MA 203 NUMERICAL METHODS

NUMERICAL ANALYSIS OF PARACHUTE LANDING AND TIME TAKEN

Problem Statement

Parachute landing is the controlled descent and landing of a person or object using a parachute. Parachutes are a crucial tool in various fields, including military, aviation, sports, and emergency situations, as they allow individuals to safely descend from an aircraft or other elevated positions. It involves descending from great heights to the ground, relying on a canopy-like parachute to safely control the fall. Parachute landing is utilized in various activities, including skydiving, military airborne operations, search and rescue operations, firefighting, cargo drops, and space exploration.

The successful execution of a parachute landing involves a careful orchestration of techniques and maneuvers to ensure a safe and accurate descent from an airborne position to the desired landing area. The efficiency and safety of parachute landings are of paramount importance, directly impacting mission success, the well-being of individuals involved, and the overall effectiveness of operations. By understanding the factors affecting the landing, appropriate safety measures and training protocols can be developed to mitigate risks and ensure the safety of parachutists.

To analyze the accuracy and precision of parachute landing coordinates, we need to assess various parameters such as weather conditions, parachute design, altitude, and wind patterns. In this project, we delve into the realm of parachute landing, focusing on refining existing techniques and introducing innovative solutions to enhance precision, control, and safety during the descent phase. Precision in parachute landings is essential to ensure the safety of personnel and equipment and to facilitate mission objectives.

Why is it better to use numerical methods to analyze parachute landings?

Using numerical methods for analyzing parachute landings is advantageous due to their ability to simulate complex physical interactions accurately. Parachute dynamics involve intricate variables like air pressure, wind patterns, and material properties, making analytical solutions impractical. Numerical simulations employ algorithms to approximate these complex equations, offering precise insights into parachute behavior under diverse conditions. They enable the study of multiple influencing factors, such as altitude, wind speed, and parachute design, providing a comprehensive understanding of landing coordinates. Moreover, numerical simulations facilitate iterative testing, allowing for the optimization of deployment parameters for improved precision and safety. The accuracy and flexibility of numerical methods is essential to ensure data-driven decision-making and advancements in parachute technology, ultimately enhancing the effectiveness and safety of parachute landings.

Numerical Method used to solve the above problem:

1. Runge-Kutta Method

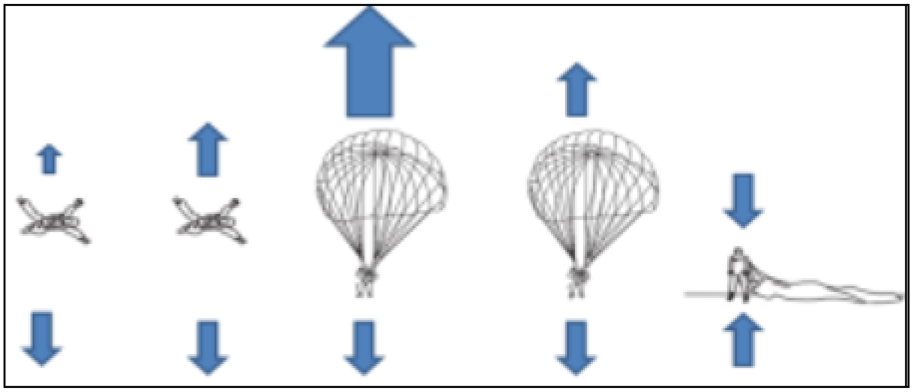


Figure 1: Parachute landing

The specific objectives of the project are presented below:

1. Develop a physical model of the parachute.
2. Make suitable assumptions and make the problem tractable.
3. Derive the governing equations from first principles.
4. Formulate the initial and boundary conditions.
5. Choose the correct set of property values and parameters.
6. Adopt judicious numerical schemes to conduct a numerical analysis.
7. Use appropriate numerical methods to conduct a numerical analysis
8. Write a computer program to solve the equations derived and obtain solutions
9. Determine the landing coordinates of the parachute and time taken to reach the ground.
10. Conduct parametric studies to explore the effects of altitude, wind speed, parachute design, and others.
11. Compare the computed values with the analytical results that are used in daily life.

1. Physical Model

Consider a jumper of certain mass who jumps from a flying airplane with a specific velocity from a height above the ground. After jumping from the aircraft, he travels for a while and opens the parachute. Later, he lands on the ground safely with the help of the parachute. The whole journey is not just a simple free fall, there are many parameters such as air resistance, drag force, parachute drag, and wind force which influence the whole trajectory, landing coordinates, velocity, and time taken by the person. This path is not a one-dimensional fall; rather we consider it as a 3-D model taking into account the forces acting on it in all the directions.

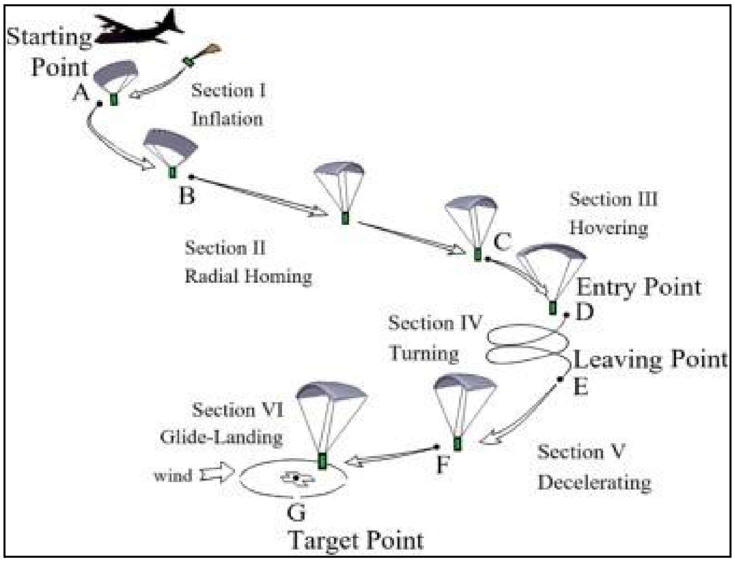


Figure 2: The whole path of the landing

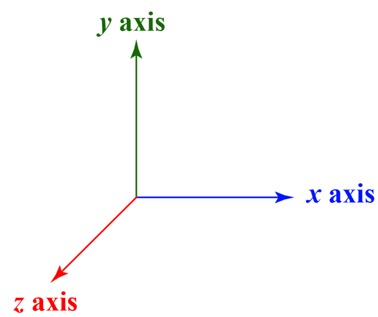
2. Assumptions

We have to develop mathematical equations from scratch to evaluate the forces and then use them in computing the landing coordinates, time taken, etc. We consider some assumptions to simplify the problem.

1. We don't consider the time taken to open the parachute.
2. We consider that the parachute opens immediately and the drag force of the parachute takes effect.
3. The jumper leaves the airplane without any rotation; he jumps straight out of it.
4. The jumper lands on the ground without any bending or twisting.
5. We also do not consider complex landings, such as landing on mountains, trees, etc.
6. We assume that the density of air doesn't change throughout the path.
7. We also consider average wind velocity without any fluctuations in the wind caused by external factors.
8. Assuming uniform gravity throughout the motion (independent of height) .
9. Assuming the parachute maintains a stable and predictable orientation throughout the descent, neglecting dynamic instabilities or significant disturbances .

3. Governing Equations

We are writing the equations considering the aircraft is moving along the y direction, the air resistance acting on the jumper is in the x direction (out of the plane), and the vertical direction is along the z- axis.



To find the landing coordinates of the parachute and to develop a mathematical model, we consider the forces acting on it in the x, y, and z directions.

1. **Forces acting on it in the x direction:**

   It is influenced by cross-wind which is along the x direction

$$F_x = ma_x$$
$$a_x = \triangle v_x / t$$
$$\triangle v_x = (w - v_0)$$
$$F_x = m(w - v_0)/t$$
$$F_x = b(w - v_0)$$

2. **Forces acting along y-direction:**

   We assume the effect of air resistance in the y-direction.
   The corresponding equations are:

$$F_y = -bv$$

   The negative sign indicates that the force is opposite to the motion.

3. **Forces in the z-direction:**

   It is the vertical direction from the aircraft to the ground. Gravitational force and drag force due to the parachute is acted on the jumper.

$$Drag\ force\ (D) = C_d \rho v^2 A/2$$

Parameters used:

1. m = Mass of the jumper
2. F_x = Force acting in the x direction
3. a_x = Acceleration of the jumper in the direction of the motion of the plane
4. w = Speed of the crosswind
5. V_0 = It is the velocity of the jumper
6. t = Time
7. F_y = Force of air resistance in the y direction
8. b = Drag coefficient due to air resistance
9. D = Drag force
10. C_d = Drag coefficient of parachute (value = 1.75)
11. ρ = Density of air (value = 1.225 kg/cubic meters)
12. A = Area of the parachute

Calculating the time taken by the parachute to land:

To find the total time taken, we need to find the terminal velocity of the jumper when the parachute is deployed. He attains terminal velocity when the drag force of the parachute is equal to his weight (mg).

$$mg = D$$

$$\tfrac{1}{2} * C_d \rho v^2 A = mg$$

We get,

$$V_t = (2mg / C_d \rho A)^{1/2}$$

The total time taken is equal to the time spent during the fall after opening the parachute and the time spent before opening the parachute during the free fall.

Time spent during the fall after opening the parachute is equal to :

$$T_2 = V_t * height$$

Where height is the vertical distance to the ground from the parachute opening.

Determining the landing coordinates:

As we assume vertical direction as z, to find the landing coordinates, we need x and y direction forces.

In the x-direction:

$$F_x = b(w - v_0) = ma$$

$$bw - bv_0 = ma$$

$$ma + bv_0 = bw$$

Now, to solve for x:

$$ma + bv = bw$$
$$v = dx/dt$$
$$a = dx^2/dt^2$$

Rewriting the equations in terms of x,

$$mx'' + bx' = bw$$

In the y-direction:

$$F_y = -bv = ma;$$
$$-bv = ma$$
$$v = dy/dt$$
$$a = dy^2/dt^2$$

Re-writing the equations,

$$bv + ma = 0$$

$$my'' + by' = 0$$

4. Boundary conditions while solving the equations:

Boundary conditions are essential to define the behavior of a system at its boundaries or edges. In the context of solving the equations describing the motion of a jumper with a parachute using numerical methods (such as Runge-Kutta), boundary conditions typically refer to the initial conditions and any specific conditions related to the problem that need to be satisfied at the boundaries of the domain.

The boundary conditions can be defined as follows:

1. These are the conditions at the start of the simulation (when the jumper is just starting the descent).
    - $v_{x,0}$: Initial velocity in the x-direction.
    - $v_{y,0}$: Initial velocity in the y-direction.
    - x_0 : Initial x-coordinate (horizontal position).
    - y_0 : Initial y-coordinate (vertical position).

2. Landing or Terminal Conditions: These are conditions that define when the simulation should stop, often related to the landing of the jumper.

   When $y \leq 0$, the solution is terminated when the jumper's altitude becomes zero or negative, indicating they have landed.

5. Solution Methodology:

From the equations we derived in the previous sections, we got two ODE's for x and y respectively.
It is non-homogenous ODE of order 2. First we need to initialize all the boundary conditions. Then it is converted to two first order ODE's. Consider some velocity while jumping and then go through the problem. Runge-Kutta method is applied to these equations in computer code to find the x and y values by giving parameters certain values.

6. Analytical solution:

In Y direction:

Using the characteristic equation $\mathbf{mD^2 + bD = 0,}$ the roots are $\mathbf{D = 0}$ and $\mathbf{D = -(b/m).}$
So, the general form of the solution is $\mathbf{y(t) = C_1e^0 + C_2e^{-(bt/m)}}$, where constants C1 and C2 are found by using the initial conditions.
These conditions are at time t = 0; the time at which the parachute is opened, and air resistance is no longer negligible.
For the equation, as we initially considered y_initial coordinate as 0 while jumping we assume y(0) = 0 and initial velocity as v.

In X direction:

In the x-direction, the differential equation is a non - homogeneous, therefore it's homogeneous equation can be written as: $\mathbf{mx'' + bx' = 0}$ after which the non-homogeneous equation $\mathbf{mx'' + bx' = bw}$ is solved. Using the characteristic equation $\mathbf{mD^2 + bD = 0,}$ the roots are $\mathbf{D = 0}$ and $\mathbf{D = -(b/m).}$
So, the general form of the solution is $\mathbf{x_i = C_1e^0 + C_2e^{-(bt/m)}}$, where constants C1 and C2 are found by using the initial conditions. Now, for the particular solution, we solve for x_p of the non-homogenous equation.
Solution will be $x = x_0 + x_p$
To find constants, we use x(0) = 0 as we initialized earlier. Then take a value of initial velocity in x direction.

7. Numerical Solution:

To solve for the coordinates,the first step we used was to express the differential equation in a system of first order ODEs, and then we integrated using the Runge-Kutta method.

- The given equation is: $mx'' + bx' = bw$
- We can rewrite this as system of first order ODEs, and we get:
  Let $v = x' \Rightarrow v' = (bw-bv)/m$
- Using the Runge Kutta method to solve the system numerically,
  Where:

  h=step size
  k1, k2, k3, k4=intermediate values for v
  l1, l2, l3, l4=intermediate values for x

- Using the following steps,we iterated the equation using Runge Kutta method:

1) Initialization:

   We set the initial conditions to x_0, v_0 and the step size to h and the final time to t_{final}.

2) Iterative numerical integration(for each time step t):

   a) Calculating k1 and l1 using initial values:

      $k1= (bw-bv)/m$
      $l1=v$

   b) Calculating k2 and l2 using intermediate values:
      As x is dependent on v hence we should increment in terms of k1, k2, k3, k4

      $k2=(bw-b(v+k1*h/2))/m$
      $l2=(v+k1*h/2)$

   c) Calculating k3 and l3 using intermediate values:

      $k3=(bw-b(v+k2*h/2))/m$
      $l3=(v+k2*h/2)$

   d) Calculating k4 and l4 using intermediate values:

      $k4=(bw-b(v+k3*h/2))/m$
      $l4=(v+k3*h)$

   e) Updating v and x using the weighted sum of the k values and l values:

      $v_{next}= v+h*(k1+2k2+2k3+k4)/6$
      $x_{next}=x+h*(l1+2l2+2l3+l4)/6$

   f) Updating v and x for the next iteration:

      $v=v_{next}$
      $x=x_{next}$

   g) Updating time to t=t+h

3) Stopping Criteria:
   We have to stop when t exceeds t_{final}.

Using the same method for calculating y.
The output will give the x and y landing coordinates of the parachute.

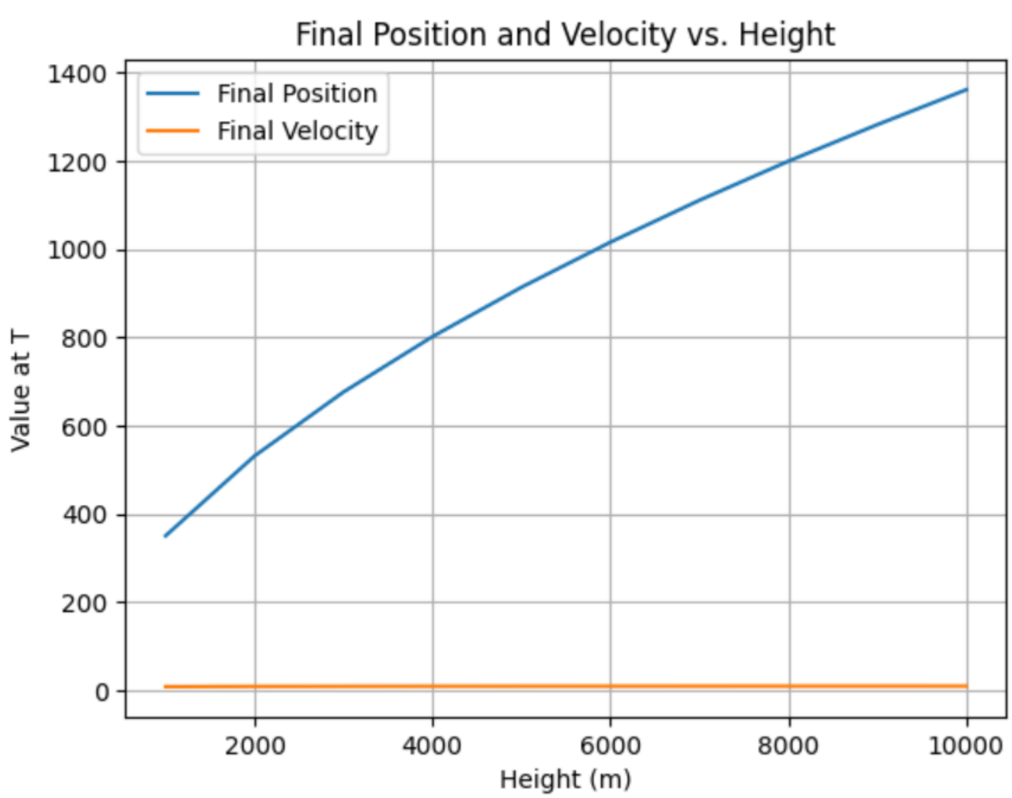
8. Algorithm used:

To solve the problem via computer program, a code was written in python. Steps followed in the code are:

1. Initialize x_initial and y_initialize as 0 and 0.
2. Consider initial velocity in x and y directions as some values.
3. Define all the parameters and give them any values.
4. Here, we need to find both x and y values hence two separate codes where the equations differ by constant bw.
5. This ODE can be solved using two equations for solving v and x which are dependent on each other.
6. Implement the Runge-Kutta method to find corresponding v and x for each iteration.
7. Stop these iterations when t = T.
8. Implementing the same for y using different equations we get y-coordinate.
9. These values of x and y are the landing coordinates.

9. Results and Discussion:

;

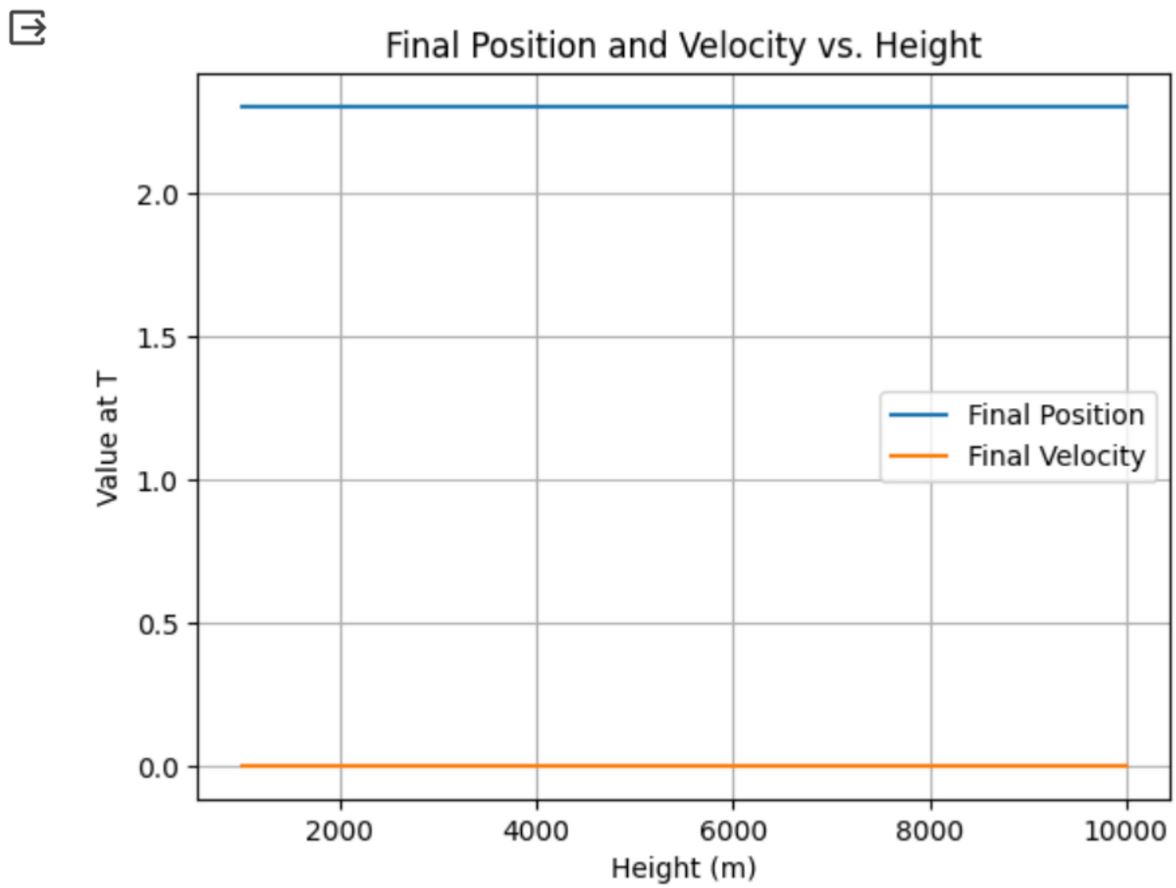


The above plot will have two lines:

Final Position (X) vs. Height: This line will show how the final position of the parachute changes with different initial heights.
Final Velocity vs. Height: This line will show how the final velocity of the parachute changes with different initial heights.

Typically, for a falling object in a vacuum (without air resistance), the final velocity would increase linearly with the square root of the height. However, in this case, with air resistance taken into account, the relationship will likely be more complex and nonlinear.

Based on the physics of the system, the plot is expected to show a trend where both final position and final velocity increase with increasing height. However, the rate of increase is not linear due to the complex dynamics of the system, including air resistance,gravity, and other system parameters.



The above plot will have two lines:

Final Position (Y) vs. Height: This line will show how the final position of the object changes with different initial heights.
Final Velocity vs. Height: This line will show how the final velocity of the object changes with different initial heights.

The plot will show that as the initial height increases, the final position and final velocity of the falling object will also increase.
Here are a few potential reasons that might explain the linear appearance of the graph:

1. Linear Approximation of Air Resistance: The calculation of air resistance might be approximated in such a way that the overall behavior of the system appears linear for the given range of heights.

2. Linear Regime of Behavior: The system being simulated might exhibit linear-like behavior within the given height range due to specific conditions or parameters chosen for the simulation.

3. Simplified Numerical Integration: The numerical integration scheme used for simulation might be a simple scheme that could approximate the behavior in a linear manner.

10. References:

1. Fig. 6-General flight profile for current ram-air type airdrop. (n.d.). ResearchGate. https://www.researchgate.net/figure/General-flight-profile-for-current-ram-air-type-airdrop_fig5_330449967

2. Parachute motion. (2015, November 16). IGCSE Physics. https://hgphysics.com/parachute-motion/

3. https://airborne-sys.com/wp-content/uploads/2016/10/aiaa-2005-1624.pdf

Name of the Group Members	Roll Number
1. Gangannagudem Siri	22110083
2. SriJahnavi Chinthalapudi	22110063
3. Pasala Greeshma	22110182
4. Ankeshwar Ruthesha	22110024
5. Mekala Sonalika	22110151