Assignment 4

(MA6.102) Probability and Random Processes, Monsoon 2025

Release date: 29 September 2025, Due date: 6 October 2025

INSTRUCTIONS

- Discussions with other students are not discouraged. However, all write-ups must be done individually with your own solutions.
- Any plagiarism when caught will be heavily penalised.
- Be clear and precise in your writing.

Problem 1. Consider a random variable X with the following two-sided exponential PDF

$$f_X(x) = \begin{cases} p\lambda \mathsf{e}^{-\lambda x}, & \text{if } x \geq 0, \\ (1-p)\lambda \mathsf{e}^{\lambda x}, & \text{if } x < 0, \end{cases}$$

where λ and p are scalars with $\lambda > 0$ and $p \in [0, 1]$. Find the mean and the variance of X.

Problem 2. Prove that two random variables X and Y (either both continuous or both discrete) are independent if and only if $F_{XY}(x, y) = F_X(x)F_Y(y)$, for all x, y.

Problem 3. Two points are chosen randomly and independently from the interval $[a, b]$ according to a uniform distribution. Find the expected distance between the two points.

Problem 4. (a) For any discrete random variable X and any event A such that $P(A) > 0$, show that

$$\mathbb{E}[X|A] = \frac{\mathbb{E}[\mathbb{1}_A X]}{P(A)},$$

where $\mathbb{1}_A$ is the indicator random variable of event A.

(b) X denotes the sum of outcomes obtained by rolling a die twice and A_i is the event that the first die shows i, for $i \in [1:6]$. Compute $\mathbb{E}[X|A_i]$, for $i \in [1:6]$.

Problem 5. Find the PDF, the mean and the variance of the random variable X with the CDF

$$F_X(x) = \begin{cases} 1 - \frac{a^3}{x^3}, & \text{if } x \geq a, \\ 0, & \text{if } x < a, \end{cases}$$

where a is a positive constant.

Problem 6. The joint PDF of X and Y is given by

$$f_{X,Y}(x, y) = c(y^2 - x^2)\mathsf{e}^{-y}, \; 0 < y < \infty, -y \leq x \leq y.$$

Find c and the marginal PDFs f_X and f_Y.

Problem 7. Let X_1, X_2, X_3 be independent continuous random variables with common PDF f_X. Express $P(X_1 < X_2 < X_3)$ as an integral involving f_X and evaluate its value.