EC5.102: Information and Communication

(Lec-5)

Channel coding-1

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Summary of the last class

Recap

- Kraft inequality
- Show that: For Prefix codes, $L(C) \geq H(X)$
- Statement of source coding theorem

Source coding theorem

- Notation: $X, X^n, \mathcal{X}, \mathcal{X}^n, H(X)$
- Encoding: For $x^n \in \mathcal{X}^n$ codeword is $f^n(x^n)$
- Decoding: Codeword $f^n(x^n)$ is decoded as $g^n(f^n(x^n)) := \hat{x}^n$
- Decoding error: $\hat{x}^n \neq x^n$
- Statement of source coding theorem:
  - Achievability: If average length $L(C) > H(X)$, then there exists a sequence of codes $\{f^{(n)}, g^{(n)}\}$ of average length $L(C)$ such that

$$P_e^{(n)} := \mathbb{P}\left[\hat{X}^n \neq X^n\right] \to 0 \quad \text{as} \quad n \to \infty$$

  - Converse:

    If $L(C) < H(X)$, then $P_e^{(n)} > 0$ for any n.

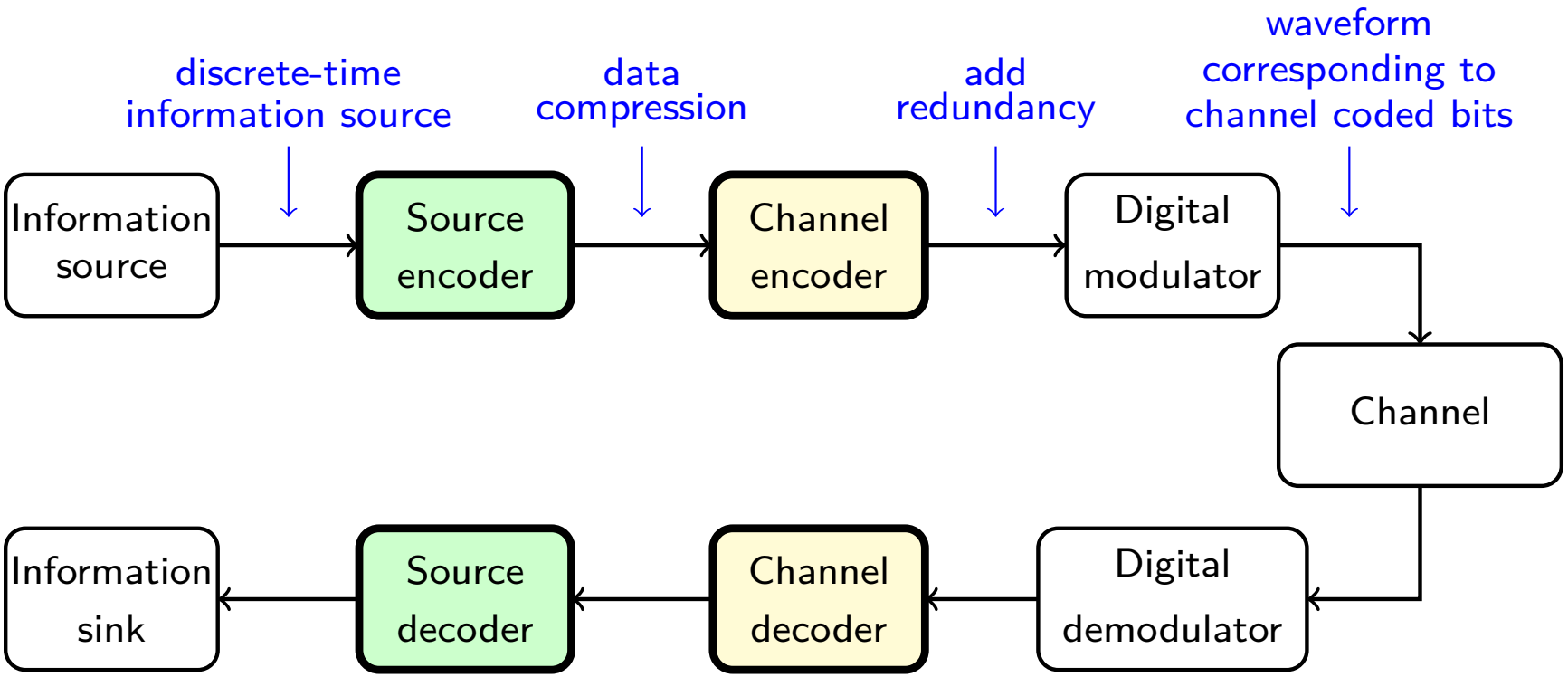
Reference Books

- Thomas M. Cover and Joy A. Thomas, *"Elements of Information Theory"*, Wiley India press, Edition 2.
- R. Roth, *"Introduction to Coding Theory"*, Cambridge University Press, 2007.
- S. Lin and D.J. Costello, *"Error Control Coding"*, Pearson, 2011.

Block diagram of a digital communication system

Digital communication system

- Block diagram of digital communication system



Information theory provide guidelines to design source & channel coding.

What are "Channel Codes"?

(also called as "Error Correcting Codes (ECC) or Forward Error Correction (FEC) codes")

What are channel codes?

Alice

Channel

Bob

Can Alice do "something" so that Bob is able to interpret her message possibly after doing "some processing"?

Introduction to channel codes

- Our focus: Messages are sequence of bits (bitstream/bit-sequence)
- Transmitted bit-sequence gets "corrupted" by the channel.
  - What do you mean by "corrupted"? How to define a "channel"?
  - Example of a channel model: Binary erasure channel (BEC)
- Let us design a channel code....!
  - Alice wants to send either the message "0" or the message "1" to Bob.
  - Channel model: BEC
  - Can you help Alice to design a mechanism so that Bob able to interpret the message?
- These are repetition (REP) codes!

Example: Channel codes

- Let us design another channel code.
- Example of a channel model: Binary symmetric channel (BSC)
- Suppose the message is “00” or “01” or “10” or “11”.
- Consider the situation when one of the bit is flipped by the channel.
- Can you help Alice to design a mechanism so that Bob able to “detect” “one-bit” error?
- These are single-parity check codes (SPC)!
- **Questions:**
  - Will I be able to “correct” the bit using this code? Justify.
  - Will this code detect two-bit errors?
  - Can you design SPC code of length n?

Introduction to channel codes

- Can I design a code that is better than a repetition code or a single parity check code?
- When do I say one code is better than the other?
- How to quantify whether a code is good or bad?
- What sorts of processing a receiver can do?
- What about real-life channel models?
- Any other questions?

Motivation for studying error correcting codes

- Error-correcting codes are a fundamental tool for protecting data from noise introduced by the channel.
- They appear naturally throughout computer science, electrical engineering, maths, and physics.

Applications of channel codes

Applications of channel codes

Cyclic codes	• Burst error correction • Internet data packets: CRC code
Convolutional codes	• Mobile communication • Satellite communication: Serially concatenated convolutional and Reed-Solomon codes
LDPC codes	• Satellite transmission of digital television • Various telecommunication standards • Wireless communication: 4G, 5G, Wi-Fi 802.11
Turbo codes	• Wireless communication: 3G, 4G, LTE, IEEE 802.16 (WiMAX) standard • NASA missions: Mars Reconnaissance Orbiter
Polar codes	• Wireless communication: 5G
Reed-Solomon codes	• Wireless communication: 5G • Storage devices

Applications of channel codes

- Coding for data storage
- Distributed machine learning
- Cryptography based of channel codes
- Storing data on non-volatile memories
- Designing fault tolerant systems
- Studying structure of DNA sequences