Assignment 3

EC5.201 Signal Processing

The Continuous-Time Fourier Transform & Sampling
Deadline: (Theory) 8th Sep '25, 11.55PM IST

Instructions:

- All the questions are compulsory
- The questions contain both theory (analytical) and practical (MATLAB coding) parts
- The submission format is as follows:
  - **Theory (on Moodle)** – PDF file containing the handwritten theory assignment solutions
  - **Lab (on GitHub)**
    - **Code** – folder containing all the codes
    - **Images** – folder containing all the .png images
    - **Report** – PDF/text file containing observations for the MATLAB section

  The naming convention for the code and image files is q<qn no.> _ <sub-part no.>.
- **Late submission:** For the theory component, a 10% penalty per day will be applicable (accepted up to at most 3 days after the deadline). No late submissions will be accepted for the lab component.

Question 1: CTFT

1. Compute the Fourier transform and sketch the magnitude and phase spectra of each of the following signals:

   (a) $x(t) = \delta(t - t_0)$

   (b) $x(t) = \begin{cases} 1, & |t| < T_0 \\ 0, & |t| \geq T_0 \end{cases}$

   (c) $x(t) = e^{-\alpha t}u(t)$, $\alpha \in \mathbb{C}$, $\mathrm{Re}(\alpha) > 0$ and $\mathrm{Im}(\alpha) \neq 0$.
   [Hint: Assume the form $\alpha = a + jb$.]

   (d) $x(t) = e^{-\alpha|t|}$, α is real and positive.

   (e) $x(t) = \dfrac{\sin^2(\pi t)\cos(\pi t)}{\pi t^2}$
   Refrain from directly computing the transform. Instead, use the properties of CTFT.
   [Hint: You might need to simplify the signal first.]

2. Prove that the Fourier transform $X(\omega)$ of a real and odd signal $x(t)$ is purely imaginary.

MATLAB:

Define a function `continuousFT` as follows,

```
function X = continuousFT(t, x, a, b, w)
    % evaluates the continuous-time Fourier transform of given signal
    %
    % t - symbolic variable
    % x - signal whose FT is to be computed (function of t)
    % a,b - signal is equal to x in interval [a,b] and zero outside
    % w - vector of frequencies where FT is to be computed
    %
    % X - vector containing FT of x for each frequency in w

    %% code goes here
end
```

The vector `w` is supposedly a continuous-frequency vector, i.e., it is supposed to contain a continuous set of frequency values. Define it accordingly.

Write a script to compute the CTFT of signal (1*b*) given above, taking $T = \{1, 0.5, 2\}$. Make a 2×3 plot showing the time-domain signals and their magnitude spectra.

What property of Fourier transform is at play here? Mention clearly in your report.

Question 2: Inverse CTFT

Find the signal corresponding to each of the following Fourier transforms.

(a) $X_1(\omega) = \dfrac{1}{\lambda + j\omega}$

(b) $X_2(\omega) = \delta(\omega + T_0) + \delta(\omega - T_0)$

(c) $X(\omega) = X_1(\omega)X_2(\omega)$

Question 3: Sampling

1. Consider the impulse train

$$p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_0)$$

.

   (a) Find the Fourier series of $p(t)$.

   (b) Find the Fourier transform of $p(t)$. [Hint: Recall the synthesis equation.]

2. Consider the system which has $x(t)$ as input and $x_p(t) = x(t)p(t)$, where $p(t)$ is the impulse train defined above. Take $x(t) = \cos\omega_0 t$ and $T = 1/3$.

   (a) Sketch $X_p(\omega)$ for $-10\pi \leq \omega \leq 10\pi$ for the following values of ω_0:

      i. $\omega_0 = \pi$

      ii. $\omega_0 = 3\pi$

      iii. $\omega_0 = 5\pi$

   (b) For which of the preceding values of ω_0 is $x_p(t)$ identical? What can you infer from this observation?

MATLAB:

Consider the signal $x(t) = \cos(5\pi t) + \sin(10\pi t)$.

(a) Use the time-grid `t_fine = 0:0.001:2` and plot the signal using the `plot()` command. Ensure that the ticks on the axes are accurate.

(b) Sample the signal with interval `Ts = 0.1s` and denote the sampled signal `x[n] = x(nTs)`. In the same figure as above, plot `x[n]` using the `stem()` command. You might have to generate the time vector appropriately.

Question 4: Reconstruction

MATLAB:

1. Here, you will use the inbuilt `interp1()` function from MATLAB for reconstruction of a signal `x(t)` from its samples `x[n]`.

   (a) Use the same signal as above, $x(t) = \cos(5\pi t) + \sin(10\pi t)$. Make a 2×2 plot and repeat the above plot in the top-left panel.

   (b) Perform *zero-hold interpolation* based reconstruction of `x(t)`. You will have to give the appropriate value for the `method` parameter of the `interp1()` function. Note that reconstruction should be done over the time-grid `t_fine` defined above. Plot the reconstructed signal in the second panel.

   (c) Similarly, perform *linear interpolation* based reconstruction of `x(t)` and plot it in the third panel.

   (d) Recall *band-limited interpolation* based reconstruction of `x(t)`:

   $$x_r(t) = \sum_{n=-\infty}^{\infty} x(nT_s) \frac{\omega_c T_s}{\pi} \frac{\sin(\omega_c(t - nT_s))}{\omega_c(t - nT_s)}$$

   Define a function `sinc_recon` as follows,

   ```
   function xr = sinc_recon(n, xn, Ts, t_fine)
       % finds approximate sinc interpolated signal
       %
       % n - integer locations of the samples x[n]
       % xn - the sampled signal x[n] = x(nT_s)
       % Ts - sampling interval
       % t_fine - time-grid for reconstruction of xr
       %
       % xr - reconstructed signal over t_fine

       %% code goes here
   end
   ```

   Remember to restrict over the time interval $[0, 2]$ and use a cut-off frequency $\omega_c = \omega_s/2$. You might need to take care of divide-by-zero errors as well.
   Plot the reconstructed signal in the final panel.

   (e) For each of the three interpolation methods above, compute the maximum absolute error (MAE) between the original and reconstructed signal in the interval $[0.25, 1.75]$. Comment on the quality on reconstruction in your report.

2. Consider the signal $x(t) = \cos(5\pi t)$.

(a) Consider samples of `x(t)` for the sampling intervals (i) `Ts = 0.1s`, (ii) `Ts = 0.2s`, (iii) `Ts = 0.3s`, (iv) `Ts = 0.4s`.

(b) Perform band-limited interpolation from the above samples over the interval $[0, 2]$. Make a 2×2 plot and plot the signals, one panel for each `Ts`.

(c) What are your observations as the sampling interval is changed? Write clearly in the report.