Information And Communication

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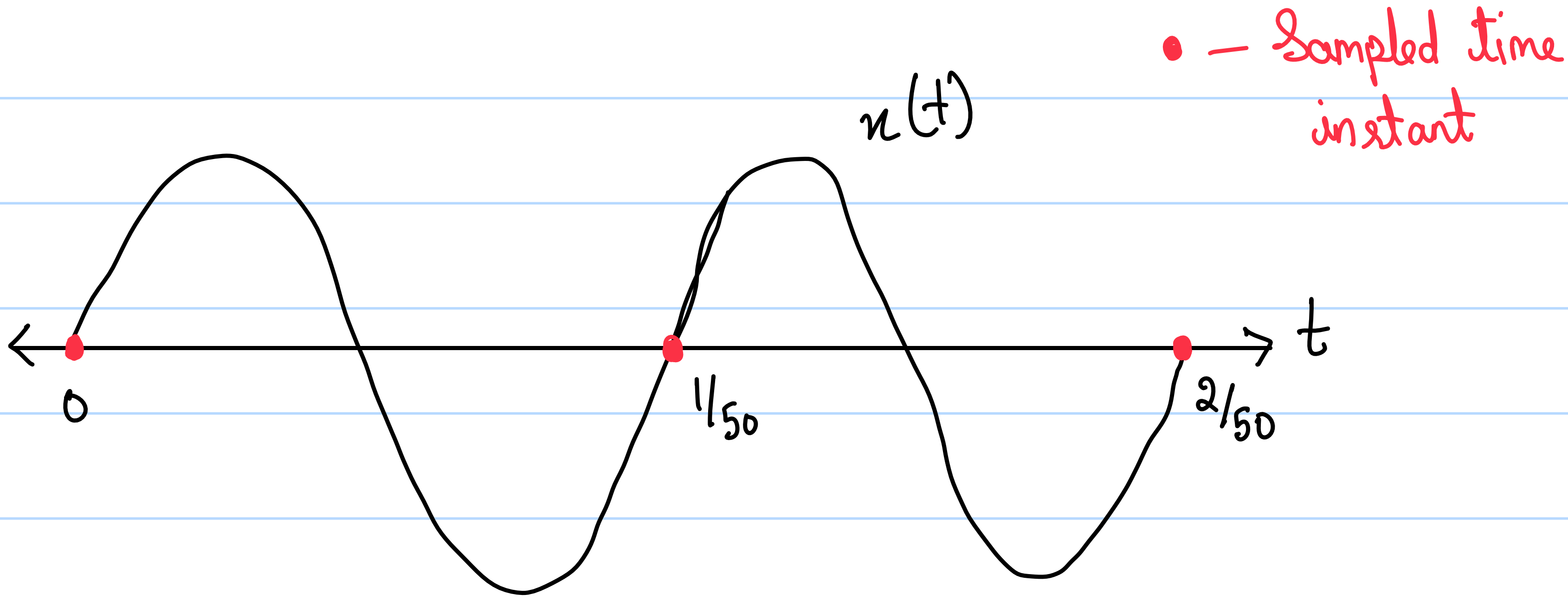
Q1. $x(t) = \sin(100\pi t)$

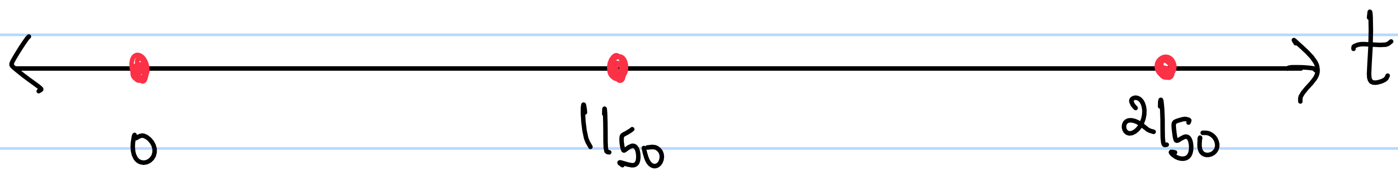
$$T = \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{100\cancel{\pi}} = \frac{1}{50}\ s$$

1) $f_s = 50\ Hz$

$$T_s = \frac{1}{50}$$



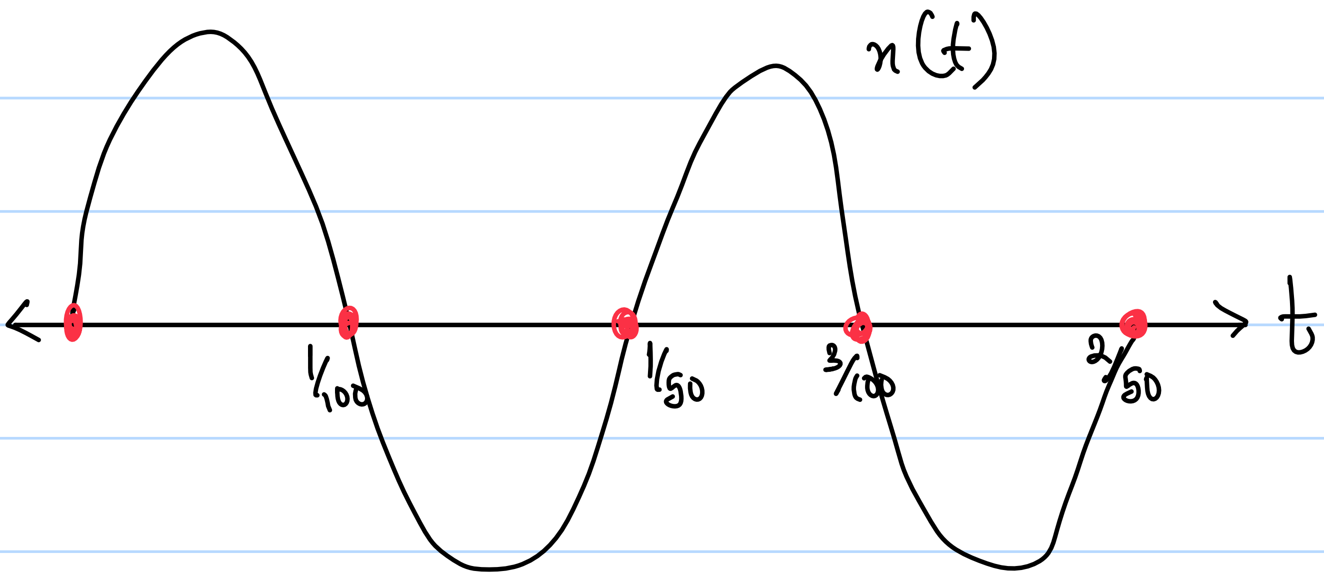
Sampled Signal :-



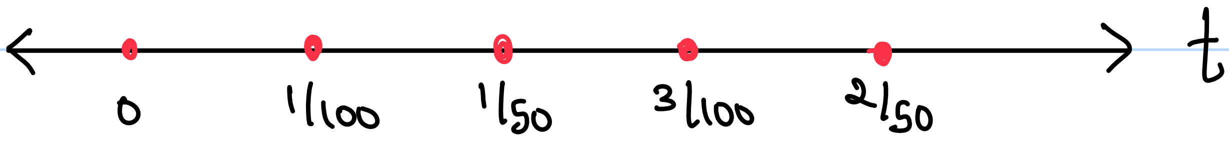
Since the frequency of the sampler exactly matches that of the sine wave, it repeatedly measures the exact same amplitude (which happens to be zero).

2) $f_s = 100\ Hz$

$T_s = \frac{1}{100}\ s$



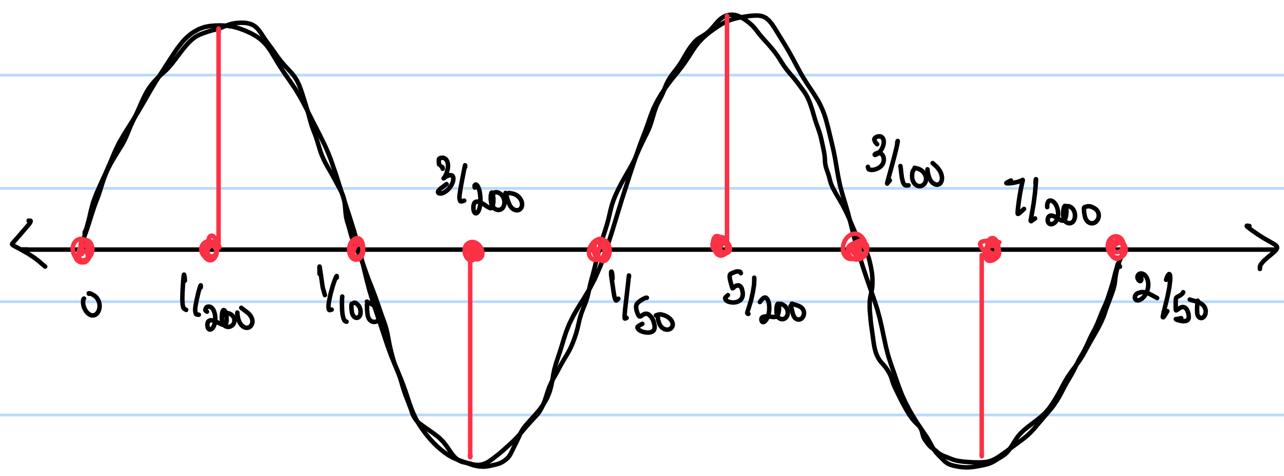
Sampled Signal :-



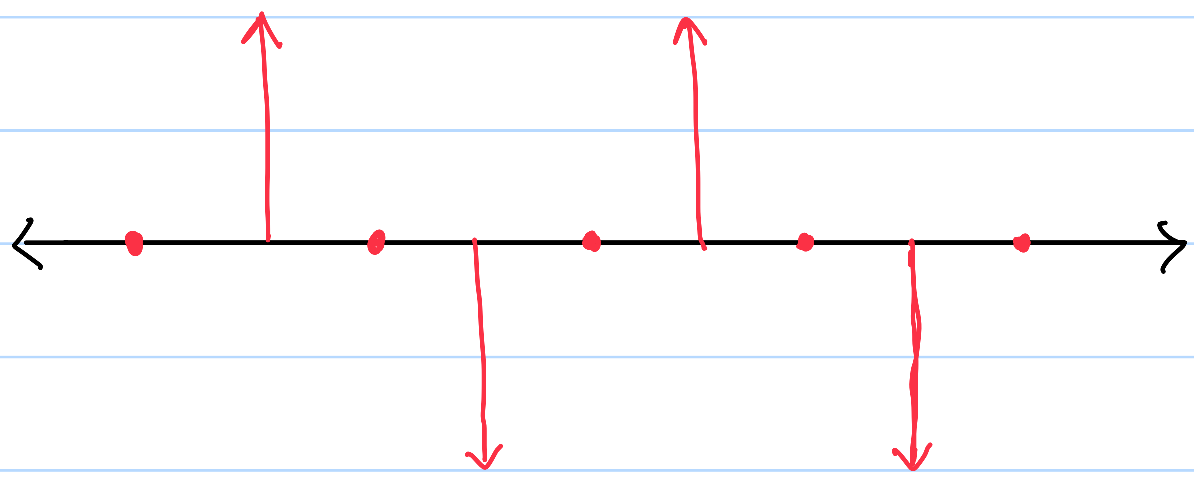
Due to the nature of the sine wave, the amplitude measured during each sample still is zero. However due to the higher sample rate, the data points are denser.

3) $f_s = 200\ Hz$

$T_s = \frac{1}{200}\ s$



Sampled Signal :-



This signal sort of mimics the behaviour of a sinusoidal signal, thereby proving the fact that as the frequency of sampling increases, the amount of information lost is reduced.

Q2. No. of levels $= 2^8 = 256$

$$Q_{noise} = \frac{\Delta^2}{12}, \quad \Delta = \frac{\text{Signal Range}}{\text{No. of levels}}$$

i) $x(t) = 2\cos(t)$

Signal Range $= |2-(-2)| = 4$

$$\Delta = \frac{4}{256} = \frac{2^2}{2^8} = \frac{1}{2^6} = \frac{1}{64}$$

$$Q_{noise} = \frac{\left(\frac{1}{64}\right)^2}{12} = \frac{1}{64^2 \times 12} = \underline{\underline{2.034 \times 10^{-5}}} \text{ dB}$$

2) $x(t) = -|\sin(t)|$

$$\text{Signal Range} = |0-(-1)| = 1$$

$$\Delta = \frac{1}{256}$$

$$Q_{noise} = \frac{\left(\frac{1}{256}\right)^2}{12} = \underline{1.271 \times 10^{-6} \text{ dB}}$$

3) $x(t) = \begin{cases} t, & -T/2 < t < T/2 \\ -t+T, & T/2 < t < 3T/2 \end{cases}$

$$t = -t + T$$
$$2t = T$$
$$t = T/2$$

$$\text{Range} = \left|\frac{T}{2} - 0\right| = T/2$$

$$\Delta = \frac{T/2}{256} = \frac{T}{512}$$

$$Q_{noise} = \frac{\left(\frac{T}{512}\right)^2}{12} = \underline{\underline{3.179\,T^2 \times 10^{-7}}}$$

Q3. No. of days b/w 6^{th} August 2022 and 18^{th} November 2022
$= 105$ days

If the weather on 18^{th} November 2022 must be humid,

1. Same weather must continue for 105 days : p^{105}

2. Weather must change only twice : ${}^{105}C_2\, p^{103} (1-p)^2$

3. Weather must change only 4 times : ${}^{105}C_4\, p^{101} (1-p)^4$

$\vdots$

57. Weather must change only 104 times : ${}^{105}C_{104}\, p (1-p)^{104}$

$$\therefore P = p^{105} + {}^{105}C_2\, p^{103} (1-p)^2 + {}^{105}C_4\, p^{101} (1-p)^4 \cdots\cdots + {}^{105}C_{104} (1-p)^{104} \cdot p$$

(ie, only the odd terms of the expansion $(p+(1-p))^{105}$

$$= \frac{(p+(1-p))^{105} + (p-(1-p))^{105}}{2}$$

$$= \frac{1 + (p-1+p)^{105}}{2}$$

$$= \frac{1 + (2p-1)^{105}}{2}$$

Q4. In a group of n people, to calculate the probability that 2 people share the same birthday, we can consider the complement event.

No. of possible birthday combinations $= (365)^n$

No. of birthday combinations st they are all unique $= \frac{(365)!}{(365-n)!}$

$$P' = \frac{\frac{(365)!}{(365-n)!}}{365^n}$$

$$\Rightarrow \quad P = 1 - \frac{\frac{(365)!}{(365-n)!}}{365^n}$$

$$= \quad P = \frac{365^n - \frac{(365)!}{(365-n)!}}{365^n}$$

Using this formula, set $n = 23$,

$$\Rightarrow \quad P = \frac{365^{23} - \frac{365!}{(365-23)!}}{365^{23}} \approx \underline{0.507297}$$

Q5.

1) $m = 1$

A is the event st $\max\{X, Y\} = 1$

B is the event st $\min\{X, Y\} = 2$

If B has occured, it implies $\min(X, Y) = 2$

$\Rightarrow \max(X, Y) \geq 2$ always

$\therefore \max(X, Y) \neq 1$

$\therefore P(A|B) = 0$

2) $m = 2$

A is the event st $\max(X, Y) = 2$

B is the event st $\min(X, Y) = 2$

$\Rightarrow$ The event $A|B$ is $(X, Y) = (2, 2)$

$P((2,2)) = 1/12$

3) $m = 3$

A is the event st $\max(X, Y) = 3$

B is the event st $\min(X, Y) = 2$

$\therefore A|B \Rightarrow (X, Y) = (2, 3)$ or $(3, 2)$

$\Rightarrow P(A|B) = \frac{1}{12} + \frac{1}{12} = \frac{1}{6}$

Q6. If the man was telling the truth the first time, it implies that the first card was a red Ace.

Let T be the event st the man was telling the truth the first time.

Let A_R be the event st the first card is a red Ace.

Let A_2 be the event st the second card is an Ace.

$$P(T|A_2) = \frac{P(A_2|T)\,P(T)}{P(A_2)}$$

$$= \frac{P(A_2|T)\,P(T)}{P(A_2|T)\,P(T) + P(A_2|\bar{T})\,P(\bar{T})}$$

$$= \frac{3/51 \times 3/4}{3/51 \times 3/4 + \left(\left(3/51 \times \frac{1}{4}\right) + \left(4/51 \times \frac{1}{4}\right)\right)}$$

$$= \frac{\frac{9}{51 \times 4}}{\frac{9}{51\times 4} + \frac{7}{51 \times 4}}$$

$$= \frac{9}{9+7}$$

$$\Rightarrow \underline{\underline{P(T|A_2) = \frac{9}{16}}}$$

Q7. $P(i) = c\lambda^i / i!$, $i = 0, 1, 2, \dots$, $\lambda > 0$

a) $P(X=0) = c(0)^i / 0! = \underline{\underline{0}}$

b) $P(X>2) = \dfrac{c\lambda^3}{3!} + \dfrac{c\lambda^4}{4!} + \dfrac{c\lambda^5}{5!}$

W.k.t,

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \dots\dots = e^x$$

$$\Rightarrow \quad 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \dots\dots = e^\lambda$$

$$= \quad \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \dots\dots = e^\lambda - 1$$

$$= \quad \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \dots\dots = (e^\lambda - 1) - \left(\lambda + \frac{\lambda^2}{2!}\right)$$

$$= \quad \frac{c\lambda^3}{3!} + \frac{c\lambda^4}{4!} + \dots = c\left[(e^\lambda - 1) - \left(\lambda + \frac{\lambda^2}{2}\right)\right]$$

$$\Rightarrow P(X>2) = \underline{\underline{c\left[e^\lambda - 1 - \lambda - \frac{\lambda^2}{2}\right]}}$$

Q8. X is a Poisson RV of parameter λ

$$\Rightarrow \quad P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\frac{dP(X=k)}{d\lambda} = \frac{k\lambda^{k-1} e^{-\lambda}}{k!} - \frac{\lambda^k e^{-\lambda}}{k!} = 0$$

$$= \left(\frac{e^{-\lambda}}{k!}\right)\left(k\lambda^{k-1} - \lambda^k\right) = 0$$

$$= \left(\frac{\lambda^{k-1} e^{-\lambda}}{k!}\right)(k-\lambda) = 0$$

$$\Rightarrow \lambda = 0, \quad \lambda = k$$

$$\frac{d^2(P(X=k))}{d\lambda^2} = \frac{k(k-1)\lambda^{k-2} e^{-\lambda}}{k!} - \frac{k\lambda^{k-1} e^{-\lambda}}{k!}$$

$$+ \frac{k\lambda^{k-1} e^{-\lambda}}{k!} - \frac{\lambda^{k} e^{-\lambda}}{k!}$$

$$\lambda = k, \quad \frac{d^2 P}{d\lambda^2} = \frac{k(k-1)k^{k-2} e^{-k}}{k!} - \cancel{\frac{k e^{-\lambda}}{k!}}$$

$$+ \cancel{\frac{k e^{-\lambda}}{k!}} - \frac{k^k e^{\lambda}}{k!}$$

$$\frac{d^2 P}{d\lambda^2} = \frac{(k-1)k^{k-1} e^{-k}}{k!} - \frac{k^k e^{\lambda}}{k!}$$

$$= \cancel{\frac{k e^{-k}}{k!}} - \frac{k^{k-1} e^{-k}}{k!} - \cancel{\frac{k^k e^{\lambda}}{k!}}$$

$$\frac{d^2 P}{d\lambda^2} = -\frac{k^{k-1} e^{-k}}{k!} < 0$$

$\Rightarrow$ $\underline{\lambda = k}$ is the maxima of $P(X=k)$.