Information and Communication — Problem Set 4 — February 23, 2025

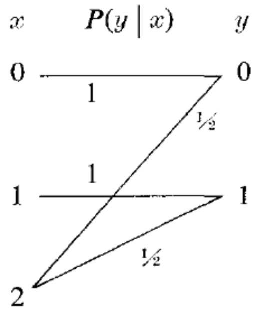
1. Let X, Y, Z be a joint random variables distributed as $P_{XYZ}(.)$. Prove the following inequalities and find conditions for equality.
   (a) $I(X, Y; Z) \geq I(X; Z)$.
   (b) $H(X, Y|Z) \geq H(X|Z)$.
   (c) $H(X, Y, Z) - H(X, Y) \leq H(X, Z) - H(X)$.

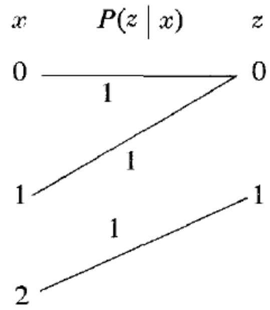
2. Frequency p_n of the n^{th} most frequent word in English is roughly approximated by

$$p_n = \begin{cases} \frac{0.1}{n} & n \in 1, 2, \ldots, 12367 \\ 0 & n > 12367 \end{cases}.$$

   If we assume that English is generated by picking words at random according to this distribution, what is the entropy of English (per word)?

3. A source X produces letters from a three-symbol alphabet with the probability assignment $P_X(0) = 0.25, P_X(1) = 0.25, P_X(2) = 0.50$. Each source letter X is directly transmitted through two channels simultaneously with outputs Y and Z and the transition probabilities indicated below in the figure. Calculate $H(X), H(Y), H(Z), H(Y, Z), I(X; Y), I(X; Z)$.





4. (a) Let X be a discrete random variable. Show that the entropy of a function of X is less than or equal to the entropy of X. (Hint: Expand the entropy of $H(X; g(X))$ using chain rule of entropy).
   (b) Show that if $H(Y|X) = 0$, then Y is a function of X i.e., for all x with $p(x) > 0$, there is only one possible value of y with $p(x, y) > 0$.

5. Calculate the entropy of a geometric random variable.