Information And Communication

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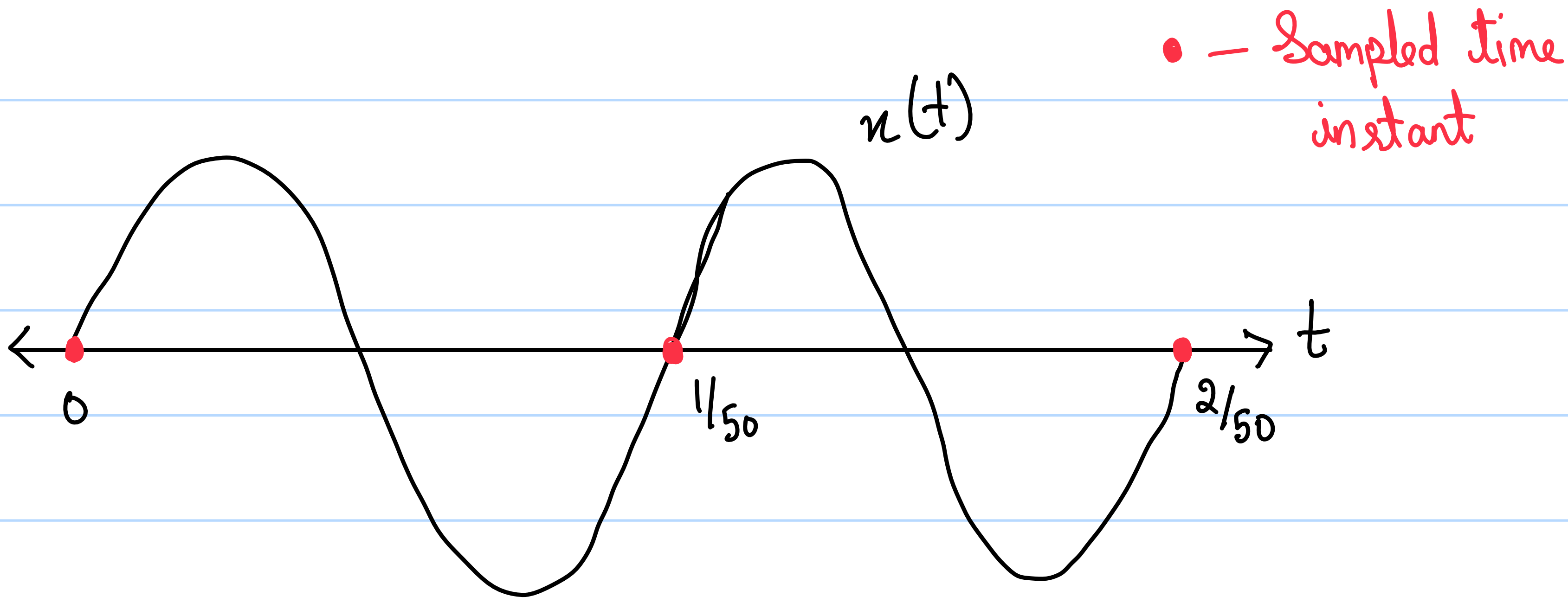
Q1. $x(t) = \sin(100\pi t)$

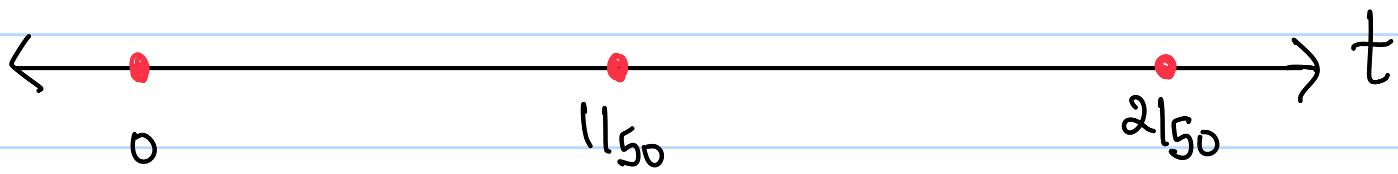
$$T = \frac{2\pi}{\omega} = \frac{2\cancel{\pi}}{100\cancel{\pi}} = \frac{1}{50}\ s$$

1) $f_s = 50\ Hz$

$$T_s = \frac{1}{50}$$



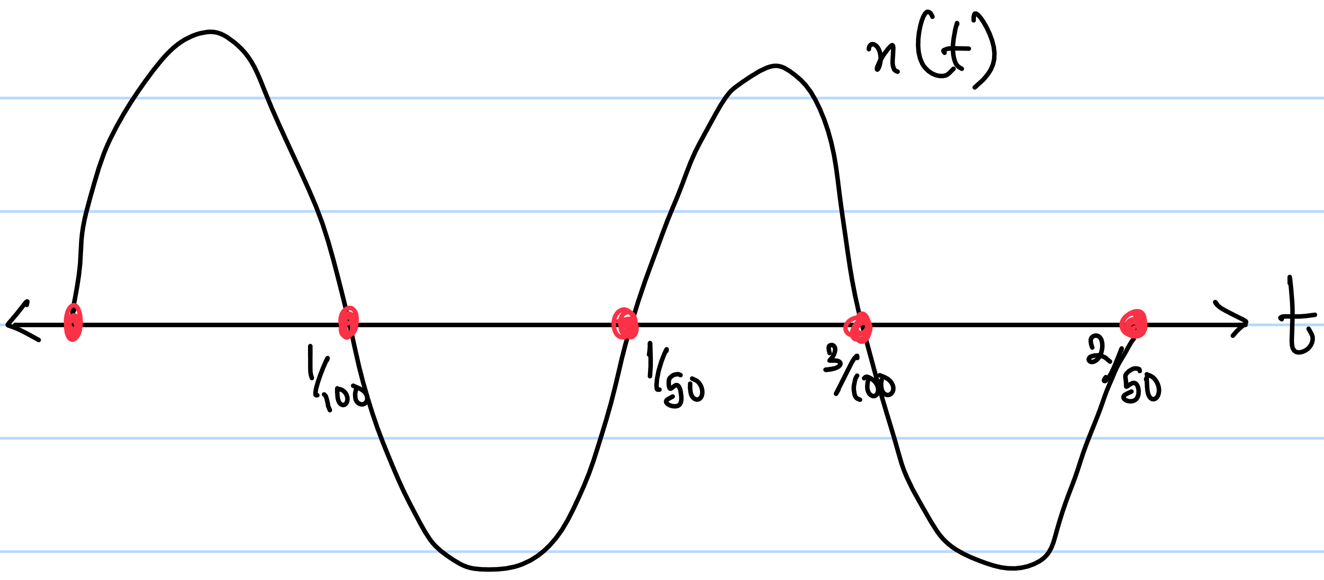
Sampled Signal :-



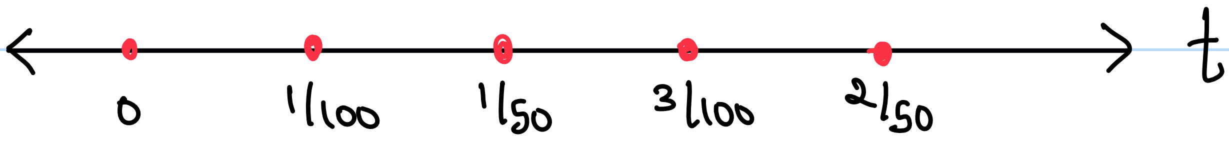
Since the frequency of the sampler exactly matches that of the sine wave, it repeatedly measures the exact same amplitude (which happens to be zero).

2) $f_s = 100\ Hz$

$T_s = \frac{1}{100}\ s$



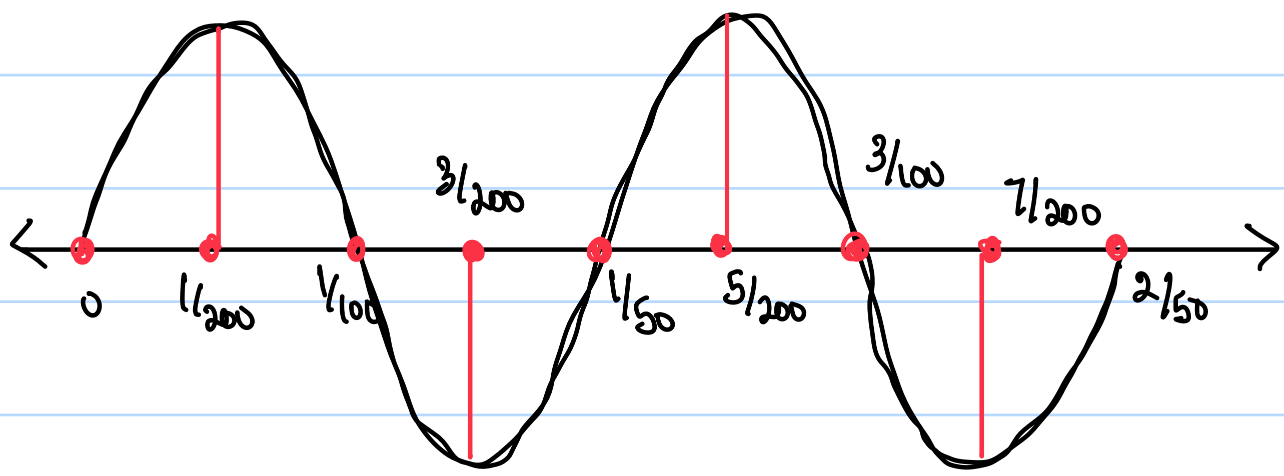
Sampled Signal :-



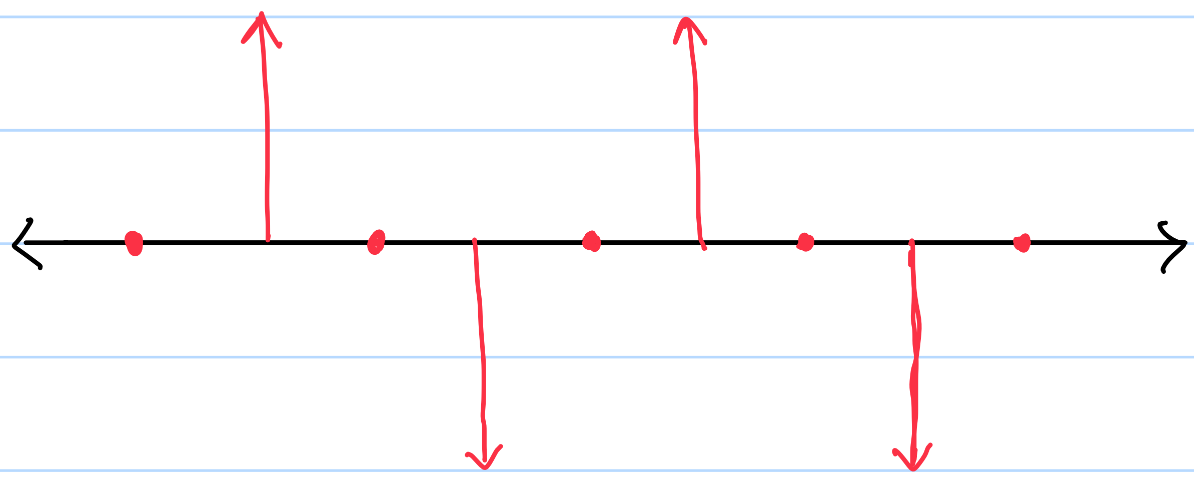
Due to the nature of the sine wave, the amplitude measured during each sample still is zero. However due to the higher sample rate, the data points are denser.

3) $f_s = 200\ Hz$

$T_s = \frac{1}{200}\ s$



Sampled Signal :-



This signal sort of mimics the behaviour of a sinusoidal signal, thereby proving the fact that as the frequency of sampling increases, the amount of information lost is reduced.

Q2. No. of levels $= 2^8 = 256$

$$Q_{noise} = \frac{\Delta^2}{12}, \quad \Delta = \frac{\text{Signal Range}}{\text{No. of levels}}$$

i) $x(t) = 2\cos(t)$

Signal Range $= |2-(-2)| = 4$

$$\Delta = \frac{4}{256} = \frac{2^2}{2^8} = \frac{1}{2^6} = \frac{1}{64}$$

$$Q_{noise} = \frac{\left(\frac{1}{64}\right)^2}{12} = \frac{1}{64^2 \times 12} = \underline{\underline{2.034 \times 10^{-5}}} \ dB$$

2) $x(t) = -|\sin(t)|$

$$\text{Signal Range} = |0-(-1)| = 1$$

$$\Delta = \frac{1}{256}$$

$$Q_{noise} = \frac{\left(\frac{1}{256}\right)^2}{12} = \underline{1.271 \times 10^{-6} \ dB}$$

3) $x(t) = \begin{cases} t, & -T/2 < t < T/2 \\ -t+T, & T/2 < t < 3T/2 \end{cases}$

$$t = -t + T$$

$$2t = T$$

$$t = T/2$$

$$\text{Range} = \left|\frac{T}{2} - 0\right| = T/2$$

$$\Delta = \frac{T/2}{256} = \frac{T}{512}$$

$$Q_{noise} = \frac{\left(\frac{T}{512}\right)^2}{12} = \underline{\underline{3.179 T^2 \times 10^{-7}}}$$

Q3. No. of days b/w 6^{th} August 2022 and 18^{th} November 2022
= 104 days

If the weather on 18^{th} November 2022 must be humid,

1. Same weather must continue for 104 days : p^{104}

2. Weather must change only twice : ${}^{104}C_2\, p^{102}(1-p)^2$

3. Weather must change only 4 times : ${}^{104}C_4\, p^{100}(1-p)^4$

⋮

57. Weather must change only 104 times (ie all days) : ${}^{104}C_{104} \cdot (1-p)^{104}$

$$\therefore P = p^{104} + {}^{104}C_2\, p^{100}(1-p)^2 + {}^{104}C_4\, p^{100}(1-p)^4 \cdots\cdots + {}^{104}C_{104}(1-p)^{104}$$

(ie, only the odd terms of the expansion $(p+(1-p))^{104}$

$$= \frac{(p+(1-p))^{104} + (p-(1-p))^{104}}{2}$$

$$= \frac{1 + (p-1+p)^{104}}{2}$$

$$= \frac{1 + (2p-1)^{104}}{2}$$

Q4. In a group of n people, to calculate the probability that 2 people share the same birthday, we can consider the complement event.

No. of possible birthday combinations $= (365)^n$

No. of birthday combinations st they are all unique $= \dfrac{(365)!}{(365-n)!}$

$$P' = \frac{\frac{(365)!}{(365-n)!}}{365^n}$$

$$\Rightarrow \quad P = 1 - \frac{\frac{(365)!}{(365-n)!}}{365^n}$$

$$= \quad P = \frac{365^n - \frac{(365)!}{(365-n)!}}{365^n}$$

Using this formula, set $n = 23$,

$$\Rightarrow \quad P = \frac{365^{23} - \frac{365!}{(365-23)!}}{365^{23}} \approx \underline{0.507297}$$

Q5. $n(\text{Sample Space}) = 4 \times 4 = 16$

$B = \{(2,2), (2,3), (2,4), (3,2), (4,2)\}$

$P(B) = 5/16$

1) $m = 1$

A is the event st $\max(X, Y) = 1$

$\Rightarrow A = \{(1,1)\}$

$A \cap B = \phi$

$\Rightarrow P(A \cap B) = 0 \Rightarrow \underline{\underline{P(A|B) = 0}}$

2) $m = 2$

A is the event st $\max(X, Y) = 2$

$\Rightarrow A = \{(1,2), (2,1), (2,2)\}$

$A \cap B = \{(2,2)\}$

$\Rightarrow P(A \cap B) = 1/16$, $P(A|B) = \frac{1/16}{5/16} = \underline{\underline{1/5}}$

3) $m = 3$

A is the event st $\max(X, Y) = 3$

$\Rightarrow A = \{(1,3), (2,3), (3,3), (3,2), (3,1)\}$

$A \cap B = \{(2,3)(3,2)\}$

$\Rightarrow P(A \cap B) = 2/16 = 1/8$, $P(A|B) = \frac{1/8}{5/16} = \underline{\underline{2/5}}$

Q6. Let the event of the man telling the truth in the first draw be represented as T_1.

Let the event of drawing an Ace in the second draw be A_2.

$$P(T_1|A_2) = \frac{P(A_2|T_1)\,P(T_1)}{P(A_2)}$$

$$P(T_2) = 3/4 = P(T_1)$$

$$P(A_2|T_1) = \frac{P(A_2 \cap T_1)}{P(T_1)}$$

$A_2 \cap T_1$: The man got an Ace in the second draw and also was telling the truth in the first draw.

But it is not given if the man was telling the truth or not in the second draw.

$$\Rightarrow P(A_2 \cap T_1) = P(A_2 \cap T_1 \cap T_2) + P(A_2 \cap T_1 \cap \overline{T_2})$$

$A_2 \cap T_1 \cap T_2$: Ace in the second draw and truth in both the draws,

This implies that the second card is an Ace of Hearts, which implies that the first card is an Ace of Diamonds.

$$\Rightarrow P(A_2 \cap T_1 \cap T_2) = \frac{1}{52} \times \frac{1}{51} = \frac{1}{52 \times 51}$$

$A_2 \cap T_1 \cap \overline{T_2}$: Ace in the second draw and false statement in the second draw, and truth in the first draw.

Since the first draw was the truth, a red Ace was drawn.

Since the second statement was false, the card was an Ace from a black suit.

$$\Rightarrow \quad P(A_2 \cap T_1 \cap \overline{T_2}) = \frac{2}{52} \times \frac{2}{51} = \frac{4}{52 \times 51}$$

$$\Rightarrow P(A_2 \cap T_1) = \frac{1}{52 \times 51} + \frac{4}{52 \times 51} = \frac{5}{52 \times 51}$$

$$P(T_1) = \frac{3}{4}$$

$$P(A_2 | T_1) = \frac{\frac{5}{52 \times 51}}{\frac{3}{4}} = \frac{5 \times 4}{52 \times 51 \times 3}$$

To Calculate $P(A_2)$

$$P(A_2) = P(A_2 \cap T_1) + P(A_2 \cap \overline{T_1})$$

$$P(A_2 \cap \overline{T_1}) = P(A_2 \cap \overline{T_1} \cap \overline{T_2}) + P(A_2 \cap \overline{T_1} \cap \overline{T_2})$$

$A_2 \cap \overline{T_1} \cap T_2$: Ace in the second draw, lie in the first draw and truth in the second draw.

Just like before, this implies the second card is Ace of Hearts.

Since he was lying in the first draw, the first card is not a red Ace.

$$\Rightarrow P(A_2 \cap \overline{T_1} \cap \overline{T_2}) = \frac{50}{52} \times \frac{1}{51} = \frac{50}{52 \times 51}$$

$A_2 \cap \overline{T_1} \cap \overline{T_2}$: Ace in the second draw, lie in both draws.

This implies that the second draw is not a Ace of Hearts and the first draw is not a red Ace

If the first draw was a black Ace,

$$P(A_2 \cap \overline{T_1} \cap \overline{T_2} \cap A_{B1}) = \frac{2}{52} \times \frac{2}{51} = \frac{4}{52 \times 51}$$

Else,

$$P(A_2 \cap \overline{T_1} \cap \overline{T_2} \cap \overline{A_{B1}}) = \frac{50}{52} \times \frac{3}{51} = \frac{150}{52 \times 51}$$

$$P(A_2 \cap \overline{T_1} \cap \overline{T_2}) = P(A_2 \cap \overline{T_1} \cap \overline{T_2} \cap A_{B1}) + P(A_2 \cap \overline{T_1} \cap \overline{T_2} \cap \overline{A_{B1}})$$

$$= \frac{154}{52 \times 51}$$

$$P(A_2 \cap \overline{T_1}) = \frac{50}{52 \times 51} + \frac{154}{52 \times 51} = \frac{204}{52 \times 51}$$

$$P(A_2) = \frac{5}{52 \times 51} + \frac{204}{52 \times 51} = \frac{209}{52 \times 51}$$

$$P(T_1 | A_2) = \frac{P(A_2 | T_1)\, P(T_1)}{P(A_2)}$$

$$= \frac{\frac{5 \times \cancel{4}^{1}}{52 \times 51 \times \cancel{3}_{1}} \times \frac{\cancel{3}^{1}}{\cancel{4}_{1}}}{\frac{209}{52 \times 51}} = \frac{5}{209}$$

Q7. $P(i) = c\lambda^i / i!$, $i = 0, 1, 2, \ldots$, $\lambda > 0$

a) $P(X=0) = c\lambda^0/0! = c(1)/(1) = \underline{\underline{c}}$

b) $P(X>2) = \frac{c\lambda^3}{3!} + \frac{c\lambda^4}{4!} + \frac{c\lambda^5}{5!}$

W.k.t,

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots\cdots = e^x$$

$$\Rightarrow \quad 1 + \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \cdots\cdots = e^\lambda$$

$$= \quad \lambda + \frac{\lambda^2}{2!} + \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} + \cdots\cdots = e^\lambda - 1$$

$$= \quad \frac{\lambda^3}{3!} + \frac{\lambda^4}{4!} \cdots\cdots = (e^\lambda - 1) - \left(\lambda + \frac{\lambda^2}{2!}\right)$$

$$= \quad \frac{c\lambda^3}{3!} + \frac{c\lambda^4}{4!} + \cdots = c\left[(e^\lambda - 1) - \left(\lambda + \frac{\lambda^2}{2}\right)\right]$$

$$\Rightarrow P(X>2) = \underline{\underline{c\left[e^\lambda - 1 - \lambda - \frac{\lambda^2}{2}\right]}}$$

Q8. X is a Poisson RV of parameter λ

$$\Rightarrow \quad P(X=k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

$$\frac{dP(X=k)}{d\lambda} = \frac{k\lambda^{k-1} e^{-\lambda}}{k!} - \frac{\lambda^k e^{-\lambda}}{k!} = 0$$

$$= \left(\frac{e^{-\lambda}}{k!}\right)\left(k\lambda^{k-1} - \lambda^k\right) = 0$$

$$= \left(\frac{\lambda^{k-1} e^{-\lambda}}{k!}\right)(k-\lambda) = 0$$

$$\Rightarrow \lambda = 0, \quad \lambda = k$$

$$\frac{d^2(P(X=k))}{d\lambda^2} = \frac{k(k-1)\lambda^{k-2} e^{-\lambda}}{k!} - \frac{k\lambda^{k-1} e^{-\lambda}}{k!} + \frac{k\lambda^{k-1} e^{-\lambda}}{k!} - \frac{\lambda^{k} e^{-\lambda}}{k!}$$

$\lambda = k$,

$$\frac{d^2P}{d\lambda^2} = \frac{k(k-1)k^{k-2} e^{-k}}{k!} - \cancel{\frac{k e^{-\lambda}}{k!}} + \cancel{\frac{k e^{-\lambda}}{k!}} - \frac{k^{k} e^{\lambda}}{k!}$$

$$\frac{d^2P}{d\lambda^2} = \frac{(k-1)k^{k-1} e^{-k}}{k!} - \frac{k^{k} e^{\lambda}}{k!}$$

$$= \cancel{\frac{k e^{-k}}{k!}} - \frac{k^{k-1} e^{-k}}{k!} - \cancel{\frac{k^{k} e^{\lambda}}{k!}}$$

$$\frac{d^2P}{d\lambda^2} = -\frac{k^{k-1} e^{-k}}{k!} < 0$$

$\Rightarrow$ $\underline{\lambda = k}$ is the maxima of $P(X=k)$.