Signal Processing – Assignment - 4

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Q3. Digital Storage:

Sampling rate : 44.1 kHz, Bitrate : 32 bits

a) Sampling time period $= \frac{1}{44.1 \times 10^3}$ s $= 2.267 \times 10^{-5}$ s

Length of each song = 5 min = 300 s

$\Rightarrow$ No. of samples $= \frac{300}{2.267 \times 10^{-5}} = 1.323 \times 10^7$ Samples

$\Rightarrow$ Size of each channel $= 32 \times 1.323 \times 10^7 = 4.2336 \times 10^8$ bits

$\Rightarrow$ Size of each song $= 2 \times 4.2336 \times 10^8 = 8.4672 \times 10^8$ bits

$\Rightarrow$ Total size occupied $= 7 \times 8.4672 \times 10^8 = 5.92704 \times 10^9$ bits

b) Since the quality of the audio must be the same, the no. of bits used for each sample must be the same (32 bits).

Since the audio signals have a maximum frequency, we can sample at a lower rate of 24 kHz (as per Nyquist theorem, sampling frequency must be atleast twice the max. frequency, ie, $12 \times 2 = 24$ kHz)

Sampling time period $= \frac{1}{24 \times 10^3} = 4.167 \times 10^{-5}$ s

No. of samples $= \frac{300}{4.167 \times 10^{-5}} = 7.199 \times 10^6$

Size taken by each song $= 2 \times 32 \times 7.199 \times 10^6 = 4.6073 \times 10^8$ bits

Total size occupied $= 7 \times 4.6073 \times 10^8 = 3.22511 \times 10^9$ bits

No. of bits saved $= (5.92704 - 3.22511) \times 10^9$ bits
$= 2.70193 \times 10^9$ bits

Proportion of space saved $= \frac{2.70193 \times 10^9}{5.92704 \times 10^9} \times 100 \simeq 45.586\%$

$\therefore$ Using a reduced sampling frequency of 24 kHz reduces the space needed by 45.586 %.

Q4. Continuous-time to Discrete-time :-

a) To Prove: If the frequency f_s is above the Nyquist rate for $x(t)$, then it is still above the Nyquist frequency for $y(t)$, where $y(t) = x(t) * h(t)$, $\forall$ LTI systems.

The o/p of any LTI system $y(t)$ is denoted as $y(t) = x(t) * h(t)$.

In the Fourier Domain, the equation becomes,

$$Y(\omega) = X(\omega) \cdot H(\omega)$$

Since $x(t)$ is band-limited, $X(\omega) = 0 \;\forall\; |\omega| \geq B$, $B \in \mathbb{R}$.

(Let the bandwidth of $x(t)$ be B)

$$\Rightarrow Y(\omega) = 0 \;\forall\; |\omega| \geq B$$

$\Rightarrow$ $y(t)$ is band-limited with maximum possible bandwidth B.

Since the Nyquist frequency of a band-limited signal depends on its bandwidth only and both $x(t)$ and $y(t)$ have a maximum possible bandwidth of B, f_s will still be above the Nyquist rate for $y(t)$.

b) <u>To Find:</u> $h[n]$ such that $y[n] = h[n] * x[n]$, where $y[n]$ and $x[n]$ are the sampled $y(t)$ and $x(t)$.

Let

$$\begin{matrix} y[n] & & Y_D(\omega) \\ x[n] & \xleftrightarrow{\text{DTFT}} & X_D(\omega) \\ h[n] & & H_D(\omega) \end{matrix}$$

$$\Rightarrow \quad Y_D(\omega) = X_D(\omega)\, H_D(\omega)$$

Let the sampled version of $x(t)$ be $x_s(t)$.

$$x_s(t) = \sum_{k\in\mathbb{Z}} x(t)\,\delta(t - n/f_s)$$

The o/p of $x_s(t)$ is,

$$y_s(t) = h(t) * x_s(t)$$

$$y_s(t) = \int_{-\infty}^{\infty} h(\tau)\, x_s(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) \sum_{n\in\mathbb{Z}} x(t-\tau)\,\delta(t - n/f_s - \tau)\, d\tau$$

$$= \sum_{n\in\mathbb{Z}} \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\,\delta(t - n/f_s - \tau)\, d\tau$$

$$= \sum_{n\in\mathbb{Z}} h(t - n/f_s)\, x(t - (t - n/f_s))$$

$$y_s(t) = \sum_{n\in\mathbb{Z}} x(n/f_s)\, h(t - n/f_s)$$

$$\Rightarrow y_s(t) = \sum_{n\in\mathbb{Z}} x[n]\, h(t - n/f_s) \qquad \left[x[n] = x(nT_s) = x\left(\frac{n}{f_s}\right)\right]$$

W.k.t, $y[m] = y_s(mT_s) = y_s(m/f_s)$

$$\Rightarrow y[m] = \sum_{n\in\mathbb{Z}} x[n]\, h(m/f_s - n/f_s)$$

$$\Rightarrow y[m] = \sum_{n\in\mathbb{Z}} x[n]\, h((m-n)/f_s)$$

$$\Rightarrow y[m] = x[m] * h(m/f_s)$$

Also, for discrete-time systems we know that,

$$y[m] = x[m] * h_D[m]$$

On comparing, we see $h_D[m] = h(m/f_s)$

$\therefore$ $h[n] = h(n/f_s)$, ie, the impulse response in discrete-time is the sampled signal of the continuous-time impulse response.