Information and Communication: Pset 1.

Basic, definitions.

Question 1:

A box contains 3 red balls, 2 green balls, and 1 blue ball. Two balls are drawn out of the box (simultaneously).

1. Define the sample space for this experiment.
2. Define the event space for the event that at least one red ball is drawn.
3. What is the probability of the event that at least one red ball is drawn?

Question 2:

A fair coin is tossed three times. What is the probability of getting at least two heads?

Question 3:

A fair six-sided die is rolled twice. Let event A be defined as the event that the sum of the two rolls is greater than 7. Let event B be defined as the event that at least one of the rolls shows a 4.

1. Calculate P(A) the probability that the sum of the two rolls is greater than 7.
2. Calculate $P(B)$, the probability that at least one of the rolls shows a 4
3. Calculate $P(A \mid B)$, the conditional probability that the sum is greater than 7 given that at least one roll shows a 4.

Question 4:

Let A and B be two independent events where P(A)=0.4 and $P(B)$=0.5. What is the probability that at least one of the events occurs?

Conditional Probablity

Question 5:
A standard deck of 52 playing cards contains 26 red cards and 26 black cards. If two cards are drawn without replacement, what is the conditional probability that both cards are red given that the first card drawn was red?

Question 6:
[**Monty Hall Problem**]

Law of total probability:

Questions 7:
A factory produces items from two machines: Machine I produces items with a defect rate of 2%, while Machine II has a defect rate of 5%. If Machine I produces 70% of the items and Machine II produces the remaining items, what is the overall defect rate for items produced by both machines?

Bayes theorem

Question 8:
In a certain population, 2% of individuals have a specific disease. A test for the disease has a sensitivity of 90% (true positive rate) and a specificity of 95% (true negative rate). If a randomly selected individual tests positive, what is the probability that they actually have the disease?

Question 9:

A bag contains three coins:

1. One is a **fair coin** (probability of heads = 0.50.
2. One is a **biased coin** that lands heads 75% of the time.
3. One is a **two-headed coin** (always lands heads).

You randomly pick one coin from the bag and flip it twice. Both flips result in heads.

What is the probability that the coin you picked is the two-headed coin?

(Q.1) Q (1) Sample space : $^6C_2 = 15$ (2 balls from set of 6 balls)

(2) Event space : →

{$(R_1, G_1), (R_1, G_2)$: ...}

a) Exactly 1 red ball → $^3C_1 \times ^3C_1$ } → 12

b) Both balls are red → $^3C_2$

(3) P(atleast 1 red ball) = 12/15

(Q.2) Sample space → $2^3 = 8$

event space → A = {HHH, HHT, HTH, THH}

$P(A) = 4/8 = 1/2$

(Q.3) (a) Total possible outcomes = 36

1. sum = 8 → 5 outcomes   10 → 3 outcomes   12 → 1 outcome

= 9 → 4 outcomes   11 → 2 outcomes

∴ favourable = 15

$P(A) = 15/36$

(b) P(Neither die show a 4) = $5/6 \times 5/6 = 25/36$ (= P(A))

∴ P(atleast one 4) = 1 − P(A) = 11/36

(c) $P(A|B) = \frac{P(A \cap B)}{P(B)}$,

calcu → $A \cap B$ → $x+y > 7$, $x > 3$ → (4,4), (4,5), (4,6), (5,4), (6,4)

Event space → 5 outcomes ⇒ $P(A \cap B) = 5/36$

∴ $P(A|B) = \frac{5/36}{11/36} = 5/11$

Q.4 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= 0.4 + 0.5 - 0.4 \times 0.5$

Q.5 A → both cards are Red
B → First card is Red.

$P(B) = \frac{26}{52} = \frac{1}{2}$

$P(A \cap B) = P(B) \times P(\text{Second card is Red} / B)$

$= \frac{1}{2} \times \frac{25}{51} = \frac{25}{102}$

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{25/102}{1/2} = \frac{25}{51}$

Q.6 Link

Q.7 → D → event that an indivisual has disease
T^+ → event that the test result are +ve

$P(D|T^+) = \frac{P(T^+/D) \times P(D)}{P(T^+)}$

$P(D) = 0.02$
$P(D^c) = 0.98$
$P(T^+/D) = 0.9$
$P(T^-/D^c) = 0.95$
$P(T^+/D^c) = 1 - P(T^-/D^c) = 0.05$

$P(T^+) = P(T^+/D) \cdot P(D) + P(T^+/D^c) \cdot P(D^c)$
$P(T^+) = 0.9 \times 0.02 + 0.05 \times 0.98$
$P(T^+) = 0.067$
$P(D|T^+) = \frac{0.9 \times 0.02}{0.067} \approx 0.2687.$

Q-8 $C_1 \rightarrow$ event that 2-headed coin selected

$C_2 \rightarrow$ — // — biased coin selected

$C_3 \rightarrow$ — // — fair coin selected

$H_2 \rightarrow$ 2 heads are observed.

$$P(C_1/H_2) = \frac{P(H_2/C_1)\cdot P(C_1)}{P(H_2)}$$

$P(C_1) = P(C_2) = P(C_3) = 1/3$

$P(H_2/C_1) = 1$

$P(H_2/C_2) = 0.75 \times 0.75 = 0.5625$

$P(H_2/C_3) = 0.5 \times 0.5 = 0.25$

$P(H_2) = P(H_2/C_1)\cdot P(C_1) + P(H_2/C_2)\cdot P(C_2) + P(H_2/C_3)\cdot P(C_3)$

$= 1 \times 1/3 + 1/3 \times 0.5625 + \frac{1}{3} \times 0.25$

$P(H_2) = 0.6042$

$$P(C_1/H_2) = \frac{P(H_2/C_1)\,P(C_1)}{P(H_2)} = \frac{1 \times 1/3}{0.6042} = 0.5528$$

Q-9. $M_1 \Rightarrow$ event that item is produce by Machine 1

$M_2 \rightarrow$ —— " ——————— Machine 2

$D \rightarrow$ item is defective.

$P(M_1) = 0.7$, $P(D/M_1) = 0.02$

$P(M_2) = 0.3$ $P(D/M_2) = 0.05$

$P(D) = P(D/M_1) \cdot P(M_1) + P(D/M_2) \cdot P(M_2)$

$P(D) = 0.029$.