Confidence interval for phi_2

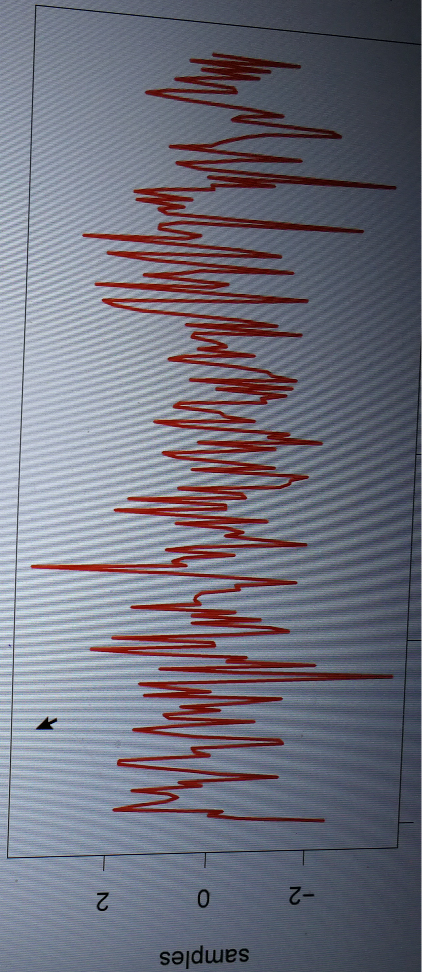
```
##         ar2        ar2
## -0.06775436 0.30562809
```

The original value of ϕ_2 that is 0.1 falls within the 95% confidence interval for estimates ϕ_2 by MLE.

Question 3

Generate 200 observations of a seasonal $ARIMA(0,0,1)\times(0,0,1)_{12}$ model with coefficients $\Theta = 0.6$ and $\theta = 0.3$ by using arima.sim(). Plot sample ACF and PACF and also theoretical ACF and PACF. Which patterns can you see at the theoretical ACF and PACF? Are they repeated at the sample ACF and PACF?

This is a seasonal model with only MA terms. The model can be written as $X_t = (1+\Theta_1 B^{12})(1+\theta_1 B)w_t$
Thus $X_t = w_t + \theta_1 B w_t + \Theta_1 B w_{t-12} + \theta_1 \Theta_1 B w_{t-13}$



~~S12 D log log~~

S12 D d log dxt = diff(dlogdxt, 12) — doing seasonal diff

dlogdxt = diff(logdxt) — doing normal diff

tseries :: adf.test(dlogxt)

acf and pacf or ~~residuals~~ residuals (fit)

hist(residuals) should be normal after the fit

xt reconstruction

inverse of diff

xtfor = cumsum(exp(log dxt for $pred)) + tail(xt, n=1)

plot(xt)

lines(length(xt)+c(1:20), xtfor), ~~col~~ =

Question 3

a) generate

) ok.

$$z_t = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} z_{t-1} + e_t$$

$x_t = [1 \ \ \theta_1] z_t$

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Question 3

a) generate

b) ok.

c) $$z_t = \begin{bmatrix}\phi_1 & \phi_2 \\ 1 & 0\end{bmatrix} z_{t-1} + e_t$$

$$x_t = [1 \ \ \theta_1] z_t .$$

$$\begin{bmatrix} z_t^1 \\ z_t^2 \end{bmatrix} = \begin{bmatrix}\phi_1 & \phi_2 \\ 1 & 0\end{bmatrix} \begin{bmatrix} z_{t-1}^1 \\ z_{t-1}^2 \end{bmatrix} + \begin{bmatrix} w_t \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} z_t^1 \\ z_t^2 \end{bmatrix} = \begin{bmatrix}\phi_1 z_{t-1}^1 + \phi_2 z_{t-1}^2 + w_t \\ z_{t-1}^1 + 0\end{bmatrix}$$

Hence $z_t^2 = z_{t-1}^1 = B z_t^1$ and.

$w_t = z_t^1 - \phi_1 z_{t-1}^1 - \phi_2 z_{t-1}^2$ and so.

$(1 - \phi_1 B - \phi_2 B^2) z_t^1 = w_t$.

also $x_t = z_t^1 + \theta_1 z_t^2 = (1 + B\theta_1) z_t^1$

$\therefore$ $x_t = (1 + B\theta_1) z_t^1$

$x_t = (1 + B\theta_1)(1 - \phi_1 B - \phi_2 B^2)^{-1} w_t$.

$(1 - \phi_1 B - \phi_2 B^2) x_t = (1 + B\theta_1) w_t$,

so we conclude

~~zt~~ x_t ←

c) No because $x_t = z_t^1 + \theta_1 z_t^2$ and unless $\theta_1 = 0$, $x_t \neq z_t^1$

d) $e_t = \begin{bmatrix} w_t \\ 0 \end{bmatrix}$

$y_t = [1 \quad 0]\, z_t + v_t$

$z_t = \begin{bmatrix} \phi_1 & \phi_2 \\ 1 & 0 \end{bmatrix} z_{t-1} + w_t$

$E[e_t] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$E[e_t e_t^T] = E\begin{bmatrix} w_t w_t & 0 \\ 0 & 0 \end{bmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

e) v_t = rnorm(n, 0, 1)
$y_t = v_t + x_t$.

A = t(c(1, theta_1)) ;
F = matrix (c (phi_1 , 1, phi_2, 0), nrow = 2) ;
Q = matrix (c(1, 0, 0, 0) , nrow = 2)
Sig 0 = matrix(c(100, 0, 0, 100), nrow = 2)

Mu0 = t(c(0, 0))

2tf = kaloutput[1, ,]

cQ = sqrt (Q)

cR = 1

g) $y_t = [1 \quad 0]\, z_t + \mu + v_t$ (v_t initially)

$v_t \sim N(\mu_t + R)$

$v_t' \sim N(0, R)$

$y_t' = y_t - \mu$.

change the line 4
insted of $x_t - C m_{t|t-1}$
write $x_t - \mu_t - C m_{t|t-1}$

here μ_t is an observation mean.

h) 1) filtering kal \$ xf(1, ,) + theta_1 * kal \$ kal\$ xf[2, ,]

2) kal \$ xf[1, ,]

3) kal \$ xf[2, ,]
kal \$ pf [1, 1,]

i) ARIMA (2, 1, 1)
$(1-B)(1-\phi_1 B - \phi_2 B^2) x_t = (1+\theta_1 B)\, w_t$