Lecture 11 Generative Modeling

Recap

- Expected Value
- Correlation
- Multivariate Gaussian distributions
- Intuition behind the multivariate Gaussian formula
- Mahalanobis distance
- Minimum Covariance Determinant

Classification problem

- You have a bottle of wine with missing label
- Is it from winery 1, 2 or 3?

Data set

- Training set: records for 130 bottles
  - Winery 1: 10%
  - Winery 2: 50%
  - Winery 3: 40%
- 13 features
  - Alcohol, Malic acid, Flavanoids etc.
- Test set: records of 48 bottles

Univariate generative approach: Fit Gaussian

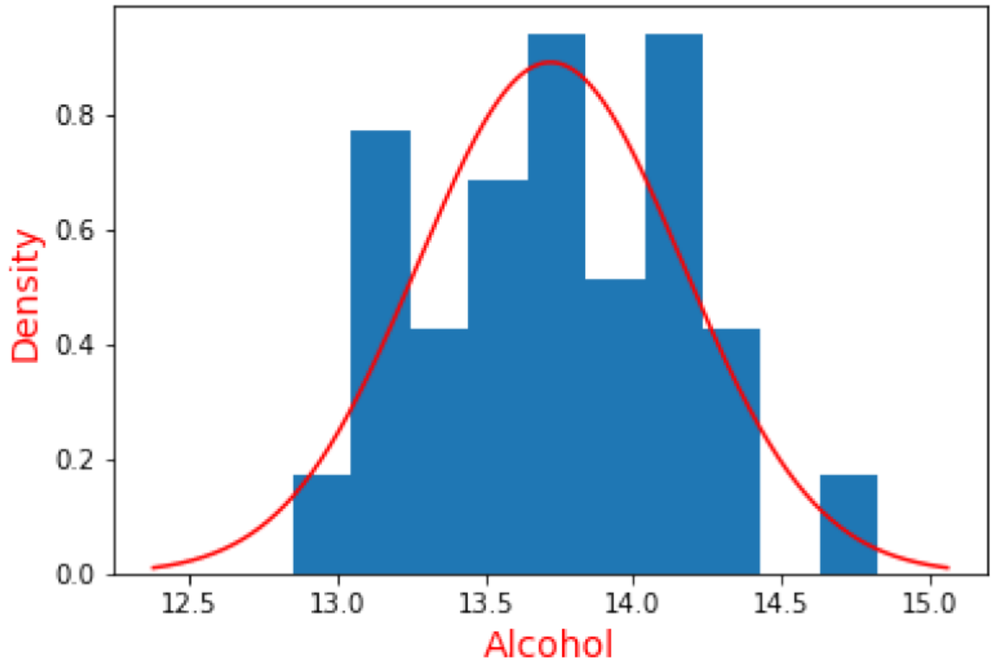
- Find mean and variance for each winery
- Fit 3 different Gaussian distributions for 3 wineries

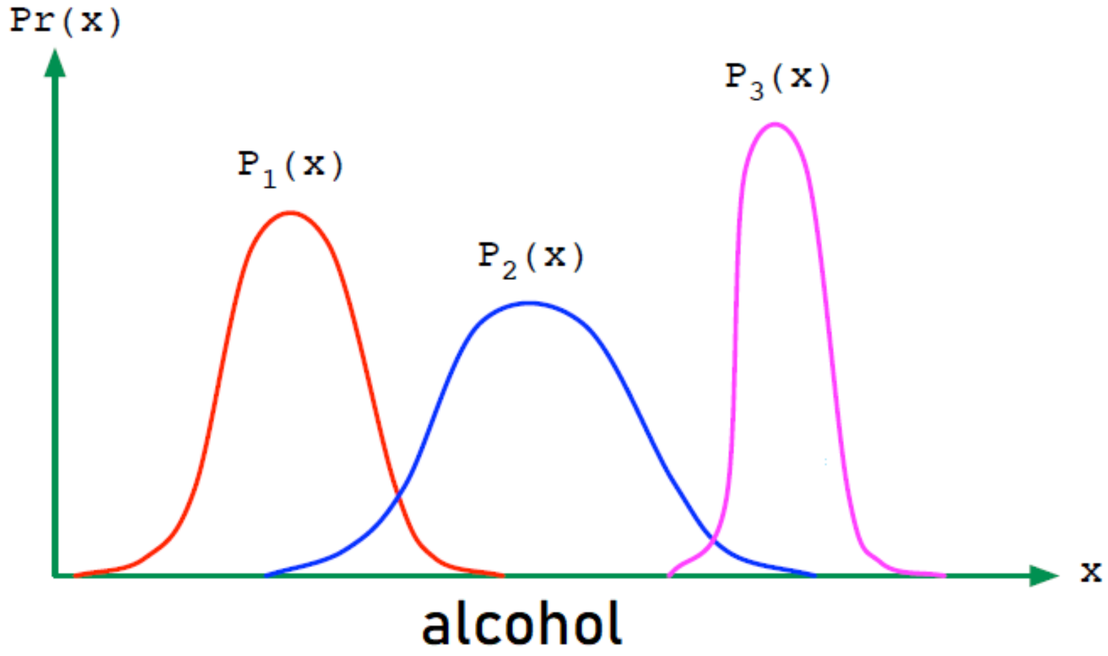
$$X \sim \mathcal{N}(\mu, \sigma^2) \qquad \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$$

$$X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$$

$$X_3 \sim \mathcal{N}(\mu_3, \sigma_3^2)$$





Univariate generative approach: baby steps

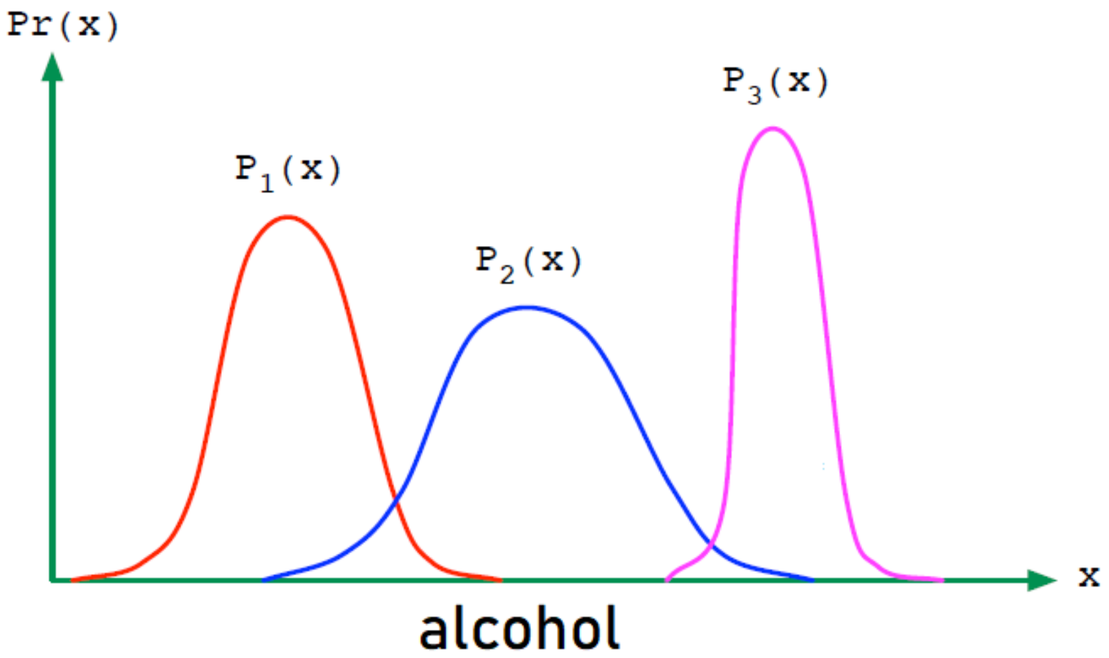
- After fitting 3 different Gaussian distributions

$$\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-1}{2}(\frac{x-\mu}{\sigma})^2}$$

$$X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$$

$$X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$$

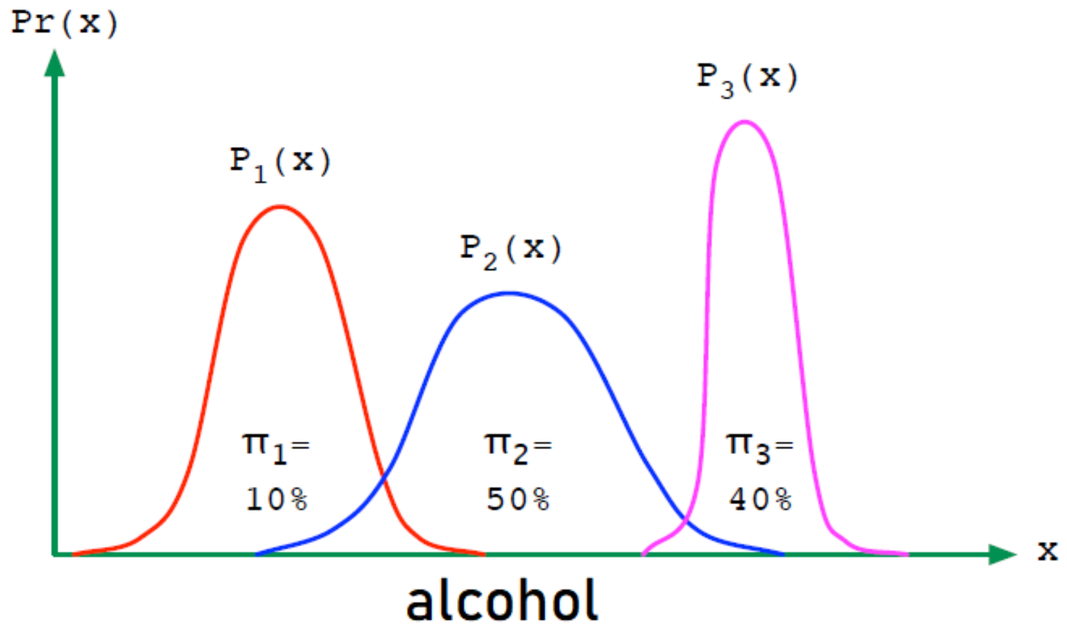
$$X_3 \sim \mathcal{N}(\mu_3, \sigma_3^2)$$



- For any new x-test, plugin x-test (alcohol) into equation
- Get likelihoods for 3 wineries. Find max of 3 likelihoods

- Aside: Why do we call this likelihood instead of Probability?

Univariate generative approach: Using the prior



- Is this enough?
  - Max of Pr1(x), Pr2(x) & Pr3(x)
- What about relative percentages?
- Consider the intersection of 2 distributions

- Solution

Prior

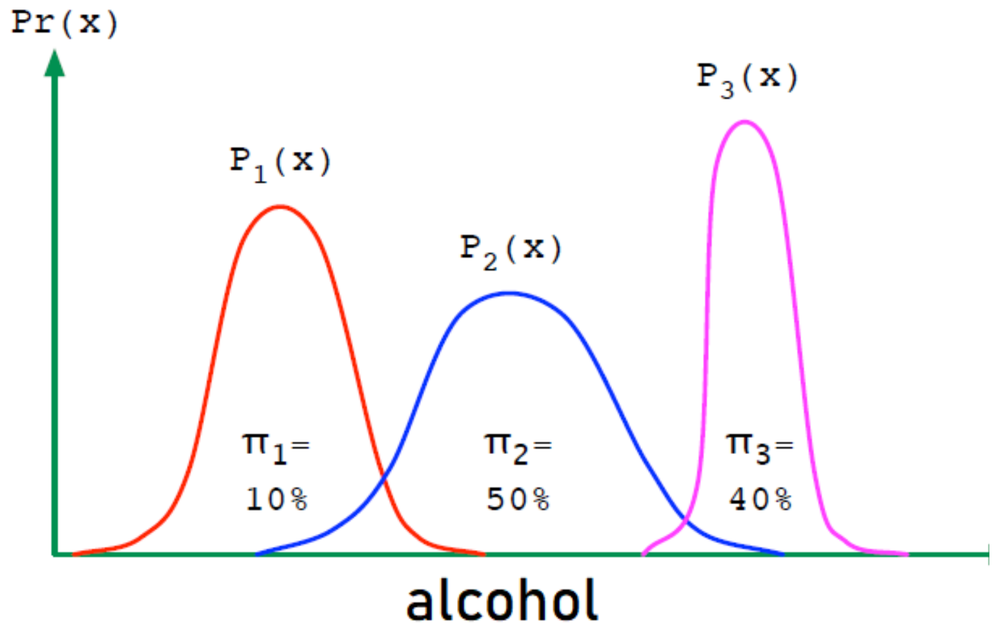
Likelihood

Max value of numerator

Posterior

$$\Pr(y = j|x) = \frac{\Pr(y = j)\Pr(x|y = j)}{\Pr(x)} = \frac{\pi_j P_j(x)}{\Pr(x)}$$

Univariate generative approach: results



- Test error = 29% with one feature
- How about we use 2 features?
- Same Bayes formula

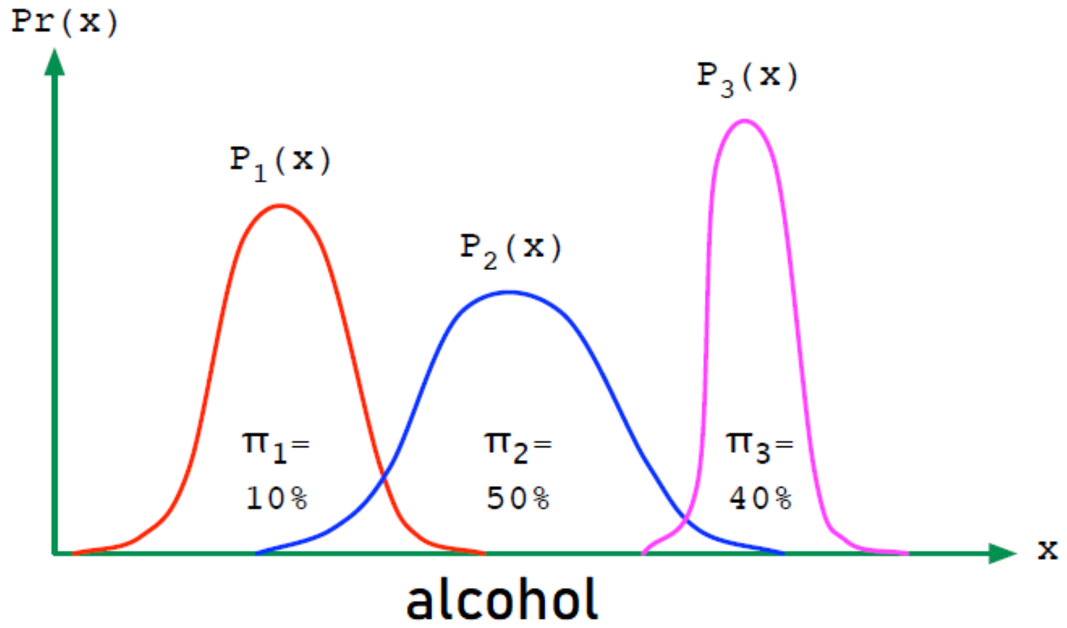
x is vector. [x1, x2, x3…]

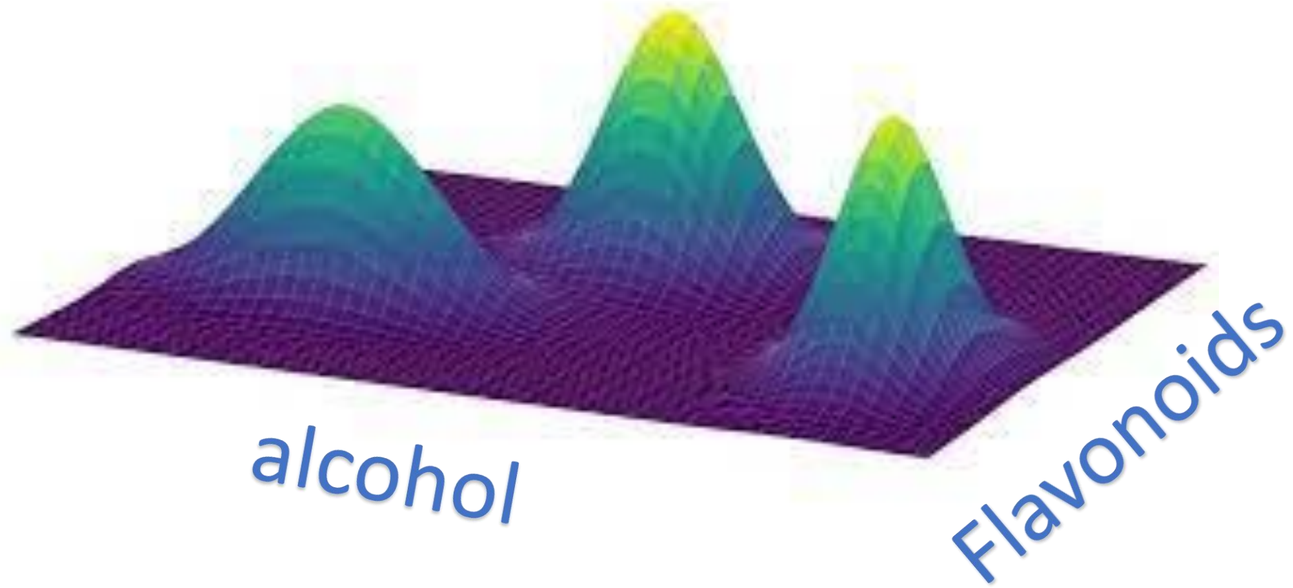
$$\Pr(y=j|x) = \frac{\Pr(y=j)\Pr(x|y=j)}{\Pr(x)} = \frac{\pi_j P_j(x)}{\Pr(x)}$$

- Fit bivariate Gaussian: x1 = Alcohol, x2 = Flavonoids

$$Pr(X) = \frac{1}{\sqrt{det(\Sigma)(2\pi)^D}} e^{\frac{-1}{2}(\mathbf{X}-\mu)^T\Sigma^{-1}(\mathbf{X}-\mu)} \qquad \Sigma = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{21} & \sigma_2^2 \end{bmatrix}$$

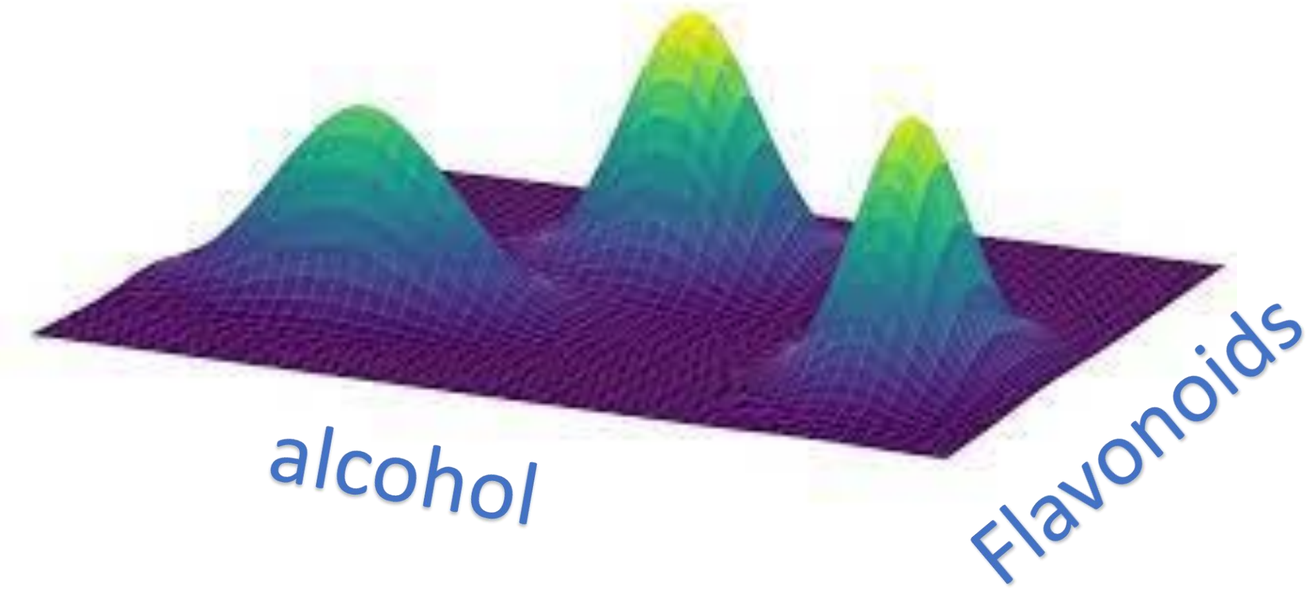
Bivariate generative approach

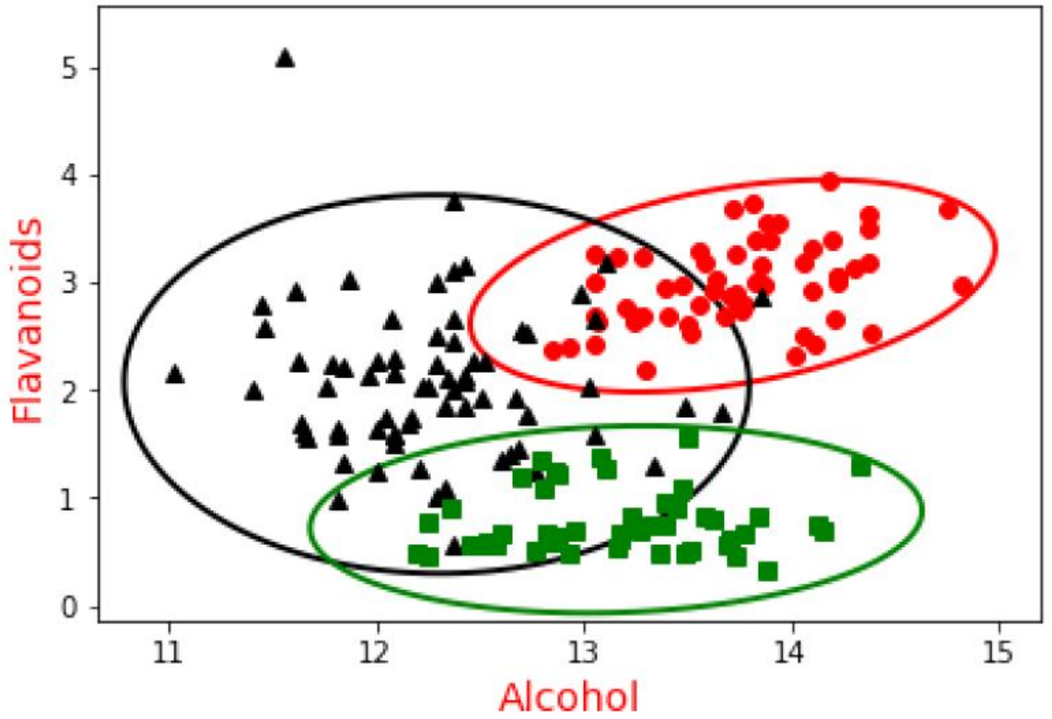




- Overlap seen in 1-D is considerably reduced in 2-D
- Better separation = Better predictions

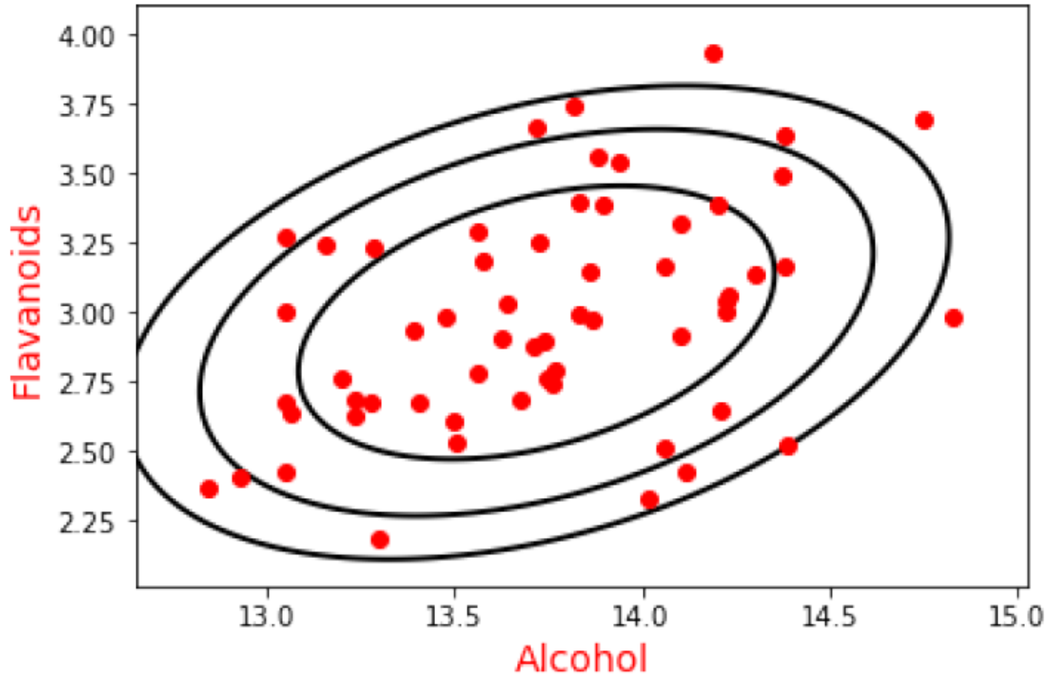
Bivariate generative approach (contd.)





- Same as Nearest Centroid model using Mahalanobis distance
- Adjusts for spread and covariance along different directions

Bivariate generative approach results



- Results: Error went from 29% to 8%
- Winery 1 class parameters

$$\mu = \begin{bmatrix} 13.7 \\ 3.0 \end{bmatrix} \quad \Sigma = \begin{bmatrix} 0.20 & 0.06 \\ 0.06 & 0.12 \end{bmatrix}$$

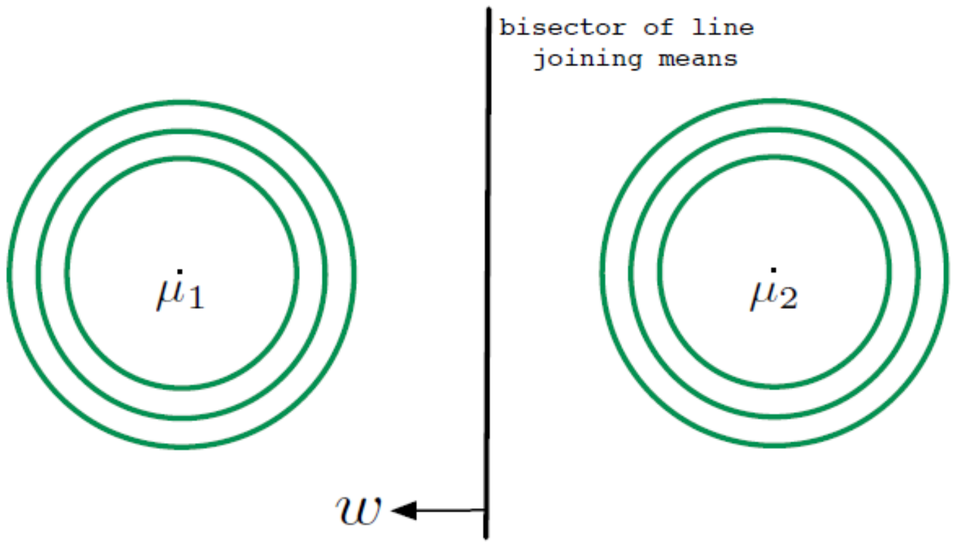
- Winery 2 & 3 class have different mean & covariance matrix
- How does the decision boundary look?

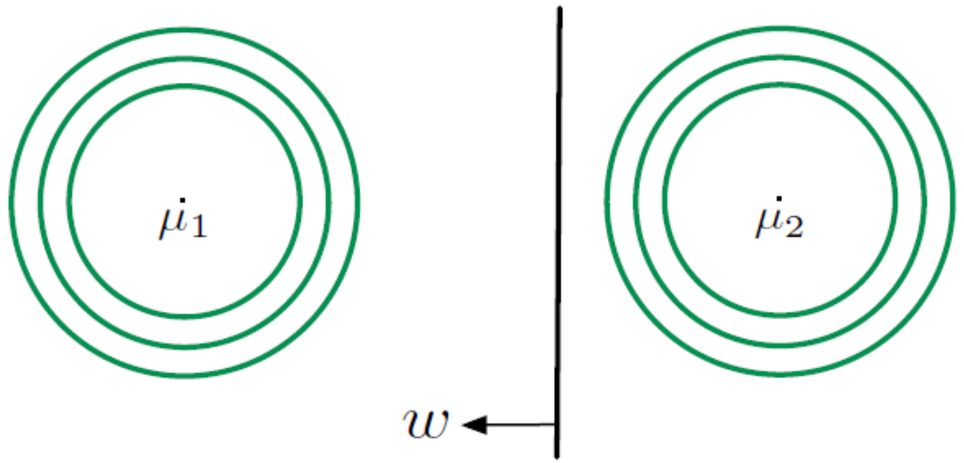
Bivariate generative: decision boundary

- Two class have same variance, same covariance
- Called: Spherical Gaussian

- Same covariance $\Sigma_1 = \Sigma_2$
- Same proportion $\pi_1 = \pi_2$

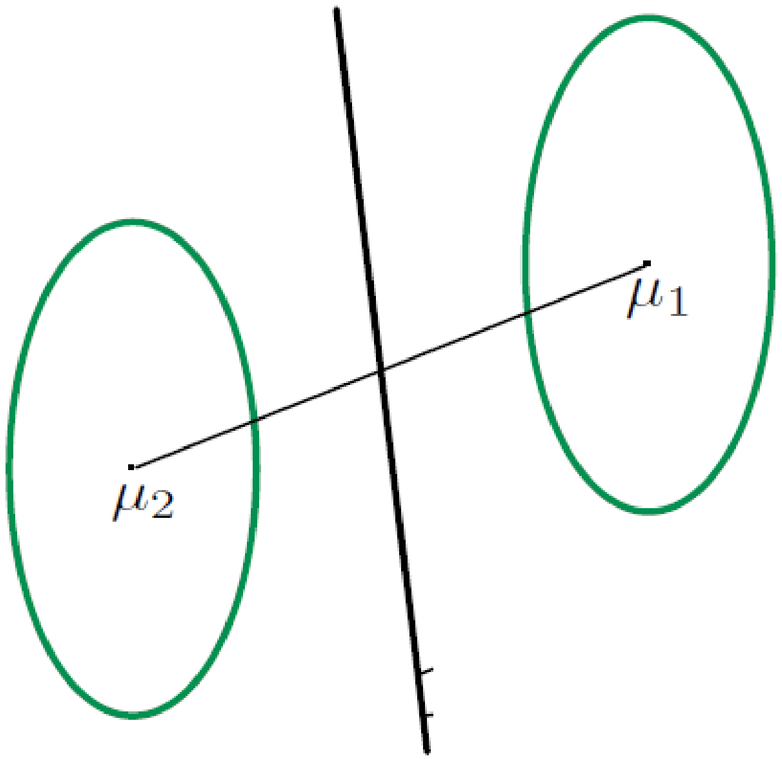
- Same covariance $\Sigma_1 = \Sigma_2$
- Diff proportion $\pi_1 > \pi_2$





Bivariate generative: decision boundary

- Two class have different variance, same covariance matrix
- Not Spherical Gaussian

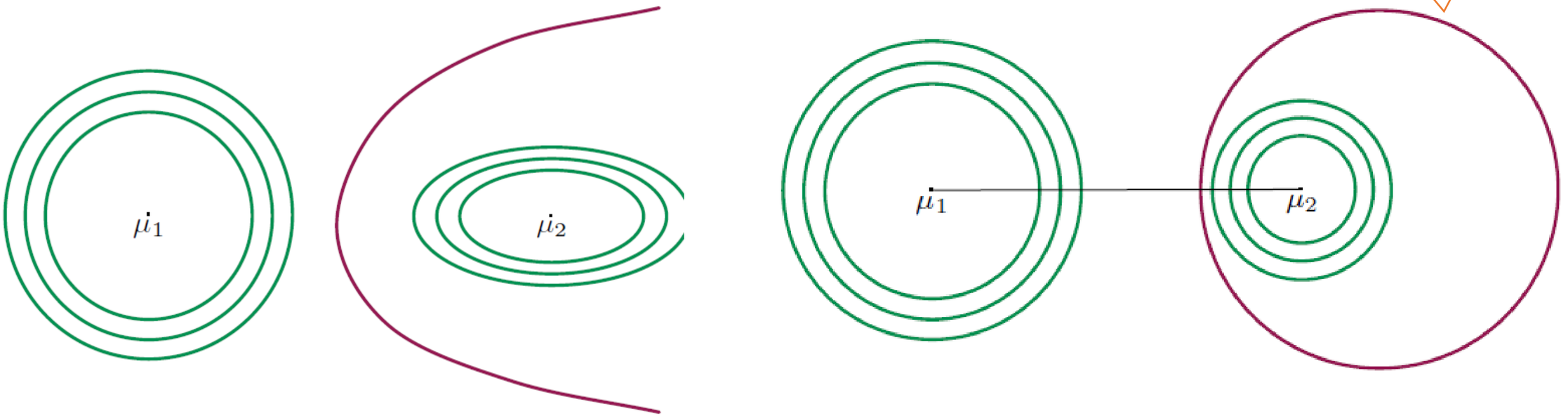


- All these linear boundaries seen till now come under Linear Discriminant Analysis (LDA)
- Second semester – MLPA subject

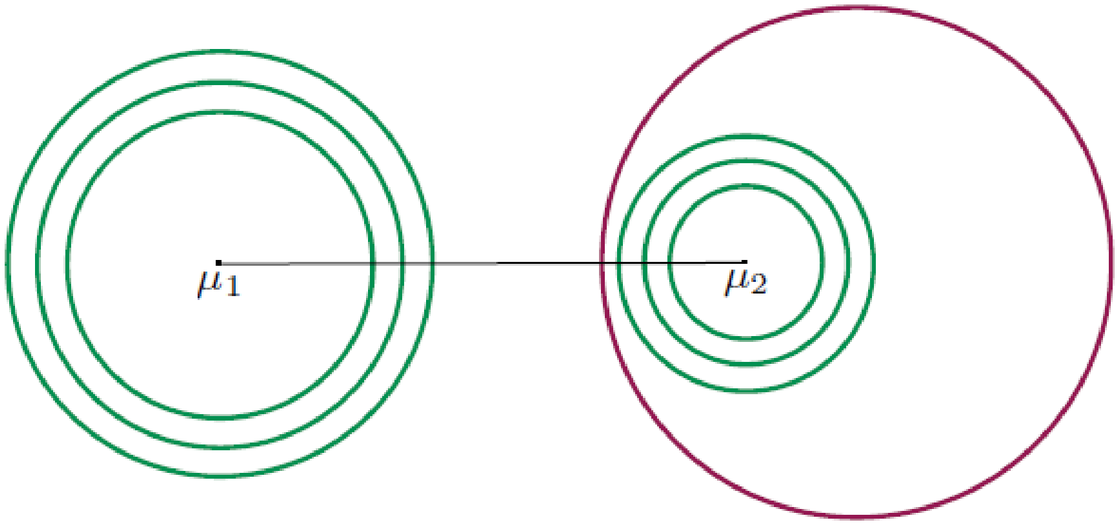
Bivariate generative: decision boundary

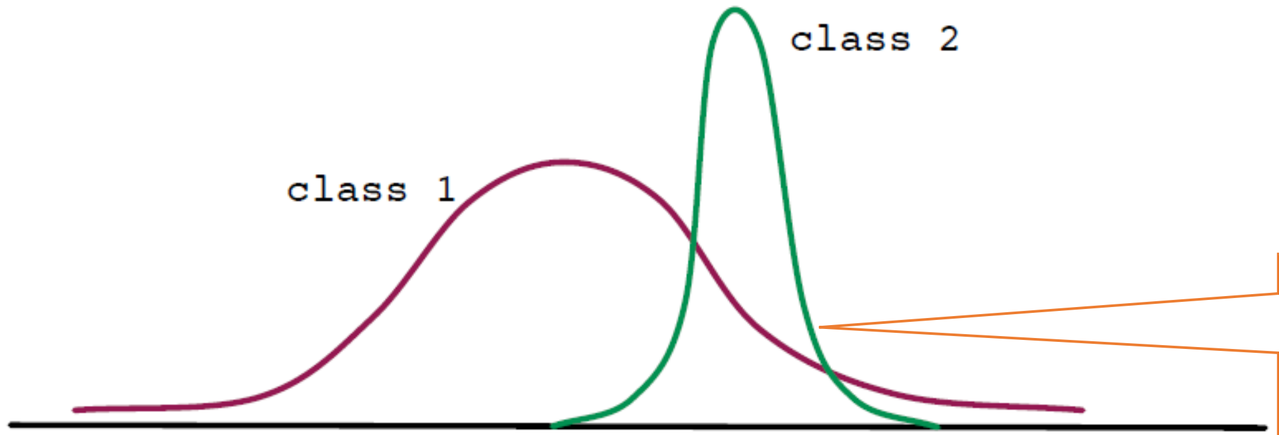
- Different covariance matrix. Quadratic decision boundary
- Quadratic Discriminant Analysis (QDA)
- Second semester – MLPA subject

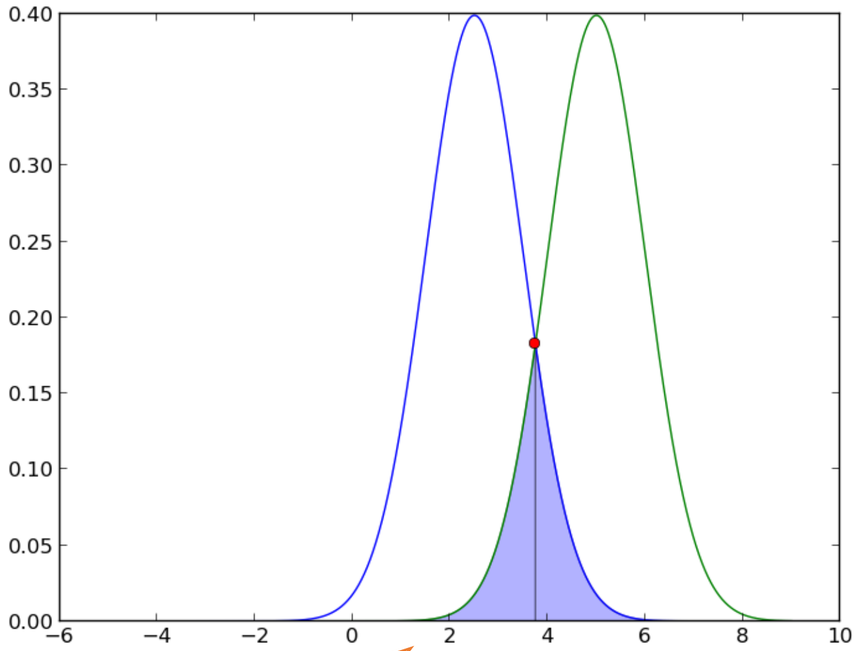
This is a bit odd. Why?



Bivariate generative: decision boundary



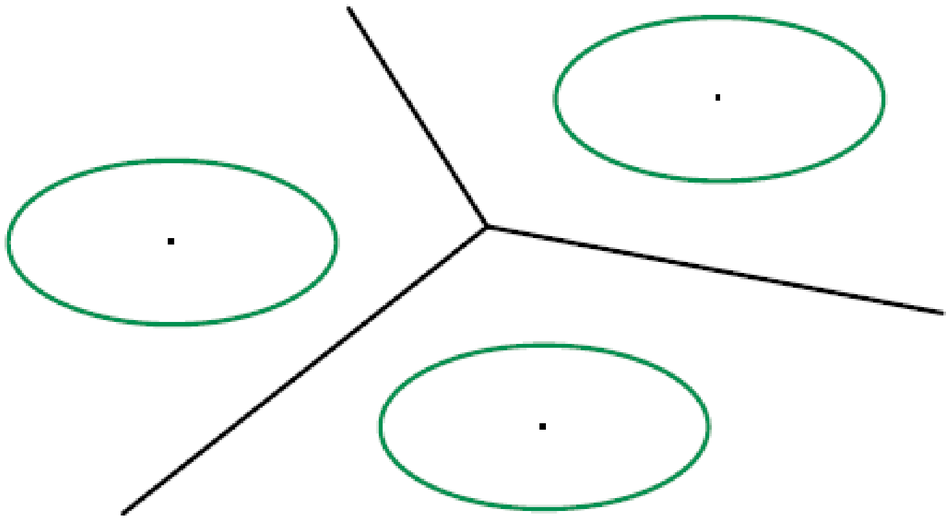




Ideal distribution for LR

If you try to fit a Logistic Regression (LR) to data like this, your performance is guaranteed to be bad

Multiclass decision boundary



Nearest Centroid v/s Gaussian comparison

Nearest Centroid	Gaussian
Eager	Eager
Batch	Batch
Parametric	Parametric
Discriminative	Generative

Naïve Bayes

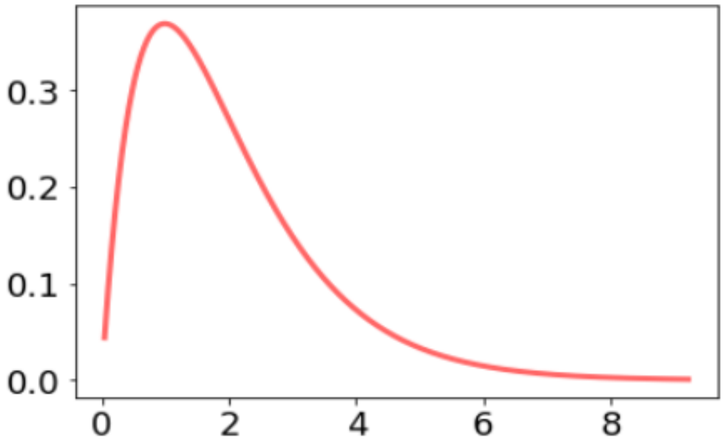
$$Pr(y|x) = \frac{Pr(x|y)Pr(y)}{Pr(x)}$$

$$Pr(y = j|x) = \frac{Pr(x|y = j)Pr(y = j)}{Pr(x)}$$

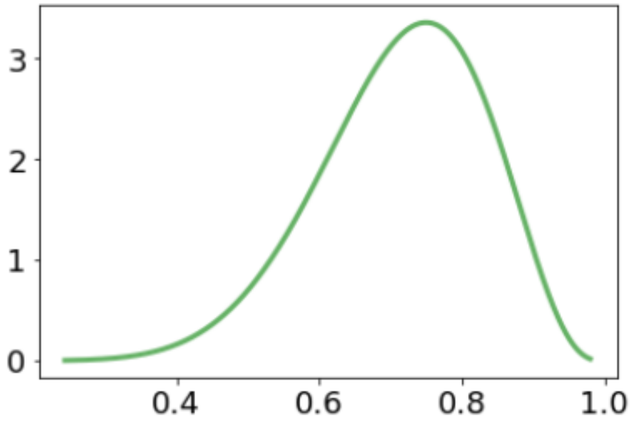
- When features are independent

$$Pr(y = j|x) = \frac{Pr(x_1|y = j)Pr(x_2|y = j)..Pr(x_n|y = j)Pr(y = j)}{Pr(x)}$$

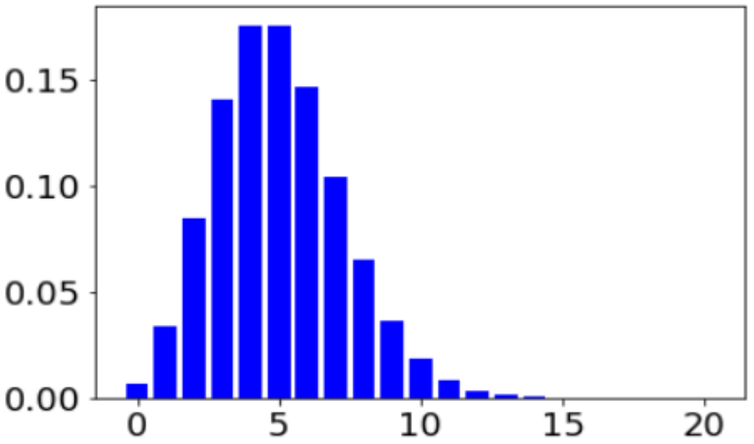
Other distributions



GAMMA



BETA



POISSON

It was the best of times, it was the worst of times, it was the age of wisdom, it was the age of foolishness, it was the epoch of belief, it was the epoch of incredulity, it was the season of Light, it was the season of Darkness, it was the spring of hope, it was the winter of despair, we had everything before us, we had nothing before us, we were all going direct to Heaven, we were all going direct the other way – in short, the period was so far like the present period, that some of its noisiest authorities insisted on its being received, for good or for evil, in the superlative degree of comparison only.

1	despair
2	evil
0	happiness
1	foolishness

CATEGORICAL

QUESTIONS

ThankYou!