Science 1 - Quantum Mechanics

→ **Electromagnetic Waves :-**

Coupled electric and magnetic oscillations moving at the speed of light and exhibits typical wave behavior.

- EM waves are created through accelerating electric charges.

$$c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = 3 \times 10^8 \ m/s$$

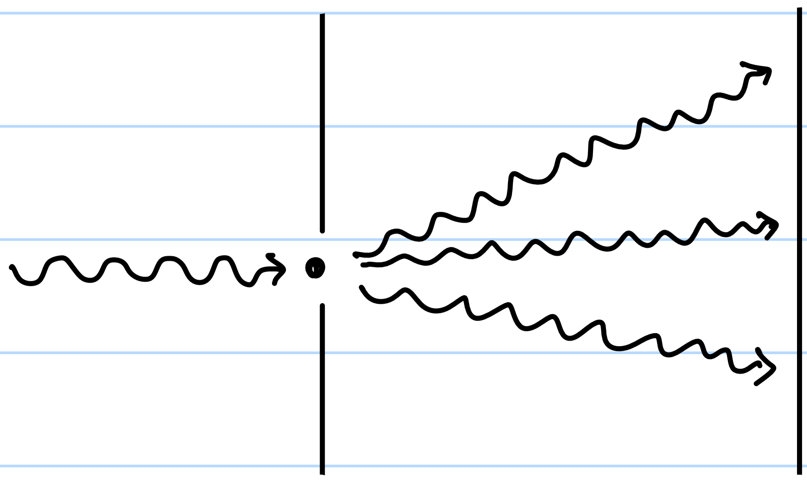
ε_0 = Permittivity of vacuum
μ_0 = Permeability of vacuum

- All waves obey the principle of superposition,

When 2 or more waves of a similar nature pass through a point at the same time, then the instantaneous amplitude is equal to the sum of the instantaneous amplitude of the individual waves.

- When multiple waves interact with each other as per the principle of superposition, the phenomenon is termed as interference.

- Another characteristic behavior of waves is diffraction, which is the spreading out of waves when passing through a slit.

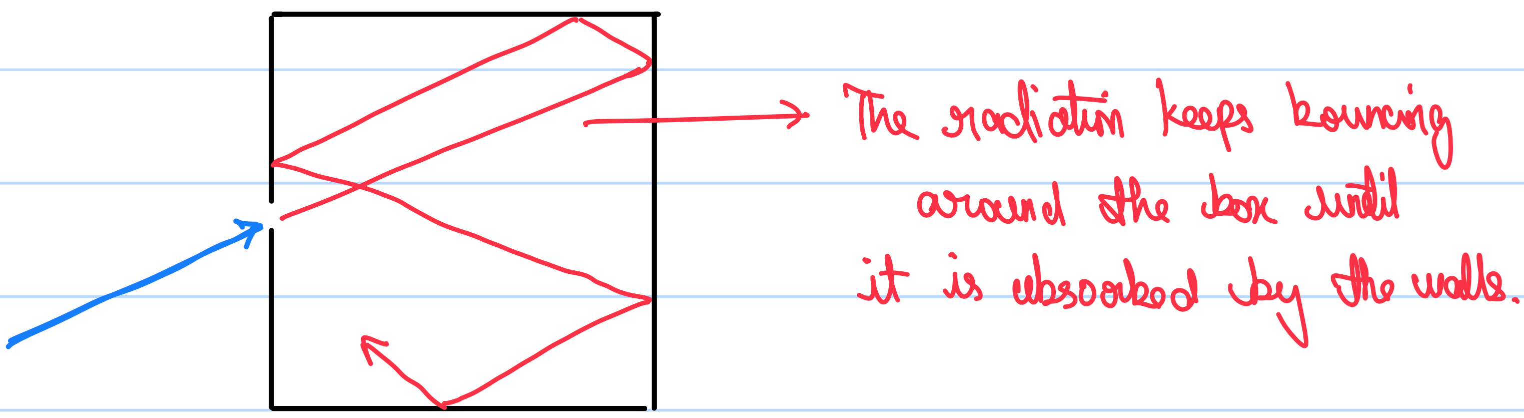


Diffraction

- The characteristic light-dark pattern we see in Young's double slit experim-ent is due to the interference between the 2 diffraction patterns formed by the 2 slits.

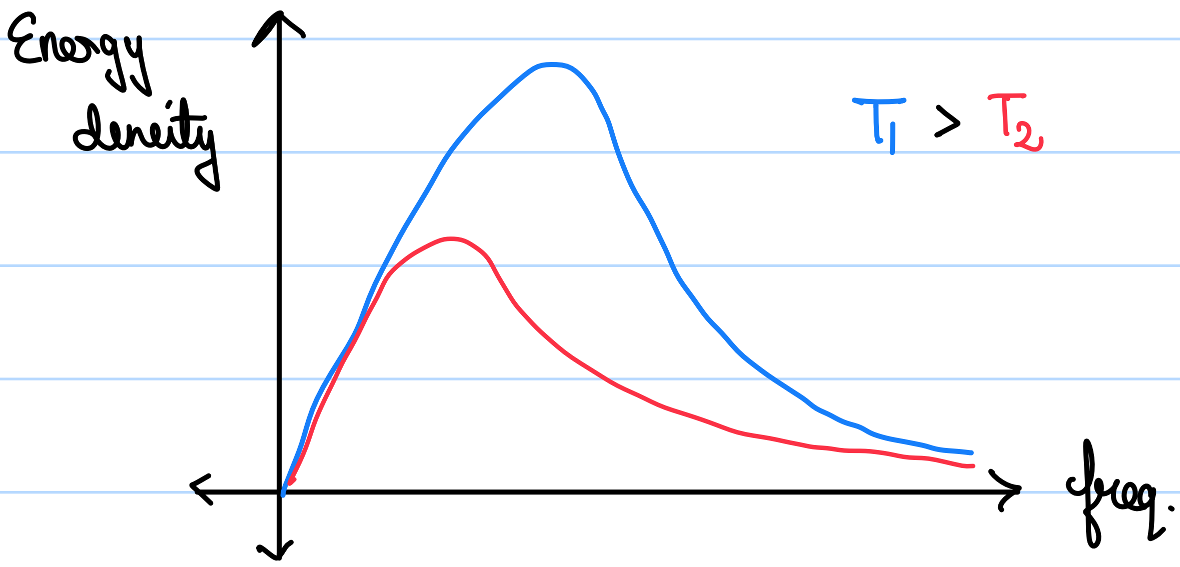
→ **Blackbody Radiation :-**

- A blackbody is an ideal object that absorbs all EM waves incident on it, regardless of its frequency.
- At thermal equilibrium, any body emits the same amount of energy at the same rate as the energy absorbed by it.
- A hole in the wall of a hollow object is an approximation of a black body.



• The Ultraviolet Catastrophe :-

- Experimentally the spectrum of the energy emitted by a blackbody at different temperatures is found as below.



- The explanation for the above spectrum is as follows,

1. In the hollow box example, the multiple reflections of the radiation interfere with each other and cause standing waves within the box.

   The density of the standing waves of frequency ν is found to be,

$$G(\nu)\,d\nu = \frac{8\pi\nu^2\,d\nu}{c^3}$$

2. The energy per standing wave is,

$$E = kT \qquad k - \text{Boltzmann Constant.}$$

3. So, the energy density per frequency is given by

$$\boxed{u(\nu)\,d\nu = \frac{8\pi\nu^2 kT}{c^3}\,d\nu}$$ (Rayleigh-Jeans formula)

This above formula is derived using classical mechanics, but shows an obvious problem.

As per Rayleigh-Jeans, $u(\nu) \propto \nu^2$

Therefore, as $\nu \longrightarrow \infty$, $u(\nu) \longrightarrow \infty$

But empirically we know as $\nu \longrightarrow \infty$, $u(\nu) \longrightarrow 0$

- Planck Radiation Formula :-

Planck came up with a ad hoc formula to explain the above discrepancy.

$$\boxed{u(\nu)\,d\nu = \frac{8\pi h\nu^3\,d\nu}{c^3\left(e^{h\nu/kT}-1\right)}}$$

A consequence of this equation is that the energies of the oscillating standing waves, must be a discrete distribution such that,

$$\boxed{E = nh\nu}$$

- This same result applies to light as well. (since light is an EM wave), where it is stated that light is made up of photons, each having energy $h\nu$ and zero rest mass.

- The photons and their behavior (photoelectric effect, pair generation, etc) make up the particle nature of light.

★ Wave Nature of Light :-

→ De-Broglie Wavelength :-

- The momentum of a photon is given by,

$$p = \frac{h\nu}{c} = \frac{h}{\lambda}$$

$$\Rightarrow \boxed{\lambda = \frac{h}{p}}$$

- De-Broglie stated that the above formula is a general property of any moving object, not only of light. So, the De-Broglie wavelength of a moving body is given by,

$$\lambda = \frac{h}{\gamma m v} \qquad \left(\gamma = \frac{1}{\sqrt{1 - v^2/c^2}} \right)$$

- This wave behavior is termed as matter wave, and they don't represent the oscillation of any physical quantity - they represent waves of probability.

→ Wave function (ψ) :-

- The oscillating quantity in matter waves is the wave function ψ of the object, which is a function on coordinates x, y, z and time t, ($\psi(x, y, z, t)$) such that

$$|\psi^2(x,y,z,t)| \propto \text{Probability of finding the object at } (x,y,z) \text{ at time } t.$$

→ Particle Diffraction :-

- Diffraction is a phenomenon that is characteristic of wave nature, and as per Classical mechanics, not possible for particles.