Differentiation

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1 If y=f$\left(\frac{2x-1}{x^2+1}\right)$ and f'(x) = $\sin x^2$, then $\frac{dy}{dx}$ = ...

2 $f_r(x)$,$g_r(x)$, $h_r(x)$, r = 1,2,3 are polynomials in x such that $f_r(a) = g_r(a)$=$h_r(a)$ r = 1,2,3 and

$$F(x) = \begin{vmatrix} f_1(x) & f_2(x) & f_3(x) \\ g_1(x) & g_2(x) & g_3(x) \\ h_1(x) & h_2(x) & h_3(x) \end{vmatrix}$$

Then F'(x) at x = a is ....

3 If f(x) = $\log_x$(ln x), then $f'(x)$ at x = e is ....

4 The derevative of $sec^{-1}\left(\frac{1}{2x^2-1}\right)$ with respect to $\sqrt{1-x^2}$ at x = $\frac{1}{2}$ is ....

5 If f(x) = $|x-2|$ and g(x) = f[f(x)], then g'(x) = ... for x>20

6 If xe^{xy} = y + $\sin^2$x, then at x=0, $\frac{dy}{dx}$ = ...

7 The derivavtive of an even function is always an odd function.

8 If y^2 = P(x), a polynomial of degree 3, then $2\frac{d}{dx}\left(y^3\frac{d^2y}{dx^2}\right)$, equals .....

(a) P'''(x)+P'(x)
(b) P''(x)P'''(x)
(c) P(x)P'''(x)
(d) constant

9 Let f(x) be a quadratic expression which is positive for all the real values of x. If g(x) = f(x)+f'(x)+f''(x), then for any real x,

(a) g(x)<0
(b) g(x)>0
(c) g(x) = 0
(d) g(x)≥0

10 If y = $\sin x^{\tan x}$ then $\frac{dy}{dx}$ is equals to

(a) $\sin x^{\tan x}(1+\sec^2 x \log\sin(x)$
(b) $\tan x(\sin x)^{\tan x-1}.\cos x$
(c) $\sin x^{\tan x}\sec^2 x\log\sin x$
(d) $\tan x(\sin x)^{\tan x-1}$

11 $x^2+y^2 = 1$

(a) yy''-2y'^2+1 = 0
(b) yy''+y'^2+1 = 0
(c) yy''+y'^2-1 = 0
(d) yy''+2y'^2+1=0

12 Let f:(0 ∞) → R and F(x) = $\int_0^x$ f(t)dt. If F(x^2) = x^2(1+x), then f(4) equals

(a) $\frac{5}{4}$
(b) 7
(c) 4
(d) 0

13 If y is a function of x and log$(x+y)$-2xy = 0, then the value of y'(0) is equals to

(a) 1
(b) -1
(c) 2
(d) 0

14 If f(x) is a twice diffferentiable function and given that
f(1) = 1;f(2) = 4,f(3) = 9, then
(a) f''(x)=2 for ∀ x ∈ (1,3)
(b) f''(x)=f'(x) = 5 for some x ∈ (2,3)
(c) f''(x)=3 for ∀ x ∈ (1,3)
(d) f''(x) =2 for some x ∈ (1,3)

15 $\frac{d^2x}{dy^2}$ equals

(a) $\left(\frac{d^2y}{dx^2}\right)^{-1}$
(b) -$\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$
(c) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$
(d) -$\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$

16 Let g(x)=logf(x) where f(x) is twice differentiable positive function on (0, ∞) such that f(x+1) = xf(x). Then, for N=1,2,3,.......
g''$\left(N+\frac{1}{2}\right)$-g''$\left(\frac{1}{2}\right)$ =
g''$\left(N+\frac{1}{2}\right)$ - g''($\frac{1}{2}$ =

(a) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N-1)^2}\right\}$
(b) $4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N-1)^2}\right\}$
(c) $-4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N+1)^2}\right\}$
(d) $4\left\{1+\frac{1}{9}+\frac{1}{25}+......+\frac{1}{(2N+1)^2}\right\}$

17 Let f:[0, 2] → R be a function which is continuous on [0,2] and is differntiable on(0,2) with f(0) = 1. Let $F(x) = \int_0^{x^2} f(\sqrt{t})dt$ for x ∈ [0,2]. If F'(x) = f'(x) for all x ∈ (0,2), then F(2) equals

a e^2 - 1 (c) e - 1
(b) e^4 - 1 (d) e^4

18 Let f:R → R, g:R → R and h : R → R be differentiable functions such that f(x)=x^2+3x+2, g(f(x))=x and h(g(g(x)))=x for all x ∈ R. Then

(a) g'(2) = $\frac{1}{15}$ (c) h(0) = 16
(b) h'(1) = 666 (d) h(g(3)) = 36

19 For every twice differentiable function f:R → [-2,2] with $(f(0))^2 + (f(0))^2 = 85$, which of the following statement(s) is True?

(a) There exist r,s ∈ R, where r<s, such that f is on the open interval (r,s)
(b) There exists x_0 →(-4,0) such that $|f'(x_0)| \leq 1$
(c) $\lim_{x\to\infty}$ = 1
(d) There exists α → (-4, 4) such that f(α)+f'(α) = 0 and f'(α) ≠ 0

20 For any positive integer n, define f_n:(0, ∞)→R as $f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right)$ for all x∈ (0,∞).
Here, the inverse trignometric function $\tan^{-1}$(x) assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$
Then, which of the following statements are True?
(a) $\sum_{j=1}^{5} \tan^2 f_j(0)) = 55$
(b) $\sum_{j=1}^{10} (1+f_j'(0))\sec^2(f_j'(0)) = 10$
(c) For any fixed positive integer n, $\lim_{x\to\infty} \tan(f_n(x)) = \frac{1}{n}$
(d) For any fixed positive integer n, $\lim_{x\to\infty} \sec^2(f_n(x)) = 1$

21 Let f:(0,π)→R be a twice differentiable function such that $\lim_{t\to\infty} \frac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$ for all x∈ $(0,\pi)$. If $\frac{\pi}{6}$ = -$\frac{\pi}{12}$, then which of the following statement(s) are True?

(a) $f\left(\frac{\pi}{4}\right) = \frac{\pi}{4\sqrt{2}}$
(b) f(x)< $\frac{x^4}{6} - x^2$ for all x∈ $(0,\pi)$
(c) There exist $\alpha \in (0,\pi)$ such that f'(α) = 0
(d) $f'\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right) = 0$

22 Find the derivative of $\sin(x^2+1)$ with respect to x from first principle.

23 Find the derivative of

$$F(x) = \begin{cases} \frac{x-1}{2x^2-7x+5}, & \text{when } x \neq 1 \\ -\frac{1}{3}, & \text{when } x = 1 \end{cases}$$

24 Given y = $\frac{5x}{3\sqrt{(1-x)^2}}$ + $\cos^2$(2x+1); Find $\frac{dy}{dx}$.

25 $y = e^{x\sin x^3} + (\tan x)^x$. Find $\frac{dy}{dx}$

26 Let f be a twice differentiable function such that
f''(x) = -f(x) and f'(x) = g(x), h(x) = $[f(x)]^2 + [g(x)]^2$ find h(10) if h(5) =11

27 If α be a repeated root of a quadratic equation f(x) = 0 and A(x), B(x) and C(x) be polynomi-

als of degree 3,4 and 5 respectively, then show that

$$\begin{vmatrix} A(x) & B(x) & C(x) \\ A(\alpha) & B(\alpha) & C(\alpha) \\ A'(\alpha) & B'(\alpha) & C'(\alpha) \end{vmatrix}$$

is divisible by f(x), where prime denotes the derivatives.

28 If x = $\sec\theta$ - $\cos\theta$ and y= $\sec^n\theta$ - $\cos^n\theta$, then show that $(x^2+4)\left(\frac{dy}{dx}\right)^2 = n^2(y^2+4)$.

29 Find $\frac{dy}{dx}$ at x = -1, when

$(\sin y)^{\sin(\frac{\pi}{2}x)} + \frac{\sqrt{3}}{2}\sec^{-1}(2x) + 2^x\tan(ln(x+2))$ = 0

30 y = $\frac{ax^2}{(x-a)(x-b)(x-c)} + \frac{bx}{(x-b)(x-c)} + \frac{c}{(x-c)}$+1, prove that $\frac{y'}{y} = \frac{1}{x}\left(\frac{a}{a-x} + \frac{b}{b-x} + \frac{c}{c-x}\right)$.

31 Let f(x) = 2+$\cos x$ for all real x.

STATEMENT-1: For each real t, there exist a point c in [t,t+π] such that f'(c) = 0 because
STATEMENT-2 f(t) = f(t+2π) for each real t.

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation of Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation of Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

32 Let f and g be real valued functions defined on interval (-1,1) such that g"(x) is continuous, g(0) ≠ 0. g'(0) = 0, g" ≠ 0, and f(x) = g(x)$\sin x$

STATEMENT-1: $\lim_{x\to 0}$ [g(x)cot x - g(0)cosec x] = f"(0) and
STATEMENT-2: f'(0) = g(0)

(a) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation of Statement-1
(b) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation of Statement-1
(c) Statement-1 is True, Statement-2 is False
(d) Statement-1 is False, Statement-2 is True.

33 If the function f(x) =$x^3 + e^{\frac{x}{2}}$ and g(x) =f^{-1}(x), then the value of g'(1) is

34 Let f(θ) = $\sin\left(\tan^{-1}\left(\frac{\sin\theta}{\sqrt{\cos 2\theta}}\right)\right)$, where $-\frac{\pi}{4} < \theta < \frac{\pi}{4}$, Then the value of $\frac{d}{d(\tan\theta)}$(f(θ)) is

35 If y = (x+ $\sqrt{1+x^2}$)^n, then (1+x^2)$\frac{d^2y}{dx^2}$+x$\frac{dy}{dx}$ is
(a) n^2y
(b) -n^2y
(c) -y
(d) 2x^2y

36 If f(y) =e^y, g(y) = y; y>0 and

F(t) = $\int_0^t$ f(t-y)g(y)dy, then
(a) F(t) = te^{-t}
(b) F(t) = 1-te^{-t}(1+t)
(c) F(t) = e^t-(1+t)
(a) F(t) = te^t

37 f(x) = x^n, then the value of

f(1)-$\frac{f'(1)}{1!} + \frac{f"(1)}{2!} + \frac{f"'}{3!}$ + ...$\frac{(-1)^n f^n((1)}{n!}$ is

(a) 1 (b) 2^n (c) 2^n-1 (d) 0

38 Let f(x) be a polynomial function of second degree. If f(1) = f(-1) and a,b,c are in A.P then f'(a),f'(b),f'(c) are in

(a)Arthemetic-Geometric progression
(b)A.P
(c)G.P

(d)H.P

39 If $e^{y+e^{y}+e^{y+...\infty}}$, x>0, then $\frac{dy}{dx}$

(a) $\frac{1+x}{x}$ (b) $\frac{1}{x}$ (c) $\frac{1-x}{x}$ (d) $\frac{x}{1+x}$

40 The value of a for which sum of the squares of the roots of the equation x^2-(a-2)x-a-1 = 0 assume the least value is

(a) 1 (b) 0 (c) 3 (d) 2

41 If the roots of the equation x^2-bx+c = 0 be two consecutive integers, then b^2-4ac equals

(a) -2 (b) 3 (c) 2 (d) 1

42 let f:R→R be a differentiable function having f(2) = 6, f'(2) = $\frac{1}{48}$ Then $\lim_{x\to f(x)} \int_6^{f(x)} \frac{4t^3}{x-2}$dt equals

(a) 24 (b) 36 (c) 12 (d) 18

43 The set of points where f(x) =$\frac{x}{1+|x|}$ is differentiable is

(a) $(-\infty, 0) \cup (0, \infty)$ (c) $(-\infty, \infty)$
(b) $(-\infty, -1) \cup (-1, \infty)$(d) $(0, \infty)$

44 If $x^m.y^n = (x+y)^{m+n}$, then $\frac{dy}{dx}$ is

(a) $\frac{y}{x}$ (b) $\frac{x+y}{xy}$ (c) xy (d) $\frac{x}{y}$

45 Let y be an implicit function of x defined by $x^{2x} - 2x^x coty - 1 = 0$. Then y'(1) equals

(a) 1 (b) log 2 (c) -log 2 (d) -1

46 Let f:(-1,1)→R be a differentiable function with f(0) = -1 and f'(0) = 1. Let g(x)=$[f(2f(x)+2))]^2$ Then g'(0) =

(a) -4 (b) 0 (c) -2 (d) 4

47 $\frac{d^2x}{dx^y}$ equals:

(a) -$\left(\frac{d^2y}{dx^2}\right)^{-1}\left(\frac{dy}{dx}\right)^{-3}$ (c) -$\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-3}$
(b) $\left(\frac{d^2y}{dx^2}\right)\left(\frac{dy}{dx}\right)^{-2}$ (d) $\left(\frac{d^2y}{dx^2}\right)^{-1}$

48 If y= sec($\tan^{-1} x$), then $\frac{dy}{dx}$ at x = 1 is equals to:

(a) $\frac{1}{\sqrt{2}}$ (b) $\frac{1}{2}$ (c) 1 (d) $\sqrt{2}$

49 If g is the inverse of a function f and $f^{-1}(x) = \frac{1}{1+x^5}$ then g'(x) is equals to:

(a) $\frac{1}{1+(g(x))^5}$ (c) 1+x^5
(b) 1+$(g(x))^5$ (d) $5x^4$

50 If x = -1 and x = 2 are extreme points of f(x) =$\alpha log|x| + \beta x^2 + x$ then

(a) $\alpha = 2, \beta = -\frac{1}{2}$ (c) $\alpha = -6, \beta = \frac{1}{2}$
(b) $\alpha = 2, \beta = \frac{1}{2}$ (d) $\alpha = -6, \beta = -\frac{1}{2}$

51 If for x∈ $\left(0, \frac{1}{4}\right)$, the derivative of $tan^{-1}\left(\frac{6x\sqrt{x}}{1-9x^3}\right)$ is $\sqrt{x}$.g(x), then g(x) equals:

(a) $\frac{3}{1+9x^3}$ (c) $\frac{3x\sqrt{x}}{1-9x^3}$
(b) $\frac{9}{1+9x^3}$ (d) $\frac{3x}{1-9x^3}$