Hall Ticket Number:

I/IV B.Tech (Regular\Supplementary) DEGREE EXAMINATION

April, 2022 **Common to all branches**

First Semester **Linear Algebra and ODE**

Time: Three Hours **Maximum:** 70 Marks

Answer Question No.1 compulsorily. (1X14 = 14 Marks)

Answer ONE question from each unit. (4X14=56 Marks)

1 Answer all questions (1X14=14 Marks)

a) Write any two elementary row operations.

b) Define Characteristic equation of a matrix.

c) Write any two Properties of Eigen Values.

d) Find the integrating factor of $\frac{dx}{dy}+\frac{3}{y}x = \frac{1}{y^2}$.

e) Write Bernoulli's Equation form.

f) Solve $\frac{dy}{dx}=e^{3x-2y}$

g) Find the general solution of $(D^2-2)^2y = 0$.

h) Find the P.I of $\frac{d^2y}{dx^2}+\frac{dy}{dx}=e^{3x}$.

i) Find the particular integral of $(D^2+a^2)y = \sin ax$.

j) Write the Wronskian of cos2x, sin2x.

k) Find the Laplace transform of a^t.

l) Find L[t^2].

m) Write Convolution theorem.

n) Find L[$\cos(2t+3)$].

UNIT I

2\. a) Using Gauss – Jordan method , find the inverse of a matrix A = $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$. 7M

b) Investigate for what values of λ *and* μ *the* simultaneous equations 2x + 3y + 5z = 9 ; 7x + 3y -2z = 8 ; 2x +3y + λ z = μ have (i) No Solution (ii) a unique solution (iii) an infinite number of solutions . 7M

(OR)

3\. a) Using Cayley – Hamilton theorem for the matrix A = $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ find its inverse. 7M

b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. 7M

UNIT II

4\. a) Solve $(1+y^2)dx+(x-e^{-\tan^{-1}y})dy=0$ 7M

b) If the air is maintained at 30°C and temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. 7M

(OR)

5\. a) Solve $x\log x\frac{dy}{dx}+y = \log x^2$. 7M

b) Solve $ye^{xy}dx+(xe^{xy}+2y)dy = 0$. 7M

UNIT III

6\. a) Solve $\frac{d^2x}{dt^2}+n^2x = k\cos(nt+\alpha)$. 7M

b) Solve by the method of variation of parameters $\frac{d^2y}{dx^2}+y = \tan x$. 7M

(OR)

7\. a) Solve $(D^2-4D+3)y = \sin 3x\cos 2x$. 7M

b) Solve $\frac{d^4y}{dx^4}+8\frac{d^2y}{dx^2}+16y = 0$. 7M

UNIT IV

8\. a) Find the Laplace transform of $\frac{e^{-at}-e^{-bt}}{t}$. 7M

b) Using Convolution theorem find the inverse Laplace transform of $\frac{s^2}{(s^2+a^2)^2}$. 7M

(OR)

9\. a) Find the Laplace transform of the following functions 7M
(i) $e^{3t}\sin 2t$ (ii) $e^t\ (\cos 2t+\sinh 2t)$

b) Solve $\frac{d^2y}{dt^2}+2\frac{dy}{dt}-3y = \sin t$ if y(0) = y'(0) = 0,by Laplace transform method. 7M

BAPATLA ENGINEERING COLLEGE, BAPATLA
DEPARTMENT OF MATHEMATICS
I/IV B.Tech (Regular/Supplementary) DEGREE EXAMINATION

Linear Algebra And ODE Scheme of Evaluation — April, 2022

20CB/CE/CS/DS/EC/EE/EI/IT/ME 101 (MA01) — First Semester

1. (a). Elementary row operations:

(i). The interchange of any two rows.

(ii). The multiplication of any row by a non-zero number.

(iii). The addition of a constant multiple of the elements of any row to the corresponding elements of any other row.

(b). If A is any square matrix of order n, the matrix $A-\lambda I$, where I is the n^{th} order unit matrix. The determinant of this matrix equated to zero, $|A-\lambda I|=0$ is called the characteristic equation of A.

(c). Properties of Eigen values:

(i) The sum of the eigen values of a matrix is its trace.

(ii) The product of the eigen values of a matrix A is equal to its determinant.

(iii) Any square matrix A and its transpose A^T have the same eigen values.

(d). Given DE is $\frac{dx}{dy}+\frac{3}{y}x=\frac{1}{y^2}$

Integrating factor, $IF = e^{\int \frac{3}{y}dy} = e^{3\ln y} = \underline{y^3}$

(e). The Bernoulli's equation form is $\frac{dy}{dx}+Py=Qy^n$.
where P, Q are functions of x.

(f). $\frac{dy}{dx} = e^{3x-2y}$

$\Rightarrow e^{2y}\,dy = e^{3x}\,dx$

$\Rightarrow \int e^{2y}\,dy = \int e^{3x}\,dx$

$\Rightarrow \frac{e^{2y}}{2} = \frac{e^{3x}}{3}+c$

$\Rightarrow 3e^{2y} = 2e^{3x}+c$, is the solution.

(g). Given DE is $(D^2-2)^2 y = 0$.

It's auxiliary equation is $(D^2-2)^2=0 \Rightarrow D^2 = 2, 2$

$\Rightarrow D = \pm\sqrt{2}, \pm\sqrt{2}$.

Hence CS is $y = (c_1 + c_2 x)\, e^{\sqrt{2}x} + (c_3 + c_4 x)\, e^{-\sqrt{2}x}$.

(h). Given DE is $\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{3x}$.

$PI = \frac{1}{D^2+D} e^{3x} = \frac{1}{3^2+3} e^{3x} = \frac{e^{3x}}{12}$.

(i) $PI = \frac{1}{D^2+a^2} \sin ax = \frac{x}{2D} \sin ax = \frac{x}{2}\int \sin ax \, dx = \frac{x}{2}\left[-\frac{\cos ax}{a}\right] = \frac{-x\cos ax}{2a}$.

(j). The wronskian of $\cos 2x$, $\sin 2x$ is

$$W(\cos 2x, \sin 2x) = \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix} = 2.$$

(k). $L(a^t) = L(e^{t \ln a}) = \frac{1}{s-\ln a}$.

(l). $L(t^2) = \frac{2}{s^3}$.

(m). Convolution theorem: If $L^{-1}\{\bar{f}(s)\} = f(t)$, and $L^{-1}\{\bar{g}(s)\} = g(t)$, then

$L^{-1}\{\bar{f}(s)\, \bar{g}(s)\} = F * G = \int_0^t f(u)\, g(t-u)\, du$.

(n). $L[\cos(2t+3)]$

$= L[\cos 2t \cos 3 - \sin 2t \sin 3]$

$= \cos 3 \; L(\cos 2t) - \sin 3 \; L(\sin 2t)$

$= \cos 3 \left(\frac{s}{s^2+2^2}\right) - \sin 3 \left(\frac{2}{s^2+2^2}\right)$

$= \frac{s\cos 3 - 2\sin 3}{s^2+4}$.

UNIT-I

2(a). Given $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

consider $[A \mid I_3] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 3 \\ 0 & 1 & 0 & 1 & 3 & -3 \\ 0 & 0 & 1 & -2 & -4 & -4 \end{array}\right]$.

$R_2 - R_1$; $R_3 + 2R_1$ $\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 2 & -6 & -1 & 1 & 0 \\ 0 & -2 & -2 & 2 & 0 & 1 \end{array}\right]$ $R_2/2$ $\sim \left[\begin{array}{ccc|ccc} 1 & 1 & 3 & 1 & 0 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & -2 & -2 & 2 & 1 & 0 \end{array}\right]$

$R_1 - R_2$; $R_3 + 2R_2$ $\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & -4 & 1 & 1 & 1 \end{array}\right]$ $R_3/-4$ $\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 6 & 3/2 & -1/2 & 0 \\ 0 & 1 & -3 & -1/2 & 1/2 & 0 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$

$R_1 - 6R_3$; $R_2 + 3R_3$ $\sim \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & 1 & 3/2 \\ 0 & 1 & 0 & -5/4 & -1/4 & -3/4 \\ 0 & 0 & 1 & -1/4 & -1/4 & -1/4 \end{array}\right]$

$\therefore A^{-1} = \begin{bmatrix} 3 & 1 & 3/2 \\ -5/4 & -1/4 & -3/4 \\ -1/4 & -1/4 & -1/4 \end{bmatrix} = \frac{-1}{4}\begin{bmatrix} -12 & -4 & -6 \\ 5 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix}$

2(b). Given system of equations are
$$2x + 3y + 5z = 9$$
$$7x + 3y - 2z = 8$$
$$2x + 3y + \lambda z = \mu.$$

The given system can be represented in the matrix form $AX = B$.
where $A = \begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & \lambda \end{bmatrix}$; $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$.

consider $K = [A \mid B] = \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$ $2R_2 - 7R_1$; $R_3 - R_1$ $\sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15 & -39 & -47 \\ 0 & 0 & \lambda-5 & \mu-9 \end{array}\right]$

The given system have
(i) No solution when $\lambda = 5$ & $\mu \neq 9$
(ii) a unique solution when $\lambda \neq 5$ & any μ.
(iii) an infinite no. of solutions when $\lambda = 5$ & $\mu = 9$.

(OR)

3(a). Given $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & 1\cancel{-\lambda} \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0.$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0.$$

By cayley-Hamilton theorem, every square matrix satisfies its own characteristic equation.

i.e, $A^3 - 6A^2 + 9A - 4I = 0.$

Multiplying with A^{-1} on both sides, we get

$$A^2 - 6A + 9I - 4A^{-1} = 0.$$

$$\Rightarrow A^{-1} = \frac{1}{4}(A^2 - 6A + 9I)$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left\{ \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right\}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

3(b). Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 2-\lambda & 0 & 1 \\ 0 & 2-\lambda & 0 \\ 1 & 0 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 11\lambda - 6 = 0$$

$$\Rightarrow \lambda = 1, 2, 3.$$

$\therefore$ The eigen values of A are $\lambda = 1, 2, 3.$

when $\lambda = 1$, the corresponding eigen vector is $X_1 = (1, 0, -1)$

when $\lambda = 2$, the corresponding eigen vector is $X_2 = (0, 1, 0)$

when $\lambda = 3$, the corresponding eigen vector is $X_3 = (1, 0, 1)$.

UNIT-II

4(a). Given DE is $(1+y^2)\,dx + (x - e^{-\tan^{-1}y})\,dy = 0$.

$$\Rightarrow (1+y^2)\,dx = (e^{-\tan^{-1}y} - x)\,dy$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{-\tan^{-1}y}}{1+y^2}$$

comparing this DE with $\frac{dx}{dy} + P(y)\,x = Q(y)$.

Here $P(y) = \frac{1}{1+y^2}$; $Q(y) = \frac{e^{-\tan^{-1}y}}{1+y^2}$.

$$IF = e^{\int P(y)\,dy} = e^{\int \frac{1}{1+y^2}dy} = e^{\tan^{-1}y}$$

The solution is $x(IF) = \int Q(y)\,(IF)\,dy + c$

$$\Rightarrow x\,e^{\tan^{-1}y} = \int \frac{e^{-\tan^{-1}y}}{1+y^2}\,e^{\tan^{-1}y}\,dy + c$$

$$\Rightarrow x\,e^{\tan^{-1}y} = \int \frac{1}{1+y^2}\,dy + c$$

$$\Rightarrow \boxed{x\,e^{\tan^{-1}y} = \tan^{-1}y + c}$$

4(b). By Newton's law of cooling, The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

i.e, $\frac{d\theta}{dt} = -K(\theta - \theta_0)$, where K is constant.

$$\Rightarrow \theta = \theta_0 + c\,e^{-Kt}$$

Given conditions, when $t=0$, $\theta = 80^\circ c$ —— ①

when $t=12$, $\theta = 60^\circ c$ —— ②

Temperature of the air, $\theta_0 = 30^\circ c$.

$$\therefore \theta = 30 + c\,e^{-Kt}$$

From ①, $80 = 30 + c\,e^{-K(0)} \Rightarrow c = 50$

From ②, $60 = 30 + 50\,e^{-12K} \Rightarrow K = -\frac{1}{12}\ln\left(\frac{3}{5}\right)$

$$\therefore \theta = 30 + 50\,e^{\frac{t}{12}\ln\left(\frac{3}{5}\right)}$$

The temperature of the body after 24 minutes is

$$\theta = 30 + 50\, e^{\frac{24}{12}\ln\left(\frac{3}{5}\right)}$$

$$\Rightarrow \theta = 48^\circ c.$$

(OR)

5(a). Given DE is $x \log x \frac{dy}{dx} + y = \log x^2$.

$$\Rightarrow \frac{dy}{dx} + \frac{y}{x \log x} = \frac{2\log x}{x \log x}$$

$$\Rightarrow \frac{dy}{dx} + \left(\frac{1}{x\log x}\right) y = \frac{2}{x}$$

comparing this DE with $\frac{dy}{dx} + P(x)\,y = Q(x)$.

Here $P(x) = \frac{1}{x \log x}$; $Q(x) = \frac{2}{x}$

$$IF = e^{\int P(x)\,dx} = e^{\int \frac{1}{x\log x}\,dx} = e^{\int \frac{1/x}{\log x}\,dx} = e^{\log(\log x)} = \log x.$$

The solution is $y(IF) = \int Q(x)\,(IF)\,dx + c$

$$\Rightarrow y(\log x) = \int \frac{2}{x}(\log x)\,dx + c$$

$$\Rightarrow y(\log x) = (\log x)^2 + c$$

$$\Rightarrow \boxed{y = \log x + \frac{c}{\log x}}$$

5(b). Given differential equation is

$$y e^{xy}\,dx + (x e^{xy} + 2y)\,dy = 0.$$

comparing the given differential equation with $M\,dx + N\,dy = 0$.

Here $M = y e^{xy}$; $N = x e^{xy} + 2y$

$$\Rightarrow \frac{\partial M}{\partial y} = e^{xy} + xy\,e^{xy} \; ; \; \frac{\partial N}{\partial x} = e^{xy} + xy\,e^{xy}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Thus the equation is exact and its solution is

$$\int_{(y \text{ constant})} M\,dx + \int (\text{Terms of } N \text{ not containing } x)\,dy = c$$

$$\Rightarrow \int_{(y \text{ constant})} y e^{xy}\,dx + \int 2y\,dy = c$$

$$\Rightarrow \boxed{e^{xy} + y^2 = c}$$

UNIT-III

6(a). Given differential equation is $\frac{d^2x}{dt^2} + n^2x = K\cos(nt+\alpha)$

Given equation in symbolic form is $(D^2+n^2)x = K\cos(nt+\alpha)$

It's AE is $D^2+n^2=0 \Rightarrow D = \pm ni$

Thus $CF = c_1\cos nt + c_2\sin nt$.

$$PI = \frac{1}{D^2+n^2} K\cos(nt+\alpha)$$

$$= K \cdot \frac{t}{2D}\cos(nt+\alpha)$$

$$= \frac{Kt}{2}\int \cos(nt+\alpha)\,dt = \frac{Kt}{2n}\sin(nt+\alpha).$$

Hence CS is $\boxed{x = c_1\cos nt + c_2\sin nt + \frac{Kt}{2n}\sin(nt+\alpha)}$

6(b). Given differential equation is $\frac{d^2y}{dx^2} + y = \tan x$.

Given equation in symbolic form is $(D^2+1)y = \tan x$.

It's AE is $D^2+1=0 \Rightarrow D = \pm i$

Thus $CF = c_1\cos x + c_2\sin x$.

Here $y_1 = \cos x$; $y_2 = \sin x$; $X = \tan x$

And $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} \cos x & \sin x \\ -\sin x & \cos x \end{vmatrix} = 1.$

By the method of variation of parameters,

$$PI = -y_1 \int \frac{y_2 X}{W} dx + y_2 \int \frac{y_1 X}{W} dx$$

$$= -\cos x \int \frac{\sin x \tan x}{1} dx + \sin x \int \frac{\cos x \tan x}{1} dx$$

$$= -\cos x \int (\sec x - \cos x) dx + \sin x \int \sin x \, dx$$

$$= -\cos x \left[\log(\sec x + \tan x) - \sin x \right] + \sin x (-\cos x)$$

$$= -\cos x \log(\sec x + \tan x).$$

Hence the CS is $\boxed{y = c_1 \cos x + c_2 \sin x - \cos x \log(\sec x + \tan x)}$

(OR)

7(a). Given DE is $(D^2 - 4D + 3) y = \sin 3x \cos 2x$.

It's AE is $D^2 - 4D + 3 = 0 \Rightarrow D = 1, 3$.

Thus $CF = c_1 e^x + c_2 e^{3x}$

$$PI = \frac{1}{D^2 - 4D + 3} \sin 3x \cos 2x$$

$$= \frac{1}{D^2 - 4D + 3} \cdot \frac{1}{2} (\sin 5x + \sin x)$$

$$= \frac{1}{2} \left[\frac{1}{-5^2 - 4D + 3} \sin 5x + \frac{1}{-1^2 - 4D + 3} \sin x \right]$$

$$= \frac{1}{2} \left[\frac{-1}{4D + 22} \sin 5x + \frac{1}{2 - 4D} \sin x \right]$$

$$= \frac{1}{4} \left[\frac{-1}{2D + 11} \left(\frac{2D - 11}{2D - 11} \right) \sin 5x + \frac{1}{1 - 2D} \left(\frac{1 + 2D}{1 + 2D} \right) \sin x \right]$$

$$= \frac{1}{4} \left[\frac{11 - 2D}{4D^2 - 121} \sin 5x + \frac{1 + 2D}{1 - 4D^2} \sin x \right]$$

$$= \frac{1}{4} \left[\frac{11 \sin 5x - 2D \sin 5x}{4(-5^2) - 121} + \frac{\sin x + 2D \sin x}{1 - 4(-1^2)} \right]$$

$$= \frac{1}{4} \left[\frac{1}{221} (10 \cos 5x - 11 \sin 5x) + \frac{1}{5} (\sin x + 2 \cos x) \right]$$

$$\Rightarrow PI = \frac{1}{884} (10 \cos 5x - 11 \sin 5x) + \frac{1}{20} (\sin x + 2 \cos x).$$

Hence CS is $\boxed{y = c_1 e^x + c_2 e^{3x} + \frac{1}{884} (10 \cos 5x - 11 \sin 5x) + \frac{1}{20} (\sin x + 2 \cos x)}$

7(b). Given differential equation is $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$.

Given DE in symbolic form is $(D^4 + 8D^2 + 16)y = 0$.

It's AE is $D^4 + 8D^2 + 16 = 0$

$\Rightarrow (D^2+4)^2 = 0$

$\Rightarrow D^2 = -4, -4$

$\Rightarrow D = \pm 2i, \pm 2i$.

Hence CS is $\boxed{y = (c_1 + c_2 x)\cos 2x + (c_3 + c_4 x)\sin 2x}$

UNIT-IV

8(a).
we have, $L\left(\frac{f(t)}{t}\right) = \int_s^\infty \bar{f}(s)\,ds$

$$\Rightarrow L\left[\frac{e^{-at} - e^{-bt}}{t}\right] = \int_s^\infty \left(\frac{1}{s+a} - \frac{1}{s+b}\right) ds$$

$$= \left(\log(s+a) - \log(s+b)\right]_s^\infty$$

$$= \left[\log\left(\frac{s+a}{s+b}\right)\right]_s^\infty$$

$$= \left(\log\left(\frac{1+a/s}{1+b/s}\right)\right]_s^\infty$$

$$= \log 1 - \log\left(\frac{1+a/s}{1+b/s}\right)$$

$$\Rightarrow \boxed{L\left(\frac{e^{-at} - e^{-bt}}{t}\right) = \log\left(\frac{s+b}{s+a}\right)}$$

8(b). $$L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right) = L^{-1}\left[\frac{s}{s^2+a^2} \cdot \frac{s}{s^2+a^2}\right]$$

$$= \cos at * \cos at$$

$$= \int_0^t \cos au \cos a(t-u)\,du$$

$$\Rightarrow L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right) = \int_0^t \cos au \, \cos(at-au)\, du$$

$$= \frac{1}{2}\int_0^t \left[\cos at + \cos(2au-at)\right] du$$

$$= \frac{1}{2}\left[u \cos at + \frac{\sin(2au-at)}{2a}\right]_0^t$$

$$= \frac{1}{2}\left[t \cos at + \frac{\sin at}{2a} - \frac{\sin(-at)}{2a}\right]$$

$$= \frac{1}{2}\left(t\cos at + \frac{\sin at}{a}\right)$$

$$\Rightarrow \boxed{L^{-1}\left(\frac{s^2}{(s^2+a^2)^2}\right) = \frac{1}{2a}(at\cos at + \sin at)}$$

(OR)

9.(a) (i) $L(\sin 2t) = \frac{2}{s^2+2^2}$

By first shifting property, $L(e^{3t}\sin 2t) = \frac{2}{(s-3)^2+4}$

$$\Rightarrow L(e^{3t}\sin 2t) = \frac{2}{s^2-6s+13}.$$

(ii) $L(\cos 2t + \sinh 2t) = \frac{s}{s^2+2^2} + \frac{2}{s^2-2^2}.$

By first shifting property,

$$L\left(e^{t}(\cos 2t + \sinh 2t)\right) = \frac{s-1}{(s-1)^2+4} + \frac{2}{(s-1)^2-4}.$$

$$\Rightarrow L\left[e^{t}(\cos 2t + \sinh 2t)\right] = \frac{s-1}{s^2-2s+5} + \frac{2}{s^2-2s-3}.$$

9(b). Given DE is $\frac{d^2y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t$; $y(0) = y'(0) = 0$.

$$\Rightarrow y'' + 2y' - 3y = \sin t$$

Taking Laplace transformation on both sides, we get

$$L(y''+2y'-3y) = L(\sin t)$$

$$\Rightarrow L(y'') + 2L(y') - 3L(y) = L(\sin t)$$

$$\Rightarrow \left(s^2 L(y) - s\,y(0) - y'(0)\right) + 2\left(s\,L(y) - y(0)\right) - 3L(y) = \frac{1}{s^2+1}$$

$$\Rightarrow L(y)\,(s^2+2s-3) = \frac{1}{s^2+1}$$

$$\Rightarrow L(y) = \frac{1}{(s^2+1)(s-1)(s+3)}.$$

let $\dfrac{1}{(s^2+1)(s-1)(s+3)} = \dfrac{A}{s-1} + \dfrac{B}{s+3} + \dfrac{Cs+D}{s^2+1}$

$$\Rightarrow 1 = A(s+3)(s^2+1) + B(s-1)(s^2+1) + (Cs+D)(s-1)(s+3)$$

If $s=1$, $1 = A(4)(2) \Rightarrow A = \frac{1}{8}$

If $s=-3$, $1 = B(-4)(10) \Rightarrow B = \frac{-1}{40}$.

comparing s^3 coefficient on both sides, $A+B+C=0 \Rightarrow C = \frac{-1}{10}$.

comparing constant terms on both sides, $3A-B-3D=1 \Rightarrow D = \frac{-1}{5}$.

$$\therefore L(y) = \frac{1}{8(s-1)} - \frac{1}{40(s+3)} + \frac{-\frac{1}{10}s - \frac{1}{5}}{s^2+1}$$

$$\Rightarrow y = \frac{1}{8}L^{-1}\left(\frac{1}{s-1}\right) - \frac{1}{40}L^{-1}\left(\frac{1}{s+3}\right) - \frac{1}{10}L^{-1}\left(\frac{s}{s^2+1}\right) - \frac{1}{5}L^{-1}\left(\frac{1}{s^2+1}\right)$$

$$\Rightarrow \boxed{y = \frac{1}{8}e^{t} - \frac{1}{40}e^{-3t} - \frac{1}{10}(\cos t + 2\sin t)}$$

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