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CIS 575: Intro to Algorithm Analysis

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1.)

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 \quad a = 4, b = 2, r = \log_2 4$$

$$f(n) = n^2 \in (n^2 \lg n) \rightarrow T(n) \in \Theta(n^2 \lg n)$$

$$T(n) = 4T\left(\frac{n}{2}\right) + N \quad a = 4, b = 2, r = \log_2 4$$

$$T(n) \in \Theta(n^2)$$

$$T(n) = T(n+1) + n^2 \quad a = 1, b = 1, r = \log_1 1$$

$$T(n) \in \Theta(n^2)$$

2.)

1.) $T\left(\frac{n}{2}\right) + 3$

2.) $T(n) \in \Theta(\lg(n))$

3.)

Handwritten derivation of the recurrence relation $T(n) = T\left(\frac{n}{3}\right) + T\left(\frac{n}{3}\right) + n$ for $n \geq 3$.

The derivation shows the steps to find the constant c in the master theorem:

$$\begin{aligned} T(n) &= T\left(\frac{n}{3}\right) + T\left(\frac{n}{3}\right) + n \quad n \geq 3 \\ \rightarrow T(n) &\leq c\left(\frac{n}{3}\right) \lg\left(\frac{n}{3}\right) + c\left(\frac{n}{3}\right) \lg\left(\frac{n}{3}\right) + n \\ &\leq c\left(\frac{n}{3}\right) \lg(n) + c\left(\frac{n}{3}\right) \lg(2n) + n \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} \lg(2n) + n \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3}(\lg(2) + \lg(n)) + n \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} \lg(2) + \frac{2}{3} \lg(n) + n \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} \lg(2) + n \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} cn + n \leq cn \lg(3n) \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} cn + n \leq cn(\lg 3 + \lg n) \\ &= cn\left(\frac{1}{3} \lg(n)\right) + \frac{2}{3} cn + n \leq cn(\lg 3) + cn(\lg n) \\ &\quad \frac{2}{3} + \frac{1}{3} \leq \lg(3) + \lg(n) \\ &\quad \frac{1}{3} \leq \lg(3) - \frac{2}{3} \\ &\quad \frac{1}{3} \leq 0.9183 \\ &\quad c \geq 1.08897 \end{aligned}$$