

Sol: R.E can be divided into two subparts.

1.  $L_1$  denotes strings which begin with 00 or 11.
2.  $L_2$  denotes strings which ends with 00 or 11.

R.E for  $L_1$  is  $(00+11)(0+1)^*$

R.E for  $L_2$  is  $(0+1)^*(00+11)$

$\therefore (00+11)(0+1)^* + (0+1)^*(00+11)$

Conversion from DFA to R.E :-

Theorem:-

If  $L = L(A)$  for some DFA, A. then there is some regular expression  $R$  such that  $L = L(R)$ .

$\therefore A = (Q, \Sigma, \delta, q_0, f)$

Proof:-

Basics:-

R.E  $R_{ij}^{(k)}$  is a R.E whose language is set of strings  $w$  such that,  $w$  is label of the path from state  $i$  to  $j$  in  $A$ .

And path has no intermediate node whose number greater than  $k$ .

To construct an expression  $R_{ij}^{(k)}$  starting at  $k=0$  and finally reaching at  $k=n$ .

Where  $n$  indicates no. of states.

For  $k=0$  there are only two kinds of paths that meet the condition.

1. An arc from state  $i$  to state  $j$ .
2. Path of length 0 that consists of only some node

'i'.  $i \neq j$

then

(a). If there is no symbol,  $R_{ij}^{(0)} = \phi$

(b). If there is exactly one symbol: i.e. 'a' then

$$R_{ij}^{(0)} = a.$$

(c). If there are symbols  $a_1, a_2, \dots, a_k$  then

$$R_{ij}^{(0)} = a_1 + a_2 + \dots + a_k.$$

3. A path of length 0 that consists only some node

'i'.  $i = j$

(a). ~~Take~~ If there is no symbol then  $R_{ij}^{(0)} = \phi + \epsilon = \epsilon$ .

(b). If there is exactly one symbol i.e. 'a' then

$$R_{ij}^{(0)} = \epsilon + a$$



$k=0$

(0). If there are symbols  $a_1, a_2, \dots, a_k$  then

$$R_{ij}^{(0)} = \epsilon + a_1 + a_2 + \dots + a_k.$$

### Induction:

Suppose there is a path from state 'i' to state 'j' that goes to no state higher than  $k$ .

There are two possible cases to consider.

(1). The path that doesn't go through the state  $k$  at all. Then the label of the path is  $R_{ij}^{(k-1)}$ .

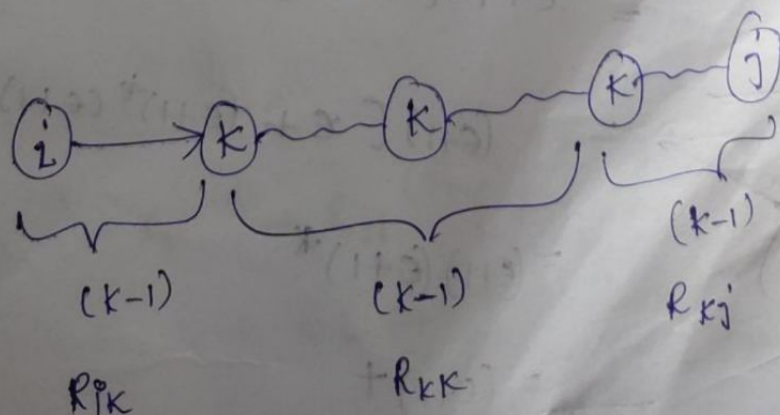
(2). The path goes to the state 'k' at least once then we can break the path into several pieces.

To combine expressions of path is as follows:

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)}$$

$$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} \cdot (R_{kk}^{(k-1)})^* \cdot R_{kj}^{(k-1)}$$

~~$R_{ij}^{(k)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} + R_{ik}^{(k-1)} \cdot R_{kk}^{(k-1)} + R_{ik}^{(k-1)} \cdot R_{kk}^{(k-1)} \cdot R_{kj}^{(k-1)} + \dots$~~

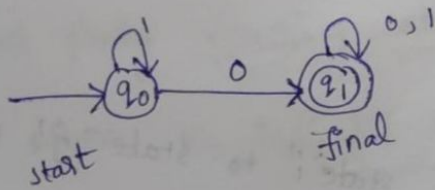


Consider the following sequence

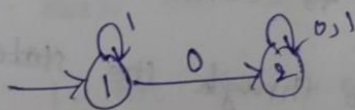
$1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, 10946, 17711, 28657, 46368, 75025, 121393, 196418, 317811, 514229, 832040, 1346269, 2178309, 3524578, 5702887, 9227465, 14930352, 24157817, 39088169, 63245986, 102334155, 165580141, 267914296, 433494441, 701408737, 1134903178, 1836311925, 2971215053, 4807526981, 7778732050, 12586269025, 20365598583, 32951280097, 53317065922, 86267571272, 139583544755, 225851433717, 365435296163, 591286729879, 956733219056, 1558014150135, 2514752137761, 4072766386896, 6587480514641, 10658536376501, 17241363537816, 27905691984617, 45156838262133, 73090791596544, 118238142443961, 191349044481395, 309587186934936, 500936227116431, 810465271980608, 1311962499106539, 2122378160062144, 3434340659272683, 5556718819334827, 9009059478536971, 14561308147811798, 23570358616348769, 38131666764160567, 61702024911972365, 100073376624019842, 161775401535992209, 261848778159912051, 423624179695904360, 685400580855816411, 1109024760015718462, 1794425340871634873, 2903449100927353284, 4697874441799074155, 7601399542720708027, 12295273954680862182, 19896673496400936209, 32191947451081644181, 52088620947482580390, 84280568400184224571, 136369188347666804761, 220649709295149385151, 357018897642836189912, 577668606937985575063, 934687396580824964975, 1512356003518764154136, 2447043399456649729111, 3959429402975473884086, 6406472802434238603197, 10365902205413712487283, 16772375007848151190480, 27138277213261863677673, 43910652221100015167863, 71048927234368166358343, 114959579455469981526216, 186008506679770196684079, 300968086135140158210322, 486976592814910354894301, 787944678950050513104623, 1274921271084960671314924, 2062895949935110825419547, 3337817221019071338524471, 5399713170954182159839418, 8737529891973193498363889, 14137243062927375657503307, 22874762954881558055867196, 37011906016808933654171083, 59849149079730509311675279, 96861055096638442965836362, 156700204176368952277507641, 253549259276100462189182903, 410250263452469405144019265, 663799522628570357421527168, 1074049786080939762565546933, 1737849048709509119686574091, 2811898834788448882208101024, 4549747883497957991773675115, 7361646718207467103981776139, 11911394601697425095655451254, 19273041485195382297637226373, 31184436186892807391618997627, 50457477672088189489256224000, 81631518858280611780874195177, 132088996430368801270130421181, 213720515288649413059404616180, 345808414149018214839535037361, 559528929429667627898939653541, 905337343578686041938474690722, 1464866272908353669837414344303, 2370203616487040291775988934844, 3835070889395626331613403325565, 6205274505882666521489392260309, 10035345395278292853102795585874, 16240619901160959374588187846173, 26275965296439252227690983431987, 42516585197600211599279171278160, 68792550493769463826870154710147, 111309135691369675426068126008311, 180091686185139089252938280718458, 291390821876508764678996406726569, 471482508061647853931934587434927, 762873329938156618608030994161486, 1234355837999784472540065581600413, 1997229167937941331151996575735340, 3231585005937725803691962157335753, 5228814173935667134843958733071093, 8460399180073392938535920890406846, 13689213353911059072379879623478039, 22149612533884451210915790513879132, 35838825887795510283295660337357171, 57988438421679961494211450851236303, 93827264309475471777507161188603474, 151815702731155433271718612040039777, 245642967040635404969225773228643081, 397458669771800836240944385268646858, 643091636812436241210170158496689939, 1040549306584236077451114543765336800, 1683640943396672318661284702262026739, 2724190250000908559872400246027366539, 4407831193397580878533684948289403278, 7132021443408489438406085194316770817, 11539852636806070317039669142606174095, 18671874080213659755445754336922944912, 30211726716620130072485423479539119007, 48883598796833789827931177816462063919, 79095475413453920580416931196001182926, 127979074210287610408348109012463246845, 207074549623741530986280040208464430761, 335053623834029141394628149120465613687, 542028173457770672382908189328929044448, 877081800291801813777536338449394658135, 1424109973749571956160444527778313702583, 2301191774041373828538352866107708360721, 3725291747841175785308797393886022062904, 6026483521882549613847149260003730363625, 9751675269723725399155946653889752426529, 15778158791606275012993125913893482789154, 25529834051329000412148072567783235212783, 41307992842935275425141198481676717998937, 66837826894264275437289271049460193211720, 108145819737199540662430469531136911109657, 174983646631463816099619740580607104321377, 283129466368663356762050210111744015431034, 458113113000127172861669950692351119752411, 741232579361590529023720160804058235183488, 1199345692361717691885380371496409354935905, 1940578271723308214909100532190460474688393, 3139923964085025906794480903686869830624298, 5080502235808334121703581435877330305312691, 8210426199893360028497962339564190135937089, 13290928435701694150201543775441520941250780, 21501350635595024271699525115318851076562871, 34792279071296714421901488890760371217812960, 56293629706891738693599414006079222294375831, 91085908778188452915500892896839593512188791, 147379538485080181609100306892918815726564622, 238465447263271634524601199789758331038752413, 385844985748359816143701506682677146765317005, 62431043301163145066830270647243607780407041, 100915541876000126681200421315511322356938742, 163346585177163271748030691962754930137345783, 264262126978163416829231113278266252524284525, 427608712155326688577261805240977582661630308, 692214838132490105406492918519242835185975091, 1119823550287816793985754831759220417847605400, 1812038388420306900392247750278463253033580491, 2931861938708123694378002581937683670881185891, 4743900327128430595270250332216147023914766382, 7675762265826556500648252914153830694795947273, 12419662592954987195918503246370013718710723655, 20095424858881537796566756160523844412625671028, 32515087451836524992485259374673858131336394683, 52510512310718062789052015535197692544061425711, 85025599762554587781537274910871550675397819494, 137536112073272650570590290446069243219459245205, 222561624135827238352127565361260835763856664699, 360097736209099890922717855807330079083255910904, 582659360344927129274845421168590924847112575603, 942757096553926919297563276975920903930368486507, 1525416456898854048572408708144511828777481062110, 2468173553452780967869971984120432732707863548617, 3993640010351635016442380692264944561485344610727, 6461813563804415984312352676385377294193208159344, 10455453574156046000754733368649321855678552770071, 16917267137960461985067086045034700149871760929415, 27372720712116508005821819413684021905550313699486, 44290087849076969990888905458718722055422074628901, 71662808561193477996710724872752723960972388258387, 115952896410270447987599630331471446016394462887288, 187615694971463425984310355204224170977366851145675, 303568591381733873971909985535695617033761314032963, 491184286353204301956220335740919788011128165178638, 794752877734938175928130321276615405044890079211601, 1285937164088142477904350657017535193056018244389239, 2080689941823080653832480982794154598099908323600878, 3366627105911223131736831639811690091155926567990117, 5447317047734303785569312622605844689255834891591095, 8813944153645526917306144262417534780411761459581212, 14261261191379830602875456885023379469667596351172307, 23075205345025357519181603767440914250079357810753514, 37336466536405188121957059652464293719746954161925821, 60411671881430545641138663419905208069826311972679335, 97748138417835733763095723072369401789573266134605156, 158159810299266279404234386492274610859409578107284491, 255907948717102013167329109564644012648982846241889646, 414067758916368292571563496056918623508392424349174101, 669975707633470305738892605621562636157375270590863747, 1084943466550838598310456001678481259665767694939037848, 1754919174184308904049348607300043895823142965530901595, 2839862640735147492359804608978525155488910660470939443, 4594781814919456396409153216278569051312053626001841038, 7434693629838603808768357825256694206790964251472780486, 12029475444758060105177511041535263258102917877474621524, 19464169074596663913945868866791957464893882128947402010, 31493644519354724019123379908327220722996799996422023534, 50957813593951387933069248775119178187890682125369425544, 82451458113306111952192628683446408910887482121791449078, 133409271707257499885261877458565587098778164247160874622, 215860729820563611837454506142012005909665846372452300196, 349270001527821111722716383600577593008444010619613174818, 564530731348384723559170890742589598918109857002065475014, 913800732876205835281887274343167191926553867621678649832, 1478331464224590558841058165085756790844663724623744124846, 2392132196090796394122945439428923982771217592245422774678, 3870463660315386952963903604514680773615781316869166919524, 6262595856606183347086807214033604756387002633714589094052, 10133060516921569299050710828048285530002784250583755288076, 16395656373527752646137518042081886286389786884308344382128, 26528716890449321945188228870130171816392571134892100670204, 43054373263977074591325746912211958092782357919275444952332, 69572589154426396536513975782342129909174929054167545904664, 112627062418403471127839722694554087901957286973443090857000, 182199651572830867664353698476896217811132216027610636761664, 294826714091234338792193421171450305713089492991053727618664, 476926365664065206456547119648346523524221709018664364280328, 771753080755299545248740540820796829237311202009718091901092, 1248679446419364751705287660469143352761532911028382456181420, 2020332527174664297153828201389939182098844113038094548082512, 3268911973594029048859115861859082534860377024066477006264004, 5289244490768693346012944063249021716959221137104571554346516, 8558156464362722394872059925108104251819598161171048560610532, 13847400955131415740884903988357125968778819298275620114957048, 22405547419494138135757863913465230220598417460446668675513580, 36253048374625553876642767891822356189377236758722288790470636, 58658595794119691912400631805287586409975654219168957466084216, 94911644168745245789043399697109942599352890977891246256554852, 153570239962864937701444031502397529009328545197060203722639068, 248481884131610183490487421199507471608681440396251407479293136, 401952123294475121191931452701805000618009985593311611201932204, 65043390742608530468241887380131247222669142698956301867122544, 105238603072056042587435032650311747284469141258287463087215760, 170281993814664572955676919930442994507138283957243764954338304, 275520596886720615543111952580754741791607425215531228041554064, 450802590701385188498788872511197736298745708172772992996892368, 726323187588105804041890825091952478090353133388304221093784736, 1177125778289491092530679697603150214389108841561077213090677072, 1903448965877596896572570522695102692479461975949381434184461808, 3080574744167087989103249220308252906868570817510458647275138880, 4984023710044684885675819743003355609347932793459839081460600688, 8064598454211772874779068963311608516216503610970297728735739568, 13048622164256457760454888706314964125564436404430136810171479248, 21113210618468230635233957669626572641780939915400434538907218928, 34161832782724688395688846375941536767345376319830571349078698408, 55275043391192919030922804045568109409126316235230905887985917328, 89436876173917607426611650421509646176471692555061807236971835728, 144711919565110526457534454467077755585598009090292712523957753056, 234148795739028133884146104888587364762069691645354519760929588800, 378860715304138660341680559355665120347667700735647232284887341808, 613009511043166794225826664244252485119737392381001752045816930608, 996868226347305454567507223600917605467405093116648984330704272416, 1610877737390472248793333887845170090587142485497650736376521203024, 2607745963737777703360841111445337696054547578614299472753042475440, 4218623701128250002154174999290507786641690064111950209129563678464, 6826369664865927705515016110735845482696237642726249681882606153904, 11045033366094177707669191109025633269337927706848200891012169832368, 17871403030960105413184207219761478752034165349574450572894776086304, 28916436397054283120853398328787112021372093056422651463906945918672, 467878394270143885340376055485485907734062584060071020$



Convert the following DFA into R.E.



$$1^* 0 (0+1)^*$$



1 at  $k=0$

$$R_{11}^{(0)} = \epsilon + 1$$

$$R_{12}^{(0)} = 0$$

$$R_{21}^{(0)} = \phi$$

$$R_{22}^{(0)} = \epsilon + 0 + 1$$

at  $k=1$

$$R_{ij}^{(1)} = R_{ij}^{(k-1)} + R_{ik}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)}$$

$$R_{11}^{(1)} = R_{11}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)}$$

$$R_{21}^{(1)} = \epsilon + 1 + \epsilon + 1 (\epsilon + 1)^* (\epsilon + 1)$$

$$R_{22}^{(1)} = (\epsilon + 1) [\epsilon + (\epsilon + 1)^* (\epsilon + 1)]$$

$$= (\epsilon + 1) (\epsilon + 1)^*$$

$$= (\epsilon + 1)^+$$

$$R_{11}^{(1)} = 1^*$$

$\left. \begin{aligned} \epsilon + R \cdot R^+ &= R^+ \\ R \cdot R^+ &= R^+ \end{aligned} \right\}$

$$\begin{aligned}
 R_{12}^{(1)} &= R_{1j}^{(k-1)} + R_{1k}^{(k-1)} (R_{kk}^{(k-1)})^* R_{kj}^{(k-1)} \\
 &= R_{12}^{(0)} + R_{11}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)} \\
 &= 0 + (\epsilon+1) (\epsilon+1)^* 0
 \end{aligned}$$

$i=1$   
 $j=2$   
 $k=1$

$$\begin{aligned}
 &\overline{(\epsilon+1)^* \cdot 0} \\
 &\overline{1^* \cdot 0}
 \end{aligned}$$

$$\begin{aligned}
 &\overline{R^* = R^*} \\
 &\overline{(\epsilon+1)^* = R^*}
 \end{aligned}$$

$$\begin{aligned}
 &= 0 (\epsilon + (\epsilon+1) (\epsilon+1)^*) \\
 &= (\epsilon + (\epsilon+1) (\epsilon+1)^*) 0 \\
 &= (\epsilon+1)^* \cdot 0
 \end{aligned}$$

$$\boxed{R_{12}^{(1)} = 1^* \cdot 0} //$$

$$R_{21}^{(1)} = R_{21}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{11}^{(0)}$$

$i=2$   
 $j=1$   
 $k=1$

$$\begin{aligned}
 &= \phi + \phi (\epsilon+1)^* (\epsilon+1) \\
 &= \phi [\epsilon + (\epsilon+1)^* (\epsilon+1)]
 \end{aligned}$$

$$\begin{aligned}
 &= \phi [\epsilon + R] \\
 &= \phi R^*
 \end{aligned}$$

$$\begin{aligned}
 &\epsilon + (\epsilon+1)^* (\epsilon+1) \\
 &= R^*
 \end{aligned}$$

$$= \phi 1^*$$

$$R_{22}^{(1)} = R_{22}^{(0)} + R_{21}^{(0)} (R_{11}^{(0)})^* R_{12}^{(0)}$$

$i=2$   
 $j=2$   
 $k=1$

$$\begin{aligned}
 &= 0 + \phi (\epsilon+1)^* (0) \\
 &= 0 [\epsilon + \phi (\epsilon+1)^*]
 \end{aligned}$$

$$\begin{aligned}
 &= (0 + \epsilon + 1) // \\
 &= \epsilon + 0 + 1
 \end{aligned}$$

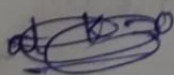
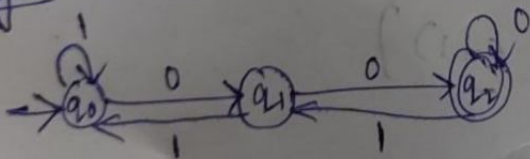


at  $K=2$ .

$$\begin{aligned}
 R_{12}^{(2)} &= R_{1j}^{(K-1)} + R_{1K}^{(K-1)} (R_{KK}^{(K-1)})^* R_{Kj}^{(K-1)} \\
 &= R_{12}^{(1)} + R_{12}^{(1)} (R_{22}^{(1)})^* R_{22}^{(1)} \\
 &= 1*0 + 1*0 (E+0+1)^* (E+0+1) \\
 &= 1*0 [E + (E+0+1)^* (E+0+1)] \\
 &= 1*0 [E + (0+1)^* (E+0+1)] \\
 \therefore ((E+R)^* &= R^*) \\
 &= 1*0 [E + (0+1)^*] \\
 &= 1*0 (0+1)^*
 \end{aligned}$$

4/11/22

Friday

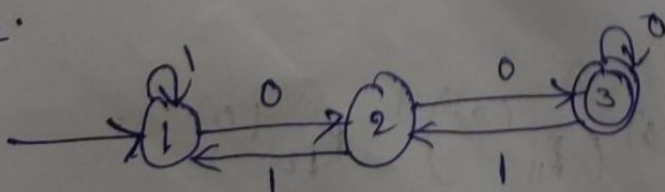


At  $K=0$

$$R_{11}^{(0)} = E+1$$

$$R_{12}^{(0)} = 0$$

$$R_{13}^{(0)} = \phi$$



10/11/22

Thurs

## Table filling Algorithm:-

Finding distinguishable pairs in DFA

Table filling algorithm is a recursive discovery of the distinguishable pair of

$$A = (Q, \Sigma, \delta, q_0, f)$$

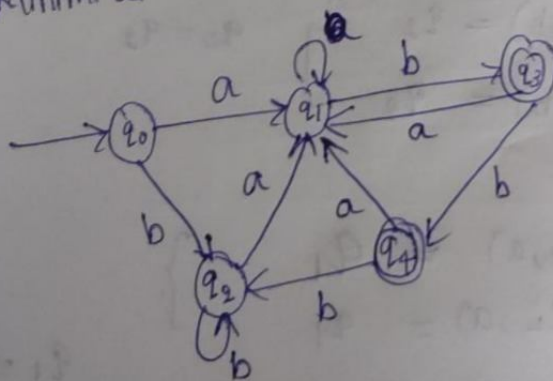
### Basics:-

If 'p' is an accepting state and 'q' is non-accepting state then the pair (p, q) is distinguishable.

### Induction:-

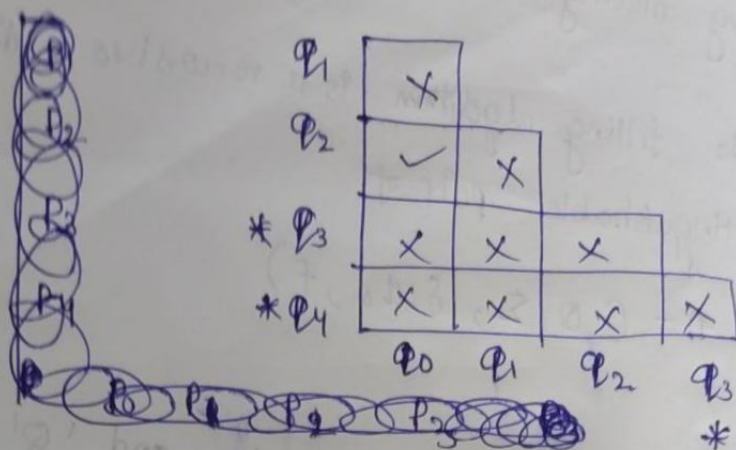
Let 'p' and 'q' be two states such that for some input symbol 'a', if  $\delta(p, a) = r$  and  $\delta(q, a) = s$  are two distinguishable pairs (r, s) then p, q are distinguishable states.

①. Minimize the following DFA.





Step 1: Mark final and non-final states as distinguishable  
(\*)



Step 2: Identify the remaining states are distinguishable or not.

$$\begin{aligned}
 (q_0, q_1) &= \begin{aligned} &\delta(q_0, a) = q_1 \\ &\delta(q_1, a) = q_1 \\ &\delta(q_0, b) = q_2 \\ &\delta(q_1, b) = q_3 \end{aligned} \left. \vphantom{\begin{aligned} &\delta(q_0, a) = q_1 \\ &\delta(q_1, a) = q_1 \\ &\delta(q_0, b) = q_2 \\ &\delta(q_1, b) = q_3 \end{aligned}} \right\} q_0 \neq q_1
 \end{aligned}$$

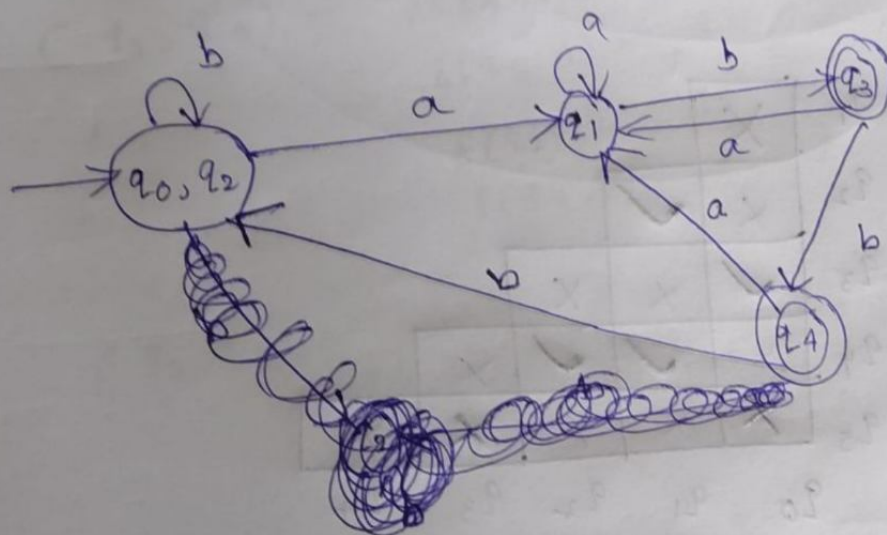
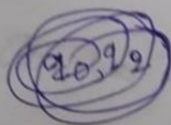
$$\begin{aligned}
 (q_0, q_2) &= \begin{aligned} &\delta(q_0, a) = q_1 \\ &\delta(q_2, a) = q_1 \\ &\delta(q_0, b) = q_2 \\ &\delta(q_2, b) = q_2 \end{aligned} \left. \vphantom{\begin{aligned} &\delta(q_0, a) = q_1 \\ &\delta(q_2, a) = q_1 \\ &\delta(q_0, b) = q_2 \\ &\delta(q_2, b) = q_2 \end{aligned}} \right\} q_0 = q_2
 \end{aligned}$$

$$\begin{aligned}
 (q_1, q_2) &\Rightarrow \begin{aligned} &\delta(q_1, a) = q_1 \\ &\delta(q_2, a) = q_1 \\ &\delta(q_1, b) = q_3 \\ &\delta(q_2, b) = q_2 \end{aligned} \left. \vphantom{\begin{aligned} &\delta(q_1, a) = q_1 \\ &\delta(q_2, a) = q_1 \\ &\delta(q_1, b) = q_3 \\ &\delta(q_2, b) = q_2 \end{aligned}} \right\} q_1 \neq q_2
 \end{aligned}$$

hable

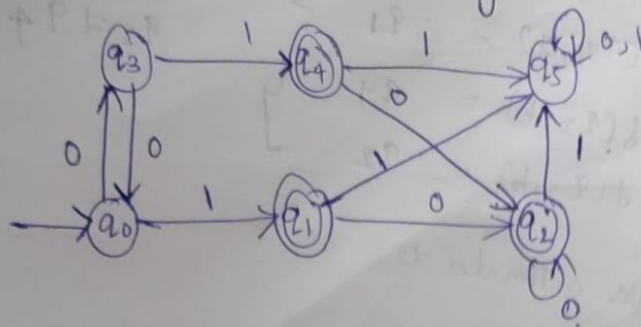
$$\begin{aligned}
 (q_3, q_4) &\Rightarrow \left. \begin{aligned} \delta(q_3, a) &= q_1 \\ \delta(q_4, a) &= q_1 \end{aligned} \right\} \\
 &\quad \left. \begin{aligned} \delta(q_3, b) &= q_4 \\ \delta(q_4, b) &= q_2 \end{aligned} \right\} \quad q_3 \neq q_4
 \end{aligned}$$

∴ Minimized finite automata is





1. Minimize the following DFA :-



Step 1: Mark final and non-final states as distinguishable

* q1	X				
* q2	X	✓			
q3	✓	X	X		
* q4	X	✓	✓	X	
q5	X	X	X	X	X
	q0	q1	q2	q3	q4
		*	*		*

Step 2: Identify the remaining states are distinguishable.

$$\begin{aligned}
 (q_0, q_5) = & \left. \begin{aligned} \delta(q_0, 0) &= q_3 \\ \delta(q_5, 0) &= q_5 \end{aligned} \right\} q_0 \neq q_5 \\
 & \left. \begin{aligned} \delta(q_0, 1) &= q_1 \\ \delta(q_5, 1) &= q_5 \end{aligned} \right\}
 \end{aligned}$$

$$\begin{aligned}
 (q_0, q_3) = & \left. \begin{aligned} \delta(q_0, 0) &= q_3 \\ \delta(q_3, 0) &= q_0 \end{aligned} \right\} \\
 & \left. \begin{aligned} \delta(q_0, 1) &= q_1 \\ \delta(q_3, 1) &= q_4 \end{aligned} \right\}
 \end{aligned}$$

$\therefore q_0 = q_3$  ✓  
From next step.

ie  $q_1 = q_4$

$$(q_1, q_4) = \left. \begin{array}{l} \delta(q_1, 0) = q_2 \\ \delta(q_4, 0) = q_2 \\ \delta(q_1, 1) = q_5 \\ \delta(q_4, 1) = q_5 \end{array} \right\}$$

$$q_1 = q_4 \checkmark$$

$$(q_1, q_2) = \left. \begin{array}{l} \delta(q_1, 0) = q_2 \\ \delta(q_2, 0) = q_2 \\ \delta(q_1, 1) = q_5 \\ \delta(q_2, 1) = q_5 \end{array} \right\}$$

$$q_1 = q_2 \checkmark$$

$$(q_2, q_4) = \left. \begin{array}{l} \delta(q_2, 0) = q_2 \\ \delta(q_4, 0) = q_2 \\ \delta(q_2, 1) = q_5 \\ \delta(q_4, 1) = q_5 \end{array} \right\}$$

$$q_2 = q_4 \checkmark$$

$$(q_3, q_5) = \left. \begin{array}{l} \delta(q_3, 0) = q_0 \\ \delta(q_5, 0) = q_5 \\ \delta(q_3, 1) = q_4 \\ \delta(q_5, 1) = q_5 \end{array} \right\}$$

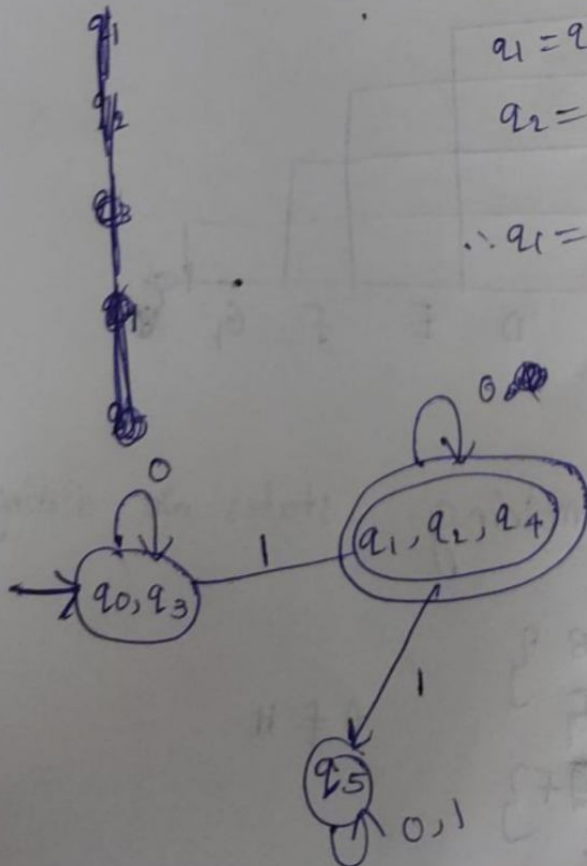
$$q_3 \neq q_5$$

$$\therefore q_1 = q_4, q_0 = q_3$$

$$q_1 = q_2$$

$$q_2 = q_4$$

$$\therefore q_1 = q_2 = q_4$$



$$= q_4$$