Optimal Electric Network Distribution

Samuel Rico Felipe Gómez Gregorio Bermúdez

EAFIT University

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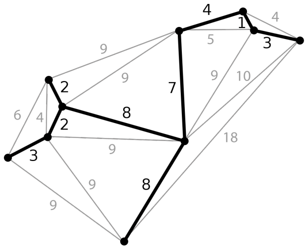
Introduction (Motivation)

- High demand and cost in making an electric network
- Reduce contamination by optimizing cable length
- Strengthen reliability and reduce loss of information
- Enhancing Antioquia's communication infrastructure, foster economic growth, bridge the digital division between urban and rural areas, and improve access to essential services for all its citizens

Basic concepts

- A **tree** is an **undirected graph** in which any two **vertices** are connected by exactly one path.
- A **spanning tree** T of an **undirected graph** G is a **subgraph** that is a **tree** which includes all of the **vertices** of G.
- A **minimum spanning tree (MST)** is a subset of the **edges** of a **weighted and undirected graph** that connects all the **vertices** together, without any **cycles** and with the minimum possible total edge weight.



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Case study

- We chose to optimize the cable length in an electric network in Caracolí, Antioquia that contains 170 utility poles.
- First, we got the information of the electric network from EPM.

Case study

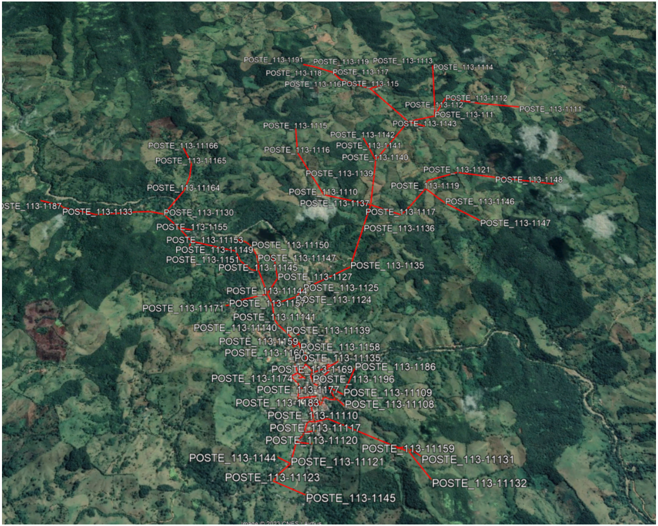

Figure: Original network.

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Subtour Elimination Formulation

$$
\begin{aligned}
\min_{x} \quad & \sum_{(i,j)\in E} \phi_{ij} x_{ij} && (1)\\
\text{s.t.} \quad & \sum_{(i,j)\in E} x_{ij} = n-1 && (2)\\
& \sum_{(i,j)\in E: i\in S: j\in S} x_{ij} \leq |S| - 1, \quad \forall S \subset V, S \neq V, S \neq \emptyset && (3)\\
& x_{ij} \in 0, 1, \quad \forall (i,j) \in E && (4)
\end{aligned}
$$

(3) is known as Subtour elimination constraint. Any subset of k vertices must have at most $k-1$ edges contained in that subset.

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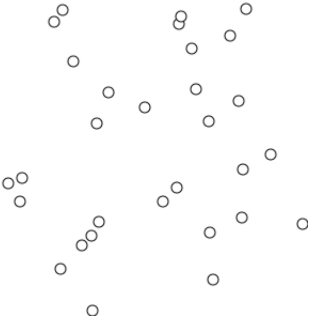
Methodology

- We modeled the electric network in Python, using each post's exact location to create a complete graph, where each vertex represented a utility pole and each edge represented a connection between two different poles.
- At first, we tried using a solver in Python (PuLP) to find the MST by solving the Subtour Elimination formulation. Nevertheless due to the set of constrains of our approach the problem is intractable.
- Finally, we used Kruskal's, Boruvka's and Prim's algorithms to convert the complete graph into the minimum spanning tree of the electric network.

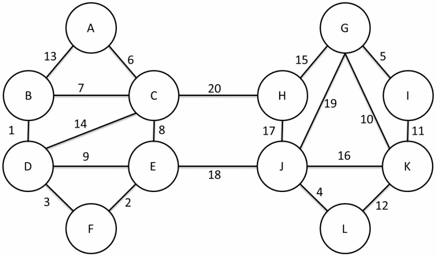
Kruskal's algorithm

1. Sort all edges of given graph $G = (V, E)$ in increasing order of their edge weights.
2. Set the expansion tree $T = (V, \emptyset)$.
3. For each edge (u, v), if $T \cup (u, v)$ doesn't form a cycle, then include (u, v) in T.
4. Return the Minimum Spanning Tree T.



Boruvka's algorithm

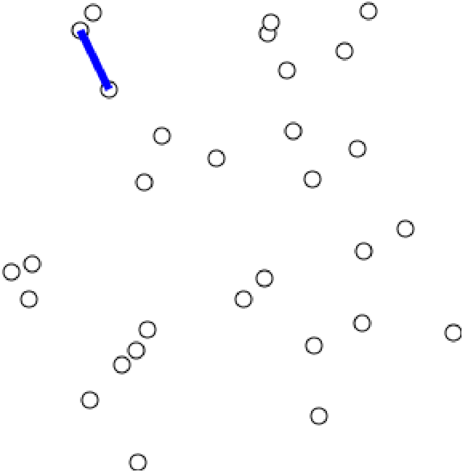
1. The input is a connected, weighted and undirected graph.
2. Initialize all nodes (vertices) is individual components or sets.
3. Initialize the MST as an empty graph.
4. If there is more than one component in the original graph, then the algorithm continues as it follows:
    4.1 Find the minimum cost edge that connects the component to any other component.
    4.2 Add the chosen edge to the MST is has not been already added.
5. Return the minimum spanning tree.



Prim's algorithm

1. Determine an arbitrary vertex as the starting node of the MST.
2. Find all edges connecting the vertex in the MST to a vertex that is not yet included in the MST.
3. Find the minimum weighted edge among all the edges found in the step before.
4. Add the edge to the MST if it does not form any cycle and the vertex to the set of vertices included in the MST.
5. Return the minimum spanning tree (MST).
   5.1 Find the minimum cost edge that connects the component to any other component.
   5.2 Add the chosen edge to the MST is has not been already added.
6. Return the minimum spanning tree.

Prim's algorithm animation



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Output

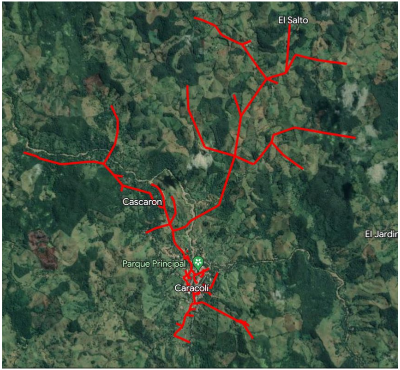

Figure: Original Network (22987m)

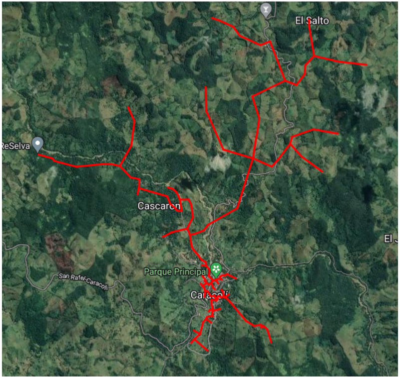

Figure: MST (20923m)

Results

Algorithms			
	Kruskal	Boruvka	Prim
Distance (m)	20923.278	20923.278	20923.278
Execution Time (ms)	159.92	33.38	12.31
Complexity	O(E(log(EV))	O(ElogV)	O(ElogV)

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Conclusions

- Optimized distance.
  - Distance reduced in about 10% (Around 2 km).
  - Cost and Material Reduction.
  - Safe data transmission and communication through network.
- Possible restrictions overlooked.
- Ideal for local telecommunications companies looking to impact in rural areas.

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References

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