18MA001

**Hall Ticket Number:**

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**I/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION**

**December, 2019** **Common to all branches**

**First Semester** **Linear algebra and ODE**

**Time:** Three Hours **Maximum:** 50 Marks

*Answer Question No.1 compulsorily.* (1X10 = 10 Marks)

*Answer ONE question from each unit.* (4X10=40 Marks)

1. Answer all questions (1X10=10 Marks)

(a) Find the rank of $A=\begin{bmatrix}1 & 2 & 3\\ -2 & 0 & 5\end{bmatrix}$ 1M

b) When will the system contain only unique solution in non- homogeneous equations? 1M

c) Find the Eigen values of $\begin{bmatrix}1 & 2 & 5\\ 0 & 3 & -4\\ 0 & 0 & 4\end{bmatrix}$ 1M

d) Solve $\frac{dy}{dx}=x^2e^{-2y}$ 1M

e) What is the standard form of Bernoulli's Equation? 1M

f) Write Growth Equation. 1M

g) Solve $\frac{d^2y}{dt^2}+5\frac{dy}{dt}+6y=0$ 1M

h) Find PI of $(D^2-4)y=e^{2x}$ 1M

i) Define Laplace transform 1M

j) Find $L^{-1}\left(\frac{s^2-4}{s^3}\right)$ 1M

**UNIT I**

2. a) Find inverse of the matrix $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 3 & -3\\ -2 & -4 & -4\end{bmatrix}$. 5M

b) Solve the equations $4x+2y+z+3w=0, 6x+3y+4z+7w=0$ *and* $2x+y+w=0$ 5M

**(OR)**

3. a) Find Eigen values and Eigen vectors of the matrix $\begin{bmatrix}8 & -6 & 2\\ -6 & 7 & -4\\ 2 & -4 & 3\end{bmatrix}$ 5M

b) Verify Cayley-Hamilton theorem for $A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}$ 5M

18MA001

## UNIT II

4. a) Solve $\dfrac{dy}{dx} = \text{Sin}(x+y) + \text{Cos}(x+y)$ 5M

b) Solve $(x+1)\dfrac{dy}{dx} = y + e^{3x}(x+1)^2$ 5M

**(OR)**

5. a) Solve $\left(x^4 - 2xy^2 + y^4\right)dx - \left(2x^2y - 4xy^3 + \text{Sin}\,y\right)dy = 0$ 5M

b) If the temperature of the air is $30^0$C and the substance cools from $100^0$C to $70^0$C in 15 minutes. Then what will be the time for getting the temperature $40^0$C. 5M

## UNIT III

6. a) Solve $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = (1-e^x)^2$ 5M

b) Solve $\dfrac{d^2y}{dx^2} - 6\dfrac{dy}{dx} + 9y = \dfrac{e^{3x}}{x^2}$ by using variation of parameters 5M

**(OR)**

7. a) Solve $\left(D^2 - 2D + 1\right)\ y = x\,e^x \sin x$ 5M

b) An unchanged condenser C is charged by applying e.m.f $E\text{Sin}\,(\dfrac{t}{\sqrt{LC}})$ through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plate is 5M

$$\frac{EC}{2}\left[\text{Sin}\,\frac{t}{\sqrt{LC}} - \frac{t}{\sqrt{LC}}\,\text{Cos}\,\frac{t}{\sqrt{LC}}\right]$$

## UNIT IV

8. a) (i). Find $L\left[t^3e^{-3t}\right]$ 2M

(ii). Find $L\left(\dfrac{e^t \sin t}{t}\right)$ 3M

b) Find $L^{-1}\left(\dfrac{2s^2-1}{(s^2+1)(s^2+4)}\right)$ 5M

**(OR)**

9. a) Using Convolution theorem, find $L^{-1}\left[\dfrac{1}{(s+1)(s+3)}\right]$ 5 M

b) By the method of transforms, solve $\dfrac{d^2x}{dt^2} - 2\dfrac{dx}{dt} + x = e^t$ given $x(0) = 2$ *and* $x'(0) = -1$ 5 M

I/IV B.Tech (Regular) Degree Examination

Paper: Linear algebra and ODE (Sem-I)

Paper code: 18MA001 December 2019

**Scheme of Evaluation**

Common to all branches

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1. Answer ONE question

(a) given that $A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 5 \end{bmatrix}$

The second order minor $\begin{vmatrix} 1 & 2 \\ -2 & 0 \end{vmatrix} = 4 \neq 0$

The largest order of non-vanishing minor is 2

$\therefore$ rank(A) = 2

(or)

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -2 & 0 & 5 \end{bmatrix}$$

$R_2 \to R_2 + 2R_1$

$$\sim \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 11 \end{bmatrix}$$

It is in echelon form, the no of non-zero rows = 2

$\therefore \rho(A) = 2$

(b) $\rho(A) = \rho(A:B) = n$

then the system of non-homogeneous equations has only unique solution.

(c) The eigenvalues of $\begin{bmatrix} 1 & 2 & 5 \\ 0 & 3 & -4 \\ 0 & 0 & 4 \end{bmatrix}$ are 1, 3 and 4

because it is an upper triangular matrix.

(d) given that $\frac{dy}{dx} = x^2 e^{-2y}$

$\Rightarrow \frac{dy}{e^{-2y}} = x^2 dx$

$\Rightarrow e^{2y} dy = x^2 dx$

Integrating on bothsides

$$\frac{e^{2y}}{2} = \frac{x^3}{3} + c$$

$$\Rightarrow 3e^{2y} = 2x^3 + 2c$$

$$\Rightarrow e^{2y} = \frac{2}{3}x^3 + \frac{2}{3}c$$

$$\Rightarrow 2y = \log\left[\frac{2}{3}x^3 + \frac{2}{3}c\right]$$

$$\therefore y = \frac{1}{2}\log\left[\frac{2}{3}x^3 + \frac{2}{3}c\right]$$

e) The standard form of Bernoulli's equation is

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

f) The growth equation is $\frac{dN}{dt} = KN$ $\left[\because \frac{dN}{dt} \propto N\right]$

g) The Auxiliary equation is $D^2 + 5D + 6 = 0$

$$\Rightarrow D(D+3) + 2(D+3) = 0$$

$$\Rightarrow (D+2)(D+3) = 0$$

$$\therefore D = -2 \ \& \ -3$$

$$\therefore \ C.S(y) = C_1 e^{-2t} + C_2 e^{-3t}$$

h) To find P.I

$$y = \frac{e^{2x}}{D^2 - 4}$$

if $D = 2$, it goes to $\infty$

$$y = x\,\frac{1}{2D}e^{2x} = \frac{x}{4}e^{2x}$$

i) Let $f(t)$ be a function of $t$ defined for all $t \geq 0$. Then the laplace transform of $f(t)$, denoted by $L[f(t)]$, is defined as $L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$ provided that the integral exists, $s$ is a parameter which may be real or Complex.

(j) $L^{-1}\left[\frac{s^2-4}{s^3}\right] = L^{-1}\left[\frac{1}{s} - \frac{4}{s^3}\right]$

$= L^{-1}\left[\frac{1}{s}\right] - L^{-1}\left[\frac{4}{s^3}\right]$

$= 1 - 4 \cdot \frac{t^2}{2} = 1 - 2t^2$

## UNIT-I

2a) given $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$

write the given matrix $A$ and the identity matrix side by side ie $[A_{3\times3} \mid I_{3\times3}]$

$$\left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 1 & 3 & -3 & 0 & 1 & 0 \\ -2 & -4 & -4 & 0 & 0 & 1 \end{array}\right]$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 + 2R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 & 1 \end{array}\right]$$

$R_2 \to \frac{R_2}{2}$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & -2 & -2 & 2 & 0 & 1 \end{array}\right]$$

$R_1 \to R_1 - R_2$, $R_3 \to R_3 + 2R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & \frac{-1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$$

$R_3 \to R_3/-4$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & \frac{3}{2} & -\frac{1}{2} & 0 \\ 0 & 1 & -3 & -\frac{1}{2} & \frac{1}{2} & 0 \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & \frac{-1}{4} \end{array}\right]$$

$R_1 \to R_1 - 6R_3,\ R_2 \to R_2 + 3R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & \frac{3}{2} \\ 0 & 1 & 0 & -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ 0 & 0 & 1 & -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{array}\right]$$

$$\therefore A^{-1} = \begin{bmatrix} 3 & 1 & \frac{3}{2} \\ -\frac{5}{4} & -\frac{1}{4} & -\frac{3}{4} \\ -\frac{1}{4} & -\frac{1}{4} & -\frac{1}{4} \end{bmatrix}$$

2

b) The given equations are

$$4x + 2y + z + 3w = 0$$
$$6x + 3y + 4z + 7w = 0$$
$$2x + y + w = 0$$

The system can be written as $AX = 0$

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

The coefficient matrix $A = \begin{bmatrix} 4 & 2 & 1 & 3 \\ 6 & 3 & 4 & 7 \\ 2 & 1 & 0 & 1 \end{bmatrix}$

Reducing the coefficient matrix A to echelon form,

we get $R_2 \to R_2 - \frac{3}{2}R_1,\ R_3 \to R_3 - \frac{1}{2}R_1$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 5/2 & 5/2 \\ 0 & 0 & -1/2 & -1/2 \end{bmatrix}$$

$R_2 \to \frac{2}{5}R_2, \; R_3 \to -2R_3$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$\sim \begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$\rho(A) = 2$, number of unknown $= 4$

Since $r < n$, we get non-trivial solution which gives infinite number of solutions

$$\begin{bmatrix} 4 & 2 & 1 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Expanding $4x + 2y + z + 3w = 0$

$z + w = 0$

$n - r = 4 - 2 = 2$, the arbitrary values are to be given to two variables

let $w = K_1$, then $z = -K_1$

also let $y = K_2$ then $4x + 2K_2 - K_1 + 3K_1 = 0 \Rightarrow x = -\frac{1}{2}(K_1 + K_2)$

$\therefore$ The solution is $X = \begin{bmatrix} -\frac{1}{2}(K_1 + K_2) \\ K_2 \\ -K_1 \\ K_1 \end{bmatrix}$

Let $K_1 = 1, K_2 = 1$ $\quad X = \begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$

(8)

a) The given matrix $(A) = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The ch equation is $|A - \lambda I| = 0$

$$\begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\therefore \lambda = 0, 3 \ \& \ 15$$

Case(i) when $\lambda_1 = 0$

Let $X_1$ be the eigen vector corresponding to the eigen value $\lambda_1 = 0$

$\Rightarrow (A - \lambda_1 I) X_1 = 0$ i.e $(A - 0I) X_1 = 0$

$$A X_1 = 0 \Rightarrow \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$$

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix} \underset{\sim}{\xrightarrow[R_2 \to 8R_2 + 6R_1]{R_3 \to 4R_3 - R_1}} \begin{bmatrix} 8 & -6 & 2 \\ 0 & 20 & -20 \\ 0 & -10 & 10 \end{bmatrix} \underset{\sim}{\xrightarrow{R_3 \to 2R_3 + R_2}} \begin{bmatrix} 8 & -6 & 2 \\ 0 & 20 & -20 \\ 0 & 0 & 0 \end{bmatrix}$$

$r = 2, n = 3$, The system has non-trivial solution

we can fix $n - r = 3 - 2 = 1$ variable.

Here $20x_2 - 20x_3 = 0 \Rightarrow x_2 = x_3$

Let $x_1 = K_1$, $x_2 = 2K_1 = x_3$

$$\therefore X_1 = \begin{bmatrix} K_1 \\ 2K_1 \\ 2K_1 \end{bmatrix} \qquad \text{when } K_1 = 1;\ X_1 = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

Case(ii) when $\lambda_2 = 3$

Let $X_2$ be the eigen vector corresponding to the eigen value

$\lambda_2 = 3 \Rightarrow (A - \lambda_2 I) X_2 = 0$ i.e $(A - 3I) X_2 = 0$

$$\begin{bmatrix} 5 & -6 & 2 \\ -6 & 4 & -4 \\ 2 & -4 & 0 \end{bmatrix} \xrightarrow[R_3 \to 5R_3 - 2R_1]{R_2 \to 5R_2 + 6R_1} \sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & -16 & -8 \\ 0 & -8 & -4 \end{bmatrix} \xrightarrow[R_3 \to R_3/4]{R_2 \to R_2/4} \sim \begin{bmatrix} 5 & -6 & 2 \\ 0 & -4 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

Here $r=2$, $n=3$, $r<n$

The system has non-trivial solution, we can fin $n-r=3-2=1$ variable

$-4x_2-2x_3=0 \Rightarrow x_3=2x_2$, Let $x_2=K_1$
$x_3=-2K_1$

$5x_1-6x_2+2x_3=0 \Rightarrow x_1=2K_1$

$\therefore X_2 = \begin{bmatrix} 2K_1 \\ K_1 \\ -2K_1 \end{bmatrix}$ ; Let $K_1=1$, $X_2 = \begin{bmatrix} 2 \\ 1 \\ -2 \end{bmatrix}$

Case(iii) when $\lambda_3=15$

Let $X_3$ be the eigen vector corresponding to the eigen value $\lambda_3=15$

$\ni (A-\lambda_3 I)X_3=0$ i.e $(A-15I)X_3=0$

$$\begin{bmatrix} -7 & -6 & 2 \\ -6 & -8 & -4 \\ 2 & -4 & -12 \end{bmatrix} \xrightarrow[R_3 \to 7R_3 + 2R_1]{R_2 \to 7R_2 - 6R_1} \sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & -20 & -40 \\ 0 & -40 & -80 \end{bmatrix} \xrightarrow{R_3 \to R_3 + 2R_2} \sim \begin{bmatrix} -7 & -6 & 2 \\ 0 & -20 & -40 \\ 0 & 0 & 0 \end{bmatrix}$$

$r=2$, $n=3$ and $r<n$, the system has non-trivial solution

we can fin $n-r=3-2=1$ variable

$\cancel{4+2\gamma}$ $x_2+2x_3=0$, Let $x_3=K_1$ then $x_2=-2K_1$

$-7x_1-6x_2+2x_3=0 \Rightarrow x_1=2K_1$

$\therefore X_3 = \begin{bmatrix} 2K_1 \\ -2K_1 \\ K_1 \end{bmatrix}$ ; Let $K_1=1$, $X_3 = \begin{bmatrix} 2 \\ -2 \\ 1 \end{bmatrix}$

3 (b) given $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The ch equation is $|A-\lambda I|=0$

i.e $\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$

$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$

Caley-Hamilton Theorem Says that
Every square matrix satisfies it's own charecteristic eq.

To show that $A^3 - 6A^2 + 9A - 4I = 0$

$$A^2 = A \cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} ; \quad A^3 = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 22 & -21 & 22 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4I = 0$

Hence Caley-Hamilton Theorem is verified.

## UNIT-II

4
a) $\dfrac{dy}{dx} = \sin(x+y) + \cos(x+y) \longrightarrow (1)$

Let $x + y = t$

$1 + \dfrac{dy}{dx} = \dfrac{dt}{dx} \Rightarrow \dfrac{dy}{dx} = \dfrac{dt}{dx} - 1$

Substituting in (1), we get

$\dfrac{dt}{dx} - 1 = \sin t + \cos t$

$\Rightarrow \dfrac{dt}{dx} = 1 + \sin t + \cos t$ i.e $dx = \dfrac{dt}{1 + \sin t + \cos t}$

Integrating on both sides

$x = \displaystyle\int \frac{1}{1 + \sin t + \cos t} dt + c$, putting $t = 2\theta$
$\Rightarrow dt = 2 d\theta$

$= \displaystyle\int \frac{2\, d\theta}{1 + \sin 2\theta + \cos 2\theta} + c = \int \frac{\sec^2\theta}{1 + \tan\theta} d\theta + c$

$= \log(1 + \tan\theta) + c$

$\therefore x = \log\left(1 + \tan\left(\frac{x+y}{2}\right)\right) + c$

④ b. $(x+1)\frac{dy}{dx} - y = e^{3x}(x+1)^2$

$\Rightarrow \frac{dy}{dx} - \frac{y}{x+1} = e^{3x}(x+1)$

Here $P = -\frac{1}{x+1}$ ; $I.F = e^{\int P dx} = e^{\int -\frac{1}{x+1} dx} = e^{-\log(1+x)} = \frac{1}{1+x}$

The sol is

$$y \, I.F = \int e^{3x}(x+1) \, I.F \, dx + C$$

$$y \cdot \frac{1}{1+x} = \int e^{3x}(x+1) \cdot \frac{1}{x+1} dx + C$$

$$\Rightarrow \frac{y}{1+x} = \frac{e^{3x}}{3} + C$$

$$\therefore y = (1+x)\left(\frac{e^{3x}}{3} + C\right)$$

⑤ a $(x^4 - 2xy^2 + y^4)dx - (2x^2y - 4xy^3 + \sin y)dy = 0$

Here $M = x^4 - 2xy^2 + y^4$, $N = -2x^2y + 4xy^3 - \sin y$

$$\frac{\partial M}{\partial y} = -4xy + 4y^3 = \frac{\partial N}{\partial x}$$

$\therefore$ It is an exact diff. equation

The solution is

$$\int_{y=c} M \, dx + \int_{\substack{\text{not containing} \\ x \text{ terms}}} N \, dy = c$$

$$\int (x^4 - 2xy^2 + y^4)dx + \int (-\sin y)dy = c$$

$$\Rightarrow \frac{x^5}{5} - 2 \cdot \frac{x^2}{2} y^2 + xy^4 + \cos y = c$$

$\therefore x^5 - 5x^2y^2 + 5xy^4 + 5\cos y = c'$ where $c' = 5c$

(5) b. By Newton's law of Cooling

The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself. i.e $\frac{dT}{dt} = -K(T-T_0)$ where K is Constant

we have $T = T_0 + Ce^{-Kt}$

given Conditions, when $t=0$, $T=100°c$ — (1)

when $t=15$, $T=70°c$ — (2)

From (1), $100 = 30 + C \Rightarrow C = 70$

From (2), $70 = 30 + 70e^{-K(15)} \Rightarrow K = 0.0373$

The temparature is reduced to 40°c

$$40 = 30 + 70\, e^{-0.0373t}$$

$$\Rightarrow t = 52 \text{ mts}$$

(6) a $\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = (1-e^x)^2$

given equation in auxiliary form is $D^2 + D + 1 = 0$

$$\therefore D = \frac{-1 \pm \sqrt{1^2 - 4.1.1}}{2.1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = -\frac{1}{2} \pm \frac{\sqrt{3}i}{2}$$

Thus $C.F = e^{-x/2}\left[C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right]$

To find P.I

$$P.I = \frac{1}{D^2+D+1}(1-e^x)^2 = \frac{1}{D^2+D+1}(1-2e^x+e^{2x})$$

$$= \frac{1}{D^2+D+1}(e^{0x} - 2e^x + e^{2x}) = \frac{1}{0^2+0+1}e^{0x} - \frac{2}{1^2+1+1}e^x + \frac{1}{2^2+2+1}e^{2x}$$

$$= 1 - \frac{2}{3}e^x + \frac{e^{2x}}{7}$$

$$\therefore CS = C.F + P.I = e^{-x/2}\left[C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right] + 1 - \frac{2}{3}e^x + \frac{e^{2x}}{7}$$

(10)

(6) b $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$

The Auxiliary equation is $D^2 - 6D + 9 = 0 \Rightarrow (D-3)^2 = 0$
$\Rightarrow D = 3, 3$

$$C.F = (C_1 + C_2 x)e^{3x} = C_1 e^{3x} + C_2 x e^{3x}$$

Let $y_1 = e^{3x}$, $y_2 = xe^{3x}$

$$W(y_1, y_2) = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}(1+3x) \end{vmatrix} = (1+3x)e^{6x} - 3xe^{6x} = e^{6x} \neq 0$$

$$P.I = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$$

$$= -e^{3x} \int \frac{xe^{3x}\left(\frac{e^{3x}}{x^2}\right)}{e^{6x}} dx + xe^{3x} \int \frac{1}{x^2} dx = -e^{3x}\log x - e^{3x} = -e^{3x}[1 + \log x]$$

$$\therefore C.S = C_1 e^{3x} + C_2 x e^{3x} - e^{3x}(1 + \log x)$$

(7) a $(D^2 - 2D + 1)y = xe^x \sin x$

The Auxiliary equation is $D^2 - 2D + 1 = 0 \Rightarrow (D-1)^2 = 0 \Rightarrow D = 1, 1$

$$\therefore C.F = (C_1 + C_2 x)e^x$$

$$P.I \quad \frac{1}{D^2 - 2D + 1} xe^x \sin x = e^x \frac{1}{(D+1)^2 - 2(D+1) + 1} x \sin x$$

$$= e^x \frac{1}{D^2 + 2D + 1 - 2D - 2 + 1} x \sin x = e^x \frac{1}{D^2} x \sin x = e^x \frac{1}{D} \int x \sin x \, dx$$

$$= e^x \frac{1}{D}[x(-\cos x) - 1(-\sin x)] = e^x \frac{1}{D}[-x\cos x + \sin x]$$

$$= e^x \left[\int -x\cos x\, dx + \int \sin x \, dx\right] = e^x[-(x\sin x - 1(-\cos x)) - \cos x]$$

$$= e^x[-x\sin x - \cos x - \cos x] = e^x[-x\sin x - 2\cos x]$$

$$C.S = C.F + P.I = (C_1 + C_2 x)e^x + e^x[-x\sin x - 2\cos x]$$
$$= (C_1 + C_2 x)e^x - e^x[x\sin x + 2\cos x]$$

7 b. The diff. eq is $L\frac{d^2q}{dt^2}+\frac{q}{c}=E\sin\frac{t}{\sqrt{LC}}$

The A.E is $LD^2+\frac{1}{c}=0 \Rightarrow D=\pm\frac{i}{\sqrt{LC}}$

$C.F = C_1\cos\frac{t}{\sqrt{LC}}+C_2\sin\frac{t}{\sqrt{LC}}$

$P.I = \frac{1}{LD^2+\frac{1}{c}} E\sin\frac{t}{\sqrt{LC}}$, if $D^2=-\frac{1}{LC}$ the denominator is 0

$= Et\frac{1}{2LD}\sin\frac{t}{\sqrt{LC}} = \frac{Et}{2L}\int\sin\frac{t}{\sqrt{LC}}dt = -\frac{Et}{2L}\cos\frac{t}{\sqrt{LC}}\cdot\sqrt{LC}$

$= -\frac{Et}{2}\sqrt{\frac{c}{L}}\cos\frac{t}{\sqrt{LC}}$

$q=C.S=C.F+P.I= C_1\cos\frac{t}{\sqrt{LC}}+C_2\sin\frac{t}{\sqrt{LC}} - \frac{Et}{2}\sqrt{\frac{c}{L}}\cos\frac{t}{\sqrt{LC}}$

when $t=0$, $q=0$ then $C_1=0$

$q= C_2\sin\frac{t}{\sqrt{LC}} - \frac{Et}{2}\sqrt{\frac{c}{L}}\cos\frac{t}{\sqrt{LC}}$

diff. w.r.to t, we get

$\frac{dq}{dt}=\frac{C_2}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}} - \frac{E}{2}\sqrt{\frac{c}{L}}\left\{\cos\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}\sin\frac{t}{\sqrt{LC}}\right\}$

when $t=0$, $\frac{dq}{dt}=i=0$

$\frac{C_2}{\sqrt{LC}}-\frac{E}{2}\sqrt{\frac{c}{L}}=0 \Rightarrow C_2=\frac{EC}{2}$, substituting $C_2$ in (2), q at any time t

is given by $q=\frac{EC}{2}\left\{\sin\frac{t}{\sqrt{LC}}-\frac{t}{\sqrt{LC}}\cos\frac{t}{\sqrt{LC}}\right\}$

8 a) i) $L\{t^3e^{-3t}\}$

we know that $L[t^3]=\frac{3!}{s^4}$

$L[t^3e^{-3t}]=\frac{3!}{(s+3)^4}$ [$\because$ By shifting Theorem]

$=\frac{6}{(s+3)^4}$

(iii) $L\left[e^{t}\frac{\sin t}{t}\right]$

$L[\sin t] = \frac{1}{s^2+1}$ ; $\therefore L\left[\frac{\sin t}{t}\right] = \int_s^\infty \frac{1}{s^2+1}\,ds$

$= \left[\tan^{-1}s\right]_s^\infty = \frac{\pi}{2} - \tan^{-1}s = \cot^{-1}s$

$\therefore L\left[\frac{\sin t}{t}\right] = \cot^{-1}s \Rightarrow L\left[e^{t}\frac{\sin t}{t}\right] = \cot^{-1}(s-1)$

(8)
b) $L^{-1}\left[\frac{2s^2-1}{(s^2+1)(s^2+4)}\right]$

Let $s^2=p$ ; $L^{-1}\left[\frac{2p-1}{(p+1)(p+4)}\right] = L^{-1}\left[\frac{-1}{p+1} + \frac{3}{p+4}\right]$

$= L^{-1}\left[\frac{-1}{s^2+1} + \frac{3}{s^2+4}\right] = -L^{-1}\left[\frac{1}{s^2+1}\right] + 3L^{-1}\left[\frac{1}{s^2+4}\right]$

$= -\sin t + \frac{3}{2}\sin 2t$

(9)
a) $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right]$

By Convolution Theorem $L^{-1}[\bar{f}(s)\,\bar{g}(s)] = \int_0^t f(u)\,g(t-u)\,du$

$L^{-1}\left[\frac{1}{s+3}\cdot\frac{1}{s+1}\right] = L^{-1}[\bar{f}(s)\,\bar{g}(s)]$

Here $\bar{f}(s) = \frac{1}{s+3} \Rightarrow L[f(t)] = \frac{1}{s+3} \Rightarrow f(t) = L^{-1}\left[\frac{1}{s+3}\right] = e^{-3t}$

$\bar{g}(s) = \frac{1}{s+1} \Rightarrow L[g(t)] = \frac{1}{s+1} \Rightarrow g(t) = L^{-1}\left[\frac{1}{s+1}\right] = e^{-t}$

$f(u) = e^{-3u}$ and $g(t-u) = e^{-(t-u)}$

$L^{-1}\left[\frac{1}{(s+3)}\cdot\frac{1}{(s+1)}\right] = \int_0^t e^{-3u}\cdot e^{-(t-u)}\,du = \int_0^t e^{-3u}\cdot e^{-t+u}\,du = \int_0^t e^{-t}\cdot e^{-2u}\,du$

$= e^{-t}\left[\frac{e^{-2u}}{-2}\right]_0^t = e^{-t}\left[\frac{e^{-2t}}{-2} + \frac{1}{2}\right] = \frac{1}{2}\left[e^{-t} - e^{-3t}\right]$

(9)
b) $\frac{d^2x}{dt^2} - 2\frac{dx}{dt} + x = e^t$ given $x(0)=2$ & $x'(0)=-1$

$x''(t) - 2x'(t) + x(t) = e^t$

Applying Laplace Transform on bothsides

$L[x''(t)] - 2L[x'(t)] + L[x(t)] = L[e^t]$

$s^2\bar{x}(s) - s x(0) - x'(0) - 2[s\bar{x}(s) - x(0)] + \bar{x}(s) = \frac{1}{s-1}$

$s^2\bar{x}(s) - s \times 2 - 1 - 2[s\bar{x}(s) - 2] + \bar{x}(s) = \frac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{x}(s) - 2s + 5 = \frac{1}{s-1}$

$\Rightarrow (s^2 - 2s + 1)\bar{x}(s) = \frac{1}{s-1} + 2s - 5$

$\therefore \bar{x}(s) = \frac{2s^2 - 7s + 6}{(s-1)^3}$

$L[x(t)] = \frac{A}{s-1} + \frac{B}{(s-1)^2} + \frac{C}{(s-1)^3} = \frac{2}{s-1} - \frac{3}{(s-1)^2} + \frac{1}{(s-1)^3}$

$\Rightarrow x(t) = L^{-1}\left[\frac{2}{s-1} - \frac{3}{(s-1)^2} + \frac{1}{(s-1)^3}\right]$

$= 2e^t - 3te^t + \frac{1}{2}e^t t^2$

$\therefore x(t) = e^t\left[2 - 3t + \frac{1}{2}t^2\right]$

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3/1/2020

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