

Neural Control Variates

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Following the idea proposed in 2312.08228.

U(1) Model Application

- Conflict between the U(1) model plaquette picture and the link picture.

Problem: The Action in the U(1) model is a sum of cosine of the angle of the plaquettes, but in the link picture it is the cosine of the sum of the angles of the links.

$$S_{\text{plaquette}}^{2D} = \beta(L^2 - \sum_{j=1}^L \sum_{i=1}^L \cos(\theta_{i,j, \text{ plaquette}})) \quad (1)$$

$$S_{\text{link}}^{2D} = \beta(L^2 - \sum_{j=1}^L \sum_{i=1}^L \cos(\theta_{i,j, \text{ link}}^{\mu=0} + \theta_{i+1,j, \text{ link}}^{\mu=1} - \theta_{i,j+1, \text{ link}}^{\mu=0} - \theta_{i,j, \text{ link}}^{\mu=1})) \quad (2)$$

Can we show that the plaquette picture is equivalent to the link picture in the Haar measure when we compute the partition function?

Solution: The Haar measure is invariant under the transformation of the link variables. If we choose a gauge where all link variables pointing along the time direction are rotated to the unit element which means [**Temporal Gauge**]

$$\theta_{i,j, \text{ link}}^{\mu=0} = 0 \quad \forall i, j \in [1, L]$$

Then the link action becomes

$$S_{\text{link}}^{2D} = \beta(L^2 - \sum_{j=1}^L \sum_{i=1}^L \cos(\theta_{i,j, \text{ link}}^{\mu=1} - \theta_{i+1,j, \text{ link}}^{\mu=1})) \quad (3)$$

Now we can see that the link action is equivalent to the plaquette action in the Haar measure.