

# rk4vsleapfrog

April 16, 2019

```
In [2]: import numpy as np
import matplotlib.pyplot as plt
plt.ion()

Nsteps = 1000000 #the total number of time steps
h = 1e-3 #time stepping interval value

#creating arrays to store the values of p and q
q = np.zeros(Nsteps)
p = np.zeros(Nsteps)

#variables to store temporary values during calculations
q_temp = 0
p_temp = 0

#initial value of p
p[0] = 5.0

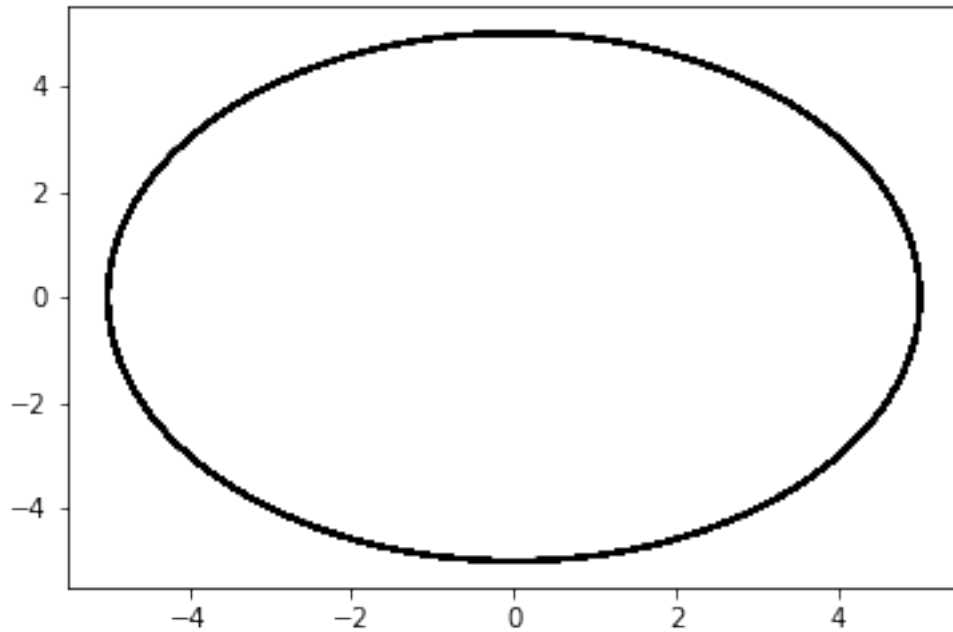
#shifting the p to half time steps by an initial forward euler.
p_temp = p[0] - 0.5*h*q[0]
p[0] = p_temp

for i in range(1,Nsteps):
    #computing the q's
    q_temp = q[i-1] + h*p[i-1]
    q[i] = q_temp

    #computing the p's
    p_temp = p[i-1] - h*q[i]
    p[i] = p_temp

plt.plot(q,p, '-k') #a line plot

q_lp = q
p_lp = p
```



## 0.1 RK4

```
In [3]: import numpy as np
import matplotlib.pyplot as plt
plt.ion()

Nsteps = 1000000 #the total number of time steps
h = 1e-3 #time stepping interval value

#creating arrays to store the values of p and q
q = np.zeros(Nsteps)
p = np.zeros(Nsteps)

#initial value of p
p[0] = 5.0

#variables to store the intermediate forces
qk1 = 0
qk2 = 0
qk3 = 0
qk4 = 0

qp1 = 0
qp2 = 0
qp3 = 0
```

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qp4 = 0

for i in range(1,Nsteps):
    qk1 = h*p[i-1]
    qk2 = h*(p[i-1] - 0.5*h*q[i-1])
    qk3 = qk2
    qk4 = h*(p[i-1] - h*q[i-1])

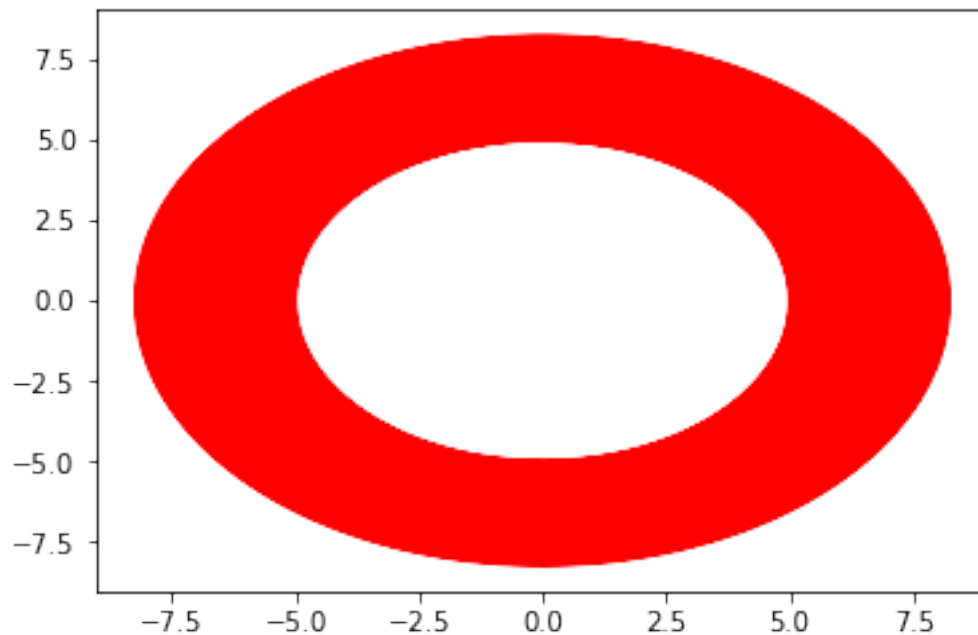
    pk1 = -h*q[i-1]
    pk2 = -h*(q[i-1] - 0.5*h*p[i-1])
    pk3 = pk2
    pk4 = -h*(q[i-1] - h*p[i-1])

    q[i] = q[i-1] + (1./6)*(qk1 + 2*qk2 + 2*qk3 + qk4)
    p[i] = p[i-1] + (1./6)*(pk1 + 2*pk2 + 2*pk3 + pk4)

plt.plot(q,p, '-r')

q_rk = q
p_rk = p

```



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In [4]: E_rk = 0.5*(p_rk**2 + q_rk**2)
        E_lp = 0.5*(p_lp**2 + q_lp**2)

        plt.plot(E_rk, '-r', label='RK4')
        plt.plot(E_lp, '-k', label='Leapfrog')

```

```
plt.title('Energy plot')
plt.ylabel('Hamiltonian')
plt.xlabel('Number of Timesteps')
plt.legend()
```

Out[4]: <matplotlib.legend.Legend at 0x7fea8b3f12e8>

