18MA001
20MA101

Hall Ticket Number:

I/IV B.Tech (Regular) DEGREE EXAMINATION

July, 2021 — **Common to all branches**

First Semester — **Linear Algebra and ODE**

Time: Three Hours — **Maximum:** 70 Marks

Answer Question No.1 compulsorily. (14X1 = 14 Marks)

Answer ONE question from each unit. (4X14=56 Marks)

1 Answer all questions. (14X1=14 Marks)

a) Define minor of a matrix.

b) The maximum value of the Rank of a 4X5 matrix is…………………

c) If A = $\begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$ then find the sum and product of the eigen values of A.

d) Write Cayley – Hamilton theorem.

e) Write the differential equation corresponding to Newton's law of cooling.

f) Find the integrating factor of $(\sqrt{1-y^2})\,dx = (\sin^{-1} y - x)\ dy$.

g) Find the particular integral of $y^{II} - y^{I} + 2y = 12$.

h) Write the Wronskian value of $y_{1,}y_2$.

i) Write the differential equation of L-R-C circuit with an emf E=E_0 Sin(wt).

j) Find the general solution of $(D^2-2)^2y = 0$.

k) Find the value of L [3^t].

l) State first shifting property for Laplace transforms.

m) Find the value of $L^{-1}\left(\frac{1}{S^2-4}\right)$.

n) Write Convolution theorem for Laplace transforms.

UNIT I

2\. a) Use Gauss-Jordan method to find the inverse of the matrix $\begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$ 7M

b) For What value of 'k' the equations $x + y + z = 1, 2x + y + 4z = k , 4x + y + 10z = k^2$ have a solution and solve completely in each case. 7M

(OR)

3\. a) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ 7M

b) Verify Cayley – Hamilton theorem for the matrix $\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and find its inverse. 7M

UNIT II

4\. a) Solve $(1 + y^2)\,dx + (x - e^{-\tan^{-1}y})\,dy = 0$. 7M

b) If the air is maintained at 30°C and the temperature of the body cools from 80°C to 60°C in 12 minutes, find the temperature of the body after 24 minutes. 7M

(OR)

5\. a) Solve $ye^{xy}dx + (xe^{xy} + 2y)dy = 0$. 7M

b) Solve $2xy' = 10x^3y^5 + y$. 7M

UNIT III

6\. a) Solve $\frac{d^2x}{dt^2} + n^2x = k\cos(nt + \alpha)$. 7M

b) Solve by the method of variation of parameters $y'' - 6y' + 9y = \frac{e^x}{x^2}$. 7M

(OR)

7\. a) Solve $y'' - 4y' + 5y = 0, y(0) = 2, y'(0) = -1$. 7M

b) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$. 7M

P.T.O.

UNIT IV

8.	a)	Find the Laplace transform of (i)$(\sin t - \cos t)^2$ (ii) $\cos(at + b)$	7M
	b)	Find the inverse Laplace transform of $\frac{1}{s^3 - a^3}$.	7M
		(OR)	
9.	a)	Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$	7M
	b)	Solve $y'' + 5y' + 6y = 5e^{2t}$, Given $y(0)=0$ and $y'(0)=0$ using Laplace transforms.	7M

BAPATLA ENGINEERING COLLEGE, BAPATLA

DEPARTMENT OF MATHEMATICS

20MA01

I/IV B.Tech (Regular) DEGREE EXAMINATION

Linear Algebra And ODE Scheme of Evaluation — July, 2021

Common to all branches — First Semester

1(a). Minor of a matrix: A minor is the determinant of the square matrix formed by deleting one row and one column from some larger square matrix.

(b). The maximum value of the Rank of a 4×5 matrix is 4.

(c). $A = \begin{bmatrix} 4 & 5 \\ 6 & 7 \end{bmatrix}$

Sum of the eigen values of $A = 4 + 7 = 11$

Product of the eigen values of $A = \begin{vmatrix} 4 & 5 \\ 6 & 7 \end{vmatrix} = 28 - 30 = -2$

(d). Cayley-Hamilton theorem: Every square matrix satisfies its own characteristic equation.

(e). The differential equation corresponding to Newton's law of cooling is $\frac{dT}{dt} = -k(T - T_0)$.

(f). $(\sqrt{1-y^2})\,dx = (\sin^{-1}(y) - x)\,dx$

$$\Rightarrow \frac{dx}{dy} = \frac{\sin^{-1}(y) - x}{\sqrt{1-y^2}}$$

$$\Rightarrow \frac{dx}{dy} + \frac{x}{\sqrt{1-y^2}} = \frac{\sin^{-1}(y)}{\sqrt{1-y^2}}$$

$$IF = e^{\int \frac{1}{\sqrt{1-y^2}} dy} = e^{\sin^{-1}(y)}.$$

(g). $PI = \frac{1}{D^2-D+2} 12 = \frac{1}{D^2-D+2} 12 e^{0x} = \frac{1}{2} 12 e^{0x} = 6.$

(h). The wronskian of y_1, y_2 is $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$.

(i) The differential equation of L-R-C circuit with an emf $E = E_0 \sin(\omega t)$ is $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{q}{C} = E_0 \sin(\omega t)$.

(j). $(D^2-2)y = 0$.

Its AE is $(D^2-2)^2 = 0$.

$\Rightarrow D = \pm\sqrt{2}, \pm\sqrt{2}$

Hence the general solution is $y = (c_1 + c_2 x)e^{\sqrt{2}x} + (c_3 + c_4 x)e^{-\sqrt{2}x}$.

(k). $L[3^t] = L[e^{t\ln 3}] = \frac{1}{s-\ln 3}$. $\left(\because L(e^{at}) = \frac{1}{s-a}\right)$.

(l). First Shifting property for Laplace transforms:

If $L\{f(t)\} = \bar{f}(s)$ then $L\{e^{at} f(t)\} = \bar{f}(s-a)$.

(m). $L^{-1}\left[\frac{1}{s^2-4}\right] = L^{-1}\left[\frac{1}{s^2-2^2}\right] = \frac{1}{2}\sinh 2t.$

(n). Convolution theorem for Laplace transforms:

If $L^{-1}[\bar{f}(s)] = f(t)$ and $L^{-1}[\bar{g}(s)] = g(t)$ then

$L^{-1}[\bar{f}(s)\,\bar{g}(s)] = f * g.$

2(a). Let $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

Write the matrix A and the identity matrix side by side.

i.e, $[A_{3\times3} \mid I_{3\times3}]$

$$= \left[\begin{array}{ccc|ccc} 2 & 3 & 4 & 1 & 0 & 0 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 1 & 2 & 4 & 0 & 0 & 1 \end{array}\right]$$

R_{13}

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 4 & 3 & 1 & 0 & 1 & 0 \\ 2 & 3 & 4 & 1 & 0 & 0 \end{array}\right]$$

$R_2 - 4R_1$; $R_3 - 2R_1$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & -5 & -15 & 0 & 1 & -4 \\ 0 & -1 & -4 & 1 & 0 & -2 \end{array}\right]$$

$\left(-\frac{1}{5}\right)R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 2 & 4 & 0 & 0 & 1 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & -1 & -4 & 1 & 0 & -2 \end{array}\right]$$

$R_1 - 2R_2$; $R_3 + R_2$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & -1 & 1 & -1/5 & -6/5 \end{array}\right]$$

$(-1)R_3$

$$\sim \left[\begin{array}{ccc|ccc} 1 & 0 & -2 & 0 & 2/5 & -3/5 \\ 0 & 1 & 3 & 0 & -1/5 & 4/5 \\ 0 & 0 & 1 & -1 & 1/5 & 6/5 \end{array}\right]$$

$R_1 + 2R_3$
$R_2 - 3R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & 4/5 & 9/5 \\ 0 & 1 & 0 & 3 & -4/5 & -14/5 \\ 0 & 0 & 1 & -1 & 1/5 & 6/5 \end{array}\right].$$

$$\therefore A^{-1} = \begin{bmatrix} -2 & \frac{4}{5} & \frac{9}{5} \\ 3 & -\frac{4}{5} & \frac{-14}{5} \\ -1 & \frac{1}{5} & \frac{6}{5} \end{bmatrix}.$$

2(b). The given system of equations are $x+y+z=1$, $2x+y+4z=k$, $4x+y+10z=k^2$.

The system of equations in matrix notation is given by $AX=B$.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix} ; \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} ; \quad B = \begin{bmatrix} 1 \\ k \\ k^2 \end{bmatrix}.$$

The augmented matrix of the given system of equations is

$$[A:B] = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & k \\ 4 & 1 & 10 & k^2 \end{array}\right]$$

R_2-2R_1 ; R_4-4R_1

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & -3 & 6 & k^2-4 \end{array}\right]$$

R_3-3R_2

$$\sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & k-2 \\ 0 & 0 & 0 & k^2-3k+2 \end{array}\right]$$

Case(i): If $k^2-3k+2 \neq 0$ then $\rho(A) \neq \rho(A:B)$. $\rho(A)=2$, $\rho(A:B)=3$.

Hence the system is inconsistent and it has no solution.

Case(ii): If $k^2-3k+2=0 \Rightarrow k=1,2$.

in such case $\rho(A)=2$, $\rho(A:B)=2$, $n=3$. $\therefore \rho(A)=\rho(A:B) < n$.

when $k=1$, the augmented matrix becomes $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Let $z=k_1$, then $y=1+2k_1$, $x=-3k_1$.

$\therefore X = \begin{bmatrix} -3k_1 \\ 1+2k_1 \\ k_1 \end{bmatrix}$ be the solution of given system of equations.

when $k=2$, the augmented matrix becomes $\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

Let $z=k_2$, then $y=2k_2$, $x=1-3k_2$.

$\therefore X = \begin{bmatrix} 1-3k_2 \\ 2k_2 \\ k_2 \end{bmatrix}$ be the solution of given system of equations.

3 (a). Let $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$

$$\Rightarrow \begin{vmatrix} 8-\lambda & -6 & 2 \\ -6 & 7-\lambda & -4 \\ 2 & -4 & 3-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 18\lambda^2 + 45\lambda = 0$$

$$\Rightarrow \lambda = 0, 3, 15.$$

when $\lambda = 0$, the corresponding eigen vector $X_1 = (1,2,2)$.

when $\lambda = 3$, the corresponding eigen vector $X_2 = (2,1,-2)$

when $\lambda = 15$, the corresponding eigen vector $X_3 = (2,-2,1)$.

3(b). Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

The characteristic equation of A is $|A - \lambda I| = 0$.

$$\Rightarrow \begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0.$$

By cayley - Hamilton theorem, every square matrix satisfies its own characteristic equation.

i.e, $A^3 - 6A^2 + 9A - 4I = 0$.

$$A^2 = A \cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^3 = A^2 \cdot A = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 22 & -21 & 22 \end{bmatrix}$$

$\therefore A^3 - 6A^2 + 9A - 4I = 0$.

Hence cayley- Hamilton theorem is verified.

$$A^3 - 6A^2 + 9A - 4I = 0.$$

Multiplying with A^{-1} on both sides, we get

$$A^2 - 6A + 9I - 4A^{-1} = 0$$

$$\Rightarrow A^{-1} = \frac{1}{4}\left(A^2 - 6A + 9I\right)$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

UNIT-II

4(a). $(1+y^2)\,dx + \left(x - e^{-\tan^{-1}y}\right)dy = 0.$

$$\Rightarrow (1+y^2)\,dx = \left(e^{-\tan^{-1}y} - x\right)dy$$

$$\Rightarrow \frac{dx}{dy} = \frac{e^{-\tan^{-1}y}}{1+y^2} - \frac{x}{1+y^2} \Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{e^{-\tan^{-1}y}}{1+y^2}.$$

Comparing this DE with $\frac{dx}{dy} + P(y)x = Q(y)$

Here $P(y) = \frac{1}{1+y^2}$; $Q(y) = \frac{e^{-\tan^{-1}(y)}}{1+y^2}$

$$IF = e^{\int P(y)\,dy} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1}(y)}.$$

The solution is

$$x\,(IF) = \int Q(y)\,(IF)\,dy + C$$

$$\Rightarrow x\,e^{\tan^{-1}y} = \int \frac{e^{-\tan^{-1}y}}{1+y^2}\cdot e^{\tan^{-1}y}\,dy + C$$

$$\Rightarrow x\,e^{\tan^{-1}y} = \int \frac{1}{1+y^2}\,dy + C$$

$$\Rightarrow x\,e^{\tan^{-1}y} = \tan^{-1}y + C.$$

4(b). By Newtons law of cooling,

The temperature of a body changes at a rate which is proportional to the difference in temperature between that of the surrounding medium and that of the body itself.

i.e, $\frac{dT}{dt} = -k(T - T_0)$, where k is constant.

$$\Rightarrow T = T_0 + ce^{-kt}.$$

Given conditions, when $t=0$, $T = 80°c$ —— ①

when $t = 12$, $T = 60°c$ —— ②

Temperature of the air, $T_0 = 30°c$.

$$\therefore T = 30 + ce^{-kt}$$

From ①, $80 = 30 + c \Rightarrow c = 50.$

From ②, $60 = 30 + 50\, e^{-12k} \Rightarrow k = \frac{-1}{12}\ln\left(\frac{3}{5}\right).$

$$\therefore T = 30 + 50\, e^{\frac{t}{12}\ln\left(\frac{3}{5}\right)}.$$

The temperature of the body after 24 minutes is

$$T = 30 + 50\, e^{\frac{24}{12}\ln\left(\frac{3}{5}\right)}$$

$$\Rightarrow T = 48°c.$$

(OR)

5(a). Given differential equation is

$$y\, e^{xy}\, dx + (x\, e^{xy} + 2y)\, dy = 0.$$

Comparing the given differential equation with $M\,dx + N\,dy = 0$.

Here $M = y\, e^{xy}$; $N = x\, e^{xy} + 2y$.

$$\frac{\partial M}{\partial y} = e^{xy} + xy\, e^{xy} \quad ; \quad \frac{\partial N}{\partial x} = e^{xy} + xy\, e^{xy}$$

$$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

Thus equation is exact and its solution is

$$\underset{(y\ \text{constant})}{\int} M\,dx + \int (\text{Terms of } N \text{ not containing } x)\, dy = c.$$

$$\Rightarrow \underset{(y\ \text{constant})}{\int} y e^{xy}\, dx + \int 2y\, dy = c$$

$$\Rightarrow \boxed{e^{xy} + y^2 = c}$$

5(b). Given differential equation is

$$2x\, y' = 10x^3y^5 + y$$

$$\Rightarrow 2x\frac{dy}{dx} - y = 10x^3y^5.$$

$$\Rightarrow \frac{1}{y^5}\frac{dy}{dx} - \frac{1}{2x}\cdot\frac{1}{y^4} = 5x^2$$

$$\Rightarrow -\frac{1}{4}\frac{dz}{dx} - \frac{1}{2x} z = 5x^2 \quad \left(\text{Let } \frac{1}{y^4} = z \Rightarrow \frac{-4}{y^5}\frac{dy}{dx} = \frac{dz}{dx} \Rightarrow \frac{1}{y^5}\frac{dy}{dx} = \frac{-1}{4}\frac{dz}{dx}\right)$$

$$\Rightarrow \frac{dz}{dx} + \frac{2}{x} z = -20x^2.$$

$$IF = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2.$$

The solution is $z(IF) = \int -20x^2\,(IF)\,dx + c$

$$\Rightarrow \frac{1}{y^4}(x^2) = \int -20\, x^2 (x^2)\, dx + c$$

$$\Rightarrow \frac{x^2}{y^4} = -4x^5 + c.$$

$$\Rightarrow \boxed{x^2 + (4x^5 + c)\, y^4 = 0}$$

UNIT- III

6(a). Given differential equation is $\frac{d^2x}{dt^2} + n^2x = k \cos(nt+\alpha)$.

Given equation in symbolic form is $(D^2+n^2)x = k\cos(nt+\alpha)$

(i) To find CF: Its AE is $D^2+n^2=0 \Rightarrow D = \pm ni$

Thus $CF = c_1 \cos nt + c_2 \sin nt$.

(ii) To find PI:

$$PI = \frac{1}{D^2+n^2} k\cos(nt+\alpha)$$

$$= k \frac{t}{2D}\cos(nt+\alpha)$$

$$= \frac{kt}{2}\int \cos(nt+\alpha)\,dt$$

$$= \frac{kt}{2}\frac{\sin(nt+\alpha)}{n}.$$

Thus $PI = \frac{kt}{2n}\sin(nt+\alpha)$

Hence the CS is $y = c_1\cos nt + c_2 \sin nt + \frac{kt}{2n}\sin(nt+\alpha)$.

6(b). Given differential equation is $y'' - 6y' + 9y = \frac{e^x}{x^2}$.

In symbolic form, $(D^2-6D+9)y = \frac{e^x}{x^2}$.

AE is $D^2-6D+9=0 \Rightarrow (D-3)^2=0 \Rightarrow D = 3, 3.$

$\therefore CF = (c_1+c_2x)e^{3x}$.

Here $y_1 = e^{3x}$; $y_2 = xe^{3x}$; and $X = \frac{e^x}{x^2}$

$$\therefore W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = \begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & e^{3x}+3xe^{3x} \end{vmatrix} = e^{6x}$$

Thus $PI = -y_1\int \frac{y_2 X}{W}dx + y_2\int\frac{y_1 X}{W}dx.$

$$\Rightarrow PI = -e^{3x}\int \frac{x e^{3x}}{e^{6x}} \cdot \frac{e^{x}}{x^2}\, dx + x e^{3x}\int \frac{e^{3x}}{e^{6x}} \cdot \frac{e^{x}}{x^2}\, dx$$

$$= -e^{3x}\int \frac{1}{x e^{2x}}\, dx + x e^{3x}\int \frac{1}{x^2 e^{2x}}\, dx.$$

(OR)

7(a). $y'' - 4y' + 5y = 0$, $y(0) = 2$, $y'(0) = -1$.

Given equation in symbolic form is $(D^2 - 4D + 5)y = 0$.

AE is $D^2 - 4D + 5 = 0 \Rightarrow D = 2 \pm i$

Hence the solution is $y = e^{2x}[c_1 \cos x + c_2 \sin x]$

Given $y(0) = 2$, $\therefore c_1 = 2$

And $y' = 2e^{2x}[c_1 \cos x + c_2 \sin x] + e^{2x}[-c_1 \sin x + c_2 \cos x]$

Also given $y'(0) = -1$, $2c_1 + c_2 = -1$

$\Rightarrow c_2 = -5.$

Hence the particular solution is $y = e^{2x}[2\cos x - 5 \sin x]$.

7(b). $(D^2 - 4D + 3)y = \sin 3x \cos 2x.$

(i) <u>To find CF</u>: Its AE is $D^2 - 4D + 3 = 0$

$\Rightarrow D = 1, 3.$

$\therefore CF = c_1 e^{x} + c_2 e^{3x}.$

(ii) <u>To find PI</u>:

$$PI = \frac{1}{D^2 - 4D + 3} \sin 3x \cos 2x.$$

$$\Rightarrow PI = \frac{1}{D^2-4D+3} \frac{1}{2}(\sin 5x + \sin x)$$

$$= \frac{1}{2}\left[\frac{1}{-5^2-4D+3}\sin 5x + \frac{1}{-1^2-4D+3}\sin x\right]$$

$$= \frac{1}{2}\left[\frac{-1}{4D+22}\sin 5x + \frac{1}{2-4D}\sin x\right]$$

$$= \frac{1}{4}\left[\frac{-1}{2D+11}\left(\frac{2D-11}{2D-11}\right)\sin 5x + \frac{1}{1-2D}\left(\frac{1+2D}{1+2D}\right)\sin x\right]$$

$$= \frac{1}{4}\left[\frac{11-2D}{4D^2-121}\sin 5x + \frac{1+2D}{1-4D^2}\sin x\right]$$

$$= \frac{1}{4}\left[\frac{11\sin 5x - 2D\sin 5x}{4(-5^2)-121} + \frac{\sin x + 2D\sin x}{1-4(-1^2)}\right]$$

$$= \frac{1}{4}\left[\frac{1}{221}(10\cos 5x - 11\sin 5x) + \frac{1}{5}(\sin x + 2\cos x)\right]$$

$$\Rightarrow PI = \frac{1}{884}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(\sin x + 2\cos x)$$

Hence CS is $y = c_1 e^{x} + c_2 e^{3x} + \frac{1}{884}(10\cos 5x - 11\sin 5x) + \frac{1}{20}(\sin x + 2\cos x)$

UNIT-IV

8(a). (i) $L[(\sin t - \cos t)^2]$

$$= L[\sin^2 t + \cos^2 t - 2\sin t\cos t]$$

$$= L[1 - \sin 2t]$$

$$= L(1) - L(\sin 2t)$$

$$= \frac{1}{s} - \frac{2}{s^2+2^2}$$

$$= \frac{s^2-2s+4}{s(s^2+4)}$$

8(a). (ii) $L[\cos(at+b)]$

$$= L[\cos at \cos b - \sin at \sin b]$$

$$= \cos b \; L(\cos at) - \sin b \; L(\sin at)$$

$$= \cos b \left(\frac{s}{s^2+a^2}\right) - \sin b \left(\frac{a}{s^2+a^2}\right)$$

$$= \frac{s\cos b - a\sin b}{s^2+a^2}.$$

8(b). $L^{-1}\left[\frac{1}{s^3-a^3}\right].$

consider $\frac{1}{s^3-a^3} = \frac{1}{(s-a)(s^2+as+a^2)} = \frac{A}{s-a} + \frac{Bs+C}{s^2+sa+a^2}$ (say)

$$\Rightarrow \quad 1 = A(s^2+as+a^2) + (Bs+c)(s-a).$$

$$\Rightarrow \quad 1 = A(s^2+as+a^2) + (Bs^2 - Bas + cs - ac)$$

comparing s^2 coefficient on both sides : $0 = A+B \Rightarrow A = -B.$

comparing s coefficient on both sides : $0 = Aa - Ba + C$

$\Rightarrow 2aA + C = 0$ —— ①

comparing constant term on both sides: $a^2A - aC = 1$ —— ②

solving equations ① & ②, we get $C = \frac{-2}{3a}$, $B = \frac{-1}{3a^2}$, $A = \frac{1}{3a^2}.$

$$\therefore \frac{1}{s^3-a^3} = \frac{1}{3a^2(s-a)} + \frac{\frac{-1}{3a^2}s - \frac{2}{3a}}{s^2+as+a^2}.$$

$$\Rightarrow L^{-1}\left[\frac{1}{s^3-a^3}\right] = \frac{1}{3a^2}L^{-1}\left[\frac{1}{s-a}\right] - \frac{1}{3a^2}L^{-1}\left[\frac{s}{s^2+as+a^2}\right] - \frac{2}{3a}L^{-1}\left[\frac{1}{s^2+as+a^2}\right].$$

$$\Rightarrow L^{-1}\left[\frac{1}{s^3-a^3}\right] = \frac{1}{3a^2} L^{-1}\left[\frac{1}{s-a}\right] - \frac{1}{3a^2} L^{-1}\left[\frac{s}{(s+\frac{a}{2})^2+\frac{3a^2}{4}}\right] - \frac{2}{3a} L^{-1}\left[\frac{1}{(s+\frac{a}{2})^2+\frac{3a^2}{4}}\right]$$

$$= \frac{1}{3a^2} L^{-1}\left[\frac{1}{s-a}\right] - \frac{1}{3a^2} L^{-1}\left[\frac{s+a/2}{(s+\frac{a}{2})^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]$$

$$+ \frac{1}{3a^2} L^{-1}\left[\frac{a/2}{(s+\frac{a}{2})^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right] - \frac{2}{3a} L^{-1}\left[\frac{1}{(s+\frac{a}{2})^2+\left(\frac{\sqrt{3}a}{2}\right)^2}\right]$$

$$= \frac{1}{3a^2} e^{at} - \frac{1}{3a^2} e^{-\frac{at}{2}} \cos\left(\frac{\sqrt{3}\,at}{2}\right) + \frac{1}{3a^2}\,\frac{a}{2}\,\frac{2}{\sqrt{3}a}\, e^{-\frac{at}{2}} \sin\left(\frac{\sqrt{3}\,at}{2}\right)$$

$$- \frac{2}{3a} \cdot \frac{2}{\sqrt{3}a} \sin\left(\frac{\sqrt{3}\,at}{2}\right)$$

$$= \frac{1}{3a^2} e^{at} - \frac{1}{3a^2} e^{-\frac{at}{2}} \cos\left(\frac{\sqrt{3}\,at}{2}\right) + \frac{1}{3\sqrt{3}a^2} e^{-\frac{at}{2}} \sin\left(\frac{\sqrt{3}\,at}{2}\right)$$

$$- \frac{4}{3\sqrt{3}a^2} e^{-\frac{at}{2}} \sin\left(\frac{\sqrt{3}\,at}{2}\right)$$

$$\Rightarrow L^{-1}\left[\frac{1}{s^3-a^3}\right] = \frac{1}{3a^2}\left[e^{at} - e^{-\frac{at}{2}}\left\{\cos\left(\frac{\sqrt{3}\,at}{2}\right) + \sqrt{3}\sin\left(\frac{\sqrt{3}\,at}{2}\right)\right\}\right]$$

9 (a). $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$

By convolution theorem, $L^{-1}[\bar{f}(s)\ \bar{g}(s)] = f * g$

$$\Rightarrow L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = L^{-1}\left[\frac{s}{(s^2+a^2)} \cdot \frac{1}{(s^2+a^2)}\right]$$

$$= L^{-1}\left(\frac{s}{s^2+a^2}\right) * L^{-1}\left(\frac{1}{s^2+a^2}\right)$$

$$= \cos at * \frac{1}{a}\sin at$$

$$= \frac{1}{a}\int_0^t \cos au \ \sin(at-au)\,du$$

$$\Rightarrow L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{1}{2a}\int_0^t [\sin at - \sin(2au-at)]\, du$$

$$= \frac{1}{2a}\left[\sin at\,(u) + \frac{\cos(2au-at)}{2a}\right]_0^t$$

$$= \frac{1}{2a}\left[t \sin at + \frac{\cos at}{2a} - \left(0 + \frac{\cos(-at)}{2a}\right)\right]$$

$$= \frac{t}{2a}\sin at.$$

$$\therefore L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right] = \frac{t}{2a}\sin at.$$

9(b). $y'' + 5y' + 6y = 5e^{2t}$; $y(0) = 0$, $y'(0) = 0$.

Given differential equation is $y'' + 5y' + 6y = 5e^{2t}$

Taking Laplace transform on both sides

$$L[y'' + 5y' + 6y] = L[5e^{2t}]$$

$$\Rightarrow L(y'') + 5L(y') + 6L(y) = 5 L(e^{2t})$$

$$\Rightarrow [s^2\bar{y}(s) - s\,y(0) - y'(0)] + 5[s\bar{y}(s) - y(0)] + 6\bar{y}(s) = 5\frac{1}{s-2}$$

$$\Rightarrow (s^2 + 5s + 6)\,\bar{y}(s) = \frac{5}{s-2}$$

$$\Rightarrow \bar{y}(s) = \frac{5}{(s-2)(s+2)(s+3)}$$

Let $\frac{5}{(s-2)(s+2)(s+3)} = \frac{A}{s-2} + \frac{B}{s+2} + \frac{C}{s+3}$.

$$\Rightarrow 5 = A(s+2)(s+3) + B(s-2)(s+3) + C(s-2)(s+2)$$

If $s = 2$, $A = \frac{1}{4}$

If $s = -2$, $B = \frac{-5}{4}$

If $s = -3$, $C = 1$.

$$\therefore \; L[y(t)] = \bar{y}(s) = \frac{1}{4(s-2)} - \frac{5}{4(s+2)} + \frac{1}{s+3}$$

Taking the inverse Laplace transform on both sides, we get

$$y(t) = L^{-1}\left[\frac{1}{4(s-2)} - \frac{5}{4(s+2)} + \frac{1}{s+3}\right]$$

$$\Rightarrow y(t) = \frac{1}{4} L^{-1}\left(\frac{1}{s-2}\right) - \frac{5}{4} L^{-1}\left(\frac{1}{s+2}\right) + L^{-1}\left(\frac{1}{s+3}\right)$$

$$\Rightarrow y(t) = \frac{1}{4} e^{2t} - \frac{5}{4} e^{-2t} + e^{-3t}$$

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