20CB/CE/CS/DS/EC/EE/EI/IT/ME 101(MA01)

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I/IV B.Tech (Supplementary) DEGREE EXAMINATION

Firs	st Se	,	Common to all branches Linear Algebra and ODE Maximum: 70 Marks						
Answer Question No.1 compulsorily. Answer ONE question from each unit. 1 Answer all questions		ONE question from each unit.	(1X14 = 14 Marks) (4X14=56 Marks) (1X14=14 Marks)						
	a) b)	Define rank of a matrix. When the systems of equations has an infinite number of solutions.	CO1						
	c)	If $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 3 & 5 \\ 0 & () & -2 \end{bmatrix}$ then the Eigen values of A^2 are	COI						
	d)	Find the order of the differential equation $(1 + y_1^2)^{\frac{3}{2}} = C y_2$.	CO2						
	e)	Find the integrating factor of the differential equation $\frac{dy}{dx} + y \cos x = \frac{\sin 2x}{2}$.	CO2						
	f)	Write general form of a Line ir differential equation of first order.	CO2						
	g)	Solve $y^{11} - 4y^{1} - 5y = 0$.	CO3						
	h)	Find the P.I of $(1)^2 - 16$ y = $\sin 2x$.	CO3						
	i)	Find the Wror skian of cos2x,sin2x.	CO3						
	j)	Find Inverse Laplace transform of $\frac{1}{(s+a)(s+b)}$.	CO4						
	k)	Find the value of L [3^t].	CO4						
	1)	Find $L[e^{at}]$	CO4						
	m)	Write Convolution theorem for Laplace transforms.	CO4						
	n)	State change of scale property.	CO4						
		UNIT I							
2.	a)	Use Gauss-Jordan method, find the inverse of a matrix $A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \end{bmatrix}$.	COI 7M						

2.	a)		[8	4	3]	COL	7M
		Use Gauss-Jorda i method, find the inverse of a matrix A =					
					1		
	10.3	Find the values of to tend the for which the equations $v + o$		~~~		 diameter 4	

Find the values of 'a 'and 'b' for which the equations x+ay+z=3, x+2y+2z=b, x+5y+3z = 9 are consistent. When will the equations have a unique solution.

3. a) Verify Cayley – familton theorem for the matrix
$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$
 and find its inverse.

b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

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UNIT II

- 4. a) Solve $xy(1+x_1/2)\frac{dy}{dx} = 1$. CO₂ 7M
 - b) The number N of bacteria in a culture grow at a rate proportional to N. The value of N will CO2 7Mwas initially 100 and increased to 332 in one hour. What would be value of N after $1\frac{1}{2}$ hours?.

- 5. a) Solve $(1+y^2)dx + (x-e^{-\tan^{-1}y})dy = 0$. CO₂ 7M
 - b) Solve $(x^2 ay) dx = (ax y^2) dy$. CO₂ 7M

UNIT III

- 6. a) Solve $\frac{d^2y}{dx^2} + 4y = \sin x$. CO₃ 7M
 - b) $solve \frac{d^2y}{dx^2} + a^2y = \cot ax$ by the method of variation of parameters. CO₃ 7M

(OR)

- 7. a) Solve $\frac{d^2y}{dx^2} + 16y = x \sin 3x$. b) Solve $(D^2 + 5D + 6)y = x^2 + 2x + 4$. CO₃ 7M
 - CO3 7M

UNIT IV

- Find the Laplace transform of $\left(\sqrt{t} \frac{1}{\sqrt{t}}\right)^3$ CO4 7M 8.
 - Apply Convolution theorem to evaluate $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$. CO4 7M

- (OR) Find the Laplace transform of $f(t) = |t 1| + |t + 1|, t \ge 0$ CO4 7M
 - CO₄ 7M Solve $\frac{d^2x}{dt^2} + 9x = \cos 2t$, i'x (0) =1, x ($\frac{\pi}{2}$) = -1 by transform method.



DEPARTMENT OF MATHEMATICS

I/N B Tech (Supplementary) DEGREE EXAMINATION
Linear Algebra And ODE Scheme of Evaluation RO (B/CE/CS/ DS/EC/EE/EI/IT/ME IOI (MAOI)

1(a). Rank of a matrix: A matrix is said to be of nank n when

(i) it has at least one non-zero minot of order 17, and

- (a) every minor of order higher than it vanishes
- (b). S(A) = S(A:B) < n, then the system of equations has an infinite number of solutions.

(1). The eigen values of A^2 are 1,4,9.

(d) Given DE is $(1+y_1^2)^{3/2} = cy_2$.

(e). The IF of the DE $\frac{dy}{dx} + y \cos x = \frac{\sin 2x}{2}$ is $e = e^{-\frac{x}{2}}$

(f). The general form of a linear DE of first order is $\frac{dy}{dx} + P(x) y = Q(x).$

(g). Given DE is y"-4y'-5y=0. It's symbolic form is $(D^{2}-4D-5)y=0$ AE is (D-5)(D+1)=0

:. cs is y= q ex + & ex

(h). $PT = \frac{1}{D^2-16}$ sinex = $\frac{1}{-2^2-16}$ sinex = $\frac{-1}{20}$ sinex.

(i). Wronskian of corex, sinex is $w(corex, sinex) = \begin{vmatrix} corex & sinex \\ -2sinex & 2corex \end{vmatrix} = 2$.

(i) $\vec{L}\left[\frac{1}{(S+\alpha)(S+b)}\right] = \vec{L}\left[\frac{1}{a-b}\left\{\frac{1}{S+b} - \frac{1}{S+a}\right\}\right] = \frac{1}{a-b}\left[\frac{1}{e^{bt}} - \frac{1}{e^{at}}\right]$

(k).
$$\lfloor (3^{t}) \rfloor = \lfloor (e^{t \ln 3}) \rfloor = \frac{1}{S - \ln 3}$$

(1)
$$L\left(e^{\alpha t}\right) = \frac{1}{s-\alpha}$$

(i)
$$L = \overline{s-a}$$

(ii) $L = \overline{s-a}$
(iii) $L = \overline{s-a}$
(iii) $L = \overline{s-a}$
(iii) $L = \overline{s-a}$
(iv) $L =$

(n). If
$$L[f(t)] = \overline{f}(s)$$
 then $L[f(at)] = \frac{1}{\alpha} \overline{f}(\frac{s}{\alpha})$.

UNIT-I

2(a) Given
$$A = \begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{bmatrix}$$

convider $[A: T_3] = \begin{bmatrix} 8 & 4 & 3 & | & 1 & 0 & 0 \\ 2 & 1 & 1 & | & 0 & 1 & 0 \\ 1 & 2 & 1 & | & 0 & 0 & 1 \end{bmatrix}$

$$\sim \begin{bmatrix} 1 & 0 & 0 & | \frac{1}{3} & -2\frac{1}{3} & -\frac{1}{3} \\ 0 & 0 & 1 & | & 1 & 4 & 0 \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & -2 & -1 \\ 1 & -5 & 2 \\ -1 & 4 & 0 \end{bmatrix}.$$

2(b). Given equations are x+ay+z=3, x+2y+2z=b, x+5y+3z=9. Augmented matrix, [A:B] = [1 a 1 3]

$$R_2 - R_1$$
; $R_3 - R_1$
 $\sim \begin{bmatrix} 1 & a & 1 & 3 \\ 0 & 2 - a & 1 & b - 3 \\ 0 & 5 - a & 2 & 6 \end{bmatrix}$

$$(z-a) R_3 - (5-a) R_2$$

$$\sim \begin{bmatrix}
1 & \alpha & 1 & 3 \\
0 & 2-\alpha & 1 & b-3 \\
0 & 0 & -\alpha-1 & 27-9\alpha-5b+\alpha b
\end{bmatrix}$$

If a \(-1 \), b has any value, equations will be consistent and have a unique solution.

(OR)

3(a). Let
$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

The characteristic equation of A is
$$|A - \lambda I| = 0$$

$$\Rightarrow \begin{vmatrix} 2 - \lambda & -1 & 1 \\ -1 & 2 - \lambda & -1 \\ 1 & -1 & 2 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$$

$$\Rightarrow \lambda^3 - 6\lambda^2 + 9\lambda - 4 = 0$$

By cayley- Hamilton theorem, Every square matrix satisfies its own characteristic equation. i.e, $A^3 - 6A^2 + 9A - 4I = 0$.

$$A^{2} = A \cdot A = \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix}$$

$$A^{3} = A^{2} A = \begin{bmatrix} 22 & -21 & 21 \\ -21 & 22 & -21 \\ 22 & -21 & 22 \end{bmatrix}$$

:. A³-6A²+9A-4I=0. Hence cayley- Hamilton theorem is verified.

3(b). Let
$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

The characteristic equation of A is
$$|A - \lambda \Sigma| = 0$$

$$\Rightarrow \begin{vmatrix} 8 - \lambda & -6 & 2 \\ -6 & 7 - \lambda - 4 \\ 2 & -4 & 3 - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \lambda^3 - 18 \lambda^2 + 45 \lambda = 0.$$

$$\Rightarrow \lambda = 0, 3, 15$$

when $\lambda=0$, the coursending eigen vector, $X_1=(1,2,2)$ when 1=3, the corresponding eigen vector, $x_2=(2,1,-2)$ when $\lambda=15$, the corresponding eigen vector, $X_3=\{2,-2,1\}$.

4(a). Given
$$xy(1+xy^2)\frac{dy}{dx}=1$$

$$\Rightarrow \frac{dx}{dy} - xy = x^2y^3$$

$$\Rightarrow \frac{dx}{dy} - \frac{dy}{dy} - \frac{dy}{dy} = y^3 - 0 ; let \frac{1}{x} = z \Rightarrow \frac{-1}{x^2} \frac{dx}{dy} = \frac{dz}{dy}$$

$$0 \Rightarrow \frac{d^2}{dy} + y^2 = -y^3$$
, which is leibnitz's linear in 2.

IF = e /y dy = e 1/2

The solution is
$$7(IF) = \int (-y^3)(IF) dy + C$$

$$\Rightarrow 7 e^{3/2} = \int (-y^3) e^{3/2} dy + C$$

$$= \int (-y^3) e^{3/2} dy + C$$

$$= -2 \int t e^t dt + C$$

$$= -2\int te^{-t} dt - e^{-t} dt + c$$

$$= -2 \left(+ e^{2} - e^{2} \right)$$

$$\Rightarrow z = (z - y^{\nu}) + (e^{-y/2})$$

4(b). Let 'N' be the amount of substance at the time t. By law of natural growth and Decay dN & N.

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let = t

= y dy = dt

In initial condition, t=0, N=100.

Also given N=332 when t=1hr.

$$\therefore 332 = 100 e^{K} = 1 K = \log (3.32)$$

we have to find N when $t = 1\frac{1}{2} hr = \frac{3}{2} hrs.$

$$N = 100 e^{(3.32) \cdot \frac{3}{2}}$$

$$= 100 e (3.32)^{3/2}$$

$$= 100 e (3.32)^{3/2}$$

$$= 100 \left(3.32 \right)^{3/2}$$

How
$$P = \frac{1}{1+y^2}$$
 and $Q = \frac{1}{2} \frac{1+y^2}{1+y^2}$ which is linear in Z .

Thus the solution of DE is
$$x(IF) = \int Q(IF) dy + c$$

 $\Rightarrow x e^{\tan^2 y} = \int \frac{e^{-\tan^2 y}}{1+y^2} \cdot e^{\tan^2 y} dy + c$
 $\Rightarrow x e^{\tan^2 y} = \tan^2 y + c$ is the solution

5(b). Given DE is
$$(x^2-ay) dx = (ax-y^2) dy$$
.

$$\Rightarrow (x^2-ay) dx + (y^2-ax) dy = 0$$

Here
$$M = x^2 - ay$$
 and $N = y^2 - ax$

$$= \frac{\partial M}{\partial y} = -a \quad \text{and} \quad \frac{\partial N}{\partial x} = -a \qquad \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Thus the equation is exact and its solution is M dx + \ (Terms of N not containing 2) dy = C (y constant)

$$\Rightarrow \int (x^2 - ay) dx + \int y^2 dy = c$$

$$\Rightarrow y \text{ constant}$$

=)
$$\frac{x^3}{3} - axy + \frac{y^3}{3} = c$$

$$\Rightarrow x^3 + y^3 - 3axy = c. \text{ is the solution.}$$

G(a). Given DE is
$$\frac{dy}{dx^2} + 4y = \sin x$$

$$PI = \frac{1}{D^2+4} \sin x = \frac{1}{-1^2+4} \sin x = \frac{1}{3} \sin x$$

$$\exists \ \ \mathcal{J} = C_1 \ \ \text{col} \ 2x + C_2 \ \ \text{Sin} \ 2x + \frac{1}{3} \ \ \text{Sin} \ x \,.$$

6(b). Given DE is
$$\frac{dy}{dx^2} + ay = \cot ax$$
.

Auxiliary equation is
$$D'+\alpha'=0 \Rightarrow D=\pm\alpha i$$

$$CF = C_1 \cos x + C_2 \sin ax$$

Here $y_1 = \cos ax$, $y_2 = \sin ax$, $X = \cot ax$. And $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = a$.

$$PT = -y_1 \int \frac{y_2 x}{w} dx + y_2 \int \frac{y_1 x}{w} dx$$

$$= -\cos \alpha x \int \frac{\sin \alpha x \cdot \cot \alpha x}{\alpha} dx + \sin \alpha x \int \frac{\cos \alpha x \cdot \cot \alpha x}{\alpha} dx$$

$$= -\frac{\cos ax}{a} \int \cos ax \, dx + \frac{\sin ax}{a} \int \frac{\cos^2 ax}{\sin ax} \, dx$$

$$= -\frac{1}{a^{\nu}} \cos x + \frac{\sin ax}{a} \int (\csc ax - \sin ax) dx$$

$$= -\frac{1}{a^{\nu}} (\cos x + \sin ax + \frac{1}{a^{\nu}} \sin ax + \frac{1}{a^{\nu}} (\cos ax - \cot ax) + \frac{1}{a^{\nu}} (\cos ax + \sin ax)$$

$$= -\frac{1}{a^{\nu}} (\cos ax + \sin ax + \frac{1}{a^{\nu}} \sin ax + \frac{1}{a^{\nu}} (\cos ax - \cot ax) + \frac{1}{a^{\nu}} (\cos ax - \cot ax)$$

(OR)

7(a). Given DE is
$$\frac{d^2y}{dx^2} + 16y = x \sin 3x$$
.

$$PI = \frac{1}{D716} (x \sin 3x)$$

$$= \frac{1}{D^{2}+16} \left(x \cdot \text{I-P-of } e^{3ix} \right)$$

= I.P. of
$$\left[\frac{1}{D^2+16} \times e^{3ix}\right]$$

= I.P. of
$$\left[e^{3ix} \frac{1}{(D+3i)+16}x\right]$$

=
$$\mathbf{T} \cdot \mathbf{P} \cdot \mathbf{O} \cdot \left[e^{3ix} \frac{1}{7} \left[1 + \frac{6iD}{7} + \frac{D}{7} \right] (x) \right]$$

=
$$\mathbb{T} \cdot P \cdot \text{ of } \left[\frac{1}{7} \left(\cos 3x + i \sin 3x \right) \left(x - \frac{6i}{7} \right) \right]$$

$$= \frac{1}{7} \left(x \sin 3x - \frac{6}{7} \cos 3x \right)$$

Hence the (S is
$$y = CF + PI$$

$$= y = c_1 (\sigma s + x + c_2 s in + x + \frac{1}{7} (x s in 3x - \frac{6}{7} c \sigma s 3x)$$

7(b). Given DE is
$$(D^{2}+5D+6)y=x^{2}+2x+4$$

Auxiliary equation is $D'+5D+6=0 \Rightarrow (D+2)(D+3)=0 \Rightarrow D=-2,-3$.

$$cF = c_1 e^{-2x} + c_2 e^{-3x}$$

$$PI = \frac{1}{(D+2)(D+3)} (x^{2}+2x+4)$$

$$= \left(\frac{1}{D+2} - \frac{1}{D+3}\right) \left(\frac{\chi^2 + 2\chi + 4}{2}\right)$$

$$= \left[\frac{1}{D+2} - \frac{1}{D+3}\right]^{-1} (x^{2}+2x+4) - \frac{1}{3} (1+\frac{D}{3})^{-1} (x^{2}+2x+4)$$

$$= \frac{1}{2} (1+\frac{D}{2})^{-1} (x^{2}+2x+4) - \frac{1}{3} (1+\frac{D}{3})^{-1} (x^{2}+2x+4)$$

$$= \frac{1}{2} (1+\frac{D}{2})^{-1} (x^{2}+2x+4) - \frac{1}{3} (1+\frac{D}{3})^{-1} (x^{2}+2x+4)$$

$$= \frac{1}{2} \left(1 + \frac{1}{2} \right)^{-1} \left(x^{2} + 2x + 4 \right) - \frac{1}{3} \left(1 + \frac{1}{3} \right) \left(x^{2} + 2x + 4 \right)$$

$$= \frac{1}{2} \left[1 - \frac{1}{2} + \frac{1}{4} \right] \left(x^{2} + 2x + 4 \right) - \frac{1}{3} \left[1 - \frac{1}{3} + \frac{1}{4} \right] \left(x^{2} + 2x + 4 \right)$$

$$= \frac{1}{2} \left[1 - \frac{1}{2} + \frac{1}{4} \right] \left(x^{2} + 2x + 4 \right)$$

$$= \frac{1}{2} \left(1 - \frac{1}{2} + \frac{1}{4} \right)$$

$$= \frac{1}{2} \left(x^{2} + x + \frac{7}{2} \right) - \frac{1}{3} \left(x^{2} + \frac{1}{3} x + \frac{32}{9} \right)$$

$$= \frac{1}{2} \left(x^{2} + x + \frac{7}{2} \right) - \frac{1}{3} \left(x^{2} + \frac{1}{3} x + \frac{32}{9} \right)$$

$$= \frac{1}{2} \left(x^{2} + x + \frac{1}{2} \right) - \frac{1}{3} \left(x^{3} + x + \frac{3}{2} \right) - \frac{1}{3} \left(x^{2} + \frac{3}{3} x + \frac{3}{4} \right)$$
Hence the cs is $y = c_{1} e^{-2x} + c_{2} e^{-3x} + \frac{1}{2} \left(x^{2} + x + \frac{3}{2} \right) - \frac{1}{3} \left(x^{2} + \frac{3}{3} x + \frac{3}{4} \right)$

8(b)
$$L^{-1} \left[\frac{s}{(s^2 + \alpha^2)^3} \right]$$

$$= L^{-1} \left[\frac{s}{s^2 + \alpha^2} \right]$$

$$= \int cosau \frac{sina(t-u)}{a} du$$

$$= \frac{1}{2a} \int \left[sinat - sin(eau-at) \right] dt$$

$$= \frac{1}{2a} \left[u \sin t + \frac{1}{2a} \cos(eau-at) \right] dt$$

$$= \frac{1}{2a} \left[(t \sin t + \frac{1}{2a} \cos t) - \frac{1}{2a} \cos(-at) \right]$$

$$= \frac{1}{2a} \left[(t \sin t + \frac{1}{2a} \cos t) - \frac{1}{2a} \cos(-at) \right]$$

$$= \frac{1}{2a} \left[(t \sin t + \frac{1}{2a} \cos t) - \frac{1}{2a} \cos(-at) \right]$$

Hence $L^{-1} \int \frac{S}{(S^{+}a^{+})^{2}} dt = \frac{t}{ea} \sin at$

(OR)

Given $f(t) = |t-1| + |t+1|, t \ge 0$. Given function is equivalent to $f(t) = \begin{cases} 2, 0 \le t < 1 \\ zt, t \ge 1. \end{cases}$ $= \int_{-\infty}^{\infty} e^{-st} (z) dt + \int_{-\infty}^{\infty} e^{-st} (zt) dt$ $= 2\left(\frac{e^{-St}}{-S}\right)^{1} + 2\left[+\left(\frac{e^{-St}}{-S}\right) - \left(\frac{e^{-St}}{(-S)^{\nu}}\right)\right]^{2\nu}$ $=2\left(\frac{\overline{e}^{S}}{-S}+\frac{1}{S}\right)+2\left[\frac{0-\overline{e}^{S}}{-S}-\frac{0-\overline{e}^{S}}{S^{r}}\right]$

9(b). Given DE is $\frac{d^{\frac{1}{2}}}{dt^{\frac{1}{2}}} + 9x = (0.02t, x(0) = 1, x(\frac{\pi}{2}) = -1.$

 $= \frac{2}{c} \left[1 + \frac{e^{5}}{c} \right]$

Since x'(0) is not given, we assume x'(0)=a.

Taking the Laplace transforms of both sides of the equation L(x'') + 9 L(x) = L(coszt)

$$\Rightarrow s^{\nu} \overline{x} - s x(0) - x'(0) + 9 \overline{x} = \frac{s}{s+4}$$

$$\Rightarrow (s^{2}+9) \overline{\lambda} = S+Q+\frac{S}{s^{2}+4}$$

$$\Rightarrow (S+9) = \frac{S+\alpha}{S+9} + \frac{S}{(S+4)(S+9)} = \frac{\alpha}{S+9} + \frac{1}{5} \cdot \frac{S}{S+4} + \frac{4}{5} \cdot \frac{S}{S+9}$$

$$\Rightarrow \pi = \frac{S+\alpha}{S+9} + \frac{S}{(S+4)(S+9)} = \frac{\alpha}{S+9} + \frac{1}{5} \cdot \frac{S}{S+4} + \frac{4}{5} \cdot \frac{S}{S+9}$$

$$\Rightarrow x = \frac{a}{3} \sin 3t + \frac{1}{5} \cot 2t + \frac{4}{5} \cot 3t$$

when
$$t = \pi/2$$
, $-1 = -\frac{\alpha}{3} - \frac{1}{5}$ (or) $\frac{\alpha}{3} = \frac{4}{5}$.

Hence the solution is $x = \frac{1}{5}$ (coset + 4 sinst + 4 cosst).

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