Communication Theory

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→ **Communication :**

The transmission of information from a source to a destination via a transmission medium.

- Information transfer can be across space or time (time → storage)

$$\underbrace{\text{Source} \longrightarrow \text{Transmitter}}_{\text{Message Signal}} \longrightarrow \underbrace{\text{Channel}}_{\text{Distortion \& Noise}} \longrightarrow \underbrace{\text{Reciever} \longrightarrow \text{Destination}}_{\text{Estimation of Message}}$$

→ **Channel :-**

- Guided Channels - Point to point channel. ex: Wired communication, waveguide.

- **Challenges :-**

1) **Spectrum :** A channel can allow only a finite bandwidth of frequencies, given by its frequency response.

- The finite bandwidths exist for both wired and wireless channels

- Low Radio Frequencies : Large Antenna Heights
  High Radio Frequencies : High attenuations (abv 5 GHz → point 2 point)

- The wireless spectrum is also divided between different techno-logies and applications. ex:

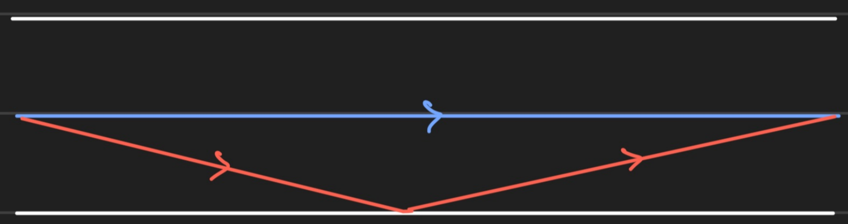
  AM : 0.535 – 1.7 MHz

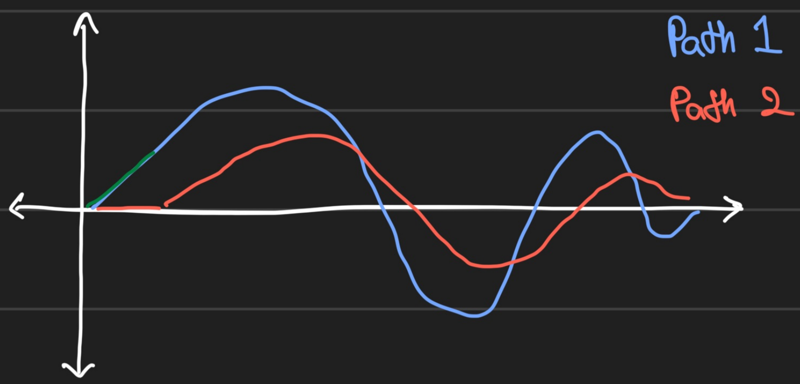
2) <u>Noise</u> : A channel can also distort the signal

- Can be thermal, man-made EMI, powerline interferences.

- Due to the random nature, it is modelled as AWGN (additive white gaussian noise)

- Multipath is also a problem,





3) Time Variance :

We often model channels as LTI systems, ie, linear and time-invariant systems.

- But most channels, especially wireless, is time variant.

- Channel Capacity :-

The capacity of a AWGN channel is given by,

$$C = B \log_2 (1 + SNR)$$

B – Channel Bandwidth, $SNR = P/N$ (Signal to Noise Ratio)

P – Signal Power, N – Noise Power

→ Communication Systems :-

- Analog Message Signal – Continuous-time signals which takes a continuous range of values.

- Digital Message Signal – Discrete-time signal with a discrete range of values.

- Analog Communication Systems :-

Source ⟶ Modulator → Channel → Demodulator → Destination

- Modulation is the shifting of the signal to a higher frequency for transmission and multiplexing.

  (Multiplexing - Different signals are shifted to different bands so that they can be transmitted at once)

- Analog communication is the obvious thing to do, but it is not very efficient.

Digital Communication Systems :-

- The message signal is digital, ie, a signal that can be converted into 0's and 1's. (need not be binary signals always)

Information Source ⟶ Source Encoder ⟶ Channel Encoder ⟶ Modulator ⟶ Channel ⟶ Demodulator ⟶ Channel Decoder ⟶ Source Decoder ⟶ Destination

- Source Coding : Obtains digital representation of the information and removes any redundancy within the information. ex: Compression

- Channel Coding : Adds redundancy to the signal for error correction due to channel noise and distortion. ex: Repetition Coding

- Digital Modulation : Converts the bit-stream into a waveform suitable for modulation.

- Digital systems are robust against distortion and noise.

- Digital systems can also use regenerative repeaters, ie, devices within the channel that remove the distortion until that point (similar to resetting the signal to its original form)

- Digital Systems can do error correction using Shannon's theorem, and they can also be encrypted.

- The processor devices are digital as well.

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Information Theory - Source Coding
Error Correcting Codes - Channel Coding
Wireless Communications - Modulation & Transmission

☆ Logistics :-

- Textbook :- Intro. To Comm. Systems, U. Madhow

- Grading :-

  Classwork + Surprise Quiz + Q1 + Q2 - 20%

  Midsem - 20 %

Assignment and Quiz - 20%

Endsem - 40%

* 2 : Overview of Signals and Systems :-

- The notation for the step function is given as $\mathbb{I}_{[0,\infty)}$, ie, an indicator function.

- Sinusoids : $A\cos(2\pi f_0 t + \theta)$. Modulation varies A, f_0 and θ to convey information. (If A, f_0, θ are known, then no info can be passed)

- Impulse Response - Response of the system to an impulse signal. Used to study LTI systems.

- Rectangular form :

$$s(t) = A_c \cos(2\pi f_0 t) - A_s \sin(2\pi f_0 t)$$

Where,

$$A_c = A\cos\theta \quad , \quad A_s = A\sin\theta$$

Using Euler's formula,

$$Ae^{j\theta} = A\cos\theta + jA\sin\theta = A_c + jA_s$$

$$A = \sqrt{A_c^2 + A_s^2} \quad , \quad \theta = \tan^{-1}(A_s / A_c)$$

- Complex Exponentials : $A e^{j(2\pi f_0 t + \theta)} = \alpha e^{j 2\pi f_0 t}$, $\alpha = A e^{j\theta}$

These are Eigensignals to any LTI system.

Usually, α is varied to encode the information and $e^{j2\pi f_0 t}$ is known (carrier wave)

- <u>Inner Product</u> :-

$$\langle s, r \rangle = \sum_{i=-\infty}^{\infty} s[i]\, r^*[i] = r^H s$$ (H – Hermition | Conj. Transpose) (For digital signals)

$$\langle s, r \rangle = \int_{-\infty}^{\infty} s(t)\, r^*(t)\, dt$$ (For analog signals)

- <u>Energy of a Signal</u> :-

$$E_s = \|s\|^2 = \langle s, s \rangle = \int_{-\infty}^{\infty} s(t)\, s^*(t)\, dt$$

Q. Find the energy of :

i) $s(t) = 2\mathbb{1}_{[0,T]} + j\mathbb{1}_{[T/2, 2T]}$

ii) $s(t) = e^{-3|t| + j2\pi t}$

i) $s(t) = 2\,\mathbb{1}_{[0,T]} + j\,\mathbb{1}_{[T/2,2T]}$

$$= \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ 2+j, & t \in [\frac{T}{2}, T] \\ j, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow |s(t)| = \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ \sqrt{3}, & t \in [\frac{T}{2}, T] \\ 1, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow E_s = \int_0^{T/2} 2^2\, dt + \int_{T/2}^{T} \sqrt{3}^2\, dt + \int_T^{2T} 1^2\, dt$$

$$= 4\left(\frac{T}{2}\right) + 3\left(T - \frac{T}{2}\right) + 1\,(2T - T)$$

$$= 2T + \frac{3T}{2} + T$$

$$= \frac{11T}{2}$$

- Power of a Signal :-

The power of a signal is defined as the time average of its energy computed over a large time interval.

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2\, dt = \overline{|s(t)|^2}$$

- DC Value :-

The DC value of a signal is the value of the Fourier Transform of the signal at $f = 0$.

Q. Compute P_s and DC value for a real sinusoid $s(t) = A\cos(2\pi f_0 t + \theta)$ where $A > 0$, $\theta \in [0, 2\pi]$, $f_0 \in \mathbb{R}$

$$P_s = \lim_{T_0 \to \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A\cos(2\pi f_0 t + \theta)\, dt$$

'ill do it later

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★ 3 : Baseband and Passband Representations :-

- Multiplexing : Encoding multiple pieces of information in the same signal using different freq. ranges.

→ Fourier Transform :-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega)\, e^{j\omega t}\, d\omega$$

- Parseval's Identity :-

$$\|u\|^2 = \langle u,u\rangle = \int_{-\infty}^{\infty} |u(t)|^2 \, dt = \int_{-\infty}^{\infty} |U(j\omega)|^2 \, d\omega$$

→ Shared Medium :-

- Many practical channels are shared medium, ie, used by multiple devices at once.

- Time Division Multiple Access (TDMA) : Divides the multiple information into packets, which are then sent at different instances in time.

  ie, Each device is given a time slot within which they can communicate.

- Frequency Division Multiple Access (FDMA) : Assigns each type of information its own frequency band, ie, each device is allowed to communicate in a particular bandwidth only.

Device $\triangleq$ Transmitter - Reciever Pair

- We can perform TDMA and FDMA at the same time (ex: Statistic-al Multiplexing) to increase the user capacity of the medium.

→ **Energy Spectral Density and Bandwidth :-**

- **ESD $(E_v(f))$** : For a signal $v(t)$, the ESD $E_v(f)$ is the energy of the output of the filter divided by the width Δf in the limit $\Delta f \rightarrow 0$

↳ filter of width Δf, centered at f.

$$U(f) \longrightarrow \boxed{H(f)} \longrightarrow Y(f) = U(f)\,H(f)$$

↳ $H(f)$ = Shifted rect of width Δf centered at f

Prove that $E_v(f) = |U(f)|^2$

$$\int_{-\infty}^{\infty} |y(t)|^2\,dt = \int_{-\infty}^{\infty} |Y(f)|^2\,df = \int_{-\infty}^{\infty} |U(f)H(f)|^2\,df$$

$$= \int_{f-\frac{\Delta f}{2}}^{f+\frac{\Delta f}{2}} |U(f)|^2\,df \approx |U(f)|^2\,\Delta f \quad \text{(for small } \Delta f)$$

$$\Rightarrow \quad E = |U(f)|^2\,\Delta f$$

$$\Rightarrow \quad \underline{E_v(f) = |U(f)|^2}$$

- Bandwidth :-

- For bandlimited signals, bandwidth is the range of frequencies beyond which the spectrum of the signal is zero.

- For real-valued signals, $U(-f) = U^*(f)$. So only positive frequencies count for real valued signals, the bandwidth is one sided.

- For non-bandlimited signals, the energy-containment bandwidth is the size of the smallest band that contains a certain fraction of the signal's energy. (usually 99%)

- Or, it is the band over which $U(f)$ stays within a certain range about its peak value (ex:- 3dB bandwidth).

Note: $W \operatorname{sinc}(Wt) \xleftrightarrow{f} \mathbb{1}_{[-\frac{W}{2}, \frac{W}{2}]}(f)$

Find the bandwidth of 1) $u(t) = \operatorname{sinc}(2t)$ 2) $u(t) = \mathbb{1}_{[2,4]}(t)$, t is in microseconds.

1) $u(t) = \operatorname{sinc}(2t) \Rightarrow U(f) = \frac{1}{2}\mathbb{1}_{[-1,1]}(f)$

$\Rightarrow$ Bandwidth of the signal = $[-1, 1]$ MHz.

2) $u(t) = \mathbb{1}_{[2,4]}(t)$

Let $x(t) = u(t+3) = \mathbb{1}_{[-1,1]}(t)$

$\Rightarrow X(f) = 2\,\text{sinc}(2f)$

$\Rightarrow U(f) = 2\,\text{sinc}(2f)\, e^{-j6\pi f}$

$\Rightarrow |U(f)| = |X(f)|$

$$E_u = \int_{-\infty}^{\infty} |U(f)|^2 df = \int_{-\infty}^{\infty} |u(t)|^2 dt = \int_{2}^{4} dt = 2$$

$\Rightarrow$ 99% of $E_u = 1.98$

$$\Rightarrow 2\int_0^W |2\,\text{sinc}(2f)|^2 df = 1.98 \xrightarrow{\text{on solving}} \underline{W = 5.1 \text{ MHz}}$$

- Any analysis that needs to occur in passband, the same analysis can be carried out by shifting the signal into baseband.

Baseband – Spectrum centered around DC ($f = 0$)

Passband – Spectrum centered around a non-zero frequency.

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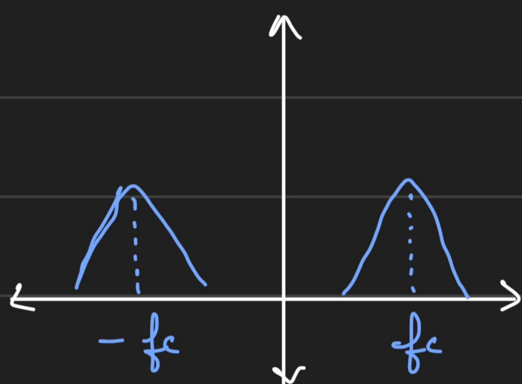
- Calculations are easier in baseband since it can be sampled into a digital signal at a low sampling frequency and passed into a general purpose DSP.

- $\text{Baseband} \underset{\text{Demodulation}}{\overset{\text{Modulation}}{\rightleftarrows}} \text{Passband}$

→ **Modulation and Demodulation :-**

- Method 1: Multiplication with a sinusoid,

$$u_p(t) = u(t)\cos(2\pi f_c t) \longleftrightarrow \frac{1}{2}\left(U(f-f_c) + U(f+f_c)\right)$$



For a sine wave,

$$u_p(t) = u(t)\sin(2\pi f_c t) \longleftrightarrow \frac{1}{2j}\left(M(f-f_c) - M(f+f_c)\right)$$

- Using both the sine and cosine carrier waves,

$$u_p(t) = \underbrace{u_c(t)\cos(2\pi f_c t)}_{\text{in-phase component}} - \underbrace{u_s(t)\sin(2\pi f_c t)}_{\text{quadrature component}}$$

$u_c(t)$ and $u_s(t)$ are different baseband signals of bandwidth atmost W, and $f_c > W$. They hold all the information that is to be transmitted.

The carrier waves are determinate, ie, cannot transfer information.

- The inphase (I) and quadrature (Q) components are orthogonal, so the signal can carry 2 independent information at the same time (one in I, one in Q)

Mathematically, if $a(t) = u_c(t)\cos(2\pi f_c t)$, $b(t) = u_s(t)\sin(2\pi f_c t)$, then,

$$\langle a(t), b(t) \rangle = 0$$

(a, b ∈ ℝ and continuous)

Proof:

$$\langle a(t), b(t)\rangle = \int_{-\infty}^{\infty} a(t)\, b^*(t)\, dt = \int_{-\infty}^{\infty} a(t)\, b(t)\, dt$$

$$a(t)\,b(t) = u_c(t)\, u_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$x(t) = a(t)\,b(t) = \underbrace{u_c(t)\, u_s(t)}_{p(t)} \sin(4\pi f_c t) \cdot \frac{1}{2}$$

$$x(t) = \frac{p(t) \sin(4\pi f_c t)}{2} \xleftrightarrow{FT} \underbrace{\frac{P(f-2f_c) + P(f+2f_c)}{4j}}_{X(f)}$$

$$P(f) = U_c(f) * U_s(f)$$

If U_c has bandwidth W_1 and U_s has bandwidth W_2, then P has bandwidth $W_1 + W_2$.

$$f_c > W_1 \,.\; f_c > W_2 \Rightarrow 2f_c > W_1 + W_2$$

$\Rightarrow$ $P(f-2f_c)$, $P(f+2f_c)$ are passband spectra.

$$\Rightarrow \underline{X(0) = 0} \Rightarrow \int_{-\infty}^{\infty} x(t)\, dt = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} a(t)\,b(t) = 0$$

$$\Rightarrow \underline{\langle a(t), b(t)\rangle = 0}$$

- To demodulate the passband signal, we can again multiply the signal by the same sinusoid and passing the o/p through a LPF.

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

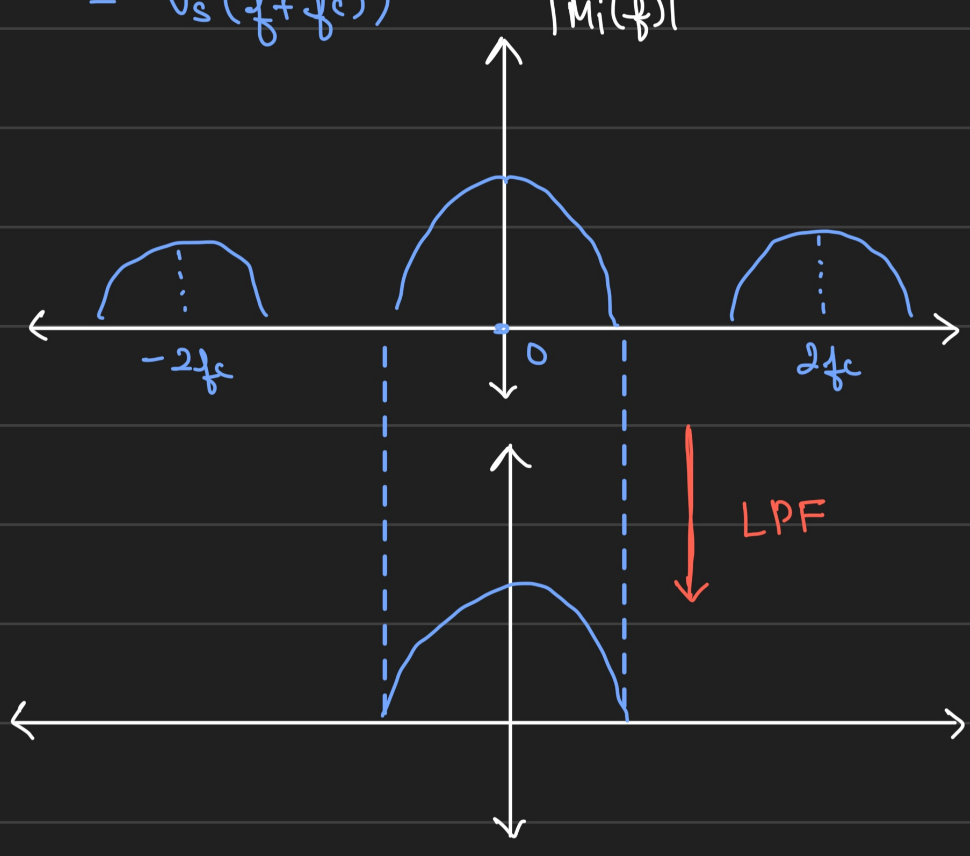
$$m_i(t) = 2u_p(t)\cos(2\pi f_c t)$$

$$= 2u_c(t)\cos^2(2\pi f_c t) - 2u_s(t)\sin(2\pi f_c t)\cos(2\pi f_c t)$$

$$= \cancel{2}u_c(t)\left(\frac{1+\cos(4\pi f_c t)}{\cancel{2}}\right) - u_s(t)\sin(4\pi f_c t)$$

$$= u_c(t) + u_c(t)\cos(4\pi f_c t) - u_s(t)\sin(4\pi f_c t)$$

$$\Rightarrow M_i(f) = U_c(f) + \frac{1}{4}\left(U_c(f-2f_c) + U_c(f+f_c)\right) + \frac{1}{4j}\left(U_s(f-f_c) - U_s(f+f_c)\right)$$



$\Big\}\ \underline{U_c(f)}$

IIIly

$$m_q(t) = -2j\, v_p(t) \sin(2\pi f_c t)$$

$$= -2j\left(v_c(t) \cos(2\pi f_c t) \sin(2\pi f_c t) - v_s(t) \sin^2(2\pi f_c t)\right)$$

$$= -j\, v_c(t) \sin(4\pi f_c t) + v_s(t)\left(1 - \cos(4\pi f_c t)\right)$$

$$\Rightarrow \quad M_q(f) = -j\left(\frac{1}{4j}\left(V_c(f-2f_c) - V_c(f+4f_c)\right)\right.$$
$$\left. + V_s(f) - \frac{1}{4}\left(V_s(f-2f_c) + V_s(f+2f_c)\right)\right)$$

$$\downarrow \text{LPF}$$

$$\underline{M_q(f) = V_s(f)}$$

- $v_p = (v_c, v_s)$, is termed as two-dimensional modulation.

- Using polar coordinates,

$$e(t) = \sqrt{v_c^2(t) + v_s^2(t)} \rightarrow \text{Envelope}$$

$$\theta(t) = \tan^{-1}\left(\frac{v_s(t)}{v_c(t)}\right) \rightarrow \text{Phase}$$

$$\Rightarrow \quad v(t) = e(t) \cos(2\pi f_c t + \theta(t)) \longrightarrow \text{Polar Envelope.}$$

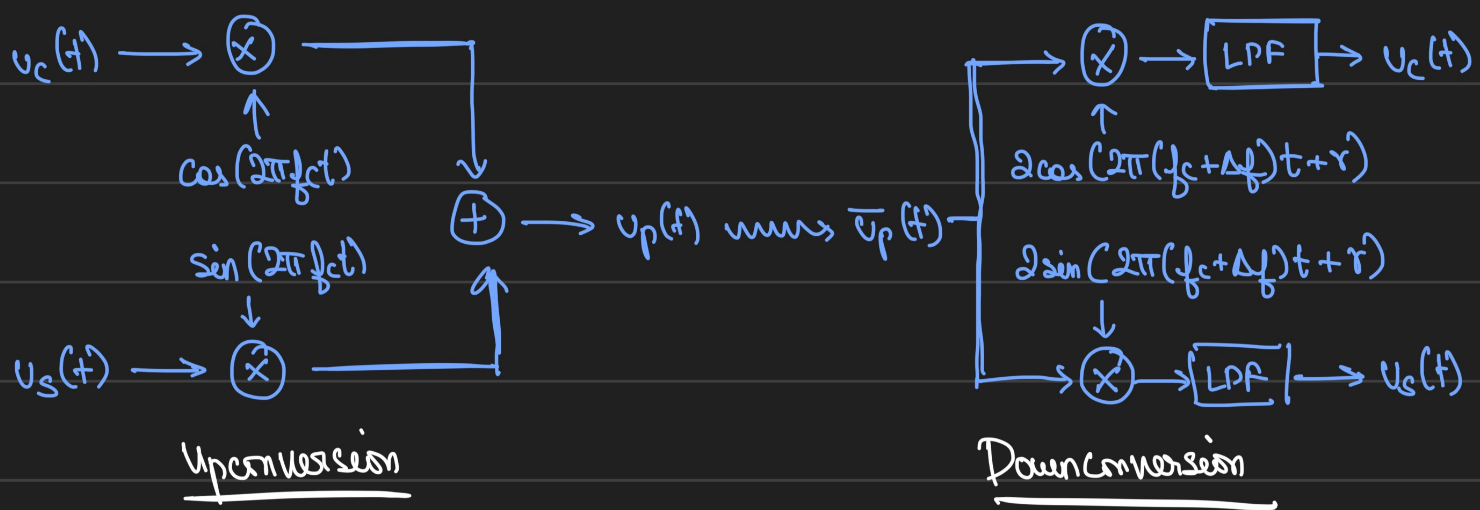
$$\Rightarrow \quad v(t) = v_c(t) + j\, v_s(t) = e(t) \exp(j\theta(t)) \rightarrow \text{Complex Envelope.}$$

The passband signal $v_p(t)$ can be expressed as $v_p(t) = \mathrm{Re}\left(v(t) e^{j2\pi f_c t}\right)$

$$\mathrm{Re}\left\{u(t)e^{j2\pi f_c t}\right\} = \mathrm{Re}\left\{\left(u_c(t)+j u_s(t)\right)\left(\cos(2\pi f_c t) + j\sin(2\pi f_c t)\right)\right\}$$

$$= \underline{u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)} = u_p(t)$$

- **Effect of Frequency and Phase Offset :-**



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Let $\phi(t) = 2\pi\Delta f t + \gamma$, then

$$u_p(t)\, 2\cos(2\pi f_c t + \phi(t))$$

$$= 2\cos(2\pi f_c t + \phi(t))\left(u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)\right)$$

$$= 2u_c(t)\cos(2\pi f_c t + \phi(t))\cos(2\pi f_c t) - 2u_s(t)\cos(2\pi f_c t + \phi(t))\sin(2\pi f_c t)$$

$$= u_c(t)\left(\cos(4\pi f_c t + \phi(t)) + \cos\phi(t)\right) - u_s(t)\left(\sin(4\pi f_c t + \phi(t)) - \sin\phi(t)\right)$$

$$\downarrow \text{LPF}$$

$$u_c(t)\cos\phi(t) + u_s(t)\sin\phi(t) = \tilde{u}_c(t)$$

|||ly $\tilde{u}_s(t) = -u_c(t)\sin\phi(t) + u_s(t)\cos\phi(t)$

- The spectrum of $u_p(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$ is,

$$u_p(t) = \mathrm{Re}\{u(t)e^{j2\pi f_c t}\}$$
$$= \frac{1}{2}\left(u(t)e^{j2\pi f_c t} + \left(u(t)e^{j2\pi f_c t}\right)^*\right) \leftarrow \text{Conjugate}$$
$$= \frac{1}{2}\left(u(t)e^{j2\pi f_c t} + \overset{*}{u}(t)e^{-j2\pi f_c t}\right)$$

$$\Rightarrow U_p(f) = \frac{1}{2}\left(U(f-f_c) + \overset{*}{U}(-(f+f_c))\right)$$

$$= U_p(f) = \frac{U(f-f_c) + \overset{*}{U}(-f-f_c)}{2}$$

Q. Write the proof for $\check{v}_s(t)$

$$\check{v}_s(t) = -2u_p(t)\sin(2\pi f_c t + \phi(t))$$
$$= -2\left(u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)\right)\sin(2\pi f_c t + \phi(t))$$
$$= -2u_c(t)\cos(2\pi f_c t)\sin(2\pi f_c t + \phi(t)) + 2u_s(t)\sin(2\pi f_c t)\sin(2\pi f_c t + \phi(t))$$
$$= -u_c(t)\left(\sin(4\pi f_c t + \phi(t)) + \sin\phi(t)\right) + u_s(t)\left(\cos\phi(t) + \cos(4\pi f_c t + \phi(t))\right)$$

$$\Big\downarrow \text{LPF}$$

$$\Rightarrow -v_c(t)\sin\phi(t) + u_s(t)\cos\phi(t) = \check{v}_s(t)$$

- Conversion b/w Passband and Baseband :-

- Baseband to Passband :-

$u(t) \longrightarrow$ Original Complex Signal.

$$c(t) = u(t)e^{j2\pi f_c t} \ , \ c^*(t) = u^*(t)e^{-j2\pi f_c t} = c_2(t)$$

$$\Rightarrow C(f) = U(f-f_c) \ , \ C_2(f) = U^*(-f-f_c)$$

$$\Rightarrow u_p(t) = \frac{1}{2}(c(t) + c^*(t)) \Rightarrow U_p(f) = \frac{1}{2}\left(U(f-f_c) + U^*(-f-f_c)\right)$$

- Passband to Baseband :-

$U_p(f) \longrightarrow$ Spectrum of Modulated Signal.

$$C(f) = 2U_p^+(f) = \begin{cases} 2U_p(f) \ , & f > 0 \\ 0 \ , & ow \end{cases}$$

$$u(t) = c(t)e^{-j2\pi f_c t} \Rightarrow U(f) = C(f+f_0) = 2U_p^+(f+f_0)$$

- Passband Filtering :-

$$v_p(t) \xrightarrow{h_p(t)} y_p(t) \qquad v(t) \xrightarrow{h(t)} y(t)$$

$y(t)$ and $y_p(t)$ are equivalent (ie, the same type of operation has been performed not that the signals are equal)

$$v_p(t) = Re\{v(t)e^{j2\pi f_c t}\}$$
$$y_p(t) = Re\{y(t)e^{j2\pi f_c t}\}$$
$$h_p(t) = Re\{h(t)e^{j2\pi f_c t}\}$$

→ not exactly true, there is some scaling happening.

In baseband, we can use a DSP and perform operation like convolution much more easily.

$$Y(f) = 2Y_p^+(f+f_c) = 2V_p^+(f+f_c)H_p^+(f+f_c)$$
$$= \frac{1}{2}V(f)H(f)$$

$$\Rightarrow h_p(t) = \frac{1}{2}Re\{h(t)e^{j2\pi f_c t}\}$$

✯ Analog Communication Technologies (AM, FM, PM) :-

- Recieving and transmission of signals is still in the analog domain, especially in RF and high frequency applications.

- Let $m(t)$ be the message signal with frequency response $M(f)$. For a real signal,

$$M(f) = M^*(-f)$$

- There are 3 ways of encoding information in the complex envelope, Amplitude, Phase and Frequency.

- **Amplitude Modulation :-**

- Consider a message signal as,

$$m(t) = A_m \cos(2\pi f_m t)$$

$$\Rightarrow M(f) = \frac{A_m}{2}\left(\delta(f+f_m) + \delta(f-f_m)\right)$$

- In DSB-SC (Double Sideband Suppressed Carrier)

$$u_{DSB}(t) = m(t) \underbrace{A \cos(2\pi f_c t)}_{\text{Carrier Wave}}$$

$$\Rightarrow \quad v_{DSB}(t) = A_m \cos(2\pi f_m t)\, A \cos(2\pi f_c t) \quad (f_c > f_m)$$

$$\Rightarrow \quad V_{DSB}(f) = \frac{A_m A}{4}\Big(\delta(f+f_m+f_c) + \delta(f+f_m-f_c) + \delta(f-f_m+f_c) + \delta(f-f_m-f_c)\Big)$$