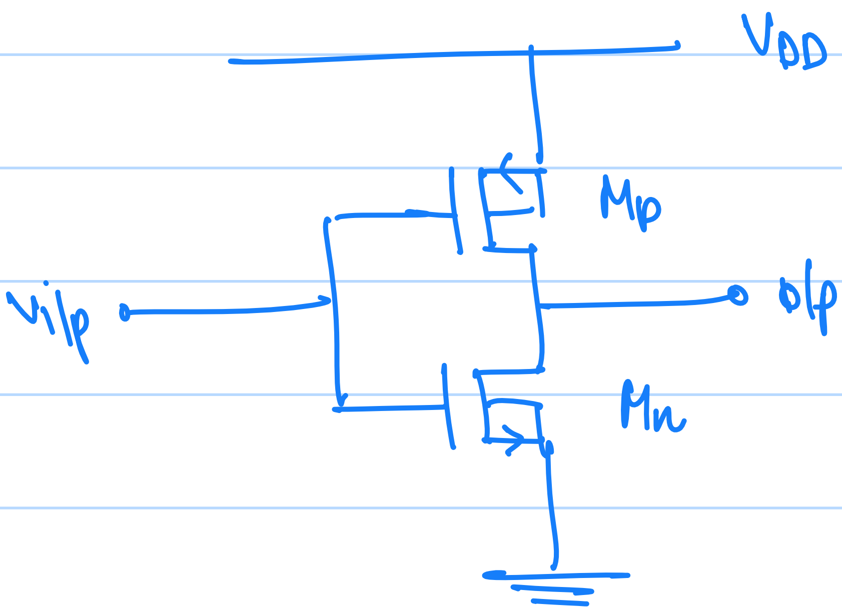


Assumptions

<u>Case 1:</u> $0 \leq V_{i/p} < V_{Tn}$

$M_n$ : $V_{GSn} = V_{i/p} - 0 = \underline{\underline{0}} < V_{Tn}$

$\therefore$ $M_n$ is in cutoff

$M_p$ : $|V_{GSp}| = |V_{i/p} - V_{DD}| = V_{DD}$

$V_{DD} > |V_{Tp}|$

$V_{DS} = V_{o/p} - V_{DD} \approx V_{DD} - V_{DD} = \underline{\underline{0}}$

$|V_{GSp}| > |V_{Tp}| \Rightarrow |V_{GSp}| - |V_{Tp}| > 0$

$\Rightarrow |V_{GSp}| - |V_{Tp}| > V_{DS}$

$\therefore$ $M_p$ is in linear mode.

Since $M_n$ is in cutoff and $M_p$ is in linear mode.

$I_D = \underline{\underline{0}} \Rightarrow V_{o/p} = V_{DD}$ (No V drop across $M_p$)

**Case 2:** $V_{Tn} \leq V_{i/p} < V_{DD}/2 \quad (V_{o/p} \approx V_{DD})$

$M_n$ : $V_{GSn} = V_{i/p} - 0 = V_{i/p} > V_{Tn}$

$V_{DSn} = V_{o/p} - 0 \approx V_{DD} \quad (V_{o/p} \approx V_{DD})$

$\Rightarrow \quad V_{DD} > V_{DD}/2 - V_{Tn} > V_{i/p} - V_{Tn}$

$\Rightarrow \quad V_{DD} > V_{i/p} - V_{Tn}$

$\Rightarrow \quad V_{DS} > V_{GS} - V_{Tn} \longrightarrow$ Saturation

$M_p$ : $|V_{GSp}| = |V_{i/p} - V_{DD}| = V_{DD} - V_{i/p} \quad (V_{DD} > V_{i/p})$

$V_{DD} - V_{i/p} \geq V_{DD} - V_{Tn} \geq |V_{Tp}| \quad (V_{DD} \geq 2V_{Tn})$

$\Rightarrow \quad |V_{GSp}| \geq |V_{Tp}|$

$V_{DSp} = V_{o/p} - V_{DD} \approx V_{DD} - V_{DD} = 0$

$\Rightarrow \quad |V_{DSp}| \leq |V_{GSp}| - |V_{Tp}| \longrightarrow$ Linear Mode

$\Rightarrow$ PMOS is in Linear , NMOS is in Saturation

$$I_{Dp} = K_p\left((V_{SG} - |V_{Tp}|)V_{SD} - \frac{1}{2}V_{SD}^2\right)$$

$$= K_p\left((V_{DD} - V_{i/p} - |V_{Tp}|)(V_{DD} - V_{o/p}) - \frac{1}{2}(V_{DD} - V_{o/p})^2\right)$$

$$I_{DN} = \frac{1}{2}K_n(V_{GS} - V_{Tn})^2$$

$$= \frac{1}{2}K_n(V_{i/p} - V_{Tn})^2$$

$$\Rightarrow \cancel{K_p}\left((V_{DD} - V_{i/p} - |V_{Tp}|)(V_{DD} - V_{o/p}) - \frac{1}{2}(V_{DD} - V_{o/p})^2\right) = \frac{1}{2}\cancel{K_n}(V_{i/p} - V_{Tn})^2$$

$$\Rightarrow \quad 2\left(V_{DD} - V_{i}|_p - |V_{Tp}|\right)\left(V_{DD} - V_{o}|_p\right) - \left(V_{DD} - V_{o}|_p\right)^2 = \left(V_{i}|_p - V_{Tn}\right)^2$$

$$V_{DD} - V_{o}|_p \longrightarrow y \qquad V_{i}|_p - V_{Tn} \longrightarrow x$$

$$\Rightarrow \quad 2\left(V_{DD} - (x + V_{Tn}) - |V_{Tp}|\right) y - y^2 = x^2$$

$$\Rightarrow \quad y^2 - 2\left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right) y + x^2 = 0$$

$$y = \frac{2\left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right) \pm \sqrt{4\left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2 - 4x^2}}{2}$$

$$= V_{DD} - x - V_{Tn} - |V_{Tp}| \pm \sqrt{\left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2 - x^2}$$

$$= V_{DD} - x - V_{Tn} - |V_{Tp}| \pm \sqrt{\left(V_{DD} - x - V_{Tn} - |V_{Tp}| - x\right)\left(V_{DD} - x - V_{Tn} - |V_{Tp}| + x\right)}$$

$$y = V_{DD} - x - V_{Tn} - |V_{Tp}| \pm \sqrt{\left(V_{DD} - V_{Tn} - |V_{Tp}|\right)\left(V_{DD} - V_{Tn} - |V_{Tp}| - 2x\right)}$$

$$V_{DD} - V_{o}|_p = V_{DD} - V_{i}|_p + \cancel{V_{Tn}} - \cancel{V_{Tn}} - |V_{Tp}| \pm \sqrt{\left(V_{DD} - V_{Tn} - |V_{Tp}|\right)\left(V_{DD} - V_{Tn} - |V_{Tp}| - 2V_{i}|_p + 2V_{Tn}\right)}$$

$$V_{o}|_p = V_{i}|_p + |V_{Tp}| \pm \sqrt{\left(V_{DD} - V_{Tn} - |V_{Tp}|\right)\left(V_{DD} - 2V_{i}|_p + V_{Tn} - |V_{Tp}|\right)}$$

Case 3: $V_{i/p} \approx V_{DD}/2$

$M_n$ : $V_{GSn} = V_{i/p} > V_{Tn}$

$V_{DSn} \approx V_{DD}/2 > V_{DD}/2 - V_{Tn} \approx V_{i/p} - V_{Tn}$

$\Rightarrow V_{DSn} > V_{GSn} - V_{Tn}$

$M_n$ is in Saturation

$M_p$ : $V_{SGp} = V_{DD} - V_{i/p} \approx V_{DD}/2$

$V_{DD}/2 > |V_{Tp}|$

$V_{SDp} \approx V_{DD} - V_{DD}/2 = V_{DD}/2 > V_{DD}/2 - |V_{Tp}|$

$\Rightarrow V_{SDp} > V_{SGp} - |V_{Tp}|$

$M_p$ is in Saturation

PMOS and NMOS are both in Saturation,

$$I_{Dp} = \frac{K_p}{2}\left(V_{SGp} - |V_{Tp}|\right)^2$$
$$= \frac{K_p}{2}\left(V_{DD} - V_{i/p} - |V_{Tp}|\right)^2$$

$$I_{Dn} = \frac{K_n}{2}\left(V_{GSn} - V_{Tn}\right)^2$$
$$= \frac{K_n}{2}\left(V_{i/p} - V_{Tn}\right)^2$$

$$I_{Dp} = I_{Dn}$$

$$\Rightarrow \frac{K_p}{\cancel{2}}\left(V_{DD} - V_{i/p} - |V_{Tp}|\right)^2 = \frac{K_n}{\cancel{2}}\left(V_{i/p} - V_{Tn}\right)^2$$

$$V_{DD} - V_{i/p} - |V_{Tp}| = \sqrt{\frac{K_n}{K_p}}\left(V_{i/p} - V_{Tn}\right)$$

$$V_{DD} - |V_{Tp}| + \sqrt{\frac{K_n}{K_p}}\, V_{Tn} = V_{i/p}\left(\sqrt{\frac{K_n}{K_p}} + 1\right)$$

$$\Rightarrow \quad V_{i/p} = \frac{V_{DD} - |V_{Tp}| + \sqrt{\frac{K_n}{K_p}}\, V_{Tn}}{\sqrt{\frac{K_n}{K_p}} + 1}$$

$$\Rightarrow \quad V_M = \frac{V_{DD} - |V_{Tp}| + \sqrt{\frac{K_n}{K_p}}\, V_{Tn}}{\sqrt{\frac{K_n}{K_p}} + 1}$$

For Saturation in PMOS and NMOS,

$V_{DSn} \geq V_{GSn} - V_{Tn}$ $\qquad$ $V_{SDp} \geq V_{SGp} - |V_{Tp}|$

$V_{o/p} \geq V_{i/p} - V_{Tn}$ $\qquad$ $V_{DD} - V_{o/p} \geq V_{DD} - V_{i/p} - |V_{Tp}|$

$\qquad V_{o/p} \leq V_{i/p} + |V_{Tp}|$

$\therefore\ V_{o/p} \in [V_{i/p} - V_{Tn},\ V_{i/p} + |V_{Tp}|]$

<u>Case 4:</u> $V_M < V_{i/p} < V_{DD} - |V_{Tp}|$ $\quad (V_{o/p} \approx 0)$

$M_n$ : $V_{GSn} = V_{i/p} > V_{Tn}$

$V_{DSn} = V_{o/p} = 0 < V_{i/p} - V_{Tn}$

$\Rightarrow V_{DSn} < V_{GSn} - V_{Tn}$

$M_n$ is in linear mode,

$M_p$ : $V_{SGp} = V_{DD} - V_{i/p}$

$V_{SDp} = V_{DD} - V_{o/p} \approx V_{DD} > V_{DD} - V_{i/p}$

$\Rightarrow V_{SDp} > V_{SGp} - |V_{Tp}|$

$M_p$ is in Saturation mode

$$I_{Dp} = \frac{K_p}{2}\left(V_{SGp} - |V_{Tp}|\right)^2$$

$$= \frac{K_p}{2}\left(V_{DD} - V_{i/p} - |V_{Tp}|\right)^2$$

$$I_{Dn} = K_n\left((V_{GS} - V_{Tn})V_{DS} - \frac{1}{2}V_{DS}^2\right)$$

$$= K_n\left((V_{i/p} - V_{Tn})(V_{o/p}) - \frac{1}{2}(V_{o/p})^2\right)$$

$$I_{Dp} = I_{Dn}$$

$$= \frac{K_p}{2}\left(V_{DD} - V_{i/p} - |V_{Tp}|\right)^2 = K_n\left((V_{i/p} - V_{Tn})(V_{o/p}) - \frac{1}{2}(V_{o/p})^2\right)$$

Take $x \longrightarrow V_{i/p} - V_{Tn}$, $y \longrightarrow V_{o/p}$

$$\Rightarrow \left(V_{DD} - (x + V_{Tn}) - |V_{Tp}|\right)^2 = 2\left(xy - \frac{1}{2}y^2\right)$$

$$= \left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2 = 2xy - y^2$$

$$\Rightarrow y^2 - 2xy + \left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2 = 0$$

$$\Rightarrow y = \frac{2x \pm \sqrt{4x^2 - 4\left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2}}{2}$$

$$= y = x \pm \sqrt{x^2 - \left(V_{DD} - x - V_{Tn} - |V_{Tp}|\right)^2}$$

$$y = x \pm \sqrt{\left(x + V_{DD} - x - V_{Tn} - |V_{Tp}|\right)\left(x - V_{DD} + x + V_{Tn} + |V_{Tp}|\right)}$$

$$y = x \pm \sqrt{\left(V_{DD} - V_{Tn} - |V_{Tp}|\right)\left(2x - V_{DD} + V_{Tn} + |V_{Tp}|\right)}$$

$$\Rightarrow V_{o/p} = V_{i/p} - V_{Tn} \pm \sqrt{(V_{DD} - V_{Tn} - |V_{Tp}|)(2V_{i/p} - 2V_{Tn} - V_{DD} + V_{Tn} + |V_{Tp}|)}$$

$$\Rightarrow V_{o/p} = V_{i/p} - V_{Tn} - \sqrt{(V_{DD} - V_{Tn} - |V_{Tp}|)(2V_{i/p} - V_{DD} - V_{Tn} + |V_{Tp}|)}$$

**Case 5:** $V_{i/p} \in (V_{DD} - |V_{Tp}|, V_{DD}]$ $(V_{o/p} = 0)$

$M_n$ : $V_{GSn} = V_{i/p}$

$V_{DSn} = 0 < V_{GSn} - V_{Tn}$

$\Rightarrow$ $M_n$ is in linear mode

$M_p$ : $V_{SGp} = V_{DD} - V_{i/p} < V_{DD} - (V_{DD} - |V_{Tp}|)$

$\Rightarrow V_{SGp} < |V_{Tp}|$

$\therefore$ $M_p$ is in cutoff

Since $M_p$ is in cutoff, $I_p = 0$.

Since $M_n$ is in linear, $V_{o/p} = 0$, (No voltage drop across $M_n$)