Assigment - 2

3.22 a) b. $x(t)$ is periodic with $T = 6$

For $t \in [-2, 4]$

$$x(t) = \begin{cases} 2+t, & t \in [-2,-1) \\ 1, & t \in [-1,1) \\ 2-t, & t \in [1,2] \\ 0, & t \in [2,4] \end{cases}$$

$$x(t) = \sum_{k=\mathbb{Z}} a_k e^{jk\omega t}$$

$$\Rightarrow a_k = \frac{1}{T}\int x(t) e^{-jk\omega t} dt$$

$$= \frac{1}{6}\left[\int_{-2}^{-1}(2+t)e^{-jk\omega t} dt + \int_{-1}^{1} e^{-jk\omega t} dt + \int_{1}^{2}(2-t)e^{-jk\omega t} dt\right]$$

$$= \frac{1}{6}\left[\int_{-2}^{-1} 2e^{-jk\omega t} dt + \int_{-2}^{-1} te^{-jk\omega t} dt + \int_{-1}^{1} e^{-jk\omega t} dt\right.$$

$$\left. + \int_{1}^{2} 2e^{-jk\omega t} dt - \int_{1}^{2} te^{-jk\omega t} dt\right]$$

$$= \frac{1}{6}\left[2\left[\frac{-1}{j\omega k} e^{-jk\omega t}\right]_{-2}^{-1} + \left[t\int e^{-jk\omega t} dt - \int 1 \int e^{-jk\omega t} dt\, dt\right]_{-2}^{-1}\right.$$

$$+ \left[\frac{-1}{j\omega k} e^{-jk\omega t}\right]_{-1}^{1} + 2\left[\frac{-1}{j\omega k} e^{-jk\omega t}\right]_{1}^{2}$$

$$\left. - \left[t\int e^{-jk\omega t} dt - \int 1 \int e^{-jk\omega t} dt\, dt\right]_{1}^{2}\right]$$

$$= \frac{1}{6}\left[2\left(\frac{e^{2jk\omega} - e^{jk\omega}}{j\omega k}\right) + \left[\frac{-te^{-jk\omega t}}{j\omega k} - \left(\frac{-1}{j\omega k}\right)^2 e^{-jk\omega t}\right]_{-2}^{-1}\right.$$

$$+ \left(\frac{e^{jk\omega} - e^{-jk\omega}}{j\omega k}\right) + 2\left(\frac{e^{-jk\omega} - e^{-2jk\omega}}{j\omega k}\right)$$

$$\left. - \left[\frac{-te^{-jk\omega t}}{j\omega k} - \left(\frac{-1}{jk\omega}\right)^2 e^{-jk\omega t}\right]_{1}^{2}\right]$$

$$= \frac{1}{6}\left[\frac{2e^{2jk\omega}}{jk\omega} - \frac{2e^{jk\omega}}{jk\omega} + \left(\frac{e^{jk\omega}}{jk\omega} - \frac{e^{jk\omega}}{(jk\omega)^2}\right)\right.$$

$$\left. - \left(\frac{2e^{2jk\omega}}{jk\omega} - \frac{e^{2jk\omega}}{(jk\omega)^2}\right)\right] + \frac{e^{jk\omega}}{jk\omega} - \frac{e^{-jk\omega}}{jk\omega}$$

$$+ \frac{2e^{-jk\omega}}{jk\omega} - \frac{2e^{-2jk\omega}}{jk\omega} - \left[\left(\frac{-2e^{-2jk\omega}}{jk\omega}\right.\right.$$

$$\left.\left. - \frac{e^{-2jk\omega}}{(jk\omega)^2}\right) - \left(\frac{-e^{-jk\omega}}{jk\omega} - \frac{e^{-jk\omega}}{(jk\omega)^2}\right)\right]$$

$$= \frac{1}{6}\left[\cancel{\frac{2e^{2jk\omega}}{jk\omega}} - \cancel{\frac{2e^{jk\omega}}{jk\omega}} + \cancel{\frac{e^{jk\omega}}{jk\omega}} - \frac{e^{jk\omega}}{(jk\omega)^2} - \cancel{\frac{2e^{2jk\omega}}{jk\omega}}\right.$$

$$+ \frac{e^{2jk\omega}}{(jk\omega)^2} + \cancel{\frac{e^{jk\omega}}{jk\omega}} - \cancel{\frac{e^{-jk\omega}}{jk\omega}} + \cancel{\frac{2e^{-jk\omega}}{jk\omega}}$$

$$- \cancel{\frac{2e^{-2jk\omega}}{jk\omega}} + \cancel{\frac{2e^{-2jk\omega}}{jk\omega}} + \frac{e^{-2jk\omega}}{(jk\omega)^2} - \cancel{\frac{e^{-jk\omega}}{jk\omega}}$$

$$\left. - \frac{e^{-jk\omega}}{(jk\omega)^2}\right]$$

$$= \frac{1}{6}\left[\frac{-e^{jk\omega}}{(jk\omega)^2} + \frac{e^{2jk\omega}}{(jk\omega)^2} + \frac{e^{-2jk\omega}}{(jk\omega)^2} - \frac{e^{-jk\omega}}{(jk\omega)^2}\right]$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{3}$$

$$\Rightarrow a_k = \frac{1}{6\left(\frac{jk\pi}{3}\right)^2}\left(e^{2jk\omega} - e^{jk\omega} + e^{-2jk\omega} - e^{-jk\omega}\right)$$

$$\boxed{a_k = \frac{3}{2(jk\pi)^2}\left(e^{jk\frac{2\pi}{3}} - e^{jk\frac{\pi}{3}} + e^{-2jk\frac{\pi}{3}} - e^{-jk\frac{\pi}{3}}\right)}$$

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d. $x(t)$ is periodic with $T = 2$

For $t \in [0, 2)$

$$x(t) = \begin{cases} \delta(t), & t = 0 \\ -2\delta(t-1), & t = 1 \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{k \in \mathbb{Z}} a_k e^{jk\omega t}$$

$$a_k = \frac{1}{T}\int_0^2 x(t) e^{-jk\omega t} dt$$

$$= \frac{1}{T}\int_0^2 (\delta(t) e^{-jk\omega t} - 2\delta(t-1) e^{-jk\omega t}) dt$$

$$= \frac{1}{2}(e^{-jk\omega(0)} - 2e^{-jk\omega(1)})$$

$$\omega = \frac{2\pi}{2} = \pi$$

$$\boxed{a_k = \frac{1}{2}(1 - 2e^{-jk\omega})} \Rightarrow a_k = \frac{1}{2}(1 - 2e^{-j\pi k})$$

$$\boxed{a_k = \frac{1}{2}(1 - 2\cos \pi k)}$$

e. $x(t)$ is periodic with $T = 6$.

For $t \in [-3, 3]$

$$x(t) = \begin{cases} 1, & t \in [-2, -1] \\ -1, & t \in [1, 2] \\ 0, & \text{otherwise} \end{cases}$$

$$x(t) = \sum_{k \in \mathbb{Z}} a_k e^{jk\omega t} \Rightarrow a_k = \frac{1}{T}\int^{T} x(t) e^{-jk\omega t} dt$$

$$a_k = \frac{1}{6}\int_{-3}^{3} x(t)\, e^{-jk\omega t}\, dt$$

$$= \frac{1}{6}\left[\int_{-2}^{-1} e^{-jk\omega t}\, dt - \int_{1}^{2} e^{jk\omega t}\, dt\right]$$

$$\omega = \frac{2\pi}{T} = \frac{\pi}{3}$$

$$a_k = \frac{1}{6}\left[\int_{-2}^{-1} e^{-j\frac{k\pi}{3}t}\, dt - \int_{1}^{2} e^{-j\frac{k\pi}{3}t}\, dt\right]$$

$$= \frac{1}{6}\left[\left(\frac{-3}{jk\pi}\right)\left[e^{-j\frac{k\pi}{3}t}\right]_{-2}^{-1} - \left(\frac{-3}{jk\pi}\right)\left[e^{-j\frac{k\pi}{3}t}\right]_{1}^{2}\right]$$

$$= \frac{1}{6}\left(\frac{-3}{jk\pi}\right)\left[\left(e^{j\frac{k\pi}{3}t} - e^{2j\frac{k\pi}{3}}\right) - \left(e^{-2j\frac{k\pi}{3}} - e^{-j\frac{k\pi}{3}}\right)\right]$$

$$\boxed{a_k = \frac{-1}{2jk\pi}\left(e^{j\frac{k\pi}{3}t} - e^{2j\frac{k\pi}{3}} - e^{-2j\frac{k\pi}{3}} + e^{-j\frac{k\pi}{3}}\right)}$$

b) $x(t)$ is periodic with $T=2$ and,

$$x(t) = e^{-t} \text{ for } -1 < t < 1$$

$$x(t) = \sum a_k e^{jk\omega t} \cancel{dt} \Rightarrow a_k = \frac{1}{T}\int^{T} x(t)\, e^{-jk\omega t}\, dt$$

$$a_k = \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-jk\omega t}\, dt \qquad \omega = \frac{2\pi}{T} = \pi$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-t} e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\int_{-1}^{1} e^{-(jk\pi+1)t}\, dt$$

$$= \frac{1}{2}\left(\frac{-1}{jk\pi+1}\right)\left[e^{-(jk\pi+1)t}\right]_{-1}^{1}$$

$$= \frac{-1}{2(jk\pi+1)}\left(e^{-(jk\pi+1)} - e^{jk\pi+1}\right)$$

$$\boxed{a_k = \frac{e^{jk\pi+1} - e^{-(jk\pi+1)}}{2(jk\pi+1)}}$$

c) $x(t)$ has period 4.

$$x(t) = \begin{cases} \sin \pi t, & 0 \le t \le 2 \\ 0, & 2 < t \le 4 \end{cases}$$

$$x(t) = \sum a_k e^{jk\omega t} \Rightarrow a_k = \frac{1}{T}\int^{T} x(t)\, e^{-jk\omega t}\, dt$$

$$\sin \pi t = \frac{e^{j\pi t} - e^{-j\pi t}}{2}(-j) = \frac{e^{-j\pi t} - e^{j\pi t}}{2}(j)$$

$$\Rightarrow a_k = \frac{1}{4}\int_0^2 j\,\frac{e^{-j\pi t} - e^{j\pi t}}{2}\, e^{-j\frac{k\pi}{2}t}\, dt$$

$$= \frac{j}{8}\left[\int_0^2 e^{-j\pi t - j\frac{k\pi}{2}t}\, dt - \int_0^2 e^{j\pi t - j\frac{k\pi}{2}t}\, dt\right]$$

$$= \frac{j}{8}\left[\int_0^2 e^{t\left(-j\pi - j\frac{k\pi}{2}\right)} dt - \int_0^2 e^{t\left(j\pi - \frac{jk\pi}{2}\right)} dt\right]$$

$$= \frac{j}{8}\left\{\frac{-1}{j\pi + \frac{jk\pi}{2}}\left[e^{-t\left(j\pi + j\frac{k\pi}{2}\right)}\right]_0^2 - \frac{1}{j\pi - \frac{jk\pi}{2}}\left[e^{t\left(j\pi - \frac{jk\pi}{2}\right)}\right]_0^2\right.$$

$$a_k = \frac{j}{8}\left(\frac{-2}{2j\pi + jk\pi}\left(e^{-2(j\pi + j\frac{k\pi}{2})} - 1\right)\right.$$

$$\left. - \frac{2}{2j\pi - jk\pi}\left(e^{2(j\pi - j\frac{k\pi}{2})} - 1\right)\right)$$

$$= \frac{j}{8}\left(\frac{2}{j\pi}\right)\left(\frac{-1}{2+k}\left(e^{-2(j\pi + j\frac{k\pi}{2})} - 1\right) - \frac{1}{2-k}\left(e^{2(j\pi - j\frac{k\pi}{2})} - 1\right)\right)$$

$$= \frac{1}{4\pi}\left(\frac{1 - e^{-2(j\pi + j\frac{k\pi}{2})}}{2+k} - \frac{e^{2(j\pi + j\frac{k\pi}{2})} - 1}{2-k}\right)$$

$$= \frac{1}{4\pi}\left(\frac{1 - e^{-2\pi j}e^{-jk\pi}}{2+k} - \frac{e^{j2\pi}e^{jk\pi} - 1}{2-k}\right)$$

$$\boxed{a_k = \frac{1}{4\pi}\left(\frac{1 - e^{-jk\pi}}{2+k} - \frac{e^{jk\pi} - 1}{2-k}\right)}$$

3.25 a) $x(t) = \cos(4\pi t)$, $\omega = 4\pi$, $T = 1/2$

$$x(t) = \sum_{k \in \mathbb{Z}} a_k e^{jk\omega t} \Rightarrow a_k = \frac{1}{T}\int_T x(t) e^{-jk\omega t}\, dt$$

$$\Rightarrow a_k = 2\int_0^{\frac{1}{2}} \cos(4\pi t) e^{-jk4\pi t}\, dt \qquad \cos(4\pi t) = \frac{e^{j4\pi t} + e^{-j4\pi t}}{2}$$

$$= \cancel{2}\int_0^{\frac{1}{2}} \frac{e^{j4\pi t} + e^{-j4\pi t}}{\cancel{2}} \cdot e^{-jk4\pi t}\, dt$$

$$= \int_0^{\frac{1}{2}} e^{j4\pi t} \cdot e^{-jk4\pi t}\, dt + \int_0^{\frac{1}{2}} e^{-j4\pi t} e^{-jk4\pi t}\, dt$$

$$= \int_0^{\frac{1}{2}} e^{(j4\pi - jk4\pi)t}\, dt + \int_0^{\frac{1}{2}} e^{-(j4\pi + jk4\pi)t}\, dt$$

$$= \frac{1}{(j4\pi - jk4\pi)}\left[e^{(j4\pi - jk4\pi)t}\right]_0^{\frac{1}{2}} + \left(\frac{-1}{j4\pi + jk4\pi}\right)\left[e^{-(j4\pi + jk4\pi)t}\right]_0^{1/2}$$

$$= \frac{1}{j4\pi - jk4\pi}\left(e^{\frac{j4\pi - jk4\pi}{2}} - 1\right) + \left(\frac{-1}{j4\pi + jk4\pi}\right)\left(e^{-\frac{j4\pi + jk4\pi}{2}} - 1\right)$$

$$= \frac{1}{j4\pi - jk4\pi}\left(e^{j2\pi}e^{-jk2\pi} - 1\right) + \left(\frac{-1}{j4\pi + jk4\pi}\right)\left(e^{-j2\pi}e^{-jk2\pi} - 1\right)$$

$$= \frac{1}{j4\pi - jk4\pi}(1-1) + \left(\frac{-1}{j4\pi + jk4\pi}\right)(1-1)$$

$\Rightarrow \boxed{a_k = 0}$, $\boxed{\text{But indeterminate for } k = 1 \text{ \& } k = -1}$

But since w.k.t $\cos 4\pi t = \frac{1}{2}e^{j4\pi t} + \frac{1}{2}e^{-j4\pi t}$

$$\Rightarrow \cos 4\pi t = \frac{1}{2}e^{j4\pi(1)t} + \frac{1}{2}e^{j4\pi(-1)t}$$

$$\Rightarrow \boxed{a_1 = \frac{1}{2} \text{ \& } a_{-1} = \frac{1}{2}}$$

b) $y(t) = \sin(4\pi t)$, $w = 4\pi$, $T = \frac{1}{2}$

$y(t) = \sum b_k e^{jkwt} \Rightarrow b_k = \frac{1}{T}\int^{T} y(t)e^{-jkwt}\,dt$

$b_k = 2\int_0^{1/2} \sin(4\pi t)e^{-jk4\pi t}\,dt$ $\quad \sin(4\pi t) = -j\left(\frac{e^{j4\pi t} - e^{-j4\pi t}}{2}\right)$

$$= \cancel{2}\int_0^{1/2} \frac{(-j)\left(e^{j4\pi t} - e^{-j4\pi t}\right)}{\cancel{2}} e^{-j4k\pi t}\,dt$$

$$b_k = \int_0^{1/2} (-j)\left(e^{(j4\pi - jk4\pi)t} - e^{(-j4\pi - jk4\pi)t}\right) dt$$

$$= (-j)\left(\frac{1}{j4\pi - jk4\pi}\right)\left[e^{(j4\pi - jk4\pi)t}\right]_0^{1/2} - (-j)\left(\frac{-1}{j4\pi + jk4\pi}\right)\left[e^{-(j4\pi + jk4\pi)t}\right]_0^{1/2}$$

$$= \frac{-j}{j4\pi - jk4\pi}\left(e^{j2\pi - jk2\pi} - 1\right)$$

$$- \frac{j}{j4\pi + jk4\pi}\left(e^{-(j2\pi + jk2\pi)} - 1\right)$$

$$\Rightarrow b_k = \frac{-j}{j4\pi - jk4\pi}(1-1) - \frac{j}{j4\pi + jk4\pi}(1-1)$$

$\Rightarrow$ $\boxed{b_k = 0 \text{ but indeterminate for } k = 1 \text{ \& } -1}$

w.k.t

$$\sin(4\pi t) = \frac{-j}{2} e^{j4\pi t} + \frac{j}{2} e^{j4\pi(-1)t}$$

$$\Rightarrow \boxed{b_1 = \frac{-j}{2} \quad b_{-1} = \frac{j}{2}}$$

c) $z(t) = x(t)\, y(t)$

Let FS coeffs of $z(t)$ be c_k.

By multiplication property of FS coefficients,

~~$c_k = \sum a_l b_{k-l}$~~

~~Since only~~

$$c_k = \sum_{l \in \mathbb{Z}} a_l b_{k-l}$$

But since a_1, a_{-1} and b_1, b_{-1} are non-zero

$$\left.\begin{aligned} C_0 &= a_1 b_{-1} + a_{-1} b_1 \\ C_2 &= a_1 b_1 \\ C_{-2} &= a_{-1} b_{-1} \end{aligned}\right\}$$ ~~is~~ are the only non zero ~~coeffs~~ of combination $z(t)$

$$C_0 = \left(\frac{1}{2}\right)\left(\frac{j}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{-j}{2}\right) = \frac{j}{4} - \frac{j}{4} = \underline{\underline{0}}$$

$$C_2 = \left(\frac{1}{2}\right)\left(\frac{-j}{2}\right) = \frac{-j}{4}$$

$$C_{-2} = \left(\frac{1}{2}\right)\left(\frac{j}{2}\right) = \frac{j}{4}.$$

$$\therefore C_K = \begin{cases} -j/4, & k = 2 \\ j/4, & k = -2 \\ 0, & \text{otherwise} \end{cases}$$

d) $z(t) = x(t) y(t)$

$$= \cos 4\pi t \cdot \sin 4\pi t$$

$$= \frac{1}{2} \sin 8\pi t$$

$$\frac{1}{2} \sin 8\pi t = \frac{1}{2}\left(\frac{e^{j8\pi t} - e^{-j8\pi t}}{2}\right)(-j)$$

$$= \frac{-j}{4}\left(e^{j8\pi t} - e^{-j8\pi t}\right)$$

$$= \frac{-j}{4} e^{j(2)4\pi t} + \frac{j}{4} e^{j(-2)4\pi t}$$

$\Rightarrow$ $C_2 = \frac{-j}{4}$, $C_{-2} = \frac{j}{4}$ (C_k is FS coeffs of $z(t)$)

This matches with the result of the prev. qn.

3.26 $a_k = \begin{cases} 2, & k = 0 \\ j\left(\frac{1}{2}\right)^{|k|}, & \text{otherwise} \end{cases}$ $x(t) = \sum a_k e^{jk\omega t}$

a) For $x(t)$ to be real

$x(t) = x^*(t)$ where $x^*(t)$ is conjugate of $x(t)$

$x^*(t) = \sum a_k^* e^{-jk\omega t}$

$\Rightarrow$ $a_k^* = a_{-k}$ $\longrightarrow$ On comparing FS reps of $x(t)$ and $x^*(t)$

$a_k = j\left(\frac{1}{2}\right)^{|k|}$

$a_k^* = -j\left(\frac{1}{2}\right)^{|k|}$ $a_{-k} = j\left(\frac{1}{2}\right)^{|k|}$

$a_k^* \neq a_{-k}$ $\Rightarrow$ $x(t)$ is NOT real.

b) For $x(t)$ to be even

$x(t) = x(-t)$

$x(-t) = \sum a_k e^{jk\omega(-t)} = \sum a_k e^{j(-k)\omega t}$

$\Rightarrow$ $a_k = a_{-k}$ $\longrightarrow$ On comparing FS reps of $x(t)$ and $x(-t)$

$a_k = j\left(\frac{1}{2}\right)^{|k|}$, $a_{-k} = j\left(\frac{1}{2}\right)^{|k|}$ $\Rightarrow$ $a_k = a_{-k}$

$\Rightarrow$ $x(t)$ is even.

c) $\frac{d}{dt} x(t) = \frac{d}{dt} \Sigma a_k e^{jwkt}$

$= \Sigma a_k \left(\frac{d}{dt} e^{jwkt}\right)$

$= \Sigma a_k (jwk) e^{jwkt}$

If $y(t) = \frac{d}{dt} x(t)$ and its FS coeffs are b_k, then

$b_k = jwk . a_k$

For $y(t)$ to be even,

$b_k = b_{-k}$

$b_k = jwk \left(j \left(\frac{1}{2}\right)^{|k|}\right)$ $b_{-k} = -jwk \left(j\left(\frac{1}{2}\right)^{|k|}\right)$

$\Rightarrow b_k \neq b_{-k} \Rightarrow \frac{d}{dt} x(t)$ is NOT even

3.34. $h(t) = e^{-4|t|}$

W.k.t for an LTI system with impulse response $h(t)$,
The FS coefficients of the response signal for any periodic input is,

$$b_k = a_k H(k\omega j)$$, where a_k is FS coeffs of $x(t)$

Where $H(\omega j)$ is the Fourier Transform of $h(t)$.

$$H(\omega j) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$$

$$= \int_{-\infty}^{\infty} e^{-4|t|} e^{-j\omega t} dt$$

$$= \int_{0}^{\infty} e^{-4t} e^{-j\omega t} dt + \int_{-\infty}^{0} e^{4t} e^{-j\omega t} dt$$

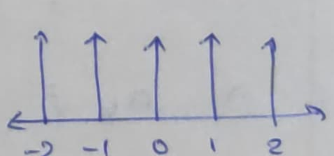
$$= \frac{-1}{4+j\omega} \left[e^{-(4+j\omega)t} \right]_0^{\infty} + \frac{1}{4-j\omega} \left[e^{(4-j\omega)t} \right]_{-\infty}^{0}$$

$$= \frac{-1}{4+j\omega} [0-1] + \frac{1}{4-j\omega} [1-0]$$

$$= \frac{1}{4+j\omega} + \frac{1}{4-j\omega}$$

a) $x(t) = \sum_{n \in \mathbb{Z}} \delta(t-n)$, $T = 1$

$$a_k = \frac{1}{1} \int_{-1/2}^{1/2} \sum_{n \in \mathbb{Z}} \delta(t-n) e^{-j\frac{2\pi}{T_1}kt} dt$$



$$= \int_{-1/2}^{1/2} \delta(t) e^{-j2\pi kt} dt$$

$\Rightarrow a_k = e^{-j2\pi k(0)}$ By sifting property of $\delta(t)$

$\Rightarrow a_k = 1 \ \forall k \in \mathbb{N}$

$b_k = a_k H(k\omega_0 j)$ (b_k is FS coeffs of $y(t)$)

$= 1\left(\frac{1}{4+k\omega j} + \frac{1}{4-k\omega j}\right)$

$$\boxed{b_k = \frac{1}{4+k\omega j} + \frac{1}{4-k\omega j}}$$

b) $x(t) = \sum_{n\in\mathbb{Z}} (-1)^n \delta(t-n)$, Period = 2

$\omega = \pi$

$$a_k = \frac{1}{2}\int_0^2 \sum_{n\in\mathbb{Z}} (-1)^n \delta(t-n)\, e^{-jk\pi t}\, dt$$

$$= \frac{1}{2}\left[\int_0^1 \delta(t)\, e^{-jk\pi t}\, dt + \int_1^2 \delta(t-1)\, e^{-jk\pi t}\, dt\right]$$

$$= \frac{1}{2}\left[e^{-jk\pi(0)} + e^{-jk\pi(1)}\right]$$

$$= \frac{1}{2}\left(1 + \cancel{e^{jk}} \cos \pi k\right)$$

$$\Rightarrow a_k = \begin{cases} 1, & k \text{ is even} \\ 0, & k \text{ is odd} \end{cases}$$

$$\Rightarrow b_K = \begin{cases} \frac{1}{4+k\omega j} + \frac{1}{4-k\omega j}, & k \text{ is } \cancel{\text{odd}} \text{ even} \\ 0, & k \text{ is odd.} \end{cases}$$

c) Period of $x(t) = 2$,

In $t \in (-1, 1)$

$$x(t) = \begin{cases} \frac{1}{2}, & t \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ 0, & \text{otherwise} \end{cases}$$

$$a_K = \frac{1}{2}\int_{-1}^{1} x(t)\, e^{-j\left(\frac{2\pi}{2}\right)kt}\, dt$$

$$= \frac{1}{2}\int_{-1/2}^{1/2} \frac{1}{2} e^{-j\pi kt}\, dt = \frac{1}{4}\int_{-1/2}^{1/2} e^{-j\pi kt}\, dt$$

$$= \frac{1}{4}\left[e^{-j\pi kt}\right]_{-1/2}^{1/2} \cdot \left(\frac{-1}{j\pi k}\right)$$

$$= \frac{-1}{4kj\pi}\left(e^{-j\pi/2\, k} - e^{j\pi/2\, k}\right)$$

$$= \frac{-1}{4k\cancel{j}\pi}\left(-\cancel{j}\sin\left(\frac{k\pi}{2}\right) - \cancel{j}\sin\left(\frac{k\pi}{2}\right)\right)$$

$$= \frac{2\sin\left(\frac{k\pi}{2}\right)}{4\pi k}$$

$$\Rightarrow a_K = \frac{\sin\frac{k\pi}{2}}{2\pi k} = \begin{cases} \frac{1}{2\pi k}, & k = 4n+1 \\ \frac{-1}{2\pi k}, & k = 4n-1 \quad \forall n \in \mathbb{N} \\ 0, & k = 2n \end{cases}$$

$$\Rightarrow b_k = \begin{cases} \frac{1}{2\pi k}\left(\frac{1}{4+k\omega j} + \frac{1}{4-k\omega j}\right), & k = 4n+1 \\ \frac{-1}{2\pi k}\left(\frac{1}{4+k\omega j} + \frac{1}{4-k\omega j}\right), & k = 4n-1 \quad \forall n \in \mathbb{N} \\ 0, & k = 2n \end{cases}$$

3.41. <u>Given:</u>

- $a_k = a_{k+2}$
- $a_k = a_{-k} \longrightarrow$ Even signal
- $\int_{-1/2}^{1/2} x(t)\,dt = 1$
- $\int_{1}^{2} x(t)\,dt = 2$
- $T = 3 \Rightarrow \omega = \frac{2\pi}{3}$

$$x(t) = \sum a_k e^{jk\frac{2\pi}{3}t}$$

$\cancel{a_k e^{jk\frac{2\pi}{3}t}}$ since $a_k = a_{k+2}$.

For the $(k+2)^{th}$ Harmonic, (Let it be A_{k+2}

$$A_{k+2} = a_{k+2} \cdot e^{j(k+2)\frac{2\pi}{3}t}$$

$$A_{k+2} = a_{k+2}\, e^{jk\frac{2\pi}{3}t}\, e^{j\frac{4\pi}{3}t}$$

$$\Rightarrow A_{k+2} = A_k e^{j\frac{4\pi}{3}t}$$

$$\Rightarrow A_k = A_{k-2} e^{j\frac{4\pi}{3}t}$$

Replacing A_k with $A_{k-2} e^{j\frac{4\pi}{3}t}$, we get

$$x(t) = x(t) e^{j\frac{4\pi}{3}t} \quad \text{(since there are infinite } A_k\text{)}$$

$$\Rightarrow x(t) = 0 \quad \text{or} \quad e^{j\frac{4\pi}{3}t} = 1$$

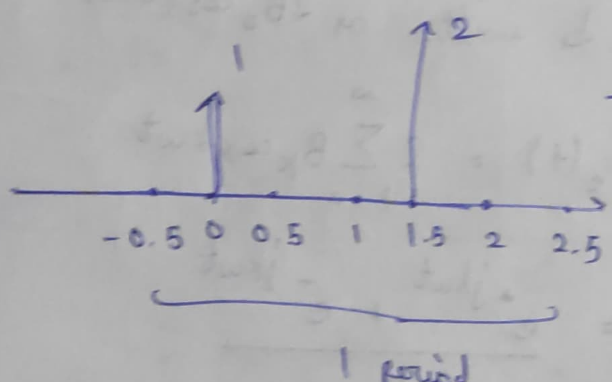
$$\Rightarrow \frac{\cancel{4}^{2}\pi}{3}t = \cancel{2}n\cancel{\pi} \quad \text{or} \quad t=0$$

$$\underline{t = \frac{3n}{2}} \quad \text{or} \quad t = 0$$

$\Rightarrow x(t)$ is non zero for $t = 0, \frac{3}{2}, 3, \frac{9}{2}, 6 \ldots$ — (1)

$$\int_{-1/2}^{1/2} x(t)\,dt = 1 \quad \& \quad \int_{1}^{2} x(t)\,dt = 2$$

$\Rightarrow$



$\rightarrow$ Using the integrals and (1)

$\Rightarrow$ For $t \in (-0.5, 2.5)$

$$\hookrightarrow x(t) = \delta(t) + 2\delta(t - \tfrac{3}{2})$$

$$\Rightarrow \boxed{x(t) = \delta(t+3k) + 2\delta(t - \tfrac{3}{2} + 3k) \quad \forall\, k \in \mathbb{Z}.}$$

3.45. $x(t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega t - 2\sum_{k=1}^{\infty} C_k \sin k\omega t$

$x(t)$ is real and periodic.

a) Even parts of $x(t)$: Terms of $x(t)$ that are equal in $x(t)$ and $x(-t)$.

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega t - 2\sum_{k=1}^{\infty} C_k \sin k\omega t$$

$$x(-t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega t + 2\sum_{k=1}^{\infty} C_k \sin k\omega t$$

On comparing $x(t)$ and $x(-t)$, we get the even part of $x(t)$, ~~B~~, $[x_e(t)]$ as,

$$x_e(t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega t$$

a_0 can be written as $2B_0 \cos(0\omega t)$

$$\Rightarrow x_e(t) = 2\sum_{k=0}^{\infty} B_k \cos k\omega t$$

$$\cos k\omega t = \frac{e^{jk\omega t} + e^{-jk\omega t}}{2}$$

$$\Rightarrow x_e(t) = \cancel{2}\sum_{k=0}^{\infty} B_k \left(\frac{e^{jk\omega t} + e^{-jk\omega t}}{\cancel{2}}\right)$$

$$x_e(t) = \sum_{k=0}^{\infty} B_k e^{jk\omega t} + \sum_{k=0}^{\infty} B_k e^{-jk\omega t}$$

$$\Rightarrow x_e(t) = \sum_{k=0}^{\infty} B_k e^{jk\omega t} + \sum_{k=-\infty}^{0} B_{|k|} e^{jk\omega t}$$

$$\Rightarrow x_e(t) = \sum_{k=-\infty}^{\infty} B_{|k|} e^{jk\omega t}$$

$$\therefore \alpha_k = B_{|k|}$$

Odd Parts of $x(t)$: Terms of $x(t)$ that stay the same ~~in~~ in $-x(-t)$

$$x(t) = a_0 + 2\sum_{k=1}^{\infty} B_k \cos k\omega t - 2\sum_{k=1}^{\infty} C_k \sin k\omega t$$

$$-x(-t) = -a_0 - 2\sum_{k=1}^{\infty} B_k \cos k\omega t - 2\sum_{k=1}^{\infty} C_k \sin k\omega t$$

$$\Rightarrow x_o(t) = -2\sum_{k=1}^{\infty} C_k \sin k\omega t$$

$$= 2\sum_{k=1}^{\infty} C_k(-\sin k\omega t)$$

$$-\sin k\omega t = \frac{(e^{jk\omega t} - e^{-jk\omega t})j}{2}$$

$$\Rightarrow x_o(t) = \cancel{2}\sum_{k=1}^{\infty} C_k \frac{(e^{jk\omega t} - e^{-jk\omega t})j}{\cancel{2}}, \quad \underline{C_0 = 0}$$

$$= \sum_{k=1}^{\infty} jC_k e^{jk\omega t} - \sum_{k=1}^{\infty} jC_k e^{-jk\omega t}$$

$$= \sum_{k=1}^{\infty} jC_k e^{jk\omega t} + \sum_{k=1}^{\infty} j(-C_k) e^{-jk\omega t}$$

$$= \sum_{k=1}^{\infty} jC_k e^{jk\omega t} + \sum_{k=-\infty}^{0} j(-C_{|k|}) e^{jk\omega t}$$

$$= \cancel{\sum_{k=-\infty}^{0} j(C_k \ldots) e^{jk\omega t}}$$

$$\therefore \; \beta_K = \begin{cases} jC_K, & k > 0 \\ -jC_{|K|}, & k < 0 \\ 0, & k = 0 \end{cases}$$

b) $\alpha_K = B_{|K|}$

$\Rightarrow \underline{\alpha_K = \alpha_{-K}}$

$B_K = jC_K$

$B_{-K} = -jC_{|K|} = -jC_K$

$\Rightarrow \underline{B_K = -B_{-K}}$

c) $x(t) = a_0 + 2\sum_{K=1}^{\infty}\left[B_K \cos\left(\frac{2\pi kt}{3}\right) - C_K \sin\left(\frac{2\pi kt}{3}\right)\right]$

$z(t) = d_0 + 2\sum_{K=1}^{\infty}\left[E_K \cos\left(\frac{2\pi kt}{3}\right) - F_K \sin\left(\frac{2\pi kt}{3}\right)\right]$

$y(t) = 4(a_0 + d_0) + 2\sum_{K=1}^{\infty}\left[\left[B_K + \frac{1}{2}E_K\right]\cos\frac{2\pi kt}{3} + F_K \sin\left(\frac{2\pi kt}{3}\right)\right]$

$a_0 = \int_0^3 x(t)e^{-j\omega_0 t}\,dt$ $\qquad$ $d_0 = \int_0^3 x(t)e^{-j\omega_0 t}\,dt$

$= \int_0^3 x(t)\,dt$ $\qquad$ $= \int_0^3 x(t)\,dt$

$= \frac{1}{2}$ $\qquad$ $= 0$

$4(a_0 + d_0) = 4\left(\frac{1}{2}+0\right) = \underline{\underline{2}}$

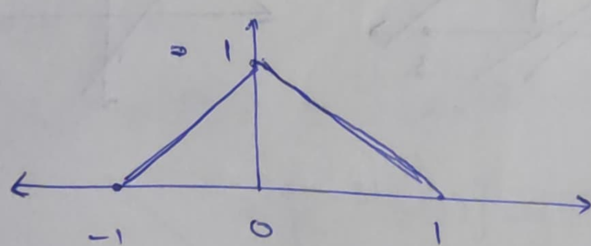
$\Rightarrow y(t) = 2 + \text{Even}(x(t)) + \frac{1}{2}\text{Even}(z(t)) \mp \text{Odd}(z(t)).$

$\cancel{x(t)} = \boxed{t \in [-3,3]} \Rightarrow$

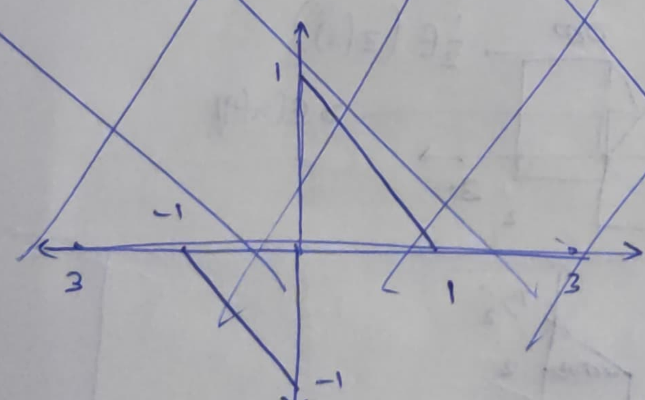
$x(t) = \begin{cases} 2(1-t), & t \in [0,1] \\ 0, & \text{otherwise} \end{cases}$ $\quad x(-t) = \begin{cases} 2(1+t), & t \in [-1,0] \\ 0, & \text{otherwise} \end{cases}$

$z(t) = \begin{cases} 2, & t \in [0,1] \\ -2, & t \in (1,2] \\ 0, & \text{otherwise} \end{cases}$ $\quad z(-t) = \begin{cases} 2, & t \in [-1,0] \\ -2, & t \in [-2,-1) \\ 0, & \text{otherwise} \end{cases}$

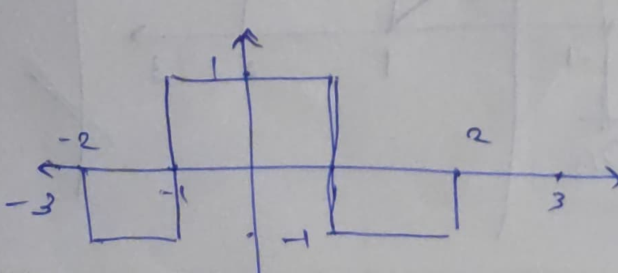
$\text{Even}(x(t)) = \frac{1}{2}[x(t) + x(-t)] = \begin{cases} 1-t, & t \in [0,1] \\ 1+t, & t \in [-1,0) \\ 0, & \text{otherwise} \end{cases}$



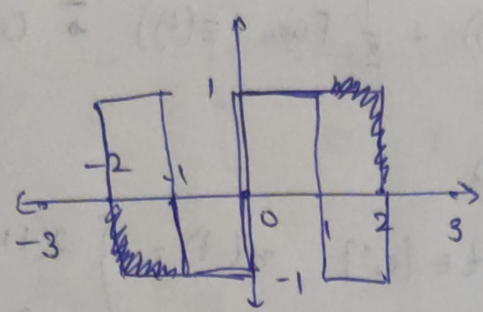
$\text{Odd}(x(t)) = \frac{1}{2}[x(t) - x(-t)] = \begin{cases} 1-t, & t \in [0,1] \\ -(1+t), & t \in [-1,0) \\ 0, & \text{otherwise} \end{cases}$



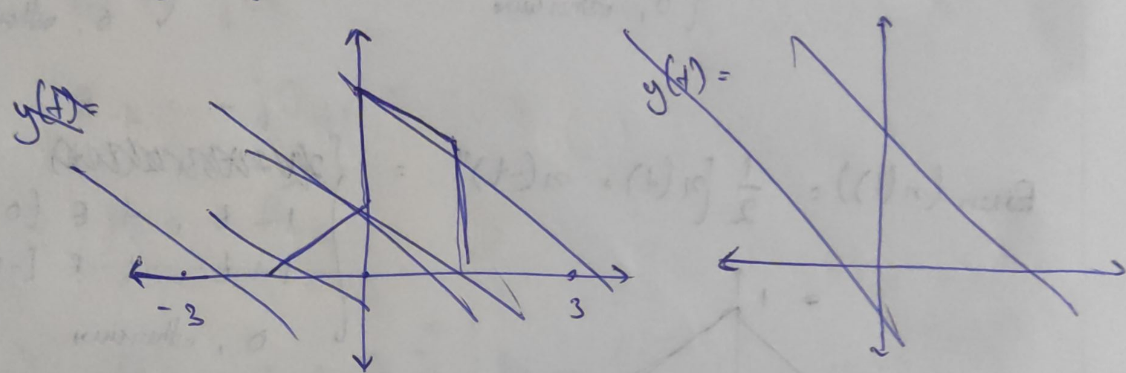
$\text{Even}(z(t)) = \frac{1}{2}(z(t) + z(-t)) = \begin{cases} 1, & t \in [-1,1] \\ -1, & |t| \in (1,2] \\ 0, & \text{otherwise} \end{cases}$

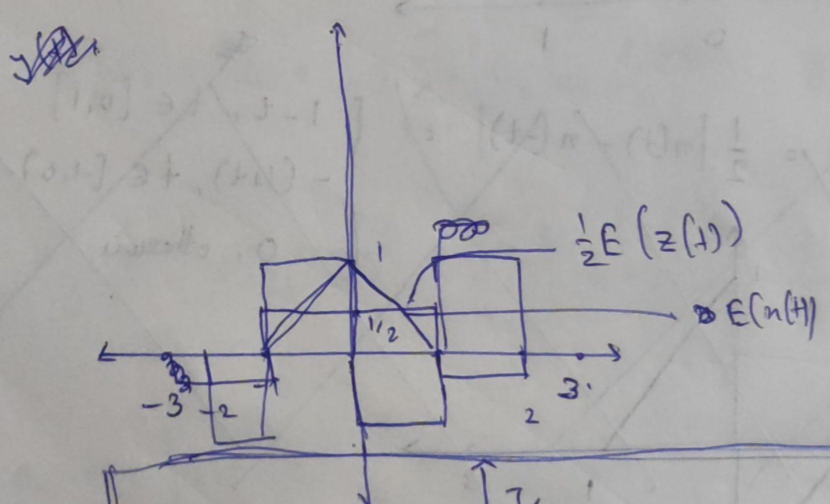


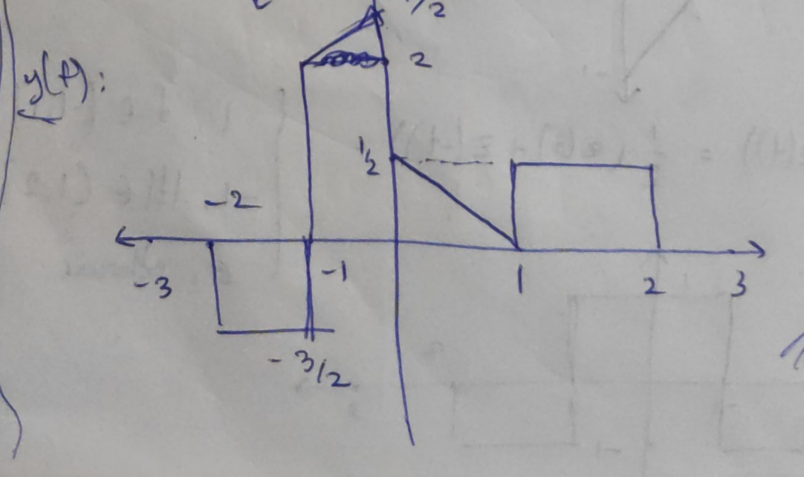
Odd $z(t) = \frac{1}{2}\left(z(t) - z(-t)\right) = \begin{cases} 1 \in [0,2] \\ -1 \in [-2,0] \\ 0, \text{ otherwise} \end{cases}$



$\Rightarrow$ By principle of superposition, the resultant signal for $t \in [-3,3]$







$\Rightarrow$

II) Laplace Transform :-

9.21. a) $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

w.k.t LT of $e^{-at}u(t) = \frac{1}{s+a}$, ROC = $\{s \in \mathbb{C} : Re(s) > -a\}$

$\rightarrow \boxed{X(s) = \frac{1}{s+2} + \frac{1}{s+3}}$, ROC = $\{s \in \mathbb{C} : Re(s) > -2\} \cap$
$\{s \in \mathbb{C} : Re(s) > -3\}$

$\Rightarrow \boxed{ROC = \{s \in \mathbb{C} : Re(s) > -2\}}$

b) $x(t) = e^{-4t}u(t) + e^{-5t}(\sin 5t)\,u(t)$

$$= e^{-4t}u(t) + e^{-5t}\left[\frac{(-j)(e^{j5t} - e^{-j5t})}{2}\right]u(t)$$

$$= e^{-4t}u(t) + \frac{(-j)}{2}e^{(5j-5)t}u(t) + \frac{j}{2}e^{-(5+5j)t}u(t)$$

$$= \frac{1}{s+4} + \left(\frac{-j}{2}\right)\frac{1}{s-5j+5} + \left(\frac{j}{2}\right)\frac{1}{s+5j+5}$$

$$\text{ROC} = \{s \in \mathbb{C} : \mathrm{Re}(s) > -4\}$$

c) $x(t) = e^{2t}u(-t) + e^{3t}u(-t)$

$$X(s) = \frac{1}{s-2} + \frac{1}{s-3}, \quad \text{ROC} = \{s \in \mathbb{C} : \mathrm{Re}(s) < 2\}$$

d) $x(t) = te^{-2|t|} = \begin{cases} te^{-2t}, & t \geq 0 \\ te^{2t}, & t < 0 \end{cases}$

$$\Rightarrow X(s) = \int_{0}^{\infty} te^{-2t} e^{-st} dt + \int_{-\infty}^{0} te^{2t} e^{-st} dt$$

$$= \int_{0}^{\infty} te^{-(2+s)t} dt + \int_{-\infty}^{0} te^{(2-s)t} dt$$

$$= \left[t \int e^{-(2+s)t} dt - \int \int e^{-(2+s)t} dt \cdot dt \right]_{0}^{\infty}$$

$$+ \left[t \int e^{(2-s)t} dt - \int \int e^{(2-s)t} dt \cdot dt \right]_{-\infty}^{0}$$

$$= \left[\frac{-te^{-(2+s)t}}{2+s} - \frac{e^{-(2+s)t}}{(2+s)^2} \right]_{0}^{\infty}$$

$$+ \left[\frac{te^{(2-s)t}}{2-s} - \frac{e^{(2-s)t}}{(2-s)^2} \right]_{-\infty}^{0}$$

$$\lim_{x \to \infty} t e^{-at} \quad (a > 0)$$

$$= \lim_{x \to \infty} \frac{t}{e^{at}} = \lim_{x \to \infty} \frac{1}{a e^{at}} = 0 \quad \text{(L-Hopital Rule)}$$

$$\Rightarrow \left[0 - \left(0 - \frac{1}{(s+2)^2} \right) \right] + \left[\left(0 - \frac{1}{(2-s)^2} \right) - 0 \right]$$

$$= \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2}$$

$Re(s) + 2 > 0$

$\&\ 2 - Re(s) > 0$

$= Re(s) > -2$

$Re(s) < 2$

$$\Rightarrow \boxed{X(s) = \frac{1}{(s+2)^2} - \frac{1}{(s-2)^2} \quad \text{ROC} : \{ s \in \mathbb{C} : Re(s) \in (-2, 2) \}}$$

e) $x(t) = |t| e^{-2|t|} = \begin{cases} t e^{-2t}, & t \geq 0 \\ -t e^{2t}, & t < 0 \end{cases}$

$$\Rightarrow X(s) = \int_0^\infty t e^{-2t} e^{-st} \, dt + \int_{-\infty}^0 -t e^{2t} e^{-st} \, dt$$

$$= \int_0^\infty t e^{-(2+s)t} \, dt - \int_{-\infty}^0 t e^{(2-s)t} \, dt$$

$$= \left[t \int e^{-(2+s)t} \, dt - \int \int e^{-(2+s)t} \, dt \, dt \right]_0^\infty$$

$$- \left[t \int e^{(2-s)t} \, dt - \int \int e^{(2-s)t} \, dt \, dt \right]_{-\infty}^0$$

$$= \left[\frac{-t e^{-(2+s)t}}{2+s} - \frac{e^{-(2+s)t}}{(2+s)^2} \right]_0^{\infty} - \left[\frac{t e^{(2-s)t}}{2-s} - \frac{e^{(2-s)t}}{(2-s)^2} \right]_{-\infty}^{0}$$

$$= \left[(0-0) - \left(0 - \frac{1}{(2+s)^2}\right) \right] - \left[\left(0 - \frac{1}{(2-s)^2}\right) - (0-0) \right]$$

$$= \frac{1}{(2+s)^2} + \frac{1}{(2-s)^2}$$

$Re(s) > -2$ &
$Re(s) < 2$

$$\Rightarrow \boxed{X(s) = \frac{1}{(2+s)^2} + \frac{1}{(2-s)^2}, \quad ROC : \{ s \in \mathbb{C} : Re(s) \in (-2, 2) \}}$$

f) $x(t) = |t| e^{2t} u(-t) = \begin{cases} 0, & t \geq 0 \\ -t e^{2t}, & t < 0 \end{cases}$

$$X(s) = \int_{-\infty}^{0} -t e^{2t} e^{-st} dt$$

$$= -\int_{-\infty}^{0} t e^{(2-s)t} dt$$

$$= \left[t \int e^{(2-s)t} dt - \iint e^{(2-s)t} dt \cdot dt \right]_0^{-\infty}$$

$$= \left[\frac{t e^{(2-s)t}}{2-s} - \frac{e^{(2-s)t}}{(2-s)^2} \right]_0^{-\infty}$$

$$= \left[(0-0) - \left(0 - \frac{1}{(2-s)^2}\right) \right], \quad 2 - Re(s) < 0 \Rightarrow Re(s) > 2$$

$$\boxed{X(s) = \frac{1}{(2-s)^2}, \quad ROC: \{s \in \mathbb{C} : Re(s) > 2\}}$$

g) $x(t) = \begin{cases} 1, & t \in [0,1] \\ 0, & \text{otherwise} \end{cases}$

$$X(s) = \int_0^1 e^{-st} dt = \left[\frac{-1}{s} e^{-st} \right]_0^1$$

$$= \left[\frac{-1}{s} e^{-s} - \left(\frac{-1}{s}\right) \right]$$

$$\boxed{X(s) = \left[-\frac{1}{s} e^{-s} + \frac{1}{s} \right]} \quad ROC: \{s \in \mathbb{C}\}$$

h) $x(t) = \begin{cases} t, & 0 \le t \le 1 \\ 2-t, & 1 \le t \le 2 \end{cases}$

$$\Rightarrow X(s) = \int_0^1 t e^{-st}\,dt + \int_1^2 (2-t) e^{-st}\,dt$$

$$= \left[t \int e^{-st} dt - \int\int e^{-st} dt \cdot dt \right]_0^1$$

$$- \left[(2-t) \int e^{-st} dt + \int\int e^{-st} dt\, dt \right]_1^2$$

$$= \left[-\frac{t}{s} e^{-st} - \left(\frac{1}{s}\right)^2 e^{-st} \right]_0^1 - \left[-\frac{(2-t)}{s} e^{-st} \right.$$

$$\left. + \left(\frac{1}{s}\right)^2 e^{-st} \right]_1^2$$

$$= \left[\left(-\frac{e^{-s}}{s} - \frac{e^{-s}}{s^2} \right) - \left(0 - \left(\frac{1}{s}\right)^2 \right) \right] - \left[\left(0 + \frac{e^{-2s}}{s^2} \right) \right.$$

$$\left. - \left(-\frac{e^{-s}}{s} + \frac{e^{-s}}{s^2} \right) \right]$$

$$= -\frac{e^{-s}}{s} - \cancel{\frac{e^{-s}}{s^2}} + \frac{1}{s^2} - \frac{e^{-2s}}{s^2} - \frac{e^{-s}}{s} + \cancel{\frac{e^{-s}}{s^2}}$$

$$\boxed{X(s) = -\frac{2e^{-s}}{s} - \frac{e^{-2s}}{s^2} + \frac{1}{s^2}, \quad \text{ROC}: \{s \in \mathbb{C}\}}$$

i) $x(t) = \delta(t) + u(t)$

$$X(s) = \int_{-\infty}^{\infty} (\delta(t) + u(t)) e^{-st}\,dt$$

$$= \int_{-\infty}^{\infty} \delta(t) e^{-st}\,dt + \int_{-\infty}^{\infty} u(t) e^{-st}\,dt$$

$$= e^{-s(0)} + \int_0^{\infty} e^{-st}\,dt$$

$$= 1 + \left[\left(\frac{-1}{s} \right) e^{-st} \right]_0^{\infty}$$

$$= 1 + \left[0 - \left(\frac{-1}{s} \right) \right] \quad Re(s) > 0$$

$$\boxed{X(s) = 1 + \frac{1}{s}, \ Roc: \{ s \in \mathbb{C} : Re(s) > 0 \}}$$

j) $x(t) = \delta(3t) + u(3t)$

Time Scaling does not affect $\delta(t)$ and $u(t)$ signals,

$\Rightarrow x(t) = \delta(t) + u(t)$

$$\Rightarrow \boxed{X(s) = 1 + \frac{1}{s}, \ Roc: \{ s \in \mathbb{C}, Re(s) > 0 \}}$$

9.26. $y(t) = x_1(t-2) * x_2(-t+3)$

$x_1(t) = e^{-2t} u(t)$, $x_2(t) = e^{-3t} u(t)$

$\downarrow$

$X_1(s) = \dfrac{1}{s+2}$ R: $\{ s \in \mathbb{C} : Re(s) > -2 \}$

$X_2(s) = \dfrac{1}{s+3}$ R: $\{ s \in \mathbb{C} : Re(s) > -3 \}$

Due to time shifting

$X_1(s) = \dfrac{e^{-s(2)}}{s+2}$ Roc: $\{ s \in \mathbb{C}, Re(s) > -2 \}$

$\longrightarrow x_1(t-2)$

$X_2(s) = \dfrac{e^{-s(3)}}{s+3}$ Roc: $\{ s \in \mathbb{C}, Re(s) > -3 \}$

$\longrightarrow x_2(t - -3)$

Due to time reversal

$x_2(-(t-3))$, $X_2(s) \Rightarrow \frac{1}{|-1|} X_2\left(\frac{s}{-1}\right)$

$$= X_2(s) = \frac{e^{-(-s)(3)}}{-s+3}, \quad \text{ROC } \{s \in \mathbb{C}, \text{Re}(s) > -3\}$$

By convolution property,

$$Y(s) = X_1(s) \cdot X_2(s)$$

$$\boxed{Y(s) = \left(\frac{e^{-2s}}{s+2}\right)\left(\frac{e^{3s}}{3-s}\right) \quad \text{ROC} : R \text{ st}\left(\{\text{Re}(s) > -2\} \subseteq R\right)}$$

9.29. a) $x(t) = e^{-t}u(t)$

$$X(s) = \frac{1}{s+1}, \quad \text{ROC} : \{s \in \mathbb{C} : \text{Re}(s) > -1\}$$

$$h(t) = e^{-2t}u(t)$$

$$H(s) = \frac{1}{s+2}, \quad \text{ROC} : \{s \in \mathbb{C} : \text{Re}(s) > -2\}$$

b) By convolution property of LT,

$Y(s) = X(s) \cdot H(s)$, (Since $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t-\tau)\, d\tau$

or $y(t) = x(t) * h(t)$)

$\downarrow$

Response of LTI system

$$Y(s) = \frac{1}{(s+1)(s+2)}$$

$$\text{ROC} = R \text{ st}\left(\{s \in \mathbb{C} : \text{Re}(s) > -1\} \subseteq R\right)$$

c) $Y(s) = \dfrac{1}{(s+1)(s+2)}$

$= \dfrac{(s+2)-(s+1)}{(s+1)(s+2)}$

$Y(s) = \dfrac{1}{s+1} - \dfrac{1}{s+2}$

ROC = R et $\{s \in \mathbb{C} : Re(s) > -1\} \subseteq R$

By observation, we can figure out that

$\Rightarrow \boxed{y(t) = e^{-t}u(t) - e^{-2t}u(t)}$

(Since if $x(t) = e^{-at}u(t)$

$X(s) = \dfrac{1}{s+a}$,

ROC: $\{s \in \mathbb{C} : Re(s) > -a\}$)

d) $y(t) = x(t) * h(t)$

$= \int_{-\infty}^{\infty} x(\tau) h(t-\tau)\, d\tau$

$= \int_{-\infty}^{\infty} e^{-\tau}u(\tau)\, e^{-2(t-\tau)} u(t-\tau)\, d\tau$

$= e^{-2t} \int_{-\infty}^{\infty} e^{-\tau} u(\tau) e^{2\tau} u(t-\tau)\, d\tau$

$t - \tau \geq 0$

$t \geq \tau$

$= e^{-2t} \int_{0}^{\infty} e^{\tau} u(t-\tau)\, d\tau$

$= e^{-2t} \int_{0}^{t} e^{\tau}\, d\tau$ (for $t \geq 0$)

$= e^{-2t} \left[e^{\tau} \right]_0^t$

$$= e^{-2t} e^{t} - e^{-2t}$$

$$\Rightarrow y(t) = e^{-t} - e^{-2t} \quad (\text{for } t > 0)$$

$$\Rightarrow \boxed{y(t) = (e^{-t} - e^{-2t})\, u(t)}$$

$u(t)$ takes care of $t > 0$ condition as ~~integral~~ value of integration is 0 if $t \leq 0$.

$\therefore$ Matches with part (c)