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Teaching session Assignment 2:-

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~~Exam~~ Assignment 1:-

Using the example solved during lecture 6, find the state space model representation of ~~of~~ for a multiplicative seasonal arima model.

$$ARIMA\ (p, d, q) \times (P, D, Q)_s$$

This process can be written using the backshift operator as...

$$\Phi^P(B^s)\ \phi^p(B)(1-B^s)^D(1-B)^d \cdot x_t = \Theta^Q(B^s) \cdot \theta^q(B)\ w_t$$

$$\Rightarrow \Phi^P(B^s) \cdot \phi^p(B)(1-B^s)^D(1-B)^d \cdot x_t \cdot [\Theta^Q(B^s) \cdot \theta^q(B)]^{-1} = w_t$$

$$\text{let } z_t = [\Theta^Q(B^s) \cdot \theta^q(B)]^{-1} \cdot x_t$$

$$\Rightarrow \left.\begin{cases} x_t = \Theta^Q(B^s) \cdot \theta^q(B) \cdot z_t & (AR) \\ w_t = \Phi^P(B^s) \cdot \phi^p(B) \cdot (1-B^s)^D(1-B)^d \cdot z_t & (MA) \end{cases}\right\} \rightarrow (1)$$

Now the state space model is given by..

$$\left.\begin{cases} z_t = A \cdot z_{t-1} + e_t \\ x_t = C \cdot z_t + r_t \end{cases}\right\} \rightarrow (2) \qquad \begin{matrix} e_t \sim N(0, R) \\ r_t \sim N(0, Q) \end{matrix}$$

Comparing equation (1) and (2) we get that the first term is AR and the second term is ~~r~~ for MA.

We now need to find matrix A and C in order to determine the state space equivalence.

②

From Harvey representation of ARIMA

$$F = \begin{pmatrix} \phi_1 & 1 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & 0 & 1 & 0 & \cdots & & 0 \\ & & \cdots & & & & \vdots \\ & & & & & & 1 \\ \phi_d & 0 & 0 & 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$\therefore \left\{ Z_t = \underbrace{F \cdot Z_{t-1}}_{AR} + \underbrace{\omega_t \cdot \begin{pmatrix} 1 \\ \theta_1 \\ \theta_2 \\ \vdots \\ \theta_{ds-1} \end{pmatrix}}_{MA} \right\} \rightarrow ③$$

$d = \max(p, q+1)$

$$\left\{ X_t = (1; 0 \; 0 \cdots 0)^T \cdot Z_t \right\} \rightarrow ④$$

We can rewrite the expression such that.

C matrix $= [1 \; \theta_1 \; \theta_2 \; \cdots \; \theta_{ds-1}]$

$$A \text{ matrix} = \begin{bmatrix} \phi_1 & 1 & 0 & 0 & \cdots & 0 \\ \phi_2 & 0 & 1 & \cdots & & \vdots \\ \vdots & & \ddots & & & \vdots \\ & & & & & 1 \\ \phi_d & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}$$

The dimension of matrix A will be.

$\max(p + Sp + d + S \cdot D, \; S \cdot Q + q)$