4033/5033 Assignment: Weighted Least Square

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$$\min_{\beta} \sum_{i=1}^{n} w_i \cdot (x_i^T \beta - y_i)^2, \tag{1}$$

$W \cdot (X \cdot \beta - Y)^2 \ W \cdot (X \cdot \beta - Y)^T (X \cdot \beta - Y)$

$(X \cdot \beta - Y)^T \cdot W \cdot (X \cdot \beta - Y)$

$(\beta^T X^T - Y^T) \cdot W \cdot (X \cdot \beta - Y)$

$(\beta^T X^T W - Y^T W) \cdot (X \cdot \beta - Y)$

$\beta^T X^T W X \beta - \beta^T X^T W Y - Y^T W X \beta - Y^T W Y$

$\beta^T X^T W X \beta - 2(X^T W Y) - Y^T W Y$

$\frac{\partial}{\partial \beta} \text{WLS} = (X^T W X + (X^T W X)^T) \beta - 2(X^T W Y)$

$\frac{\partial}{\partial \beta} \text{WLS} = 0$

$2(X^T W X) \beta - 2(X^T W Y) = 0$

$2\left[(X^T W X) \beta - (X^T W Y) \right] = 0$

$(X^T W X) \beta - X^T W Y = 0$

$(X^T W X) \beta = X^T W X$

$\beta = (X^T W X)^{-1} X^T W Y$

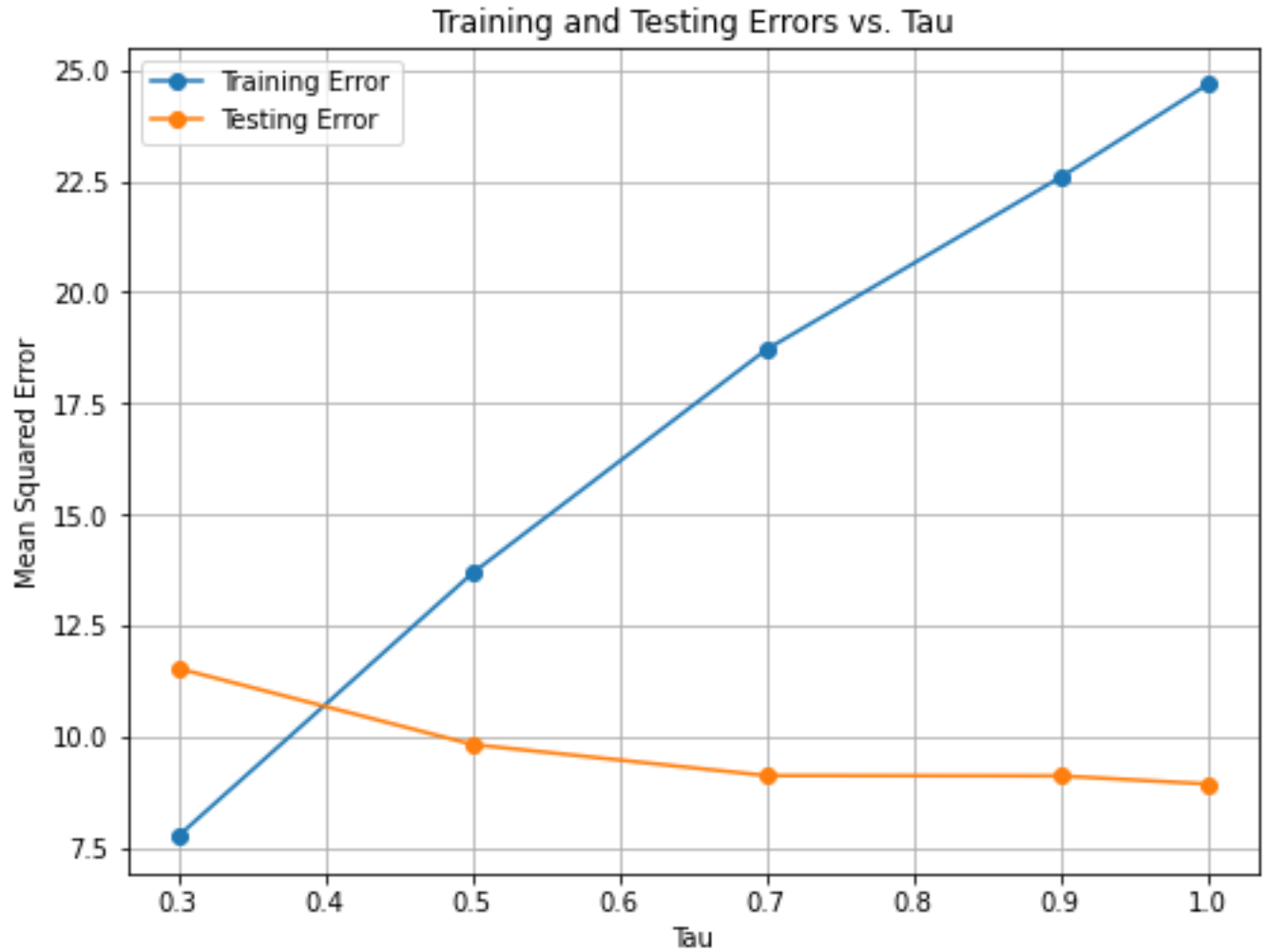Training and Testing Errors vs. Tau

Table 1. Performance of the Weighted Least Square Solution

	$w_\ell = 1, w_h = 1$	$w_\ell = 1, w_h = 10$	$w_\ell = 1, w_h = 50$	$w_\ell = 1, w_h = 0.1$
Testing Error	8.93018	21.93918	79.75695	7.62928
Error on high-risk group	1.44544	14.45444	72.27221	0.14454
Error on low-risk group	7.48473	7.48473	7.48473	7.48473