Bus impedance building algorithm

Consider the power system shown in Fig. 1. The values marked are p.u. impedances. The p.u. reactances of the generator 1 and 2 are 0.15 and 0.075 respectively. Compute the bus impedance matrix of the generator – transmission network.

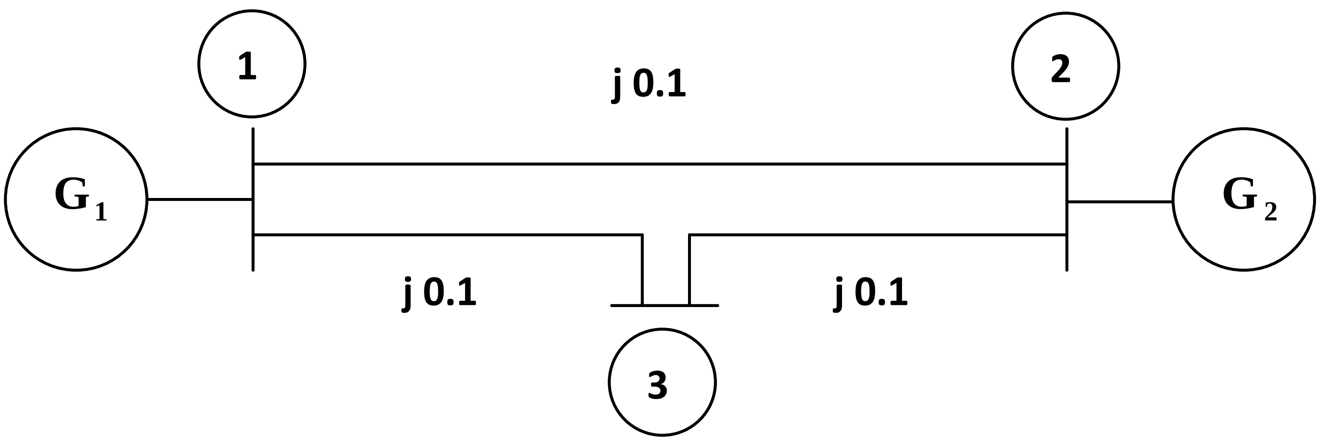


Fig. 1

The ground bus is numbered as 0 and it is taken as reference bus. The p.u. impedance diagram is shown in Fig. 2.

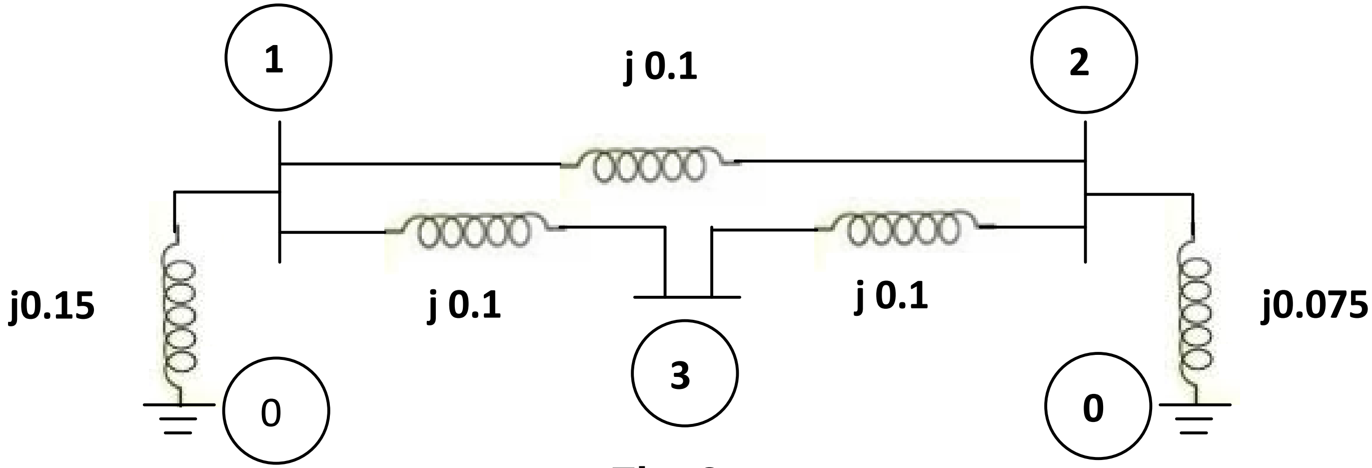


Fig. 2

Its bus admittance matrix can be obtained as

$$Y_{Bus} = -j\begin{bmatrix} 26.6667 & -10 & -10 \\ -10 & 33.3333 & -10 \\ -10 & -10 & 20 \end{bmatrix}$$

One way of finding its bus impedance matrix, Z_{Bus}, is to invert the above bus admittance matrix. If the number of buses is more, it is difficult to get direct inverse. Alternatively Z_{Bus} matrix can be obtained by constructing it adding the elemrnt one by one.

The network graph of this power system is shown in Fig. 3.

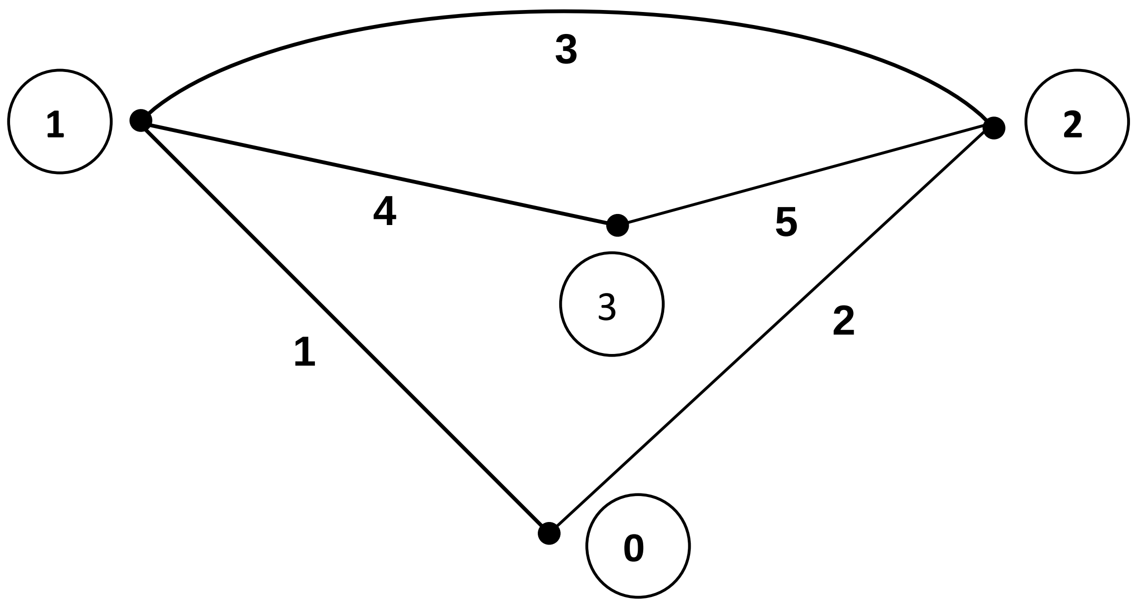


Fig. 3

The network consisting of elements 1, 2 and 3 is a partial network with buses 0, 1 and 2. To this if element 4 is added, the network will be as shown in Fig. 4.

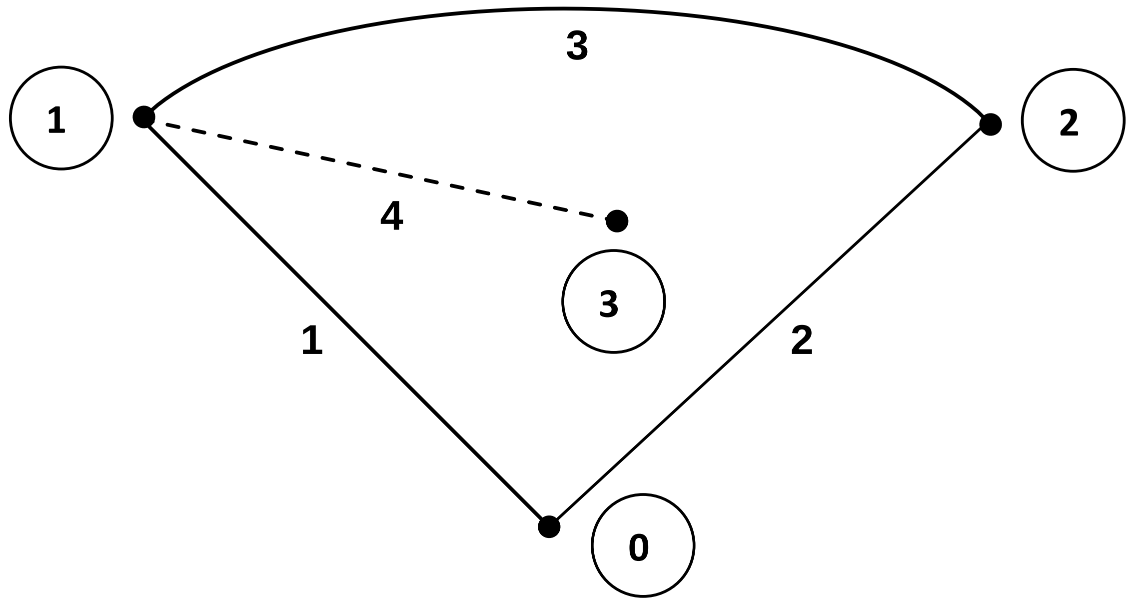


Fig. 4

Now a new bus 3 is created. The added element is a BRANCH. For the next step, network with elements 1,2,3 and 4 will be taken as partial network. This contains buses 0,1,2 and 3

When element 5 is added to this, the network will be as shown in Fig. 4.

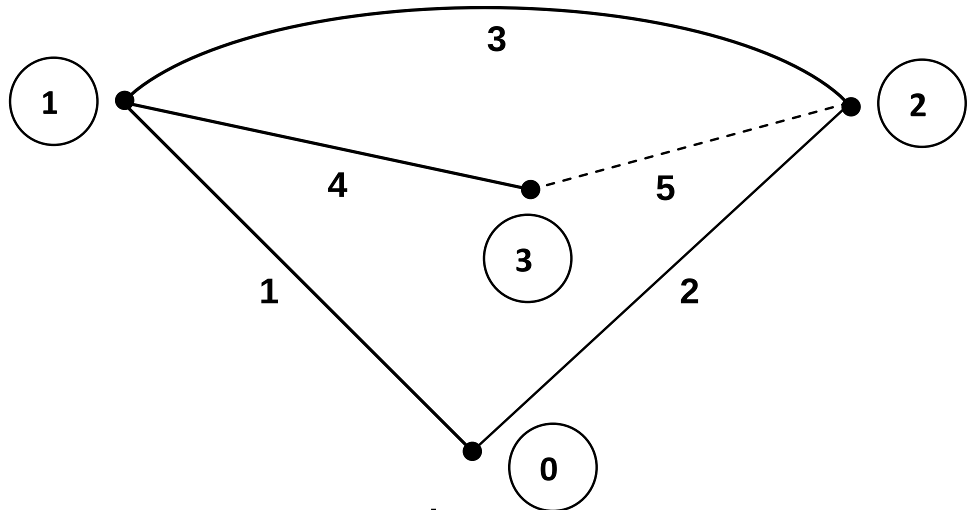
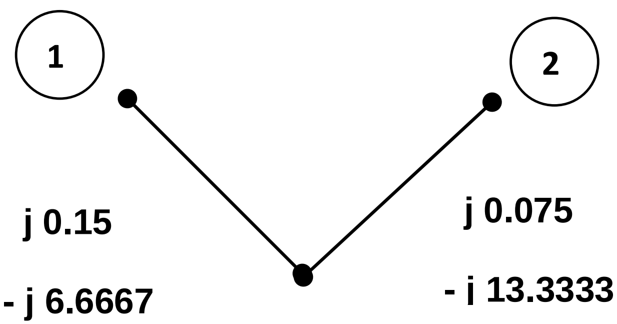


Fig. 4

In this case, no new bus is created and the added element links buses 2 and 3 and hence it is called a LINK.

If we consider elements 1 and 2 alone, its Y_{Bus} is given by

$$Y_{Bus} = -j\begin{bmatrix} 6.6667 & 0 \\ 0 & 13.333 \end{bmatrix}$$



and hence its Z_{Bus} is given by

$$Z_{Bus} = j \quad \begin{array}{cc} & \begin{array}{cc} 1 & 2 \end{array} \\ \begin{array}{c} 1 \\ 2 \end{array} & \begin{bmatrix} 0.15 & 0 \\ 0 & 0.075 \end{bmatrix} \end{array}$$

Note that these element values are the values of the shunt impedances at buses 1 and 2. This result can be extended to more number of buses also. The above Z_{Bus} corresponds to the partial network shown in Fig. 5.

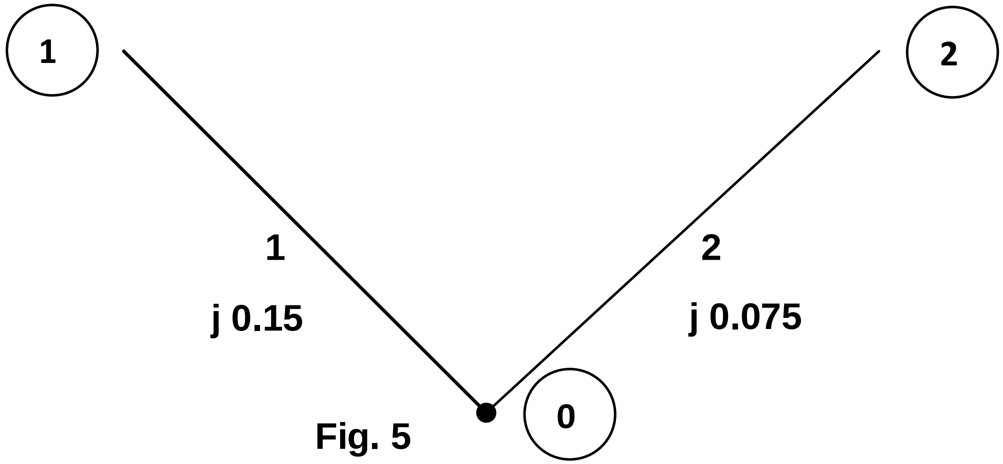


Fig. 5

Now add element 4. This it from bus 1 to 3 with an impedance j 0.1. The added element is a <u>branch.</u> It is from the existing bus 1 and it creates a new bus 3 as shown in Fig. 6. The modified Z_{Bus} matrix is

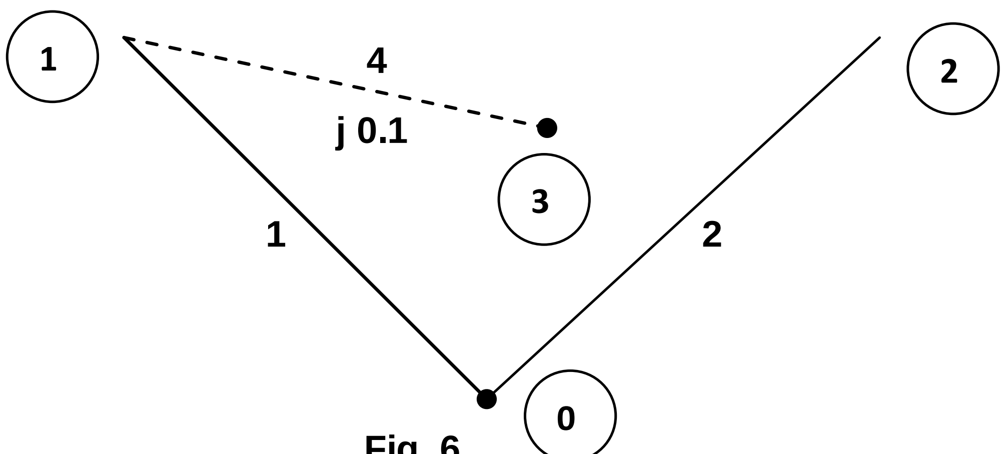


Fig. 6

$$Z_{Bus} = j \quad \begin{array}{c c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{cc|c} 0.15 & 0 & 0.15 \\ 0 & 0.075 & 0 \\ \hline 0.15 & 0 & 0.25 \end{array}\right] \end{array}$$

Now add element 3. It is from bus 1 to 2 with an impedance of j 0.1. Since it is linking two of the existing buses 1 and 2 as shown in Fig. 7, it is a Link between buses 1 and 2.

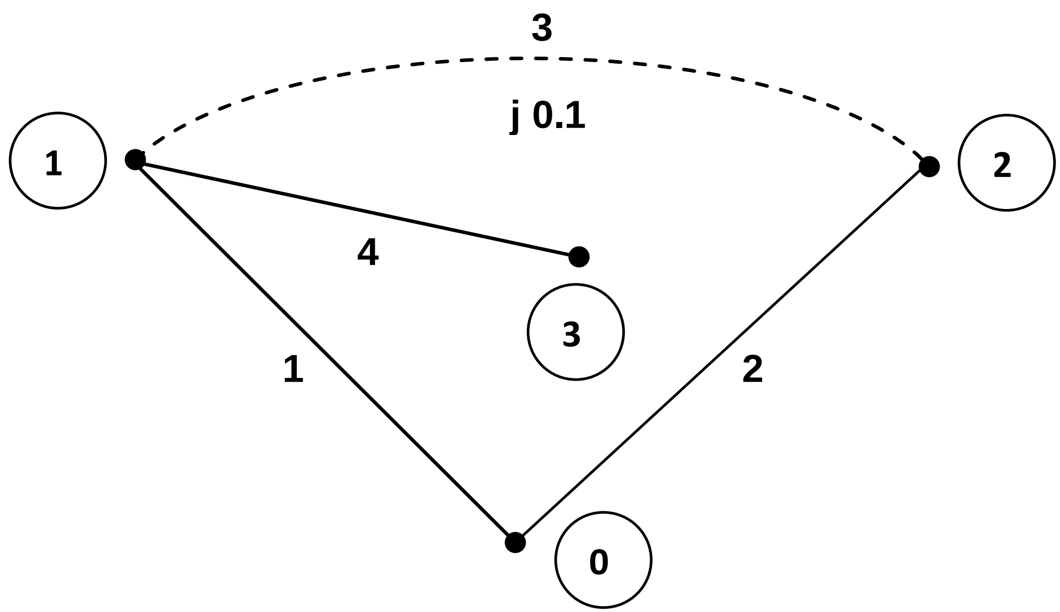


Fig. 7

$$Z_{Bus} = j\begin{bmatrix} 0.15 & 0 & 0.15 \\ 0 & 0.075 & 0 \\ 0.15 & 0 & 0.25 \end{bmatrix}$$

The modified Z_{Bus} matrix with ℓ th bus is

$$Z_{Bus} = j\quad \begin{array}{c|c} & \begin{matrix} 1 & 2 & 3 & \lambda \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ \lambda \end{matrix} & \left[\begin{array}{ccc|c} 0.15 & 0 & 0.15 & 0.15 \\ 0 & 0.075 & 0 & -0.075 \\ 0.15 & 0 & 0.25 & 0.15 \\ \hline 0.15 & -0.075 & 0.15 & 0.325 \end{array}\right] \end{array}$$

Eliminating bus ℓ

$$Z_{Bus} = j\quad \begin{array}{cc} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \end{matrix} & \begin{bmatrix} 0.0808 & 0.0346 & 0.0808 \\ 0.0346 & 0.0577 & 0.0346 \\ 0.0808 & 0.0346 & 0.1808 \end{bmatrix} \end{array}$$

Now add element 5. It is from bus 2 to 3 with an impedance of j 0.1. Since it is linking two of the existing buses 2 and 3 as shown in Fig. 8, it is a Link.

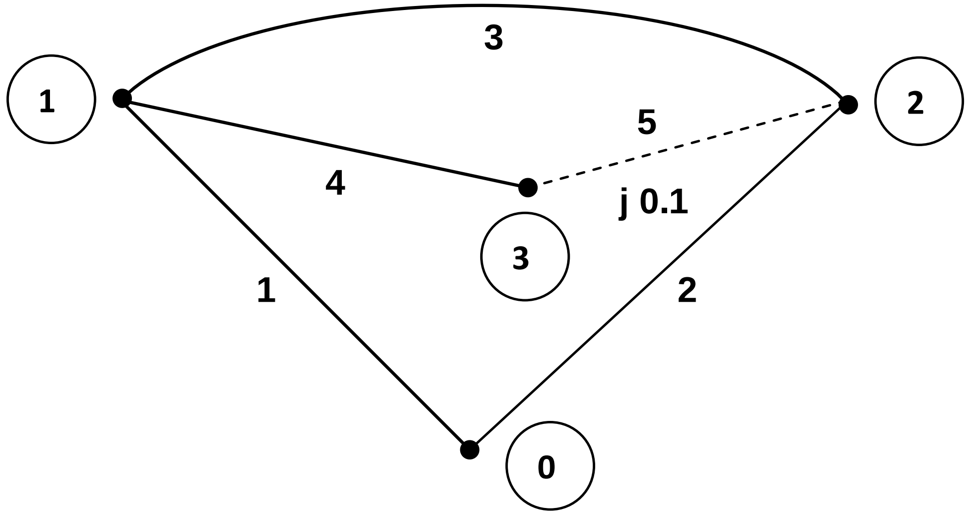


Fig. 8

The modified Z_{Bus} matrix with ℓ th bus is

$$Z_{Bus} = j \quad \begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & \lambda \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ \lambda \end{array} & \left[\begin{array}{ccc|c} 0.0808 & 0.0346 & 0.0808 & -0.0462 \\ 0.0346 & 0.0577 & 0.0346 & 0.0231 \\ 0.0808 & 0.0346 & 0.1808 & -0.1462 \\ \hline -0.0462 & 0.0231 & -0.1462 & 0.2692 \end{array}\right] \end{array}$$

$$
Z_{Bus} = j \quad \begin{array}{c} 1 \\ 2 \\ 3 \\ \lambda \end{array} \left[\begin{array}{ccc|c} 0.0808 & 0.0346 & 0.0808 & -0.0462 \\ 0.0346 & 0.0577 & 0.0346 & 0.0231 \\ 0.0808 & 0.0346 & 0.1808 & -0.1462 \\ \hline -0.0462 & 0.0231 & -0.1462 & 0.2692 \end{array} \right]
$$

Eliminating bus ℓ the final bus impedance matrix is obtained as

$$
Z_{Bus} = j \quad \begin{array}{cc} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \left[\begin{array}{ccc} 0.0729 & 0.0386 & 0.0557 \\ 0.0386 & 0.0557 & 0.0471 \\ 0.0557 & 0.0471 & 0.1014 \end{array} \right] \end{array}
$$

The correctness of the result can be verified by multiplying the values of the two matrices Z_{Bus} and Y_{Bus}.