EC5.203 Communication Theory I (3-1-0-4):

Lectures 3 and 4
Baseband and Passband Representations

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Jan. 12, 2026 and
and Jan. 20, 2026

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Reference

- Chapter 2 (Madhow)
  - Sec. 2.6-2.8: Energy Spectral Density, Bandwidth, Structure of passband signal

Recap

Indicator Function

- S&S: The step function was given by

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- *CT-1*: $u(t)$ represent CT signal and $u[n]$ represent DT signal while step function is used for

$$I_{[0,\infty)}(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

- Indicator function

$$I_A(x) = \begin{cases} 1 & x \in A \\ 0 & \text{otherwise} \end{cases}$$

Sinusoidal Signal

- Sinusoids

$$s(t) = A\cos(2\pi f_0 t + \theta) \quad \textbf{Polar Form}$$

  where $A > 0$ is the amplitude, f_0 is the frequency, and $\theta \in (0, 2\pi]$ is the phase.

- Sinusoids with known A, f_0, and θ cannot carry information.

- Modulation varies one or more of these parameters to convey information.

- Sinusoid can also be written as

Rectangular form

$$s(t) = A_c \cos 2\pi f_0 t - A_s \sin 2\pi f_0 t$$

  where $A_c = A\cos\theta$ and $A_s = A\sin\theta$ are real numbers. Using Euler's formula

$$Ae^{j\theta} = A_c + jA_s \quad \textbf{Complex number}$$

  where $A = \sqrt{A_c^2 + A_s^2}$ and $\theta = \tan^{-1}(A_s/A_c)$.

Complex Exponential

- Complex exponentials

$$s(t) = Ae^{2\pi f_0 t + \theta} = \alpha e^{2\pi f_0 t}$$

where $\alpha = e^{\theta t}$ a complex number that contains both the amplitude and phase information.

Inner Product

- The inner product for two $m \times 1$ complex vectors $\mathbf{s} = (s[1], \ldots, s[m])^T$ and $\mathbf{r} = (r[1], \ldots, r[m])^T$ is given by

$$< \mathbf{s}, \mathbf{r} >= \sum_{i=1}^{m} s[i] r^*[i] = \mathbf{r}^H \mathbf{s}.$$

  where $(.)^H$ denotes Hermitian operation with vector $(\mathbf{r})^H$ being conjugate transpose of vector $\mathbf{r}$.

- Similarly, we define the inner product of two possibly complex-valued signals $s(t)$ and $r(t)$ as follows

$$< s, r >= \int_{-\infty}^{\infty} s(t) r^*(t) \, dt$$

Energy and Norm

- The energy E_s of a signal

$$E_s = ||s||^2 \ = < s, s > = \int_{-\infty}^{\infty} |s(t)|^2 \ dt$$

- If $E_s = 0$, then it means that $s(t)$ must be zero *almost everywhere* which is also true for the norm of a vector.

Power

- The power of a signal $s(t)$ is defined as the time average of its energy computed over a large time interval

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt$$

- Finite energy signals have zero power.

Time average

- Time average of signal $g(t)$ is denoted by

$$\bar{g} = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} g(t)dt$$

- Using the above notation, power of signal is given by

$$P_s = \overline{|s(t)|^2}$$

while the DC value of $s(t)$ is $\overline{s(t)}$.

Fourier Series

- A periodic signal $x(t)$ can be represented in terms of Fourier series as

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{jk\omega_0 t}$$

where Fourier series coefficients are given by

$$a_k = \frac{1}{T_0} \int_{T_0} x(t) e^{-jk\omega_0 t} dt$$

- A periodic signal $u(t)$ can be represented in terms of Fourier series as

$$u(t) = \sum_{n=-\infty}^{\infty} u_n e^{j2\pi n f_0 t}$$

where Fourier series coefficients are given by $\omega = 2\pi f$

$$u_k = \frac{1}{T_0} \int_{T_0} u(t) e^{-j2\pi k f_0 t} dt$$

Fourier Transform

- Any aperiodic and finite duration signal $x(t)$ can be represented using Fourier transform

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}\, d\omega$$

Inverse Fourier Transform

where

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\, dt$$

Fourier Transform

- A aperiodic signal $x(t)$ can be represented in terms of Fourier transform as

$$u(t) = \int_{-\infty}^{\infty} U(f)e^{j2\pi ft} df$$

where Fourier transform is given by

$\omega = 2\pi f$

$$U(f) = \int_{-\infty}^{\infty} u(t)e^{-j2\pi ft} dt$$

Properties of Fourier Transform

- Linearity: For arbitrary complex numbers α and β,

$$\alpha u(t) + \beta v(t) \longleftrightarrow \alpha U(f) + \beta V(f)$$

- Time delay

$$u(t - t_0) \longleftrightarrow U(f)e^{-j2\pi f t_0}$$

- Frequency shift

$$U(f - f_0) \longleftrightarrow u(t)e^{j2\pi f_0 t}$$

- Differentiation in time domain

$$x(t) = \frac{du(t)}{dt} \longleftrightarrow j2\pi f U(f)$$

Properties of Fourier Transform

- Step Function: For $v(t) = I_{[0,\infty)}$

$$V(f) \longleftrightarrow \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$$

- Integration:

$$u(t) = \int_{-\infty}^{t} x(t)\ dt = x(t) * v(t)$$

$$U(f) = X(f)V(f) = \frac{X(f)}{j2\pi f} + \overline{u}\delta(f)$$

Properties of Fourier Transform

- Parseval's identity:

$$< u, v > = \int_{-\infty}^{\infty} u(t) v^*(t) dt = \int_{-\infty}^{\infty} U(f) V^*(f) df$$

- For $u = v$,

$$||u||^2 = < u, u > = \int_{-\infty}^{\infty} |u(t)|^2 dt \ = \int_{-\infty}^{\infty} |U(f)|^2 df$$

Properties of Fourier Transform

- Convolution in time domain:

$$y(t) = u(t) * v(t) = (u * v)(t) \longleftrightarrow Y(f) = U(f)H(f)$$

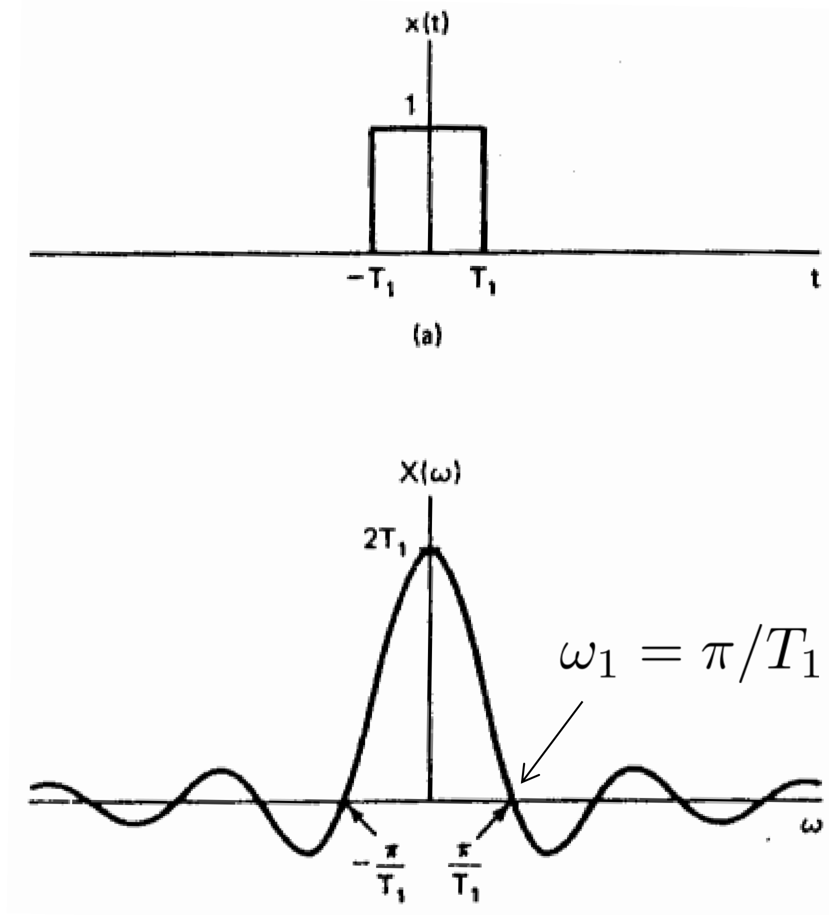
- Multiplication in time domain:

$$y(t) = u(t)v(t) \longleftrightarrow Y(f) = (U * H)(f) = U(f) * V(f)$$

Rectangular pulse and Sinc function: *S&S*

- Find Fourier transform for the signal

$$x(t) = \begin{cases} 1 & |t| < T_1 \\ 0 & |t| > T_1 \end{cases}$$



$$X(j\omega) = 2\frac{\sin \omega T_1}{\omega}$$

Rectangular pulse and Sinc function

- For a rectangular pulse $u(t) = I_{[-T/2,T/2]}(t)$ of duration T. Its Fourier transform is given by

$$U(f) = T\text{sinc}(fT) = T\frac{\sin(\pi fT)}{\pi fT}$$

- The Fourier transform is denoted by

$$I_{[-T/2,T/2]}(t) \longleftrightarrow T\text{sinc}(fT)$$

- Using duality, the other Fourier transform pair is

$$I_{[-W/2,W/2]}(f) \longleftrightarrow W\text{sinc}(Wt)$$

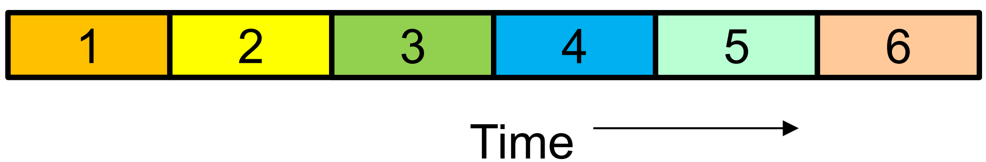
Today's Class

Energy Spectral Density and Bandwidth

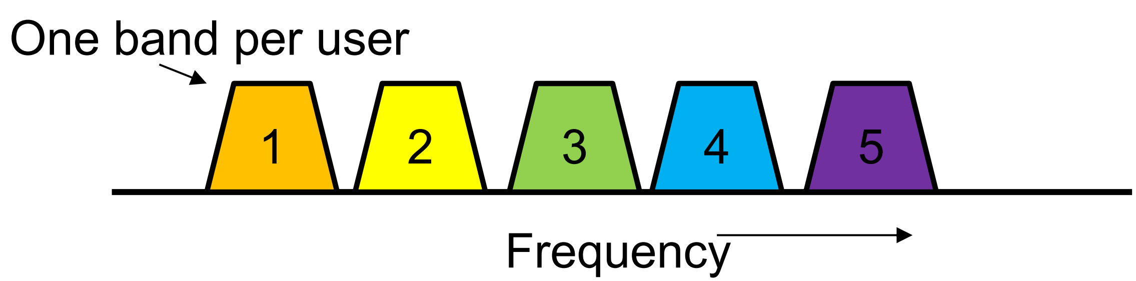
Motivation

- Shared physical medium

- Example of Time Division Multiple Access (TDMA)



- Example of Frequency Division Multiple Access (FDMA)

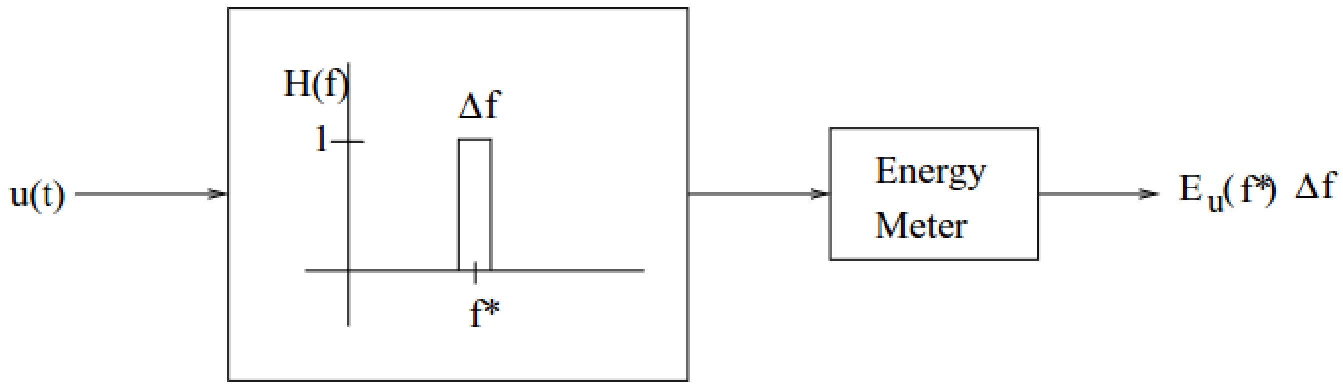


Motivation

- Communication channels have frequency-dependent characteristics
- Wireless spectrum is precious
- A broadcast and a shared medium
- Spectral characterization of the transmitted signal is important
  - Time limited signal has infinite bandwidth and vice-versa.

Energy Spectral Density

- Energy spectral density $E_u(f)$ of a signal $u(t)$ is defined to be the energy at the output of the filter divided by the width Δf in the limit as $\Delta(f) \to 0$.



- Prove that $E_u(f) = |U(f)|^2$

Energy Spectral Density

- Consider an ideal narrowband filter with transfer function

$$H_{f^*}(f) = \begin{cases} 1 & f^* - \Delta f < f < f^* + \Delta f \\ 0 & \text{otherwise} \end{cases}$$

- The frequency response of output $y(t)$

$$Y(f) = U(f)H(f) = \begin{cases} U(f) & f^* - \Delta f < f < f^* + \Delta f \\ 0 & \text{otherwise} \end{cases}$$

- By Parseval's identity, the energy at the output of the filter is

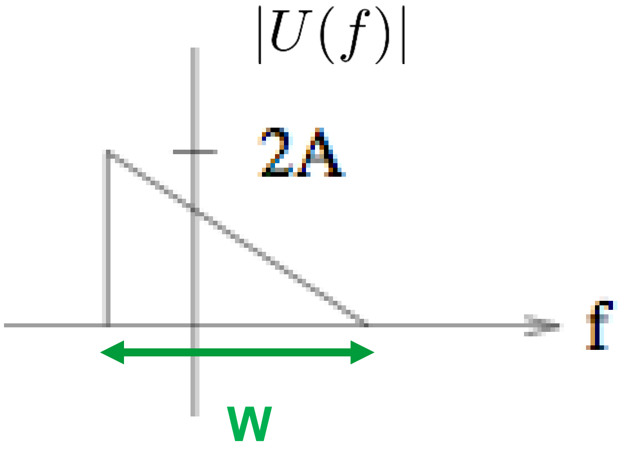
$$\int_{-\infty}^{\infty} |Y(f)|^2 \, df = \int_{f^*-\Delta/2}^{f^*+\Delta/2} |U(f)|^2 \, df \approx |U(f)|^2 \Delta f$$

- Equating this to output of energy meter, we get the desired result.

Bandwidth for bandlimited signals

Several definitions depending on the application

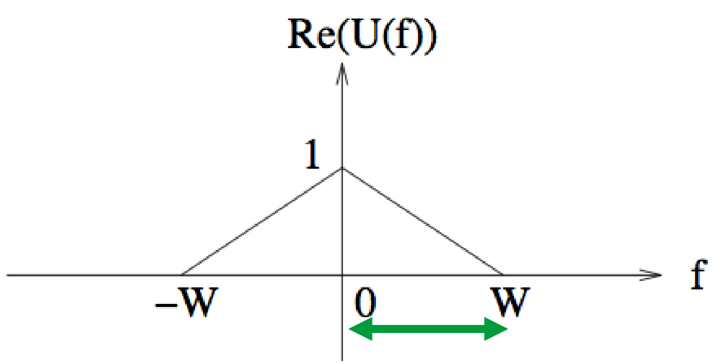
- The bandwidth of a strictly bandlimited signal $u(t)$ is defined to be the size of the band of frequencies occupied by $U(f)$.

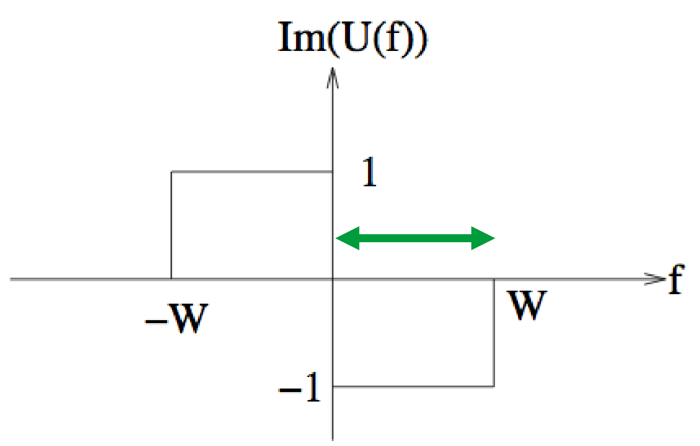


Bandwidth for bandlimited signal: *Real Signal*

Several definitions depending on the application

- For physical signals (real-valued), $U(-f) = U^*(f)$.
- Only positive frequencies count when computing bandwidth for physical (real-valued) signals. Also called one-sided bandwidth.

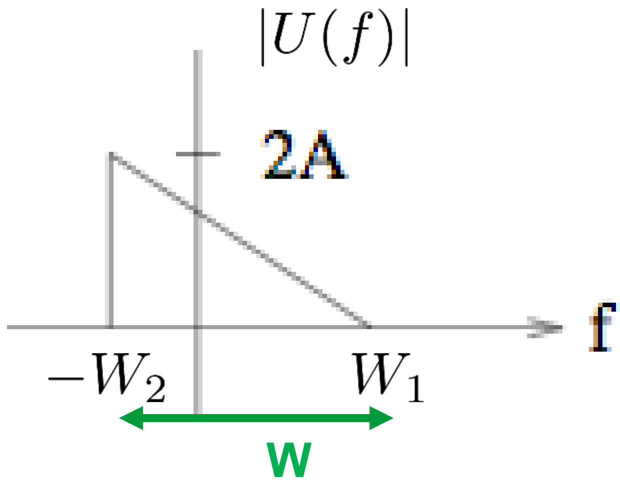




Bandwidth for bandlimited signals: complex

Several definitions depending on the application

- The bandwidth of a strictly bandlimited signal $u(t)$ is defined to be the size of the band of frequencies occupied by $U(f)$.

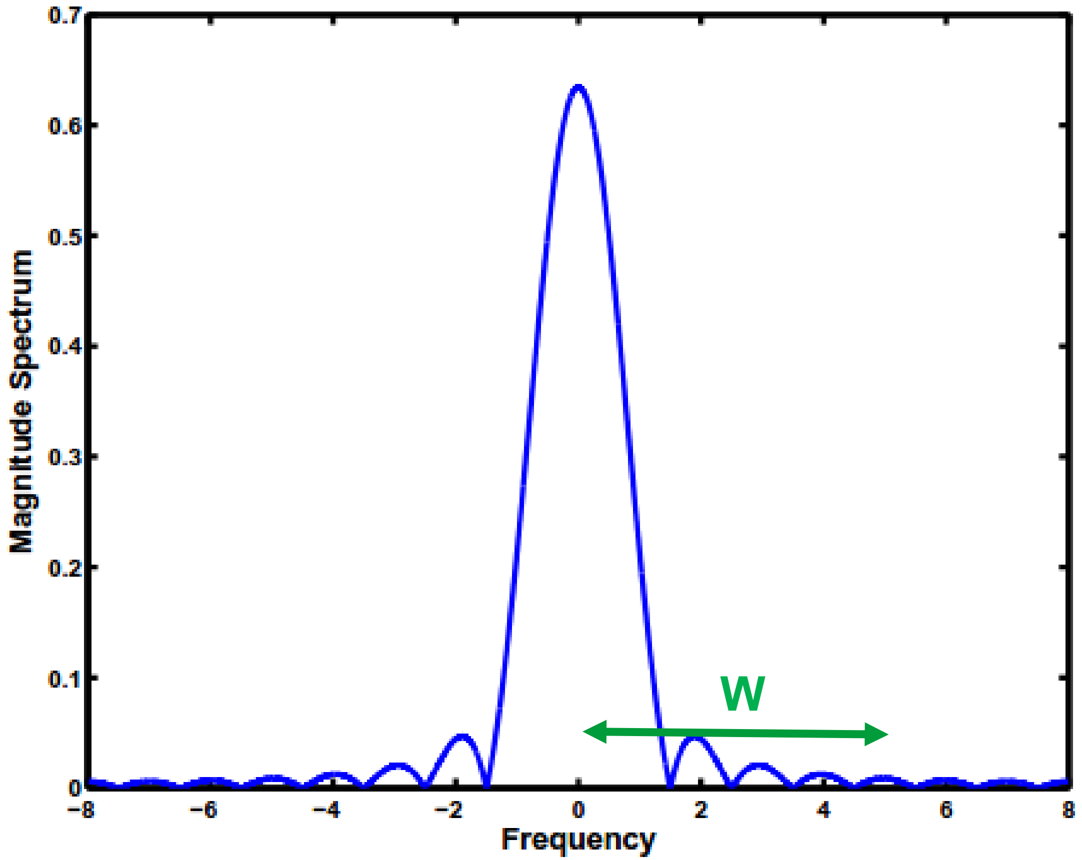


- The bandwidth of complex-valued signals is defined as the size of the frequency band it occupies over both positive and negative frequencies.

Bandwidth for not-bandlimited signal

Several definitions depending on the application

- The bandwidth of signal $u(t)$ which is not bandlimited is defined in terms of energy-containment bandwidth: the size of the smallest band which contains specified fraction of the signal energy.

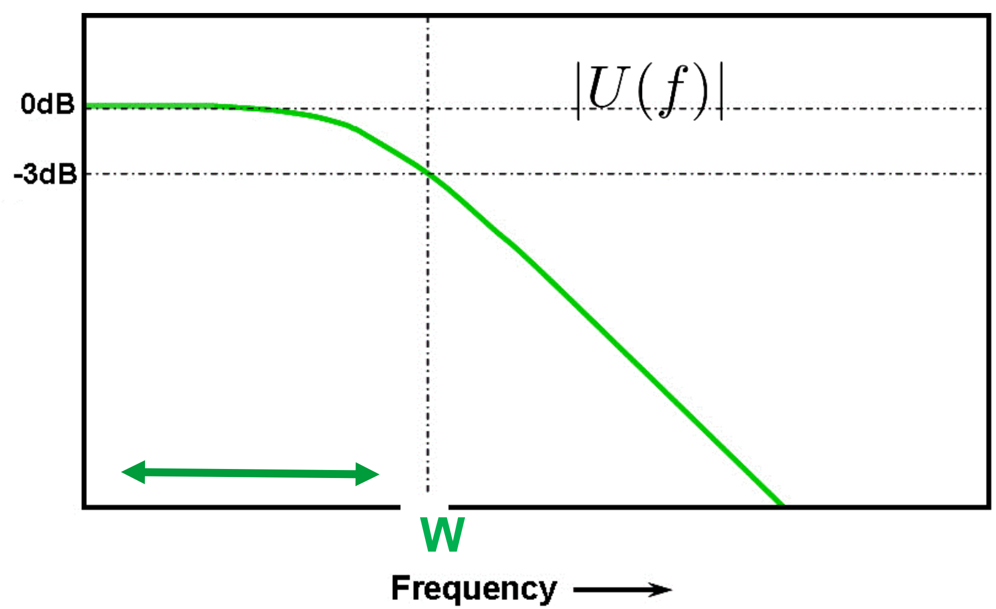


99% Energy

Bandwidth for not-bandlimited signal

Several definitions depending on the application

- The bandwidth of signal $u(t)$ which is not bandlimited can be defined in terms of band over which $|U(f)|^2$ is within some fraction of its peak value.
- For example, if the fraction is 0.5, the bandwidth is called 3-dB bandwidth.



Poll: Bandwidths?

- Are the following signals strictly bandlimited?

  1. Sinusoid (a)Yes (b) No
  2. Finite duration rectangular signal (a)Yes (b) No
  3. Sinc signal (a) Yes (b) No
  4. Impulse function (a) Yes (b) No

- How will you calculate the bandwidths of the following signals?

  1. Sinusoid (a)Energy containment (b) Finite (c) Cannot be done
  2. Finite duration rectangular signal (a)Energy containment (b) Finite (c) Cannot be done
  3. Sinc signal (a)Energy containment (b) Finite (c) Cannot be done
  4. Impulse function (a)Energy containment (b) Finite (c) Cannot be done

Example 3

- Find the bandwidth for $u(t) = \mathrm{sinc}(2t)$ where the time is in microseconds.
- Find the bandwidth for the timelimited waveform $u(t) = I_{(2,4)}(t)$, where the unit of time is in microseconds.

Rectangular pulse and Sinc function

- For a rectangular pulse $u(t) = I_{[-T/2,T/2]}(t)$ of duration T. Its Fourier transform is given by

$$U(f) = T\text{sinc}(fT) = T\frac{\sin(\pi fT)}{\pi fT}$$

- The Fourier transform is denoted by

$$I_{[-T/2,T/2]}(t) \longleftrightarrow T\text{sinc}(fT)$$

- Using duality, the other Fourier transform pair is

$$I_{[-W/2,W/2]}(f) \longleftrightarrow W\text{sinc}(Wt)$$

Baseband and Passband

The Complex Baseband Representation

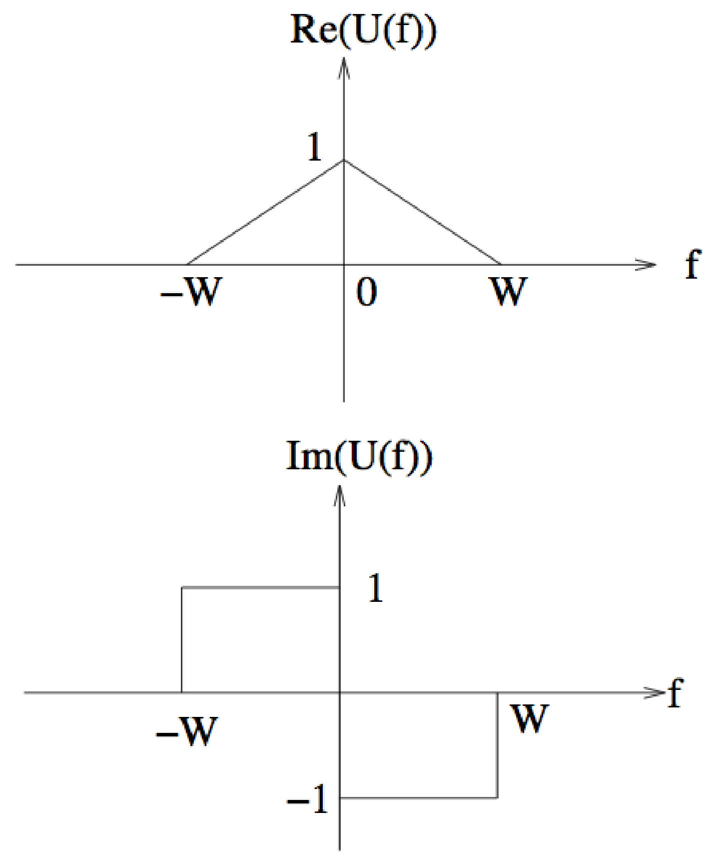
Baseband Signal

- A signal $u(t)$ is said to be baseband if the signal energy is concentrated in a band around DC and

$$U(f) \approx 0, \quad |f| > W$$

for some W.

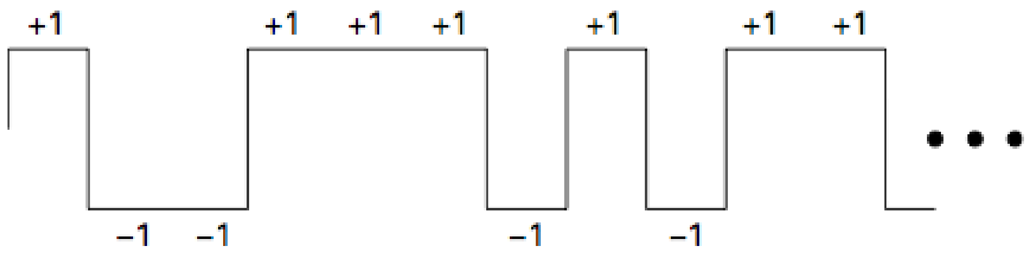
- Example: Information sources



Examples of baseband signals

- Speech and audio are baseband signals

- Two level digital signals over wired connection without modulation (also called line coding)



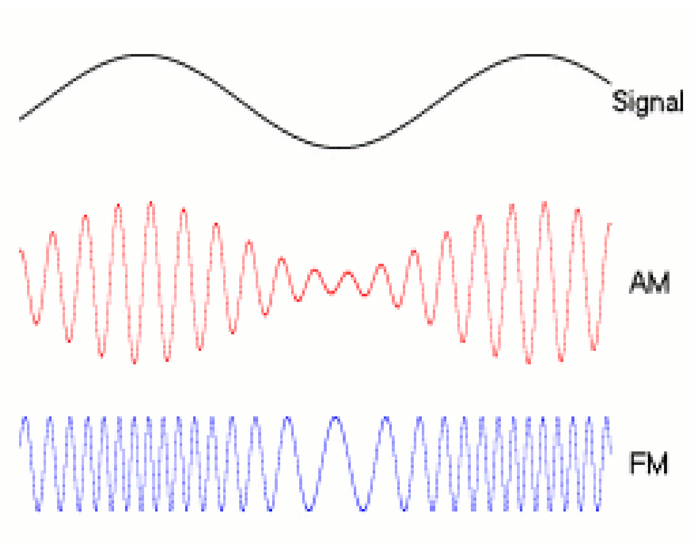
Passband Signal

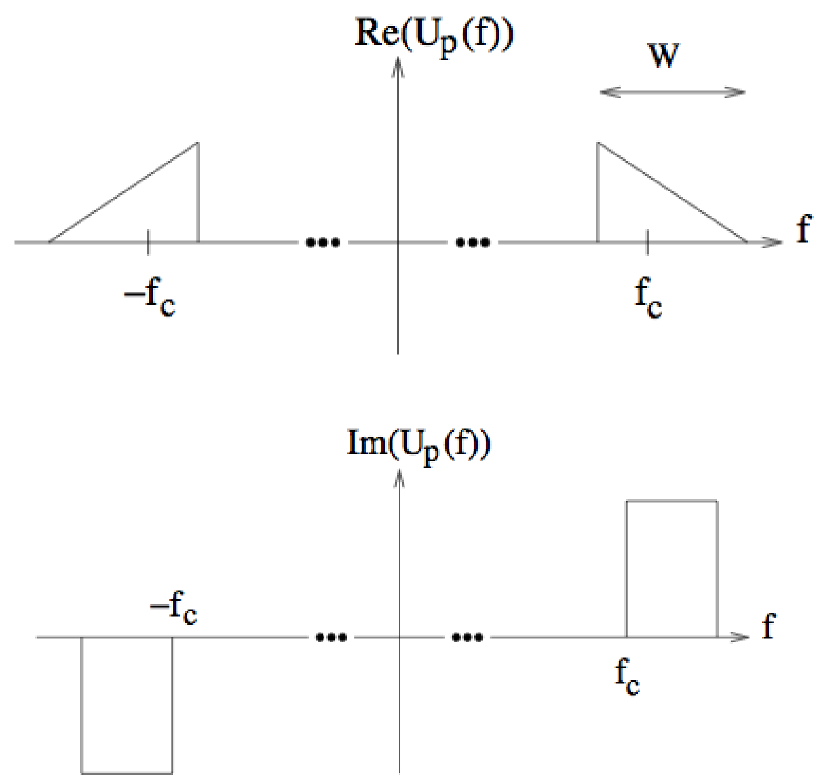
- A signal $u(t)$ is said to be passband if the signal energy is concentrated in a band away from DC with

$$U(f) \approx 0, \quad |f \pm f_c| > W$$

for some $f_c > W > 0$. Typically $f_c \gg W$

- Example: Wireless signal



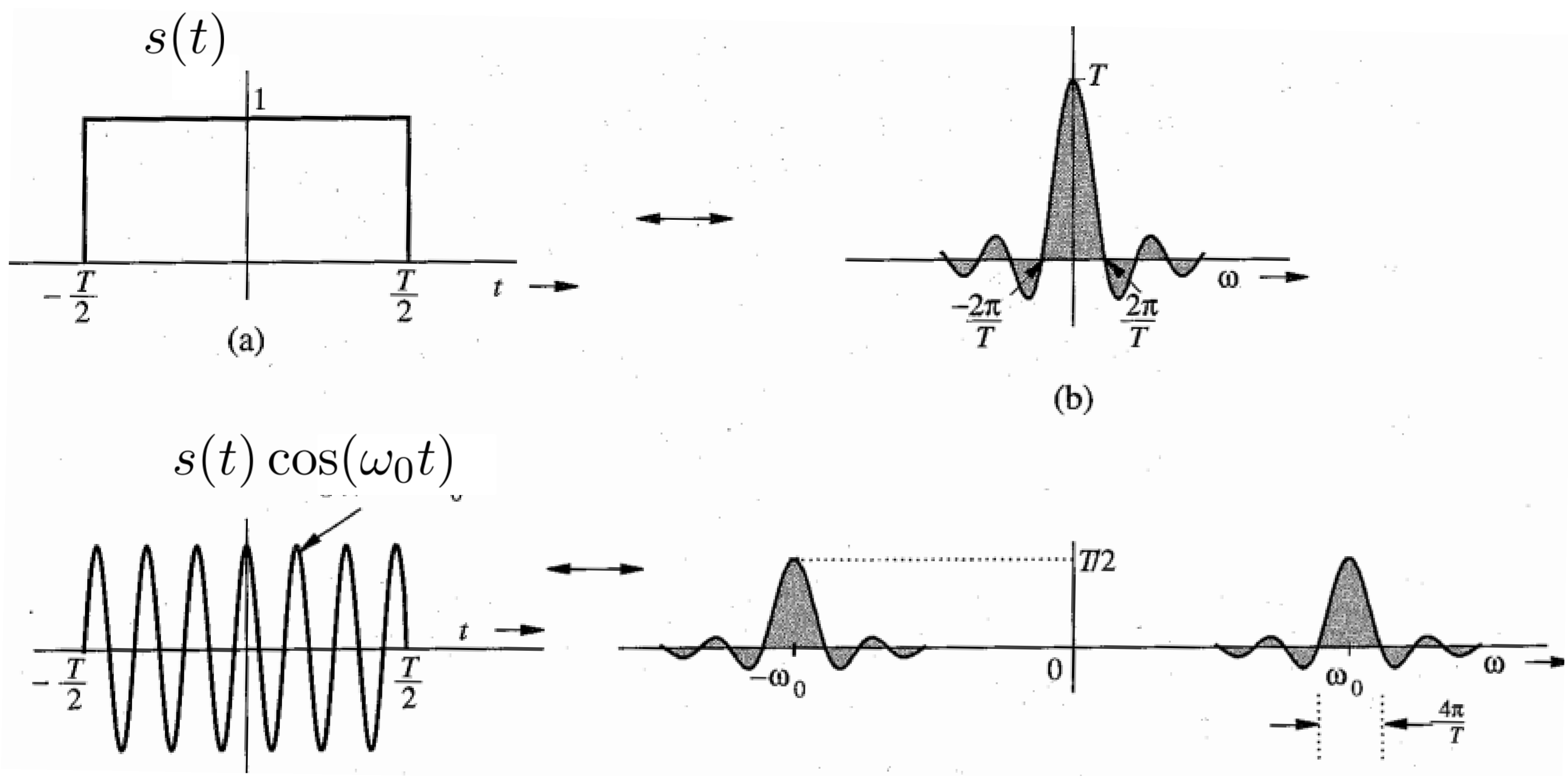


Example of Passband

- Consider this signal

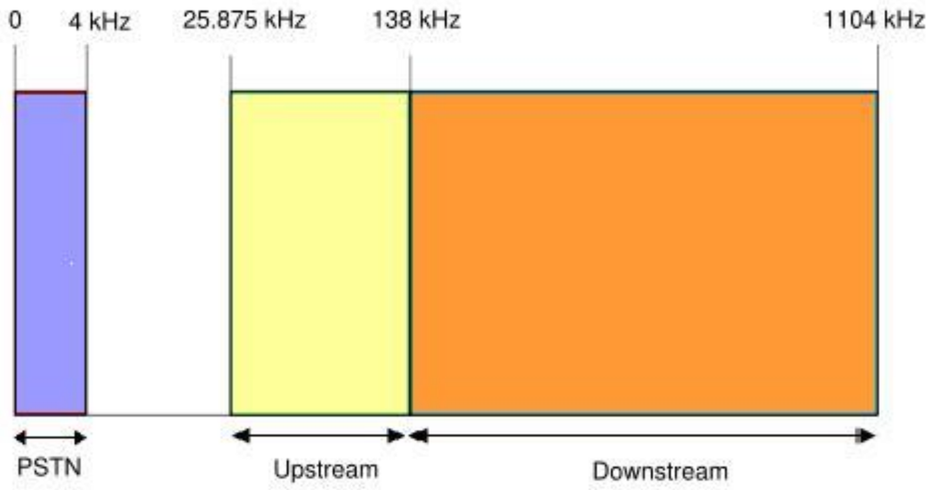
$$u_{\mathrm{p}}(t) = I_{[-T/2,T/2]}(t)\cos(2\pi f_0 t)$$

where $\omega_0 = 2\pi f_0 t$



Example of Passband in Wired Systems

- Digital Subscriber Line (DSL) is a technology, where high-speed data transmission coexists with voice transmissions in 0-4 KHz band
- Physical media is twisted pair copper wire



https://en.wikibooks.org/wiki/Communication_Networks/DSL

Differences between Baseband and Passband

- Baseband

  1. Original signal generated from the message source without any modulation of high frequency carrier
  2. Concentrated near zero frequency
  3. Can carry only one message signal at a time
  4. Examples: Ethernet, landline cables, network cables, coaxial cables.

- Passband

  1. Refers to modulated signal mostly
  2. Away from DC and concentrated around carrier frequency
  3. Multiple signals can be multiplexed simultaneously
  4. Examples: Wireless signal

Baseband Channel

- A channel modeled as a LTI system is said to be baseband if its transfer function $H(f)$ has support concentrated around DC and satisfies

$$H(f) \approx 0, \quad |f| > W$$

  for some W.

- Example: Wired channels

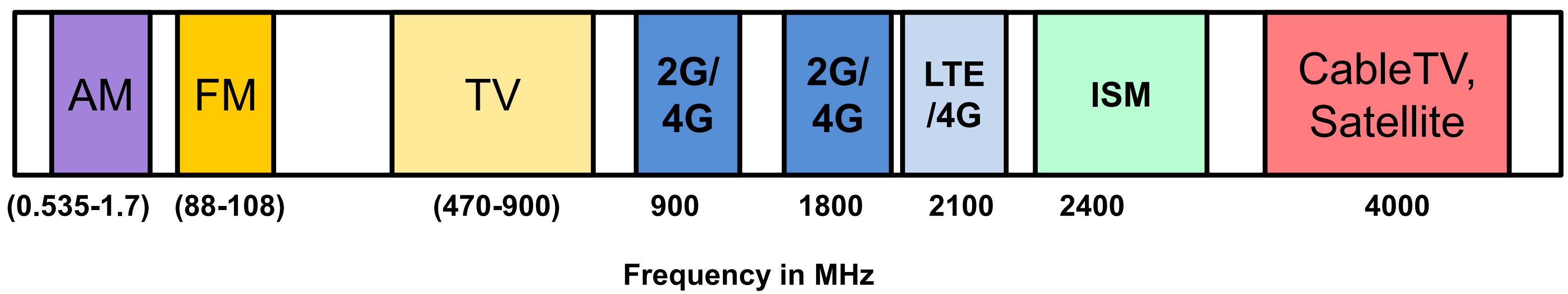
Passband Channel

- A channel modeled as a LTI is said to be passband if its transfer fuction $H(f)$ satisfies

$$H(f) \approx 0, \quad |f \pm f_c| > W$$

for some $f_c > W > 0$.

- Example: Wireless Channel



Baseband and Passband Signals/Channels

- Channels are often modeled as LTI systems: Signal passes through channel and then noise is added
- Channels allocated/described typically in terms of frequency bands: Signals have to be designed for corresponding frequency band
- Complex baseband representation of passband systems: Unified treatment of baseband and passband systems.

Questions?

Structure of Passband Signal

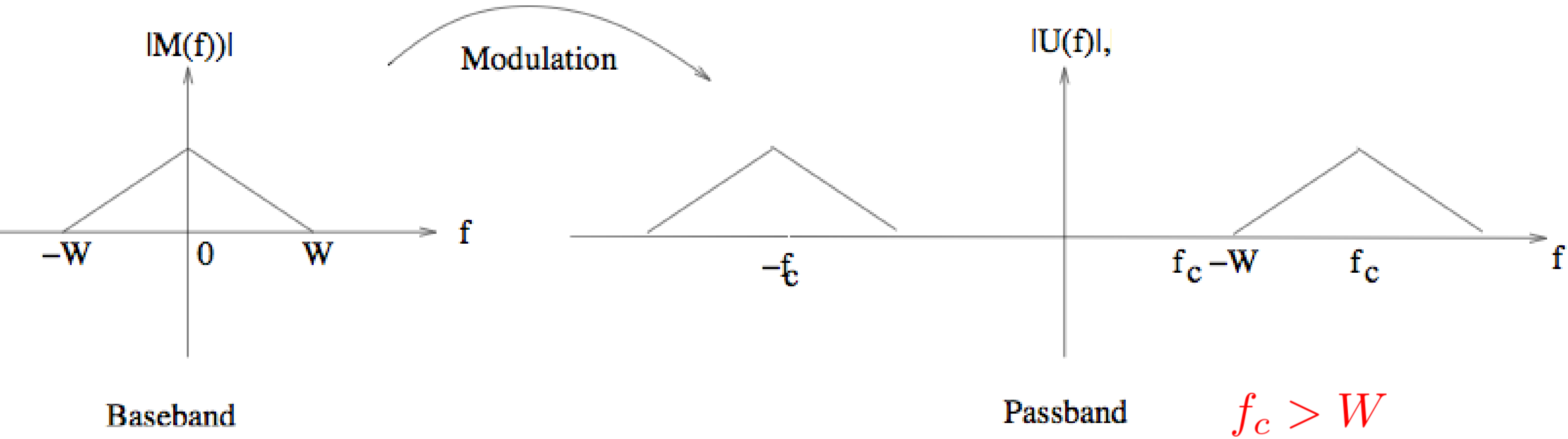
Motivation

- Passband is important for different communication systems
- Need to understand how to design transmitted signal from the information and recover the same from the received signal

Modulation or Upconversion

- Let $m(t)$ be the message signal of bandwidth W to be sent over passband channel around f_c called as carrier frequency.
- One method: multiply by a sinusoid at f_c

$$u_{\mathrm{p}}(t) = m(t)\cos(2\pi f_c t) \longleftrightarrow \frac{1}{2}\left(M(f - f_c) + M(f + f_c)\right)$$



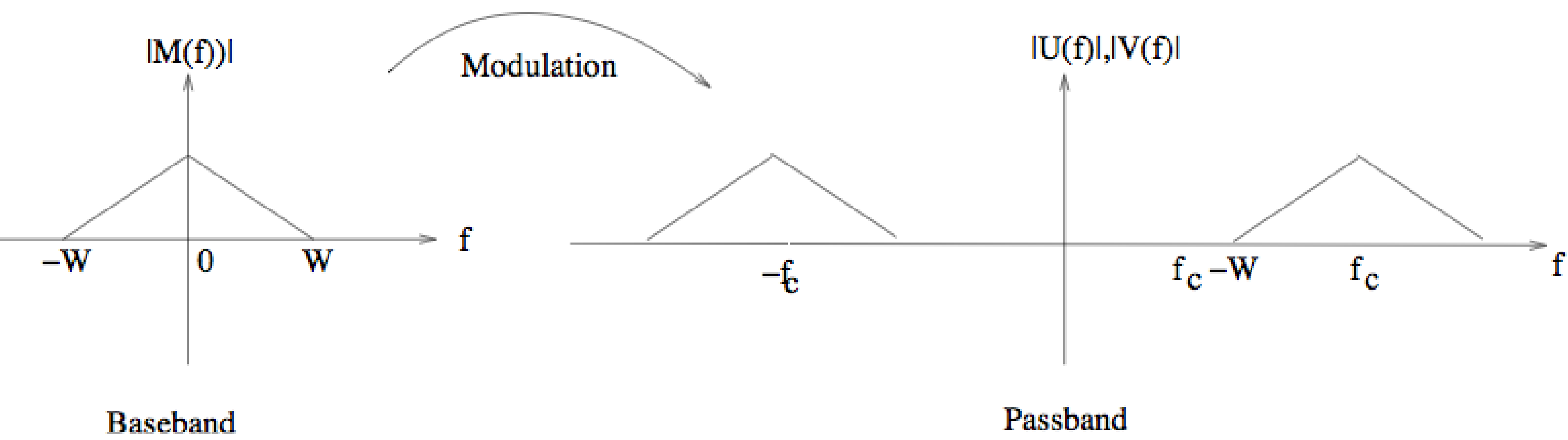
$f_c > W$

Modulation or Upconversion

- You can also use a sine

$$v_{\mathrm{p}}(t) = m(t)\sin(2\pi f_c t) \longleftrightarrow \frac{1}{2j}\left(M(f-f_c) - M(f+f_c)\right).$$

- Magnitude spectrum for $v_{\mathrm{p}}(t)$ will be same as that for $u_{\mathrm{p}}(t)$.



Modulation: *I and Q components*

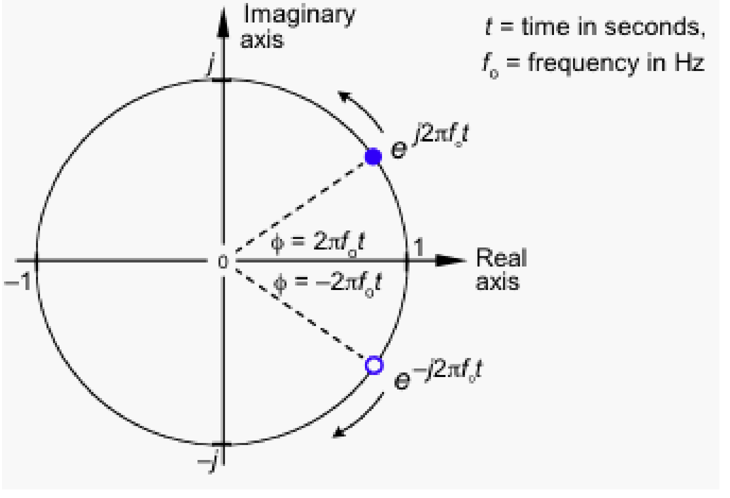
- If we use both cosine and sine carriers, we can construct a passband signal of the form

$$u_{\mathrm{p}}(t) = \boxed{u_{\mathrm{c}}(t)} \cos(2\pi f_c t) - \boxed{u_{\mathrm{s}}(t)} \sin(2\pi f_c t)$$

where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

In-phase or I component

Quadrature or Q component



Modulation: *I and Q components*

- If we use both cosine and sine carriers, we can construct a passband signal of the form

$$u_{\mathrm{p}}(t) = \boxed{u_{\mathrm{c}}(t)}\,\boxed{\cos(2\pi f_c t)} - \boxed{u_{\mathrm{s}}(t)}\,\boxed{\sin(2\pi f_c t)}$$

where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

Carry all the information

Carry NO information

Orthogonality of I and Q channels

- Show that the passband waveforms $a_{\mathrm{p}}(t) = u_{\mathrm{c}}(t)\cos(2\pi f_c t)$ and $u_{\mathrm{b}}(t) = u_{\mathrm{s}}(t)\sin(2\pi f_c t)$ are orthogonal

$$< a_{\mathrm{p}}, b_{\mathrm{p}} >= 0$$

  i.e., I and Q components of $u_{\mathrm{p}}(t)$ are orthogonal.

- Advantage: we can send separate information on I and Q at the same time!.

Demodulation or Downconversion

- If we use both cosine and sine carriers, we can construct a passband signal of the form

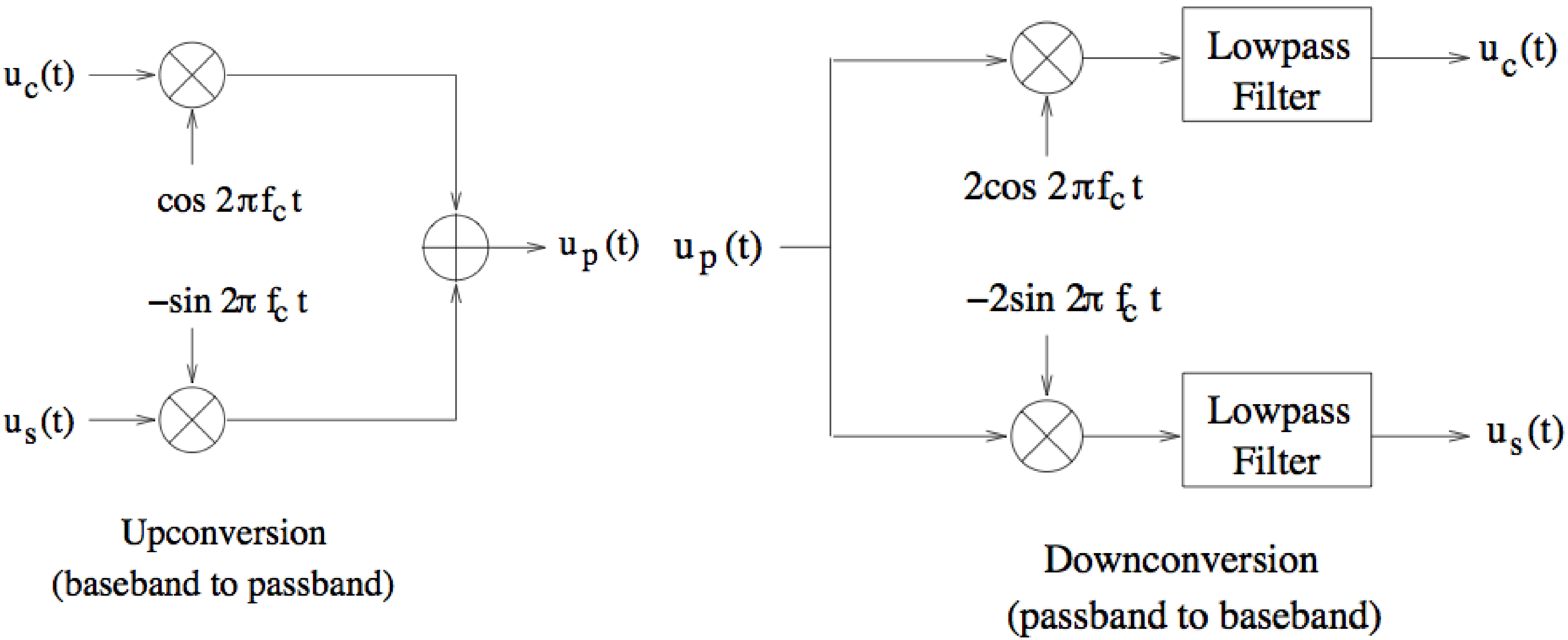
$$u_{\mathrm{p}}(t) = u_{\mathrm{c}}(t)\cos(2\pi f_c t) - u_{\mathrm{s}}(t)\sin(2\pi f_c t)$$

  where $u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, with $f_c > W$.

- Show that downconversion from passband to baseband can be done by lowpass filtering of these two components

$$\begin{aligned} &2u_{\mathrm{p}}(t)\cos(2\pi f_c t) \\ -&2u_{\mathrm{p}}(t)\sin(2\pi f_c t) \end{aligned}$$

Upconversion and Downconversion: *Block Diagrams*



Envelope and Phase

- $u_\mathrm{p} \equiv (u_\mathrm{c}, u_\mathrm{s})$ is called two-dimensional modulation.
- I and Q components corresponds to two dimensional waveforms in rectangular coordinates.
- Converting them to polar coorinates, we get

$$e(t) = \sqrt{u_c^2(t) + u_s^2(t)}$$
$$\theta(t) = \tan^{-1}\left(\frac{u_\mathrm{s}(t)}{u_\mathrm{c}(t)}\right)$$

  where $e(t) \geq 0$ is the envelope and $\theta(t)$ is the phase.
- Here

$$u_\mathrm{c}(t) = e(t)\cos\theta(t)$$
$$u_\mathrm{s}(t) = e(t)\sin\theta(t)$$

Complex Envelope

- A two dimensional point can be mapped to a complex number.
- The complex envelope $u(t)$ of the passband signal $u_p(t)$ is defined as

$$u(t) = u_c(t) + ju_s(t) = e(t)e^{j\theta(t)}$$

- Show that the passband signal $u_p(t)$ can be expressed in terms of the complex envelope $u(t)$ as

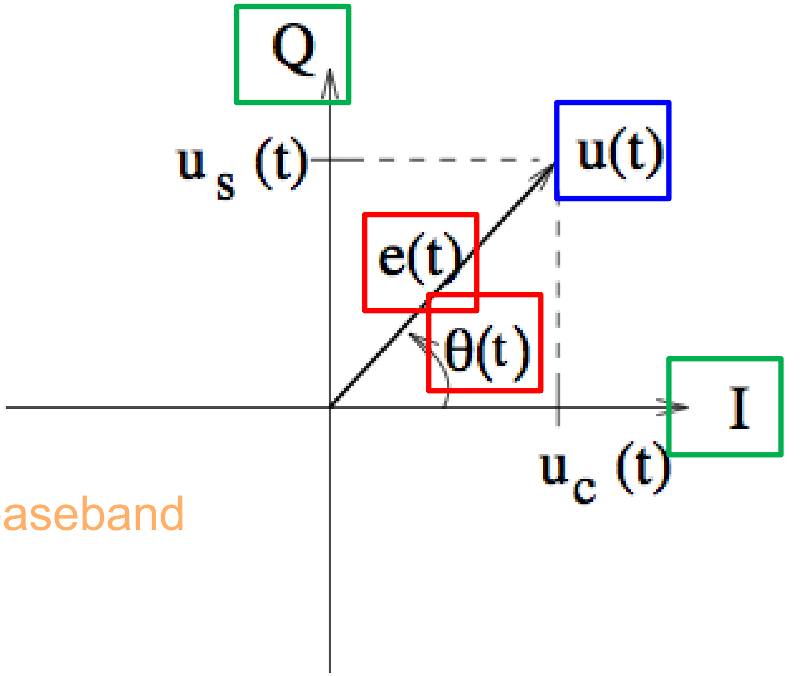
$$u_p(t) = \mathrm{Re}(u(t)e^{j2\pi f_c t})$$

Relationship between three representations

- Two-dimensional modulation: Passband signal u_p can be mapped to a pair of real baseband signals.

- The three representations are

  - Rectangular Coordinates I and Q
  - Envelope and Phase $e(t)$ and $\theta(t)$
  - Complex number or envelope $u(t)$



Information resides in complex baseband

Time domain expressions for a passband signal

- In terms of I and Q components

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

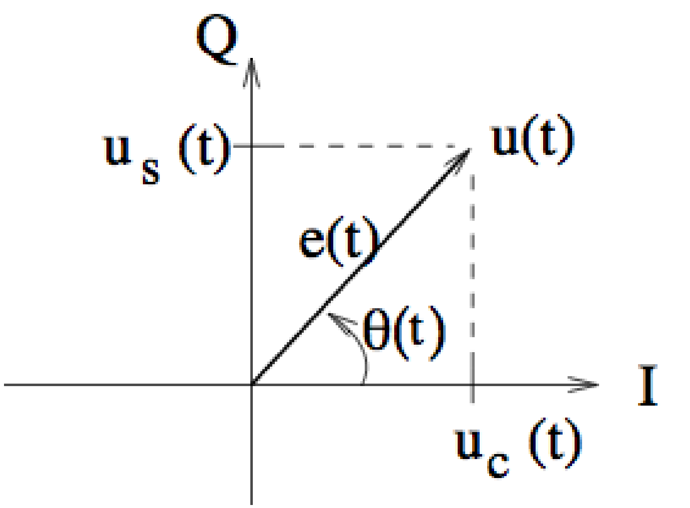
- In terms of envelope and phase

$$u_p(t) = e(t)\cos(2\pi f_c t + \theta(t))$$

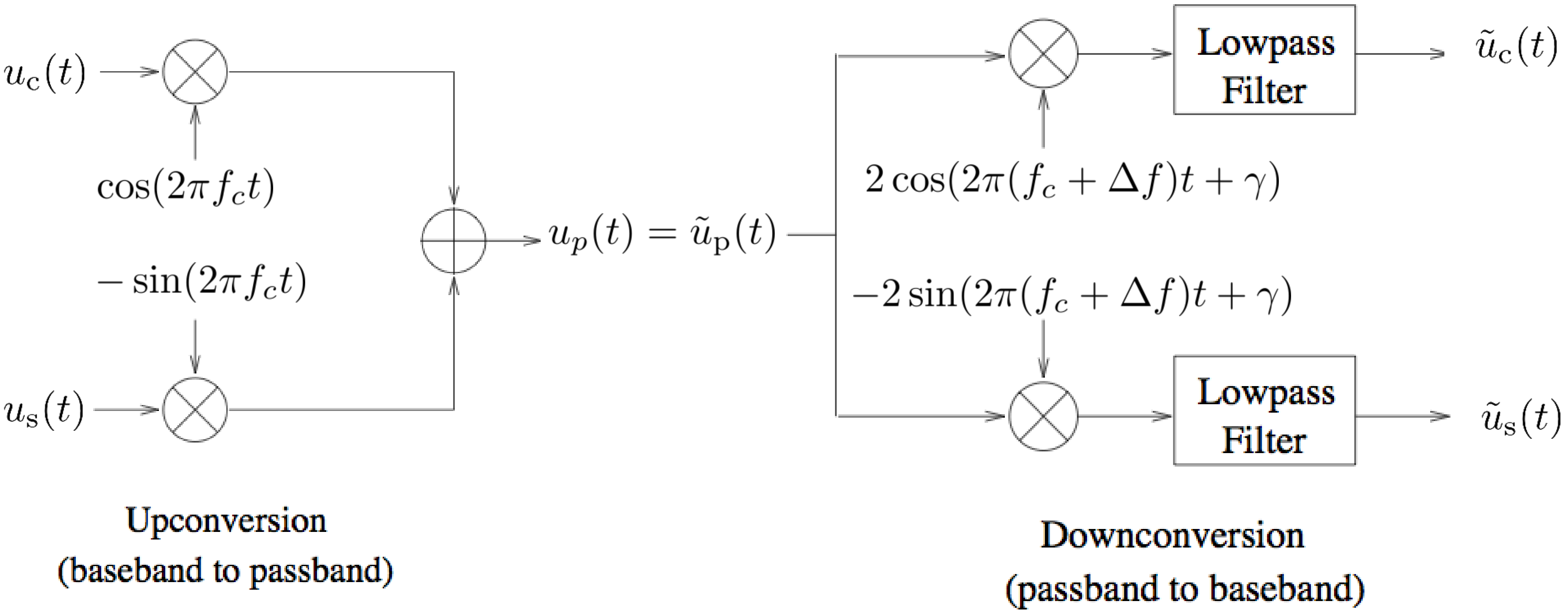
- In terms of complex envelope

$$u_p(t) = \mathrm{Re}(u(t)e^{j2\pi f_c t})$$

Starting from one representation, can derive the rest based on the relations depicted in the figure



Effect of Frequency and Phase Offset



- Show that in this case

$$\tilde{u}_c(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_s(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

where $\phi(t) = 2\pi\Delta f t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

Coherent Detection: Synchronization

- Frequency offset and phase offset cause cross-interference between I and Q components

$$\tilde{u}_{\mathrm{c}}(t) = u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t)$$
$$\tilde{u}_{\mathrm{s}}(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$$

  where $\phi(t) = 2\pi\Delta t + \gamma$ is the phase offset resulting from frequency offset Δf and the phase offset γ.

- Either have tight synchronization, i.e., $\Delta f \approx 0$ and $\gamma \approx 0$.

- Compensate for the offset $u(t) = \tilde{u}(t)e^{j\phi}$.

Summary of Properties: S&S

- If $u(t)$ is complex with Fourier transform is $U(f)$, then
  - Time reversal: $u(-t) \longleftrightarrow U(-f)$
  - Time conjugation: $u^*(t) \longleftrightarrow U^*(-f)$
  - Frequency shift: $u(t)e^{j2\pi f_0 t} \longleftrightarrow U(f - f_0)$

$U(f) = a + jb \Rightarrow a - jb$

$U^*(f) =$

$Re(U(f)) = Re(U^*(-f))$

- If $u(t)$ is real
  - Conjugate symmetry in frequency domain $U(f) = U^*(-f)$
  - Even symmetry for real part of $U(f)$: $\Re\{U(f)\} = \Re\{U(-f)\}$
  - Odd symmetry for imaginary part of $U(f)$: $\Im\{U(f)\} = -\Im\{U(-f)\}$
  - Magnitude has even symmetry: $|U(f)| = |U(-f)|$
  - Angle has odd symmetry: $\angle U(f) = -\angle U(-f)$

Frequency Domain Relationship

- Show that the frequency response for

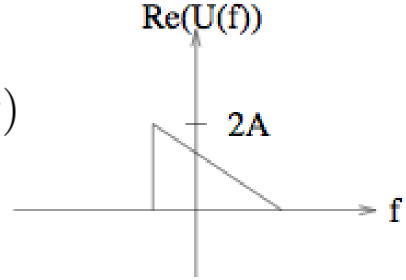
$$u_{\mathrm{p}}(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$$

is given by

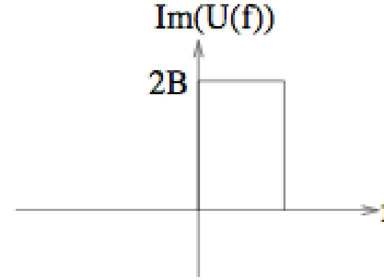
$$U_{\mathrm{p}}(f) = \frac{U(f - f_c) + U^*(-f - f_c)}{2}$$

Construct Passband from Baseband

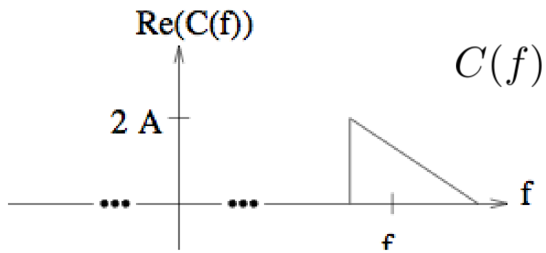
Complex signal $u(t)$



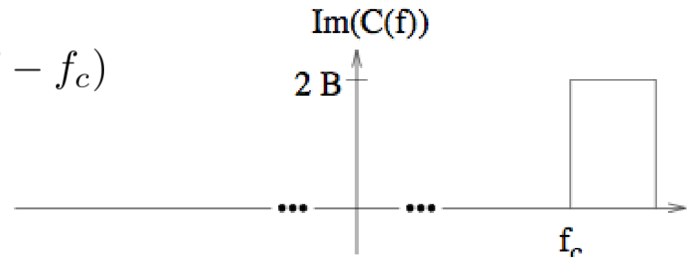
Complex $U(f)$



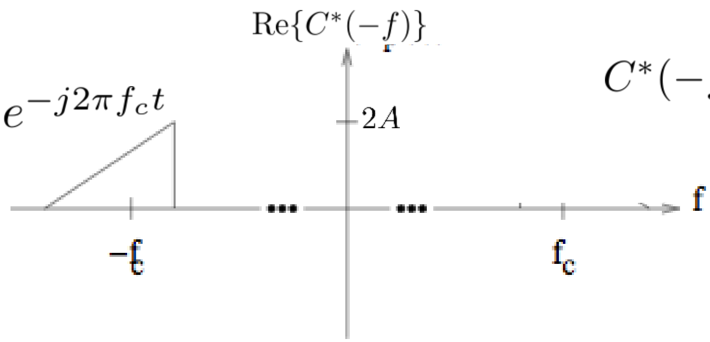
$$c(t) = u(t)e^{j2\pi f_c t}$$



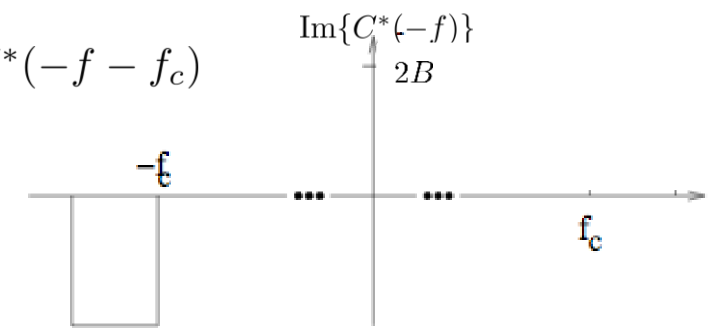
$$C(f) = U(f - f_c)$$



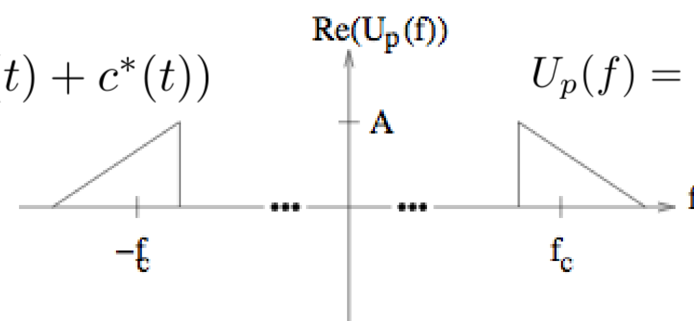
$$c^*(t) = u^*(t)e^{-j2\pi f_c t}$$



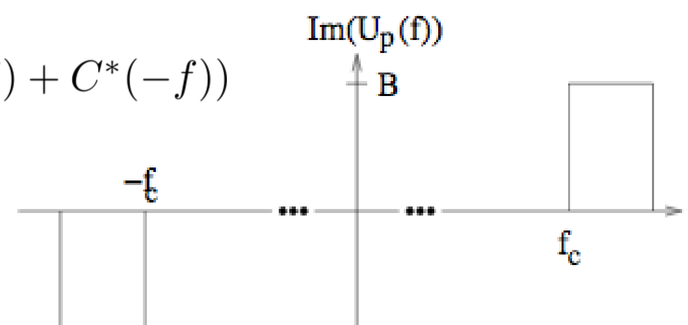
$$C^*(-f) = U^*(-f - f_c)$$



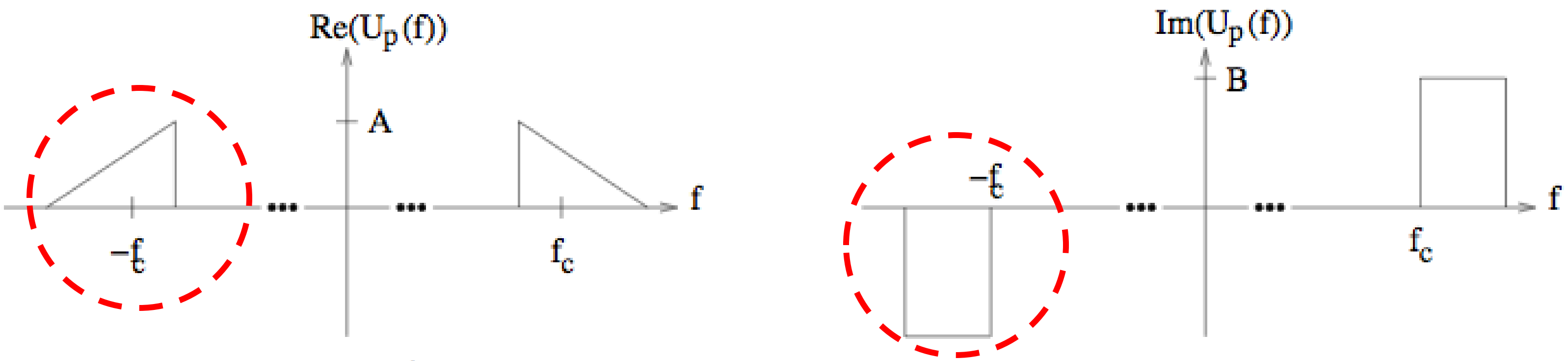
$$u_p(t) = 0.5(c(t) + c^*(t))$$



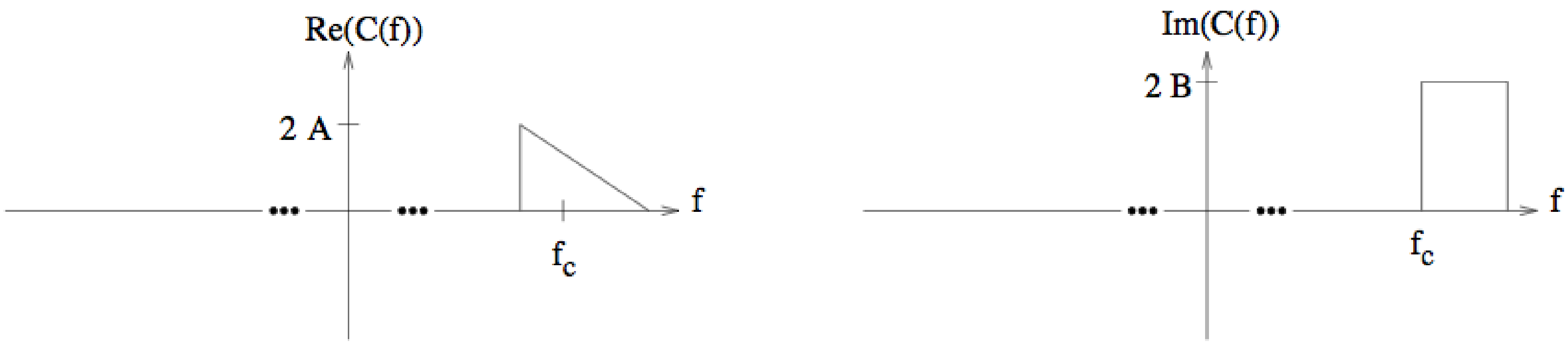
$$U_p(f) = 0.5(C(f) + C^*(-f))$$



Reconstruct Baseband from Passband



$$C(f) = 2U_p^+(f) = \begin{cases} 2U_p(f)\ , & f > 0 \\ 0\ , & f < 0 \end{cases}$$



$$u(t) = c(t)e^{-j2\pi f_c t} \leftrightarrow U(f) = C(f + f_c) = 2U_p^+(f + f_c)$$

I and Q components in frequency domain

- I and Q in time domain

$$u_{\mathrm{c}}(t) = \frac{1}{2}(u(t) + u^*(t))$$
$$u_{\mathrm{s}}(t) = \frac{1}{2j}(u(t) - u^*(t))$$

$$\updownarrow$$

- I and Q in frequency domain

$$U_{\mathrm{c}}(f) = \frac{1}{2}(U(f) + U^*(-f))$$
$$U_{\mathrm{s}}(f) = \frac{1}{2j}(U(f) - U^*(-f))$$

Passband filtering = complex baseband filtering

$$u_p(t) \longrightarrow \boxed{h_p(t)} \longrightarrow y_p(t)$$

EQUIVALENT TO
(except for a scale factor of 2)

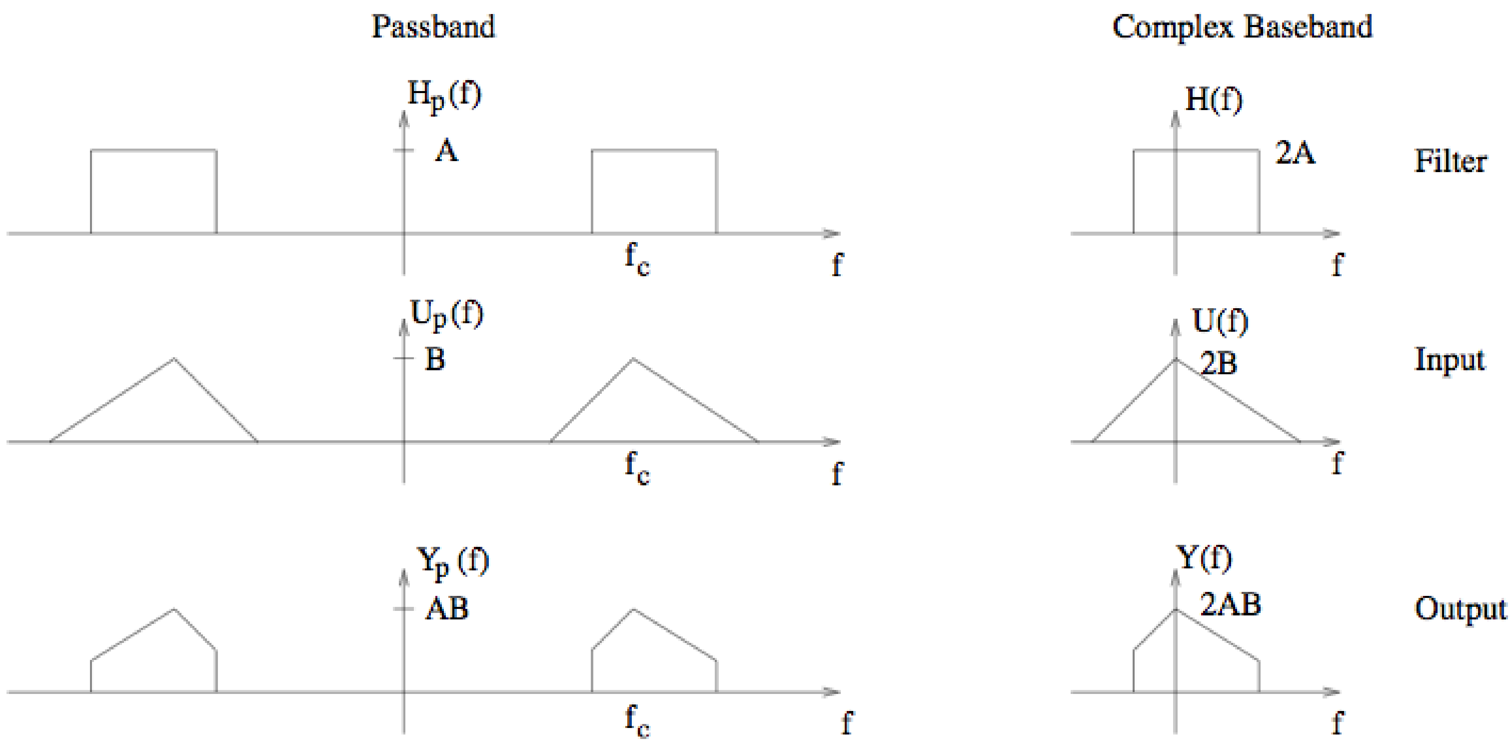
$$u(t) \longrightarrow \boxed{h(t)} \longrightarrow y(t)$$

$$u_{\mathrm{p}}(t) = \Re\{u(t)e^{j2\pi f_c t}\}$$

$$h_{\mathrm{p}}(t) = \Re\{h(t)e^{j2\pi f_c t}\}$$

$$y_{\mathrm{p}}(t) = \Re\{y(t)e^{j2\pi f_c t}\}$$

What is complex baseband representation of passband filtering?



$$Y(f) = 2Y_p^+(f + f_c) = 2U_p^+(f + f_c)H_p^+(f + f_c) = \frac{1}{2}U(f)H(f)$$

$$\boxed{Y_p(f) = U_p(f)H_p(f) \quad \equiv \quad Y(f) = \frac{1}{2}U(f)H(f)}$$

$$\boxed{y_p(t) = u_p(t) * h_p(t) \quad \equiv \quad y(t) = \frac{1}{2}u(t) * h(t)}$$

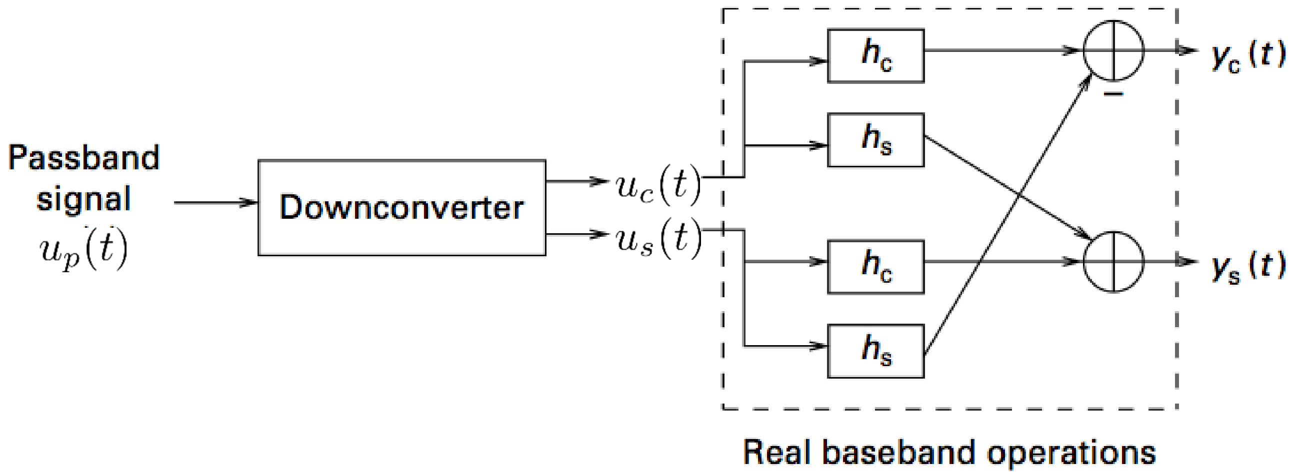
Filtering in complex baseband

- Complex-valued convolution to implement equivalent of passband filtering operation

$$y(t) = \frac{1}{2}(u * h)(t)$$

- Requires four real-valued convolutions

$$y_c = \frac{1}{2}(u_c * h_c - u_s * h_s); \qquad y_s = \frac{1}{2}(u_s * h_c + u_c * h_s);$$



$$x_{\mathrm{p}}(t) = x_{\mathrm{c}}(t)\cos(2\pi f_c t) - x_{\mathrm{s}}(t)\sin(2\pi f_c t)$$
$$x(t) = x_c(t) + jx_s(t)$$

Energy and Power

- Solve: Show that the energy of a passband signal equals that of its complex envelope, i.e.,

$$||u_p||^2 = \frac{1}{2}\left(||u_c||^2 + ||u_s||^2\right) = \frac{1}{2}||u||^2$$

- Since power is computed as time average of energy, similar relation exists for finite-power signals.

Correlation between two signals

- Correlation or inner product of two real-valued passband signals u_p and v_p is defined as

$$< u_p, v_p >= \int_{-\infty}^{\infty} u_p(t) {v^*}_{\mathrm{p}}(t)\ dt$$

- Show that

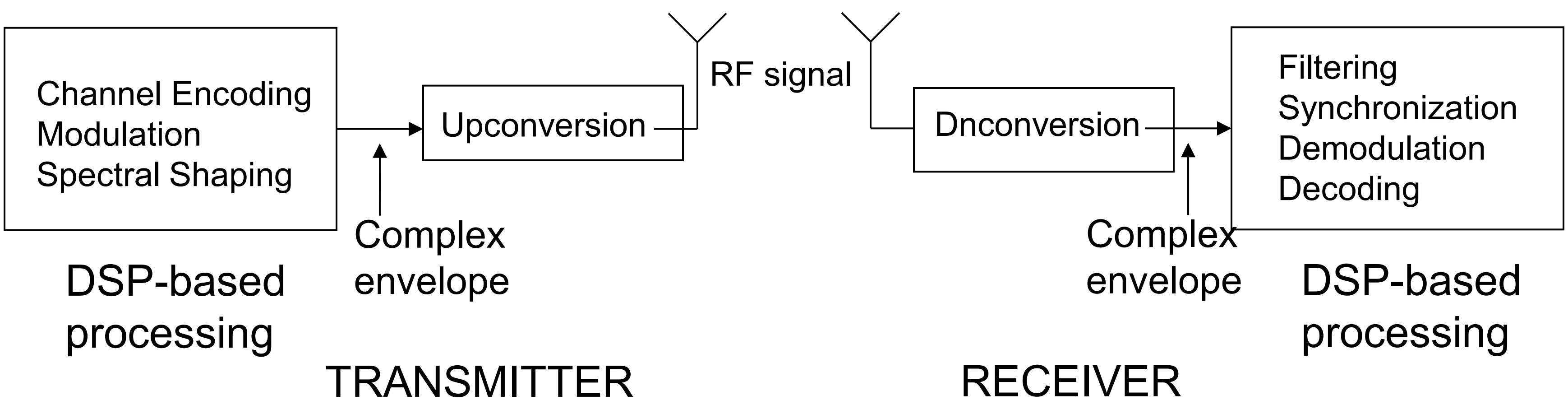
$$< u_p, v_p >= \frac{1}{2} \Re\{< u, v >\}$$

Modern transceiver architectures based on complex baseband

Modern transceiver work with the complex envelope rather than with the passband signal

- Complex baseband signals can be represented accurately by samples at a reasonable sampling rate
- Inexpensive to perform complicated DSP on the samples
- This architecture has been responsible for economics of scale in cellular and WiFi devices.
- Most of the research simulations happen in baseband.

All the action is in complex baseband for a typical wireless transceiver



Complex baseband representation: summary

- Any real-valued passband signal can be represented by a baseband signal which is in general complex-valued. This is called its complex envelope, or complex baseband representation.

- The complex envelope carries all the information in the passband signal

- Passband filtering operations can be equivalently performed in complex baseband

- Two-dimensional representation of complex envelope

  - Cartesian coordinates: A pair of real-valued baseband waveforms called the in-phase (I) and quadrature (Q) components

  - Polar coordinates: Envelope and phase waveforms

- Most of the sophisticated signal processing action in modern transceivers happens in complex baseband