A Project Report

On

Forecasting Stock Market Trend using Stochastic Geometric Brownian Motion

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Certificate

This is to certify that the project report entitled "Using the Euler-Maruyama Method for Finding a Solution to Stochastic Financial Problems" submitted by Mr. Srihari sirisipalli (HT No. 18353),Nikitha Dongsarwar (18517), Mythri Komuravelli(18217) in partial fulfillment of the requirements of the course PR-402, Project Course, embodies the work done by him/her under my supervision and guidance.

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ABSTRACT

Euler–Maruyama method is a method for the approximate numerical solution of a stochastic differential equation (SDE). It is an extension of the Euler method for ordinary differential equations to stochastic differential equations.

It is not possible to get explicit solution to many of stochastic differential equations. So, we approximate solution with numerical methods. We have based this paper around financial examples of AMAZON STOCK (1st January 2016-1st January 2017) and to see display of behaviour with varying R parameter.

We Also used Euler-Maruyama method for simulation of stochastic differential equations for payoff for a European call option and estimating value of European call option to buy the asset at the future time.

KEYWORDS:

Stochastic Differential Equations, Euler Maruyama method, Black Scholes,Expectation , Volatility,

European Call Option.

CONTENTS

CHAPTER 1

1.INTRODUCTION:

1.1 Information about Stocks:

Buying a company stock is purchasing a small share of it. People invest on the same to get a longterm benefit which they think is less value for now but has to potential to grow with the time. It's an investment that provides the long-time run and deals with long time goals with the fair objectives.

1.2

Stochastic differential equation (SDE) models play a prominent role in a range of application areas, including chemistry, biology, mechanics, economics and finance . These equations have become standard models for financial quantities such as asset prices, interest rate and their derivatives . Ito and Stratonovich put stochastic differential equations in mathematics. Ito pioneered the theory of stochastic integral and stochastic differential equations, it use in various filed such as financial mathematics. To understand the dynamics of most SDEs and their solutions, it is important to have some knowledge of probability theory as well as some mathematical/statistical principles . It is not possible to get explicit solution to many of stochastic differential equations. So, we approximate solution with numerical methods. We have based this paper around financial examples of AMAZON STOCK (1^{st} January 2016-1^{st} January 2017) .

At first, we introduce option, Brownian motion and compute discretized Brownian paths, Ito stochastic integral and stochastic differential equations. Then we survey how the Euler-Maruyama method simulate a stochastic differential equation in financial problems. stochastic differential equation in financial problems.

1.3 EULER MARUYAMA :

Euler method is a method for solving ordinary differential equations (ODEs) with a given initial value, it is named after Leonhard Euler who treated this method in his book (Institutionum calculi integral is published 1768- 70). Gisiro Maruyama in 1995 showed unique answer for stochastic differential equations with Euler approximation and he proved the mean-square convergence of the Euler approximation of the Ito process without jumps, this being one of the first papers on the approximation of Ito processes . Euler-Maruyama method is named after Leonhard Euler and Gisiro Maruyama.

The paper aims to how the Euler-Maruyama method simulate a stochastic differential equation in financial problems where analytical solutions are not feasible.

Also explore relevant derivative pricing concepts of AMAZON STOCKS derivatives trade. The specific objectives are to :

- Determine the parameters drift and volatility of the GBM, and discuss their use in forecasting stock prices, especially for cases where analytic solutions are not feasible.
- Demonstrate the use of accurate numerical approximation methods like Euler Maruyama to simulate nonlinear solutions to stochastic differential Equations (SDE) resulting from asset prices.
- Detailed empirical modelling of some candidate derivative pricing formulae in the Amazon Stock Market in subsequent work .

We used Euler-Maruyama method for simulation of stochastic differential equations for payoff for a European call option and estimating value of European call option to buy the asset at the future time.

A comparision study between Geometric Brownian Motion , Fractional Geometric Brownian Motion and Euler Maruyama Method.

BACKGROUND AND RELATED WORK:

Ito stochastic and Stratonovich integrals offer good numerical approximations to solutions of stochastic differential Equations (SDE) obtained from stock prices.They can also be used in estimating numerical solutions of other asset prices thatserve as underlying stocks of derivative products, where analytic solutions are not feasible, and the price dynamics of these underlying assets are needed in the pricing of call and put options of the derivative assets. One of the methods for evaluating of these integral equations is the use of Euler-Maruyama approximation to solutions of differential and integral equations. this paper will approach the problem by benchmarking the Nigerian Stock Market (NSM) on the Johannesburg Stock Exchange (JSE), and using comparative correlations of asset prices in the two markets to judge the degree of closeness of the markets and/or their sectors. This judgement will inform simulations of derivative prices that will fit the stylized facts of the NSMThese computational modelling under assumed scenarios requires an intimate understanding of closed-form numerical forecasting of asset prices which this paper explores. stochastic calculus is useful in many financial assets including derivatives and that they are used in solving Black-Scholes option pricing problems, market risk adjustment, and project valuation some shortcomings associated with the numerical solutions of SDEs and other differential equations, for example lack of exact solutions to linear SDEs and inability to preserve qualitative features of the exact solution. Hence, the key question for solving the NSMJSE derivative pricing problems noted above is: Under what conditions and for which numerical approximation schemes are these limitations overcome? This will facilitate suitable experimental modelling of potentially useful NSM derivatives.

Euler–Maruyama method (also called the Euler method) is a method for the approximate numerical solution of a stochastic differential equation (SDE). It is an extension of the Euler method for ordinary differential equations to stochastic differential equations.It is the most basic explicit method for numerical integration of ordinary differential equations and is the simplest Runge–Kutta method.

The Euler method is named after Leonhard Euler, who treated it in his book Institutionum calculi integralis (published 1768–1870).

Leonhard Euler (1707-1783) could single-handedly embody the mathematics of the 18th century.

CHAPTER 2:

THEORITICAL ANALYSIS:

2.1Brownian motion:

A Brownian motion, or Wiener process, over [0, T] is a random variable W (t) that depends continuously on t   [0,T] and satisfies the following conditions:

1. W(0)   0, With probability 1.

2. For 0   v   t   T the random variable given by the increment W (t) -W (v) is normally distributed N(0,t-v) and W (t)- W (v)~ N (0,1) where N(0,1) denote normally distributed random variable with zero mean and unit variance.

3. For 0   v   t   u   s   T the increments W(t) -W(v) and W(s) -W (u) are independent.

We consider discretized Brownian motion for computational purposes, where W (t) is specified at discrete t values. We set   t= T/ N (for some positive integer N). Wj Denote   twj with t j= j   t . We have W0   0 with probability 1 from condition 1, from conditions 2 and 3:

$$W_j = W_{j-1} + dW_j, \; j = 1,\dots,N,$$

2.2 STOCHASTIC DIFFERENTIAL EQUATIONS:

Stochastic differential equation (SDE) is a differential equation in which some of the terms and its solution are stochastic processes . SDEs are important in filtering problems, stochastic approaches to deterministic boundary value problems, optimal stopping, stochastic control, and financial mathematics . A general form for a stochastic differential equation is:

$$dX(t) = f(X(t))dt + g(X(t))dW(t),$$
$$X(0) = X_0,$$
$$0 \le t \le T.$$

Where W denote a Brownian motion and f and g are given functions. A solution is a stochastic process, which can be interpreted as integral equation:

$$X(t) = X_0 + \int_0^t f(X(s))ds + \int_0^t g(X(s))dW(s),$$
$$0 \le t \le T.$$

The second integral is Ito integral. If g 0 and X_0 is constant, reduces to the ordinary differential equation:

$$\frac{dX}{dt} = f(X(t)), X(0) = X_0.$$

The well- known Black–Scholes model of the asset price is described by the linear SDE :

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t),$$
$$X(0) = X_0,$$

where and are real constants.

2.3 EULER-MARUYAMA METHOD:

There is a stochastic analog of Euler's method which is known as the Euler-Maruyama method. Euler Maruyama method is constructed within the Ito integral framework To apply Euler-Maruyama method to over [0,T] we first divide the interval. Let t+ T /L (for some positive integer L) and $_J$ j t (j = 0,...,L). Our numerical approximation to X ($_j$) will be denoted X_j. The Euler-Maruyama method takes the form:

$$X_j = X_{j-1} + f(X_{j-1})\Delta t + g(X_{j-1})(W(\tau_j) - W(\tau_{j-1})),$$
$$j = 1, \ldots, L.$$

Now we want to obtain . Setting t $_j$ and t = $_{j-1}$ in , we obtain:

$$X(\tau_j) = X_0 + \int_0^{\tau_j} f(X(s))ds + \int_0^{\tau_j} g(X(s))dW(s).$$

$$X(\tau_{j-1}) = X_0 + \int_0^{\tau_{j-1}} f(X(s))ds + \int_0^{\tau_{j-1}} g(X(s))dW(s).$$

We subtract these equations:

$$X(\tau_j) = X(\tau_{j-1}) + \int_{\tau_{j-1}}^{\tau_j} f(X(s))ds + \int_{\tau_{j-1}}^{\tau_j} g(X(s))dW(s).$$

2.4 OPTIONS:

An option is a contract which gives the owner or holder the right to buy (call option) or sell (put option) a specified number of shares of an underlying asset at a fixed price (strike price, denoted as K) before a specified future date or on the specified date. The time that option expires is called expiry, denoted as T . One of the most common types of options, is European option. A European option can only be exercised at its maturity. This option gives its owner the right, to buy one unit of a particular asset at a pre-set price (K), at some fixed future time (T). If the price of the asset at time T is lower than K , the option value is zero and it will not be exercised. Otherwise, if the price of the ass et at time T exceeds K , the owner of the option may buy one unit of the asset at price K and immediately sell it at price S(T) , so gaining a profit of S (T)   K . As a result, the payoff of the European call option is:

$$(S(T)-K)^+ = \begin{cases} S(T)-K, & if\, S(T) > K, \\ 0 & , if\, S(T) \le K. \end{cases}$$

Hence, the payoff from purchasing a European call option can be represented by the C = max(S (T-K) ,0).

Black-Scholes parabolic partial differential equation which is a stochastic calculus model for option price as a function of the underlying stock price at time t:

$$\frac{\partial C}{\partial t} + \frac{1}{2}\sigma^2 S^2 \frac{\partial^2 C}{\partial S^2} + rS\frac{\partial C}{\partial S} - rC = 0,$$

where C(S ,T , τ) is the option price at time τ, r is the risk-free interest rate, S is the underlying stock price, and σ is the volatility of the stock price per annum.

We study the analytic solution to the SDE above, its associated numerical approximation, and its use in European option pricing. For a European option, which coincidently is the derivative option pricing formula proposed to be adopted for use in Nigerian Stock Market, Equation is solved under the finite boundary condition that :

$$C(0,\tau)=0, C(S,T,T)=Max\{S-K,0\}$$

$$P(S,T,T)=Max\{K-S,0\},$$

for call and put options, respectively, where T is expiration date (maturity) of the option, and K is its exercise (strike) price. Thus, the solution C(S ,T , τ) to Equations (2) and (3) above is given by the call option value derived and written as:

$$C(S,0,T)=SN\left\{\frac{\ln\left(\frac{S}{K}\right)+\left[r+\left(\frac{\sigma^2}{2}\right)\right](T-\tau)}{\sigma\sqrt{T-\tau}}\right\}$$
$$-Ke^{-r(T-t)}N\left\{\frac{\ln\left(\frac{S}{K}\right)+\left[r-\left(\frac{\sigma^2}{2}\right)\right](T-\tau)}{\sigma\sqrt{T-\tau}}\right\}$$

and N(x) is the cumulative standard normal distribution.

2.5 SIMULATIONS:

This SDE drives the stock price dynamics for a security asset in the AMAZON STOCK, with mean return μ = 0.115 and volatility (sigma) = 0.168.(From 1st January to 126 trading/woringdays) We need an actual evolution of a firm's stock price prices to approximate the solution of Equation above, after estimating the expectation and volatility parameters. For this, we use the Euler-Maruyama method to simulate the SDE by Monte-Carlo approach.

```
adj_close = data["Adj Close"][0:N1]
adj_reversed = list(reversed(adj_close))


def daily_return(adj_reversed):
    returns = []
    for i in range(0, len(adj_reversed) - 1):
        today = adj_reversed[i + 1]
        yesterday = adj_reversed[i]

        daily_return = (today - yesterday) / yesterday

        returns.append(daily_return)
    return returns


returns = daily_return(adj_reversed)
S0 = adj_reversed[0]
```

```
# In[13]:


# compute the drift(mean) and diffusion(variance) coefficients and annualize it by multiplying by 250 trading days

mu = np.mean(returns) * N1
sigma = np.std(returns) * np.sqrt(N1)
```

fig .1

fig.1 has the code to find mean daily return μ and standard deviation (sigma).

Thus, the estimated annualized volatility measure (standard deviation of Amazon stock is 0.168. Therefore, in the Black-Scholes formulation, Amazon returns is assumed to follow a normal distribution with the estimated mean $\mu = 0.115$ as the drift, and standard deviation $\sigma = 0.168$ which measures the volatility. The following code uses these statistics to closely approximate Amazons's stock price dynamics using the EM method.

As earlier stated, that Euler-Maruyama approximation is a close estimate to solutions of stochastic differential equation of interest, thus making it possible for us to adopt the same approach especially when the analytical solutions are not feasible or difficult to obtain for any SDEs of interest.

```
def Em(S0, R, mu, sigma, b, T, N):
    dt = R * (T / N)  # EM step size
    L = N / R  # intervals according to the TS
    wi = []
    wi.append(S0)
    dt = T / N
    for i in range(0, int(L)):
        delta_Wi = b[i - 1] - b[i]
        wi_new = wi[i] + mu * wi[i] * dt + sigma * wi[i] * delta_Wi
        wi.append(wi_new)
    return wi, dt
```

fig 2

fig.2 has code to Euler Maruyama

```
def BrownianMotion(seed, T, N):

    np.random.seed(seed)
    dt = T / N
    Z = np.random.randn(N)  # random variables
    Z[0] = 0

    dW = np.sqrt(dt) * Z  # single Brownian increment

    W = np.cumsum(dW)  # Brownian path
```

```
return W
```

fig.2.1

fig 3 has code to Brownian motion random variable.

exploring a choice of “R” to obtain Euler-Maruyama approximation near enough to exact solution of the stochastic differential equation.

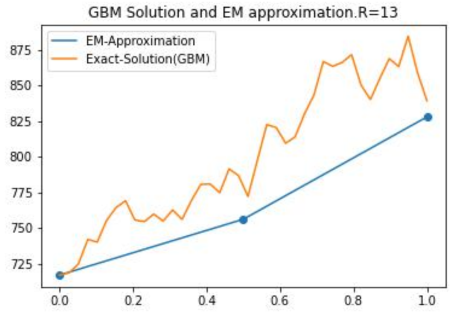


fig-4

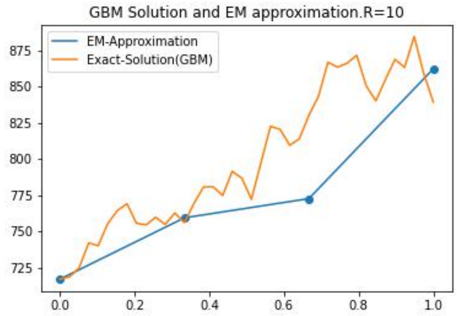


fig-5

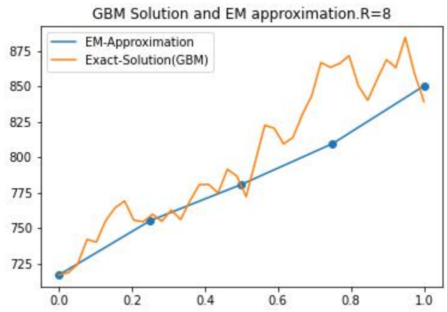


fig-6

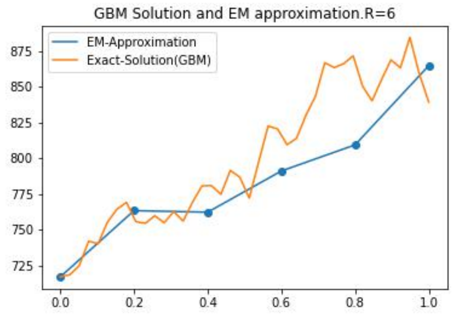


fig-7

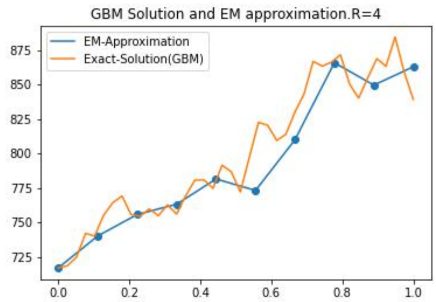


fig-8

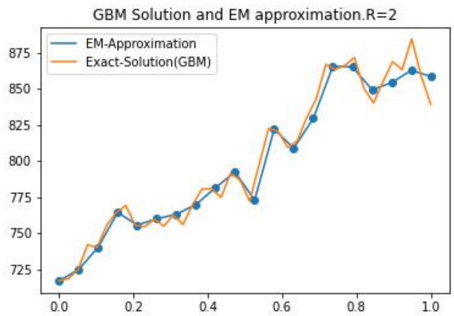


fig-9

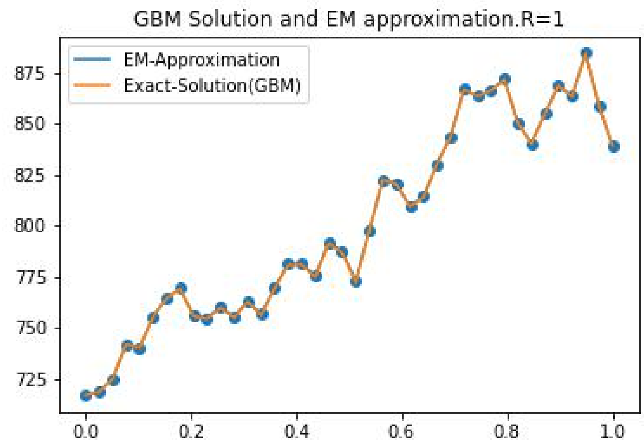


Fig-10

Fig 10 illustrates further the earlier statement that smaller “R” offers an approximate solution very close to the exact (analytical) solution of the stochastic differential equation. Thus, depending on the existence or otherwise of the analytical solution to a given differential equation of interest one can compare the two solutions and make the numerical approximation as close to the analytical solution depending on the choice(s) of the value(s) of “R”.

OPTIONS:

```
 import numpy as np
from scipy.stats import norm
N = norm.cdf
def BS_CALL(S, K, T, r, sigma):
    d1 = (np.log(S / K) + (r + sigma ** 2 / 2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    return S * N(d1) - K * np.exp(-r * T) * N(d2)
# Parameters
EM_o = []
for i in range(len(S)):
    tbegin = 0
    tend = 1
    deltat = 0.0001
    t = np.arange(tbegin, tend, deltat)
 E = K
```

```
   S0 = S[i]
 sqrtdt = np.sqrt(deltat)
N = np.zeros(10000)  # do 10000 simulations
   for i in range(0, len(N)):
       y = S0
       for j in range(1, len(t)):
           y = y + y * mu * deltat + y * sigma * random.gauss(0, np.sqrt(deltat))
       N[i] = y
N = np.maximum(0, N - E)  # payoff for call option
   EM_o.append(N.mean())
```

PYTHON CODE FOR EUROPEAN CALL OPTION

```
%Euler-Maruyama method on stochastic volatility SDE asset price.
% SDE is  dS(1) = mu*S(1) dt + S(2)*sqrt(S(1)) dW(1), S(1)_0 = Szero(1)
%         dS(2) = 1/2*(sigma_0 - S(2)) dt + 2*sqrt(S(2)) dW(2), S(2)_0 = sigma_0
% S(1) is the asset price,  S(2) represents the volatility
% European Option Price
% Vectorized across samples
clf
randn('state',1)
T = 1; N = 2^10; Delta = T/N; M = 1e+4;
mu = 0.16; Szero = 100; sigma_zero = 0.3; r = 0.05;

Xem1 = Szero*ones(M,1);
Xem2 = sigma_zero*ones(M,1);
for j = 1:N
   Winc1 = sqrt(Delta)*randn(M,1);
   Winc2 = sqrt(Delta)*randn(M,1);
   Xem1 = abs(Xem1 + Delta*mu*Xem1 + sqrt(Xem1).*Xem2.*Winc1);
   Xem2 = abs(Xem2 + Delta*1/2*(sigma_zero - Xem2) + 2*sqrt(Xem2).*Winc2);
end
Price = exp(-r)*mean(max(0,Xem1-1))
```

MATLAB CODE REFERENCE FOR EUROPEAN CALL OPTION

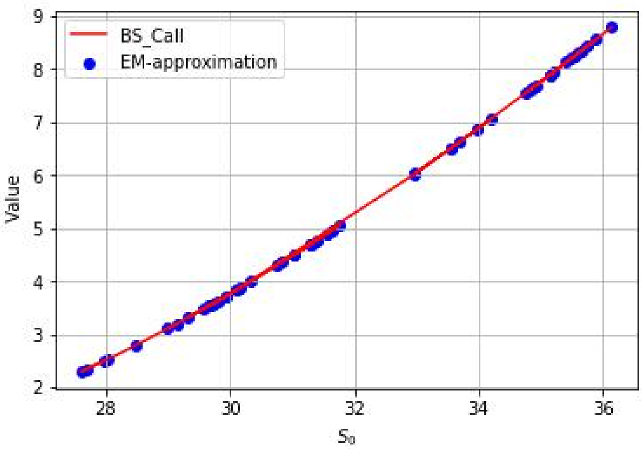


Fig.11

Fig.11 has the graph of simulation of EM approximated payoff for a European call option.

CHAPTER 3

COMPARISION STUDY OF GBM,FGBM AND EULER MARUYAMA:

3.1 GEOMETRIC BROWNIAN MOTION:

A geometric Brownian motion (GBM) (also known as exponential Brownian motion) is a continuous-time stochastic process in which the logarithm of the randomly varying quantity follows a Brownian motion (Also called a Wiener process) with drift.

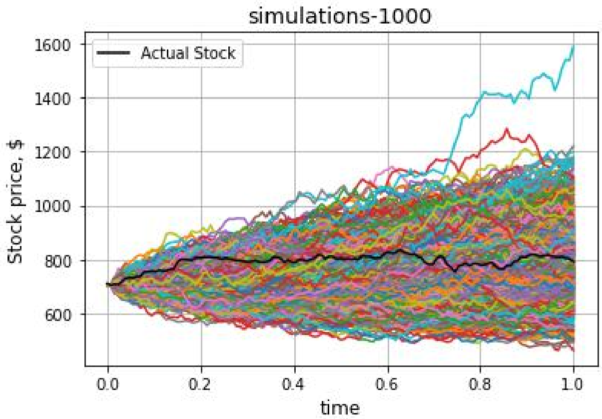
Geometric Brownian motion is used to model stock prices in the Black–Scholes model and is the most widely used model of stock price behaviour. The expected returns of GBM are independent of the value of the process (stock price), which agrees with what we would expect in reality. A GBM process only assumes positive values, just like real stock prices. In real stock prices, volatility changes over time (possibly stochastically), but in GBM, volatility is assumed constant.

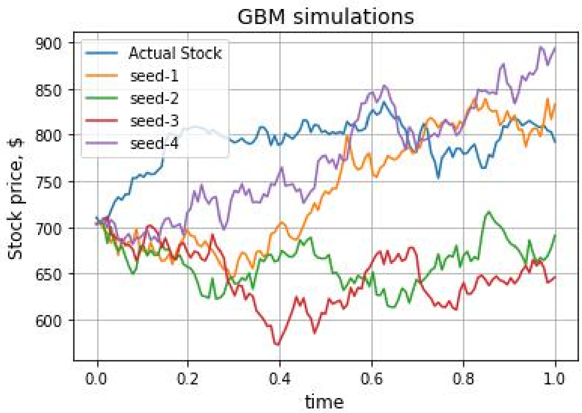
3.2 FRACTIONAL GEOMETRIC BROWNIAN MOTION:

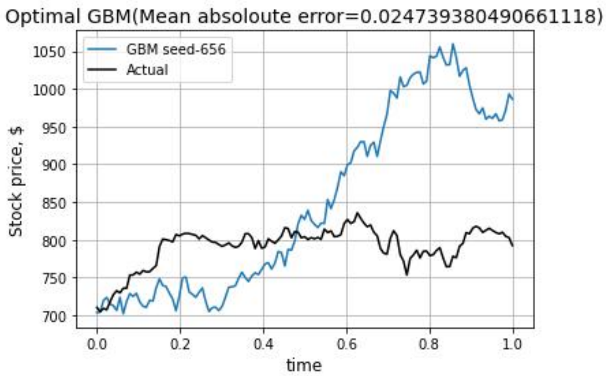
To improve the accuracy of stock-price modelling, Mandelbrot and Van Ness introduced the definition of fractional Brownian motion or fractal Brownian motion (FBM) in 1968 generalizing the BM by considering the Hurst exponent. Therefore, a new stock-price model, the geometric fractional Brownian motion (GFBM) model was published as an extension of the GBM model. The GFBM model is more general than the GBM model. It can explain more situations about the change of stock prices. The Hurst exponent indicates the intensity of long-range dependence in a time series, and it can be estimated heuristically using various methods. The GFBM model is more general than the GBM model, and it can explain more behaviours of stock price changes. Past studies have tried to analytically estimate the parameters of this model. The focus is on simulating and testing the model to assess the accuracy of the simulation with comparison to the actual prices by computing the mean absolute percentage error (MAPE) taken relative to the simulated prices, as well as to see display of persistent and anti-persistent behaviour of the given data set.

3.3 EULER-MARUYAMA METHOD:

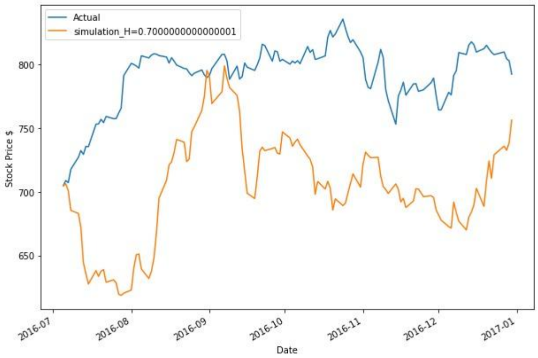
There is a stochastic analog of Euler’s method which is known as the Euler-Maruyama method. Euler Maruyama method is constructed within the Ito integral framework To apply Euler-Maruyama method to over [0,T] we first divide the interval. Let t+ T /L (for some positive integer L) and $_J$ j t (j = 0,...,L). Our numerical approximation to X ($_j$) will be denoted X $_j$.

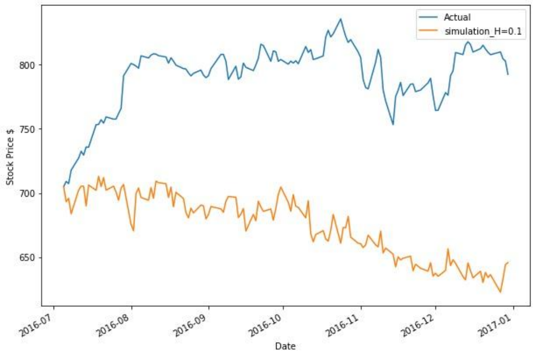


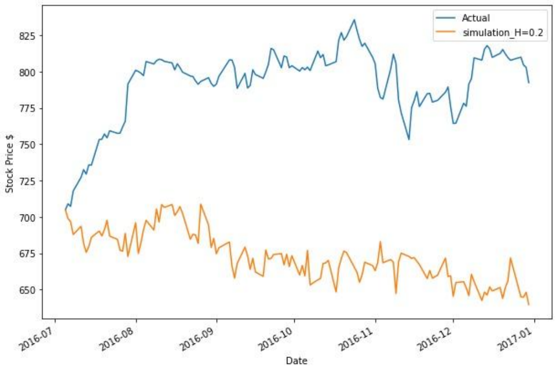




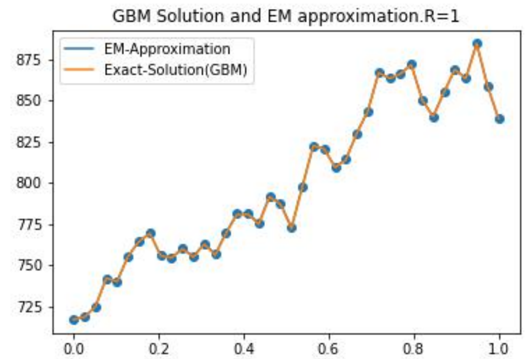
Optimal GBM solution.

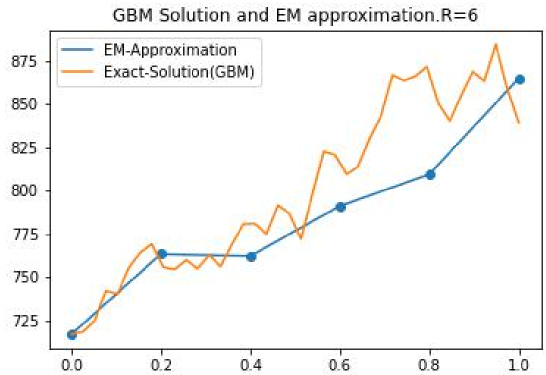






Optimal Simulation: simulation_H=0.7000000000000001 with RMSE= 96.79887671536801





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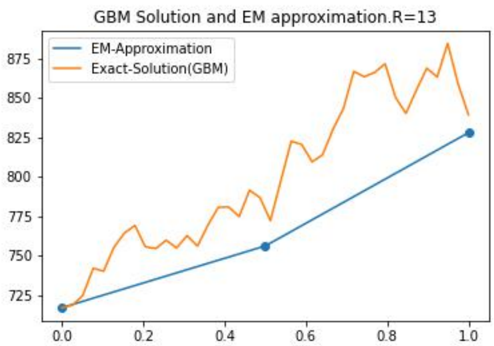

Closest approximation at R=1

CONCLUSION:

In this paper, we surveyed the applications of stochastic differential equations and Euler-Maruyama method for some financial problems. We showed that the differences between analytical solutions to derivatives pricing SDEs and their Euler-Maruyama numerical approximations, which depend on the values of the drift and volatility parameters of derivative pricing models, and the initial take off price of the asset denoted by S_0, are meaning fully negligible, generally and for the Amazon stock.

There were remarkable differences between the exact or analytical values and that of Euler-Maruyama (EM) approximations for sufficiently large value of R. For smaller values of the parameters, the plotted analytical (exact) solution coincided with that of Euler-Maruyama approximation.

Moreover, we estimated the value of European call option using Euler Maruyama to buy the asset at the future time T.

REFERENCES:

1. Desmond J. Higham. An Algorithmic Introduction to Numerical Simulation of Stochastic Differential Equations. SIAM Rev. Volume 43, Issue 3, 2001, PP. 525-546.

2. Timothy. Sauer. Numerical Solution of Stochastic Differential equations in finance. To appear, Handbook of Computational Finance, 2008, Springer.

3. Matthew, Rajotte. Stochastic Differential Equations and Numerical Application. Virginia Commonwealth University. Theses and Dissertations. 2014.

4. Peter E. Kloeden, Ekhard. Platen. A survey of numerical methods for stochastic differential equation, Stochastic Hydrology and Hydraulics. 1989, Springer.

5. Gilsing. Hagen. Shardlow, Tony. SDELab: stochastic differential equations with MATLAB. Manchester nstitute for Mathematical Science School of Mathematics. The University of Manchester. 2006.

6 Kloeden, P.E., Platen, E. and Scurz, H. (1993) Numerical Solution of Stochastic Differential Equations through Computer Experiments. Springer, Berlin.

7 Ozaki, T. (1985) Nonlinear Time-Series Models and Dynamical Systems. In: Hannan, E.J., Krishnaiah, P.R. and Rao, M.M., Eds., Handbook of Statistics, North Holland, Amsterdam, 5.

8 Ozaki, T. (1992) A Bridge between Nonlinear Time Series Models and Nonlinear Stochastic Dynamical Systems: A Local Linearization Approach. Statistica Sinica, 2, 113-135. [24] Sauer, T. (2008) Fi

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