EC5.102: Information and Communication

(Lec-11)

Modulation-2

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Recap

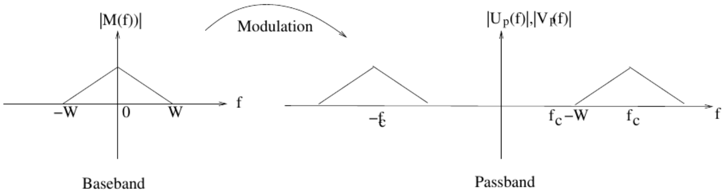
Signals and systems: Pre-requisites

- Complex numbers, complex signals
- Fourier transform (FT)
- What is FT of $\cos(2\pi f_0 t)$?
- Modulation property of FT
- Fourier transform of a real valued signal is conjugate symmetric.
- Baseband vs passband signal

Proofs of these properties will be discussed in the tutorial.

Key idea in modulation

- How to design a (high frequence signal) passband signal to "carry" information contained in the (low frequence signal) baseband signal?



- How do it? Multiply $m(t)$ it by a sinusoid at f_c.
- If we use both cosine & sine carriers, we can construct a passband signal

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$$

$u_c(t)$ and $u_s(t)$ are real baseband signals of bandwidth at most W, $f_c > W$.

- Modulation consist of encoding the message $m(t)$ in $u_c(t)$ & $u_s(t)$.

DSB-SC Amplitude modulation

DSB-SC Amplitude modulation

- Recall: A passband signal $u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t)$
- The message $m(t)$ modulates the I-component of the passband signal:

$$u_{DSB}(t) = Am(t)\cos(2\pi f_c t)$$

- As the name suggests, the amplitude of the carrier is varied according to the amplitude of the message.
- After taking FT,

$$U_{DSB}(f) = \frac{A}{2}(M(f - f_c) + M(f + f_c))$$

- Example-1: $m(t) = A_m \cos(2\pi f_m t)$
- Example-2: Arbitrary baseband $m(t)$

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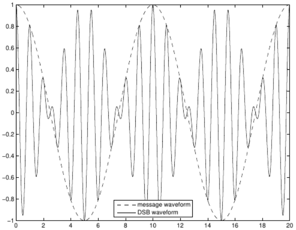
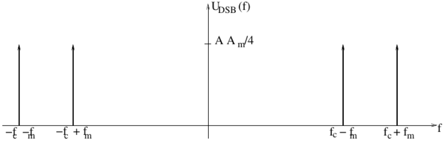


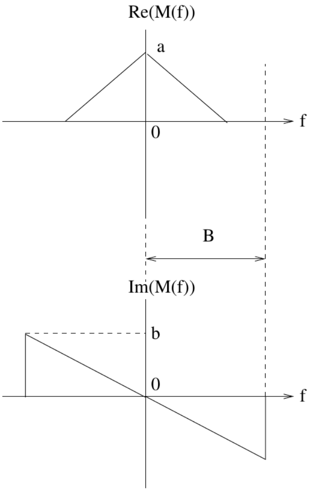
(a) DSB time domain waveform



(b) DSB spectrum

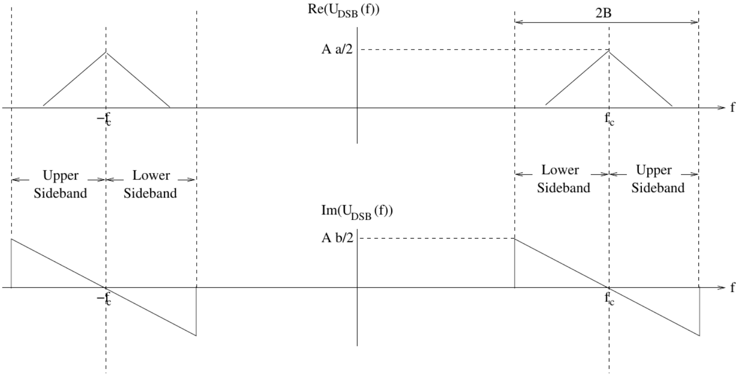
DSB-SC signal in the time and frequency domains for $m(t) = A_m \cos(2\pi f_m t)$

Example-2: Arbitrary basesband $m(t)$



Example message spectrum

Example-2: Arbitrary basesband $m(t)$



The spectrum of the passband DSB-SC signal for the message on previous slide

Comments: DSB-SC

- If $m(t)$ has a bandwidth of B, $u_{DSB}(t)$ has a bandwidth of $2B$.
- Why the name "double-side band"?
  - In some sense we have sent two bands: Upper side band & lower side band.
- Note: Information resides in one of the band and hence we are wasting bandwidth. Is it fine if we just transmit single-side band? SSB-SC
- Why the name "supressed carrier"?
  - If $m(t)$ has zero DC value, i.e, $M(0) = 0$, then there is no component at f_c.
  - So in such cases, the carrier frequency is suppressed. Hence the name suppressed carrier.
- Conventional AM: Carrier is not suppressed (Not going to discuss)
- How to demodulate DSB-SC signal? (Not going to discuss)

Comments: Analog modulation

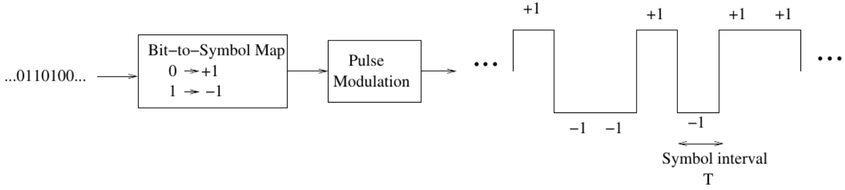
- Message: Analog signal, Carrier: Analog signal
- Other variants of AM: Single side band, Vestigial side band modulation, Conventional AM
- FM: Frequency modulation
  - "Frequency" of the carrier is varied according to message signal
- PM: Phase modulation
  - "Phase" of the carrier is varied according to message signal
- These are the first set of modulation schemes invented in early 1900's.
- Note: Key ideas of analog modulation schemes are also used in digital modulation schemes!!

Digital modulation

Our focus (BPSK, QPSK)

Pulse modulation

- Recall: Pulse modulation



- Mathematical representation of pulse modulation:

$$u(t) = \sum_n b[n]p(t - nT), \qquad (1)$$

  where each $b[n] \in \{+1, -1\}$ and $p(t)$ is the "modulating" pulse.

- Note: Waveform $u(t)$ in Eq. (1) is a baseband signal!
- Question: How to convert it to a passband signal?

BPSK

- Aim: Convert $u(t) = \sum_n b[n]p(n - nT)$ to a passband signal?
- One easy approach would be to send the passband signal

$$u_p(t) = u(t)\cos(2\pi f_c t)$$

- Picture for this modulation.
- Observe:

$$u_p(t) = \begin{cases} \cos(2\pi f_c t) \text{ if } b[n] = +1 \\ -\cos(2\pi f_c t) \text{ if } b[n] = -1 \end{cases}$$

- Since the phase of the carrier switches between two values 0 and π, this modulation scheme is termed Binary Phase Shift Keying (BPSK).

Complex envelop of a passband signal

- Recall: Passband signal is given by

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t).$$

- The complex envelope of $u_p(t)$ is given by $u(t) = u_c(t) + ju_s(t)$.
- One can consider an equivalent complex envelope: $b[n] = b_c[n] + jb_s[n]$
  - $b_c[n]$ will modulate the I-component
  - $b_s[n]$ will modulate the Q-component
- For BPSK, we have $b[n] \in \{+1, -1\}$. What will be the I and Q components for BPSK?
- We will not always ingore Q-component: Example QPSK

QPSK

- Recall: The equivalent complex envelope: $b[n] = b_c[n] + jb_s[n]$
- Let us see what happens to the passband signal when $b_c[n], b_s[n]$ each take values in $\{+1, -1\}$.

$$u_p(t) = u_c(t)\cos(2\pi f_c t) - u_s(t)\sin(2\pi f_c t).$$

with

$$u_c(t) = \sum_n b_c[n]p(t - nT) \quad \text{and} \quad u_s(t) = \sum_n b_s[n]p(t - nT)$$

- What will be $u_p(t)$ if $b_c[n] = +1$ and $b_s[n] = +1$? Similarly for other values.
- Since the phase of the carrier switches between four values $\pi/4, \pi/4, 3\pi/4$ and $-3\pi/4$, this is termed Quadrature Phase Shift Keying (QPSK).
- Constellations for BPSK, QPSK, 8-PSK, 2^m-PSK