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Teaching Session Assignment II

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Assignment 1

Using the example solved during lecture 6 find the state space representation for a multiplicative seasonal ARIMA model.

$$ARIMA(p,d,q) \times (P,D,Q)_s$$

This process can be written as in the following using backshift operator.

$$\Phi^P(B^s) \cdot \phi^p(B) (1-B^s)^D (1-B)^d \cdot x_t = \Theta^Q(B^s)\, \theta^q(B) w_t$$

$$\phi_p^r(B) = 1 - \phi_1 B - \dots \phi_p B^p \quad \text{— ①}$$

$$\Phi_P^r(B^s) = 1 - \Phi_1 B^s - \dots \Phi_P B^{P.s} \quad \text{— ②}$$

$$\theta_q^r(B) = 1 - \theta_1 B - \dots \theta_q B^q \quad \text{— ③}$$

$$\Theta_Q^r(B^s) = 1 + \Theta_1 B^s + \dots \Theta_Q B^{Q \times s} \quad \text{— ④}$$

let $r = \max(P + D + p + d,\ Q + q + 1)$

using ①, ②, ③, ④ we can write.

$$\Phi_g^r(B^s)\, \phi_g^r(B^s)\, X_t = \Theta_p^r(B^s) \cdot \theta_p^r(B) \cdot w_t$$

$$\Phi^r(B^s)\, \phi^r(B) \cdot [\Theta^r(B^s) \cdot \theta^r(B)]^{-1} x_t = w_t$$

let $[\Theta^r(B^s) \cdot \theta^r(B)]^{-1} x_t = z_t$

$$\therefore \Phi^r(B^s) \cdot \phi^r(B) \cdot z_t = w_t$$

②

$$Z_t = \begin{bmatrix} Z_t \\ Z_{t-1} \\ \vdots \\ Z_{t-r+1} \end{bmatrix}$$

$$Z_{t+1} = \underbrace{\begin{bmatrix} \phi_1 \cdot \Phi_1 & \cdots & \cdots & \phi_r \cdot \Phi_r \\ 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix}}_{r \times r} \cdot Z_t + \underbrace{\begin{bmatrix} \omega_T \\ 0 \\ 0 \\ 0 \\ \vdots \end{bmatrix}}_{r \times 1}$$

$$X_t = \begin{bmatrix} 1 & \theta_1 \Theta_1 & \cdots & \theta_{r-1} \Theta_{r-1} \end{bmatrix}$$

These equations represent the state space model for a ~~general~~ seasonal ARIMA model.