Information and Communication: Problem Set 5

Question 1

Consider the code $\{0, 01, 10, 11\}$. Answer the following questions:

(a) Is the code instantaneous? Justify your answer.

(b) Is the code uniquely decodable? Provide an example or proof.

(c) Is the code nonsingular? Explain.

Question 2

Let a discrete random variable X take values in the alphabet:

$$X = \{a, b, c, d\}$$

with the probability distribution:

$$P(X) = \left(\frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{8}\right)$$

corresponding to the symbols a, b, c, d, respectively.

(a) Design any fixed-length code.

(b) Design a code that is not uniquely decodable but has an expected length of 1.25 bits.

(c) Encode the sequence $x = (acdbac)$ using the prefix code and the non-uniquely decodable code.

(d) Based on your encoding example in (c), discuss the challenges and limitations associated with decoding when utilizing a non-uniquely decodable code, as opposed to the guaranteed unambiguous decoding offered by prefix-free codes.

Question 3

Consider the random variable

$$X = (x_1, x_2, x_3, x_4, x_5, x_6, x_7)$$

with corresponding probabilities:

$$(0.49, 0.26, 0.12, 0.04, 0.04, 0.03, 0.02)$$

(a) Find a binary Huffman code for X.

(b) Find the expected codelength for this encoding.

(c) Find a ternary Huffman code for X.

Question 4

Determine whether the following codeword lengths can be the word lengths of a 3-ary Huffman code:

(a) (1, 2, 2, 2, 2)

(b) (2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3)

Question 5

Find the binary Huffman code for the source with probabilities:

$$\left(\frac{1}{3}, \frac{1}{5}, \frac{1}{5}, \frac{2}{15}, \frac{2}{15}\right)$$

Also, argue that this code is optimal for the source with probabilities:

$$\left(\frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}, \frac{1}{5}\right)$$

Question 6

Consider the sequence:

$$S = \text{ABABABAABBA}$$

Apply the Lempel-Ziv coding algorithm to compress this sequence. Construct the dictionary and determine the encoded output.