1. The probability of getting a reservation at a restaurant if you call y days in advance is given by $1 - e^{-y}$, where $y \geq 0$. What is the minimum numbers of days that one should call in advance in order to have a probability of at least 0.95 of getting a reservation?

2. Consider a random variable X whose sample space is given by the set $S = (-1, 6)$, which is the interval $(-1, 6)$. The random variable itself is defined as follows:

$$X(s) = \begin{cases} 2 & if \quad -1 \leq s < 0 \\ 0 & if \quad s = 0 \\ 1 & if \quad 0 < s < 1 \\ 3 & if \quad 1 \leq s \leq 3 \\ 5 & if \quad 3 < s < 5 \\ 4 & if \quad s = 5 \\ 7 & if \quad 5 < s \leq 6 \end{cases} \tag{1}$$

Express the following terms in terms of probabilities of appropriate subsets of S: (i) $F_X(0)$ (ii) $F_X(3) - F_X(1)$ (iii) $F_X(3.5)$

3. Consider the random variable X with pdf $f_X(x)$ given by

$$f_X(x) = \begin{cases} A(1+x) & -1 \leq x \leq 0 \\ A(1-x) & 0 < x < 1 \\ 0 & \text{elsewhere} \end{cases} \tag{2}$$

   (a) Find A and plot $f_X(x)$.

   (b) Find the cdf $F_X(x)$.

(c) Find the point b such that $P(X > b) = \frac{1}{2}P(X \leq b)$.

4. Two identical coins are flipped simultaneously. Let X be the number of heads and Y the number of tails shown. What is the joint pmf of X and Y? What are the marginal pmfs?

5. Show that $E(I_A) = P(A)$ where I_A is the indicator random variable of the event A.

6. Let X be a random variable with the Poisson distribution. Find the conditional expectation of X given that X is an even number.

7. Show that variance of the random variable $X + a$ is the same as that of random variable X for all $a \in R$.