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Q1. Fourier Series:

$$1. x(t) = 1 + \sin(2\pi f_0 t) + 2\cos(2\pi f_0 t) + \cos(4\pi f_0 t + \frac{\pi}{4})$$

$$= 1 - j(\sin 2\pi f_0 t) + 2\cos(2\pi f_0 t) + \cos \frac{\pi}{4} \sin 4\pi f_0 t - \sin \frac{\pi}{4} \cos 4\pi f_0 t$$

$$= 1 - i \left(\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2} \right) + 2 \left(\frac{e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}}{2} \right) + \frac{1}{\sqrt{2}} \left(\frac{e^{j4\pi f_0 t} + e^{-j4\pi f_0 t}}{2} \right) + \frac{i}{\sqrt{2}} \left(\frac{e^{j4\pi f_0 t} - e^{-j4\pi f_0 t}}{2} \right)$$

$$= 1 + e^{j2\pi f_0 t} \left(-\frac{j}{2} + 1 \right) + e^{-j2\pi f_0 t} \left(\frac{j}{2} + 1 \right) + e^{j4\pi f_0 t} \left(\frac{1}{2\sqrt{2}} + \frac{j}{2\sqrt{2}} \right) + e^{-j4\pi f_0 t} \left(\frac{1}{2\sqrt{2}} - \frac{j}{2\sqrt{2}} \right)$$

Comparing with FSR $x(t) = \sum_{k \in \mathbb{Z}} c_k e^{jk\omega_0 t}$,

$$c_k = \begin{cases} 1 - \frac{j}{2}, & k = 1 \\ 1 + \frac{j}{2}, & k = -1 \\ \frac{1}{2\sqrt{2}}(1+j), & k = 2 \\ \frac{1}{2\sqrt{2}}(1-j), & k = -2 \\ 0, & \text{otherwise} \end{cases}$$

$$2. \quad x(t) = \text{rect}\left(\frac{t}{T}\right) = \begin{cases} 1 & |t| < \frac{T}{2} \\ 0 & \frac{T}{2} < |t| < \frac{3T}{2} \end{cases}, \quad T \text{ is period of } x(t)$$

$$x(t) = \sum c_k e^{j k \omega_0 t}$$

$$\Rightarrow c_k = \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-j k \omega_0 t} dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-j k \frac{2\pi}{T} t} dt$$

$$= \frac{1}{T} \int_{-\infty}^{\infty} e^{-j k \frac{2\pi}{T} t} dt$$

$$= \frac{1}{T} \left(\frac{-T}{jk2\pi} \right) \left[e^{-jk \frac{2\pi}{T} t} \right]_{-\infty}^{\infty}$$

$$= \frac{-1}{jk2\pi} \left(e^{-jk \frac{\pi i}{T}} - e^{jk \frac{\pi i}{T}} \right)$$

$$= \frac{e^{jk\pi i/T} - e^{-jk\pi i/T}}{jk2\pi}$$

$$= \frac{2j \sin(k\pi i/T)}{jk2\pi}$$

$$\Rightarrow c_k = \frac{\sin(k\pi i/T)}{k\pi} = \frac{T}{\pi} \text{sinc}\left(k\pi \frac{i}{T}\right)$$

$$3. \quad x(t) = \Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1 + \frac{2}{\tau}t, & -\frac{\tau}{2} < t < 0 \\ 1 - \frac{2}{\tau}t, & 0 < t < \frac{\tau}{2} \\ 0, & \frac{\tau}{2} < |t| < T \end{cases} \quad \tau \in [0, T)$$

$$c_k = \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) e^{-jk\omega_0 t} dt$$

$$= \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) e^{-jk\frac{2\pi}{T}t} dt$$

$$= \frac{1}{T} \left[\int_{-\tau/2}^0 \left(1 + \frac{2}{\tau}t\right) e^{-jk\frac{2\pi}{T}t} dt + \int_0^{\tau/2} \left(1 - \frac{2}{\tau}t\right) e^{-jk\frac{2\pi}{T}t} dt \right]$$

$$= \frac{1}{T} \left[\int_{-\tau/2}^{\tau/2} e^{-jk\frac{2\pi}{T}t} dt + \frac{2}{\tau} \left(\int_{-\tau/2}^0 te^{-jk\frac{2\pi}{T}t} dt - \int_0^{\tau/2} te^{-jk\frac{2\pi}{T}t} dt \right) \right]$$

$$= \frac{1}{T} \left[\int_{-\tau/2}^{\tau/2} e^{-jk\frac{2\pi}{T}t} dt + \frac{2}{\tau} \left(\frac{-\tau}{jk2\pi} \right) \left(\int_{-\tau/2}^0 -jk\frac{2\pi}{T}t e^{-jk\frac{2\pi}{T}t} dt - \int_0^{\tau/2} -jk\frac{2\pi}{T}t e^{-jk\frac{2\pi}{T}t} dt \right) \right]$$

$$= \frac{1}{T} \left[\frac{-\pi}{jk2\pi} \left[e^{-jk\frac{2\pi}{T}t} \right]_{-\tau/2}^{\tau/2} - \frac{\pi}{\tau jk\pi} \left(\left[e^{-jk\frac{2\pi}{T}t} \left(-jk\frac{2\pi}{T}t - 1 \right) \right]_0^{\tau/2} - \left[e^{-jk\frac{2\pi}{T}t} \left(-jk\frac{2\pi}{T}t - 1 \right) \right]_0^{\tau/2} \right) \left(\frac{-\tau}{jk2\pi} \right) \right]$$

$$= \frac{-1}{jk2\pi} \left(e^{-jk\pi\frac{T}{T}} - e^{jk\pi\frac{T}{T}} \right) - \frac{1}{\tau jk\pi} \left(\frac{-\tau}{jk2\pi} \right)$$

$$\left[(-1 - e^{jk\pi\frac{T}{T}} \left(jk\pi\frac{T}{T} - 1 \right)) - \left(e^{-jk\pi\frac{T}{T}} \left(-jk\pi\frac{T}{T} - 1 \right) + 1 \right) \right]$$

$$= \frac{1}{jk\pi} \left(2j \sin(k\pi \frac{\tau}{T}) \right) - \frac{T}{2\tau(k\pi)^2} \left(-1 - jk\pi \frac{\tau}{T} e^{jk\pi \frac{\tau}{T}} \right. \\ \left. + e^{jk\pi \frac{\tau}{T}} + jk\pi \frac{\tau}{T} e^{-jk\pi \frac{\tau}{T}} + e^{-jk\pi \frac{\tau}{T}} - 1 \right)$$

$$= \frac{\sin(k\pi \frac{\tau}{T})}{k\pi} - \frac{T}{2\tau(k\pi)^2} \left(-2 - jk\pi \frac{\tau}{T} \left(e^{jk\pi \frac{\tau}{T}} - e^{-jk\pi \frac{\tau}{T}} \right) \right. \\ \left. + e^{jk\pi \frac{\tau}{T}} + e^{-jk\pi \frac{\tau}{T}} \right)$$

$$= \frac{\sin(k\pi \frac{\tau}{T})}{k\pi} - \frac{T}{2\tau(k\pi)^2} \left(-2 - jk\pi \frac{\tau}{T} \left(2j \sin(k\pi \frac{\tau}{T}) \right) \right. \\ \left. + 2 \cos(k\pi \frac{\tau}{T}) \right)$$

$$= \frac{\sin(k\pi \frac{\tau}{T})}{k\pi} + \frac{T}{\tau(k\pi)^2} + \frac{\pi}{2\tau(k\pi)^2} \left(\frac{jk\pi \tau}{\pi} \right) 2j \sin(k\pi \frac{\tau}{T}) \\ - \frac{T}{2\tau(k\pi)^2} 2 \cos(k\pi \frac{\tau}{T})$$

$$= \frac{\sin(k\pi \frac{\tau}{T})}{k\pi} + \frac{T}{\tau(k\pi)^2} - \frac{\sin(k\pi \frac{\tau}{T})}{k\pi} - \frac{T}{\tau(k\pi)^2} \cos(k\pi \frac{\tau}{T})$$

$$\Rightarrow c_k = \underbrace{\frac{T}{\tau(k\pi)^2} \left(1 - \cos(k\pi \frac{\tau}{T}) \right)}_{\text{---}} + k \neq 0$$

$$c_0 = \frac{1}{T} \int_{-\tau/2}^{\tau/2} x(t) dt = \frac{1}{T} \left(\int_{-\tau/2}^0 1 + \frac{2}{\tau} t dt + \int_0^{\tau/2} 1 - \frac{2}{\tau} t dt \right) \\ = \frac{1}{T} \left(\int_{-\tau/2}^{\tau/2} dt + \frac{2}{\tau} \left(\int_{-\tau/2}^0 t dt - \int_0^{\tau/2} t dt \right) \right)$$

$$= \frac{1}{T} \left([t]_{-\tau/2}^{\tau/2} + \frac{2}{\tau} \left(\left[\frac{t^2}{2} \right]_0^{\tau} - \left[\frac{t^2}{2} \right]_0^{\tau/2} \right) \right)$$

$$= \frac{1}{T} \left(\tau + \frac{2}{\tau} \left(\left(0 - \frac{\tau^2}{8} \right) - \left(\frac{\tau^2}{8} - 0 \right) \right) \right)$$

$$= \frac{1}{T} \left(\tau + \frac{2}{\tau} \left(-\frac{\tau^2}{4} \right) \right)$$

$$\Rightarrow C_0 = \frac{\tau}{2T}$$

$$4. x(t) = \sum_{l=-\infty}^{\infty} \delta(t - lT_s)$$

$x(t)$ is an impulse train of period T_s ,

$$\Rightarrow c_k = \frac{1}{T_s} \int_{\langle T_s \rangle} x(t) e^{-jk\omega_0 t} dt$$

$$\Rightarrow c_k = \frac{1}{T_s} \int_{-T_s/2}^{T_s/2} \delta(t) e^{-jk\frac{2\pi}{T_s} t} dt$$

$$\Rightarrow c_k = \frac{1}{T_s} e^{-jk\frac{2\pi}{T_s}(0)} \quad [\text{Selection Property of Rivas Signals}]$$

$$\Rightarrow c_k = \frac{1}{T_s} \cancel{+ k}$$

Q2. Properties of LTI Systems :-

$$1. S: y[n] - 5y[n-1] + 6y[n-2] = 2x[n-1]$$

Let the DTFT of $x[n]$ and $y[n]$ be $X(\omega)$ and $Y(\omega)$.

$$\Rightarrow Y(\omega) - 5Y(\omega)e^{-j\omega} + 6Y(\omega)e^{-2j\omega} = 2X(\omega)e^{-j\omega} \quad [\text{Time Shift Property}]$$

$$\Rightarrow Y(\omega)(1 - 5e^{-j\omega} + 6e^{-2j\omega}) = X(\omega)(2e^{-j\omega})$$

$$\Rightarrow H(\omega) = \frac{2e^{-j\omega}}{1 - 5e^{-j\omega} + 6e^{-2j\omega}}$$

$$\text{Let } e^{-j\omega} = E \Rightarrow H(\omega) = \frac{2E}{1 - 5E + 6E^2}$$

$$= H(\omega) = \frac{2E}{(1-2E)(1-3E)}$$

Using partial fraction,

$$\begin{aligned} 2E &= A(1-2E) + B(1-3E) \\ &= 2E = A + B + (-2A - 3B)E \end{aligned}$$

Comparing coefficients,

$$A + B = 0 \quad -2A - 3B = 2$$

$$A = -B \rightarrow 2B - 3B = 2$$

$$A = 2 \quad \leftarrow \Rightarrow B = -2$$

$$\Rightarrow H(\omega) = \frac{2}{1-3E} - \frac{2}{1-2E}$$

$$\Rightarrow H(\omega) = 2 \left(\frac{1}{1-3e^{-j\omega}} - \frac{1}{1-2e^{-j\omega}} \right)$$

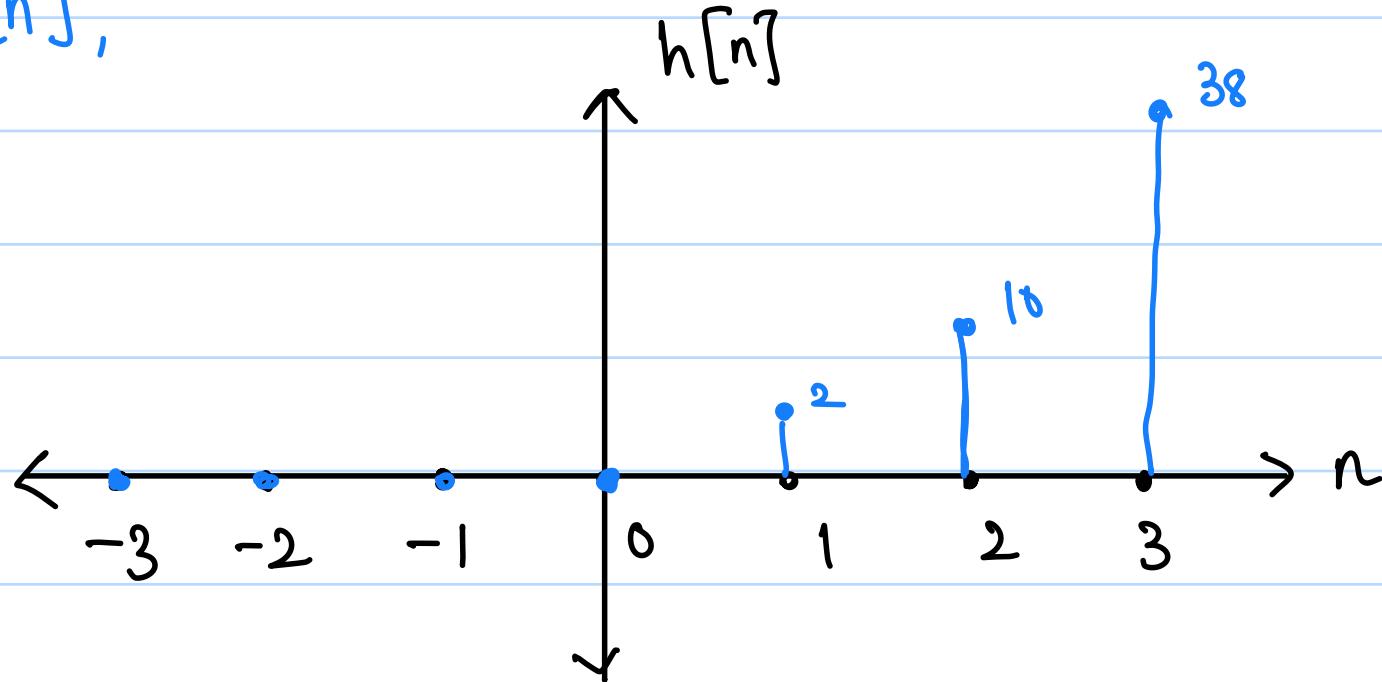
We know that,

$$a^n u[n] \xleftrightarrow{\text{DTFT}} \frac{1}{1-ae^{-j\omega}}$$

$$\Rightarrow h[n] = 2(3^n u[n] - 2^n u[n])$$

$$\Rightarrow h[n] = 2u[n](3^n - 2^n)$$

Plot of $h[n]$,



2 Let the impulse response and step response of a system H
be $h(t)$ and $s(t)$.

To prove: $s(t) = \int_{-\infty}^t h(\tau) d\tau$

We know that $v(t) = \int_{-\infty}^{\infty} \delta(t) dt$.

Also, for any ilp signal $x(t)$, the olp signal $y(t)$ is,

$$y(t) = x(t) * h(t)$$

Taking $x(t) = v(t)$, $y(t)$ will be $s(t)$

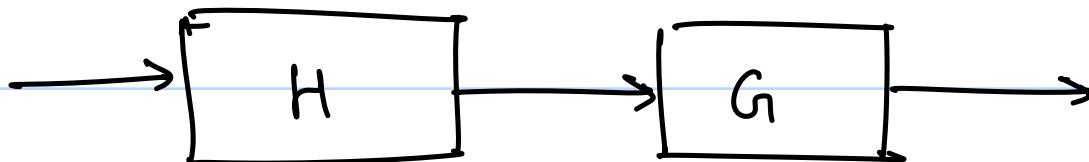
$$\Rightarrow s(t) = v(t) * h(t)$$

$$= s(t) = \int_{-\infty}^{\infty} h(\tau) \cdot v(t-\tau) d\tau$$

$$= s(t) = \underbrace{\int_{-\infty}^t h(\tau) d\tau}_{\text{---}} \quad [v(t-\tau) = 0 \# \tau > t]$$

Hence Proved.

3.



Let $x[n]$ be the ilp of H , $y_1[n]$ be the olp of H and ilp of G , $y[n]$ be the olp of G .

a) H and G are causal LTI systems.

Since H and G are LTI system.

$$y_1[n] = h_1[n] * x[n] \quad [h_1[n] = \text{impulse response of } H]$$

$$y[n] = h_2[n] * y_1[n] \quad [h_2[n] = \text{impulse response of } G]$$

$$\Rightarrow y[n] = h_2[n] * h_1[n] * x[n]$$

$$\Rightarrow \text{Impulse response of cascade} = h_2[n] * h_1[n]$$

$$\Rightarrow h_3[n] = \sum_m h_1[m] h_2[n-m]$$

Since H_1, G are causal,

$$h_1[n] = h_2[n] = 0 \quad \forall n < 0$$

$$n < 0 \Rightarrow n-m < 0 \quad \forall m > 0$$

$$\Rightarrow h_2[n-m] = 0 \quad \forall m \geq 0$$

$$\text{w.k.t } h_1[m] = 0 \quad \forall m < 0$$

$$\Rightarrow \forall n < 0, h_1[m] h_2[n-m] = 0 \quad \forall m$$

$$\Rightarrow \sum_m h_1[m] h_2[n-m] = 0 \quad \forall n < 0$$

$$\Rightarrow h_3[n] = 0 \quad \forall n < 0$$

\therefore The Cascade system is Causal.

$$\text{b) } h_3[n] = \sum_m h_1[m] h_2[n-m]$$

Since H_1, G are bounded

$$\sum_m |h_1[m]| \leq B_1, \quad \sum_m |h_2[m]| \leq B_2$$

$$\begin{aligned} \sum_n |h_3[n]| &= \sum_n \left| \left(\sum_m h_1[m] h_2[n-m] \right) \right| \\ &= \sum_n |h_3[n]| = \sum_m |h_1[m]| \sum_n |h_2[n-m]| \\ &= \sum_n |h_3[n]| = \sum_m |h_1[m]| \sum_k |h_2[k]| \quad \left[\begin{array}{l} \text{Change of variable:} \\ k = n-m \end{array} \right] \end{aligned}$$

$$\Rightarrow \sum_n |h_3[n]| \leq B_1 \cdot B_2 = B_3$$

$$\Rightarrow \underline{\sum_n |h_3[n]|} \leq B_3$$

\therefore The cascade system is also stable.

$$4. a) y[n] - \varepsilon y[n-1] = x[n]$$

Let the DTFT of $x[n]$ and $y[n]$ be $X(\omega)$ and $Y(\omega)$.

$$\Rightarrow Y(\omega) - \varepsilon Y(\omega) e^{-j\omega} = X(\omega)$$

$$\Rightarrow Y(\omega) (1 - \varepsilon e^{-j\omega}) = X(\omega)$$

$$\Rightarrow \frac{Y(\omega)}{X(\omega)} = \frac{1}{1 - \varepsilon e^{-j\omega}} = H(\omega)$$

$$\text{We know that } a^n u[n] \xrightarrow{\text{DTFT}} \frac{1}{1 - a e^{-j\omega}}$$

$$\therefore h[n] = \varepsilon^n u[n]$$

Since for a stable system,

$$\sum_{n=-\infty}^{\infty} h[n] \text{ must be finite}$$

$$\Rightarrow \sum_{n=-\infty}^{\infty} \varepsilon^n v[n] = \sum_{n=0}^{\infty} \varepsilon^n \text{ must be finite}$$

Since $\sum_{n=0}^{\infty} \varepsilon^n$ is the sum of a GP, for it to be finite,
 $\varepsilon \in [0, 1)$.

\therefore The required condition is $\varepsilon \in [0, 1)$.

b) The impulse response of the system is $h[n] = \varepsilon^n v[n]$ as derived above.

c) $h[n] = \varepsilon^n v[n]$

i) A system is memoryless if and only if $h[n] = 0 \nabla n \neq 0$.

Clearly $h[n] \neq 0 \nabla n \neq 0$, \therefore System is not memoryless.

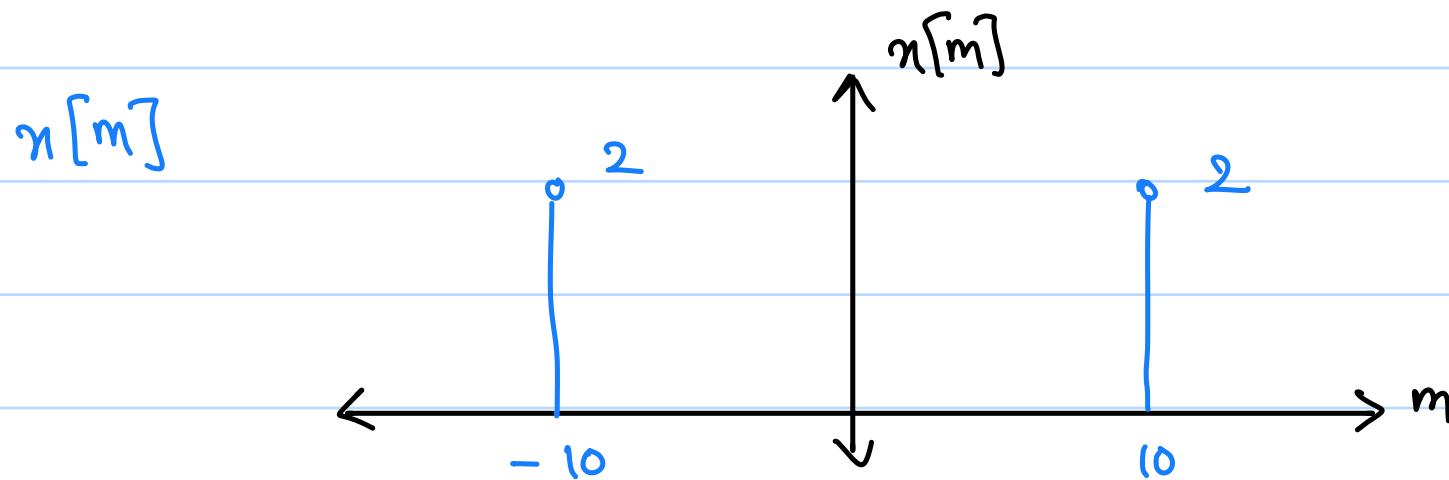
ii) $h[n] = \varepsilon^n v[n] \therefore h[n] = 0 \nabla n < 0$.

\therefore System is causal.

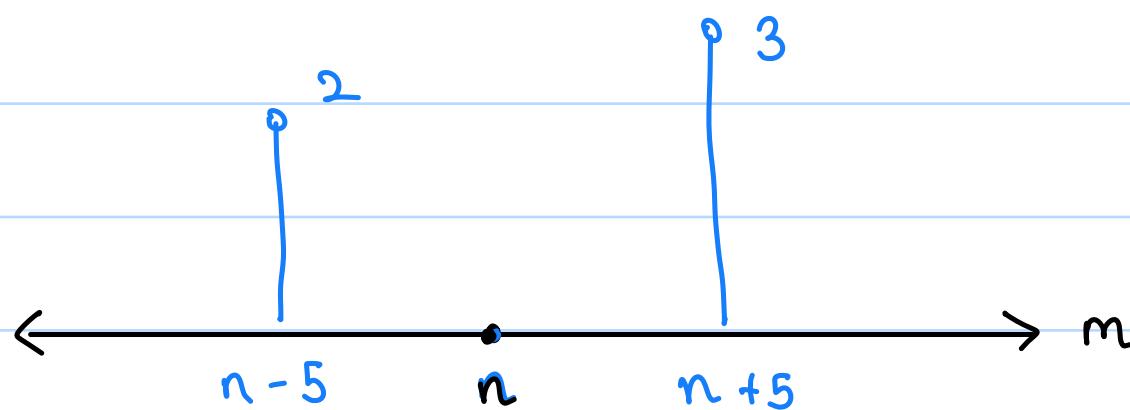
Q3. Discrete-Time Convolution :-

$$1. x[n] = 2\delta[n+10] + 2\delta[n-10]$$

$$y[n] = 3\delta[n+5] + 2\delta[n-5]$$



$y[n-m]$:



At $n+5 = -10 \Rightarrow n = -15$,

$$\sum x[m]y[n-m] = 3 \times 2 = 6$$
$$\therefore z[-15] = 6$$

At $n-5 = -10 \Rightarrow n = -5$,

$$\sum x[m]y[n-m] = 2 \times 2 = 4$$
$$\therefore z[-5] = 4$$

At $n+5 = 10 \Rightarrow n = 5$,

$$\sum x[m]y[n-m] = 3 \times 2 = 6$$
$$\therefore z[5] = 6$$

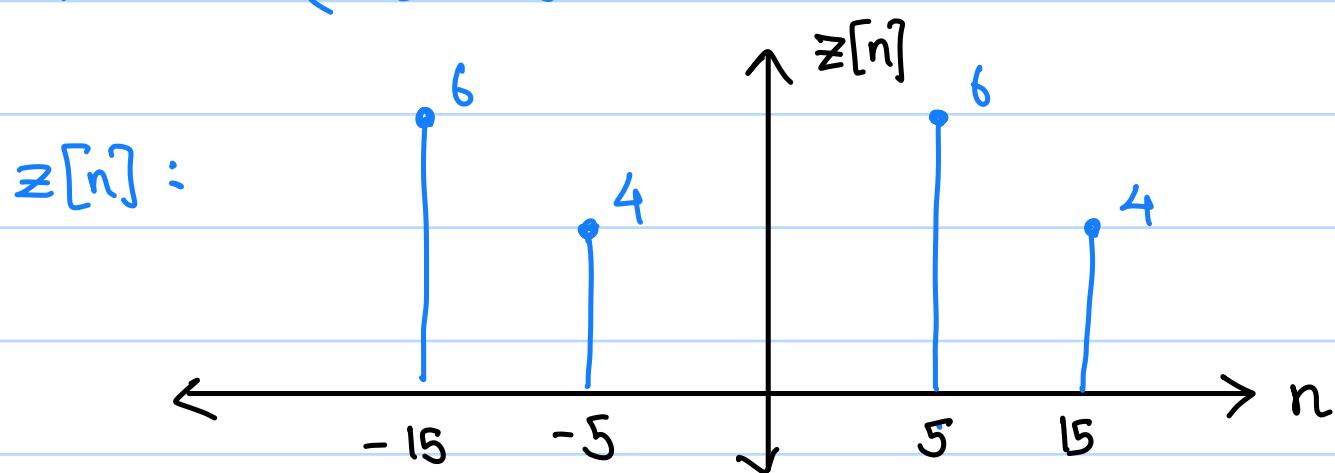
$$\text{At } n=5 \Rightarrow n=15,$$

$$\sum_{m} x[m]y[n-m] = 2 \times 2 = 4$$

$$\therefore z[15] = 4$$

$$\Rightarrow z[n] = \begin{cases} 6, & n=5 \text{ or } n=-15 \\ 4, & n=15 \text{ or } n=-5 \\ 0, & \text{otherwise} \end{cases}$$

$$\Rightarrow z[n] = 6(\delta[n-5] + \delta[n+15]) + 4(\delta[n-15] + \delta[n+5])$$

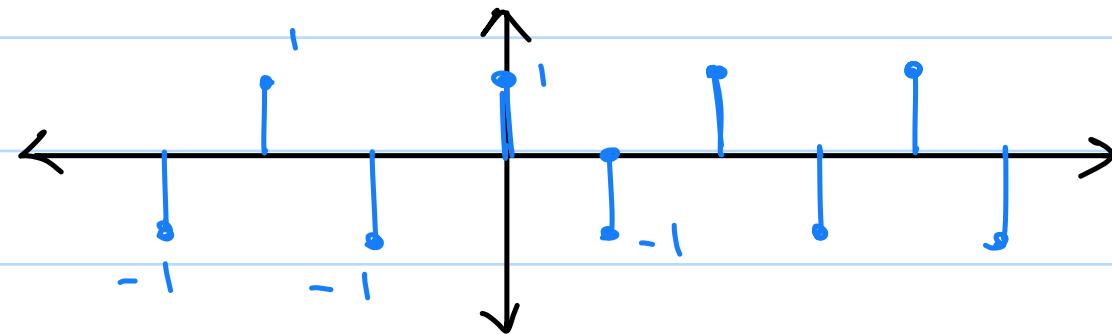


$$2. x[n] = (-1)^n$$

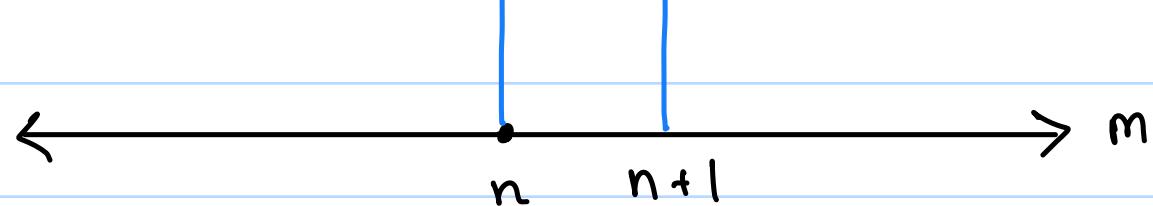
$$y[n] = \delta[n] + \delta[n-1]$$

$$z[n] = \sum_m x[m]y[n-m]$$

$$x[m]:$$



$$y[n-m]:$$



At $n = 0$,

$$x[m]y[n-m] = (1 \times 1) + (1 \times (-1)) = 1 - 1 = 0$$
$$\Rightarrow z[0] = 0$$

At $n = 1$,

$$x[m]y[n-m] = 1 \times (-1) + 1 \times 1 = -1 + 1 = 0$$
$$\Rightarrow z[1] = 0$$

Since x is periodic, the same pattern of overlaps will repeat between x and y .

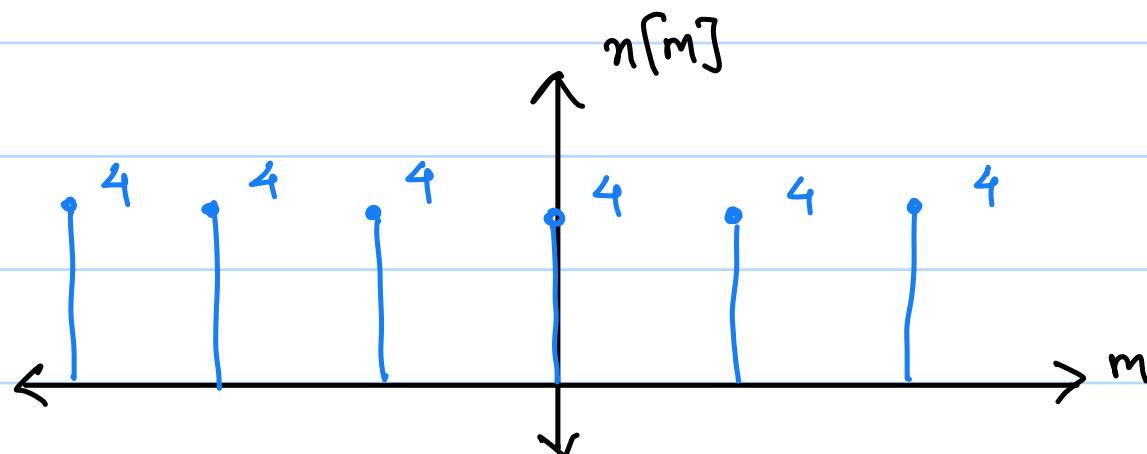
$$\therefore \underline{z[n] = 0 \neq n}$$

3. $x[n] = 4$

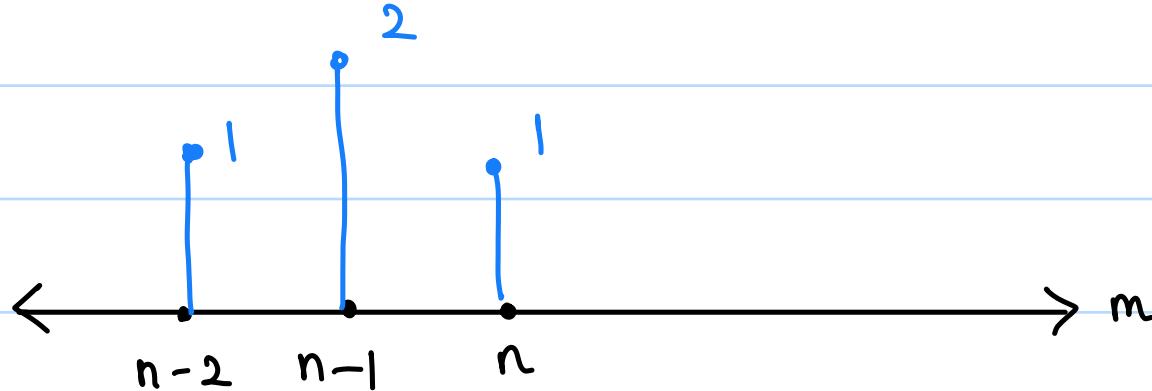
$$y[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$$

$$z[n] = \sum_m x[m]y[n-m]$$

$x[m]$:



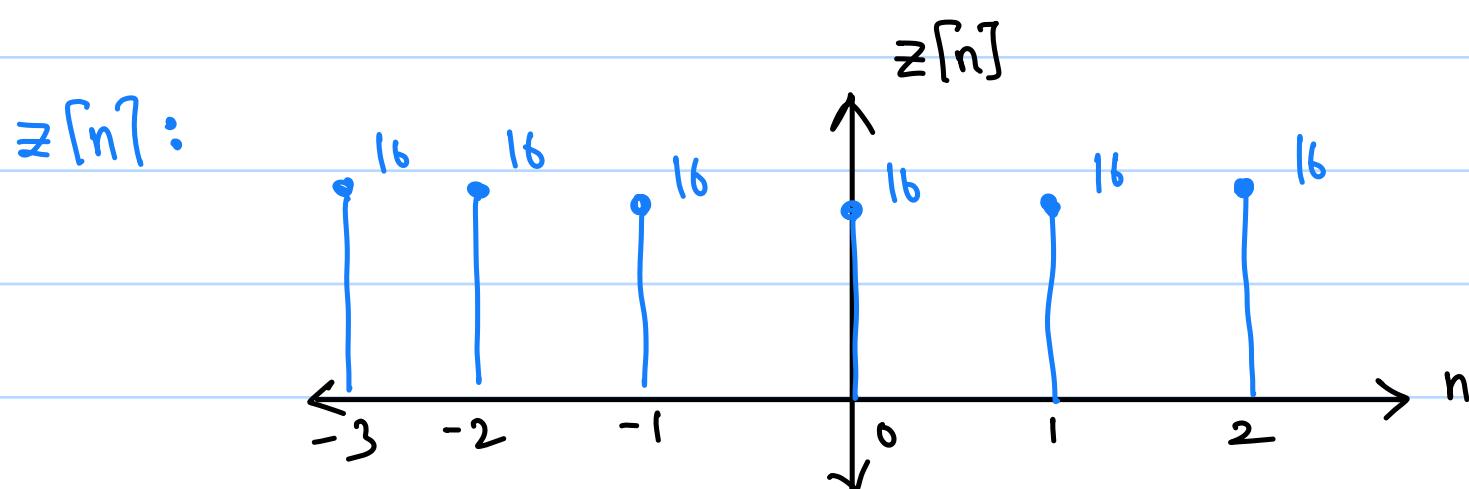
$y[n-m]$:



Since $x[n]$ is a constant, the overlap b/w $x[m]$ and $y[n-m]$ is the same $\neq n$.

$$\Rightarrow z[n] = \sum_m x[m]y[n-m] = 4(1+2+1) = 16$$

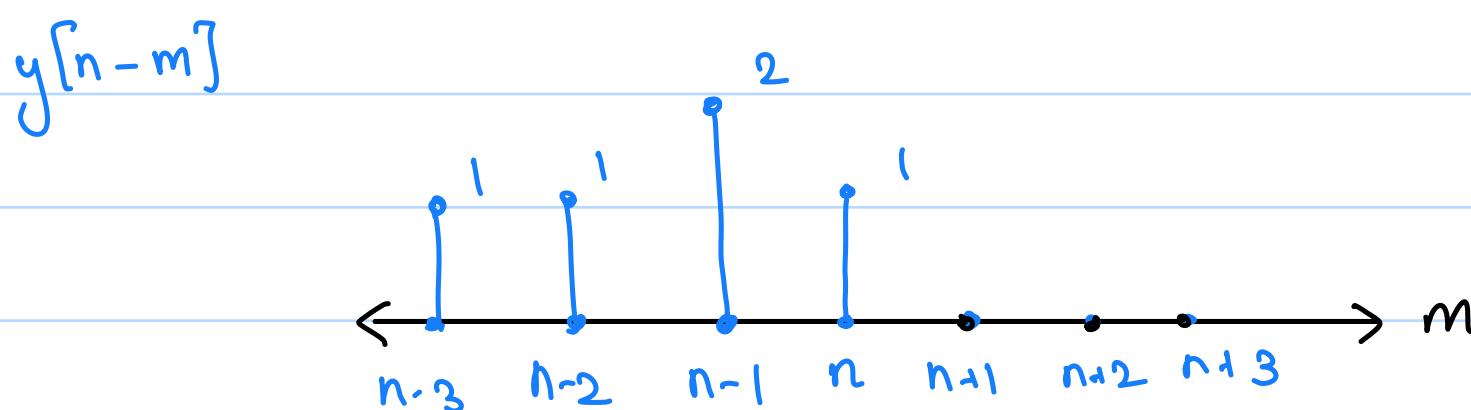
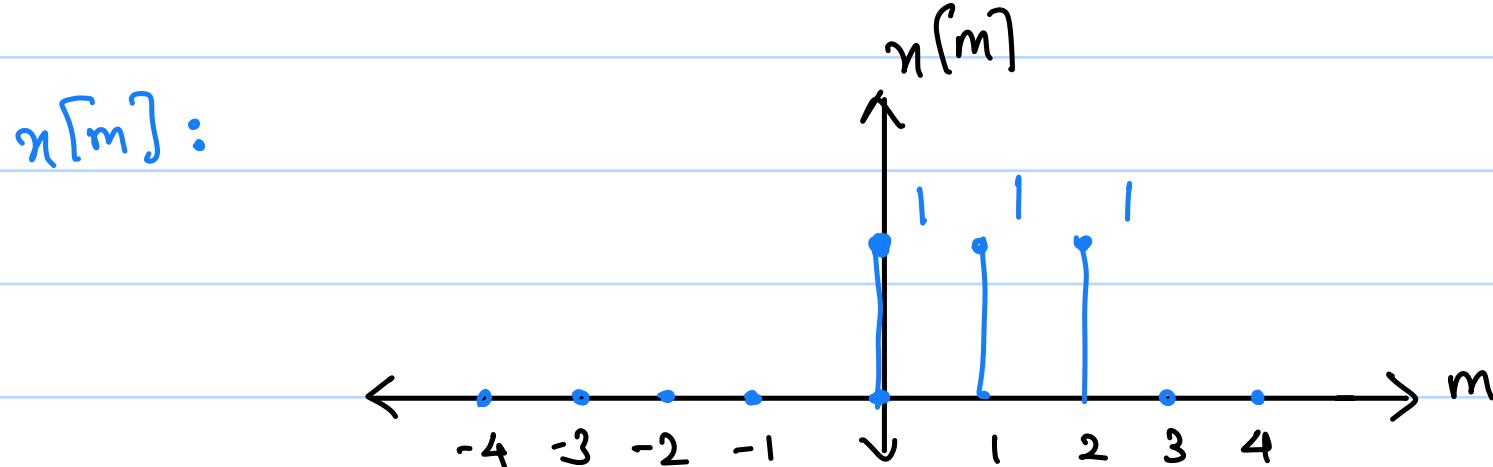
$$\Rightarrow z[n] = 16 \neq n$$



$$4. x[n] = u[n] - u[n-3]$$

$$y[n] = \begin{cases} 0, & n < 0 \\ n, & n = 0, 1, 2 \\ 1, & n \geq 3 \end{cases}$$

$$z[n] = \sum_m x[m]y[n-m]$$



At $n = 0$,

$$z[0] = 1 \times 1 = 1$$

At $n < 0$,

$$z[0] = 0$$

At $n = 1$,

$$z[1] = 2 \times 1 + 1 \times 1 = 3$$

At $n = 2$

$$z[2] = 1 \times 1 + 2 \times 1 + 1 \times 1 = 4$$

At $n = 3$

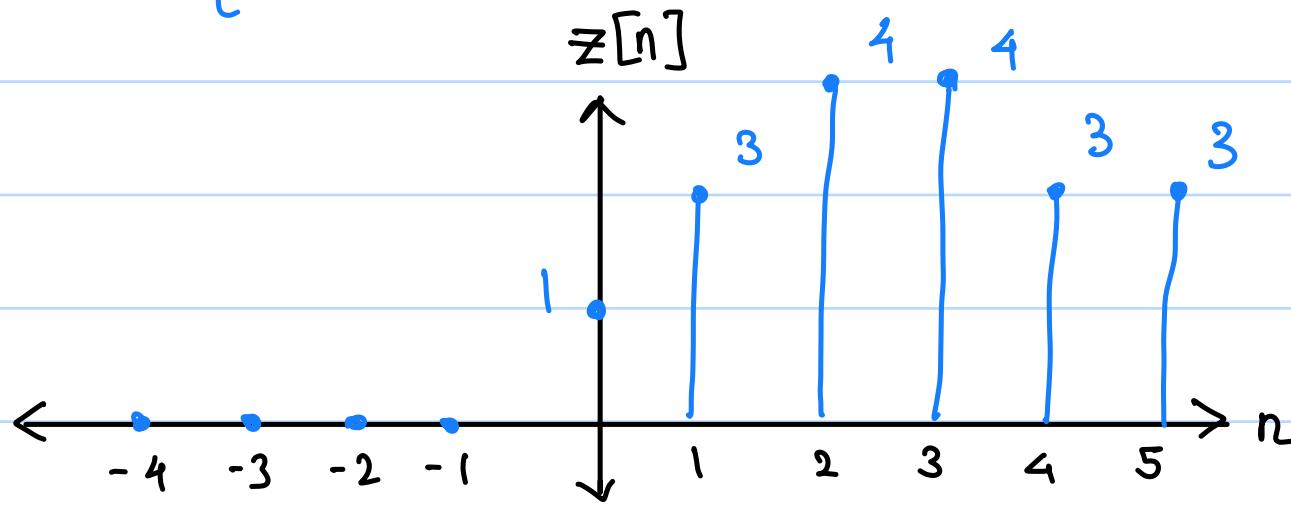
$$z[3] = 1 \times 1 + 1 \times 1 + 2 \times 1 = 4$$

At $n \geq 4$

$$z[n] = 1 \times 1 + 1 \times 1 + 1 \times 1 = 3$$

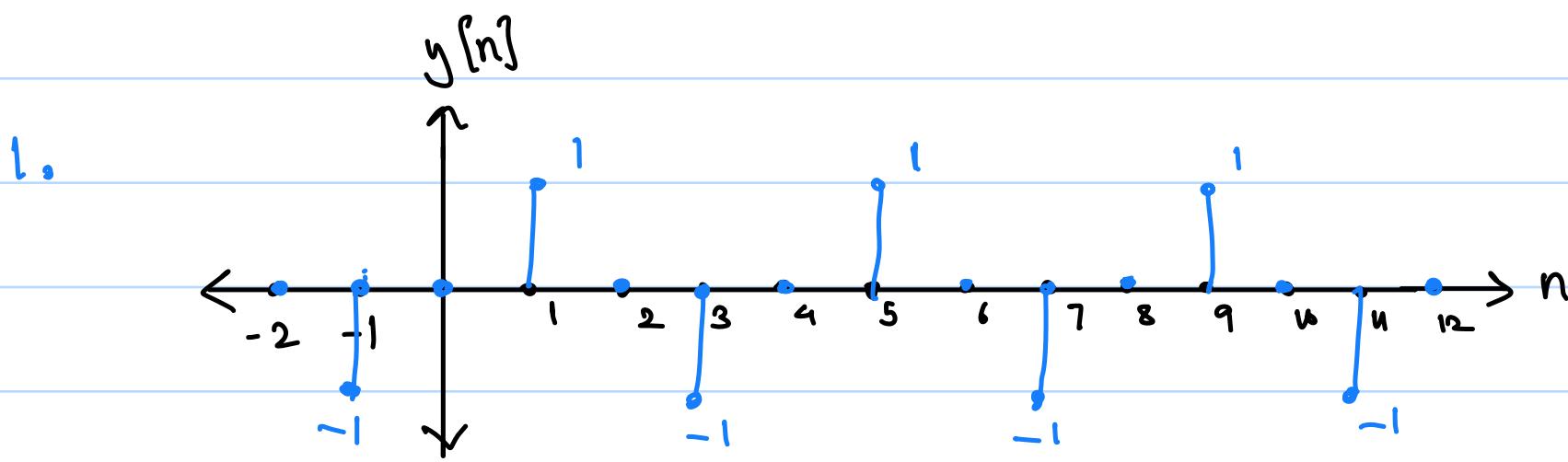
$$\therefore z[n] = \begin{cases} 0, & n < 0 \\ 1, & n = 0 \\ 3, & n = 1 \text{ or } n \geq 4 \\ 4, & n = 2 \text{ or } n = 3 \end{cases}$$

$z[n]$:



Q4. LTI Systems :-

$$h[n] = u[n] - u[n-3], \quad x[n] = \sin\left(\frac{\pi}{2}n\right)u[n]$$



$$2. \quad y[n] = h[n] * x[n]$$

$$= \sum_{m=-\infty}^{\infty} h[m] x[n-m]$$

$$= \sum_{m=-\infty}^{\infty} (u[m] - u[m-3]) \sin\left(\frac{\pi}{2}(n-m)\right) u[n-m]$$

$$\Rightarrow y[n] = \sum_{m=0}^2 \sin\left(\frac{\pi}{2}(n-m)\right) u[n-m]$$

$$\Rightarrow y[n] = \sum_{m=0}^{\min(n, 2)} \sin\left(\frac{\pi}{2}(n-m)\right)$$

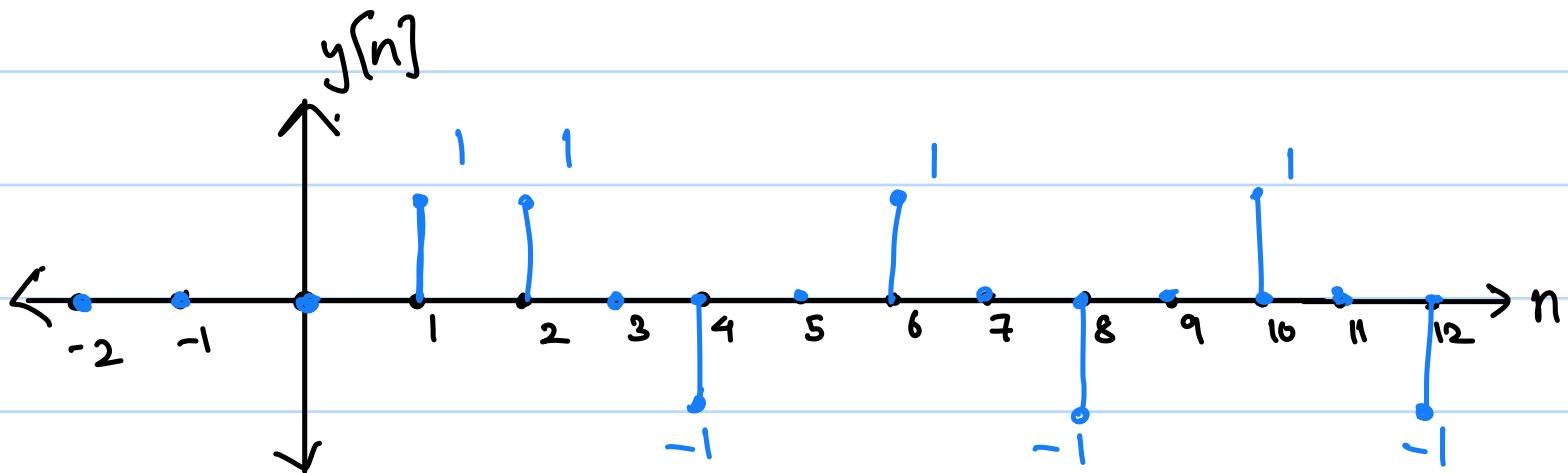
$$\text{At } n \leq 0, \quad y[n] = 0$$

$$\text{At } n=1, \quad y[1] = \sin\left(\frac{\pi}{2}(1-0)\right) + \sin\left(\frac{\pi}{2}(1-1)\right) \\ = 1 + 0 = 1$$

$$\text{At } n=2, \quad y[2] = \sin\left(\frac{\pi}{2}(2-0)\right) + \sin\left(\frac{\pi}{2}(2-1)\right) + \sin\left(\frac{\pi}{2}(2-2)\right) \\ = 0 + 1 + 0 = 1$$

$$\text{At } n \geq 3, \quad y[n] = \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-2)\right) \\ = \cancel{\sin\left(\frac{\pi}{2}n\right)} - \cos\left(\frac{\pi}{2}n\right) - \cancel{\sin\left(\frac{\pi}{2}n\right)} \\ = -\cos\left(\frac{\pi}{2}n\right)$$

$$\therefore y[n] = \begin{cases} 0, & n \leq 0 \\ 1, & n = 1 \text{ or } n = 2 \\ -\cos\left(\frac{\pi}{2}n\right), & n \geq 3 \end{cases}$$



$$3. y[n] = \sum_{m=-\infty}^{\infty} h[m] \nu[n-m]$$

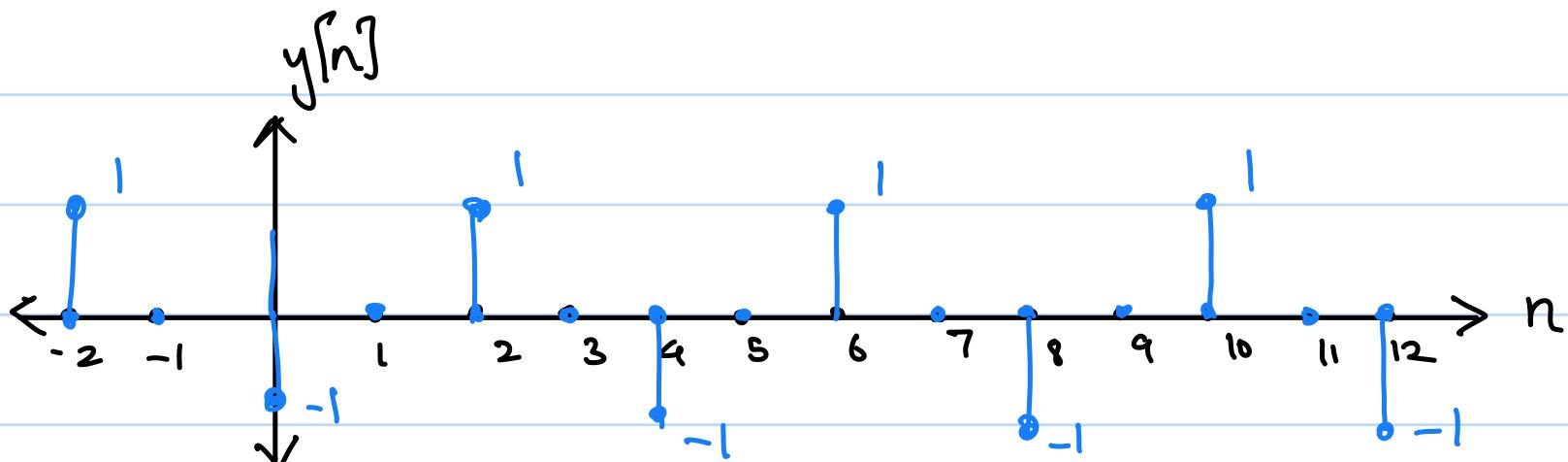
$$= \sum_{m=-\infty}^{\infty} (\nu[m] - \nu[m-3]) \sin\left(\frac{\pi}{2}(n-m)\right)$$

$$= \sum_{m=0}^2 \sin\left(\frac{\pi}{2}(n-m)\right)$$

$$= \sin\left(\frac{\pi}{2}n\right) + \sin\left(\frac{\pi}{2}(n-1)\right) + \sin\left(\frac{\pi}{2}(n-2)\right)$$

$$= \cancel{\sin\left(\frac{\pi}{2}n\right)} - \cos\left(\frac{\pi}{2}n\right) - \cancel{\sin\left(\frac{\pi}{2}n\right)}$$

$$\therefore \underline{y[n]} = -\cos\left(\frac{\pi}{2}n\right)$$



Q5. Eigeneignals :-

Let $g[n]$ be the eigeneignal of a system, then
 $y[n] = k \cdot g[n]$, where $k \in \mathbb{C}$, $y[n]$ is the output for $g[n]$ in the system.

1. $h[n] = \delta[n-4]$

$$h[n] * g[n] = k \cdot g[n]$$

$$= \sum_{m=-\infty}^{\infty} \delta[m-4] g[n-m] = k \cdot g[n]$$

$$= g[n-4] = k \cdot g[n]$$

An example of such a signal : $g[n] = 2^n$

$$2^{n-4} = k 2^n \Rightarrow \frac{1}{16} 2^n = k 2^n \rightarrow \text{Satisfied}$$

2. $h[n] = \delta[n-4] + \delta[n-2]$

$$\Rightarrow \sum_{m=-\infty}^{\infty} (\delta[m-4] + \delta[m-2]) g[n-m] = k g[n]$$

$$= g[n-4] + g[n-2] = k g[n]$$

An example of such a signal : $g[n] = 3^n$

$$\Rightarrow 3^{-4} 3^n + 3^{-2} 3^n = k 3^n$$

$$= (3^{-4} + 3^{-2}) 3^n = k 3^n \rightarrow \text{Satisfied}$$

