EC5.102: Information and Communication

(Lec-9)

Channel coding-5

(7-April-2025)

Arti D. Yardi

Email address: arti.yardi@iiit.ac.in

Office: A2-204, SPCRC, Vindhya A2, 1st floor

Calendar

- Agenda for the rest of the classes:
  - 7-April: Decoding of linear block codes
  - 10-April, 14-April: Modulation
  - 21-April: Channel capacity
  - 24-April: Introduction to cryptography
- Assignment-3:
  - To be posted by 16-April, Submission deadline 23-April, 11:59pm

Summary of the last class

Recap: LBCs

- $\mathcal{C}(n, k)$: Definition, parameters, generator matrix, parity check matrix
- Dual code, denoted by $\mathcal{C}^{\perp}(n, n-k)$ or $\mathcal{C}^{\perp}$
- Examples:
  - $\text{REP}(n, k = 1)$
  - $\text{SPC}(n, k = n - 1)$
  - $\text{Hamming}(n = 2^m - 1, k = 2^m - m - 1)$ where $m \geq 3$.
- Hamming weight of $\mathbf{w} \in \mathbb{F}_2^n$, denoted by $d_H(\mathbf{w})$
- Minimum distance of a binary LBC $\mathcal{C}$, denoted by $d_{\min}(\mathcal{C})$
- Block vs linear block code

Decoding LBCs

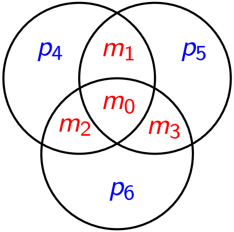
Introduction: Decoding

- Message $\mathbf{u} \rightarrow$ Codeword $\mathbf{v} \rightarrow$ Channel $\rightarrow \mathbf{y}$
- Definition of decoder
- Agenda:
  - Maximum likelihood (ML) decoding for REP-3 codes
  - Decoding of Hamming code with $(n = 7, k = 4)$
  - Standard array decoding of arbitrary LBCs

Introduction to ML decoding

- Maximum likelihood (ML) decoding for REP-3 codes (In class)

Decoding of Hamming codes of length 7



- Suppose m_0 m_1 m_2 m_3 are message bits.
- Parity p_4 is obtained using m_0 m_1 m_2 such that $m_0 + m_1 + m_2 + p_4 = 0$.
  Parity p_5 is obtained using m_0 m_1 m_3 such that $m_0 + m_1 + m_3 + p_5 = 0$.
  Parity p_6 is obtained using m_0 m_2 m_3 such that $m_0 + m_2 + m_3 + p_6 = 0$.
- Codeword is given by $[m_0 \ m_1 \ m_2 \ m_3 \ p_4 \ p_5 \ p_6]$
- **Claim: You can correct any one-bit error!** (In class)
- Decode $\mathbf{y} = \begin{bmatrix} y_0 & y_1 & y_2 & y_3 & y_4 & y_5 & y_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix}$

Standard array decoding of LBCs

Standard array

- All possible 2^n vectors are arranged in a table described below and the obtained table is called standard array.
- Coset leader $\mathbf{e}_j$ is chosen such that it has not appeared in the rows above it and has minimum weight.

2^k columns

2^{n-k} rows

$00\ldots0$	$\mathbf{v}_2$	$\cdots$	$\mathbf{v}_i$	$\cdots$	$\mathbf{v}_{2^k}$
$\mathbf{e}_2$	$\mathbf{e}_2+\mathbf{v}_2$	$\cdots$	$\mathbf{e}_2+\mathbf{v}_i$	$\cdots$	$\mathbf{e}_2+\mathbf{v}_{2^k}$
$\mathbf{e}_j$	$\mathbf{e}_j+\mathbf{v}_2$	$\cdots$	$\mathbf{e}_j+\mathbf{v}_i$	$\cdots$	$\mathbf{e}_j+\mathbf{v}_{2^k}$
$\mathbf{e}_{2^{n-k}}$	$\mathbf{e}_{2^{n-k}}+\mathbf{v}_2$	$\cdots$	$\mathbf{e}_{2^{n-k}}+\mathbf{v}_i$	$\cdots$	$\mathbf{e}_{2^{n-k}}+\mathbf{v}_{2^k}$

Decoding using standard array

- For the given noise-affected vector $\mathbf{y}$, find its location in the standard array.
- Decode $\mathbf{y}$ as $\mathbf{v}_i$ if it lies in the column of $\mathbf{v}_i$.
- When my decoding is correct? ★★ **Answer:** When true error is $\mathbf{e}_j$.
- Coset leaders $\{\mathbf{0}_n, \mathbf{e}_2, \ldots, \mathbf{e}_j, \ldots, \mathbf{e}_{2^{n-k}}\}$ are correctable error patterns.

2^k columns

2^{n-k} rows

$00\ldots0$	$\mathbf{v}_2$	$\cdots$	$\mathbf{v}_i$	$\cdots$	$\mathbf{v}_{2^k}$
$\mathbf{e}_2$	$\mathbf{e}_2 + \mathbf{v}_2$	$\cdots$	$\mathbf{e}_2 + \mathbf{v}_i$	$\cdots$	$\mathbf{e}_2 + \mathbf{v}_{2^k}$
$\mathbf{e}_j$	$\mathbf{e}_j + \mathbf{v}_2$	$\cdots$	$\boxed{\mathbf{y} = \mathbf{e}_j + \mathbf{v}_i}$	$\cdots$	$\mathbf{e}_j + \mathbf{v}_{2^k}$
$\mathbf{e}_{2^{n-k}}$	$\mathbf{e}_{2^{n-k}} + \mathbf{v}_2$	$\cdots$	$\mathbf{e}_{2^{n-k}} + \mathbf{v}_i$	$\cdots$	$\mathbf{e}_{2^{n-k}} + \mathbf{v}_{2^k}$

Decoding using standard array: Example

- Find standard array of REP-3 code and decode the received vector $\mathbf{y} = [101]$.
- $C = \{000, 111\}$

Decoding using standard array: Example

- Find standard array of SPC-3 code and decode the received vector $\mathbf{y} = [111]$.
- $C = \{000, 011, 101, 110\}$

Decoding using standard array: Example

- Consider the $(6,3)$ linear block code with generator matrix given by

$$G = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

  Find standard array of this code.

- $C = \{000000, 011100, 101010, 110001, 110110, 101101, 011011, 000111\}$

Coset leader 000000	011100	101010	110001	110110	101101	011011	000111
100000	111100	001010	010001	010110	001101	111011	100111
010000	001100	111010	100001	100110	111101	001011	010111
001000	010100	100010	111001	111110	100101	010011	001111
000100	011000	101110	110101	110010	101001	011111	000011
000010	011110	101000	110011	110100	101111	011001	000101
000001	011101	101011	110000	110111	101100	011010	000110
100100	111000	001110	010101	010010	001001	111111	100011

Understanding standard array decoding

- Recall: Coset leaders $\{\mathbf{0}_n, \mathbf{e}_2, \ldots, \mathbf{e}_j, \ldots, \mathbf{e}_{2^{n-k}}\}$ are correctable error patterns.
- For the BSC(p), if the cross-over probability $p < 1/2$, what is a good choice for coset leaders? Justify.
- For the BSC(p), if the cross-over probability $p > 1/2$, what is a good choice for coset leaders? Justify.
- Standard array decoding is ML decoding!
- A LBC $\mathcal{C}(n, k, d_{\text{min}})$ can correct all possible error patterns of weight less than or equal to $t = \lfloor d_{\text{min}} - 1 \rfloor / 2$. A code is then said to be t-error correcting code. (No proof)