THE PENNSYLVANIA STATE UNIVERSITY Test
SCHREYER HONORS COLLEGE

DEPARTMENT OF DEPARTMENT

TITLE OF THESIS

NAME
SEASON NOW

A thesis
submitted in partial fulfillment
of the requirements
for baccalaureate degrees
in Major1 and Major2
with honors in Area of Honors

Reviewed and approved* by the following:

Some Person
Professor of Something
Thesis Supervisor

Some Other Person
Professor of Something Else
Some Other Position
Honors Adviser

*Signatures are on file in the Schreyer Honors College.

Abstract

Abstract abstract abstract. Abstract? Abstract! Abstract...

Table of Contents

List of Figures

List of Tables

Acknowledgements

To Mr. Scruffles.

Chapter 1

Introduction and Historical Review

1.1 Introduction

Blah blah blah. Here is an example of how to include and cite a figure: see Figure 1.1

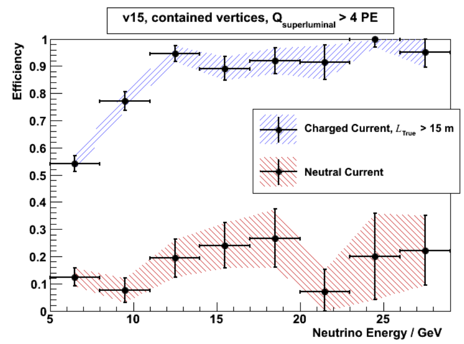

Figure 1.1: Caption to go underneath figure

1.2 Literature Review

Blah blah blah Here's an example of a table: see Table 1.1.

Parameter	**Current Best Value**
Δm^2_{21}	$7.50^{+0.19}_{-0.20} \times 10^{-3}$ eV^2
$\lvert\Delta m^2_{32}\rvert$	$2.32^{+0.12}_{-0.08} \times 10^{-5}$ eV^2
$\sin^2(\theta_{12})$	$0.857^{+0.023}_{-0.025}$
$\sin^2(2\theta_{23})$	> 0.95
$\sin^2(\theta_{13})$	0.098 ± 0.013

Table 1.1: Name of table for caption above table

Here's an example of an equation and some math:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{1.1}$$

with $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$.

Here's an example citation: [1].

1.3 Thesis Outline

Impulsive thrust approximations are often used to simply the analysis of the optimal maneuver between circular orbits. In actuality, spacecraft exhibit finite thrust arcs to perform these orbital transfers. This thesis analyzes these minimum burn time, and thus fuel consumption, thrust arcs required for these optimal orbital transfers. Requiring two separate numerical integrations, this is computationally extensive. This thesis then seeks to develop methods to drastically reduce the time required for these computations through means of a C++ implementation utilizing Boost libraries and parallelization through OpenMP. Chapter 2 will detail the finite thrust transfer problem solved within this analysis, while Chapter 3 details the implementation details which were used. Chapter 4 then details the results of these implementations, including speedup comparisons and methods to greater improve the probability of the algorithm approaching an optimal solution.

Chapter 2

Problem Statement

2.1 Problem Definition

This thesis analyzes the optimal finite thrust transfer between two circular orbits. A PSO approach is utlized to attempt to find and optimal solution. The development of this problem is derived from [2].

This transfer is developed with respect to an initial circular orbit of radius R_1 and a final circular orbit of radius R_2 subject to the conditions $R_1 > R_2$ where the parameter $\beta = R_2/R_1 > 1$. The inertial reference frame selected for this problem definition is centered at the attracting body. The corresponding coordinate frame definition is as follows: The x axis is aligned with the spacecraft at the initial time t_0 and the y axis is located in the orbital plane. The z axis is determined by the right hand rule of these two basis axis. Within this problem v_r denotes the spacecraft radial velocity, v_θ the spacecraft the horizontal component of velocity, r the radisu, and ζ the angular displacement from the x axis. The gravitational parameter of the attracting body is μ_B.

Initial conditions at time t_0 are given by

$$v_r(t_0) = 0 \quad v_\theta(t_0) = \sqrt{\mu_B/R_1} \quad r(t_0) = R_1 \quad \xi(t_0) = 0 \tag{2.1}$$

And final conditions given by

$$v_r(t_f) = 0 \quad v_\theta(t_f) = \sqrt{\mu_B/R_2} \quad r(t_f) = R_2 \tag{2.2}$$

The problem assumes a transfer trajectory of an initial thrust arc, followed by a coasting arc, and ending with a second thrust arc. Optimization of this problem corresponds to determining the thrust pointing angle function corresponding to the minimization of propellant consumption subject to the constraints of equation 2.2.

Additional assumptions are made to simplify the analysis:

1. Throughout the duration of each thrust arc maximum thrust is generated
2. Each thrust arc pointing angle is represented as a third-degree polynomial as a function of time.

T and c are used to represent the spacecraft thrust level and effective exhaust velocity respectively. The previous assumptions indicate that the thrust-to-mass ratio has the following form

$$\frac{T}{m} = \begin{cases} \dfrac{T}{m_0 - \frac{T}{c}t} = \dfrac{cn_0}{c - n_0 t} & 0 \leq t \leq t_1 \\ 0 & t_1 \leq t \leq t_2 \\ \dfrac{T}{m_0 - \frac{T}{c}(t_1 + t - t_2)} = \dfrac{cn_0}{c - n_0(t_1 + t - t_2)} & t_2 \leq t \leq t_f \end{cases} \tag{2.3}$$

Where m_0 is the initial spacecraft mass and n_0 is the initial thrust to mass ratio at t_0. The state space equations of motion for the spacecraft are as follows

$$\dot{v} = -\frac{\mu_B - r v_\theta^2}{r^2} + \frac{T}{m}\sin\delta \tag{2.4}$$

$$\dot{v_\theta} = -\frac{v_r v_\theta}{r} + \frac{T}{m}\cos\delta \tag{2.5}$$

$$\dot{r} = v_r \tag{2.6}$$

$$\dot{\xi} = \frac{v_\theta}{r} \tag{2.7}$$

where $\dfrac{T}{m}$ is given in equation 2.3 and δ is the thrust pointing angle represent by a third-order polynomial of time. The state vector used in this problem is $[x_1 \ x_2 \ x_3 \ x_4]^T = [v_r \ v_\theta \ r \ \xi]^T$.

The thurst pointing angle δ is defined as

$$\delta = \begin{cases} \zeta_0 + \zeta_1 t + \zeta_2 t^2 + \zeta_3 t^3 & 0 \leq t \leq t_1 \\ \nu_0 + \nu_1(t - t_2) + \nu_2(t - t_2)^2 + \nu_3(t - t_3)^3 & t_2 \leq t \leq t_f \end{cases} \tag{2.8}$$

Where the optimum thrust pointing angle coefficients $\{\zeta_0 \ \zeta_1 \ \zeta_2 \ \zeta_3\}$ and $\{\nu_0 \ \nu_1 \ \nu_2 \ \nu_3\}$ are determined by PSO.

During the coasting arc the problem consists of a Kepler orbit. As such, the semimajor axis a and eccentricity e of the coasting arc can be computed

$$a = \frac{\mu_B r_1}{2\mu_B - r_1(v_{r1}^2 + v_{\theta 1}^2)} \tag{2.9}$$

$$e = \sqrt{1 - \frac{r_1^2 v_{\theta_1}^2}{\mu_B a}} \tag{2.10}$$

Provided that the orbit is elliptic ($a > 0$) the true anomaly at $t_1(f_1)$ can be computed as

$$\sin f_1 = \frac{v_{r1}}{e}\sqrt{\frac{a(1-e^2)}{\mu_B}} \quad \text{and} \quad \cos f_1 = \frac{v_{\theta_1}}{e}\sqrt{\frac{a(1-e^2)}{\mu_B}} - \frac{1}{e} \tag{2.11}$$

and the eccentric anomaly E_1 as

$$\sin E_1 = \frac{\sin f_1 \sqrt{1-e^2}}{1 + e\cos f_1} \quad \text{and} \quad \cos E_1 = \frac{\cos f_1 + e}{1 + e\cos f_1} \tag{2.12}$$

The PSO parameter ΔE represents the variation in eccentric anomaly througout the coasting arc. Therefore, the eccentric enomaly at t_2 is $E_2 = E_1 + \Delta E$. True anamoly can be thus determined utilizing the results of equations in 2.12

$$\sin f_2 = \frac{\sin E_2 \sqrt{1-e^2}}{1 - e\cos E_2} \quad \text{and} \quad \cos f_2 = \frac{\cos E_2 - e}{1 - e\cos E_2} \tag{2.13}$$

Furthermore, the coasting time interval t_{co} can be calculated through Kepler's law as

$$t_{co} \triangleq t_2 - t_1 = \sqrt{\frac{a^3}{\mu_B}}[E_2 - E_1 - e(\sin E_2 - \sin E_1)] \tag{2.14}$$

The results from 2.9, 2.10, and 2.13 provide the initial conditions required to numerical integrate the spacecraft equations of motion for the second thrust arc beginning at time t_2. These initial conditions for the second thrust arc are computed as

$$v_{r_2} = \sqrt{\frac{\mu_B}{a(1-e^2)}} e \sin f_2 \tag{2.15}$$

$$v_{\theta 2} = \sqrt{\frac{\mu_B}{a(1-e^2)}}(1 + e\cos f_2) \tag{2.16}$$

$$r_2 = \frac{a(1-e^2)}{1 + e\cos f_2} \tag{2.17}$$

$$\xi_2 = \xi_1 + (f_2 - f_1) \tag{2.18}$$

Integrating the second thrust arc with these initial conditions over the second thrust arc time duration with yield the final orbital characteristics.

This system depends on the eight coefficients representing the thrust pointing angles of the first and second thrust arcs, $\{\zeta_0\ \zeta_1\ \zeta_2\ \zeta_3\}$ and $\{\nu_0\ \nu_1\ \nu_2\ \nu_3\}$ respectively, and the time intervals for the first coast arc $\Delta t_1 \triangleq t_1$, the coasting arc, Δt_{co}, and the second coast arc $\Delta t_2 \triangleq t_f - t_2$. These 11 unknowns are sought to be chosen to minimize the objective function

$$J = \Delta t_1 + \Delta t_2 \tag{2.19}$$

Optimizing the unknown coefficients to minimize 2.19 corresponds to the minimization of propellant consumption. Minimizing propellant consumption results in a maximization of the final-to-initial mass ratio given by

$$\frac{m_f}{m_0} = \frac{m_0 - \frac{T}{c}\Delta t_1 - \frac{T}{c}\Delta t_2}{m_0} = 1 - \frac{n_0}{c}(\Delta t_1 + \Delta t_2) \tag{2.20}$$

The coasting arc is required to be elliptic, therefore any particles with $a \leq 0$ are assigned an infinite value.

Since Δt_{co} can be computed using ΔE, the eccentric anomaly variation replaces the coasting time interval in the problem's 11 unkown parameters. Each particle in the swarm thus consists of the following

$$\chi = [\zeta_0 \quad \zeta_1 \quad \zeta_2 \quad \zeta_3 \quad \nu_0 \quad \nu_1 \quad \nu_2 \quad \nu_3 \quad \Delta t_1 \quad \Delta E \quad \Delta t_2]^T \tag{2.21}$$

2.2 Canonical Units Definition

2.3 Parameter Definition

2.4 Time Optimization (probably needs rewording)

Chapter 3

Methodology

3.1 Particle Swarm Optimization Overview

3.2 Problem Implementation

Stuff detailing the general rk-dopri-5 algorithm goes here

3.2.1 MATLAB

MATLAB implementation details go here e.g. ODE45, etc.

3.2.2 C++ Single Threaded

Details about Boost libraries, timers, etc. go here

3.2.3 C++ Parallelization

Details about OpenMP go Test

Chapter 4

Results

4.1 Numerical Results

4.2 Rehydration Results

4.3 Speedup

Bibliography

[1] J. Beringer et al. Review of particle physics. *Phys. Rev. D*, 86:010001, Jul 2012.

[2] Mauro Pontani and Bruce A. Conway. Particle swarm optimization applied to space trajectories. *Journal of Guidance, Control, and Dynamics*, 33(5):1429–1441, 2010.