

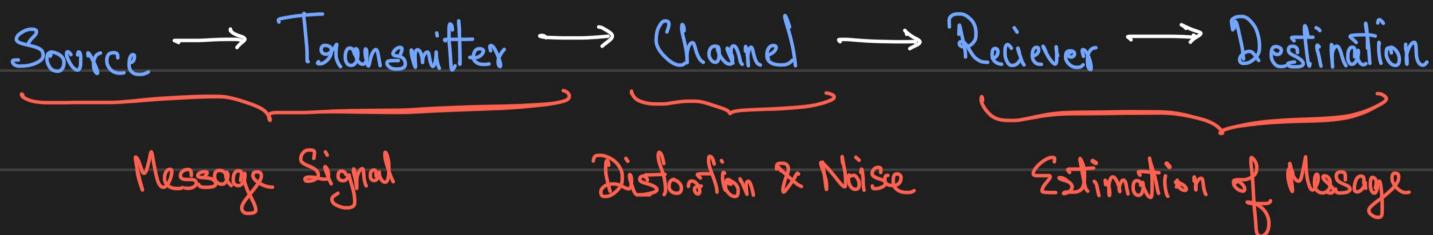
Communication Theory

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→ Communication :

The transmission of information from a source to a destination via a transmission medium.

- Information transfer can be across space or time (time → storage)



→ Channel :-

- Guided Channels - Point to point channel. ex: wired communication, waveguide.

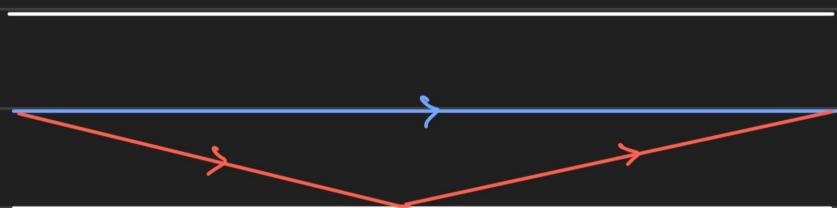
• Challenges :-

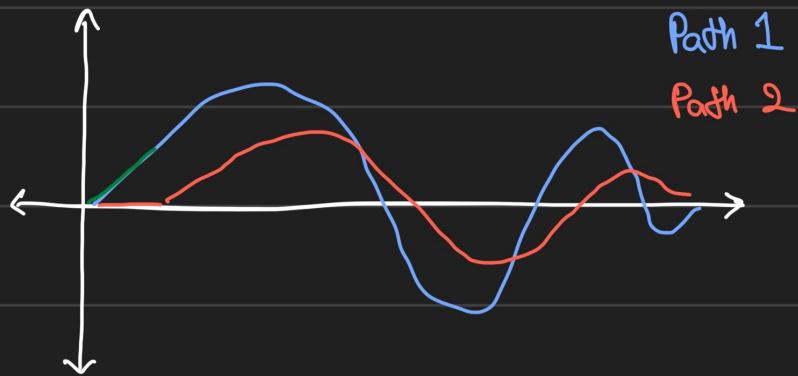
- 1) Spectrum : A channel can allow only a finite bandwidth of frequencies, given by its frequency response.

- The finite bandwidths exist for both wired and wireless channels
- Low Radio Frequencies : Large Antenna Heights
High Radio Frequencies : High attenuations (abv 5 GHz → point 2 point)
- The wireless spectrum is also divided between different technologies and applications. ex:
AM : 0.535 - 1.7 MHz

2) Noise: A channel can also distort the signal

- Can be thermal, man-made EMI, powerline interferences.
- Due to the random nature, it is modelled as AWGN (additive white gaussian noise)
- Multipath is also a problem,





3) Time Variance :-

We often model channels as LTI systems, ie, linear and time-invariant systems.

- But most channels, especially wireless, are time variant.

° Channel Capacity :-

The capacity of a AWGN channel is given by,

$$C = B \log_2 (1 + SNR)$$

B - Channel Bandwidth, $SNR = P/N$ (Signal to Noise Ratio)

P - Signal Power, N - Noise Power

→ Communication Systems :-

- Analog Message Signal - Continuous-time signals which takes a continuous range of values.
- Digital Message Signal - Discrete-time signal with a discrete range of values.
- Analog Communication Systems :-

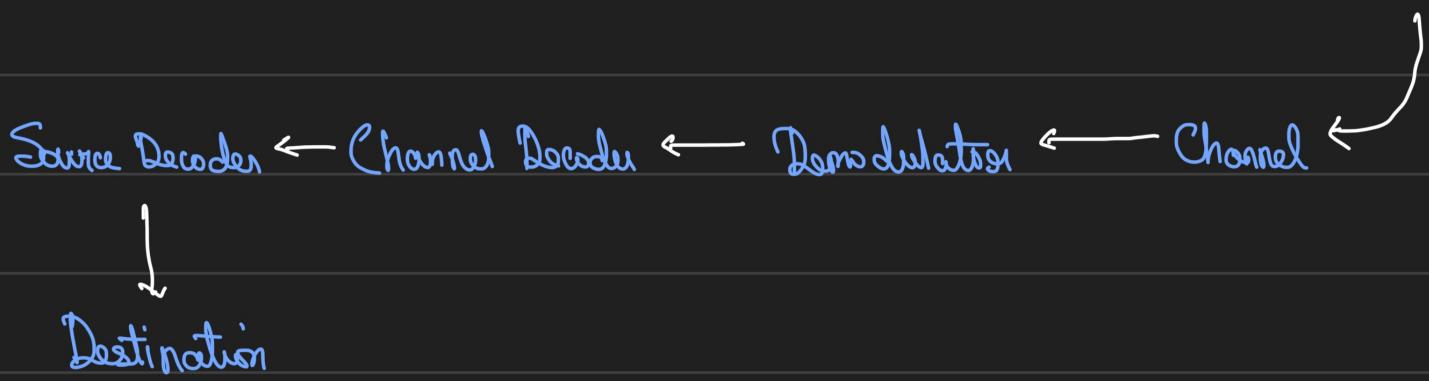
Source → Modulator → Channel → Demodulator → Destination

- Modulation is the shifting of the signal to a higher frequency for transmission and multiplexing.
(Multiplexing - Different signals are shifted to different bands so that they can be transmitted at once)
- Analog communication is the obvious thing to do, but it is not very efficient.

Digital Communication Systems :-

- The message signal is digital, i.e., a signal that can be converted into 0's and 1's. (need not be binary signals always)

Information Source → Source Encoder → Channel Encoder → Modulator



- Source Coding: Obtains digital representation of the information and removes any redundancy within the information. ex: Compression
- Channel Coding: Adds redundancy to the signal for error correction due to channel noise and distortion. ex: Repetition Coding
- Digital Modulation: Converts the bit-stream into a waveform suitable for modulation.
- Digital systems are robust against distortion and noise.

- Digital systems can also use regenerative repeaters, i.e., devices within the channel that removes the distortion until that point (similar to resetting the signal to its original form)
- Digital Systems can do error correction using Shannon's theorem, and they can also be encrypted.
- The processor devices are digital as well.

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Information Theory - Source Coding

Error Correcting Codes - Channel Coding

Wireless Communication - Modulation & Transmission

★ Logistics :-

◦ Textbook :- Intro. To Comm. Systems , U. Madhow

◦ Grading :-

Classwork + Surprise Quiz + Q1 + Q2 - 20%

Midsem - 20 %

Assignment and Quir - 20 %

Endsem - 40 %.

* 2: Overview of Signals and Systems :-

- The notation for the step function is given as $\mathbb{I}_{[0,\infty)}$, ie, an indicator function.
- Sinusoids : $A \cos(2\pi f_0 t + \theta)$. Modulation varies A, f_0 and θ to convey information. (If A, f_0, θ are known, then no info can be passed)
- Impulse Response - Response of the system to an impulse signal.
Used to study LTI systems.
- Rectangular波形 :

$$s(t) = A_c \cos(2\pi f_0 t) - A_s \sin(2\pi f_0 t)$$

Where ,

$$A_c = A \cos \theta, \quad A_s = A \sin \theta$$

Using Euler's formula,

$$A e^{j\theta} = A \cos \theta + j A \sin \theta = A_c + j A_s$$

$$A = \sqrt{A_c^2 + A_s^2}, \quad \theta = \tan^{-1}(A_s/A_c)$$

• Complex Exponentials: $A e^{j(2\pi f_0 t + \theta)} = \alpha e^{j2\pi f_0 t}$, $\alpha = A e^{j\theta}$

These are Eigensignals to any LTI system.

Usually, α is needed to encode the information and $e^{j2\pi f_0 t}$ is known (carrier wave)

• Inner Product :-

$$= \gamma^H s \quad (\text{H - Hermitian} \mid \text{Conj. Transpose})$$

$$\langle s, r \rangle = \sum_{i=-\infty}^{\infty} s[i] \gamma^*[i] \quad (\text{For digital signals})$$

$$\langle s, r \rangle = \int_{-\infty}^{\infty} s(t) \gamma^*(t) dt \quad (\text{For analog signals})$$

• Energy of a Signal :-

$$E_s = \|s\|^2 = \langle s, s \rangle = \int_{-\infty}^{\infty} s(t) \bar{s}(t) dt$$

Q. Find the energy of :

$$i) \quad s(t) = 2\mathbb{1}_{[0, T]} + j\mathbb{1}_{[T/2, 2T]}$$

$$ii) \quad s(t) = e^{-3|t| + j2\pi t}$$

$$i) s(t) = 2\mathbb{1}_{[0, \frac{T}{2}]} + j\mathbb{1}_{[\frac{T}{2}, T]}$$

$$= \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ 2+j, & t \in [\frac{T}{2}, T] \\ j, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow |s(t)| = \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ \sqrt{3}, & t \in [\frac{T}{2}, T] \\ 1, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow E_s = \int_0^{\frac{T}{2}} 2^2 dt + \int_{\frac{T}{2}}^T \sqrt{3}^2 dt + \int_T^{2T} 1^2 dt$$

$$= 4\left(\frac{T}{2}\right) + 3\left(T - \frac{T}{2}\right) + 1(2T - T)$$

$$= 2T + \frac{3T}{2} + T$$

$$= \frac{11T}{2}$$

Power of a Signal :-

The power of a signal is defined as the time average of its energy computed over a large time interval.

$$P_s = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt = \overline{|s(t)|^2}$$

o DC Value :-

The DC value of a signal is the value of the Fourier Transform of the signal at $f = 0$.

Q. Compute P_s and DC value for a real sinusoid $s(t) = A \cos(2\pi f_0 t + \theta)$
where $A > 0$, $\theta \in [0, 2\pi]$, $f_0 \in \mathbb{R}$

$$P_s = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cos(2\pi f_0 t + \theta) dt \quad \text{ill do it later}$$

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★ 3 : Baseband and Passband Representations :-

o Multiplexing : Encoding multiple pieces of information in the same signal using different freq. ranges.

→ Fourier Transform :-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} dw$$

- Poornima's Identity :-

$$\|v\|^2 = \langle v, v \rangle = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_{-\infty}^{\infty} |V(j\omega)|^2 d\omega$$

- Shared Medium :-

- Many practical channels are shared medium, ie, used by multiple devices at once.
- Time Division Multiple Access (TDMA) : Divides the multiple information into packets, which are then sent at different instances in time.
ie, Each device is given a time slot within which they can communicate.
- Frequency Division Multiple Access (FDMA) : Assigns each type of information its own frequency band, ie, each device is allowed to communicate in a particular bandwidth only.

Device \triangleq Transmitter - Receiver Pair

- We can perform TDMA and FDMA at the same time (ex: Statistical Multiplexing) to increase the user capacity of the medium.

\rightarrow Energy Spectral Density and Bandwidth :-

- ESD ($E_v(f)$): For a signal $v(t)$, the ESD $E_v(f)$ is the energy of the output of the filter divided by the width Δf in the limit $\Delta f \rightarrow 0$
 - ↪ filter of width Δf , centered at f .

$$v(f) \xrightarrow{\boxed{H(f)}} y(f) = v(f) H(f)$$

↪ $H(f) = \text{Shifted root of width } \Delta f$
centered at f

Prove that $E_v(f) = |v(f)|^2$

$$\begin{aligned} \int_{-\infty}^{\infty} |y(t)|^2 dt &= \int_{-\infty}^{\infty} |y(f)|^2 df = \int_{-\infty}^{\infty} |v(f) H(f)|^2 df \\ &= \int_{f - \frac{\Delta f}{2}}^{f + \frac{\Delta f}{2}} |v(f)|^2 df \approx |v(f)|^2 \Delta f \quad (\text{for small } \Delta f) \end{aligned}$$

$$\Rightarrow E = |v(f)|^2 \Delta f$$

$$\Rightarrow \underline{E_v(f) = |v(f)|^2}$$

o Bandwidth :-

- For bandlimited signals, bandwidth is the range of frequencies beyond which the spectrum of the signal is zero.
- For real-valued signals, $U(-f) = U^*(f)$. So only positive frequencies count for real valued signals, the bandwidth is one sided.
- For non-bandlimited signals, the energy-containment bandwidth is the size of the smallest band that contains a certain fraction of the signal's energy. (Usually 99%)
- Or, it is the band over which $U(f)$ stays within a certain range about its peak value (ex:- 3dB bandwidth).

Note: $W \sin(Wt) \xleftrightarrow{f} \mathbb{1}_{[-\frac{W}{2}, \frac{W}{2}]}(f)$

Find the bandwidth of 1) $u(t) = \sin(2t)$ 2) $u(t) = \mathbb{1}_{[2,4]}(t)$, t is in microseconds.

1) $u(t) = \sin(2t) \Rightarrow U(f) = \frac{1}{2} \mathbb{1}_{[-1,1]}(f)$

\Rightarrow Bandwidth of the signal = $[-1, 1]$ MHz.

$$2) v(t) = \mathbb{1}_{[0,4]}(t)$$

$$\text{Let } x(t) = v(t+3) = \mathbb{1}_{[-1, 1]}(t)$$

$$\Rightarrow X(f) = 2 \operatorname{sinc}(2f)$$

$$\Rightarrow V(f) = 2 \operatorname{sinc}(2f) e^{-j6\pi f}$$

$$\Rightarrow |V(f)| = |X(f)|$$

$$E_v = \int_{-\infty}^{\infty} |V(f)|^2 df = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_2^4 dt = 2$$

$$\Rightarrow 99\% \text{ of } E_v = 1.98$$

$$\Rightarrow 2 \int_0^{\omega} 2 |\operatorname{sinc}(2f)|^2 df = 1.98 \xrightarrow{\text{on solving}} \underline{\omega = 5.1 \text{ MHz}}$$

- Any analysis that needs to occur in passband, the same analysis can be carried out by shifting the signal into baseband.

Baseband - Spectrum centered around DC ($f=0$)

Passband - Spectrum centered around a non-zero frequency.

