Neural Control Variates

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College Park
2024-03-25

2024

March

March 25, 2024

Following the idea proposed in 2312.08228.

U(1) Model Application

- Conflict between the U(1) model plaquette picture and the link picture.

Problem: The Action in the U(1) model is a sum of cosine of the angle of the plaquettes, but in the link picture it is the cosine of the sum of the angles of the links.

$$S^{2D}_{\text{plaquette}} = \beta(L^2 - \sum_{\text{j}=1}^{L}\sum_{\text{i}=1}^{L}\cos(\theta_{\text{i,j, plaquette}})) \tag{1}$$

$$S^{2D}_{\text{link}} = \beta(L^2 - \sum_{\text{j}=1}^{L}\sum_{\text{i}=1}^{L}\cos(\theta^{\mu=0}_{\text{i,j, link}} + \theta^{\mu=1}_{\text{i+1,j, link}} - \theta^{\mu=0}_{\text{i,j+1, link}} - \theta^{\mu=1}_{\text{i,j, link}})) \tag{2}$$

Can we show that the plaquette picture is equivalent to the link picture in the Haar measure when we compute the partition function?

Solution: The Haar measure is invariant under the transformation of the link variables. If we choose a gauge where all link variables pointing along the time direction are rotated to the unit element which means [**Temporal Gauge**]

$$\theta^{\mu=0}_{\text{i,j, , link}} = 0 \ \ \forall \text{ i, j} \in [1, L]$$

Then the link action becomes

$$S^{2D}_{\text{link}} = \beta(L^2 - \sum_{\text{j}=1}^{L}\sum_{\text{i}=1}^{L}\cos(\theta^{\mu=1}_{\text{i,j, link}} - \theta^{\mu=1}_{\text{i+1,j, link}})) \tag{3}$$

Now we can see that the link action is equivalent to the plaquette action in the Haar measure.