Neural Control Variates

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Following the idea proposed in 2312.08228.

U(1) Model Application

• Conflict between the U(1) model plaquette picture and the link picture.

Problem: The Action in the U(1) model is a sum of cosine of the angle of the plaquettes, but in the link picture it is the cosine of the sum of the angles of the links.

$$S_{\text{plaquette}}^{2D} = \beta(L^2 - \sum_{j=1}^{L} \sum_{i=1}^{L} \cos(\theta_{i,j, \text{ plaquette}}))$$
 (1)

$$S_{\text{link}}^{2D} = \beta (L^2 - \sum_{i=1}^{L} \sum_{i=1}^{L} \cos(\theta_{i,j,\text{ link}}^{\mu=0} + \theta_{i+1,j,\text{ link}}^{\mu=1} - \theta_{i,j+1,\text{ link}}^{\mu=0} - \theta_{i,j,\text{ link}}^{\mu=1}))$$
 (2)

Can we show that the plaquette picture is equivalent to the link picture in the Haar measure when we compute the partition function?

Solution: The Haar measure is invariant under the transformation of the link variables.