Assignment 1: [Released date: 27.03.2025] [Submission Date: 06.04.2025]

Full Marks- 25

1. Obtain an orthogonal basis for the subspace R^3 spanned by the vectors

$$x=\begin{pmatrix}1\\1\\0\end{pmatrix}, y=\begin{pmatrix}3\\4\\2\end{pmatrix}$$, by Gram- Schmidt orthogonalisation process. Here the inner product is the standard inner product defined for R^n **[5]**

2. a) Prove that if A and B are square matrices and AB is invertible , then both A and B are invertible. b) Prove that if a symmetric matrix is invertible, then its inverse is symmetric also. **[2.5+2.5=5]**

3) Find the value of k for which A is invertible

a) A= $\begin{pmatrix} k & -k & 3 \\ 0 & k+1 & 1 \\ k & -8 & k-1 \end{pmatrix}$ b) A= $\begin{pmatrix} k & k & 0 \\ k^2 & 2 & k \\ 0 & k & k \end{pmatrix}$

4) Show that following vectors **{u₁, u₂, u₃}** forms an orthogonal basis for **R³** (where R is the set of all real numbers) and then find the coordinate of the vector **w** with respect to the basis **{u₁, u₂, u₃}**

$$\mathbf{u_1}=\begin{bmatrix}1\\1\\1\end{bmatrix} , \mathbf{u_2}=\begin{bmatrix}1\\-1\\0\end{bmatrix} , \mathbf{u_3}=\begin{bmatrix}1\\1\\-2\end{bmatrix} ; \mathbf{w}=\begin{bmatrix}7\\9\\10\end{bmatrix}$$

[a= $\begin{bmatrix}a_1\\a_2\\a_3\end{bmatrix}$ and b = $\begin{bmatrix}b_1\\b_2\\b_3\end{bmatrix}$,the inner product <a,b>= $a_1b_1+ a_2b_2+ a_3b_3$. **[5]**

5. (a) Determine whether the given matrix is orthogonal. If it is, then find its inverse

$$\begin{pmatrix} 1/3 & 1/2 & 1/5 \\ 1/3 & -1/2 & 1/5 \\ -1/3 & 0 & 2/5 \end{pmatrix}$$

(b) If **Q** be a 2x2 orthogonal matrix, then **Q** must have the form $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$ or

$\begin{pmatrix} a & b \\ b & -a \end{pmatrix}$