Information and Communication: Pset 2.

Defining Sample Space and Event Space:

- A standard six-sided die is rolled. Define the sample space for this experiment and calculate the probability of rolling an even number.

- In a deck of 52 playing cards, what is the sample space for drawing one card? What is the probability of drawing a heart?

- If you flip two coins, define the sample space for this experiment. How many outcomes are there where at least one coin shows heads?

Discrete Random Variables:

- Consider a discrete random variable X that can take values from the set {0, 1, 2, 3} with probabilities $P(X=0)=0.1$, $P(X=1)=0.4$, $P(X=2)=p$ and $P(X=3)=0.5$. Determine the p value and verify if X is a valid probability distribution.

Basic Application of Total Probability:

- A factory produces two types of widgets, A and B. Widget A comprises 70% of the production, and 5% of these are defective. Widget B makes up the remaining 30%, with a defect rate of 10%. What is the probability that a randomly selected widget is defective?

Conditional Probabilities in Different Scenarios:

- In a survey, 60% of respondents are male, and 40% are female. Among males, 80% prefer brand X, while among females, only 30% prefer brand X. What is the overall probability that a randomly selected respondent prefers brand X?

- An insurance company has three types of policies: Type I (50%), Type II (30%), and Type III (20%). The claim rates for these policies are respectively: 5%, 10%, and 15%. Calculate the overall claim rate across all policies.

- A person plans to go hiking based on weather conditions. The probability it rains on any given day is 0.4. If it rains, there's a 20% chance they will still go hiking; if it doesn't rain, there's an 80% chance they will go hiking. What is the overall probability that they will go hiking?

Bayes Theorem:

- A factory produces two types of products: Type A and Type B. Type A makes up 70% of production, and Type B makes up 30%. The defect rate for Type A is 2%, while for Type B it is 5%. If a randomly selected product is found to be defective, what is the probability that it was produced by Type B?

- A certain disease affects 1% of the population. A test for the disease is 90% accurate, meaning it correctly identifies 90% of those who have the disease (true positive rate) and correctly identifies 90% of those who do not have the disease (true negative rate). If a randomly selected individual tests positive for the disease, what is the probability that they actually have the disease?