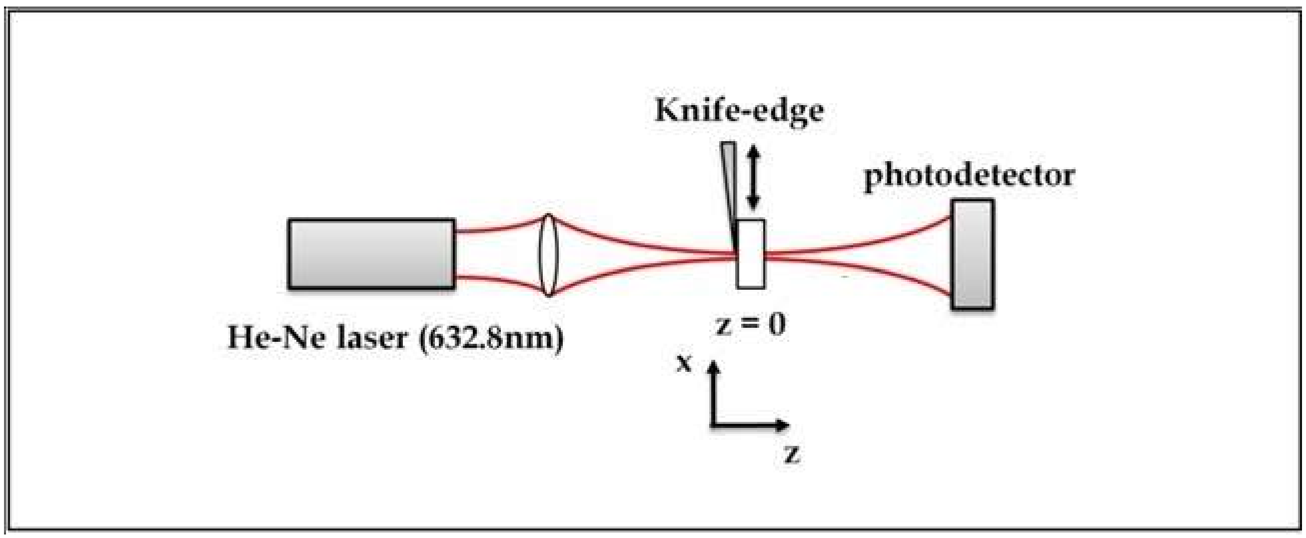
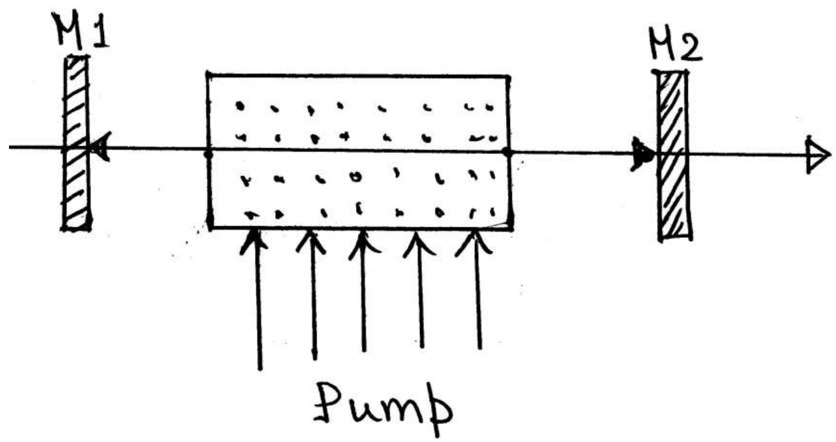
Quantitative Study of Gaussian Laser Beam using Knife Edge Technique

AIM:

- To measure the beam waist of a He-Ne laser.
- To determine the propagation factor M of the laser beam.

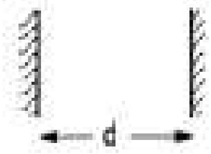


Basic components of a laser

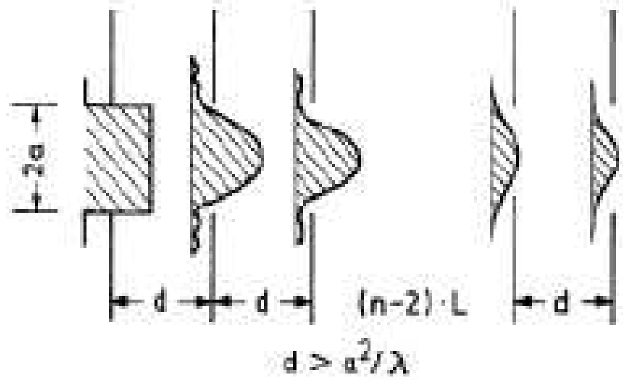


1. Active medium
2. Energy pump
3. Optical resonator

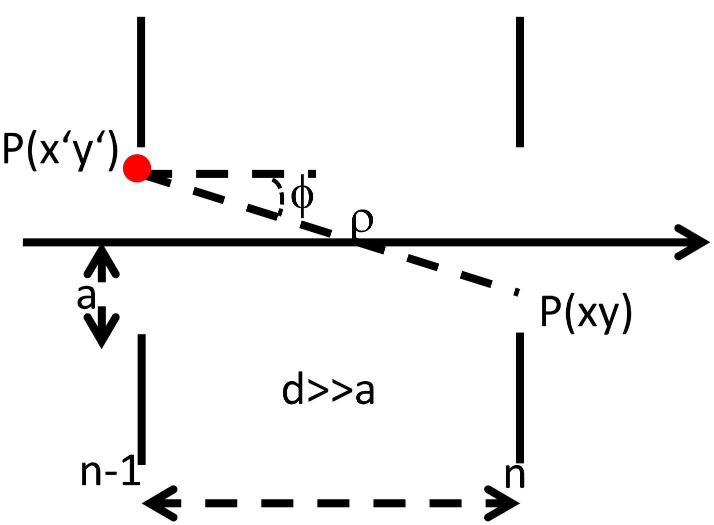
Spatial Field Distribution



Resonator



Equivalent system of equidistant apertures



From Kirchoff's diffraction theory, the field distribution of monochromatic spherical wave

$$A_n(x,y) = -\frac{i}{\lambda}\iint A_{n-1}(x',y')\frac{e^{-ik\rho}}{\rho}\cos\theta\, dx'dy'$$

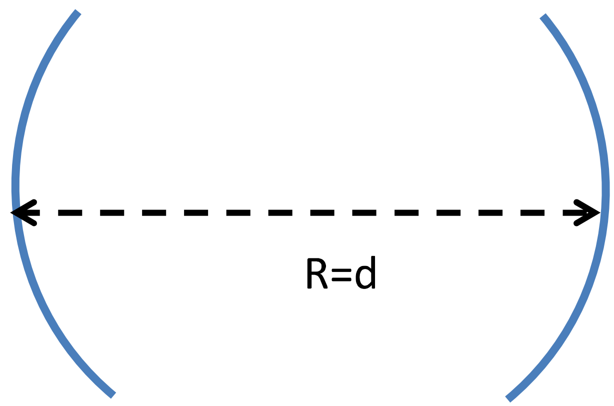
$$\rho^2 = d^2 + (x-x')^2 + (y-y')^2$$

A stationary distribution is reached if

$$A_n(x,y) = CA_{n-1}(x,y)$$

C is a constant phase factor, independent of x,y

For confocal resonator



For $a<<d$, $\rho\approx d$, $\cos\theta\sim 1$

$$\rho \approx d\left[1+\frac{1}{2}\left(\frac{x'-x}{d}\right)^2+\frac{1}{2}\left(\frac{y'-y}{d}\right)^2\right]^{1/2}$$

The stationary wave amplitude distribution

$$A_{mn}^{*}(x,y,z)=C^{*}H_m(x^{*})H_n(y^{*})\exp\left(-\frac{r^2}{w^2}\right)\exp[-i\phi(z-r,R)$$

normalization factor

Hermitian polynomial of order m

$$x^{*}=\frac{\sqrt{2x}}{w},\ y^{*}=\frac{\sqrt{2y}}{w}$$

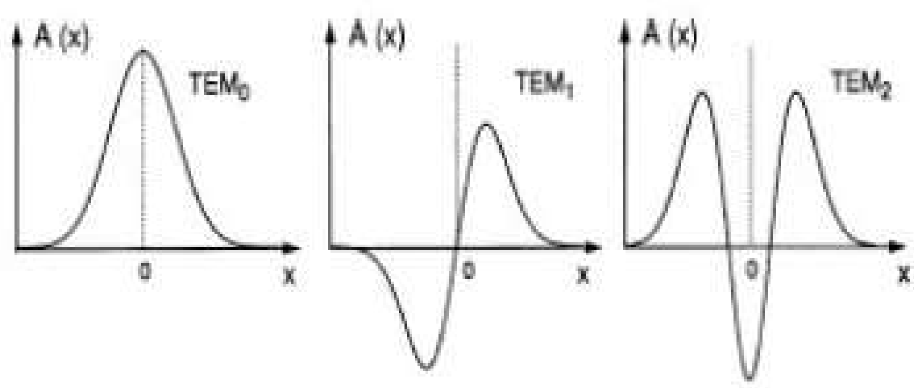
$$w^2(Z) = \frac{\lambda d}{2\pi}\left[1+\left(\frac{2Z}{d}\right)^2\right] \longrightarrow$$ determines radial intensity distribution

For the fundamental mode m=n=0 $\quad H_0(x^*) = H_0(y^*) = 1$

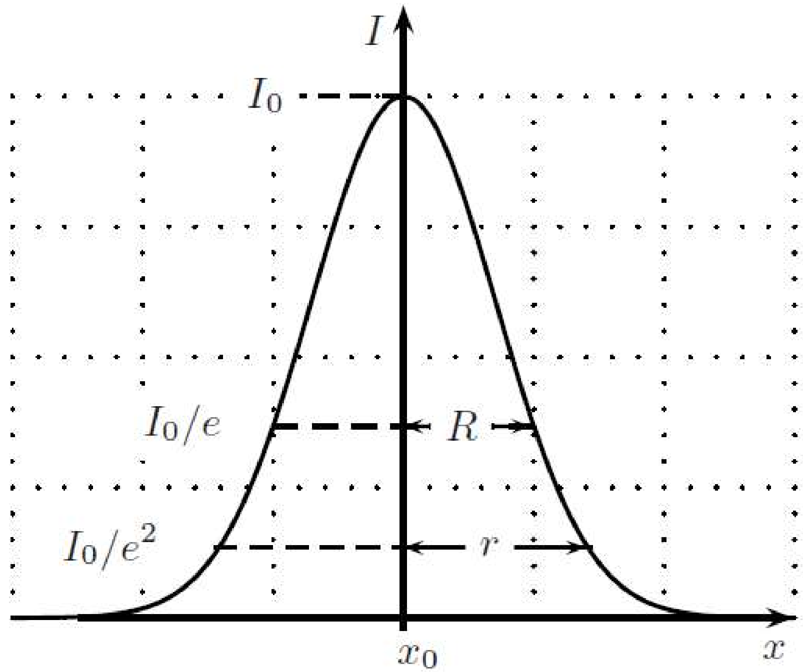
The radial intensity distribution

$$I(r) = I_0 \exp\left[-\frac{2r^2}{w^2}\right].$$

Transverse wave in 1D



Gaussian Laser beam



The beam radius : $r=w(z)$

The radius at which the intensity decreases to $1/e^2$ of its maximum value on the axis (r=0).

The beam waist= $$w_0 = \left(\frac{\lambda d}{2\pi}\right)^{1/2}$$

The smallest beam radius is the beam waist, which is located at the center Z=0.

The beam quality factor or propagation factor: M^2

$$w^2(Z) = w_0^2 + M^2\left(\frac{Z\lambda}{\pi w_0^2}\right)^{1/2}$$

Knife-Edge Method

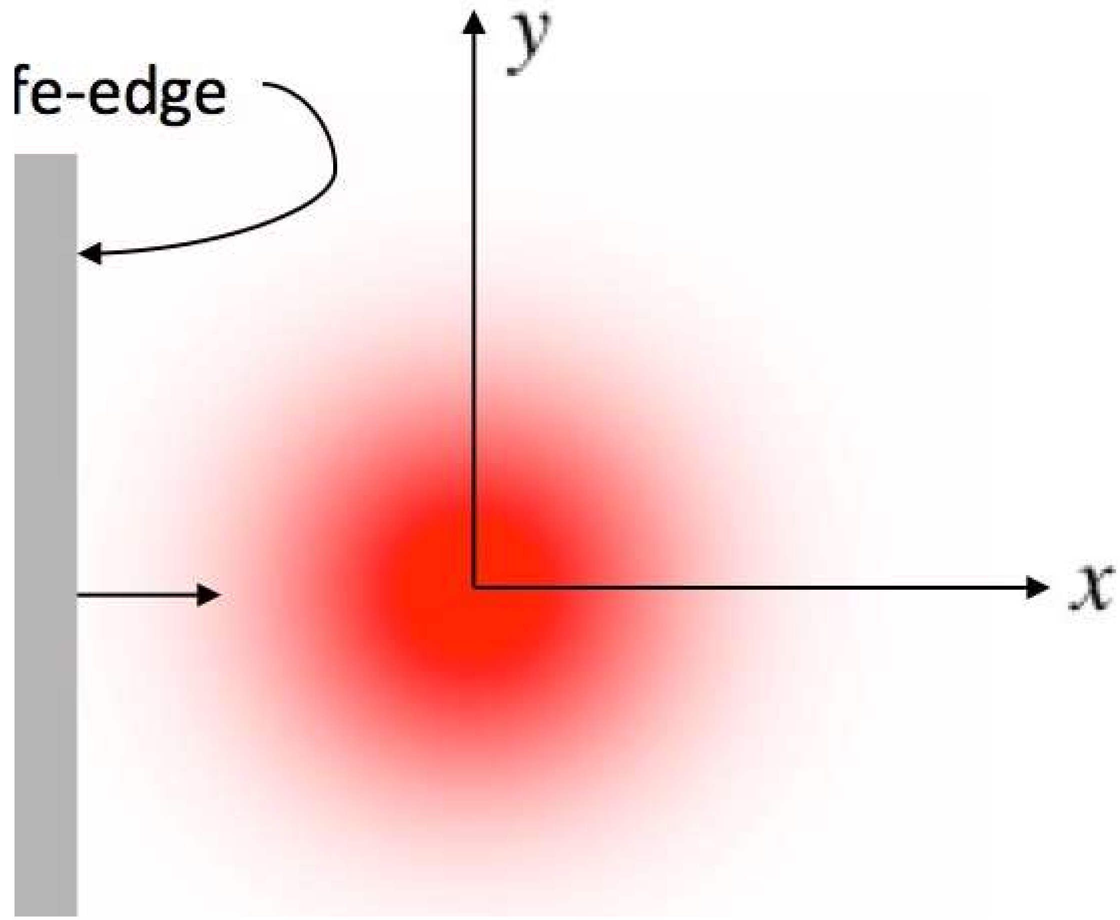
Due to the diffraction by knife edge there is an additional spreading of the laser beam radius along x or y. The measured data can be fitted by the equation,

$$I = P_0 + \frac{1}{2} P_{max}\left[1 - \mathrm{erf}\left(\frac{\sqrt{2}\,(X - X_0)}{w}\right)\right]$$

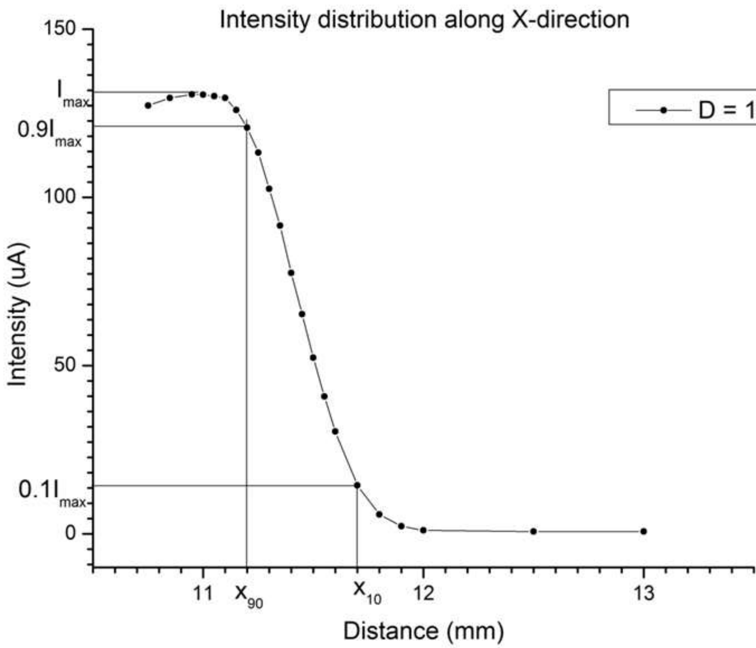
$$\mathrm{erf}(X) = \frac{2}{\sqrt{\pi}} \int_0^X e^{-u^2} du$$

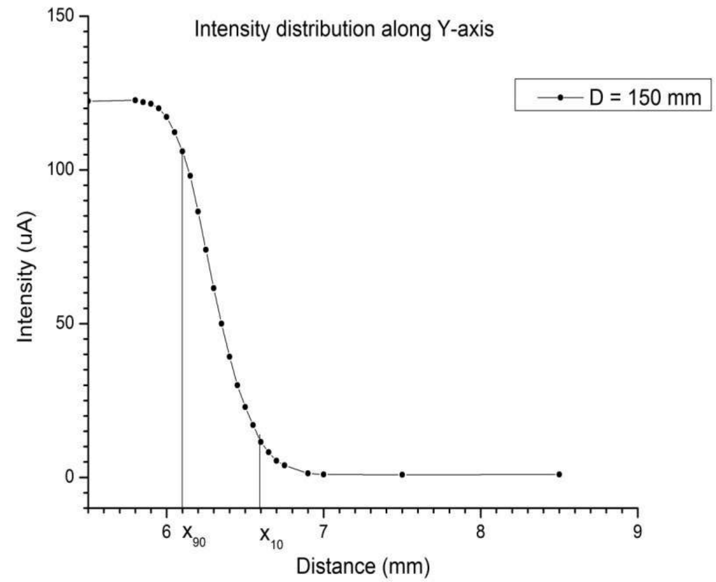
where P_0 is the background power

A two dimensional Gaussian profile integrated over the displacement of the knife edge/razor blade



fe-edge
y
x





Data courtesy: Mr. **Udvas Chattopadhyay (14PH40040)**

Ignoring P_0, at 10% power,

$$P_{10} = \frac{1}{2} P_{max} \left[1 - \mathrm{erf}\left(\frac{\sqrt{2}\, X_{10}}{W} \right) \right]$$

Or, $$\mathrm{erf}\left(\frac{\sqrt{2}\, X_{10}}{W} \right) = 0.8$$

Or, $$X_{10} = 0.64W$$

And from the symmetry of Gaussian beam,

$$\Delta X = X_{10-90} = 2 * 0.64\ W = 1.28\ W.$$

Where $X_{10\text{-}90}$ is the difference between 10% and 90% intensity

points. Beam waist at Z,

$$X_{10\text{-}90}/0.64 = 2*W = 1.5625\ X_{10\text{-}90}.$$

M^2: **the beam quality factor or propagation factor**

$\Rightarrow$ degree of variation of a beam from an ideal Gaussian beam.

$$W(Z) = W_0 \left\{ 1 + \left(M \frac{Z\lambda}{\pi W_0^2} \right)^2 \right\}^{1/2}$$

$$W(Z)^2 \approx W_0^2 + M^4 \left(\frac{Z\lambda}{\pi W_0} \right)^2$$

Now using equation

$$w(z)^2 = w_0^2\left[1 + \frac{\lambda^2 M^4 z^2}{\pi^2 w_0^4}\right]$$

Or,

$$w_0^4 - w(z)^2 w_0^2 + c^2 z^2 = 0$$

Example:

We obtain, for $D = z = 150\ mm$,

$$w_0^4 - (0.5524)^2\ w_0^2 + 150^2 c^2 = 0$$

And for $D = z = 1075\ mm$,

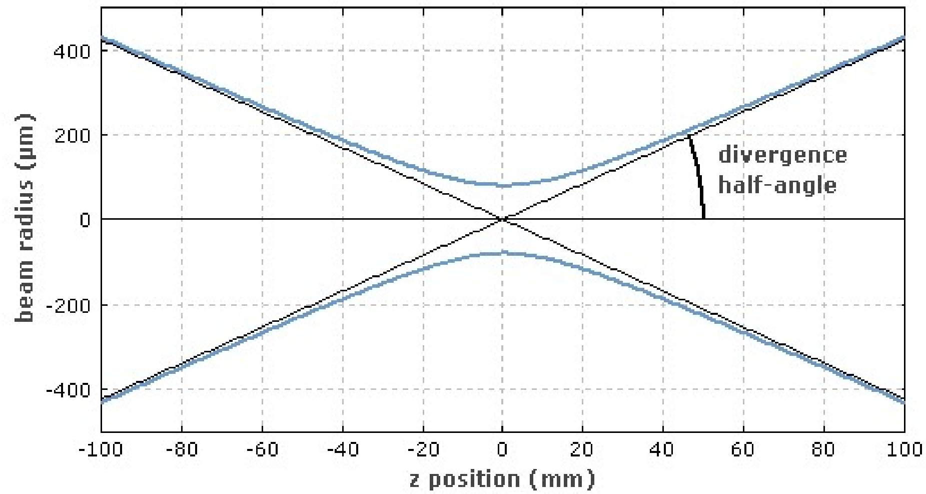
$$w_0^4 - (0.8855)^2\ w_0^2 + 1075^2 c^2 = 0$$

Solving the above two equations we get the value of
$w_0 = 0.5437\ mm$ and $c = 0.3.5396 \times 10^{-4} mm.$

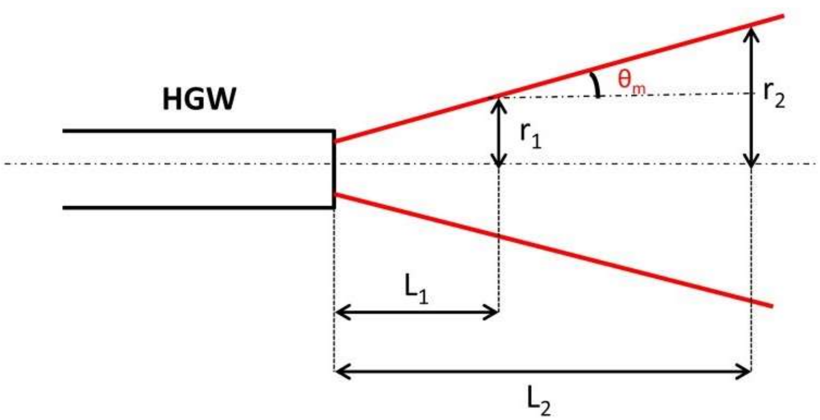
$$c = \frac{M^2 \lambda}{\pi}$$

$$\therefore M = 1.325 > 1$$

Beam divergence

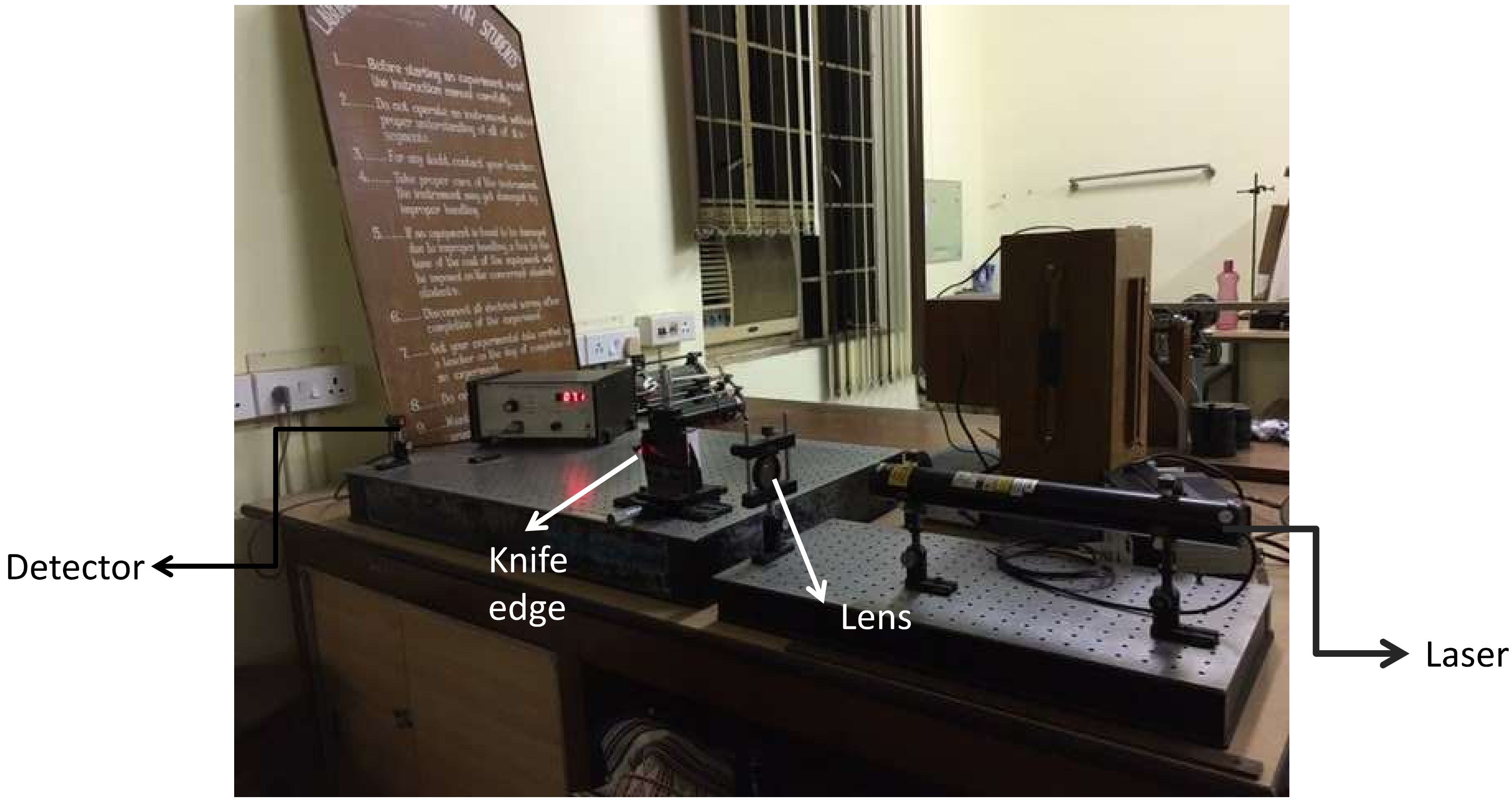


The beam divergence at a distance $Z > \frac{\pi W_0^2}{\lambda}$

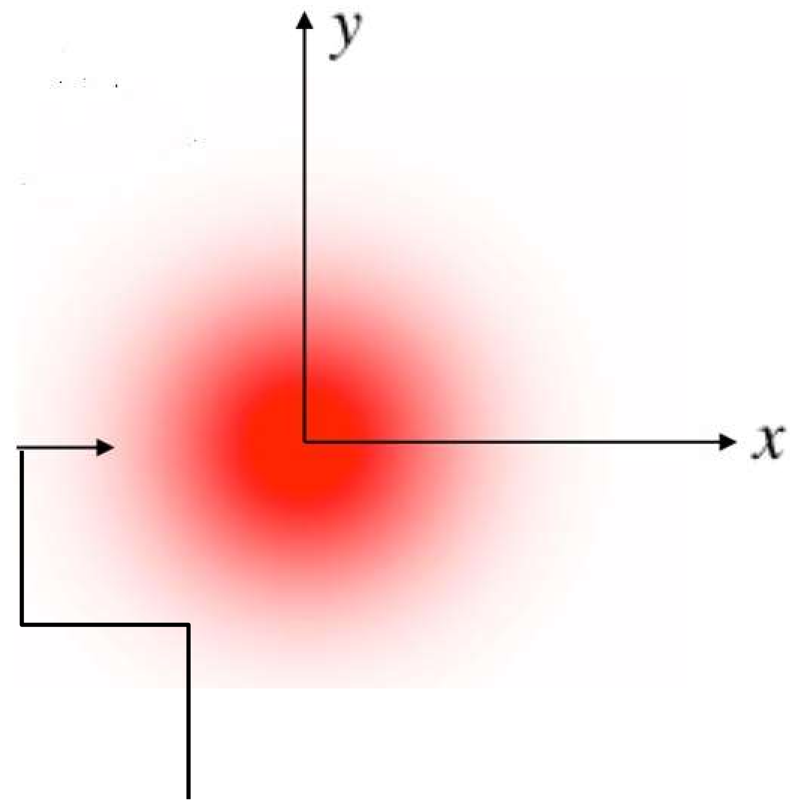


$$\theta = M^2 \frac{\lambda_0}{\pi W_0}$$

$$\theta = \frac{w_2 - w_1}{z_2 - z_1}$$


Detector
Knife edge
Lens
Laser

Knife edge



Laser beam on the screen

Other methods

Slit method

Pinhole method

Reference:

Quantitative and Qualitative Study of Gaussian Beam Visualization Techniques
J. Magnes, D. Odera, J. Hartke, M. Fountain, L. Florence, and V. Davis
http://arxiv.org/abs/physics/0605102v1