

MINIMIZATION PROJECT

Problem Description

In part 1 of this project, given the following function $F : R^2 \rightarrow R^2$

$$F = \begin{cases} x_1^3 + 2x_2^2 - 3 \\ 2x_1^2 + e^{x_2-1} - 3 \end{cases},$$

we had to solve the minimization problem

$$\min_{x \in R^2} f = \frac{1}{2} F(x)^T F(x), \quad f: R^2 \rightarrow R^2$$

with a Newton step of $x_+ = x_c - J(x_c)^{-1}F(x_c)$.

In order to solve this problem, we had to use a line-search algorithm starting from the point

$x_0 = (2, 0.5)^T$. Stopping condition was $|f| < 1 \times 10^{-10}$.

In part 2, we had to repeat all of the above for

$$F = \begin{cases} x_1^2 + x_2^2 - 2 \\ e^{x_1-1} + x_2^3 - 2 \end{cases}$$

starting from $x_0 = (-0.25, -0.5)^T$

Solution

I wrote a program using Octave. My program consisted of the following files:

- 1) ls2.m: Main program. Contains line search algorithm with backtracking.
- 2) f2.m: definition of F function.
- 3) jaco.m: definition of Jacobian function.
- 4) minif.m: definition of f function.
- 5) x.txt: Text file containing the starting vector/point.

In order to use the same program to solve both problems, I only updated files x.txt, f2.m, and jaco.m

That is, I just modified the starting point, the definition of F, and the Jacobian Matrix.

Results

Problem 1

iter	xc(1)	xc(2)	F(1)	F(2)	f	lambda
1	2.00000000e+000	5.00000000e-001	5.50000000e+000	5.60653066e+000	3.08415930e+001	1.000000
2	1.88069278e+000	8.52574222e-001	5.10578596e+000	4.93693712e+000	2.52211992e+001	0.132098
3	1.12725387e+000	1.69970465e+000	4.21039472e+000	1.55456062e+000	1.00720412e+001	1.000000
4	1.03618168e+000	1.13148562e+000	6.73039173e-001	2.87866461e-001	2.67924414e-001	1.000000
5	1.00069969e+000	1.00803044e+000	3.43512637e-002	1.08624955e-002	6.49001563e-004	1.000000
6	1.00000024e+000	1.00003217e+000	1.29412934e-004	3.31366954e-005	8.92287404e-009	1.000000
7	1.00000000e+000	1.00000000e+000	2.07017248e-009	5.17626830e-010	2.27677581e-018	1.000000

Problem 2

iter	xc(1)	xc(2)	F(1)	F(2)	f	lambda
1	-2.50000000e-001	-5.00000000e-001	-1.68750000e+000	-1.83849520e+000	3.11386043e+000	1.000000
2	-1.44005812e+000	3.77767328e-002	7.51944782e-002	-1.91279030e+000	1.83221048e+000	0.033927
3	-1.42091501e+000	7.65028558e-001	6.04268168e-001	-1.46341244e+000	1.25335800e+000	0.002500
4	-1.21015527e+000	1.02719986e+000	5.19615327e-001	-8.06477199e-001	4.60202780e-001	0.327349
5	-7.91519363e-001	1.26747130e+000	2.32986391e-001	2.02878401e-001	4.77211521e-002	1.000000
6	-7.15937688e-001	1.22276114e+000	7.71158739e-003	8.00003172e-003	6.17345438e-005	1.000000
7	-7.13748099e-001	1.22088982e+000	8.29615999e-006	1.32705726e-005	1.22467184e-010	1.000000
8	-7.13747411e-001	1.22088682e+000	9.44577749e-012	3.29056782e-011	5.86003185e-022	1.000000