FIRST SIMULATION REPORT

SUBMITTED BY: Srihari Menon | SUBMITTED TO: Dr. Cullen Blake | SUBMITTED ON:6/14/16

OBJECTIVE

To present the findings of a thermal model simulation of the problem.

THEORY

The given problem is transient in nature. The outermost wall of the problem has a time dependent boundary condition. The problem is approached in a simplified fashion and solved for mathematically here.

Le the time dependent boundary function be

$$T(R,t) = T_0 + \delta T_0 \sin(\omega t) \tag{1}$$

Using an additional symmetry boundary condition of

$$\frac{\partial T(0,t)}{\partial r} = 0 \tag{2}$$

The heat equation in cylindrical coordinates is solved for

$$\frac{\partial T}{\partial t} = \alpha \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) \tag{3}$$

(The gradient of z is dropped because it is insignificant compared to the gradient of r. The problem is solved for axisymmetrically so the gradient of θ is also dropped)

After solving for an initial condition of $T(r,0)=T_0$, the following solution is obtained:

$$T(r,t) = T_0 + \delta T_0 \sin(\omega t) + \sum_{n=1}^{\infty} \frac{2\omega\delta T_0}{\gamma_{0n} J_1(\gamma_{0n})}\left(\frac{\alpha\lambda_n^2 e^{-\alpha\lambda_n^2 t} + \alpha\lambda_n^2\cos(\omega t) + \omega\sin(\omega t)}{(\alpha\lambda_n^2)^2 + \omega^2}\right) J_0(\lambda_n r) \tag{4}$$

Where γ_{0n} is the nth zero of the Bessel function J_0, and $\lambda_n = \gamma_{0n}/R$

The detailed solution of the same is appended to this report at the end. This was solved by Dr. Michael Carchidi.

From the solution it is clear that the temperatures inside the cylinder is a function of the mean room temperature T_0, the amplitude of variation δT_0, the period of oscillation ω and the material property of thermal diffusivity α.

On varying it with time, it is realized that the rate of temperature change is a function of all the above except mean temperature T_0. On differentiating with time, then evaluating the rate of change of temperature inside with respect to that at the boundary $\left(\frac{\partial T(r,t)}{\partial T(R,t)}\right)$, it is observed that this gradient is only a function of α and ω.

Therefore it can be concluded that the rate of variation of temperature can be reduced if the variation of the room temperature is reduced both in amplitude and in period of oscillation, and if the insulation is improved, thereby reducing the thermal diffusivity. The rate of change is not a function of the mean temperature.

It is very important at this point to reiterate that this is an oversimplified interpretation of the problem: the room temperature is actually not periodic, but stochastic. Still, this serves as a reference to state the factors influencing the temperature variation.

SETUP

Geometry:

2D axisymmetric profiles of four domains: Coolant, PVC pipe, air, insulation foam. All are 7 m long.

Domain	Domain radius(mm)	Conductivity (W/m K)	Density(Kg/m^3)	Specific Heat (J/Kg K)
Coolant	0 to 3.175	0.431	1030	3914
PVC pipe	3.175 to 4.7625	0.1	1760	1170
Air	4.7625 to 7.9375	0.0256	1.225	1005
Insulation foam	7.9375 to 26.9875	0.035	96	1300

Boundary conditions:

Fluid motion: Mass flow at inlet = 8.0683e-02 kg/s; At outlet, all flow gradients=0; K-ω turbulence

Heat transfer: Temperature at mass flow inlet=289.15 K

Time dependent temperature at insulation boundary:

T=24.79+3.91cos(wt)-5.724sin(wt)-2.064cos(2wt)-5.393sin(2wt)-4.287cos(3wt)-1.001sin(3wt)-2.354cos(4wt)+2.285sin(4wt)+0.3752cos(5wt)+2.058sin(5wt)+1.042cos(6wt)+0.5306sin(6wt)+0.588cos(7wt)-0.2709sin(7wt)+0.04344cos(8wt)-0.2283sin(8wt), w=5.749e-05

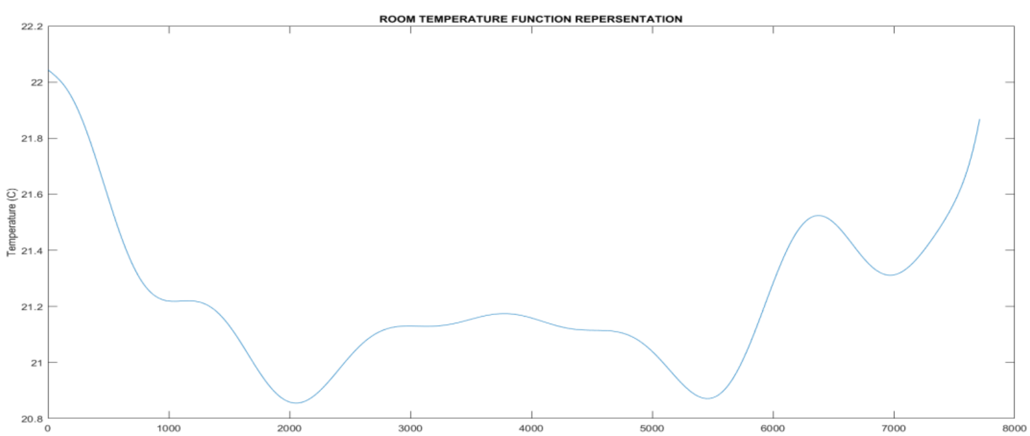
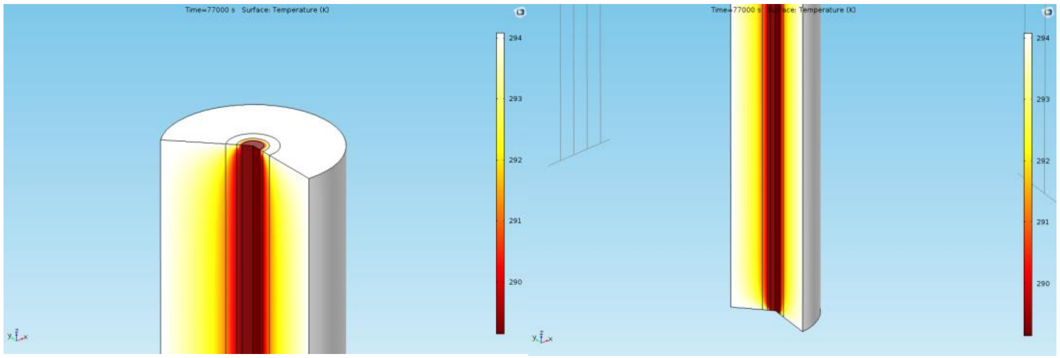


Fig. (1)

The simulation is run to simulate 77080 seconds of the flow from the beginning, in steps of 10 seconds, with an initial condition of all domains being at T=294.15 K. The temperature profile is taken from Report #3. The results of this is stated herewith.

RESULTS

The temperature contours of the flow is presented in fig(2), representing both the inlet and the outlet.

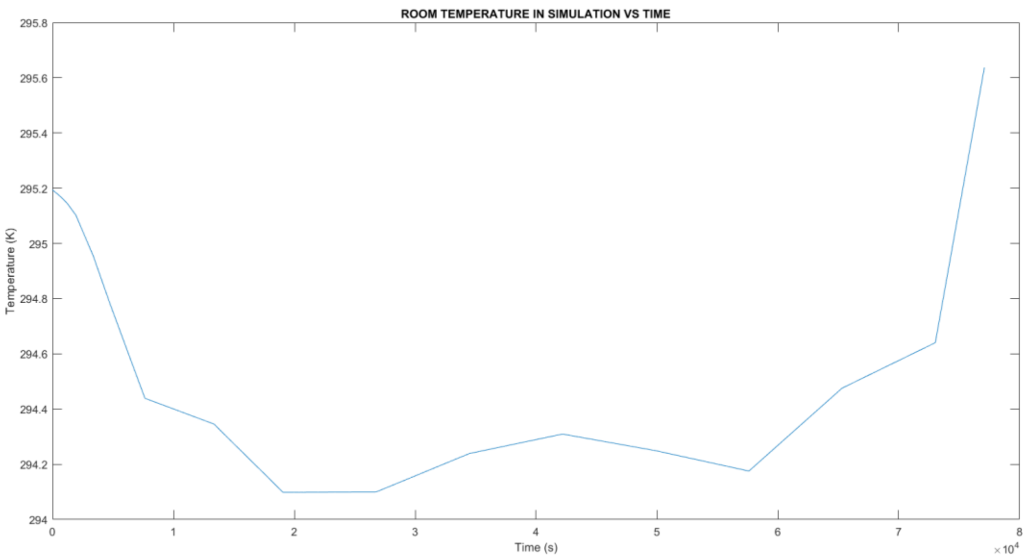


Fig(2) Inflow and outflow

Given that the radius of the tubing is ~27 mm but the length is 7000 mm, the whole tube cannot be placed in one picture. The contours show how the insulation is able to create a visible gradient of temperature, but that the air gap creates the sharpest temperature gradient, and rightfully so, since it has the least thermal conductivity. Note that convection was not considered, and that air is treated as a solid.

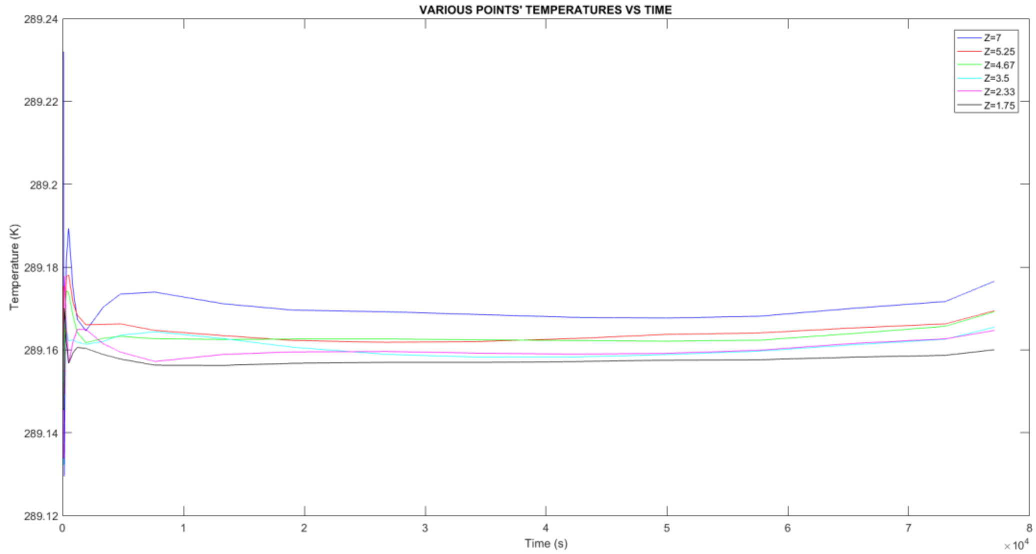
Temperature profiles:

For the given room temperature profile in fig (3), the temperature responses on various points along the coolant at the PVC wall (r=3.175 mm) is presented in fig (4).



Fig(3)

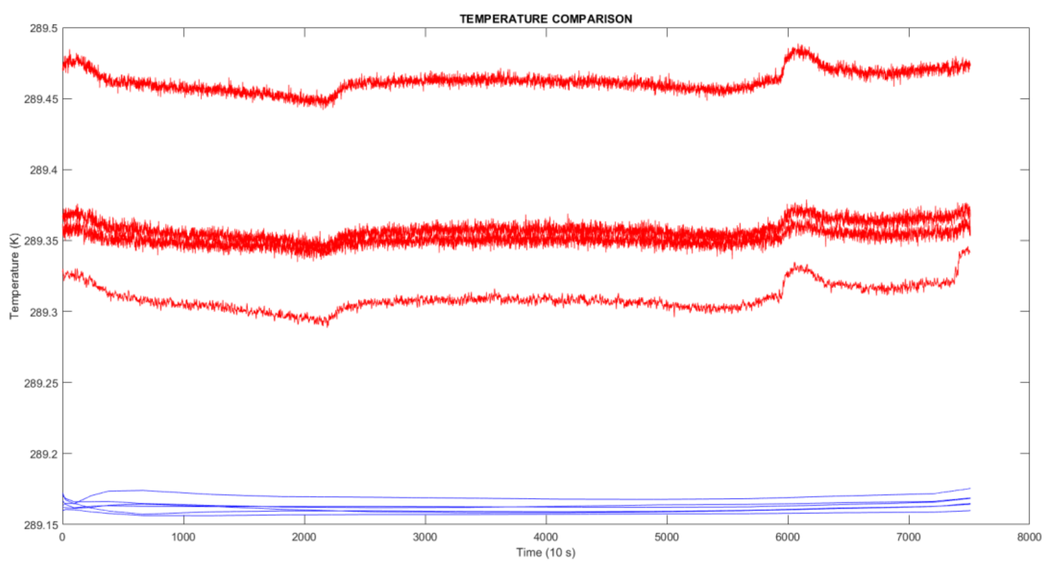
Report #4



Fig(4)

Note that the first 100 seconds of the coolant's temperatures are not shown here due to the issue of scaling and to compensate for flow stabilization. The Z represents the position at which the temperature is noted, Z=7 meaning the outlet and Z=0 meaning the inlet. Also note that the temperatures were recorded every 10 seconds and not lesser owing to memory storage issues.

A temperature comparison of the simulation and that of report #3 is presented here, the red lines pertaining to the experimental data, and the blue lines the simulation data.



Fig(5)

The micro-variations in the experimental values are due to micro-variations in room temperature. That is extremely difficult to account for the same in the simulation. The gradual dip in temperature from '0' to

'2000' is seen both in the simulations and the experiment. The bumps at ~'2500' and '6000' are missing in the simulation because the room temperature simulated for was not calibrated to impose the same. The gradual rise in temperatures by the end are seen both in the simulation and the experiment.

Quantitative observations:

It is important to observe here that the average temperature of each point is actually different from the others. Other observations are stated under:

Point(m)	Maximum(K)	Minimum(K)	Difference(K)
Z=7	289.2320	289.1295	0.1025
Z=5.25	289.1781	289.1493	0.0288
Z=4.67	289.1743	289.1466	0.0277
Z=3.5	289.1677	289.1323	0.0354
Z=2.33	289.1779	289.1337	0.0442
Z=1.75	289.1701	289.1455	0.0246

It is observed that the differences in the peaks and troughs of the simulation are greater than the tolerable limit of 0.01 K for the given temperature profile of fig(3). The differences are somewhat similar to those of the results obtained in Report #3.

CONCLUSIONS

- The simulation values somewhat agree with the experimental values, and the three occasions they don't agree at are when (1) the experimental sensors are not calibrated to a common absolute temperature, (2) it is extremely difficult to simulate micro-variations, and (3) the time step itself is too large to be able to capture changes in the room temperature formulation. However, small time steps (~1 s) create file sizes of the order of ~250 GB, and given the drive memory only so much can be stored.
- The current insulation **does not** satisfy the requirement of keeping the variation below 0.01 K. Given that the room temperature variation is beyond control, further simulations are to be done using other materials and using vacuum (radiation study), to be able to further reduce the thermal diffusivity of the insulation.