Conjugate priors

LATEST SUBMISSION GRADE

100%

1. Prior is said to be conjugate to a likelihood function if:

1 / 1 point

- ( ) the prior is from the same family of distributions as the likelihood
- ( ) the prior, the likelihood function and the posterior would be in a same family of distributions
- (x) the posterior would stay in the same family of distributions as prior
- ( ) the prior lies in the same family of distributions as the likelihood

Correct

Posterior and prior are both distributions over θ, so they can lie in the same family

2. Finding a conjugate prior is useful because:

1 / 1 point

- [x] As long as posterior will stay in the same family with prior, the integral $p(x_{new} \mid x) = \int p(x_{new} \mid \theta) p(\theta \mid x) d\theta$ which is used for prediction is also tractable

Correct

This integral is called the evidence and it can be computed analytically if prior, likelihood and posterior are known

- [ ] It leads to a better MAP estimate
- [x] We can perform analytical inference and find posterior distribution instead of taking point MAP estimate

Correct

Since posterior lies in a known family of distributions, we will be able to perform analytical inference

- [ ] It is the only prior for which it is possible to perform analytical inference

3. Out of the following pairs of priors and likelihood functions, choose those that are conjugate:

1 / 1 point

- [ ] $\mathcal{N}(\sigma_1^2 \,|\, m, s^2)$ prior over parameter σ_1^2 of $\mathcal{N}(X \,|\, \mu_1, \sigma_1^2)$ likelihood
- [x] $\mathcal{N}(\mu_1 \,|\, m, s^2)$ prior over parameter μ_1 for $\mathcal{N}(X \,|\, \mu_1, \sigma_1^2)$ likelihood

Correct

This example was discussed in a lecture

- [x] $\Gamma(\lambda \,|\, \alpha, \beta)$ prior over parameter λ of $Exp(x \,|\, \lambda)$ likelihood ($\Gamma(x, | \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ and $Exp(x \,|\, \lambda) = \lambda e^{-\lambda x}$)

Correct

Multiplying these distribution and grouping the terms will lead to gamma distribution again

- [ ] $\Gamma(\sigma_1^2 \,|\, \alpha, \beta)$ prior over parameter σ_1^2 of $\mathcal{N}(X \,|\, \mu_1, \sigma_1^2)$ likelihood

4. Which of the following prior distributions over parameter σ^2 are conjugate to likelihood $\mathcal{N}(x \,|\, \mu, \sigma^2)$?

1 / 1 point

- [ ] $Exp(\sigma^2 \,|\, \lambda) = \lambda e^{-\lambda \sigma^2}$
- [ ] $\mathcal{N}(\sigma^2 \,|\, \mu_1, \sigma_1^2)$
- [x] Inverse gamma with pdf $p(\sigma^2 \,|\, \alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)(\sigma^2)^{-\alpha-1} \exp\left(-\frac{\beta}{\sigma^2}\right)}$

Correct

Multiplying these distribution and grouping the terms will lead to normal distribution

- [x] Scaled inverse chi-squared with pdf $f(\sigma^2 \,|\, \nu, \tau) = \frac{(\tau^2\nu/2)^{\nu/2}}{\Gamma(\nu/2)} \frac{\exp\left(-\frac{\nu\tau^2}{2\sigma^2}\right)}{(\sigma^2)^{1+\nu/2}}$

Correct

Multiplying these distribution and grouping the terms will lead to normal distribution

5. **Choose the correct statements:**

1 / 1 point

- [ ] For arbitrary likelihood and prior pair, we can always perform inference and compute posterior analytically
- [x] Putting initial knowledge into prior distribution is an advantage of Bayesian approach

  ✓ **Correct**
  That's the one

- [ ] Although not for every pair of prior and likelihood there is an analytical expression for posterior, we can always find a conjugate prior in some simple family and compute posterior analytically
- [x] For some problems conjugate prior may be inadequate

  ✓ **Correct**
  That's true

6. **Imagine that you want to pat your friend's cat Becky. Cats are really random creatures.**

1 / 1 point

Becky might get grumpy and scratch you with probability p or curl up and start purring (with prob. $1-p$). You don't know Becky well yet, so you estimate prior on p to be distributed as $Beta(2,2)$. Within one evening, Becky has scratched you 6 times and only 2 times she purred. What will be the parameters for posterior distribution over p? What is the MAP-estimate for p?

Enter your answers separated by comma: e.g. if you think that correct answer is $Beta(1,0.2)$ and MAP is 3, you should enter 1,0.2,3. Express real numbers as decimals with dot as delimiter.

8.,4.,0.7

✓ **Correct**