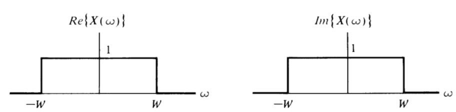
Information and Communication Assignment 3

1. Determine the Fourier transform of $x(t) = e^{-t/2}u(t)$ and sketch the following:
   (a) $|X(\omega)|$
   (b) $\angle X(\omega)$
   (c) $Re\{X(\omega)\}$
   (d) $Im\{X(\omega)\}$

2. The figure below shows the real and imaginary parts of the Fourier transform of a signal $x(t)$.



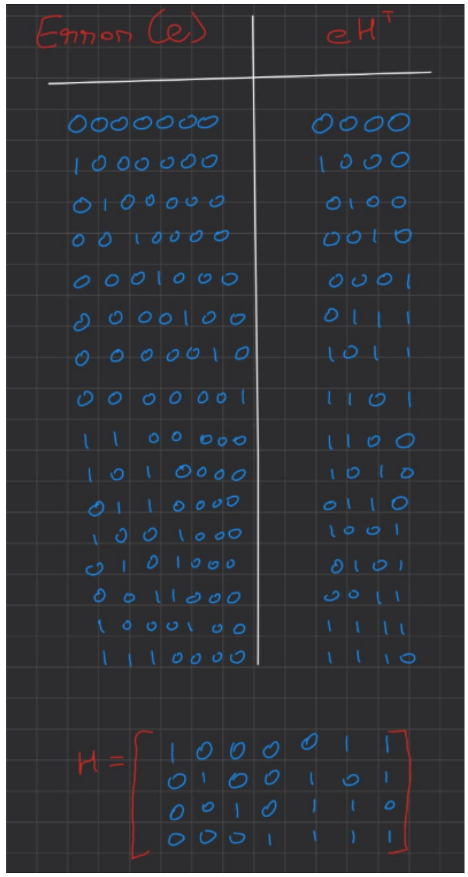
   (a) Sketch the magnitude and phase of the Fourier transform $X(\omega)$.
   (b) In general, if a signal $x(t)$ is real, then $X(-\omega) = X^*(\omega)$. Determine whether $x(t)$ is real for the Fourier transform sketched in the above figure.

3. (a) Use the modulation property to find the Fourier Transform of $x(t)\cos(2\pi f_0 t)$, where $X(f)$ is the Fourier Transform of $x(t)$. How does modulation affect the spectrum?
   (b) Let $x(t) = \text{rect}(t/2)$. Plot the spectrum of $x(t)\cos(20\pi t)$ and label key frequencies. What bandwidth is occupied by the modulated signal?

4. Standard array decoding for an [n,k] linear code requires storing all 2^n codewords, which can take up a lot of space and becomes infeasible for large n.

   Instead, consider the table below, which lists coset leaders alongside the vectors obtained by multiplying each coset leader with the transpose of a parity check matrix, i.e, eH^T.

   HINT: The received vector r can be written as c+e, where c is a codeword and e is the error pattern. $cH^T = 0$.



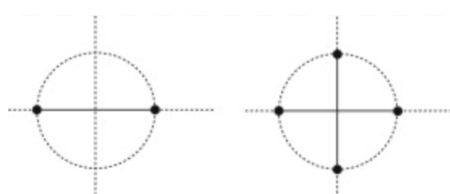
Error (e)	eH^T
0000000	0000
1000000	1000
0100000	0100
0010000	0010
0001000	0001
0000100	0111
0000010	1011
0000001	1101
1100000	1100
1010000	1010
0110000	0110
1001000	1001
0101000	0101
0011000	0011
1000100	1111
1110000	1110

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix}$$

(a) Suppose a received word **r** is given. Explain how you could use this table to identify and correct an error in **r**, without constructing a full standard array. How does this approach save space?

(b) Given that $\mathbf{r} = [0, 0, 1, 1, 0, 1, 1]$, find the corresponding codeword using the above table.

5. Refer to the figure below and answer the following:



(a) Using the constellation diagrams above, explain how BPSK and QPSK differ in terms of:

i. Number of bits represented per symbol
ii. Phase shifts used in modulation
iii. Bandwidth efficiency
iv. Susceptibility to noise

(b) Suppose both modulation schemes are operating under the same bandwidth and power conditions. Which modulation is expected to have a lower bit error rate (BER), and why?

6. Derive the mathematical expression for a BPSK modulated signal, assuming a binary data sequence $b(t) \in \{0, 1\}$ and a carrier wave $\cos(2\pi f_c t)$.

   Draw a waveform diagram showing the modulated signal for the binary sequence: [1,0,1,1,0]. Clearly indicate phase changes.

7. A message signal $m(t) = 5\cos(2\pi \cdot 1{,}000\, t)$ is used to modulate a carrier signal of the form $c(t) = 10\cos(2\pi \cdot 100{,}000\, t)$ using DSB-SC modulation.

   (a) Write the mathematical expression for the modulated signal.
   (b) Determine the bandwidth of the modulated signal.
   (c) Sketch or describe the frequency spectrum of the modulated signal, labeling key frequencies.

8. A DSB-SC modulated signal is received as:

$$s(t) = 4\cos(2\pi \cdot 10^6 t)\cos(2\pi \cdot 10^3 t)$$

   (a) Identify the message frequency and carrier frequency.
   (b) Find the bandwidth of the signal.