PARALLEL PARKING PROBLEM

Project report submitted

In partial fulfilment of the requirement for the degree of

Bachelor of Technology

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(November-2023)

CERTIFICATE

It is certified that the work contained in the project report titled "**Parallel parking problem**," by "Mooti Meghana" and "Gundeti Srivardhan" has been carried out under my supervision and that this work has not been submitted elsewhere for a degree

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November 2023.

DECLARATION

I declare that this written submission represents my ideas in my own words and where others' ideas or words have been included, I have adequately cited and referenced the sources. I also declare that I have adhered to all principles of academic honesty and integrity and have not misrepresented fabricated or falsified any idea/data/fact/source in my submission. I understand that any violation of the above will be cause for disciplinary action by the Institute and can also evoke penal action from the sources that have thus not been properly cited or from whom proper permission has not been taken when needed.

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Thank you.

With sincere regards,

Dated: 15th November, 2023.

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ABSTRACT

This study addresses the optimization of parallel parking in vehicles to enhance manoeuvrability and efficiency in urban scenarios. The challenge of seamlessly parking a vehicle between two others in limited spaces presents a complex computational and control problem. Our research focuses on the development of intelligent algorithms and control strategies to facilitate precise parallel parking manoeuvres.

The proposed approach integrates advanced contour-following algorithms and employs machine learning to analyse and interpret vehicle size data, categorized by vehicle type. This analysis allows for the calculation of corresponding contours, enabling the vehicle to recognize potential parking spaces and determine optimal trajectories for parking.

Furthermore, we investigate the impact of various factors, including vehicle size, parking space dimensions, and environmental conditions, on parallel parking performance. Considering these variables, the developed system dynamically adapts to diverse parking scenarios, ensuring robustness and reliability.

The research findings contribute to ongoing efforts in autonomous vehicle technology, enhancing urban parking efficiency, reducing traffic congestion, and promoting overall transportation sustainability. The proposed solutions aim to pave the way for future advancements in autonomous parking systems, fostering a safer and more convenient urban driving experience.

Keywords: - Parallel parking optimization, Vehicle manoeuvrability, Optimal trajectories, Contour-following algorithms, Vehicle size data, Autonomous vehicle technology.

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Chapter 1

Introduction

1.1 Introduction

Driving cars, or as a matter of fact, any of the four-wheelers is a tricky thing to do, especially if the place is over-crowded or congested for example, cities. We all might have experienced this one time or the other, where we think twice about driving into the city because the parking, much to our dismay gives us a headache. Be it the parking lots near the restaurants, malls, and every other thing there is, we often face difficulty with the Parallel Parking manoeuvre, if we ought to park our car between two already parked ones.

In the modern world where everyone wants to travel luxuriously, cars have become a major need. But as the usage of cars increased direly, the place to park has decreased, demanding either more space to do the parking or the reduction of its usage, where neither can be compromised. There are different parking requirements for different places. When there’s no underground parking, however, we are forced to park our vehicles on the street. How specialized the driver may be, they are strained to get the car parked safely, with no damage to their vehicle as well as others. The cars are arranged in a single line, with the back bumper of one car facing the front bumper of the car just adjacent to it and so on, and is done parallel to a curb if provided with one. This type of parking is called *Parallel Parking.*

Parallel parking is one of the most common modes of parking used in parking lots or parking structures. Other modes of parking are Perpendicular, angular, diagonal, inner city, and Tadem parking. Studies show that in the US, the spaces allotted for parking usually range from 8.5 to 9 feet (2.6-2.7 m) in width and 20 to 24 feet (6.1-7.3 m) long whereas typically in the UK, it goes to 7.8 feet (2.4 m) wide and 15.75 feet in length. According to CMDA norms, the standard parking space size in India, however, is 8.2 feet (2.5 m) wide and 16.33 feet (5 m) in length.

As per the IRC, the standard dimensions of a car are taken as 5 x 2.5 meters. But are all cars of the same dimensions? Not really. Does every parking lot there is, follow the same above-allotted range? Not particularly. It could mainly depend on the place or the type of building we are talking about. For example, the residential area requirements differ from Walmart parking lot requirements. But one thing that remains common throughout is the tactic used to park.

1.2 The Problem

PARALLEL PARKING MANOEUVRE:

The figure below depicts the basic idea of what parallel parking looks like. The cars are parked along the road or the curb. There is no forward movement or backwards while a vehicle is being parked or unparked which is deemed as the safest form of parking avoiding accidents. This doesn't mean there are no accidents. Most accidents in this scenario occur when the driver doesn't know how to manoeuvre. Bumping with the adjacent cars, misjudging the space, and overestimating that the limited space would be sufficient to carry out is much risky.

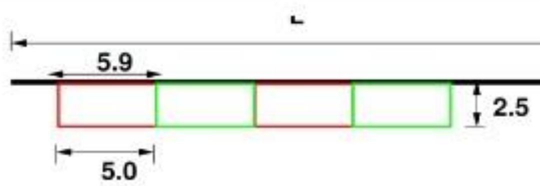


Figure showing the schematic of a parallel parking problem.

The length available to park N number of vehicles, L = $N * 5.9$

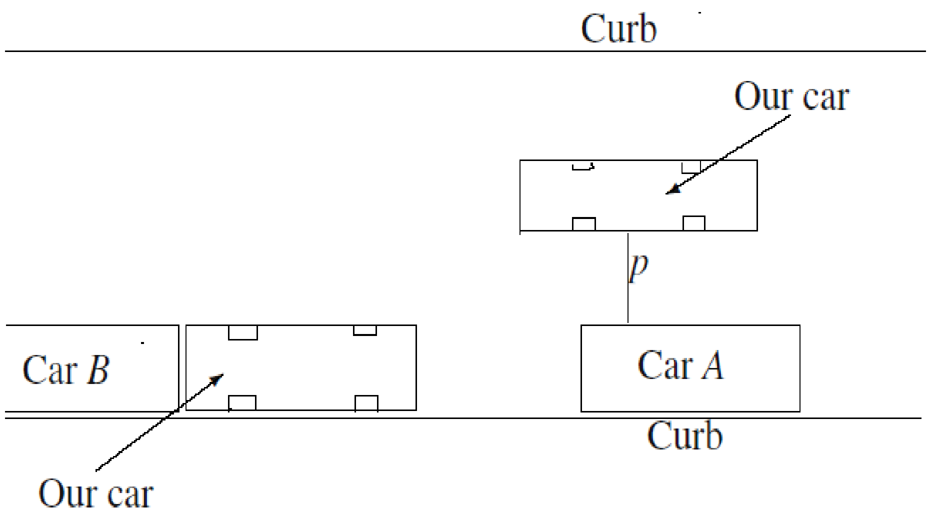
However, just like the schematic, the scenario in real life is similar and we can see that this type of parking occupies the maximum length of the curb which implies that the very least number of cars can be parked. The basic idea on how to achieve this is steering the vehicle ahead, reversing in the opposite direction, and steering it again to align it with the front and back drive.

This is just a basic or a rough estimate on how to proceed but we will get into detail as we proceed further accordingly. Beginners use reference points to align themselves with the car positioned in the front, the angle of backing needed, and when to turn the steering wheel. Reversing into the spot viz this parallel parking technique gives us an advantage of making use of a space that's not longer than the car, at least not that much.

However, the techniques might differ if we are to use two or three reference points while the backing is done. A driver, a skilled one at that can turn the steering wheel all the way to the right and to the left immediately at a particular critical point. However, such is not the case with the beginners. They result in things like collision with either of the cars, parking away from the curb, or too close to the front or rear of the vehicles situated on either side.

This could be avoided by adding an intermediate step where the backing up can be done after attaining a certain ideal angle that supports backing. They back up their wheels straight until the rear is back enough to make the final reverse possible. This is one of the main areas where the problem arises. How much angle is an ideal angle? Can it be followed for cars that are long, longer than usual? At what distance should we start the manoeuvre?

The below figure gives us the basic idea of a schematic as to how one has to visualise the situation to find the perfect solution. One has to also keep in mind that the space here can vary. It cannot be this way all the time, sometimes there would be no sufficient place at all. We have to consider all possible scenarios but for now, let us assume that this is the case.



Assume the situation as the figure depicted above. The place is between the cars A and B. All we need to do is just park it in the space. Ref: The beauty of everyday mathematics © Springer-Verlag Berlin Heidelberg 2012

Steps that are to be followed, in general:

1. Stop directly in front of Car A, let's say a distance p from it.
2. Back up until the midpoint of our car i.e., the midpoint between all four wheels, is parallel with the read end of car A.
3. And now, turn the steering wheel to its maximum so we can drive into the space. While doing so, it must be kept in mind about the movement of the car along the circular arc of an included angle α.
4. Now turn the steering wheel as far as possible in the opposite direction in order for our car to be parallel to the street one last time and move the car along the circular arc of the same included angle α, but in the opposite direction.
5. Now, drive a bit ahead so as to not hit the car B behind.

This problem's solution was reported to radios and televisions in Germany in the fall of 2003 and has been prominent ever since. There have been many famous mathematicians who worked on this problem like the British Mathematician Rebecca Hoyle. The infamous Rebecca Hoyle's formula was found leaving almost everyone berserk because of how big it was and how absurd it was, owing it to the British pound symbol used in the middle. However, she did not publish that formula on her site but a set of four formulae, that gave more questions than answers.

People criticized her formula for multiple reasons. For it has superfluous variables, has a complete formula for calculating something that isn't quite necessary and also, this set of formulae doesn't apply to the typical midsized cars.

Later on, concepts like the turning circles and the terms related to it were introduced making it much easier to come up with new formulae. The real challenge was these formulae, even though almost perfect, were vastly based on the product of theory which when applied in real-life situations may or may not work or be applicable just yet.

1.3 Scope of Work

The following are the reasons why we need to work on this, despite having found the fitting formulae:

- As per the formulae found, the distance between our car and the neighbouring car is assumed to be 0 mm, for us to get an optimal angle α that solves our parking problem, which in fact, can't be done in real time.
- Rather than wondering about the space to park or worrying about taking the right angle as indicated, its time saving and luxurious if we let the computer do the calculations. It's about time we can put an end to this problem.
- Cars will run into trouble if the said car has a long hood. The rear would have much space left and it isn't noticeable until we dent the front car's rear, damaging ours in the process.

As said earlier, technological advancements made it possible to incorporate such technology into our cars where we are just required to drop the dimensions of the car, and the computer in it does the calculations and parks the car by itself automatically with the help of sensors and the algorithm.

In this project, we aim to work on designing the algorithm. This project was chosen specifically because it has real-time applications which when worked on can create a system where either it directs the driver to park accordingly or it parks by itself, preventing mishaps and not only saving time but also money.

Chapter 2

Review Of Literature

2.1 Previous works

2.1.1 Rebecca Hoyle's formula: The start

It was all over the newspapers, this formula, that the British Mathematician Rebecca Hoyle published on her site in April 2003. At first glance, it might look too rubbish because many things aren't the way they should be. This confused everyone all around the world.

$$p = r - w/2, g) - w + 2r + b, f) - w + 2r - fg$$

$$\max\left(\left(r+\frac{w}{2}\right)^2 + f^2, \left(r+\frac{w}{2}\right)^2 + b^2\right) \text{£}\min\left(\left(2r\right)^2, \left(r+\frac{w}{2}+k\right)^2\right.$$

There were too many things here that didn't make any sense and appeared to be wrong, whatsoever. The formula appeared to have no open brackets at the very beginning of the equation, so, why did it have a closed bracket? There were too many commas in the middle of the *formula* and there's a British pound sign. What does that even mean?

This very formula was not published on her site… But there was, however, a set of four formulae that made more sense compared to the above one. They are:

$$p = r - \frac{w}{2},$$

$$g \geq w + 2r + b,$$

$$f \ \leq w + 2r - fg,$$

$$\max\left(\left(r + \frac{w}{2}\right)^2 + f^2, \left(r + \frac{w}{2}\right)^2 + b^2\right) \leq \min\ (4r^2, (r + \frac{w}{2} + k)\text{^}2.$$

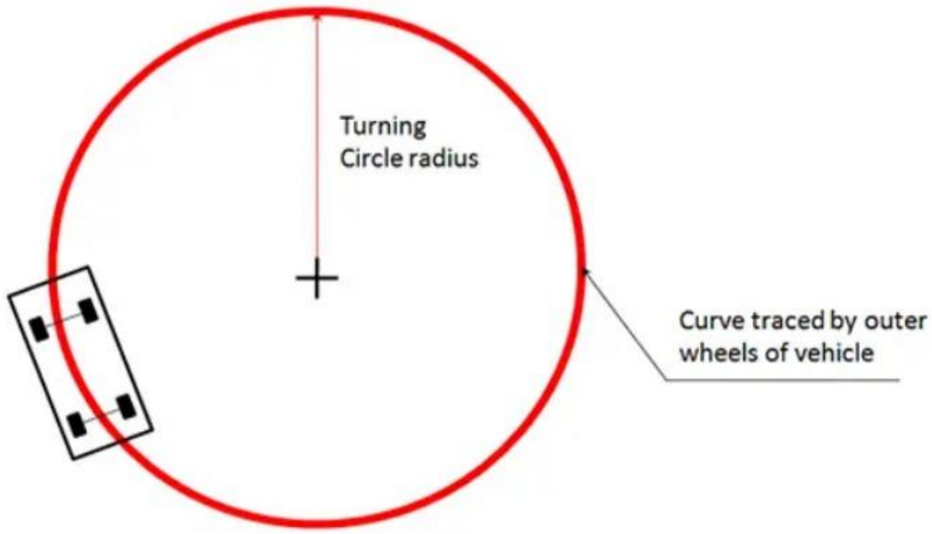
Where:

- p is the lateral distance from car A in the front.
- r be the radius of the smallest circle which ought to be created by the centre of the car, that is the centre of that rectangle that includes all four tyres.
- w is the width of our car.
- g is the size of the parking space.
- f is the distance between the midpoint of the car to its front.
- b is the distance between the midpoint of the car to its rear.
- fg is the distance in the front at the end of the parking manoeuvre.
- k is the distance from the curb at the end of the parking.

The above formula is written this way, meaningfully so. But, even if we can use these formulae in problems that involve a parking manoeuvre, these however arouse many doubts and were faced with possible criticisms. One of them is the inclusion of calculating fg which is of little to no significance, we might as well say it to be redundant. We are calculating k, sure but do we need it? Guess not. We also need to note that this shall stay theoretical and on applying it to a typical midsized car, the results would be catastrophic.

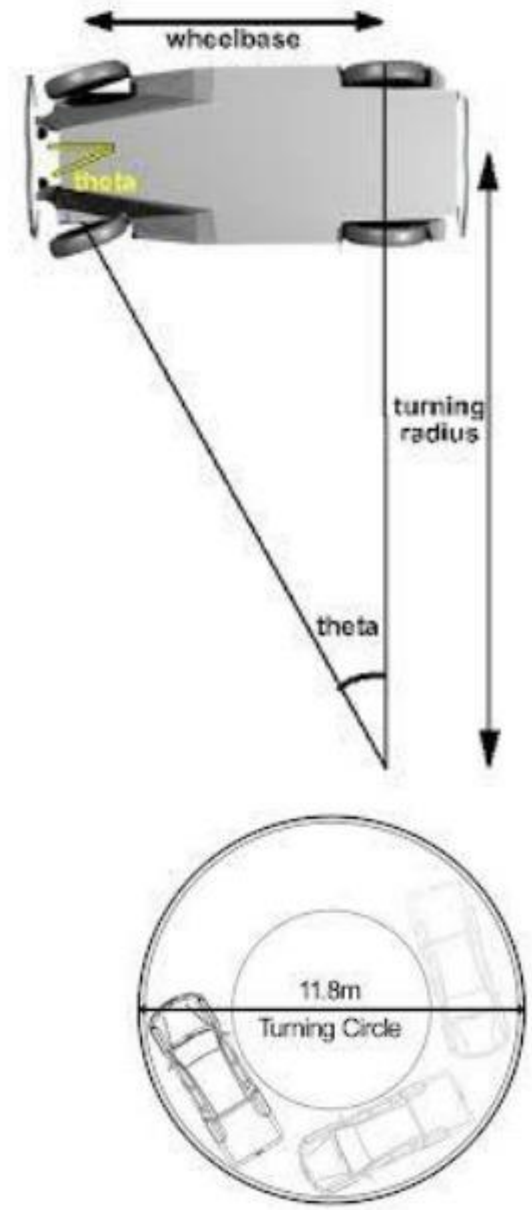
2.1.2 The Turning Circle

One would assume that a car would return to the starting point after a 360° rotation, but that's most definitely not the case. It was found that it reached a point about 30 cm from it. Although we will find something mentioned about the turning circle, the diameter is rounded to about 11m, for golf in the documents about a car. Here, we assume it to be a perfect circle.



So, What's a *Turning circle*? In an experimental setting, a turning circle is made by the outer wheels of the vehicle during a complete turn. A turning circle's radius or diameter determination is done when the vehicle turns either in the left or right direction in a lock condition. For example, if the car takes a left turn, the right-side wheels, both front and rear, become the outer wheels. Hence this circle traced by the outer wheels gives us a turning circle and its radius is the turning radius of the vehicle. A well-built car would give the same results when taken in the opposite direction.

We got everything about the turning circle and its radius. But in a truthful sense, how can we consider the centre of this circle? If we happen to think that the tires are small slits, they roll parallel to the tangents of this circle which also means, we need to consider a perpendicular to the rear wheels, to find its centre. This way it makes more sense because our car is in no way symmetrical and going our regular way of thinking might hand us wrong assumptions or presumptions. On looking at the figure given below, we would get a clear idea of how to ideate it.



The figure gives us an idea about the turning circle centre.

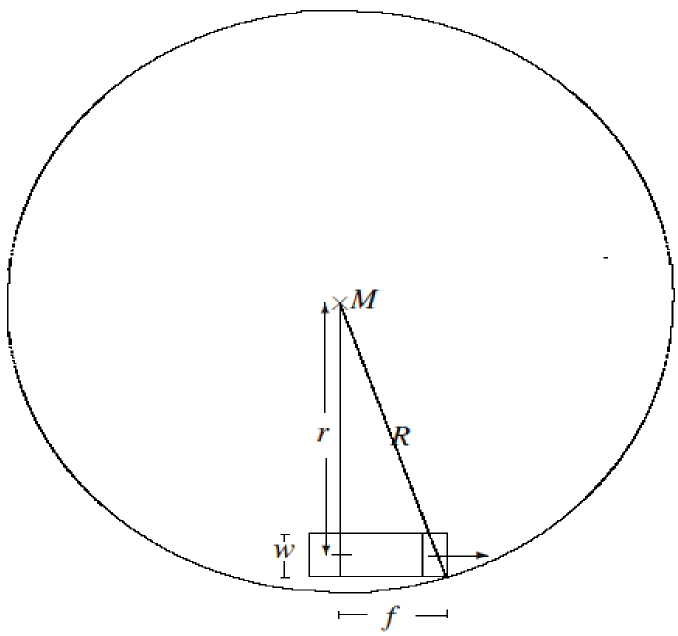
2.1.3 The Smallest Circle Possible

We need to remember that when a car takes a turn and we tend to consider the circle traced by outer wheels, we should also know that the wheels, be it front and rear, and every other point there move in a circle and no circle ever intersects. So, which point do we need to consider so the circle we get is the smallest one there is? In a way, the point following the smallest circle shouldn't ever touch the adjacent car. That's what the whole point is. If the car is turned left, then it means the rear left axle's outermost point would provide us with the smallest circle ever.

This is where our whole manoeuvring starts. On aligning our car's rear axle with the neighbouring car's front axle or the bumper, we are good to carry on with our parking manoeuvre. If we keep this in mind, we can avoid bumping into the neighbouring cars. There are several ways to make sure of it. The rear seat and right-hand door tiller should be enough to make sure that we are indeed on the right path.

2.1.4 The Effective Radius

Hoyle, however, in one of her formulae, or rather all, considered r which is not the turning circle radius but something else. We can see that in the figure given below.



By using the Pythagorean theorem, we will get,

$$r^2 + \left(\frac{w}{2}\right)^2 = R^2$$

Hence the effective radius r would be

$$r = (R^2 - f^2)^{0.5} - \frac{w}{2}$$

2.1.5 New Formulations for Parallel Parking

Rebecca Hoyle's Formulations were correct to some extent, but they were not accurate enough for all dimensions of vehicles and thus the need to formulate new formulas began with the knowledge of Turning circles. Soon new formulae have come up after thinking long enough about getting out of a parking space instead of getting into it. We start from a fixed position right behind the rear car, and where we end up is where we begin. This makes the whole process reversible and possible.

Think that our car is at the centre of a parking space adjacent to the curb. When exiting, reverse the car until it nearly touches the rear car. By treating this distance as 0 mm in our calculations (around 5 cm in actuality). Subsequently, turning the steering wheel completely to the left and navigating a curved path with an angle denoted as α; we'll delve into specifics later. Following this curved path, come to a stop, turn the steering wheel fully in the opposite direction, and traverse another curved path with the same α angle. Now, find yourself outside the parking space, running parallel to the curb.

Now, let 'd' be the distance that extends from your car to the curb, while 'x' marks the distance from the midpoint of the rear axle to a point where two turning circles intersect. If you extend this point rightward until it aligns with the line drawn through the right car, the distance from the top-right centre of your car to this line equals 'x.' This yields a total distance of 2x. Shifting the line downward by half the car's width (w/2) lands precisely at 'd.' Thus, 'd' equals 2x, and the calculation of 'x' involves the cosine of the α angle as follows

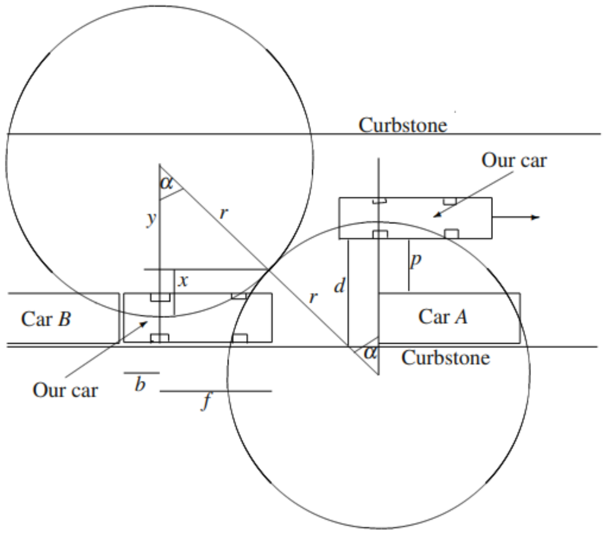
$$d = 2x = 2 * (r - y) = 2 * (r - r\cos\alpha)$$

The distance from the neighbouring car is

$$p = d - w = 2r(1 - \cos\alpha) - w$$

For the parking space length, we get

$$g \geq \sqrt{2r \cdot w + f^2} + b,$$



Hence, we get the optimal angle α that solves our parking problem, $\alpha = \cos^{-1}[\frac{2r-w}{2r}]$

We can make the parking space smaller by backing up more when leaving, just avoiding hitting the car in front. This allows the car to stand a bit closer. So, the length of the smallest parking space is influenced by this adjustment. The parking space obtained this way is

$$g \geq \sqrt{2r \cdot w + f^2} + b,$$

Final Formulations:

Hence the new formulas that were obtained are:

- Distance from the neighbouring car, p = 0.
- Circular arc and the angle i.e., α is $\alpha = \cos^{-1}\left[\frac{2r-w}{2r}\right]$
- Required length of parking space, $g \geq \sqrt{2r \cdot w + f^2} + b,$

2.1.6 Automatic Parking System

Over the past few decades, the automatic parallel parking system has grown in popularity. It was used in manually driven vehicles mainly. However, this added more problems to the already existing ones. It has limited environment sensing ability and also uses a redundant space to complete a single parking manoeuvre. It was observed that there were, indeed, two variations in the upcoming automatic driving. One being accurate perception systems to be available and second being, there would be even large number of parking rials if there's no person in the car. In these particular terms, the vehicle needs to perceive and understand if the external as well as internal spaces are large enough to carry on with the manoeuvre.

Studies revealed that the already existing systems are or rather were, designed for non-fully automatic fuel vehicles, which have some issues to be taken care of. As it has a hard time detecting real-time environmental perception, the redundancy was added, to avoid collisions. With the development of the Intelligent Transport system, intelligent vehicles are more than capable of avoiding the above drawbacks and by using advanced radar and video detection technology, the accuracy can be improved. Also, the trajectory tracking will be more precise than the manually driven vehicles or the semi-autonomous ones.

What an automatic parking system does is it steers cars into the parking but there's also a thing it doesn't do. It can't apply a brake or change the gear. If we have opted for sensors, these advanced sensors read the gaps between the cars where we want to park. There's a catch. This, however, won't activate if the place is not sufficient, which by the way, is a great prospect. When activated, it initiates the manoeuvre by taking over the steering wheel. As it is done, all we need to do is just apply the brake, shift the gear and change the attitude of the car so it sets just right. We need to be aware of the people in the surroundings and not grab the steering wheel during this.

It has its pros and cons. But it had been rather helpful in parking manoeuvres despite it not being able to apply a brake on its own. Which is why, we need to be attentive throughout. And should work towards removing the cons.

2.1.7 The Algorithm

Many further investigations led to the implementation of the algorithm formulation coupled with an automated parking system which had been implemented as a modified car. For this simulation, they have used a mobile robot car (MCR) and the system has been implemented in three steps: scanning, calculating and parking. In the very first step, the parking area is sensed by infrared sensors followed by moving of the car to the suitable space where the process can be started. The parking step manoeuvres are calculated dynamically. In this work, it has been assumed to be a non-holonomic system.

In the very first step of it, the closed-loop controls allow us to determine places that don't hit anything. In the next calculating step, if the place is found, the MCR proceeds to coordinate onto the point provided as the target position in the first step. In the last step, in the final location where the MCR is desired to be located after the parking is completed.

This approach, however, was of great use and is modelled to be used in real-life scenarios and can be used in cars with different dimensions and was experimentally studied and verified proving it efficient.

2.2 Critical Appraisal

The study helped a lot in actually understanding the gravity of the problems and what major difficulties were faced in general. It opened various streams of ideas as to what should be our thought process or inclusion of things while carrying on with our analysis. From one formulation to the other, each acted as the stepping stone for another.

It's noteworthy how in the end, the algorithm formulation came close to the perfect manoeuvring solution. It was tough and gruesome looking but ultimately it opened new opportunities and ideas to work on, as to how to simplify it further.

Chapter 3

Report on the present Investigation

3.1 Ideas

We have gone through all the theory that's mentioned in the earlier chapters and have looked into many vehicles to proceed with the experiments such as All-terrain vehicles (ATV), Turtle bot but have found all the vehicles highly deviating from the main objective of the project i.e., the parallel parking Problem and we are yet to decide whether to use the mobile robot control or not. To use it in the place of the four-wheeler for the experiments since it can act as a single medium for multiple vehicle dimensions, and it also follows the generated contour/path precisely.

Hence the parallel parking Problem, as discussed in Chapter 2 can also be solved by implementing Contour following algorithm on the car-like robot (MCR) through contour-following techniques. This method is adaptable to different car sizes; one can modify pertinent parameters, such as car length and width, within the controller software. The proposed approach effectively addresses the general parallel parking problem and is extendable to more intricate scenarios, encompassing diverse parking spaces. By incorporating hybrid sensor systems, it can offer enhanced environmental modelling for complex parking situations.

Implementing the parallel parking algorithm on an MCR involves acquiring environmental data, making decisions, and executing the task by utilizing three hardware modules:

1. The Microcontroller module
2. The sensor module
3. The motion module

Furthermore, earlier when we have gone through the entire past works of the people, a single question pertained to all of them. What if a computer arguably does an exceptional job in parking a car effectively? All we can do is just program it in a way which can effectively implement this manoeuvre. As the formulae from Chapter 2 suggested, they work well in theoretical scenarios as in practical ones, neither the parking space is the same all the time nor the dimensions of a car.

We wanted to also check if this method is valid to proceed further. One of us had an idea initially. To write an algorithm, that's not specific to a single car. Meaning, the variables in the algorithm would be the dimensions of the car, as far as the formulae are concerned. The manufacturer can make sure to drop in the exact dimensions of the car while it's being manufactured. However, there's a thing. The distance between our car and the neighbouring car, however, was taken as 0 mm. That we needed to take care of. This would also help us in proceeding further with the contour, as mentioned above.

So, throughout our project, we are going to verify the prospects and explore our ideas as to which solution would satiate and end the problem once and for all. And not only that, this needs to be available for everyone and not just the people who buy luxurious vehicles. Through and through with our investigation, we plan to conduct experiments because we don't expect them to work, all of them, but we plan on leaving no stone unturned with it.

3.2 Proceeding

We ought to run a simulation depicting our idea. It is required to further look into using a four-wheeler-like model and it is still up for deciding as to which of the type we need to use. As said earlier, we have come across way too many models to work with. While was were too redundant, we want our simulation done after assembling the four-wheeler, if we fail to obtain one that's assembled.

The parts of the vehicle that are required have also been listed, just in case. Robot wheels, Power source, Motors, Batteries etc, are necessary for us to proceed with the assembly. Furthermore, we have decided to simulate it as per the schematic of the problem, where we have two already parked vehicles and the third one needs to be accommodated safely between the two of them. We planned to do it with the following with contour on our very first trial, so we could proceed further with the project.

Chapter 4

Results and Discussions

As we have seen in the earlier parts parallel parking of vehicles, especially four-wheelers, poses a wide variety of problems to be solved ranging from complex parking contours to follow, to limited space availability, to the very consumption of time. The formulations that gave an idea of optimized contours to follow for efficient parking were also explained by Rebecca Hoyle's formulation.

From her formulation, it was made easy as to what to consider or what to omit, which terms were contributing to the mathematical formulation or which were superfluous. Accordingly, it has been modified and they have arrived at modern conclusions to solve problems by using technology.

Many people on further research found solutions for this problem like automated parking systems, intelligent vehicles, parking trajectories based on mathematical models of classical curves, and geometric methods based on the reverse procedure of retrieving a car from the parking lot, which were often based on the circular arcs and included angles. However, the major drawbacks were, that it was assumed that the space that the car needed to be parked was of the same dimensional width as the car that needed to be parked but the external obstacles were not been considered.

We would like to think every solution took a very long time to be brought into existence, keeping aside the drawbacks. If we work to avoid the drawbacks… Or if the drawbacks are kept in mind, we can develop much more promising solutions in future. The efforts of the researchers are to be appreciated because developing something as non-trivial as this is very commendable. We believe that the drawbacks that are present would make us think further to include all possible scenarios while trying to solve the problem on our own.

Chapter 5

Summary and Conclusion

From the above, it can be summarized that this problem is much bigger than it was known. Sure, skilled drivers can just *intuitively* park cars, despite its difficulty. Not everyone is that skilled. If it were true then we wouldn't be having accidents pertaining to careless or rough manoeuvring etc.

Following the solutions given, we can say that they are not useful for small parking spaces. In the original sense, earlier mentioned works have actively considered just the length of the parking space but not the width and external parking, which are as important as the earlier one in an automatic parallel parking system. In all their proceedings it has been concluded that one has to park in a single trial.

As said earlier, a skilled driver parks in a single trial. What about spaces where one trial leads to an accident and that we should, by any means, opt for n-trial parking? And what about beginners? They are to be parked in multiple trials. This brings us to the next point that a one trial parking is convenient in every sense, iff the space is large enough to accommodate the car and there's still reasonable space left between the cars. Nevertheless, an n trial parking ($n>1$) is important too, if we are to execute it in a small place that's borderline enough. Space constraints matter as well.

Perhaps, it can be concluded that there has been not enough research done about it. It's considered as a *solution*, when it is useful in every sense, in every case. And there is scope for massive improvements as well, on which we hope to work on.

Appendix

Appendix I: Glossary of Terms- Detailed information

1. Parallel Parking: The parallel parking problem involves skilfully manoeuvring a vehicle into a tight parking space between two others along a road. Drivers must navigate with precision, considering the vehicle's dimensions, spatial awareness, and steering control. Modern technology, such as parking sensors and cameras, assists in this manoeuvre. Mastering parallel parking is crucial for confident and effective urban driving.

2. Manoeuvre: A movement or series of moves requiring skill and care. It’s spelt as manoeuvre in ancient British English but in modern times, it can also be spelt as maneuver.

3. Perpendicular parking: In this parking, Vehicles are parked perpendicular to the curb. Though the maximum the width of road like the length of the car is occupied the length of the curb is deemed small. It also accommodates a maximum number of vehicles along the curb length.

4. Angular parking: The vehicles are to be parked at certain angles like 30°, 45° or 60° whichever way it is marked. As the angle increases, a greater number of vehicles can be parked this way. Here, the angle is the angle of parking.

5. Diagonal Parking: Also known as Back-in angle parking or back-in diagonal parking or reverse angle parking, involves parking a vehicle at a much steeper angle to the curb.

6. Curb: A curb is the edge where a raised sidewalk meets a street or other roadway.

7. Turning Circle: The circle formed by the outermost wheels whenever a car is turned in its full lock position.

8. Automated parking: A promising technology where a car can be parked which is often deemed to need an extra redundant space to do the manoeuvre.

9. Trajectories: A path followed by an object under a force.

10. Intelligent vehicles: An intelligent vehicle is a regular vehicle that has been equipped with advanced smart car technology. The term, however, is associated with artificial intelligence, digitalisation and self-driving technology.

Appendix II: Important Relations and Formulations

1. Rebecca Hoyle's Formulation:

$$p = r - \frac{w}{2},$$
$$g \geq w + 2r + b,$$
$$f \leq w + 2r - fg,$$
$$\max\left(\left(r+\frac{w}{2}\right)^2 + f^2\,,\ \left(r+\frac{w}{2}\right)^2 + b^2\right).$$
$$\leq \min\left(4r^2, \left(r+\frac{w}{2}+k\right)^2\right).$$

2. The Turning Circle:
   - angle is given as

     α = arc cos [(2r −w)/2r]

   - The length of the smallest possible parking space thus is given by

     $$g \geq \sqrt{2r \cdot w + f^2} + b,$$

3. The final formulation is:
   - Distance from neighbouring car, p = 0.
   - Angle of circular arc, $\alpha = \cos^{-1}$ [(2r − w)/2r].
   - Required length of parking space, $g \geq \sqrt{2rw + f^2} + b.$

Appendix III: Acronyms and Abbreviations

1. CMDA: Chennai Metropolitan Development Authority
2. ATV: All-Terrain Vehicle
3. MCR: Mobile Robot Car
4. Iff: If and only if

References

Books and Journal articles

1. Adimurthy. M, Mallinath B Kuchanur, Prasad P Sagar, Ramalingappa R Loni,: “Parallel Parking: A Review”, International Journal of Engineering Research & Technology (IJERT), Vol. 6 Issue 10, pp. 145-149, October 2017.

2. Norbert Herrmann, *The Beauty of Everyday Mathematics*, 3rd Ed., Ch. 5, pp 41-58, Copernicus books, (Springer science+ business media), United States, 2012.

3. Xuan Wang, Momiao Zhou, Zhengqiong Liu and Zhizhong Ding,: “On Minimum Parking Space Required by Automatic Parallel Parking”, Sensors MDPI 2022, 22, 795, pp. 1-3, January 2022.

Other Articles

1. Sinan Öncü and Mürvet Kırcı, A Solution to the Parallel Parking Problem (missouri.edu)
2. Automatic Parallel Parking Feature: My Car Does What
3. ENIGINEERING MECHANICS Lecture 3 (iith.ac.in)
4. Parking Studies (iitb.ac.in)