Here also the graph of the function $y = f(x)$ is approximated by a secant line but at each iteration, two most recent approximations to the root are used to find the next approximation. Also it is not necessary that the interval must contain the root.

Taking x_0, x_1 as the initial limits of the interval, we write the equation of the chord joining these as

$$y - f(x_1) = \frac{f(x_1) - f(x_0)}{x_1 - x_0}(x - x_1)$$

Then the abscissa of the point where it crosses the x-axis ($y = 0$) is given by

$$x_2 = x_1 - \frac{x_1 - x_o}{f(x_1) - f(x_0)} f(x_1)$$

which is an approximation to the root. The general formula for successive approximations is, therefore, given by

$$x_{n+1} = x_n - \frac{x_n - x_n - 1}{f(x_n) - f(x_n - 1)} f(x_n), n \geq 1.$$

Rate of Convergence. If at any interation $f(x_n) = f(x_{n-1})$, this method fails and shows that it does not converge necessarily. This is a drawback of secant method over the method of false position which always converges. But if the secant method once converges, its rate of convergence is 1.6 which is faster than that of the method of false position.

EXAMPLE 2.23

Find a root of the equation $x^3 - 2x - 5 = 0$ using the secant method correct to three decimal places.

Solution:

Let $f(x) = x^3 - 2x - 5$ so that $f(2) = -1$ and $f(3) = 16$.

$\therefore$ Taking initial approximations $x_0 = 2$ and $x_1 = 3$, by the secant method, we have

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 3 - \frac{3-2}{16+1} 16 = 2.058823$$

Now $\quad f(x_2) = -0.390799$

$$\therefore \qquad x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 2.081263$$

and $\quad f(x_3) = -0.147204$

$$\therefore \qquad x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 2.094824$$

and $\quad f(x_4) = 0.003042$

$$\therefore \qquad x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4) = 2.094549$$

Hence the root is 2.094 correct to three decimal places

EXAMPLE 2.24

Find the root of the equation $xe^x = \cos x$ using the secant method correct to four decimal places.

Solution:

Let $\quad f(x) = \cos x - xe^x = 0.$

Taking the initial approximations $x_0 = 0$, $x_1 = 1$

so that $f(x_0) = 1$, $f(x_1) = \cos 1 - e = -2.17798$

Then by the secant method, we have

$$x_2 = x_1 - \frac{x_1 - x_0}{f(x_1) - f(x_0)} f(x_1) = 1 + \frac{1}{3.17798}(-2.17798) = 0.31467$$

Now $\quad f(x_2) = 0.51987$

$$\therefore x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 0.44673 \textit{ and } f(x_3) = 0.20354$$

$$\therefore \qquad x_4 = x_3 - \frac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3) = 0.53171$$

Repeating this process, the successive approximations are $x_5 = 0.51690$, $x_6 = 0.51775$, $x_7 = 0.51776$ etc.

Hence the root is 0.5177 correct to four decimal places.

NOTE

Obs. *Comparing Examples 2.18 and 2.21, we notice that the rate of convergence in the secant method is definitely faster than that of the method of false position*