A Proportionate NLMS Algorithm for the Identification of Sparse Bilinear Forms

Constantin Paleologu*, Jacob Benesty†, Camelia Elisei-Iliescu*,
Cristian Stanciu*, Cristian Anghel*, and Silviu Ciochină*
*Department of Telecommunications, University Politehnica of Bucharest, Romania
†INRS-EMT, University of Quebec, Montreal, Canada
Email: pale@comm.pub.ro

***Abstract*—Proportionate-type algorithms are designed to exploit the sparseness character of the systems to be identified, in order to improve the overall convergence of the adaptive filters used in this context. However, when the parameter space is large, the system identification problem becomes more challenging. In this paper, we focus on the identification of bilinear forms, where the bilinear term is defined with respect to the impulse responses of a spatiotemporal model. In this framework, we develop a proportionate normalized least-mean-square algorithm tailored for the identification of such bilinear forms. Simulation results indicate the good performance of the proposed algorithm, in terms of both convergence rate and computational complexity.**

***Keywords*—adaptive filter; bilinear forms; multiple-input/single-output system; proportionate normalized least-mean-square algorithm; sparse system identification.**

I. Introduction

Sparse adaptive filters are frequently involved in different system identification problems. In this framework, most of these algorithms were developed in the context of echo cancellation, in both network and acoustic scenarios [1], [2]. Usually, they are referred to as proportionate algorithms [3]– [6], since the basic idea is to "proportionate" the algorithm's behavior, i.e., to update each coefficient of the filter independently of the others, by adjusting the adaptation step size in proportion to the magnitude of the estimated filter coefficient.

Nevertheless, the system identification problems become more challenging when the parameter space becomes larger [7]. For example, such frameworks can be found in conjunction with nonlinear acoustic echo cancellation [8], [9], where the global system resembles (to some extent) the Hammerstein model [10]. Alternatively, the problem can be reduced to the identification of a bilinear form, where the bilinear term is defined with respect to the impulse responses of a spatiotemporal model, which resembles a multiple-input/single-output (MISO) system [11], [12].

In [11], an iterative Wiener filter was developed for the identification of such bilinear forms, while [12] provides an analysis of the conventional adaptive algorithms designed for this purpose, including the normalized least-mean-square (NLMS) algorithm. In this work, we develop a proportionate NLMS algorithm for the identification of bilinear forms.

The rest of the paper is organized as follows. Section II provides an overview of the system model, together with the NLMS algorithm designed in this context. In Section III, the proposed proportionate-type algorithm is introduced. Simulation results are presented in Section IV. Finally, Section V concludes this work.

The work of Constantin Paleologu and Silviu Ciochină was supported under the Grants SeaForest 86/2016 and SenSyStar 190/2017. Camelia Elisei-Iliescu would like to acknowledge the support of ROMATSA. The work of Cristian Stanciu and Cristian Anghel was supported by European Space Agency under Grant no. 4000121222/17/NL/CBi and by University Politehnica of Bucharest under Grant no. 18/05.10.2017.

II. Bilinear Model and the NLMS-BF Algorithm

In the framework of our bilinear model, the reference signal is defined in the context of a simplified MISO system as

$$d(t) = \mathbf{h}^T \mathbf{X}(t) \mathbf{g} + w(t) = y(t) + w(t), \tag{1}$$

where t is the discrete-time index, $\mathbf{h}$ and $\mathbf{g}$ are the two impulse responses of the system of lengths L and M, respectively, $\mathbf{X}(t) = \begin{bmatrix} \mathbf{x}_1(t) & \mathbf{x}_2(t) & \cdots & \mathbf{x}_M(t) \end{bmatrix}$ is the zero-mean multiple-input signal matrix of size $L \times M$, $\mathbf{x}_m(t) = \begin{bmatrix} x_m(t) & x_m(t-1) & \cdots & x_m(t-L+1) \end{bmatrix}^T$ is a vector containing the L most recent samples of the mth $(m = 1, 2, \ldots, M)$ input signal, and $w(t)$ is the zero-mean additive noise. It is assumed that all the signals are real valued, and $\mathbf{X}(t)$ and $w(t)$ are uncorrelated. The two impulse responses, i.e., $\mathbf{h}$ and $\mathbf{g}$, correspond to the temporal and spatial parts of the system, respectively.

Based on the vectorization operation (i.e., conversion of a matrix into a vector [13]), the matrix $\mathbf{X}(t)$ of size $L \times M$ can be rewritten as a vector of length ML:

$$\text{vec}\left[\mathbf{X}(t)\right] = \begin{bmatrix} \mathbf{x}_1^T(t) & \mathbf{x}_2^T(t) & \cdots & \mathbf{x}_M^T(t) \end{bmatrix}^T = \widetilde{\mathbf{x}}(t). \tag{2}$$

Therefore, the output signal $y(t)$ results in

$$\begin{aligned} y(t) &= \text{tr}\left[\left(\mathbf{h}\mathbf{g}^T\right)^T \mathbf{X}(t)\right] = \text{vec}^T\left(\mathbf{h}\mathbf{g}^T\right) \text{vec}\left[\mathbf{X}(t)\right] \\ &= \left(\mathbf{g} \otimes \mathbf{h}\right)^T \widetilde{\mathbf{x}}(t) = \mathbf{f}^T \widetilde{\mathbf{x}}(t), \end{aligned} \tag{3}$$

where $\text{tr}[\cdot]$ denotes the trace of a square matrix, $\otimes$ is the Kronecker product, and $\mathbf{f} = \mathbf{g} \otimes \mathbf{h}$ is the spatiotemporal (or global) impulse response of length ML, which is the Kronecker product between the individual impulse responses $\mathbf{g}$ and $\mathbf{h}$. Consequently, the reference signal in (1) becomes

$$d(t) = \mathbf{f}^T \widetilde{\mathbf{x}}(t) + w(t). \tag{4}$$

The goal is to identify the temporal and spatial impulse responses, $\mathbf{h}$ and $\mathbf{g}$, with two adaptive filters $\widehat{\mathbf{h}}(t) = \left[\begin{array}{cccc} \widehat{h}_1(t) & \widehat{h}_2(t) & \cdots & \widehat{h}_L(t) \end{array} \right]^T$ and $\widehat{\mathbf{g}}(t) = \left[\begin{array}{cccc} \widehat{g}_1(t) & \widehat{g}_2(t) & \cdots & \widehat{g}_M(t) \end{array} \right]^T$, respectively. Consequently, the spatiotemporal impulse response $\mathbf{f}$ can be identified with a long filter $\widehat{\mathbf{f}}(t) = \left[\begin{array}{cccc} \widehat{f}_1(t) & \widehat{f}_2(t) & \cdots & \widehat{f}_{ML}(t) \end{array} \right]^T$, which results as $\widehat{\mathbf{f}}(t) = \widehat{\mathbf{g}}(t) \otimes \widehat{\mathbf{h}}(t)$.

Let $\eta \neq 0$ be a real-valued number. It is clear from (1) that $y(t) = \mathbf{h}^T\mathbf{X}(t)\mathbf{g} = (\mathbf{h}/\eta)^T \mathbf{X}(t)(\eta\mathbf{g})$, so that the pair $\mathbf{h}/\eta$ and $\eta\mathbf{g}$ is equivalent to the pair $\mathbf{h}$ and $\mathbf{g}$ in the bilinear form. This implies that we can only identify $\widehat{\mathbf{h}}(t)$ and $\widehat{\mathbf{g}}(t)$ up to a scaling factor. On the other hand, since $\mathbf{f} = \mathbf{g} \otimes \mathbf{h} = (\eta\mathbf{g}) \otimes (\mathbf{h}/\eta)$, the global impulse response will be identified with no scaling ambiguity. Therefore, to evaluate the identification of the temporal and spatial filters, we should use the normalized projection misalignment (NPM) [14], while the identification of the spatiotemporal filter should be evaluated with the usual normalized misalignment (NM), i.e., $\mathrm{NM}\left[\mathbf{f}, \widehat{\mathbf{f}}(t)\right] = \left\| \mathbf{f} - \widehat{\mathbf{f}}(t) \right\|^2 / \left\| \mathbf{f} \right\|^2$, where $\|\cdot\|$ denotes the Euclidean norm.

Let us consider the two adaptive filters $\widehat{\mathbf{h}}(t)$ and $\widehat{\mathbf{g}}(t)$, and the estimated signal $\widehat{y}(t) = \widehat{\mathbf{h}}^T(t-1)\mathbf{X}(t)\widehat{\mathbf{g}}(t-1)$. As a result, we can define the error signal between the desired and estimated signals as

$$\begin{aligned} e(t) &= d(t) - \widehat{y}(t) = d(t) - \left[\widehat{\mathbf{g}}(t-1) \otimes \widehat{\mathbf{h}}(t-1)\right]^T \widetilde{\mathbf{x}}(t) \\ &= d(t) - \widehat{\mathbf{f}}^T(t-1)\widetilde{\mathbf{x}}(t). \end{aligned} \tag{5}$$

An alternative way is to define two error signals:

$$e_{\widehat{\mathbf{g}}}(t) = d(t) - \widehat{\mathbf{h}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t), \tag{6}$$

$$e_{\widehat{\mathbf{h}}}(t) = d(t) - \widehat{\mathbf{g}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t), \tag{7}$$

with $\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) = \left[\widehat{\mathbf{g}}(t-1) \otimes \mathbf{I}_L\right]^T \widetilde{\mathbf{x}}(t)$ and $\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) = \left[\mathbf{I}_M \otimes \widehat{\mathbf{h}}(t-1)\right]^T \widetilde{\mathbf{x}}(t)$, where $\mathbf{I}_L$ and $\mathbf{I}_M$ are the identity matrices of sizes $L \times L$ and $M \times M$, respectively. We can notice that $e_{\widehat{\mathbf{g}}}(t) = e_{\widehat{\mathbf{h}}}(t) = e(t)$. However, for the clarity of the developments, we keep the notation from (6) and (7).

In this context, the NLMS-based algorithm for bilinear forms, namely NLMS-BF [12], is defined by the updates

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \frac{\alpha_{\widehat{\mathbf{h}}}\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)e_{\widehat{\mathbf{g}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}^T(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) + \delta_{\widehat{\mathbf{h}}}}, \tag{8}$$

$$\widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \frac{\alpha_{\widehat{\mathbf{g}}}\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t)e_{\widehat{\mathbf{h}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}^T(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) + \delta_{\widehat{\mathbf{g}}}}, \tag{9}$$

where $\alpha_{\widehat{\mathbf{h}}}$ $(0 < \alpha_{\widehat{\mathbf{h}}} < 2)$ and $\alpha_{\widehat{\mathbf{g}}}$ $(0 < \alpha_{\widehat{\mathbf{g}}} < 2)$ are the normalized step-size parameters and $\delta_{\widehat{\mathbf{h}}} > 0$ and $\delta_{\widehat{\mathbf{g}}} > 0$ are the regularization parameters [15]. For the initialization, we may choose $\widehat{\mathbf{h}}(0) = \left[\begin{array}{cccc} 1 & 0 & \cdots & 0 \end{array} \right]^T$ and $\widehat{\mathbf{g}}(0) = (1/M)\left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array} \right]^T$.

III. Proportionate NLMS-BF Algorithm

Many interesting proportionate-type algorithms can be found in the literature, e.g, see [2]– [6] and the references therein. Among them, the improved proportionate NLMS (IPNLMS) algorithm proposed in [4] represents one of the most reliable choices. This algorithm uses the ℓ_1 norm to exploit the sparsity of the impulse responses.

Following the basic idea behind proportionate-type algorithms, let us first reformulate (8) as

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \frac{\alpha_{\widehat{\mathbf{h}}}\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)e_{\widehat{\mathbf{g}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}^T(t)\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{h}}}}, \tag{10}$$

where $\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1) = \mathrm{diag}\left[\begin{array}{ccc} q_{\widehat{\mathbf{h}},1}(t-1) & \cdots & q_{\widehat{\mathbf{h}},L}(t-1) \end{array} \right]$ is a diagonal matrix, which contains the proportionate factors $q_{\widehat{\mathbf{h}},l}(t-1) > 0$ $(l = 1, 2, \ldots, L)$ that depend on the coefficients of $\widehat{\mathbf{h}}(t-1)$, and $\widetilde{\delta}_{\widehat{\mathbf{h}}} = \delta_{\widehat{\mathbf{h}}}/L$ is the regularization term.

Similarly, a proportionate term can be introduced in (9), which results in

$$\widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \frac{\alpha_{\widehat{\mathbf{g}}}\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t)e_{\widehat{\mathbf{h}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}^T(t)\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{g}}}}, \tag{11}$$

where $\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1) = \mathrm{diag}\left[\begin{array}{ccc} q_{\widehat{\mathbf{g}},1}(t-1) & \cdots & q_{\widehat{\mathbf{g}},M}(t-1) \end{array} \right]$ is an $M \times M$ diagonal matrix, with the proportionate factors $q_{\widehat{\mathbf{g}},m}(t-1) > 0$ $(m = 1, 2, \ldots, M)$ related to the coefficients of $\widehat{\mathbf{g}}(t-1)$; also, $\widetilde{\delta}_{\widehat{\mathbf{g}}} = \delta_{\widehat{\mathbf{g}}}/M$ is a regularization constant.

Depending on how the proportionate factors are chosen, we can obtain different kind of proportionate algorithms. Here, we follow the line of the IPNLMS algorithm [4], so that

$$q_{\widehat{\mathbf{h}},l}(t-1) = \frac{1-\kappa_{\widehat{\mathbf{h}}}}{2L} + (1+\kappa_{\widehat{\mathbf{h}}})\frac{\left|\widehat{h}_l(t-1)\right|}{2\left\|\widehat{\mathbf{h}}(t-1)\right\|_1}, \ 1 \leq l \leq L,$$

$$q_{\widehat{\mathbf{g}},m}(t-1) = \frac{1-\kappa_{\widehat{\mathbf{g}}}}{2M} + (1+\kappa_{\widehat{\mathbf{g}}})\frac{\left|\widehat{g}_m(t-1)\right|}{2\left\|\widehat{\mathbf{g}}(t-1)\right\|_1}, \ 1 \leq m \leq M,$$

where $\kappa_{\widehat{\mathbf{h}}}$ $(-1 \leq \kappa_{\widehat{\mathbf{h}}} < 1)$ and $\kappa_{\widehat{\mathbf{g}}}$ $(-1 \leq \kappa_{\widehat{\mathbf{g}}} < 1)$ are the parameters that control the amount of proportionality and $\|\cdot\|_1$ denotes the ℓ_1 norm. Using (10) and (11) instead of (8) and (9), respectively, we obtain the IPNLMS algorithm tailored for bilinear forms, namely the IPNLMS-BF.

At this point, several practical aspects should be outlined. First, depending on the application (or based on some a priori information about the system to be identified), we could consider only a partial proportionate algorithm, e.g., by using (10) together with (9), or (8) together with (11). Since the overall behavior is mainly controlled by the longer filter (which is usually the temporal one) [12], the first option seems to be more apparent. Second, we could tune different values of the parameters $\kappa_{\widehat{\mathbf{h}}}$ and $\kappa_{\widehat{\mathbf{g}}}$. In practice, good choices for these parameters are -0.5 or 0 [4], but they can be also set depending on the sparseness degree of the systems (when certain a priori information is available). For example, if any of these parameters goes to -1, the respective filter is not proportionately updated anymore and behaves like a regular NLMS algorithm. In this work, for the sake of generality, we do not consider such particular cases but they could be taken into account in different scenarios.

Finally, we should note that the regular IPNLMS algorithm [4] can be used to identify the spatiotemporal impulse response, $\mathbf{f}$. This algorithm results based on desired signal expressed as in (4) and the error signal from the last line of (5); consequently, its update is given by

$$\widehat{\mathbf{f}}(t) = \widehat{\mathbf{f}}(t-1) + \frac{\alpha \mathbf{Q}_{\widehat{\mathbf{f}}}(t-1)\widetilde{\mathbf{x}}(t)e(t)}{\widetilde{\mathbf{x}}^T(t)\mathbf{Q}_{\widehat{\mathbf{f}}}(t-1)\widetilde{\mathbf{x}}(t) + \delta}, \quad (12)$$

where $\mathbf{Q}_{\widehat{\mathbf{f}}}(t-1) = \text{diag}\left[\begin{array}{ccc} q_{\widehat{\mathbf{f}},1}(t-1) & \cdots & q_{\widehat{\mathbf{f}},ML}(t-1) \end{array}\right]$ is an $ML \times ML$ diagonal matrix, which contains the proportionate factors $q_{\widehat{\mathbf{f}},k}(t-1) > 0$ $(k = 1, 2, \dots, ML)$ that depend on the coefficients of $\widehat{\mathbf{f}}(t-1)$, with

$$q_{\widehat{\mathbf{f}},k}(t-1) = \frac{1-\kappa}{2ML} + (1+\kappa)\frac{\left|\widehat{f}_k(t-1)\right|}{2\left\|\widehat{\mathbf{f}}(t-1)\right\|_1}, \; 1 \leq k \leq ML,$$

where κ $(-1 \leq \kappa < 1)$ controls the amount of proportionality, while α and δ denote the normalized step-size and the regularization parameter, respectively.

Nevertheless, we should note that the solution based on the regular IPNLMS algorithm [from (12)] involves an adaptive filter of length ML, while the IPNLMS-BF algorithm [defined by the updates (10) and (11)] uses two shorter filters of lengths L and M, respectively. Consequently, a faster convergence rate is expected for the IPNLMS-BF algorithm as compared to the conventional approach (as will be also supported by the experiments provided in the next section).

IV. Simulation Results

Experiments are performed from a system identification perspective, in the context of the bilinear system identification problem described in Section II. The temporal impulse response $\mathbf{h}$ is either the first or the fourth impulse response from G168 Recommendation [16], which is padded with zeros up to the length $L = 512$. The spatial impulse response $\mathbf{g}$ is generated with an exponential decay, such that its coefficients are given by $g_m = 0.5^m$, with $m = 1, 2, \dots, M$; in all the experiments, we set $M = 4$. Then, both impulse responses are normalized such that $\|\mathbf{h}\| = \|\mathbf{g}\| = 1$. The spatiotemporal (i.e., global) impulse response results as $\mathbf{f} = \mathbf{g} \otimes \mathbf{h}$, with the length $ML = 2048$.

The input signals $x_m(t)$, $m = 1, 2, \dots, M$ are either white Gaussian noises or AR(1) processes; each AR(1) process is generated by filtering a white Gaussian noise through a first-order system $1/\left(1 - 0.8z^{-1}\right)$. The system noise $w(t)$ is an additive white Gaussian noise with the variance $\sigma_w^2 = 0.01$ or $\sigma_w^2 = 0.1$. The performance measures is the NM (computed in dB), as explained in Section II.

In the first set of experiments, the first impulse response from G168 Recommendation [16] is involved. The proposed IPNLMS-BF algorithm defined by the updates (10) and (11) is compared with the NLMS-BF counterpart [which is based on (8) and (9)] and the RLS-BF algorithm [12]. Different

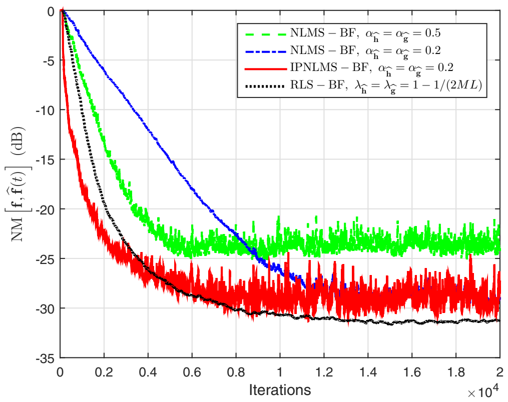

Fig. 1. Normalized misalignment of the NLMS-BF (using different normalized step-sizes), IPNLMS-BF, and RLS-BF algorithms. The temporal impulse response $\mathbf{h}$ is the first echo path from G168 Recommendation [16]. The input signals are white Gaussian noises and $\sigma_w^2 = 0.01$.

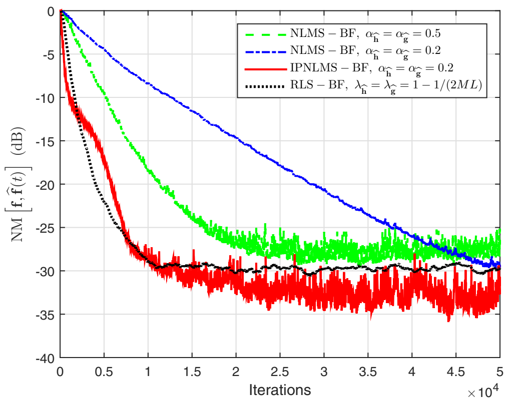

Fig. 2. Normalized misalignment of the NLMS-BF (using different normalized step-sizes), IPNLMS-BF, and RLS-BF algorithms. The temporal impulse response $\mathbf{h}$ is the first echo path from G168 Recommendation [16]. The input signals are AR(1) processes and $\sigma_w^2 = 0.01$.

normalized step-sizes are used for the NLMS-BF algorithm, with $\alpha_{\widehat{\mathbf{h}}} = \alpha_{\widehat{\mathbf{g}}} = 0.5$ providing the fastest convergence rate [12]. Also, the RLS-BF algorithm uses the forgetting factors $\lambda_{\widehat{\mathbf{h}}} = \lambda_{\widehat{\mathbf{g}}} = 1 - 1/(2ML)$. The regularization parameters of the NLMS-BF algorithm are set to $\delta_{\widehat{\mathbf{h}}} = \delta_{\widehat{\mathbf{g}}} = 20\sigma_{\widetilde{x}}^2$, where $\sigma_{\widetilde{x}}^2$ is the variance of $\widetilde{\mathbf{x}}(t)$. Consequently, the IPNLMS-BF algorithm uses $\widetilde{\delta}_{\widehat{\mathbf{h}}} = 20\sigma_{\widetilde{x}}^2/L$ and $\widetilde{\delta}_{\widehat{\mathbf{g}}} = 20\sigma_{\widetilde{x}}^2/M$; also, its specific parameters are set to $\kappa_{\widehat{\mathbf{h}}} = \kappa_{\widehat{\mathbf{g}}} = 0$.

In Fig. 1, the input signals are white Gaussian noises. As we can notice, the IPNLMS-BF algorithm with $\alpha_{\widehat{\mathbf{h}}} = \alpha_{\widehat{\mathbf{g}}} = 0.2$ outperforms the NLMS-BF algorithm in terms of convergence rate. The NLMS-BF algorithm with $\alpha_{\widehat{\mathbf{h}}} = \alpha_{\widehat{\mathbf{g}}} = 0.5$ obtains a fast convergence rate (but lower as compared to the IPNLMS-BF), paying with an increase of the misalignment level. On the other hand, the performance of the RLS-BF algorithm is similar to the proposed IPNLMS-BF. However, the computational complexity of the IPNLMS-BF algorithm is proportional to

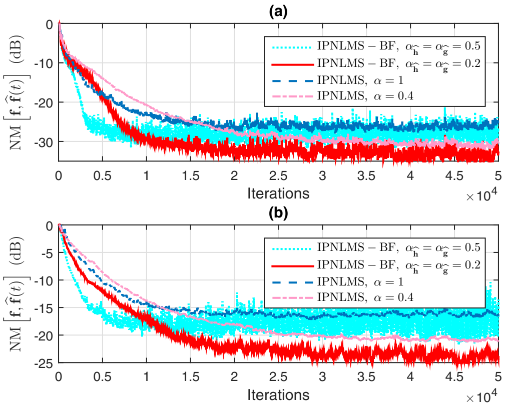

Fig. 3. Normalized misalignment of the IPNLMS and IPNLMS-BF algorithms for different values of the normalized step-size parameters. The temporal impulse response $\mathbf{h}$ is (a) the first echo path from G168 Recommendation [16] and (b) the fourth echo path from G168 Recommendation [16]. The input signals are AR(1) processes and (a) $\sigma_w^2 = 0.01$, (b) $\sigma_w^2 = 0.1$.

$\mathcal{O}(L+M)$, while the RLS-BF algorithm requires $\mathcal{O}(L^2+M^2)$ operations. Taking into account that the value of L is on the order of hundreds, the computational gain of the proposed proportionate solution is significant.

The gain becomes more apparent in Fig. 2, where the input signals are AR(1) processes. As we can see, the same conclusions apply in this case, i.e., the IPNLMS-BF algorithm outperforms the NLMS-BF counterpart (especially in terms of convergence rate), while the RLS-BF algorithm behaves similarly, but paying with an increased computational cost.

Finally, the proposed IPNLMS-BF algorithm is compared to the regular IPNLMS used for the identification of the spatiotemporal impulse response [based on the update (12)]. Different values of the normalized step-sizes are used in this experiment, such that $\alpha = \alpha_{\hat{\mathbf{h}}} + \alpha_{\hat{\mathbf{g}}}$; as shown in [12] (in the context of the NLMS-BF algorithm), the regular algorithm and its bilinear counterpart converge to the same steady-state misalignment in this case. The specific parameters of the IPNLMS-BF algorithm are the same as in the previous experiments, while the regular IPNLMS algorithm uses $\delta = 20\sigma_x^2/(ML)$ and $\kappa = 0$. Both impulse responses are involved in this experiment, also using different noise levels. The the fourth impulse response from G168 Recommendation [16] [involved in Fig. 3(a)] is less sparse as compared to the first one from the same standard [used in Fig. 3(b)].

In Figs. 3(a) and (b), the input signals are AR(1) processes. As we can notice, the IPNLMS-BF algorithm using the normalized step-sizes $\alpha_{\hat{\mathbf{h}}} = \alpha_{\hat{\mathbf{g}}}$ obtains a faster convergence rate as compared to the regular IPNLMS algorithm using $\alpha = \alpha_{\hat{\mathbf{h}}} + \alpha_{\hat{\mathbf{g}}}$, while reaching the same misalignment level (which supports the theoretical findings from [12]). As expected, larger values of the normalized step-sizes improve the convergence rate, but increase the misalignment. Concluding, the main advantage is that the proposed IPNLMS-BF algorithm uses two shorter filters of lengths L and M, while the solution based on the regular IPNLMS algorithm involves an adaptive filter of length ML. Consequently, the gain of the algorithm tailored for bilinear forms is twofold, i.e., faster convergence rate and lower computational complexity.

V. Conclusions

In this paper, we have proposed a proportionate-type algorithm tailored for the identification of bilinear forms. The bilinear term is defined in the context of a particular MISO system, with respect to the impulse responses of a spatiotemporal model. When the impulse responses to be identified are sparse, the proportionate-type algorithms represent an attractive choice, since they exploit the sparsity feature to improve the convergence performance. The proposed algorithm follows the line of the celebrated IPNLMS algorithm [4], in terms of computing the proportionate factors. The resulting IPNLMS-BF algorithm outperforms its NLMS-BF counterpart, especially in terms of convergence rate. Moreover, the proposed algorithm outperforms the regular IPNLMS, achieving a faster convergence rate but also a lower computational complexity. Therefore, it could represents an appealing choice for sparse system identification problems in the context of bilinear forms.

References

[1] J. Benesty, T. Gaensler, D. R. Morgan, M. M. Sondhi, and S. L. Gay, *Advances in Network and Acoustic Echo Cancellation*. Berlin, Germany: Springer-Verlag, 2001.

[2] C. Paleologu, J. Benesty, and S. Ciochină, *Sparse Adaptive Filters for Echo Cancellation*. Morgan & Claypool Publishers, 2010.

[3] D. L. Duttweiler, "Proportionate normalized least-mean-squares adaptation in echo cancelers," *IEEE Trans. Speech, Audio Processing*, vol. 8, pp. 508–518, Sept. 2000.

[4] J. Benesty and S. L. Gay, "An improved PNLMS algorithm," in *Proc. IEEE ICASSP*, 2002, pp. II-1881–II-1884.

[5] H. Deng and M. Doroslovački, "Proportionate adaptive algorithms for network echo cancellation," *IEEE Trans. Signal Processing*, vol. 54, pp. 1794–1803, May 2006.

[6] J. Liu and S. L. Grant, "Proportionate adaptive filtering for block-sparse system identification," *IEEE/ACM Trans. Audio, Speech, Language Processing*, vol. 24, pp. 623–630, Apr. 2016.

[7] M. Rupp and S. Schwarz, "A tensor LMS algorithm," in *Proc. IEEE ICASSP*, 2015, pp. 3347–3351.

[8] A. Stenger and W. Kellermann, "Adaptation of a memoryless preprocessor for nonlinear acoustic echo cancelling," *Signal Processing*, vol. 80, pp. 1747–1760, Sept. 2000.

[9] Y. Huang, J. Skoglund, and A. Luebs, "Practically efficient nonlinear acoustic echo cancellers using cascaded block RLS and FLMS adaptive filters," in *Proc. IEEE ICASSP*, 2017, pp. 596–570.

[10] E.-W. Bai and D. Li, "Convergence of the iterative Hammerstein system identification algorithm," *IEEE Trans. Automatic Control*, vol. 49, pp. 1929–1940, Nov. 2004.

[11] J. Benesty, C. Paleologu, and S. Ciochină, "On the identification of bilinear forms with the Wiener filter," *IEEE Signal Processing Lett.*, vol. 24, pp. 653–657, May 2017.

[12] C. Paleologu, J. Benesty, and S. Ciochină, "Adaptive filtering for the identification of bilinear forms," *Digital Signal Processing*, vol. 75, pp. 153–167, Apr. 2018.

[13] D. A. Harville, *Matrix Algebra From a Statistician's Perspective*. New York: Springer-Verlag, 1997.

[14] D. R. Morgan, J. Benesty, and M. M. Sondhi, "On the evaluation of estimated impulse responses," *IEEE Signal Processing Lett.*, vol. 5, pp. 174–176, July 1998.

[15] J. Benesty, C. Paleologu, and S. Ciochină, "On regularization in adaptive filtering," *IEEE Trans. Audio, Speech, Language Processing*, vol. 19, pp. 1734–1742, Aug. 2011.

[16] *Digital Network Echo Cancellers*, ITU-T Recommendations G.168, 2002.