Parameter Estimation

Answer 1]

Consider a random sample $(x_1, x_2, x_3 \ldots x_n)$.

$\mu = \theta_1$ (mean), $\sigma^2 = \theta_2$ (variance)

likelihood function $L(\theta_1, \theta_2) = \prod_{i=1}^{n} \frac{1}{\sqrt{2\pi\theta_2}} e^{\frac{-(x_i-\theta_1)^2}{2\theta_2}}$

$\Rightarrow$ to maximise take log on both sides

$$\ln L(\theta_1, \theta_2) = \sum_{i=1}^{n} \left[\frac{-1}{2} \ln(2\pi\theta_2) - \frac{(x_i - \theta_i)^2}{2\theta_2} \right]$$

$\Rightarrow$ Now differentiate wrt θ_1

$$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_1} = \sum_{i=1}^{n} \frac{x_i - \theta_1}{\theta_2} = 0$$

$$x_i - n\theta_1 = 0$$

$$\boxed{\theta_1 = \frac{\sum_{i=1}^{n} x_i}{n}}$$

mean.

$\Rightarrow$ Now differentiate wrt θ_2

$$\frac{d \ln L(\theta_1, \theta_2)}{d\theta_2} = \sum_{i=1}^{n} \left[\frac{-1}{2\theta_2} + \frac{(x_i - \theta_1)^2}{2\theta_2^2} \right] = 0$$

$$\frac{n}{2\theta_2} = \frac{1}{2\theta_2^2} \sum_{i=1}^{n} (x_i - \theta_1)^2.$$

$$\boxed{\theta_2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \theta_1)^2.}$$

$\hookrightarrow$ variance.

Answer 2]

Binomial distribution $B(n, \theta)$.

$$p = \theta, \quad q = 1-\theta$$

$$f(x; n, \theta) = {}^{n}C_{x} \, \theta^{x} (1-\theta)^{n-x}$$

$$L(\theta) = \prod_{i=1}^{n} {}^{n}C_{x_i} \, \theta^{x_i} (1-\theta)^{n-x_i}$$

$\Rightarrow$ taking log on both sides

$$\ln(L(\theta)) = \sum_{i=1}^{n} \left[\ln {}^{n}C_{x_i} + x_i \ln\theta + (n-x_i)\ln(1-\theta) \right].$$

$\Rightarrow$ Now differentiate wrt θ

$$\frac{d}{d\theta} \ln L(\theta) = \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n} \left[\frac{x_i}{\theta} - \frac{n-x_i}{1-\theta} \right] = 0.$$

$$\Rightarrow \sum_{i=1}^{n} \left[\frac{(1-\theta)x_i - \theta(n-x_i)}{\theta(1-\theta)} \right] = 0$$

$$\Rightarrow \sum_{i=1}^{n} (1-\theta)x_i - (n-(x_i))\theta = 0$$

$$\Rightarrow \theta \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i \cdot m$$

$$\boxed{\theta = \frac{\sum_{i=1}^{n} x_i}{n \cdot m}}$$