A Proportionate Affine Projection Algorithm for the Identification of Sparse Bilinear Forms

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***Abstract*—Identification of sparse impulse responses was addressed mainly in the last two decades with the development of the so-called "proportionate"-type algorithms. These algorithms are meant to exploit the sparseness of the systems that need to be identified, with the purpose of improving the convergence rate and tracking of the conventional adaptive algorithms used in this framework. Nevertheless, the system identification problem becomes more challenging when the parameter space is large. This issue can be addressed with tensor decompositions and modelling. In this paper, we aim to identify sparse bilinear forms, in which the bilinear term is defined with respect to the impulse responses of a spatiotemporal model. In this context, we derive a proportionate affine projection algorithm for the identification of such bilinear forms. Experimental results highlight the good behavior of the proposed solution.**

I. Introduction

In many system identification problems, the systems to be identified are sparse in nature, i.e., only a small percentage of the impulse response components have a significant magnitude, while the rest are zero or small. Consequently, sparse adaptive filters are frequently involved in such system identification problems. In this framework, most of these algorithms were developed in the context of echo cancellation [1], [2]. Usually, they are referred to as proportionate algorithms [3]– [6]. The basic idea is to "proportionately" redistribute the adaptation gain among all the coefficients, emphasizing the large ones in order to speed up their convergence and, consequently, to increase the overall convergence rate.

Adaptive filters with a large number of coefficients are usually involved in the context of sparse system identification (like echo cancellation). In addition, the system identification problems become more challenging when the parameter space is very large [7]. For example, such frameworks can be found in conjunction with nonlinear acoustic echo cancellation [8], [9], where the global system resembles (to some extent) the Hammerstein model [10]. Alternatively, the problem can be reduced to the identification of a bilinear form, where the bilinear term is defined with respect to the impulse responses of a spatiotemporal model, which resembles a multiple-input/single-output (MISO) system [11], [12].

Recently, [12] provided an analysis of the conventional adaptive algorithms designed for the identification of such bilinear forms, including the least-mean-square (LMS) and normalized LMS (NLMS) algorithms. Also, [13] introduced the improved proportionate NLMS algorithm in the context of bilinear forms, namely IPNLMS-BF. In this paper, we extend the approach to the affine projection algorithm (APA) [14] and develop a proportionate version tailored for the identification of bilinear forms. The main advantage of the APA over the NLMS algorithm consists of a superior convergence rate, especially for correlated inputs. Simulation results indicate that the proposed proportionate APA for bilinear forms outperforms the IPNLMS-BF algorithm in terms of convergence rate.

The paper is structured as follows. In Section II, we provide a system model overview, along with the NLMS-based algorithms developed in this context. Section III is dedicated to the development of the APA and its proportionate version for bilinear forms, while Section IV contains the simulation results. The conclusions are presented in Section V.

II. Bilinear Model and NLMS-Based Algorithms

Let us consider the framework of the bilinear model from [11], where we define the desired (or reference) signal in the framework of a simplified MISO system as

$$d(t) = \mathbf{h}^T \mathbf{X}(t)\mathbf{g} + w(t) = y(t) + w(t), \tag{1}$$

where t denotes the time index, the superscript ^{T} represents the transpose operator, $\mathbf{h}$ and $\mathbf{g}$ are the two system impulse responses, having the lengths L and M, respectively, $\mathbf{X}(t) = \begin{bmatrix} \mathbf{x}_1(t) & \mathbf{x}_2(t) & \cdots & \mathbf{x}_M(t) \end{bmatrix}$ is the zero-mean multiple-input signal matrix having the size $L \times M$, $\mathbf{x}_m(t) = \begin{bmatrix} x_m(t) & x_m(t-1) & \cdots & x_m(t-L+1) \end{bmatrix}^T$ is the vector that contains the most recent L samples of the mth $(m = 1, 2, \ldots, M)$ input signal, and $w(t)$ denotes the zero-mean additive noise. We assume that all signals take real values, while $\mathbf{X}(t)$ and $w(t)$ are uncorrelated. The two impulse responses $\mathbf{h}$ and $\mathbf{g}$ represent the temporal and spatial parts, respectively. As we can notice, $y(t)$ is bilinear in $\mathbf{h}$ and $\mathbf{g}$, respectively.

Based on the vectorization operation (i.e., conversion of a matrix into a vector [15]), we can rewrite the matrix $\mathbf{X}(t)$, of size $L \times M$, as a vector of length LM:

$$\mathrm{vec}\left[\mathbf{X}(t)\right] = \begin{bmatrix} \mathbf{x}_1^T(t) & \mathbf{x}_2^T(t) & \cdots & \mathbf{x}_M^T(t) \end{bmatrix}^T = \widetilde{\mathbf{x}}(t). \tag{2}$$

Consequently, the output signal $y(t)$ is

$$\begin{aligned} y(t) &= \mathrm{tr}\left[\left(\mathbf{h}\mathbf{g}^T\right)^T \mathbf{X}(t)\right] = \mathrm{vec}^T\left(\mathbf{h}\mathbf{g}^T\right)\mathrm{vec}\left[\mathbf{X}(t)\right] \\ &= \left(\mathbf{g} \otimes \mathbf{h}\right)^T \widetilde{\mathbf{x}}(t) = \mathbf{f}^T \widetilde{\mathbf{x}}(t), \end{aligned} \tag{3}$$

where $\mathrm{tr}[\cdot]$ is the trace of a square matrix, $\otimes$ is the Kronecker product, and $\mathbf{f} = \mathbf{g} \otimes \mathbf{h}$ is the global impulse response of length LM, obtained as the Kronecker product between the individual system impulse responses $\mathbf{g}$ and $\mathbf{h}$. Therefore, the desired signal in (1) becomes

$$d(t) = \mathbf{f}^T \widetilde{\mathbf{x}}(t) + w(t). \tag{4}$$

Our purpose is the identification of the temporal and spatial impulse responses $\mathbf{h}$ and $\mathbf{g}$, using two adaptive filters, $\widehat{\mathbf{h}}(t) = \begin{bmatrix} \widehat{h}_1(t) & \widehat{h}_2(t) & \cdots & \widehat{h}_L(t) \end{bmatrix}^T$ and $\widehat{\mathbf{g}}(t) = \begin{bmatrix} \widehat{g}_1(t) & \widehat{g}_2(t) & \cdots & \widehat{g}_M(t) \end{bmatrix}^T$. Hence, the global impulse response $\mathbf{f}$ can be identified using a long filter, $\widehat{\mathbf{f}}(t) = \widehat{\mathbf{g}}(t) \otimes \widehat{\mathbf{h}}(t) = \begin{bmatrix} \widehat{f}_1(t) & \widehat{f}_2(t) & \cdots & \widehat{f}_{ML}(t) \end{bmatrix}^T$.

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Let $\eta \in \mathbb{R}, \eta \neq 0$. From (1) it results immediately that $y(t) = (\mathbf{h}/\eta)^T \mathbf{X}(t)(\eta\mathbf{g})$, meaning that the pair $\mathbf{h}/\eta$ and $\eta\mathbf{g}$ is the same as the pair $\mathbf{h}$ and $\mathbf{g}$ in the bilinear form. Therefore, these systems can only be identified up to a scaling factor. However, since $\mathbf{f} = \mathbf{g} \otimes \mathbf{h} = (\eta\mathbf{g}) \otimes (\mathbf{h}/\eta)$, there is no scaling ambiguity when identifying the global impulse response. Consequently, to evaluate the performance of the individual filters identification, we can use the normalized projection misalignment (NPM), as explained in [16]. On the other hand, the global filter identification can be evaluated using the normalized misalignment (NM), which is defined as

$$\mathrm{NM}\left[\mathbf{f}, \widehat{\mathbf{f}}(t)\right] = \frac{\left\|\mathbf{f} - \widehat{\mathbf{f}}(t)\right\|_2^2}{\|\mathbf{f}\|_2^2}, \tag{5}$$

where $\|\cdot\|_2$ is the ℓ_2 norm.

If we consider two adaptive filters $\widehat{\mathbf{h}}(t)$ and $\widehat{\mathbf{g}}(t)$, then the estimated signal is $\widehat{y}(t) = \widehat{\mathbf{h}}^T(t-1)\mathbf{X}(t)\widehat{\mathbf{g}}(t-1)$. Consequently, the error signal between the desired and estimated signals can be defined as

$$\begin{aligned} e(t) &= d(t) - \widehat{y}(t) = d(t) - \left[\widehat{\mathbf{g}}(t-1) \otimes \widehat{\mathbf{h}}(t-1)\right]^T \widetilde{\mathbf{x}}(t) \\ &= d(t) - \widehat{\mathbf{f}}^T(t-1)\widetilde{\mathbf{x}}(t). \end{aligned} \tag{6}$$

In an alternative manner, we can define two error signals:

$$e_{\widehat{\mathbf{g}}}(t) = d(t) - \widehat{\mathbf{h}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t), \tag{7}$$

$$e_{\widehat{\mathbf{h}}}(t) = d(t) - \widehat{\mathbf{g}}^T(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t), \tag{8}$$

where $\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) = [\widehat{\mathbf{g}}(t-1) \otimes \mathbf{I}_L]^T \widetilde{\mathbf{x}}(t)$ and $\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) = \left[\mathbf{I}_M \otimes \widehat{\mathbf{h}}(t-1)\right]^T \widetilde{\mathbf{x}}(t)$, where $\mathbf{I}_L$ and $\mathbf{I}_M$ are the identity matrices of sizes $L \times L$ and $M \times M$, respectively. It can be seen that $e_{\widehat{\mathbf{g}}}(t) = e_{\widehat{\mathbf{h}}}(t) = e(t)$. However, for the clarity of the following developments, we prefer to keep the notation from (7) and (8). In this framework, the NLMS algorithm for bilinear forms, namely NLMS-BF [12], can be described by the updates

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \frac{\alpha_{\widehat{\mathbf{h}}}\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)e_{\widehat{\mathbf{g}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}^T(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) + \delta_{\widehat{\mathbf{h}}}}, \tag{9}$$

$$\widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \frac{\alpha_{\widehat{\mathbf{g}}}\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t)e_{\widehat{\mathbf{h}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}^T(t)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) + \delta_{\widehat{\mathbf{g}}}}, \tag{10}$$

where $\alpha_{\widehat{\mathbf{h}}}$ $(0 < \alpha_{\widehat{\mathbf{h}}} < 2)$ and $\alpha_{\widehat{\mathbf{g}}}$ $(0 < \alpha_{\widehat{\mathbf{g}}} < 2)$ represent the normalized step-size parameters, and $\delta_{\widehat{\mathbf{h}}} > 0$ and $\delta_{\widehat{\mathbf{g}}} > 0$ denote the regularization parameters. We can choose $\widehat{\mathbf{h}}(0) = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix}^T$ and $\widehat{\mathbf{g}}(0) = (1/M)\begin{bmatrix} 1 & 1 & \cdots & 1 \end{bmatrix}^T$ as initialization.

Starting from the the basic idea of proportionate-type algorithms, relation (9) can be reformulated as

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \frac{\alpha_{\widehat{\mathbf{h}}}\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t)e_{\widehat{\mathbf{g}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}^T(t)\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{h}}}}, \tag{11}$$

where

$$\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1) = \mathrm{diag}\begin{bmatrix} q_{\widehat{\mathbf{h}},1}(t-1) & \cdots & q_{\widehat{\mathbf{h}},L}(t-1) \end{bmatrix} \tag{12}$$

denotes an $L \times L$ diagonal matrix, containing the proportionate factors $q_{\widehat{\mathbf{h}},l}(t-1) \geq 0$ $(l = 1, 2, \ldots, L)$ which depend on the coefficients of $\widehat{\mathbf{h}}(t-1)$, and $\widetilde{\delta}_{\widehat{\mathbf{h}}} = \delta_{\widehat{\mathbf{h}}}/L$ denotes the regularization constant [17]. In a similar way, a proportionate term can also be introduced in (10), resulting in

$$\widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \frac{\alpha_{\widehat{\mathbf{g}}}\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t)e_{\widehat{\mathbf{h}}}(t)}{\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}^T(t)\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1)\widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{g}}}}, \tag{13}$$

where

$$\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1) = \mathrm{diag}\begin{bmatrix} q_{\widehat{\mathbf{g}},1}(t-1) & \cdots & q_{\widehat{\mathbf{g}},M}(t-1) \end{bmatrix} \tag{14}$$

denotes an $M \times M$ diagonal matrix, containing the proportionate factors $q_{\widehat{\mathbf{g}},m}(t-1) \geq 0$ $(m = 1, 2, \ldots, M)$ related to the coefficients of $\widehat{\mathbf{g}}(t-1)$; also, $\widetilde{\delta}_{\widehat{\mathbf{g}}} = \delta_{\widehat{\mathbf{g}}}/M$ denotes a regularization constant [17].

For the identification of sparse impulse responses, the improved proportionate NLMS (IPNLMS) algorithm proposed in [4] represents one of the most reliable choices. This algorithm uses the ℓ_1 norm to exploit the sparsity of the impulse response that we need to identify. Next, by following the line of the IPNLMS algorithm [4], we obtain that

$$q_{\widehat{\mathbf{h}},l}(t-1) = \frac{1-\kappa_{\widehat{\mathbf{h}}}}{2L} + (1+\kappa_{\widehat{\mathbf{h}}})\frac{\left|\widehat{h}_l(t-1)\right|}{2\left\|\widehat{\mathbf{h}}(t-1)\right\|_1},\ 1 \leq l \leq L, \tag{15}$$

$$q_{\widehat{\mathbf{g}},m}(t-1) = \frac{1-\kappa_{\widehat{\mathbf{g}}}}{2M} + (1+\kappa_{\widehat{\mathbf{g}}})\frac{|\widehat{g}_m(t-1)|}{2\|\widehat{\mathbf{g}}(t-1)\|_1},\ 1 \leq m \leq M, \tag{16}$$

where $\kappa_{\widehat{\mathbf{h}}}$ $(-1 \leq \kappa_{\widehat{\mathbf{h}}} < 1)$ and $\kappa_{\widehat{\mathbf{g}}}$ $(-1 \leq \kappa_{\widehat{\mathbf{g}}} < 1)$ are parameters that can control the amount of proportionality, while $\|\cdot\|_1$ is the ℓ_1 norm. Using (11) and (13) instead of (9) and (10), respectively, the IPNLMS algorithm tailored for the case of bilinear forms, namely the IPNLMS-BF [13] results.

Finally, it can be mentioned that the regular IPNLMS algorithm [4] could also be used for the identification of the global system impulse response, $\mathbf{f}$. This algorithm is obtained based on the reference signal expressed as in (4), together with the error signal in last line of (6), and including in its update the $ML \times ML$ proportionate matrix

$$\mathbf{Q}_{\widehat{\mathbf{f}}}(t-1) = \mathrm{diag}\begin{bmatrix} q_{\widehat{\mathbf{f}},1}(t-1) & \cdots & q_{\widehat{\mathbf{f}},ML}(t-1) \end{bmatrix}, \tag{17}$$

which is a diagonal matrix containing the proportionate factors $q_{\widehat{\mathbf{f}},k}(t-1) \geq 0$ $(k = 1, 2, \ldots, ML)$ depending on the coefficients of $\widehat{\mathbf{f}}(t-1)$. These can be evaluated as

$$q_{\widehat{\mathbf{f}},k}(t-1) = \frac{1-\kappa}{2ML} + (1+\kappa)\frac{\left|\widehat{f}_k(t-1)\right|}{2\left\|\widehat{\mathbf{f}}(t-1)\right\|_1},\ 1 \leq k \leq ML, \tag{18}$$

where κ $(-1 \leq \kappa < 1)$ controls the amount of proportionality.

However, another observation should be made, namely that the solution obtained with the regular IPNLMS algorithm contains an adaptive filter of length LM, whereas the IPNLMS-BF algorithm [using (11) and (13)] involves two short filters having lengths L and M, respectively. Hence, the IPNLMS-BF can attain a faster convergence speed in comparison to the IPNLMS algorithm [13].

III. Proportionate APA for Bilinear Forms

As compared to the LMS and NLMS algorithms, the APA [14] achieves a superior convergence rate especially for correlated inputs. Following the development of the NLMS-BF algorithm from Section II, it is straightforward to derive the APA in the bilinear context. First, let us introduce the notation $\widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}(t) = \begin{bmatrix} \widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t) & \widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t-1) & \cdots & \widetilde{\mathbf{x}}_{\widehat{\mathbf{g}}}(t-P+1) \end{bmatrix}$, $\widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}(t) = \begin{bmatrix} \widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t) & \widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t-1) & \cdots & \widetilde{\mathbf{x}}_{\widehat{\mathbf{h}}}(t-P+1) \end{bmatrix}$, and $\mathbf{d}(t) = \begin{bmatrix} d(t) & d(t-1) & \cdots & d(t-P+1) \end{bmatrix}^T$, where P denotes the projection order. Next, we define the error vectors:

$$\mathbf{e}_{\widehat{\mathbf{g}}}(t) = \mathbf{d}(t) - \widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}^T(t)\widehat{\mathbf{h}}(t-1), \tag{19}$$

$$\mathbf{e}_{\widehat{\mathbf{h}}}(t) = \mathbf{d}(t) - \widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}^T(t)\widehat{\mathbf{g}}(t-1), \tag{20}$$

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so that the updates of the APA for bilinear forms (namely APA-BF) result in

$$\widehat{\mathbf{h}}(t) = \widehat{\mathbf{h}}(t-1) + \alpha_{\widehat{\mathbf{h}}} \widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}(t) \left[\widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}^T(t) \widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}(t) + \delta_{\widehat{\mathbf{h}}} \mathbf{I}_P \right]^{-1} \mathbf{e}_{\widehat{\mathbf{g}}}(t), \tag{21}$$

$$\widehat{\mathbf{g}}(t) = \widehat{\mathbf{g}}(t-1) + \alpha_{\widehat{\mathbf{g}}} \widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}(t) \left[\widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}^T(t) \widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}(t) + \delta_{\widehat{\mathbf{g}}} \mathbf{I}_P \right]^{-1} \mathbf{e}_{\widehat{\mathbf{h}}}(t), \tag{22}$$

where $\mathbf{I}_P$ is the identity matrix of size $P \times P$. Clearly, for $P = 1$, the NLMS-BF algorithm [12] is obtained.

The proportionate approach can also be applied to the APA-BF. Following the development of the IPNLMS-BF algorithm and taking (21)–(22) into account, the updates of the improved proportionate APA for bilinear forms (namely IPAPA-BF) result in

$$\begin{aligned} \widehat{\mathbf{h}}(t) &= \widehat{\mathbf{h}}(t-1) + \alpha_{\widehat{\mathbf{h}}} \mathbf{Q}_{\widehat{\mathbf{h}}}(t-1) \widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}(t) \\ &\times \left[\widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}^T(t) \mathbf{Q}_{\widehat{\mathbf{h}}}(t-1) \widetilde{\mathbf{X}}_{\widehat{\mathbf{g}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{h}}} \mathbf{I}_P \right]^{-1} \mathbf{e}_{\widehat{\mathbf{g}}}(t), \end{aligned} \tag{23}$$

$$\begin{aligned} \widehat{\mathbf{g}}(t) &= \widehat{\mathbf{g}}(t-1) + \alpha_{\widehat{\mathbf{g}}} \mathbf{Q}_{\widehat{\mathbf{g}}}(t-1) \widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}(t) \\ &\times \left[\widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}^T(t) \mathbf{Q}_{\widehat{\mathbf{g}}}(t-1) \widetilde{\mathbf{X}}_{\widehat{\mathbf{h}}}(t) + \widetilde{\delta}_{\widehat{\mathbf{g}}} \mathbf{I}_P \right]^{-1} \mathbf{e}_{\widehat{\mathbf{h}}}(t), \end{aligned} \tag{24}$$

where $\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1)$ and $\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1)$ are defined in (12) and (14), respectively, using the proportionate factors from (15)–(16). The regularization parameters $\widetilde{\delta}_{\widehat{\mathbf{h}}}$ and $\widetilde{\delta}_{\widehat{\mathbf{g}}}$ are chosen as in the case of the IPNLMS-BF algorithm. As we can notice from (23)–(24), the IPNLMS-BF algorithm is obtained for $P = 1$, while the APA-BF results if $\mathbf{Q}_{\widehat{\mathbf{h}}}(t-1) = \mathbf{I}_L$ and $\mathbf{Q}_{\widehat{\mathbf{g}}}(t-1) = \mathbf{I}_M$.

Similar to the discussion from the end of Section II, we should outline that the regular APA [14] and its proportionate version IPAPA [18] could also be used to identify the global impulse response, $\mathbf{f}$. Based on desired signal from (4), the update of the conventional IPAPA is given by

$$\begin{aligned} \widehat{\mathbf{f}}(t) &= \widehat{\mathbf{f}}(t-1) + \alpha \mathbf{Q}_{\widehat{\mathbf{f}}}(t-1) \widetilde{\mathbf{X}}(t) \\ &\times \left[\widetilde{\mathbf{X}}^T(t) \mathbf{Q}_{\widehat{\mathbf{f}}}(t-1) \widetilde{\mathbf{X}}(t) + \delta \right]^{-1} \mathbf{e}(t), \end{aligned} \tag{25}$$

where the proportionate matrix $\mathbf{Q}_{\widehat{\mathbf{f}}}(t-1)$ is obtained based on (17)–(18), the data matrix $\widetilde{\mathbf{X}}(t)$ of size $ML \times P$ is defined as $\widetilde{\mathbf{X}}(t) = \left[\begin{array}{cccc} \widetilde{\mathbf{x}}(t) & \widetilde{\mathbf{x}}(t-1) & \cdots & \widetilde{\mathbf{x}}(t-P+1) \end{array} \right]$, the error vector results in $\mathbf{e}(t) = \mathbf{d}(t) - \widetilde{\mathbf{X}}^T(t)\widehat{\mathbf{f}}(t-1)$, while α and δ denote the normalized step-size and the regularization parameter, respectively. The update of the regular APA results from (25) when $\mathbf{Q}_{\widehat{\mathbf{f}}}(t-1) = \mathbf{I}_{ML}$, where $\mathbf{I}_{ML}$ is the identity matrix of size $ML \times ML$.

However, the solutions based on the conventional APA and IPAPA involve long adaptive filters of length ML, while the APA-BF and IPAPA-BF use two shorter filters of lengths L and M. Therefore, the APA-BF and IPAPA-BF should own a faster convergence rate as compared to the regular APA and IPAPA, respectively.

IV. Simulation Results

Simulations are performed in the framework of the MISO system described in Section II (from a system identification perspective). The temporal impulse response $\mathbf{h}$ is the first impulse response from G168 Recommendation [19], padded with zeros until the length $L = 512$. The spatial impulse response $\mathbf{g}$ is obtained by generating an exponential decay, having coefficients $g_m = 0.5^m$, with $m = 1, 2, \ldots, M$; in all experiments, we choose $M = 4$. Then, both the temporal and the spatial impulse responses are normalized, such that $\|\mathbf{h}\|_2 = \|\mathbf{g}\|_2 = 1$. The spatiotemporal system impulse response is obtained as $\mathbf{f} = \mathbf{g} \otimes \mathbf{h}$, with the length $LM = 2048$.

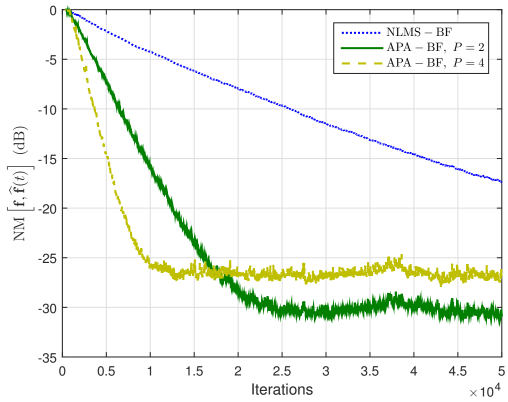

Fig. 1. Performance of the NLMS-BF and APA-BF in terms of NM. The input signals are AR(1) processes and $ML = 2048$.

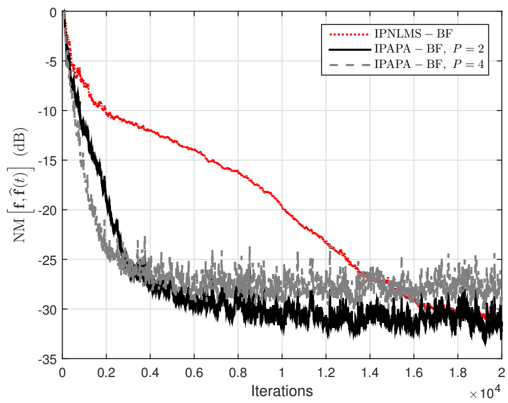

Fig. 2. Performance of the IPNLMS-BF and IPAPA-BF in terms of NM. The input signals are AR(1) processes and $ML = 2048$.

The input signals $x_m(t)$, $m = 1, 2, \ldots, M$ are generated either as white Gaussian noises or as AR(1) processes; each AR(1) process is obtained by filtering a white Gaussian noise through a first-order system $1/\left(1 - 0.8z^{-1}\right)$. The noise $w(t)$ is an additive white Gaussian noise, having the variance $\sigma_w^2 = 0.01$. The measure of the performance is the NM from (5), in dB, which evaluates the identification of the global impulse response $\mathbf{f}$.

First, the performance of the APA-BF is investigated. As compared to the NLMS-BF algorithm, the APA-BF should provide an improved convergence rate, especially for correlated inputs. This aspect is supported in Fig. 1, where the input signals are AR(1) processes, the step-sizes are set to $\alpha_{\widehat{\mathbf{h}}} = \alpha_{\widehat{\mathbf{g}}} = 0.1$, and the regularization parameters are $\delta_{\widehat{\mathbf{h}}} = \delta_{\widehat{\mathbf{g}}} = 20\sigma_{\widetilde{x}}^2$, where $\sigma_{\widetilde{x}}^2$ is the variance of $\widetilde{\mathbf{x}}(t)$. The APA-BF uses different values of the projection order, i.e., $P = 2$ and 4. As shown in Section III, the NLMS-BF algorithm is equivalent to APA-BF using $P = 1$. As we can notice in these figures, the APA-BF clearly outperforms the NLMS-BF algorithm in terms of convergence rate. The gain is significant for the APA-BF using $P = 2$, as compared to the NLMS-BF algorithm. Even if the APA-BF using $P = 4$ leads to a faster convergence rate, it also pays with an increased misalignment level. Due to this reason (and also considering the complexity issues), the APA-BF using $P = 2$ could be preferable in practice, since it achieves a proper compromise between the convergence rate and misalignment.

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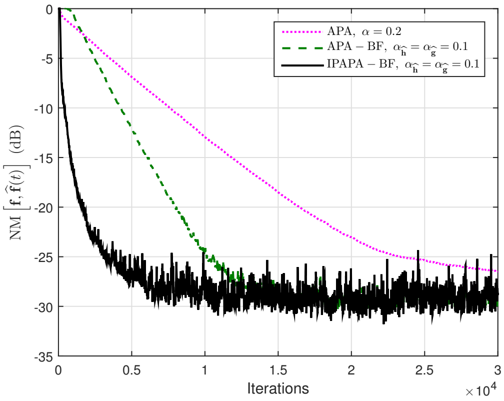

Fig. 3. Performance of the APA, APA-BF, and IPAPA-BF in terms of NM. The input signals are white Gaussian noises and $ML = 2048$.

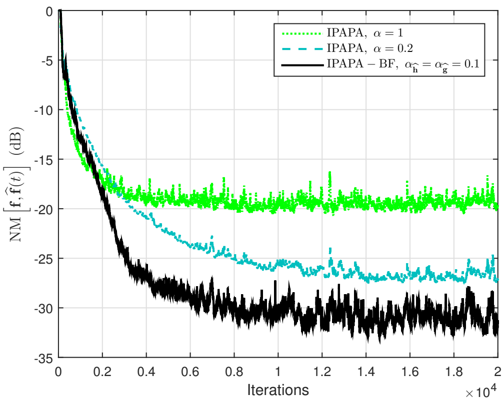

Fig. 4. Performance of the IPAPA and IPAPA-BF in terms of NM for different values of the normalized step-size parameters α, $\alpha_{\widehat{\mathbf{h}}}$, and $\alpha_{\widehat{\mathbf{g}}}$. The input signals are AR(1) processes and $ML = 2048$.

The same conclusions are valid in case of the IPAPA-BF as compared to the IPNLMS-BF algorithm. The results provided in Fig. 2 support this aspect. The step-sizes are the same as in the previous experiment, but the regularization terms are set to $\widetilde{\delta}_{\widehat{\mathbf{h}}} = 20\sigma_{\widetilde{x}}^2/L$ and $\widetilde{\delta}_{\widehat{\mathbf{g}}} = 20\sigma_{\widetilde{x}}^2/M$, and the specific parameters are $\kappa_{\widehat{\mathbf{h}}} = \kappa_{\widehat{\mathbf{g}}} = 0$. Again, the projection order $P = 2$ leads to the best compromise between the performance criteria.

In Fig. 3, the performance of the regular APA is compared to its counterparts tailored for bilinear forms, i.e., the APA-BF and IPAPA-BF. The input signals are white Gaussian noises and the projection order is $P = 2$ for all the algorithms. First, we can notice that the APA-BF outperforms the conventional APA in terms of convergence rate (as outlined in Section III). Second, the IPAPA-BF converges faster as compared to the APA-BF in case of sparse impulse responses.

Finally, the performance of the conventional IPAPA and proposed IPAPA-BF are investigated in Fig. 4. The input signals are AR(1) processes, the projection order is set to $P = 2$, and different values of the step-sizes are used. The regular IPAPA uses $\delta = 20\sigma_{\widetilde{x}}^2/(ML)$ and $\kappa = 0$. As we can notice, the IPAPA using $\alpha = 1$ achieves the fastest converge rate but a higher misalignment level. The IPAPA with $\alpha = 0.2$ improves the misalignment level, paying with a slightly slower converge rate. However, the IPAPA-BF using smaller step-sizes outperforms the conventional algorithm, achieving a convergence rate similar to the IPAPA with $\alpha = 1$, but reaching a much lower misalignment level. This performance gain supports the discussion from the end of Section III.

V. Conclusions

This paper presented a proportionate APA tailored for the identification of sparse bilinear forms, namely IPAPA-BF. This algorithm follows the line of the celebrated IPNLMS algorithm [4], in terms of computing the proportionate factors. The IPAPA-BF outperforms its non-proportionate counterpart, APA-BF, especially in terms of convergence rate. In addition, the proposed IPAPA-BF outperforms the regular IPAPA, achieving a faster convergence rate but also a lower computational complexity. Simulation results indicated the appealing performance of the proposed algorithm, in the context of bilinear sparse system identification.

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