<u>InfoComm Assignment 3</u>

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1

Given function,

$$x(t) = e^{-t/2}u(t)$$

The Fourier Transform of this function will be,

$$\begin{aligned}
X(\omega) &= \int_{-\infty}^{\infty} x(t)e^{-jwt}dt \\
&= \int_{-\infty}^{\infty} e^{-t/2}u(t)e^{-jwt}dt \\
&= \int_{0}^{\infty} e^{-t/2}e^{-jwt}dt \\
&= \int_{0}^{\infty} e^{-t(1/2+jw)}dt \\
&= \frac{-1}{\frac{1}{2}+jw}[0-1] = \frac{1}{\frac{1}{2}+jw}\left(\frac{\frac{1}{2}-jw}{\frac{1}{2}-jw}\right) = \frac{\frac{1}{2}-jw}{\frac{1}{4}+w^2} \\
\therefore X(\omega) &= \frac{0.5}{0.25+w^2} - j\left(\frac{w}{0.25+w^2}\right)
\end{aligned} \tag{1.1}$$

(a) $|X(\omega)|$ will be given by,

$$\begin{aligned}
|X(\omega)| &= \sqrt{Re(X(\omega))^2 + Im(X(\omega))^2} \\
|X(\omega)| &= \sqrt{\left(\frac{0.5}{0.25+\omega^2}\right)^2 + \left(\frac{\omega}{0.25+\omega^2}\right)^2} \\
&= \frac{\sqrt{0.25+\omega^2}}{0.25+\omega^2} \\
\therefore |X(\omega)| &= \frac{1}{\sqrt{0.25+\omega^2}}
\end{aligned}$$

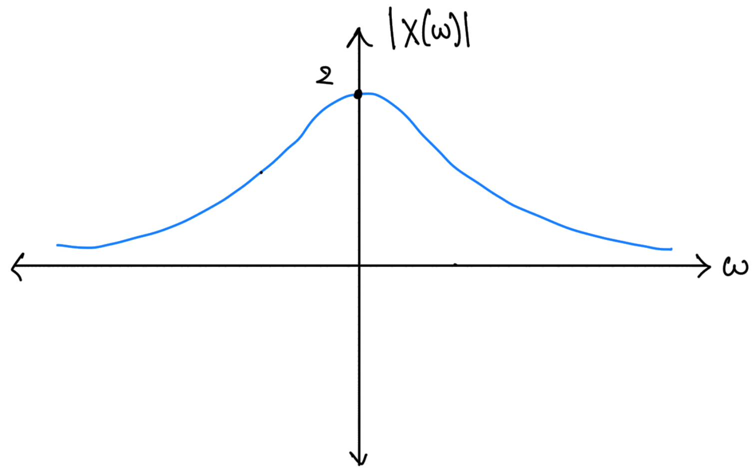

Figure 1.1: Approximate Plot of $|X(\omega)|$

(b) $\angle X(\omega)$ will be given by,

$$\begin{aligned}
\angle X(\omega) &= \tan^{-1}\left(\frac{Im(X(\omega))}{Re(X(\omega))}\right) \\
&= \tan^{-1}\left(\frac{\frac{-\omega}{0.25+\omega^2}}{\frac{0.5}{0.25+\omega^2}}\right) \\
&= \tan^{-1}\left(\frac{-\omega}{0.5}\right) \\
\therefore \angle X(\omega) &= \tan^{-1}(-2\omega)
\end{aligned}$$

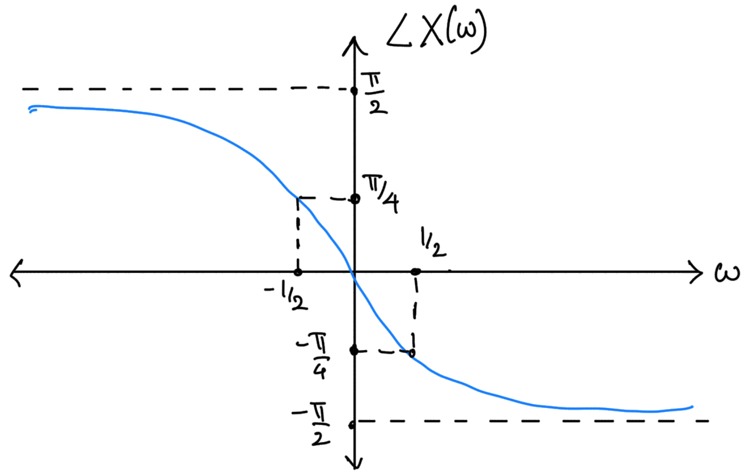

Figure 1.2: Approximate Plot of $\angle X(\omega)$

(c) $Re(X(\omega))$ is given by,

$$Re(X(\omega)) = \frac{0.5}{0.25 + \omega^2}$$

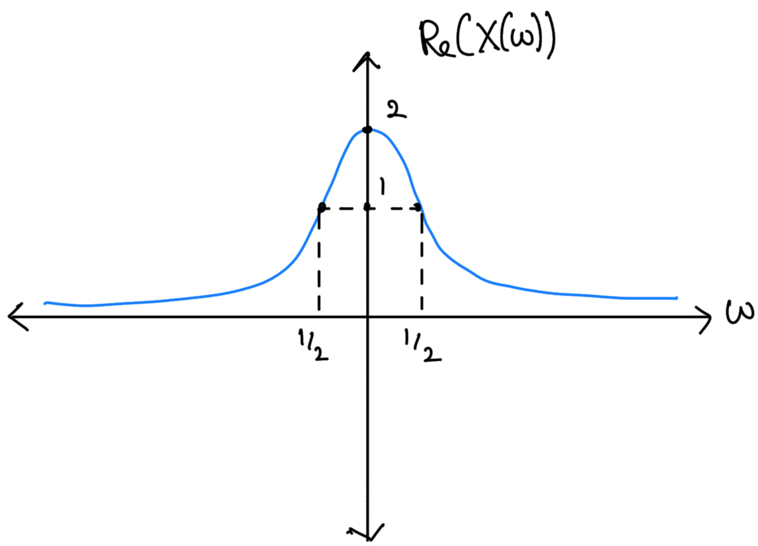

Figure 1.3: Approximate Plot of $Re(X(\omega)|)$

(d) $Im(X(\omega))$ is given by,

$$Im(X(\omega)) = \frac{-\omega}{0.25 + \omega^2}$$

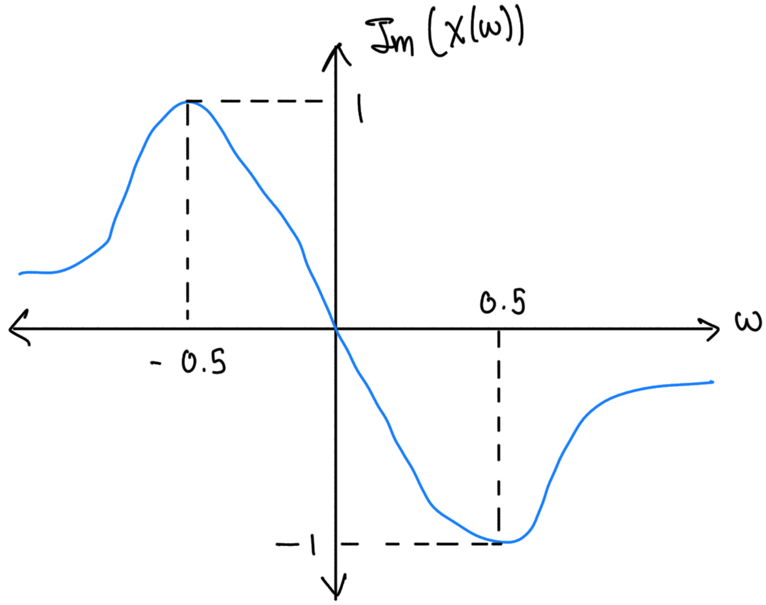

Figure 1.4: Approximate Plot of $Im(X(\omega))$

2

The function $X(\omega)$ can be represented as,

$$Re(X(\omega)) = \begin{cases} 0 & \omega \notin (-w, w) \\ 1 & \omega \in [-w, w] \end{cases}$$

$$Im(X(\omega)) = \begin{cases} 0 & \omega \notin (-w, w) \\ 1 & \omega \in [-w, w] \end{cases}$$

Using the above results, $X(\omega)$ can be defined as,

$$X(\omega) = \begin{cases} 1 + j & \omega \in [-w, w] \\ 0 & \omega \notin (-w, w) \end{cases} \tag{2.1}$$

(a) Using $X(\omega)$ as defined in 2.1, we can find phase and magnitude of $X(\omega)$ as below,

$$\begin{aligned}
|X(\omega)| &= \sqrt{Re(X(\omega))^2 + Im(X(\omega))^2} \\
\implies |X(\omega)| &= \begin{cases} 0 & \omega \notin (-w, w) \\ 2 & \omega \in [-w, w] \end{cases} \\
\angle X(\omega) &= \tan^{-1} \frac{Im(X(\omega))}{Re(X(\omega))} \\
\implies \angle X(\omega) &= \begin{cases} \frac{\pi}{4} & \omega \in [-w, w] \\ indeterminate & \omega \notin (-w, w) \end{cases}
\end{aligned}$$

(b) Using $X(\omega)$ as defined in 2.1, $X^*(\omega)$ can be defined as,

$$X^*(\omega) = \begin{cases} 1 - j & \omega \in [-w, w] \\ 0 & \omega \notin (-w, w) \end{cases} \tag{2.2}$$

If $x(t)$ is real then, $X(-\omega) = X^*(\omega)$. $\forall\ \omega \in (0, w)$ we see that,

$$\begin{aligned}
X^*(\omega) &= 1 - j \\
X(-\omega) &= 1 + j \\
\implies X^*(\omega) &\neq X(-\omega)
\end{aligned}$$

Therefore, x(t) cannot be a real signal.

3

(a) Given that the $X(f)$ is the Fourier Transform of $x(t)$. The modulation property of Fourier Transform states that,
If $Z(f)$ is the Fourier Transform of $z(t)$, then the Fourier Transform of $z(t)e^{j2\pi f_o t}$ is $Z(f - f_o)$

To find the Fourier Transform of $x(t)\cos(2\pi f_o t)$,

$$\begin{aligned}
\cos(2\pi f_o t) &= (e^{2\pi f_o t} + e^{-2\pi f_o t})/2 \\
\implies x(t)\cos(2\pi f_o t) &= x(t)(e^{2\pi f_o t} + e^{-2\pi f_o t})/2 \\
&= \frac{1}{2}(x(t)e^{2\pi f_o t} + x(t)e^{-2\pi f_o t}) \\
\therefore x(t)\cos(2\pi f_o t) &\xrightarrow{\mathcal{F}} \frac{1}{2}(X(f-f_o) + X(f+f_o))
\end{aligned}$$

(b) Given signal $x(t) = rect(t/2)$. We can rewrite $x(t)$ as,

$$x(t) = \begin{cases} 1 & t \in (-1,1) \\ 0 & otherwise \end{cases} \tag{3.1}$$

The Fourier Transform of $x(t), X(f)$, will be given by,

$$\begin{aligned}
X(f) &= \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}dt \\
&= \int_{-1}^{1} e^{-j2\pi ft}dt \\
&= \frac{-1}{j2\pi f}\left[e^{-j2\pi f} - e^{j2\pi f}\right] \\
\therefore X(f) &= \frac{1}{f\pi}\sin(2\pi f) = 2sinc(2\pi f)
\end{aligned}$$

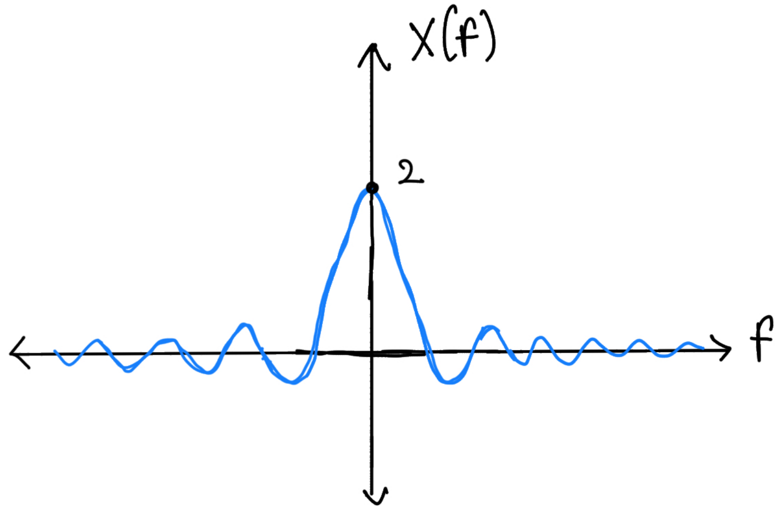

Figure 3.1: Spectrum of $X(f)$

The Magnitude spectrum $|X(f)|$ is $2sinc(2\pi f)$. Since $X(f)$ is purely real, $\angle X(f) = \tan^{-1}(0) = 0$.
By the result obtained in Question 3(a), we know that,

$$x(t)\cos(2\pi f_o t) \xrightarrow{\mathcal{F}} \frac{1}{2}(X(f-f_o)+X(f+f_o))$$

For the carrier signal, $cos(20\pi t)$, we get $f_o = 10$. Let $x(t)\cos(20\pi t) = y(t)$.

$$\begin{aligned} y(t) &\xrightarrow{\mathcal{F}} \frac{1}{2}(2sinc(2\pi(f-10)) + 2sinc(2\pi(f+10))) \\ Y(f) &= sinc(2\pi f - 20\pi) + sinc(2\pi f + 20\pi) \end{aligned}$$

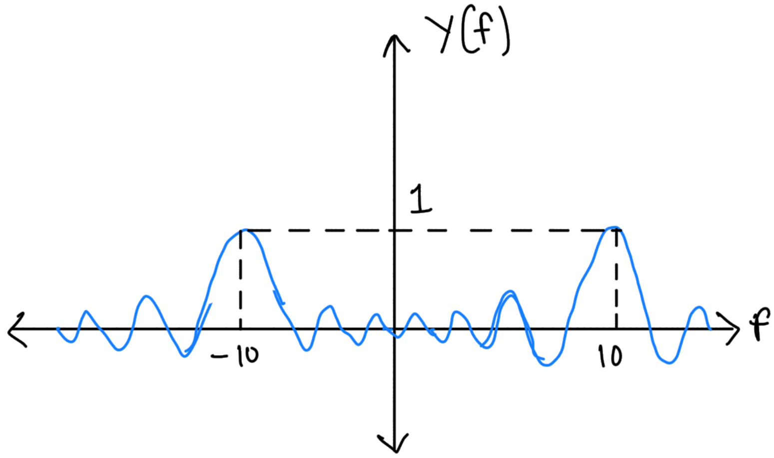

Figure 3.2: Spectrum of $Y(f)$

4

Given the error patterns e_i of [n.k], parity matrix H, and $e_i H^T$ $\forall\ i$ The given parity matrix is,

$$H_{4\times 7} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \tag{4.1}$$

We know that the order of a parity matrix is $(n-k, n)$, which will give us $n = 7, k = 3$

(a) We know that for any parity check matrix H of [n,k], we know that for any codeword c, $cH^T = \bar{0}$, and in Standard Array decoding, the recieved codeword $r = c \oplus e$

$$\begin{aligned} rH^T &= (c+e)H^T \\ &= cH^T + eH^T \\ &= eH^T \\ \implies rH^T &= eH^T \end{aligned}$$

Therefore, to decode the recieved word r, the decoding algorithm will be as follows.

1. Calculate rH^T

2. Using the result that $rH^T = eH^T$, find the corresponding eH^T and using the given table, find e
3. Calculate the codeword as $c = r + e$ ($r = c + e \implies c = r + e$ (in mod 2 addition))

This approach reduces the space required, since if there were m possible codewords and n error patterns, the Standard Array would store $m \times n$ different binary strings, but this approach would use only $2 \times n$ binary strings (n error patterns,n eH^T's)

(b) Recieved word r = [0 0 1 1 0 1 1]

$$
\begin{aligned}
rH^T &= \begin{bmatrix} 0 & 0 & 1 & 1 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\
rH^T &= \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

As per the algorithm defined in the previous question, the corresponding error pattern e will be,

$$e = [0\ 1\ 0\ 1\ 0\ 0\ 0] \tag{4.2}$$

Therefore the decoded codeword will be,

$$
\begin{aligned}
c &= r + e \\
\implies c &= [0\ 1\ 1\ 0\ 0\ 1\ 1]
\end{aligned}
$$

5

(a) Difference between BPSK and QPSK,

Property	**BPSK**	**QPSK**
Number of Bits per symbol	Has 2 different phases so can represent $\log_2(2) = 1$ bit per symbol	Has 4 different phases so can represent $\log_2(4) = 2$ bits per symbol
Phase Shifts used in Modulation	$\phi = 0, \pi$ radians, represented by Complex Envelopes 1 and -1 respectively	$\phi = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ radians, represented by Complex Envelopes $1, i, -1, -i$ respectively
Bandwidth Efficiency	Less efficient than QPSK since it can only send 1 bit per phase shift	More efficient than BPSK since it can send 2 bits per phase shift
Susceptibility to Noise	Less susceptible due to large difference between the 2 phase-shifted symbols	More susceptible than QPSK due to lesser difference between the 4 phase-shifted symbols

(b) Under similar power conditions, ie, each a symbol in BPSK has the same energy as a symbol in QPSK, therefore the distortition due to noise, per symbol, would be similar in both the modulation schemes.

As seen in (a), since each symbol in QPSK carries 2 bits, and in BPSK, each symbol carries 1 bit. Therefore, if both the modulation schemes are exposed to similar distortition, the loss of information in QPSK, per symbol, would be more.
$\therefore$ Bit error rate in QPSK is higher than bit error rate in BPSK.

6

In BPSK, the two symbols have phase shifts of 0 and π. For the carrier wave $\cos(2\pi f_c t)$, this corresponds to sinusoids,

$$\cos(2\pi f_c t) \tag{6.1}$$

$$\cos(2\pi f_c t + \pi) = -\cos(2\pi f_c t) \tag{6.2}$$

Therefore, for a bit sequence $b(t) \in \{0, 1\}$, the final BPSK modulated signal will be,

$$m(t) = \begin{cases} \cos(2\pi f_c t) & b(t) = 0 \\ -\cos(2\pi f_c t) & b(t) = 1 \end{cases} \tag{6.3}$$

The modulated signal for the bit sequence [1,0,1,1,0] will be:

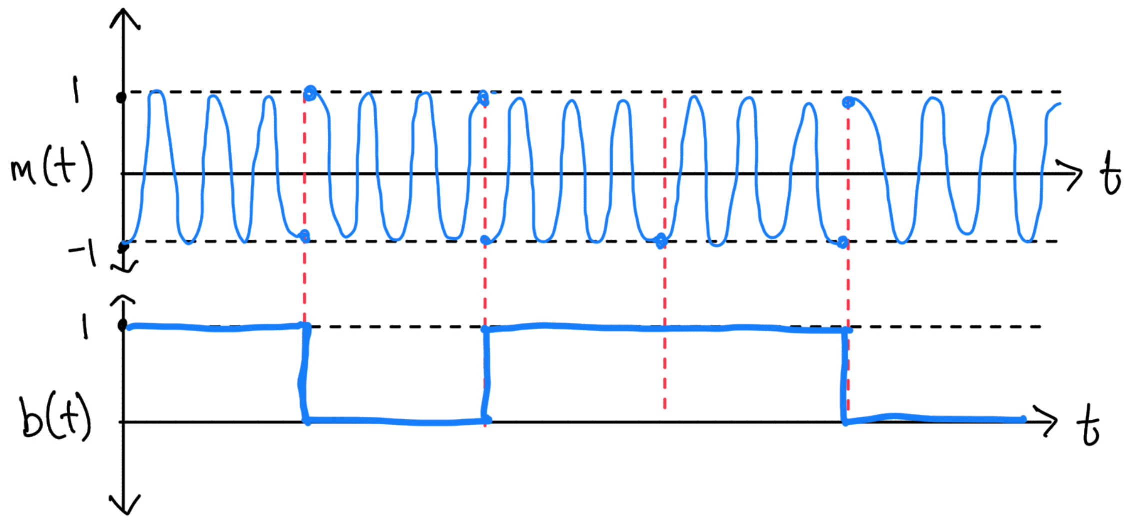

Figure 6.1: $m(t)$ for given bit sequence with the phase shifts marked in red

7

Given:
Message Signal $m(t) = 5\cos(2\pi(1000)t)$
Carrier Signal $c(t) = 10\cos(2\pi(10000)t) \implies f_c = 10000\ Hz$

(a) In DSB-SC modulation, the modulated signal (let it be $u_p(t)$) is given by,

$$\begin{aligned} u_p(t) &= m(t) \cdot c(t) \\ &= 5\cos(2\pi(1000)t) \cdot 10\cos(2\pi(10000)t) \\ &= 50/2(\cos(2\pi(1000+10000)t/2) + \cos(2\pi(1000-10000)t/2)) \\ &= 25(\cos(2\pi(5500)t) + \cos(2\pi(4500)t)) \\ \therefore u_p(t) &= 25\cos(2\pi(5500)t) + 25\cos(2\pi(4500)t) \end{aligned}$$

(b) The Bandwidth of a signal is defined as the range of frequencies a signal occupies in the spectrum.
Since $u_p(t)$ is comprised of just 2 sinusoids, of frequencies 5.5kHz and 4.5kHz, the Bandwidth of $u_p(t)$ will be $[4.5kHz,\ 5.5kHz]$

(c) We know that the Fourier Transform of a cosine is,

$$cos(2\pi f_o t) \xrightarrow{\mathcal{F}} \frac{1}{2}[\delta(f - f_o) + \delta(f + f_o)] \tag{7.1}$$

Therefore, the Fourier Transform of the 2 cosines that make up $u_p(t)$ will be,

$$25\cos(2\pi(5500)t) \xrightarrow{\mathcal{F}} \frac{25}{2}[\delta(f-5500)+\delta(f+5500)] \quad (7.2)$$
$$25\cos(2\pi(4500)t) \xrightarrow{\mathcal{F}} \frac{25}{2}[\delta(f-4500)+\delta(f+4500)] \quad (7.3)$$

By Linearity, the Fourier Transform of $u_p(t)$ will be,

$$U_p(f) = \frac{25}{2}[\delta(f-5500)+\delta(f-4500)+\delta(f+4500)+\delta(f+5500)] \quad (7.4)$$

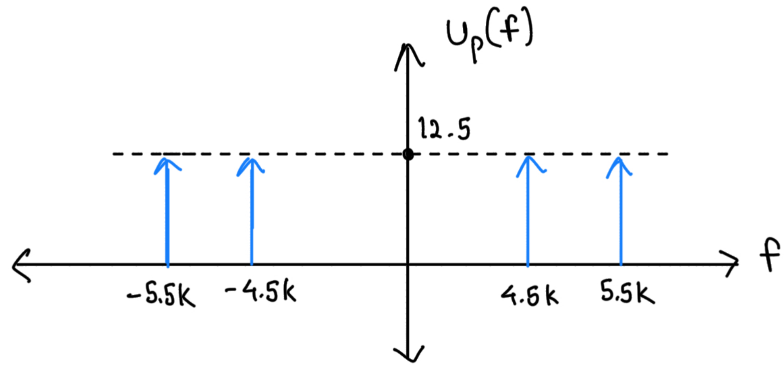

Figure 7.1: Spectrum of $u_p(t)$

8

Given DSB-SC modulation signal, $s(t) = 4\cos(2\pi 10^6 t)\cos(2\pi 10^3 t)$

(a) In DSB-SC modulation, the carrier signal is a cosine with frequency higher than the message signal. Therefore, in the given modulated signal, the carrier signal must be $A_c\cos(2\pi 10^6 t)$ and the message signal must be $A_m\cos(2\pi 10^3 t)$, where $A_c \cdot A_m = 4$.

(b) Simplifying $s(t)$,

$$\begin{aligned} s(t) &= 4\cos(2\pi 10^6 t)\cos(2\pi 10^3 t) \\ &= \frac{4}{2}(\cos(2\pi(1000000+1000)/2+\cos(2\pi(1000000-1000)/2))) \\ &= 2(\cos(2\pi(55000)t)+\cos(2\pi(45000)t)) \end{aligned}$$

Using the result mentioned in 7(b), we get the Bandwidth of $s(t)$ as $[45kHz,\ 55kHz]$