LA Assignment I - 11/1/25

1. Prove that $a \cdot b = b \cdot c \Rightarrow a = c$

Ans:

$$a \cdot b + (-a) \cdot b = b \cdot c + (-a) \cdot b$$

$$\Rightarrow b \cdot (a + (-a)) = b \cdot (c + (-a)) \rightarrow (a + (-a) = 0)$$

$$\Rightarrow b \cdot 0 = b \cdot (c + (-a))$$

$$\Rightarrow b^{-1} \cdot b \cdot 0 = b^{-1} \cdot b \cdot (c + (-a))$$

$$\Rightarrow 0 = (c + (-a)) \rightarrow (b \cdot b^{-1} = 1 \ \& \ 1 \cdot x = x)$$

$$\Rightarrow a + (-a) = c + (-a)$$

$$\Rightarrow \underline{\underline{a = c}}$$

But if $b = 0$,

$$a \cdot b = 0 \ \& \ c \cdot b = 0$$

$$\Rightarrow a \cdot b = c \cdot b$$

$\therefore$ $a \cdot b = b \cdot c \Rightarrow a = c$ or $b = 0$ (The given statement is partially true)

2. Prove that $a + b = b + c \Rightarrow a = c$

Ans

$$a + b = b + c$$

$$\Rightarrow (a + b) + (-b) = (b + c) + (-b)$$

$$\Rightarrow a + (b + (-b)) = c + (b + (-b)) \text{ (Associativity)}$$

$$\Rightarrow a + 0 = c + 0$$

$\Rightarrow \quad a = c \qquad$ ($x + 0 = x$, Additive identity)

Hence Proved.

3. Prove that any subfield of $(\mathbb{C}, +, \cdot)$ must contain every rational number.

Ans Let us take a valid subfield.
Assume that $x \in \mathbb{R}$ does not belong to the subfield

If x doesn't belong, then, by closure property, no 2 rational numbers in the subfield should add up to x.

Let r be any rational number in the subfield, then clearly,

$$r + (x - r) = x$$

Since closure must be followed and r already belongs in the subfield (by definition), it must follow that $(x - r)$ does not belong in the subfield.

$\therefore \forall\ r \in \underset{\text{subfield}}{F}$ st $r \in \mathbb{Q}$, $(x - r) \notin F$

Since r is any rational number, $(x - r)$ can represent any rational number as well.

$\therefore$ The subfield cannot contain any rational number.

But if the subfield cannot contain any rational number, the elements 0 and 1 cannot be a part of the subfield

$\therefore$ The subfield is not valid, which is a contradiction with our assumption of a valid subfield

$\therefore \nexists\ a \in \mathbb{Q}$ st $a \notin F$ Hence Proved