Signal Processing - Assignment -1

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Q1. Fourier Series:

1. $x(t) = 1 + \sin(2\pi f_0 t) + 2\cos(2\pi f_0 t) + \cos\left(4\pi f_0 t + \frac{\pi}{4}\right)$

$= 1 - j\left(j \sin 2\pi f_0 t\right) + 2\cos(2\pi f_0 t) + \cos\frac{\pi}{4}\cos 4\pi f_0 t - \sin\frac{\pi}{4}\sin 4\pi f_0 t$

$= 1 - i\left(\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2}\right) + 2\left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2}\right) + \frac{1}{\sqrt{2}}\left(\frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2}\right) + \frac{i}{\sqrt{2}}\left(\frac{e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}}{2}\right)$

$= 1 + e^{j2\pi f_0 t}\left(-\frac{j}{2} + 1\right) + e^{-j2\pi f_0 t}\left(\frac{j}{2} + 1\right) + e^{j4\pi f_0 t}\left(\frac{1}{2\sqrt{2}} + \frac{j}{2\sqrt{2}}\right) + e^{-j4\pi f_0 t}\left(\frac{1}{2\sqrt{2}} - \frac{j}{2\sqrt{2}}\right)$

Comparing with FSR $x(t) = \sum_{k \in \mathbb{Z}} c_k e^{jk\omega_0 t}$,

$$c_k = \begin{cases} 1 - \frac{j}{2}, & k = 1 \\ 1 + \frac{j}{2}, & k = -1 \\ \frac{1}{2\sqrt{2}}(1+j), & k = 2 \\ \frac{1}{2\sqrt{2}}(1-j), & k = -2 \\ 0, & \text{otherwise} \end{cases}$$

2. $x(t) = \text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1, & |t| < \frac{\tau}{2} \\ 0, & \frac{\tau}{2} < |t| < \frac{T}{2} \end{cases}$, T is period of $x(t)$

$$x(t) = \sum c_k e^{jk\omega_0 t}$$

$$\Rightarrow \quad c_k = \frac{1}{T} \int_{\langle T \rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T} \int_{-\tau/2}^{\tau/2} e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{\cancel{T}}\left(\frac{-\cancel{T}}{jk2\pi}\right)\left[e^{-jk\frac{2\pi}{T}t}\right]_{-\tau/2}^{\tau/2}$$

$$= \frac{-1}{jk2\pi}\left(e^{-jk\frac{\pi\tau}{T}} - e^{jk\frac{\pi\tau}{T}}\right)$$

$$= \frac{e^{jk\pi\tau/T} - e^{-jk\pi\tau/T}}{jk2\pi}$$

$$= \frac{2j \sin(k\pi\tau/T)}{jk2\pi}$$

$$\Rightarrow \quad c_k = \frac{\sin(k\pi\tau/T)}{k\pi} = \frac{\tau}{T}\, \text{sinc}\left(k\pi\frac{\tau}{T}\right)$$

3. $x(t) = \Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 + \frac{2}{\tau}t, & -\frac{\tau}{2} < t < 0 \\ 1 - \frac{2}{\tau}t, & 0 < t < \frac{\tau}{2} \\ 0, & \frac{\tau}{2} < |t| < T \end{cases}$ $\quad \tau \in [0, T)$

$$c_k = \frac{1}{T}\int_{\langle T\rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$= \frac{1}{T}\int_{-\tau/2}^{\tau/2} x(t)\, e^{-jk\frac{2\pi}{T}t}\, dt$$

$$= \frac{1}{T}\left[\int_{-\tau/2}^{0}\left(1+\frac{2}{\tau}t\right)e^{-jk\frac{2\pi}{T}t}\,dt + \int_{0}^{\tau/2}\left(1-\frac{2}{\tau}t\right)e^{-jk\frac{2\pi}{T}t}\,dt\right]$$

$$= \frac{1}{T}\left[\int_{-\tau/2}^{\tau/2} e^{-jk\frac{2\pi}{T}t}\,dt + \frac{2}{\tau}\left(\int_{-\tau/2}^{0} t e^{-jk\frac{2\pi}{T}t}\,dt - \int_{0}^{\tau/2} t e^{-jk\frac{2\pi}{T}t}\,dt\right)\right]$$

$$= \frac{1}{T}\left[\int_{-\tau/2}^{\tau/2} e^{-jk\frac{2\pi}{T}t}\,dt + \frac{2}{\tau}\left(\frac{-T}{jk2\pi}\right)\left(\int_{-\tau/2}^{0} -jk\frac{2\pi}{T} t e^{-jk\frac{2\pi}{T}t}\,dt\right.\right.$$

$$\left.\left. - \int_{0}^{\tau/2} -jk\frac{2\pi}{T} t e^{-jk\frac{2\pi}{T}t}\,dt\right)\right]$$

$$= \frac{1}{\cancel{T}}\left[\frac{-\cancel{T}}{jk2\pi}\left[e^{-jk\frac{2\pi}{T}t}\right]_{-\tau/2}^{\tau/2} - \frac{\cancel{T}}{\tau jk\pi}\left(\left[e^{-jk\frac{2\pi}{T}t}\left(-jk\frac{2\pi}{T}t - 1\right)\right]_{-\tau/2}^{0}\right.\right.$$

$$\left.\left. - \left[e^{-jk\frac{2\pi}{T}t}\left(-jk\frac{2\pi}{T}t - 1\right)\right]_{0}^{\tau/2}\right)\left(\frac{-T}{jk2\pi}\right)\right]$$

$$= \frac{-1}{jk2\pi}\left(e^{-jk\pi\frac{\tau}{T}} - e^{jk\pi\frac{\tau}{T}}\right) - \frac{1}{\tau jk\pi}\left(\frac{-T}{jk2\pi}\right)$$

$$\left[\left(-1 - e^{jk\pi\frac{\tau}{T}}\left(jk\pi\frac{\tau}{T} - 1\right)\right) - \left(e^{-jk\pi\frac{\tau}{T}}\left(-jk\pi\frac{\tau}{T} - 1\right) + 1\right)\right]$$

$$= \frac{1}{jk2\pi}\left(2j\sin\left(k\pi\frac{\tau}{T}\right)\right) - \frac{T}{2\tau(k\pi)^2}\left(-1 - jk\pi\frac{\tau}{T}e^{jk\pi\frac{\tau}{T}} + e^{jk\pi\frac{\tau}{T}} + jk\pi\frac{\tau}{T}e^{-jk\pi\frac{\tau}{T}} + e^{-jk\pi\frac{\tau}{T}} - 1\right)$$

$$= \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} - \frac{T}{2\tau(k\pi)^2}\left(-2 - jk\pi\frac{\tau}{T}\left(e^{jk\pi\frac{\tau}{T}} - e^{-jk\pi\frac{\tau}{T}}\right) + e^{jk\pi\frac{\tau}{T}} + e^{-jk\pi\frac{\tau}{T}}\right)$$

$$= \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} - \frac{T}{2\tau(k\pi)^2}\left(-2 - jk\pi\frac{\tau}{T}\left(2j\sin\left(k\pi\frac{\tau}{T}\right)\right) + 2\cos\left(k\pi\frac{\tau}{T}\right)\right)$$

$$= \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} + \frac{T}{\tau(k\pi)^2} + \frac{\cancel{T}}{\cancel{2}\cancel{\tau}(k\pi)^{\cancel{2}}}\left(\frac{jk\cancel{\pi}\cancel{\tau}}{\cancel{T}}\right)2j\sin\left(k\pi\frac{\tau}{T}\right) - \frac{T}{2\tau(k\pi)^2}2\cos\left(k\pi\frac{\tau}{T}\right)$$

$$= \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} + \frac{T}{\tau(k\pi)^2} - \frac{\sin\left(k\pi\frac{\tau}{T}\right)}{k\pi} - \frac{T}{\tau(k\pi)^2}\cos\left(k\pi\frac{\tau}{T}\right)$$

$$\Rightarrow \quad c_k = \frac{T}{\tau(k\pi)^2}\left(1 - \cos\left(k\pi\frac{\tau}{T}\right)\right) \quad \forall\, k \neq 0$$

$$c_0 = \frac{1}{T}\int_{\langle T\rangle} x(t)\,dt = \frac{1}{T}\left(\int_{-\tau/2}^{0} 1 + \frac{2}{\tau}t\,dt + \int_{0}^{\tau/2} 1 - \frac{2}{\tau}t\,dt\right)$$

$$= \frac{1}{T}\left(\int_{-\tau/2}^{\tau/2} dt + \frac{2}{\tau}\left(\int_{-\tau/2}^{0} t\,dt - \int_{0}^{\tau/2} t\,dt\right)\right)$$

$$= \frac{1}{T}\left([t]_{-\tau/2}^{\tau/2} + \frac{2}{\tau}\left(\left[\frac{t^2}{2}\right]_{-\tau/2}^{0} - \left[\frac{t^2}{2}\right]_{0}^{\tau/2} \right)\right)$$

$$= \frac{1}{T}\left(\tau + \frac{2}{\tau}\left(\left(0 - \frac{\tau^2}{8}\right) - \left(\frac{\tau^2}{8} - 0\right)\right)\right)$$

$$= \frac{1}{T}\left(\tau + \frac{\cancel{2}}{\cancel{\tau}}\left(-\frac{\tau^{\cancel{2}}}{\cancel{4}_{2}}\right)\right)$$

$$\Rightarrow \quad \underline{C_0 = \frac{\tau}{2T}}$$

4. $x(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT_s)$

$x(t)$ is an impulse train of period T_s,

$$\Rightarrow \quad c_k = \frac{1}{T_s}\int_{\langle T_s \rangle} x(t)\, e^{-jk\omega_0 t}\, dt$$

$$\Rightarrow \quad c_k = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \delta(t)\, e^{-jk\frac{2\pi}{T_s}t}\, dt$$

$$\Rightarrow \quad c_k = \frac{1}{T_s} e^{-jk\frac{2\pi}{T_s}(0)}$$ [Selection Property of Dirac Signals]

$$\Rightarrow \quad \underline{c_k = \frac{1}{T_s} \quad \forall\ k}$$

Q2. Properties of LTI Systems :-

1. $S:\ y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$

Let the DTFT of $x[n]$ and $y[n]$ be $X(\omega)$ and $Y(\omega)$.

$\Rightarrow\ Y(\omega) - 5Y(\omega)e^{-j\omega} + 6Y(\omega)e^{-2j\omega} = 2X(\omega)e^{-j\omega}$ [Time Shift Property]

$\Rightarrow\ Y(\omega)\left(1 - 5e^{-j\omega} + 6e^{-2j\omega}\right) = X(\omega)\left(2e^{-j\omega}\right)$

$$\Rightarrow\ H(\omega) = \frac{2e^{-j\omega}}{1 - 5e^{-j\omega} + 6e^{-2j\omega}}$$

Let $e^{-j\omega} = E \Rightarrow H(\omega) = \dfrac{2E}{1 - 5E + 6E^2}$

$$= H(\omega) = \frac{2E}{(1-2E)(1-3E)}$$

Using partial fractions,

$$2E = A(1-2E) + B(1-3E)$$

$$= 2E = A + B + (-2A - 3B)E$$

Comparing coefficients,

$A + B = 0 \qquad -2A - 3B = 2$

$A = -B \longrightarrow 2B - 3B = 2$

$A = 2 \longleftarrow \Rightarrow B = -2$

$$\Rightarrow\ H(\omega) = \frac{2}{1-3E} - \frac{2}{1-2E}$$

$$\Rightarrow \quad H(\omega) = 2\left(\frac{1}{1-3e^{-j\omega}} - \frac{1}{1-2e^{-j\omega}}\right)$$

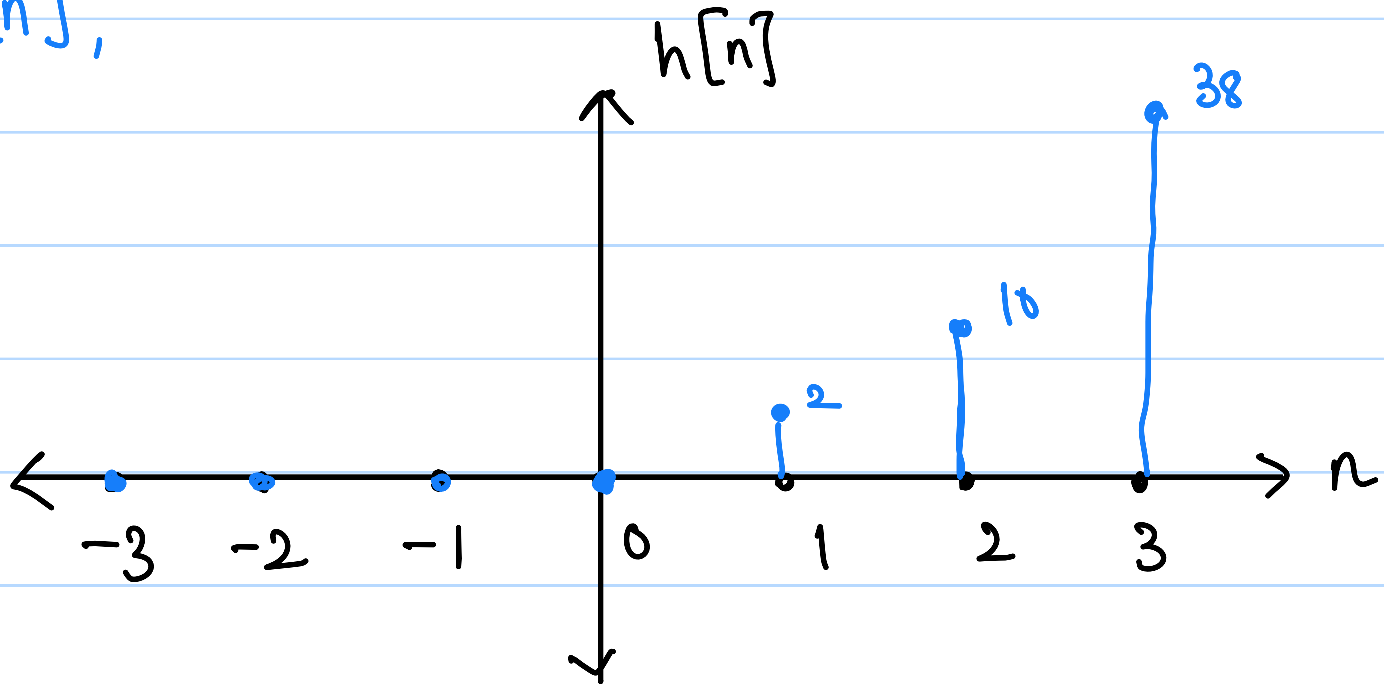
We know that,

$$a^n u[n] \xleftrightarrow{DTFT} \frac{1}{1-ae^{-j\omega}}$$

$$\Rightarrow \quad h[n] = 2\left(3^n u[n] - 2^n u[n]\right)$$

$$\Rightarrow \quad h[n] = 2u[n]\left(3^n - 2^n\right)$$

Plot of h[n],



2 Let the impulse response and step response of a system H be h(t) and s(t).

To prove:
$$s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau$$

We know that $u(t) = \int_{-\infty}^{\infty} \delta(t)\, dt$.

Also, for any i/p signal $x(t)$, the o/p signal $y(t)$ is,

$$y(t) = x(t) * h(t)$$

Taking $x(t) = u(t)$, $y(t)$ will be $s(t)$

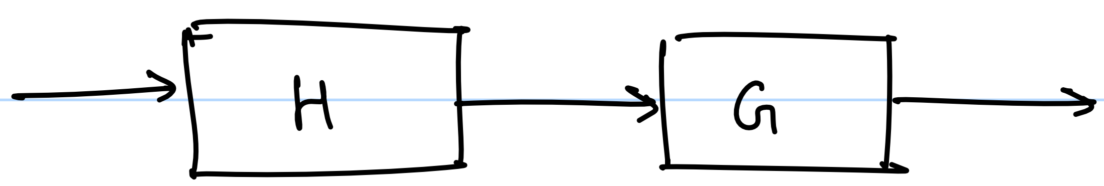
$$\Rightarrow \quad s(t) = u(t) * h(t)$$

$$= \quad s(t) = \int_{-\infty}^{\infty} h(\tau) . u(t-\tau)\, d\tau$$

$$= \quad \underline{\underline{s(t) = \int_{-\infty}^{t} h(\tau)\, d\tau}} \qquad [u(t-\tau) = 0 \ \forall \ \tau > t]$$

Hence Proved.

3.



Let $x[n]$ be the i/p of H, $y_1[n]$ be the o/p of H and i/p of G, $y[n]$ be the o/p of G.

a) H and G are causal LTI systems.

Since H and G are LTI system.

$$y_1[n] = h_1[n] * x[n] \qquad [h_1[n] = \text{impulse response of } H]$$

$$y[n] = h_2[n] * y_1[n]$$ [$h_2[n]$ = impulse response of G]

$$\Rightarrow y[n] = h_2[n] * h_1[n] * x[n]$$

$\Rightarrow$ Impulse response of cascade $= h_2[n] * h_1[n]$

$$\Rightarrow h_3[n] = \sum_m h_1[m] h_2[n-m]$$

Since H, G are causal,

$$h_1[n] = h_2[n] = 0 \quad \forall \; n < 0$$

$n < 0 \Rightarrow n - m < 0 \quad \forall \; m > 0$

$\Rightarrow h_2[n-m] = 0 \quad \forall \; m \geq 0$

w.k.t $h_1[m] = 0 \quad \forall \; m < 0$

$\Rightarrow \forall \; n < 0, \; h_1[m] h_2[n-m] = 0 \quad \forall \; m$

$$\Rightarrow \sum_m h_1[m] h_2[n-m] = 0 \quad \forall \; n < 0$$

$$\Rightarrow h_3[n] = 0 \quad \forall \; n < 0$$

$\therefore$ The cascade system is Causal.

b) $h_3[n] = \sum_m h_1[m] h_2[n-m]$

Since H, G are bounded

$$\sum_m |h_1[m]| \leq B_1, \quad \sum_m |h_2[m]| \leq B_2$$

$$\sum_n |h_3[n]| = \sum_n \left|\left(\sum_m h_1[m] h_2[n-m]\right)\right|$$

$$= \sum_n |h_3[n]| = \sum_m |h_1[m]| \sum_n |h_2[n-m]|$$

$$= \sum_n |h_3[n]| = \sum_m |h_1[m]| \sum_k |h_2[k]|$$ [Change of variable, $k = n-m$]

$$\Rightarrow \sum_n |h_3[n]| \leq B_1 \cdot B_2 = B_3$$

$$\Rightarrow \underline{\sum_n |h_3[n]| \leq B_3}$$

$\therefore$ The cascade system is also stable.

4. a) $y[n] - \varepsilon y[n-1] = x[n]$

Let the DTFT of $x[n]$ and $y[n]$ be $X(\omega)$ and $Y(\omega)$.

$$\Rightarrow Y(\omega) - \varepsilon Y(\omega) e^{-j\omega} = X(\omega)$$

$$\Rightarrow Y(\omega)(1 - \varepsilon e^{-j\omega}) = X(\omega)$$

$$\Rightarrow \frac{Y(\omega)}{X(\omega)} = \frac{1}{1-\varepsilon e^{-j\omega}} = H(\omega)$$

We know that $a^n u[n] \xleftrightarrow{\text{DTFT}} \dfrac{1}{1-ae^{-j\omega}}$

$$\therefore h[n] = \varepsilon^n u[n]$$

Since for a stable system,

$$\sum_{n=-\infty}^{\infty} h[n] \text{ must be finite}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \varepsilon^n u[n] = \sum_{n=0}^{\infty} \varepsilon^n \text{ must be finite}$$

Since $\sum_{n=0}^{\infty} \varepsilon^n$ is the sum of a GP, for it to be finite, $\varepsilon \in [0,1)$.

$\therefore$ The required condition is $\varepsilon \in [0,1)$.

b) The impulse response of the system is $h[n] = \varepsilon^n u[n]$ as derived above.

c) $h[n] = \varepsilon^n u[n]$

i) A system is memoryless if and only if $h[n] = 0 \ \forall \ n \neq 0$.

Clearly $h[n] \neq 0 \ \forall \ n \neq 0$, $\therefore$ System is not memoryless.

ii) $h[n] = \varepsilon^n u[n] \ \therefore h[n] = 0 \ \forall \ n < 0$.

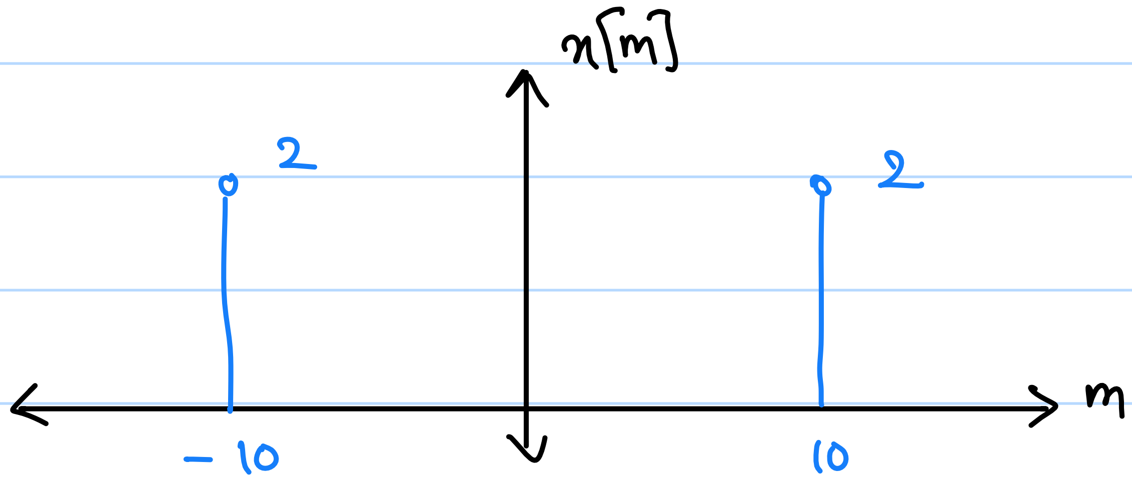
$\therefore$ System is causal.

Q3. Discrete-Time Convolution :-

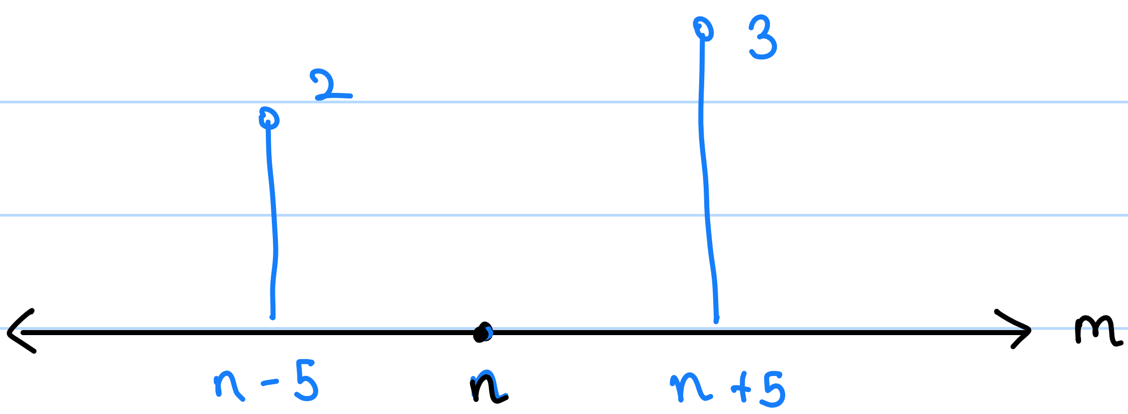
1. $x[n] = 2\delta[n+10] + 2\delta[n-10]$

   $y[n] = 3\delta[n+5] + 2\delta[n-5]$

$x[m]$



$y[n-m]$:



At $n+5 = -10 \Rightarrow n = -15$,

$$\sum x[m]y[n-m] = 3 \times 2 = 6$$

$$\therefore \; z[-15] = 6$$

At $n-5 = -10 \Rightarrow n = -5$,

$$\sum x[m]y[n-m] = 2 \times 2 = 4$$

$$\therefore \; z[-5] = 4$$

At $n+5 = 10 \Rightarrow n = 5$,

$$\sum x[m]y[n-m] = 3 \times 2 = 6$$

$$\therefore \; z[5] = 6$$

At $n-5=10 \Rightarrow n=15$,

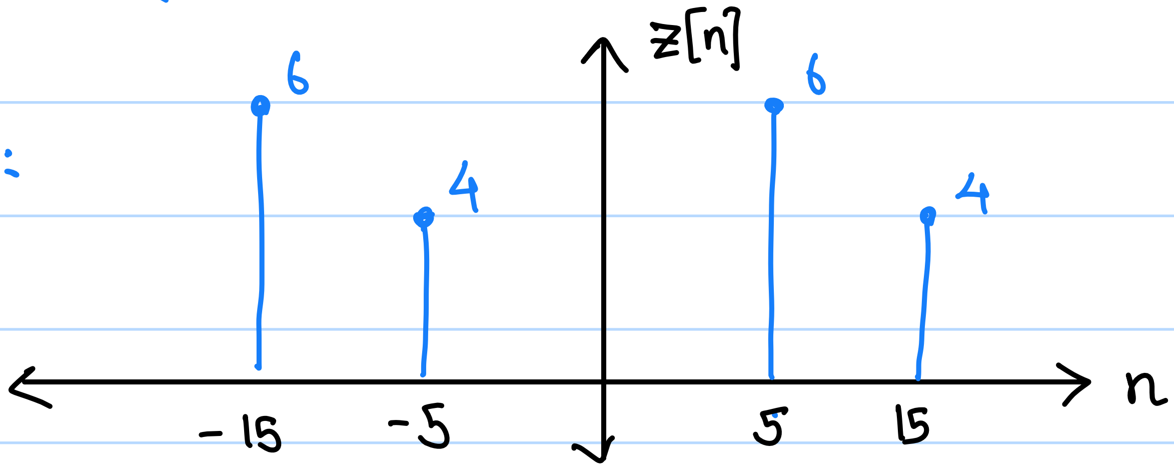
$$\sum x[m]y[n-m] = 2\times 2 = 4$$

$$\therefore z[15] = 4$$

$$\Rightarrow \quad z[n] = \begin{cases} 6, & n=5 \text{ or } n=-15 \\ 4, & n=15 \text{ or } n=-5 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow \quad z[n] = 6\left(\delta[n-5] + \delta[n+15]\right) + 4\left(\delta[n-15] + \delta[n+5]\right)$$

$z[n]$:

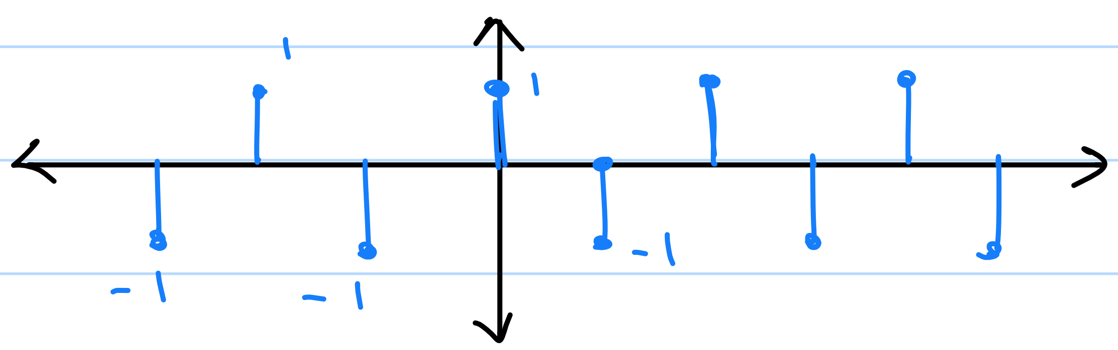


2. $x[n] = (-1)^n$

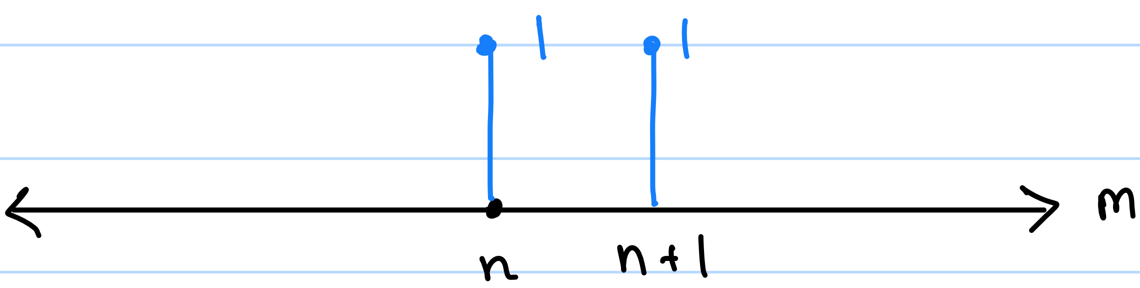
$y[n] = \delta[n] + \delta[n-1]$

$$z[n] = \sum_m x[m]y[n-m]$$

$x[m]$:



$y[n-m]$:



At $n = 0$,

$$x[m]y[n-m] = (1\times 1) + (1\times(-1)) = 1-1 = 0$$

$\Rightarrow \quad z[0] = 0$

At $n = 1$,

$$x[m]y[n-m] = 1\times(-1) + 1\times 1 = -1+1 = 0$$

$\Rightarrow \quad z[1] = 0$

Since x is periodic, the same pattern of overlaps will repeat between x and y.

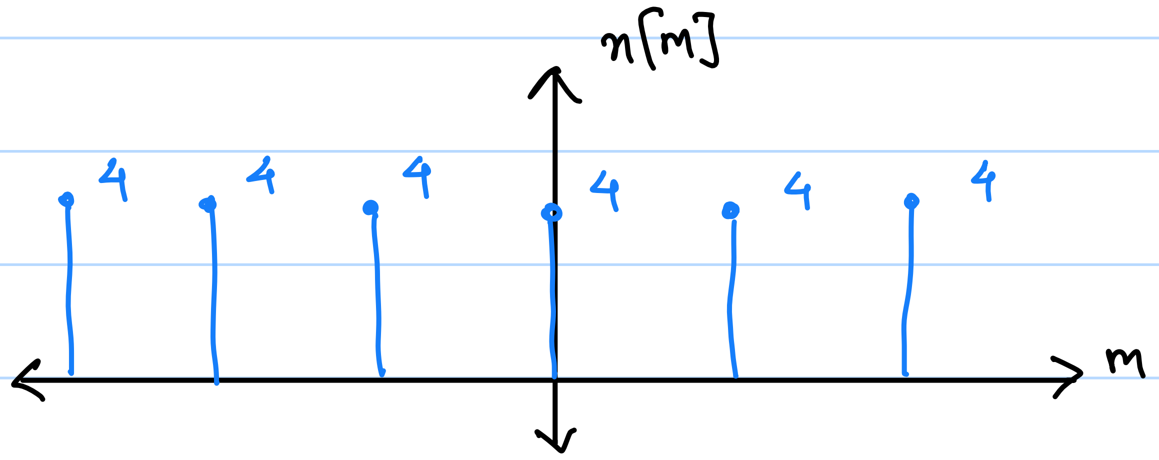
$$\therefore \quad \underline{\underline{z[n] = 0 \;\; \forall \; n}}$$

3. $x[n] = 4$

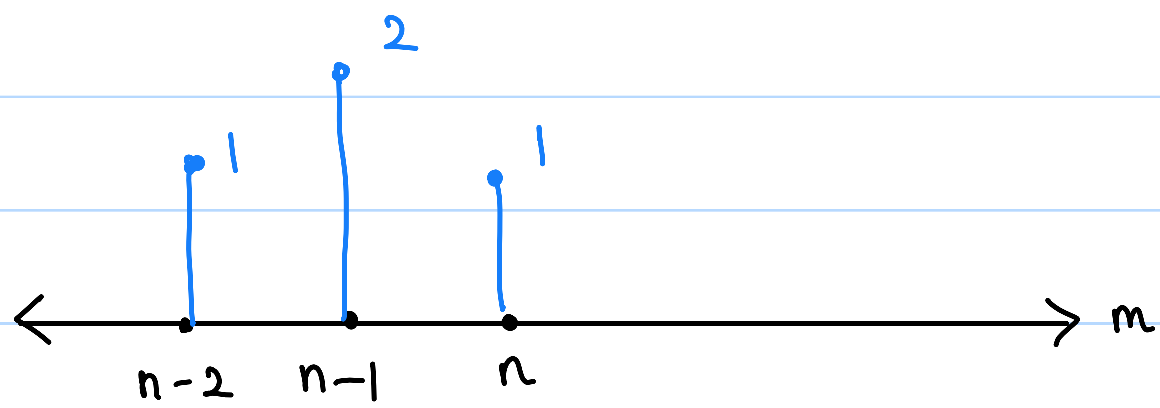
$$y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$z[n] = \sum_m x[m]y[n-m]$$

$x[m]$:



$y[n-m]$:

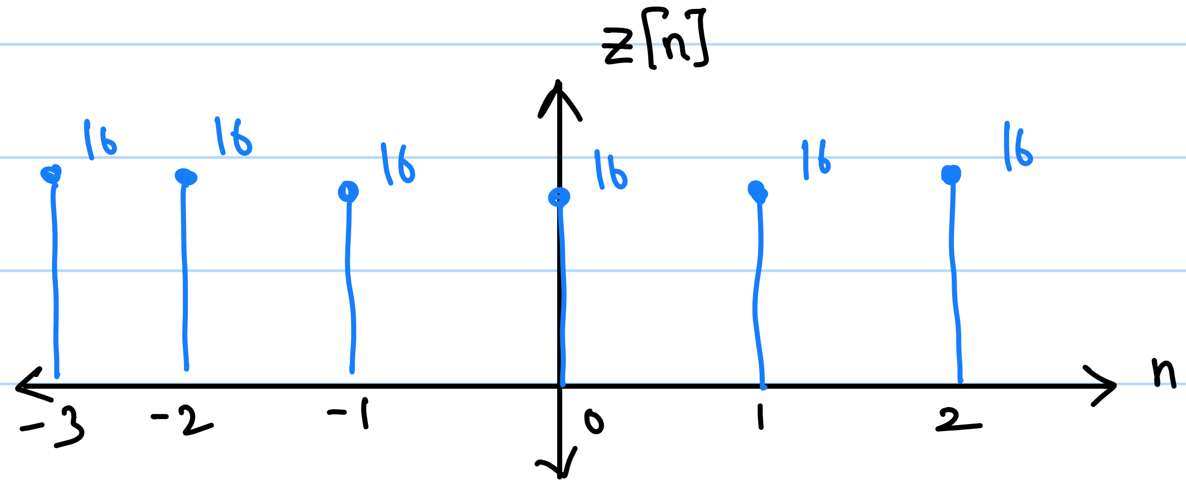


Since $x[n]$ is a constant, the overlap b/w $x[m]$ and $y[n-m]$ is the same $\forall\, n$.

$$\Rightarrow \quad z[n] = \sum_m x[m]\,y[n-m] = 4(1+2+1) = 16$$

$$\Rightarrow \quad z[n] = 16 \;\forall\, n$$

$z[n]$:

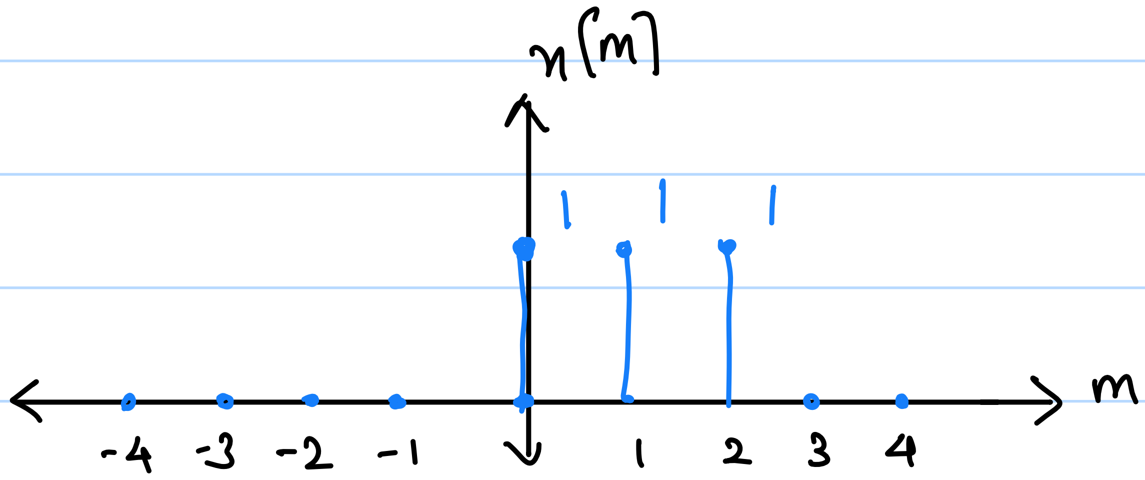


4. $x[n] = u[n] - u[n-3]$

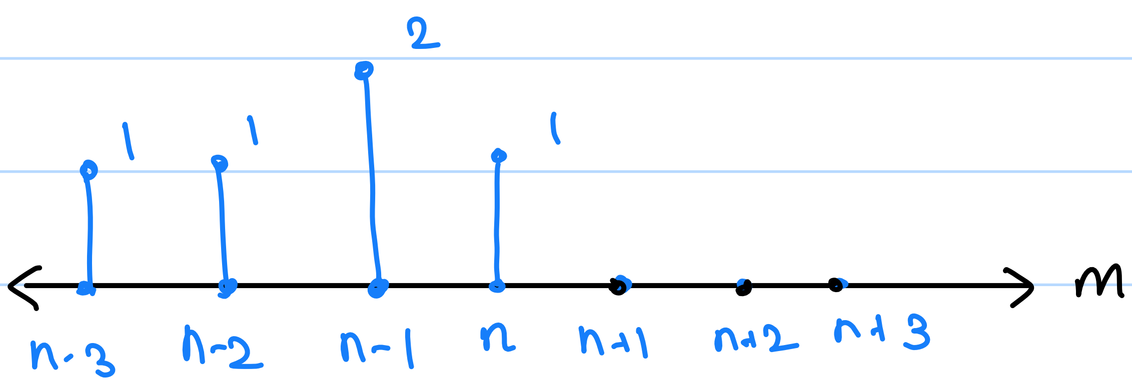
$$y[n] = \begin{cases} 0, & n < 0 \\ n, & n = 0,1,2 \\ 1, & n \geq 3 \end{cases}$$

$$z[n] = \sum_m x[m]\,y[n-m]$$

$x[m]$:



$y[n-m]$



At $n = 0$,
$$z[0] = 1 \times 1 = 1$$

At $n < 0$,
$$z[0] = 0$$

At $n = 1$,
$$z[1] = 2 \times 1 + 1 \times 1 = 3$$

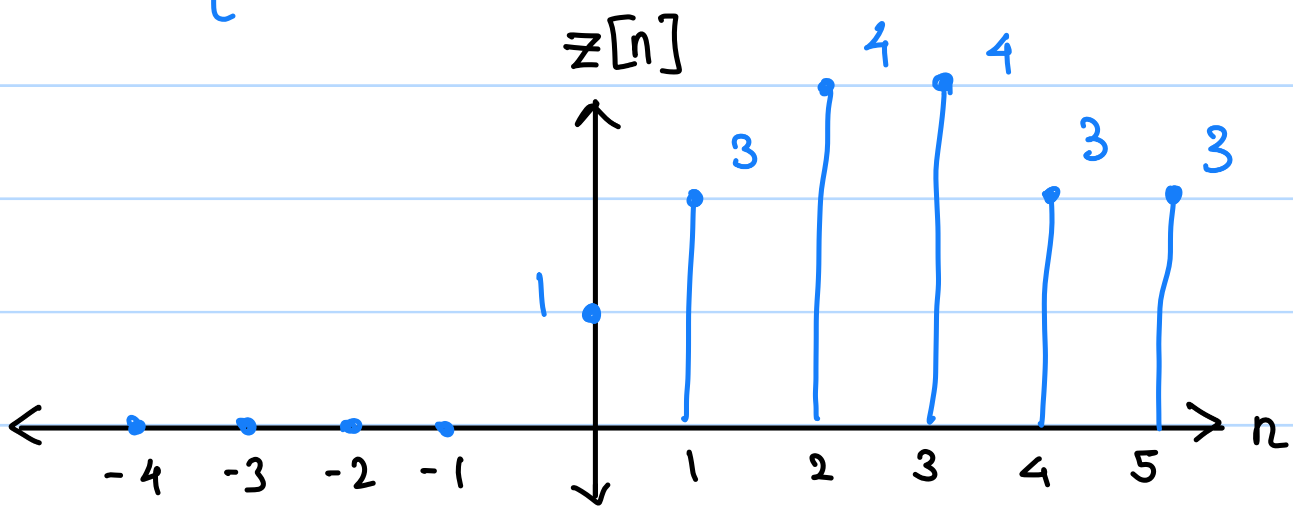
At $n = 2$
$$z[2] = 1 \times 1 + 2 \times 1 + 1 \times 1 = 4$$

At $n = 3$
$$z[3] = 1 \times 1 + 1 \times 1 + 2 \times 1 = 4$$

At $n \geq 4$
$$z[n] = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$$

$$\therefore \quad z[n] = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 3, & n = 1 \text{ or } n \geq 4 \\ 4, & n = 2 \text{ or } n = 3 \end{cases}$$

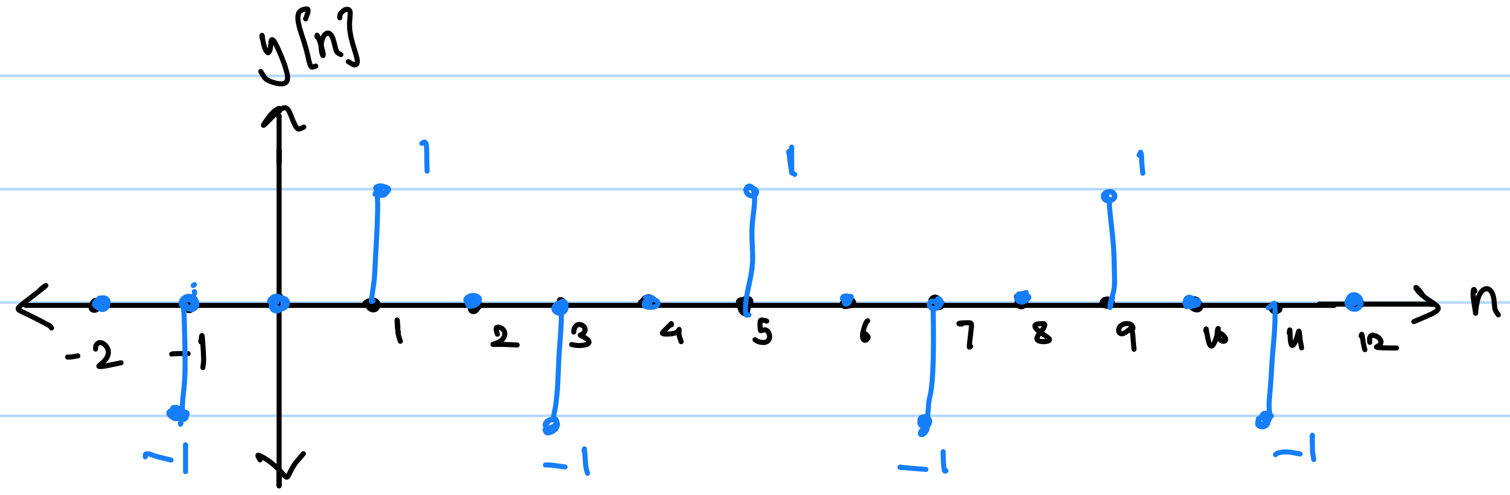
$z[n]$:



Q4. LTI Systems :-

$$h[n] = u[n] - u[n-3], \quad x[n] = \sin\left(\frac{\pi}{2} n\right) u[n]$$

1.



2. $$y[n] = h[n] * x[n]$$

$$= \sum_{m=-\infty}^{\infty} h[m]x[n-m]$$

$$= \sum_{m=-\infty}^{\infty} (u[m]-u[m-3]) \sin\left(\frac{\pi}{2}(n-m)\right) u[n-m]$$

$$\Rightarrow y[n] = \sum_{m=0}^{2} \sin\left(\frac{\pi}{2}(n-m)\right) u[n-m]$$

$$\Rightarrow y[n] = \sum_{m=0}^{\min(n,2)} \sin\left(\frac{\pi}{2}(n-m)\right)$$

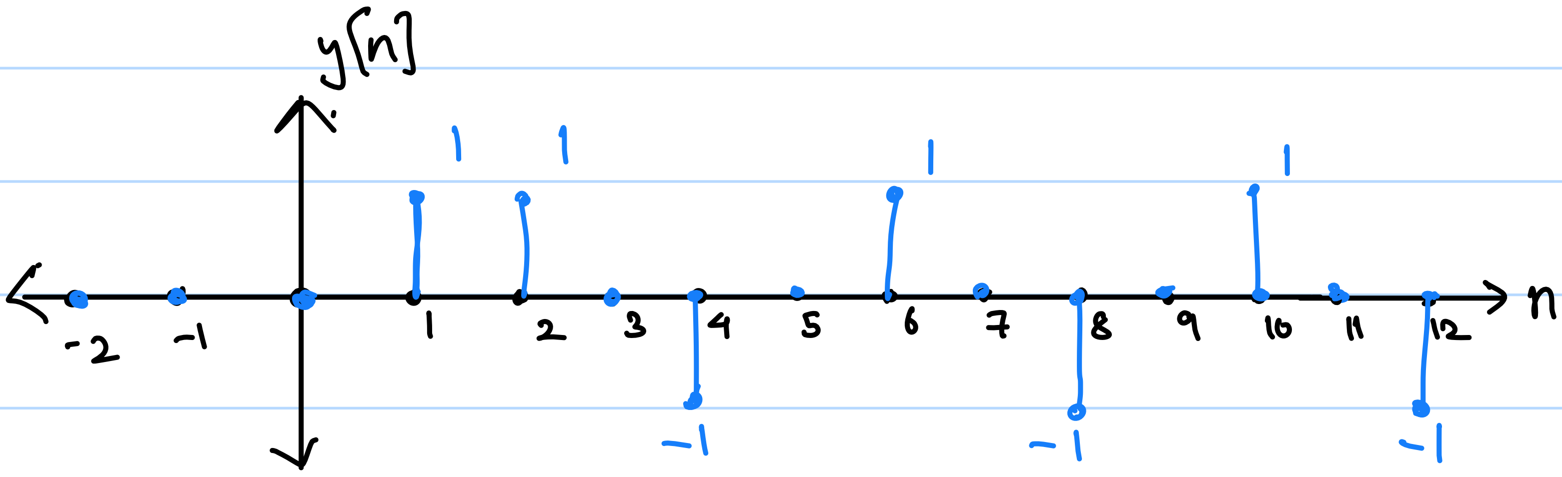
At $n \leq 0$, $y[n] = 0$

At $n=1$, $y[1] = \sin\left(\frac{\pi}{2}(1-0)\right) + \sin\left(\frac{\pi}{2}(1-1)\right)$
$= 1 + 0 = 1$

At $n=2$, $y[2] = \sin\left(\frac{\pi}{2}(2-0)\right) + \sin\left(\frac{\pi}{2}(2-1)\right) + \sin\left(\frac{\pi}{2}(2-2)\right)$
$= 0 + 1 + 0 = 1$

At $n \geq 3$, $y[n] = \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-2)\right)$
$= \cancel{\sin\left(\frac{\pi}{2}n\right)} - \cos\left(\frac{\pi}{2}n\right) - \cancel{\sin\left(\frac{\pi}{2}n\right)}$
$= -\cos\left(\frac{\pi}{2}n\right)$

$$\therefore\ y[n] = \begin{cases} 0 & , n \leq 0 \\ 1 & , n = 1 \text{ or } n = 2 \\ -\cos\left(\frac{\pi}{2}n\right) & , n \geq 3 \end{cases}$$



3. $$y[n] = \sum_{m=-\infty}^{\infty} h[m]\, x[n-m]$$

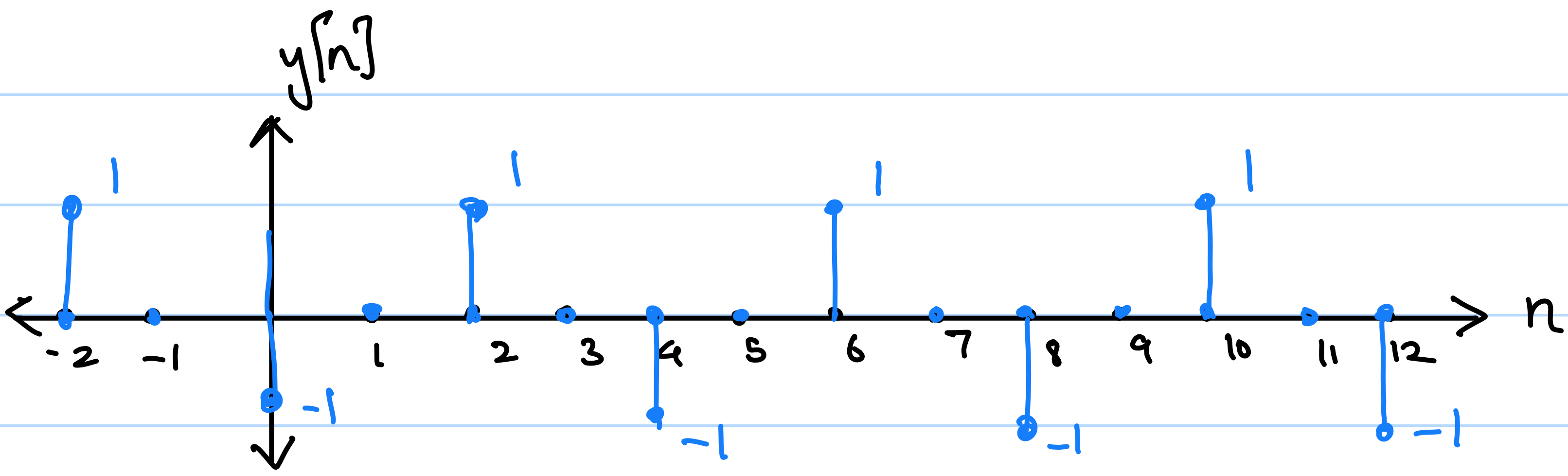
$$= \sum_{m=-\infty}^{\infty} \left(u[m] - u[m-3]\right) \sin\left(\frac{\pi}{2}(n-m)\right)$$

$$= \sum_{m=0}^{2} \sin\left(\frac{\pi}{2}(n-m)\right)$$

$$= \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-2)\right)$$

$$= \cancel{\sin\left(\frac{\pi}{2}n\right)} - \cos\left(\frac{\pi}{2}n\right) - \cancel{\sin\left(\frac{\pi}{2}n\right)}$$

$$\therefore\ \underline{y[n] = -\cos\left(\frac{\pi}{2}n\right)}$$



Q5. Eigensignals :-

Let $g[n]$ be the eigensignal of a system, then $y[n] = k \cdot g[n]$, where $k \in \mathbb{C}$, $y[n]$ is the output for $g[n]$ in the system.

1. $h[n] = \delta[n-4]$

$$h[n] * g[n] = k \cdot g[n]$$

$$= \sum_{m=-\infty}^{\infty} \delta[m-4]\, g[n-m] = k.g[n]$$

$$= g[n-4] = k.g[n]$$

An example of such a signal : $\underline{g[n] = 2^n}$

$$2^{n-4} = k2^n \Rightarrow \frac{1}{16} 2^n = k2^n \rightarrow \text{Satisfied}$$

2. $h[n] = \delta[n-4] + \delta[n-2]$

$$\Rightarrow \sum_{m=-\infty}^{\infty} (\delta[m-4] + \delta[n-2])\, g[n-m] = kg[n]$$

$$= g[n-4] + g[n-2] = kg[n]$$

An example of such a signal : $\underline{g[n] = 3^n}$

$$\Rightarrow 3^{-4} 3^n + 3^{-2} 3^n = k3^n$$

$$= (3^{-4} + 3^{-2})\, 3^n = k3^n \longrightarrow \text{Satisfied}$$