Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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5.1. The state equation and the output equation of a control system are given below :

$$\dot{\mathbf{X}} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{U} \tag{5.1.1}$$

$$\mathbf{Y} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \mathbf{X} \tag{5.1.2}$$

Then transfer function representation of the system is

5.2. **Solution:** when

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \tag{5.2.1}$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \tag{5.2.2}$$

where A,B,C,D are matrices. Then the transfer function can be find using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.3}$$

From the given state space representation of the system, we can find matrices as

$$\mathbf{A} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.4}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.5}$$

$$\mathbf{C} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \tag{5.2.6}$$

We can find the transfer function using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.7}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.8}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s-4 & -1.5 \\ 4 & s \end{pmatrix} \tag{5.2.9}$$

$$|sI - \mathbf{A}| = s(s+4) - (-4) \times (-1.5) \tag{5.2.10}$$

$$|sI - \mathbf{A}| = s^2 + 4s + 6 \tag{5.2.11}$$

and from (5.2.9)

$$Adj\,[sI - \mathbf{A}] = \begin{pmatrix} s & -1.5 \\ 4 & s+4 \end{pmatrix} \tag{5.2.12}$$

$$[sI - \mathbf{A}]^{-1} = \frac{Adj[sI - \mathbf{A}]}{|sI - \mathbf{A}|} \tag{5.2.13}$$

$$= \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix}$$

$$[sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.14}$$

$$\therefore \quad [sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.15}$$

Substituting the values of $[sI - \mathbf{A}]^{-1}.\mathbf{B}$ *and* $\mathbf{C}$ *in equation*(1

$$T(s) = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.16}$$

$$T(s) = \left(\frac{6s}{(s^2+4s+6)} + \frac{10}{(s^2+4s+6)}\right) \tag{5.2.17}$$

the transfer function representation of the system is

$$\therefore \quad \mathbf{T(s)} = \left(\frac{6s+10}{(s^2+4s+6)}\right) \tag{5.2.18}$$

verify the answer with python code https://github.com/srikanth2001/EE2227-control-systems/tree/master/codes

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