

# Information and Communication: Assignment 1

## 1 Nyquist Rate and Quantization

**Q1. (3 marks)** In this question, you will manually sample a sine wave at different sampling frequencies. Let  $x(t) = \sin(100\pi t)$ . First, plot the original signal, and then plot the sampled signal (your plot should have enough samples to show one cycle of the original sine wave) at frequencies:

1.  $f_s = 50$  Hz
2.  $f_s = 100$  Hz
3.  $f_s = 200$  Hz

Comment on the three plots and explain briefly why sampling above the Nyquist rate is required for reconstruction to be possible.

**Q2. (3 marks)** Let  $x(t)$  be an analogue signal with  $x_+$  and  $x_-$  as the maximum and minimum values assumed by the signal. This signal is digitized using a quantizer, which has 256 levels (8 bits).

**Definition (Quantization noise):** Quantization noise (or quantization error) is the difference between the actual analogue signal and its digital representation due to rounding in the quantization process. It occurs because a finite number of levels are used to approximate an infinite range of values. The formula for quantization noise power is given by:

$$Q_{\text{noise}} = \frac{\Delta^2}{12} \quad (1)$$

where  $\Delta$  is the quantization step size, given by:

$$\Delta = \frac{\text{Signal Range}}{\text{Number of Levels}} \quad (2)$$

Find the quantization noise for the following signals:

1.  $x(t) = 2 \cos(t)$
2.  $x(t) = -|\sin(t)|$
3.  $x(t) = \begin{cases} t, & -T/2 < t < T/2 \\ -t + T, & T/2 < t < 3T/2 \end{cases}$ , with period  $2T$ .

## 2 Introduction to Probability

**Q3. (3 marks)** Given that the weather on 6th August 2022 is humid, what is the probability that the weather on 18th November 2022 is humid? Assume that the weather tomorrow will be the same as today with probability  $p$ . Also, assume that the weather can only be dry or humid.

**Q4. (3 marks)** In a group of  $n$  people, What is the probability that two or more persons will have the same birthday (month and date)? Observe that in a group of 23 people at least 2 people will have same birthday with a probability of 0.5

**Q5. (3 marks)** A fair four-sided die is rolled twice. Assume that all the outcomes are equally likely. Consider  $X$  to be the result on the first die and  $Y$  to be the result on the second die. Define:

- $A = \{\max(X, Y) = m\}$
- $B = \{\min(X, Y) = 2\}$

Find  $P(A|B)$  for  $m = 1, 2, 3$ .

**Q6. (4 marks)** A man is known to tell the truth  $\frac{3}{4}$  of the time. He picks a random card and reports it as an Ace from a red suit. Then he picks up another card from the deck (without replacement) and reports it to be a Hearts card. The second card he picked is known to be an Ace. What is the probability that he was telling the truth the first time?

## 3 Discrete Random Variables

**Q7. (3 marks)** The probability mass function of a random variable  $X$  is given by  $p(i) = c\lambda^i/i!$ ,  $i = 0, 1, 2, \dots$ , where  $\lambda$  is some positive value. Find (a)  $P(X = 0)$  and (b)  $P(X > 2)$ .

**Q8. (3 marks)** Let  $X$  be a Poisson random variable with parameter  $\lambda$ . What value of  $\lambda$  maximizes  $P\{X = k\}$ ,  $k \geq 0$ ?