

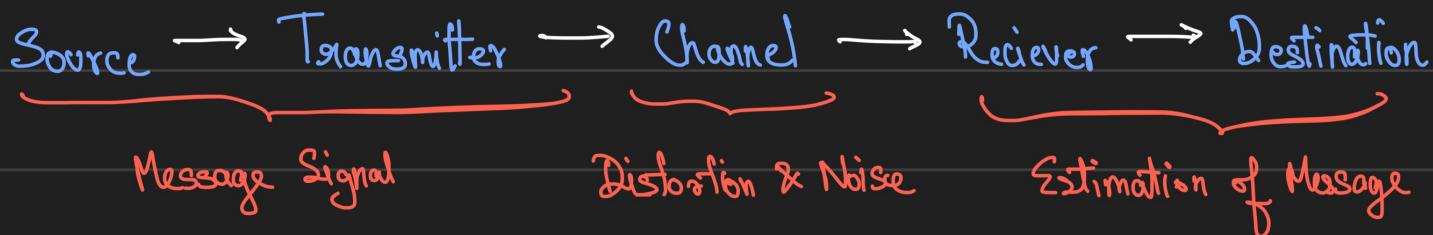
Communication Theory

5/1/26

→ Communication :

The transmission of information from a source to a destination via a transmission medium.

- Information transfer can be across space or time (time → storage)



→ Channel :-

- Guided Channels - Point to point channel. ex: wired communication, waveguide.

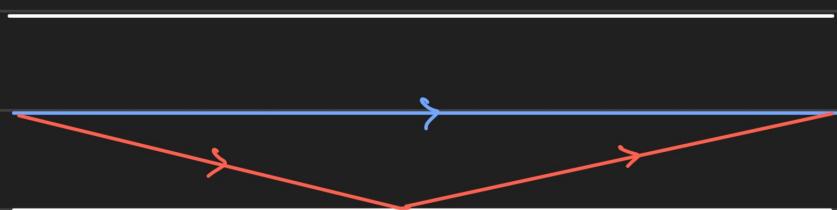
• Challenges :-

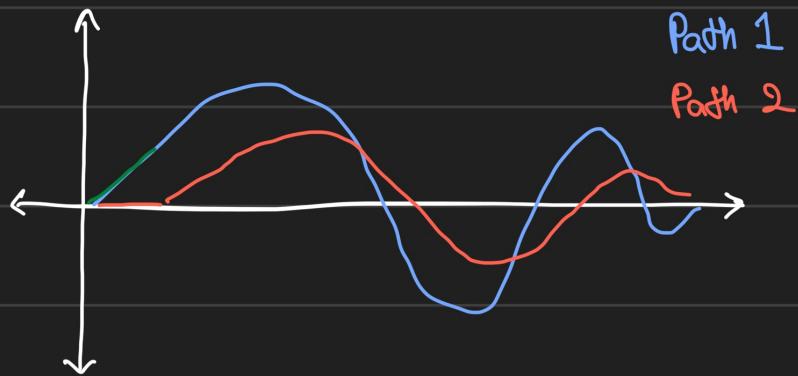
- 1) Spectrum : A channel can allow only a finite bandwidth of frequencies, given by its frequency response.

- The finite bandwidths exist for both wired and wireless channels
- Low Radio Frequencies : Large Antenna Heights
High Radio Frequencies : High attenuations (abv 5 GHz → point 2 point)
- The wireless spectrum is also divided between different technologies and applications. ex:
AM : 0.535 - 1.7 MHz

2) Noise: A channel can also distort the signal

- Can be thermal, man-made EMI, powerline interferences.
- Due to the random nature, it is modelled as AWGN (additive white gaussian noise)
- Multipath is also a problem,





3) Time Variance :-

We often model channels as LTI systems, ie, linear and time-invariant systems.

- But most channels, especially wireless, are time variant.

• Channel Capacity :-

The capacity of a AWGN channel is given by,

$$C = B \log_2 (1 + SNR)$$

B - Channel Bandwidth, $SNR = P/N$ (Signal to Noise Ratio)

P - Signal Power, N - Noise Power

→ Communication Systems :-

- Analog Message Signal - Continuous-time signals which takes a continuous range of values.
- Digital Message Signal - Discrete-time signal with a discrete range of values.
- Analog Communication Systems :-

Source → Modulator → Channel → Demodulator → Destination

- Modulation is the shifting of the signal to a higher frequency for transmission and multiplexing.
(Multiplexing - Different signals are shifted to different bands so that they can be transmitted at once)
- Analog communication is the obvious thing to do, but it is not very efficient.

Digital Communication Systems :-

- The message signal is digital, i.e., a signal that can be converted into 0's and 1's. (need not be binary signals always)

Information Source → Source Encoder → Channel Encoder → Modulator



- Source Coding: Obtains digital representation of the information and removes any redundancy within the information. ex: Compression
- Channel Coding: Adds redundancy to the signal for error correction due to channel noise and distortion. ex: Repetition Coding
- Digital Modulation: Converts the bit-stream into a waveform suitable for modulation.
- Digital systems are robust against distortion and noise.

- Digital systems can also use regenerative repeaters, i.e., devices within the channel that removes the distortion until that point (similar to resetting the signal to its original form)
- Digital Systems can do error correction using Shannon's theorem, and they can also be encrypted.
- The processor devices are digital as well.

8/1/26

Information Theory - Source Coding

Error Correcting Codes - Channel Coding

Wireless Communication - Modulation & Transmission

★ Logistics :-

◦ Textbook :- Intro. To Comm. Systems , U. Madhow

◦ Grading :-

Classwork + Surprise Quiz + Q1 + Q2 - 20%

Midsem - 20 %

Assignment and Quir - 20 %

Endsem - 40 %.

* 2: Overview of Signals and Systems :-

- The notation for the step function is given as $\mathbb{I}_{[0,\infty)}$, ie, an indicator function.
- Sinusoids : $A \cos(2\pi f_0 t + \theta)$. Modulation varies A, f_0 and θ to convey information. (If A, f_0, θ are known, then no info can be passed)
- Impulse Response - Response of the system to an impulse signal.
Used to study LTI systems.
- Rectangular波形 :

$$s(t) = A_c \cos(2\pi f_0 t) - A_s \sin(2\pi f_0 t)$$

Where ,

$$A_c = A \cos \theta, \quad A_s = A \sin \theta$$

Using Euler's formula,

$$A e^{j\theta} = A \cos \theta + j A \sin \theta = A_c + j A_s$$

$$A = \sqrt{A_c^2 + A_s^2}, \quad \theta = \tan^{-1}(A_s/A_c)$$

• Complex Exponentials: $A e^{j(2\pi f_0 t + \theta)} = \alpha e^{j2\pi f_0 t}$, $\alpha = A e^{j\theta}$

These are Eigensignals to any LTI system.

Usually, α is needed to encode the information and $e^{j2\pi f_0 t}$ is known (carrier wave)

• Inner Product :-

$$= \gamma^H s \quad (\text{H - Hermitian} \mid \text{Conj. Transpose})$$

$$\langle s, r \rangle = \sum_{i=-\infty}^{\infty} s[i] \gamma^*[i] \quad (\text{For digital signals})$$

$$\langle s, r \rangle = \int_{-\infty}^{\infty} s(t) \gamma^*(t) dt \quad (\text{For analog signals})$$

• Energy of a Signal :-

$$E_s = \|s\|^2 = \langle s, s \rangle = \int_{-\infty}^{\infty} s(t) \bar{s}(t) dt$$

Q. Find the energy of :

$$i) \quad s(t) = 2\mathbb{1}_{[0, T]} + j\mathbb{1}_{[T/2, 2T]}$$

$$ii) \quad s(t) = e^{-3|t| + j2\pi t}$$

$$i) s(t) = 2\mathbb{1}_{[0, \frac{T}{2}]} + j\mathbb{1}_{[\frac{T}{2}, T]}$$

$$= \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ 2+j, & t \in [\frac{T}{2}, T] \\ j, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow |s(t)| = \begin{cases} 2, & t \in [0, \frac{T}{2}] \\ \sqrt{3}, & t \in [\frac{T}{2}, T] \\ 1, & t \in [T, 2T] \end{cases}$$

$$\Rightarrow E_g = \int_0^{\frac{T}{2}} 2^2 dt + \int_{\frac{T}{2}}^T \sqrt{3}^2 dt + \int_T^{2T} 1^2 dt$$

$$= 4\left(\frac{T}{2}\right) + 3\left(T - \frac{T}{2}\right) + 1(2T - T)$$

$$= 2T + \frac{3T}{2} + T$$

$$= \frac{11T}{2}$$

Power of a Signal :-

The power of a signal is defined as the time average of its energy computed over a large time interval.

$$P_s = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |s(t)|^2 dt = \overline{|s(t)|^2}$$

o DC Value :-

The DC value of a signal is the value of the Fourier Transform of the signal at $f = 0$.

Q. Compute P_s and DC value for a real sinusoid $s(t) = A \cos(2\pi f_0 t + \theta)$
where $A > 0$, $\theta \in [0, 2\pi]$, $f_0 \in \mathbb{R}$

$$P_s = \lim_{T_0 \rightarrow \infty} \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} A \cos(2\pi f_0 t + \theta) dt \quad \text{ill do it later}$$

12/1/26

★ 3 : Baseband and Passband Representations :-

o Multiplexing : Encoding multiple pieces of information in the same signal using different freq. ranges.

→ Fourier Transform :-

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} dw$$

- Poornima's Identity :-

$$\|v\|^2 = \langle v, v \rangle = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_{-\infty}^{\infty} |V(j\omega)|^2 d\omega$$

- Shared Medium :-

- Many practical channels are shared medium, ie, used by multiple devices at once.
- Time Division Multiple Access (TDMA) : Divides the multiple information into packets, which are then sent at different instances in time.
ie, Each device is given a time slot within which they can communicate.
- Frequency Division Multiple Access (FDMA) : Assigns each type of information its own frequency band, ie, each device is allowed to communicate in a particular bandwidth only.

Device \triangleq Transmitter - Receiver Pair

- We can perform TDMA and FDMA at the same time (ex: Statistical Multiplexing) to increase the user capacity of the medium.

\rightarrow Energy Spectral Density and Bandwidth :-

- ESD ($E_v(f)$): For a signal $v(t)$, the ESD $E_v(f)$ is the energy of the output of the filter divided by the width Δf in the limit $\Delta f \rightarrow 0$
 - ↪ filter of width Δf , centered at f .

$$v(f) \xrightarrow{\boxed{H(f)}} y(f) = v(f) H(f)$$

↪ $H(f) = \text{Shifted root of width } \Delta f$
centered at f

Prove that $E_v(f) = |v(f)|^2$

$$\begin{aligned} \int_{-\infty}^{\infty} |y(t)|^2 dt &= \int_{-\infty}^{\infty} |y(f)|^2 df = \int_{-\infty}^{\infty} |v(f) H(f)|^2 df \\ &= \int_{f - \frac{\Delta f}{2}}^{f + \frac{\Delta f}{2}} |v(f)|^2 df \approx |v(f)|^2 \Delta f \quad (\text{for small } \Delta f) \end{aligned}$$

$$\Rightarrow E = |v(f)|^2 \Delta f$$

$$\Rightarrow \underline{E_v(f) = |v(f)|^2}$$

o Bandwidth :-

- For bandlimited signals, bandwidth is the range of frequencies beyond which the spectrum of the signal is zero.
- For real-valued signals, $U(-f) = U^*(f)$. So only positive frequencies count for real valued signals, the bandwidth is one sided.
- For non-bandlimited signals, the energy-containment bandwidth is the size of the smallest band that contains a certain fraction of the signal's energy. (Usually 99%)
- Or, it is the band over which $U(f)$ stays within a certain range about its peak value (ex:- 3dB bandwidth).

Note: $W \sin(Wt) \xleftrightarrow{f} \mathbb{1}_{[-\frac{W}{2}, \frac{W}{2}]}(f)$

Find the bandwidth of 1) $u(t) = \sin(2t)$ 2) $u(t) = \mathbb{1}_{[2,4]}(t)$, t is in microseconds.

1) $u(t) = \sin(2t) \Rightarrow U(f) = \frac{1}{2} \mathbb{1}_{[-1,1]}(f)$

\Rightarrow Bandwidth of the signal = $[-1, 1]$ MHz.

2) $v(t) = \mathbb{1}_{[0,4]}(t)$

Let $x(t) = v(t+3) = \mathbb{1}_{[-1,1]}(t)$

$\Rightarrow X(f) = 2 \operatorname{sinc}(2f)$

$\Rightarrow V(f) = 2 \operatorname{sinc}(2f) e^{-j6\pi f}$

$\Rightarrow |V(f)| = |X(f)|$

$$E_v = \int_{-\infty}^{\infty} |V(f)|^2 df = \int_{-\infty}^{\infty} |v(t)|^2 dt = \int_2^4 dt = 2$$

$\Rightarrow 99\% \text{ of } E_v = 1.98$

$$\Rightarrow 2 \int_0^{\omega} 2 |\operatorname{sinc}(2f)|^2 df = 1.98 \xrightarrow{\text{on solving}} \underline{\omega = 5.1 \text{ MHz}}$$

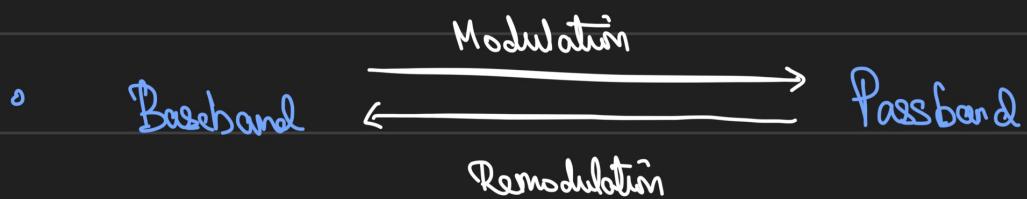
- Any analysis that needs to occur in passband, the same analysis can be carried out by shifting the signal into baseband.

Baseband - Spectrum centered around DC ($f=0$)

Passband - Spectrum centered around a non-zero frequency.

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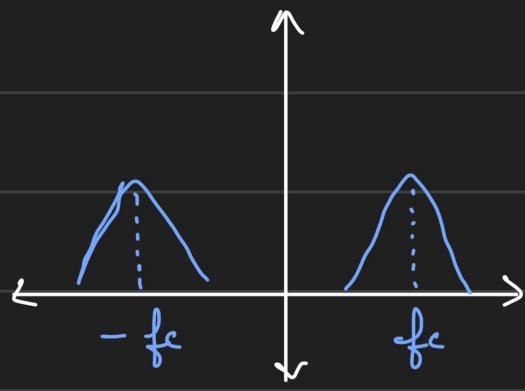
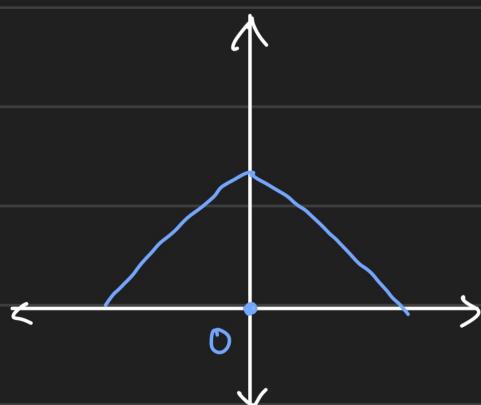
- Calculations are easier in baseband since it can be sampled into a digital signal at a low sampling frequency and passed into a general purpose DSP.



→ Modulation and Demodulation :-

- Method 1: Multiplication with a sinusoid,

$$v_p(t) = v(t) \cos(2\pi f_c t) \longleftrightarrow \frac{1}{2} (v(f-f_c) + v(f+f_c))$$



For a sine wave,

$$v_p(t) = v(t) \sin(2\pi f_c t) \longleftrightarrow \frac{1}{2j} (M(f-f_c) - M(f+f_c))$$

- Using both the sine and cosine carrier waves,

$$v_p(t) = \underbrace{v_c(t) \cos(2\pi f_c t)}_{\text{in-phase component}} - \underbrace{v_s(t) \sin(2\pi f_c t)}_{\text{quadrature component}}$$

$v_c(t)$ and $v_s(t)$ are different baseband signals of bandwidth W , and $f_c > W$. They hold all the information that is to be transmitted.

The carrier waves are determinate, ie, cannot transfer information.

- The inphase (I) and quadrature (Q) components are orthogonal, so the signal can carry 2 independent information at the same time (one in I, one in Q)

Mathematically, if $a(t) = v_c(t) \cos(2\pi f_c t)$, $b(t) = v_s(t) \sin(2\pi f_c t)$, then,

$$\langle a(t), b(t) \rangle = 0$$

($a, b \in \mathbb{R}$ and continuous)

Proof:

$$\langle a(t), b(t) \rangle = \int_{-\infty}^{\infty} a(t) b^*(t) dt = \int_{-\infty}^{\infty} a(t) b(t) dt$$

$$a(t) b(t) = v_c(t) v_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$x(t) = a(t) b(t) = \underbrace{v_c(t) v_s(t)}_{p(t)} \sin(4\pi f_c t) \cdot \frac{1}{2}$$

$$x(t) = \frac{p(t) \sin(4\pi f_c t)}{2} \quad \xleftrightarrow{\text{FT}} \quad P \frac{(f - 2f_c) + P(f + 2f_c)}{4j}$$

$$P(f) = v_c(f) * v_s(f)$$

If v_c has bandwidth W_1 and v_s has bandwidth W_2 , then

P has bandwidth $W_1 + W_2$.

$$f_c > W_1, f_c > W_2 \Rightarrow 2f_c > W_1 + W_2$$

$\Rightarrow P(f - 2f_c), P(f + 2f_c)$ are passband spectra.

$$\Rightarrow \underline{x(0) = 0} \Rightarrow \int_{-\infty}^{\infty} x(t) dt = 0$$

$$\Rightarrow \int_{-\infty}^{\infty} a(t) b(t) dt = 0$$

$$\Rightarrow \underline{\langle a(t), b(t) \rangle = 0}$$

- To demodulate the passband signal, we can again multiply the signal by the same sinusoid and passing the output through a LPF.

$$v_p(t) = v_c(t) \cos(2\pi f_c t) - v_s(t) \sin(2\pi f_c t)$$

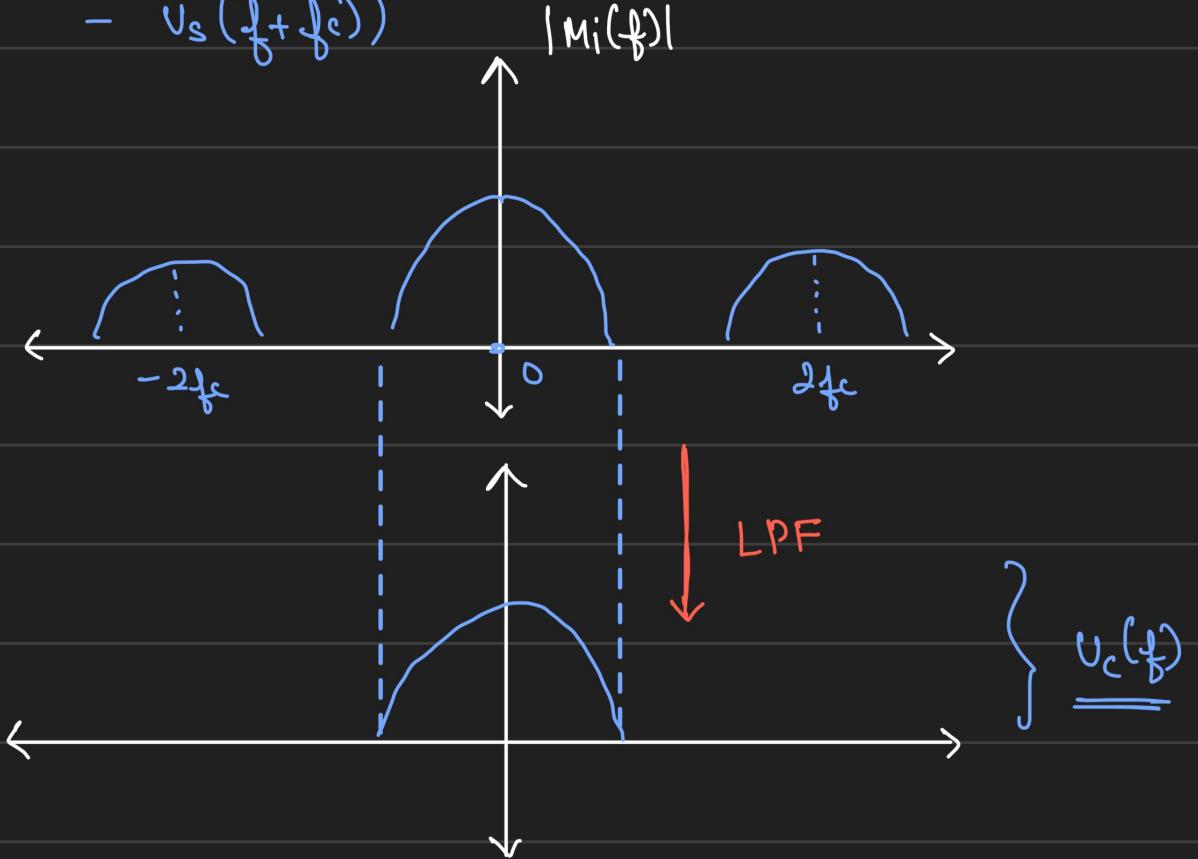
$$m_i(t) = 2v_p(t) \cos(2\pi f_c t)$$

$$= 2v_c(t) \cos^2(2\pi f_c t) - 2v_s(t) \sin(2\pi f_c t) \cos(2\pi f_c t)$$

$$= 2v_c(t) \left(1 + \frac{\cos(4\pi f_c t)}{2} \right) - v_s(t) \sin(4\pi f_c t)$$

$$= v_c(t) + v_c(t) \cos(4\pi f_c t) - v_s(t) \sin(4\pi f_c t)$$

$$\Rightarrow M_i(f) = v_c(f) + \frac{1}{4} (v_c(f-2f_c) + v_c(f+2f_c)) + \frac{1}{4j} (v_s(f-f_c) - v_s(f+f_c))$$



(11 by

$$\begin{aligned} m_q(t) &= -2j v_p(t) \sin(2\pi f_c t) \\ &= -2j (v_c(t) \cos(2\pi f_c t) \sin(2\pi f_c t) - v_s(t) \sin^2(2\pi f_c t)) \\ &= -j v_c(t) \sin(4\pi f_c t) + v_s(t)(1 - \cos(4\pi f_c t)) \end{aligned}$$

$$\Rightarrow M_q(f) = -j \left(\frac{1}{4j} (v_c(f-2f_c) - v_c(f+2f_c)) \right. \\ \left. + v_s(f) - \frac{1}{4} (v_s(f-2f_c) + v_s(f+2f_c)) \right)$$

↓ LPF

$$M_q(f) = v_s(f)$$

° $v_p = (v_c, v_s)$, is termed as two-dimensional modulation.

° Using polar coordinates,

$$e(t) = \sqrt{v_c^2(t) + v_s^2(t)} \rightarrow \text{Envelope}$$

$$\theta(t) = \tan^{-1} \left(\frac{v_s(t)}{v_c(t)} \right) \rightarrow \text{Phase}$$

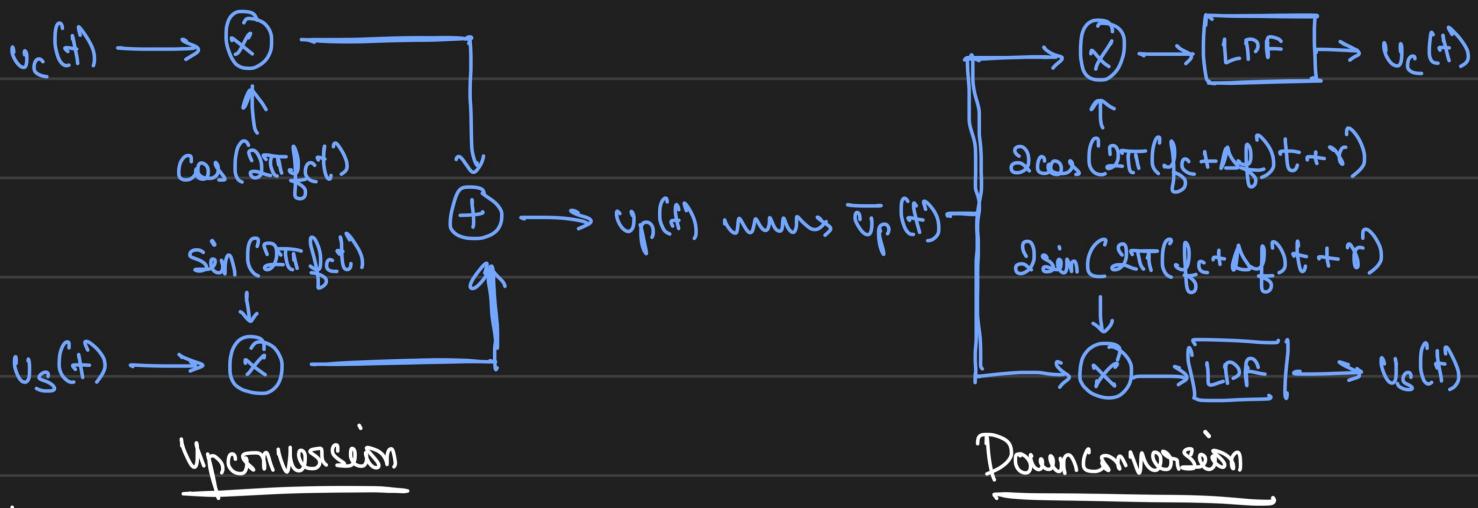
$$\Rightarrow v(t) = e(t) \cos(2\pi f_c t + \theta(t)) \longrightarrow \text{Polar Envelope.}$$

$$\Rightarrow v(t) = v_c(t) + j v_s(t) = e(t) \exp(j \theta(t)) \rightarrow \text{Complex Envelope.}$$

The passband signal $v_p(t)$ can be expressed as $v_p(t) = \operatorname{Re}(v(t) e^{j 2\pi f_c t})$

$$\begin{aligned}
 \operatorname{Re} \left\{ v(t) e^{j 2\pi f_c t} \right\} &= \operatorname{Re} \left\{ (v_c(t) + j v_s(t)) (\cos(2\pi f_c t) + j \sin(2\pi f_c t)) \right\} \\
 &= \underbrace{v_c(t) \cos(2\pi f_c t) - v_s(t) \sin(2\pi f_c t)}_{= v_p(t)} = v_p(t)
 \end{aligned}$$

Effect of Frequency and Phase Offset :-



22/11/26

Let $\phi(t) = 2\pi\Delta f t + \gamma$, then

$$\begin{aligned}
 v_p(t) &\approx \cos(2\pi f_c t + \phi(t)) \\
 &= 2 \cos(2\pi f_c t + \phi(t)) (v_c(t) \cos(2\pi f_c t) - v_s(t) \sin(2\pi f_c t)) \\
 &= 2v_c(t) \cos(2\pi f_c t + \phi(t)) \cos(2\pi f_c t) - 2v_s(t) \cos(2\pi f_c t + \phi(t)) \sin(2\pi f_c t) \\
 &= v_c(t) (\cos(4\pi f_c t + \phi(t)) + \cos \phi(t)) - v_s(t) (\sin(4\pi f_c t + \phi(t)) - \sin \phi(t))
 \end{aligned}$$

$\downarrow \text{LPF}$

$$v_c(t) \cos \phi(t) + v_s(t) \sin \phi(t) = \tilde{v}_c(t)$$

$$\text{By } \tilde{v}_s(t) = -v_c(t) \sin \phi(t) + v_s(t) \cos \phi(t)$$

• The spectrum of $v_p(t) = \operatorname{Re} \{ v(t) e^{j 2\pi f_c t} \}$ is,

$$\begin{aligned} v_p(t) &= \operatorname{Re} \{ v(t) e^{j 2\pi f_c t} \} \\ &= \frac{1}{2} \left(v(t) e^{j 2\pi f_c t} + (v(t) e^{j 2\pi f_c t})^* \right) \\ &= \frac{1}{2} \left(v(t) e^{j 2\pi f_c t} + v^*(t) e^{-j 2\pi f_c t} \right) \end{aligned}$$

Conjugate

$$\Rightarrow v_p(f) = \frac{1}{2} (v(f - f_c) + v^*(-f + f_c))$$

$$= v_p(f) = \frac{v(f - f_c) + v^*(-f - f_c)}{2}$$

Q. Write the proof for $\tilde{v}_s(t)$

$$\begin{aligned} \tilde{v}_s(t) &= -2v_p(t) \sin(2\pi f_c t + \phi(t)) \\ &= -2(v_c(t) \cos(2\pi f_c t) - v_s(t) \sin(2\pi f_c t)) \sin(2\pi f_c t + \phi(t)) \\ &= -2v_c(t) \cos(2\pi f_c t) \sin(2\pi f_c t + \phi(t)) + 2v_s(t) \sin(2\pi f_c t) \sin(2\pi f_c t + \phi(t)) \\ &= -v_c(t) \left(\sin(4\pi f_c t + \phi(t)) + \sin \phi(t) \right) + v_s(t) \left(\cos \phi(t) + \cos(4\pi f_c t + \phi(t)) \right) \end{aligned}$$

\downarrow
LDF

$$\Rightarrow -v_c(t) \sin \phi(t) + v_s(t) \cos \phi(t) = \tilde{v}_s(t)$$

• Conversion b/w Passband and Baseband :-

- Baseband to Passband :-

$v(t) \longrightarrow$ Original Complex Signal.

$$c(t) = v(t) e^{j2\pi f_c t}, \quad c^*(t) = v^*(t) e^{-j2\pi f_c t} = c_2(t)$$

$$\Rightarrow C(f) = V(f - f_c), \quad c_2(f) = V^*(-f - f_c)$$

$$\Rightarrow v_p(t) = \frac{1}{2}(c(t) + c^*(t)) \Rightarrow V_p(f) = \frac{1}{2} (V(f - f_c) + V^*(-f - f_c))$$

- Passband to Baseband :-

$V_p(f) \longrightarrow$ Spectrum of Modulated Signal.

$$C(f) = 2V_p^+(f) = \begin{cases} 2V_p(f), & f > 0 \\ 0, & \text{o.w.} \end{cases}$$

$$v(t) = c(t) e^{-j2\pi f_c t} \Rightarrow V(f) = C(f + f_c) = 2V_p^+(f + f_c)$$

o Passband Filtering :-

$$v_p(t) \xrightarrow{h_p(t)} y_p(t) \quad v(t) \xrightarrow{h(t)} y(t)$$

$y(t)$ and $y_p(t)$ are equivalent (ie, the same type of operation has been performed not that the signals are equal)

$$v_p(t) = \operatorname{Re} \{ v(t) e^{j2\pi f_c t} \}$$

$$y_p(t) = \operatorname{Re} \{ y(t) e^{j2\pi f_c t} \}$$

$$h_p(t) = \operatorname{Re} \{ h(t) e^{j2\pi f_c t} \} \rightarrow \text{not exactly true, there is some scaling happening.}$$

In baseband, we can use a DSP and perform operation like convolution much more easily.

$$\begin{aligned} Y(f) &= 2Y_p^+(f + f_c) = 2V_p^+(f + f_c) H_p^+(f + f_c) \\ &= \frac{1}{2} V(f) H(f) \end{aligned}$$

$$\Rightarrow h_p(t) = \frac{1}{2} \operatorname{Re} \{ h(t) e^{j2\pi f_c t} \}$$

* Analog Communication Technologies (AM, FM, PM) :-

° Receiving and transmission of signals is still in the analog domain, especially in RF and high frequency applications.

° Let $m(t)$ be the message signal with frequency response $M(f)$.
For a real signal,

$$M(f) = M^*(-f)$$

° There are 3 ways of encoding information in the complex envelope, Amplitude, Phase and Frequency.

° Amplitude Modulation :-

• Consider a message signal as,

$$m(t) = A_m \cos(2\pi f_m t)$$

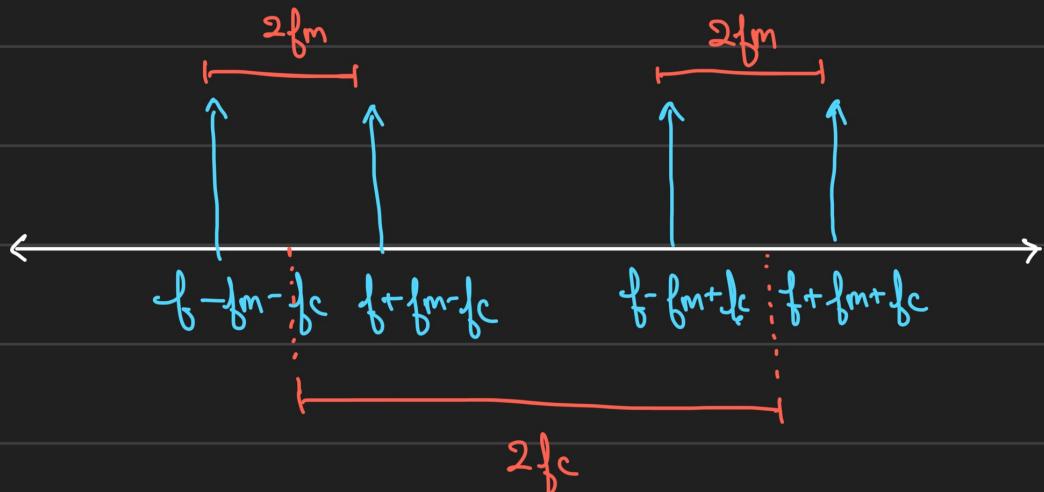
$$\Rightarrow M(f) = \frac{A_m}{2} (S(f+f_m) + S(f-f_m))$$

• In DSB-SC (Double Sideband Suppressed Carrier)

$$v_{DSB}(t) = m(t) \underbrace{A \cos(2\pi f_c t)}_{\text{Carrier Wave}}$$

$$\Rightarrow V_{DSB}(t) = A_m \cos(2\pi f_m t) A \cos(2\pi f_c t) \quad (f_c > f_m)$$

$$\Rightarrow V_{DSB}(f) = \frac{AmA}{4} \left(S(f+f_m + f_c) + S(f+f_m - f_c) + S(f-f_m + f_c) + S(f-f_m - f_c) \right)$$



27/1/26

Coherent Detection :-

- The output of the downconverter circuit

$$m(t) = A_m(t) \cos \theta_r$$

→ Delay component.

The gain component A does not affect our information gain that much. However the cosine term, due to offset b/w the transmitter and receiver, will distort the message.

- Thus, in DSB-SC, the need for coherent detection ($\theta_r \approx 0$) is high.

The cause of this offset is due to non-idealities in the oscillator circuits used in the modulation circuits.

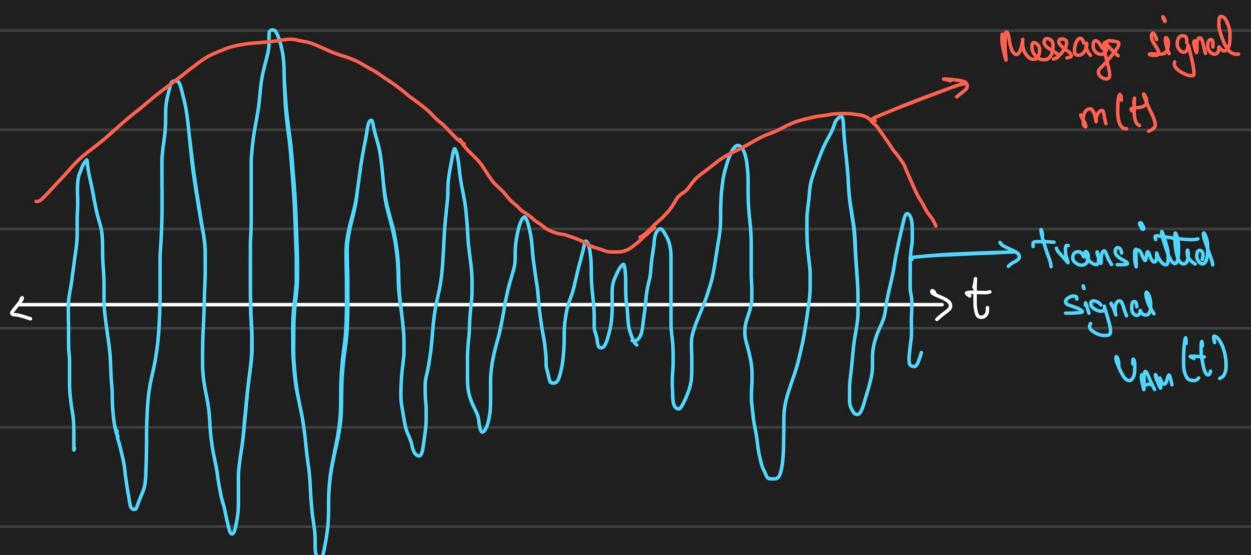
→ Conventional AM :-

- The message is encoded in the envelope of the transmitted signal.

$$v_{AM}(t) = (A_m(t) + A_c) \cos(2\pi f_c t)$$

$$= A_m(t) \cos(2\pi f_c t) + \underbrace{A_c \cos(2\pi f_c t)}_{\text{Large Carrier Component.}}$$

The envelope is $e(t) = |A_m(t) + A_c|$. If $A_m(t) + A_c > 0 \forall t$, then $m(t)$ can be recovered from $e(t)$

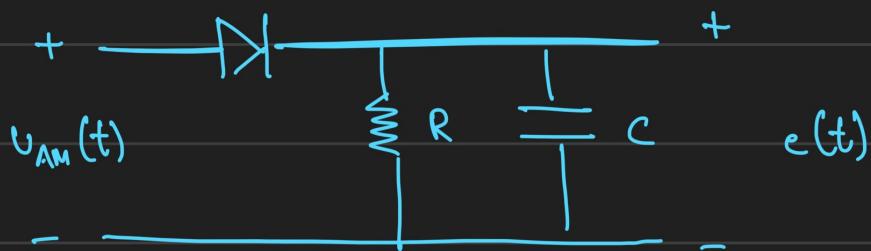


This is termed as 'Double Sideband - Strong Carrier' / Conventional AM.
(or Full Carrier - DSB-SC)

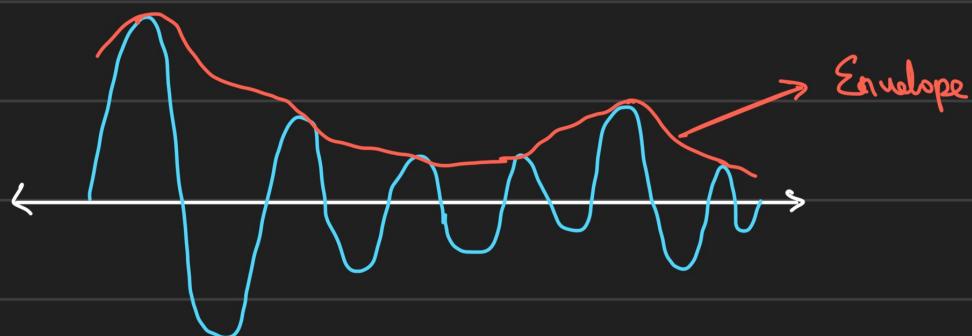
- This modulation scheme requires an envelope detector instead of an oscillator + mixer downconverting circuit.

- For conventional AM to work, Ac must be large enough such that $e(t)$ is always +ve. If not, then information will be lost.

- Envelope Detector :-



- This is similar to the filter circuit of a rectifier, ie., it latches on to the peaks and "drags" the signal near that point.



- The RC values must be such that the filter is responsive enough to catch the message envelope, but slow enough to ignore the carrier wave oscillation.

$$\Rightarrow \frac{1}{f_c} \ll \frac{1}{RC} \ll \frac{1}{B}$$

◦ Power Efficiency :-

- The main drawback of conventional AM is the power required to generate Ac.
- To prevent info,

$$A_m(t) + A_c > 0 \quad \forall t \in \mathbb{R}$$

$$\Rightarrow A \min \{m(t)\} + A_c > 0$$

$$\Rightarrow a_{\text{mod}} = \frac{A \min \{m(t)\}}{A_c} \leq \underline{\underline{1}}$$

a_{mod} - Modulation Index.

- By convention, we try to normalize the message signal such that the largest negative swing is -1.

$$m_n(t) = \frac{m(t)}{M_0} = \frac{m(t)}{|\min_t \{m(t)\}|}$$

$$\Rightarrow y_p(t) = \underbrace{B(1 + a_{\text{mod}} m_n(t)) \cos(2\pi f_c t + \theta_r)}$$

- The power efficiency of a modulated signal is ,

$$\eta = \frac{\text{Power in information signal}}{\text{Power in final modulated signal}}$$

Power of $v_{AM}(t)$ is given by ,

$$\begin{aligned}
 \overline{v_{AM}^2(t)} &= \overline{A_c^2 (1 + a_{\text{mod}} m_n(t))^2 \cos^2 2\pi f_b t} \\
 &= \overline{A_c^2 / 2 (1 + a_{\text{mod}} m_n(t))^2 (1 + \cos 4\pi f_b t)} \\
 &= \frac{A_c^2}{2} \left(\overline{1 + a_{\text{mod}} m_n(t)} \right)^2 + \frac{A_c^2}{2} \underbrace{\left(\overline{1 + a_{\text{mod}} m_n(t)} \right) \cos 4\pi f_b t}_{\text{passband signal} \Rightarrow P = 0} \\
 &= \frac{A_c^2}{2} \left(\overline{1 + a_{\text{mod}} m_n(t)} \right)^2 \\
 &= \frac{A_c^2}{2} \left(1 + \overline{a_{\text{mod}}^2 m_n^2(t)} + \overline{a_{\text{mod}}^2 m_n^2(t)} \right) \text{ since no DC component}
 \end{aligned}$$

If we assume $\overline{m_n(t)} = 0$, then

$$\overline{v_{AM}^2(t)} = \frac{A_c^2}{2} \left(1 + \overline{a_{\text{mod}}^2 m_n^2(t)} \right)$$

$$\begin{aligned}
 \overline{v(t)^2} &= \overline{(A_m(t) \cos(2\pi f_b t))^2} = \overline{(A_c a_{\text{mod}} m_n(t) \cos(2\pi f_b t))^2} \\
 &= \frac{A_c^2 a_{\text{mod}}^2 \overline{m_n^2(t)} \cos^2(2\pi f_b t)}{} \\
 &= \frac{A_c^2 a_{\text{mod}}^2 \overline{m_n^2(t)} (1 + \cos 4\pi f_b t)}{} \\
 &= \frac{A_c^2 a_{\text{mod}}^2 \overline{m_n^2(t)}}{}
 \end{aligned}$$

$$\Rightarrow \eta = \frac{a_{\text{mod}}^2 \overline{m_n^2(t)}}{1 + a_{\text{mod}}^2 \overline{m_n^2(t)}}$$

Since $a_{\text{mod}}^2 \leq 1$ and $\overline{m_n^2(t)} \leq 1$, $a_{\text{mod}}^2 \overline{m_n^2(t)} \leq 1$

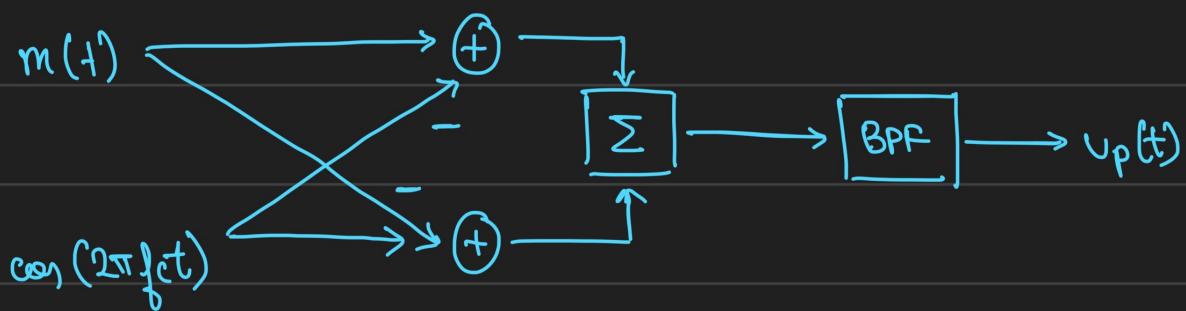
$$\Rightarrow \eta \leq \frac{1}{2}$$

- Because of the envelope detector, we do not need an oscillator at the receiver, thus removing the need for coherent detection.

Ques 1/26

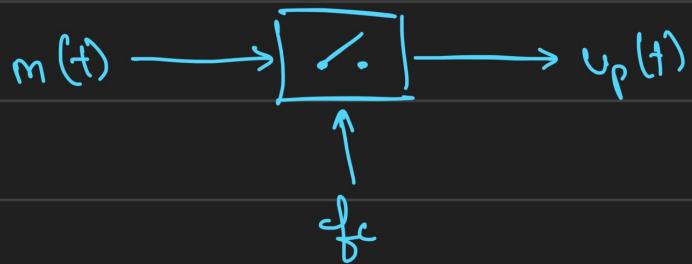
→ AM Modulators :-

◦ Non-Linear Modulators :-



- Switching:

- Instead of directly using a sinusoid, we will use a square wave, whose multiplication can be done by a switch.



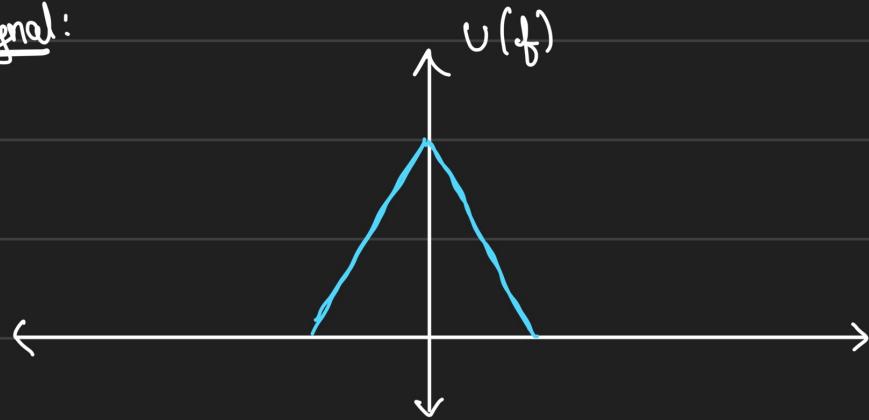
- Since the square wave is periodic, it will have multiple harmonics of frequencies fc , $2fc$, $3fc$, etc., the strongest of which is the fc part.
- The other harmonics are filtered out via a bandpass filter.

2 | 2 | 26

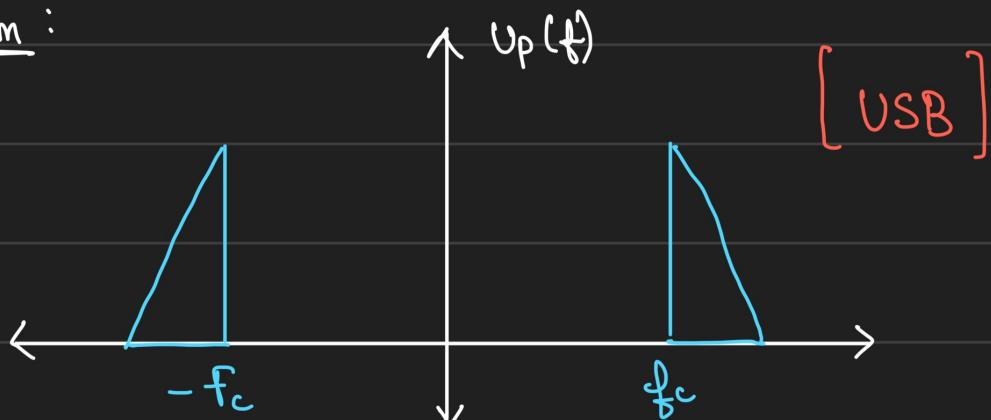
- Single Sideband Modulation :-

- Since real signals will always have symmetric spectrum, then it is possible to reconstruct the signal given only one of the sidebands.
- USB - Upper Sideband Modulation
- LSB - Lower Sideband Modulation

Original Signal:



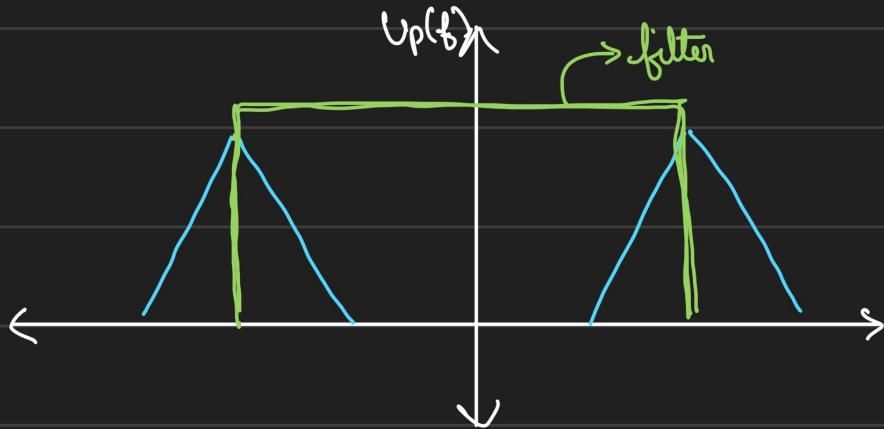
SSB Form:



Reconstruction: LPF $\{ U_p(f+f_c) + U_p(f-f_c) \}$



- But this type modulation would require the implementation of near-ideal features to cancel out one of the sidebands accurately



Such filters are implemented using the Hilbert Transform.

- Hilbert Transform :-

$$X(f) \xrightarrow{H(f) = -j \operatorname{sgn}(f)} \tilde{X}(f) = -j \operatorname{sgn}(f) X(f)$$

$$\bullet \text{ For the complex envelope } V(f) = \begin{cases} AM(f), & f \geq 0 \\ 0, & f < 0 \end{cases}$$

$$V_c(f) = \frac{V(f) + V^*(-f)}{2}$$

$$\text{For } f > 0, \quad V_c(f) = \frac{AM(f) + 0}{2} = \frac{AM(f)}{2}$$

$$f < 0, \quad V_c(f) = \frac{0 + AM(-f)}{2} = \frac{AM(f)}{2} \quad [M(f) = M(-f)]$$

$$\therefore V_c(f) = \frac{AM(f)}{2} \longrightarrow \text{In-Phase Component}$$

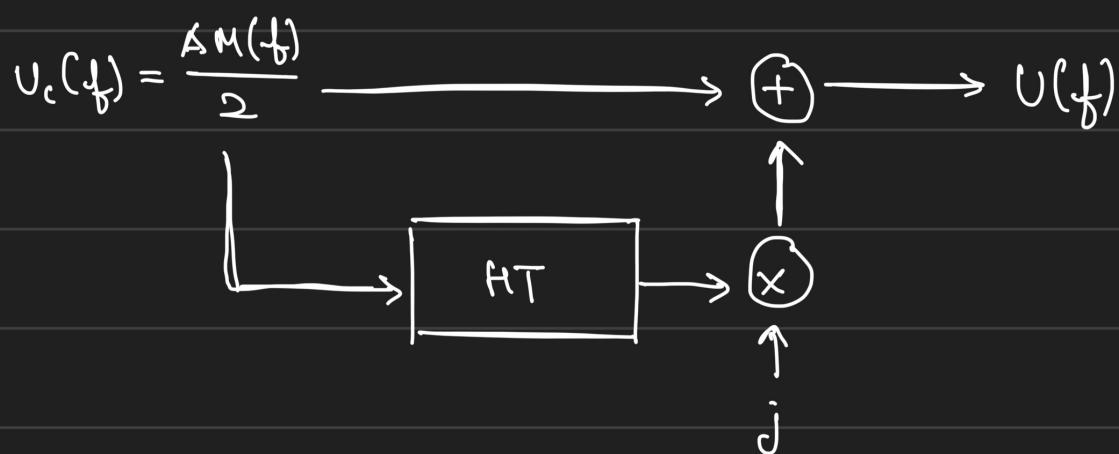
$$V_s(f) = \frac{U(f) - \bar{U}(f)}{2j}$$

$$\text{For } f > 0, \quad V_s(f) = \frac{AM(f)}{2j} = -j \frac{AM(f)}{2} = -j V_c(f)$$

$$\text{For } f < 0, \quad V_s(f) = -\frac{AM(f)}{2j} = j \frac{AM(f)}{2} = j V_c(f)$$

$$\Rightarrow V_s(f) = \underbrace{-j \operatorname{sgn}(f)}_{\text{Hilbert fn.}} V_c(f) \longrightarrow \text{Quadrature Comp.}$$

- To get the above $U(f)$, we use the Hilbert transform in the below system.



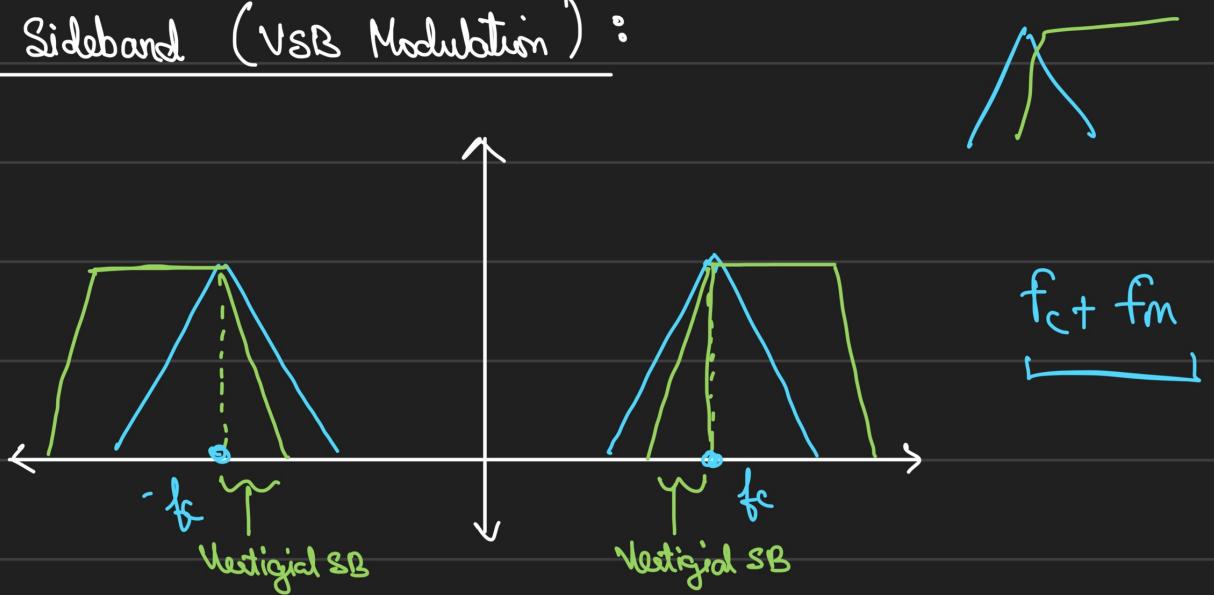
$$\text{Since } U(f) = V_c(f) + j V_s(f) \text{ and}$$

$$V_s(f) = \text{Hilbert} \{ V_c(f) \} \text{ (Shown Above)}$$

- Detection of the signals is similar to double side band, for both suppressed carrier (coherent detection) and full carrier (non-coherent envelope detection).

- Implementation of the Hilbert Transform is done using 90° phase shifters.

- Vestigial Sideband (VSB Modulation) :



- Instead of having very sharp filters as in SSB, we allow for some extra spectrum to be retained.
- Information can be detected if for the filters $H_p(f+f_c) + H_p(f-f_c)$ should be constant over the message band.

