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Control Systems

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CONTENTS 10 Oscillator 2 Abstract—This manual is an introduction to control 1 Signal Flow Graph systems based on GATE problems.Links to sample Python 1.1 Mason's Gain Formula . . . codes are available in the text. 1.2 Matrix Formula 1 Download python codes using 2 **Bode Plot** 1 svn co https://github.com/gadepall/school/trunk/ 2.1 Introduction 1 control/codes 2.2 1 Example 3 Second order System 1 1 Signal Flow Graph 3.1 Damping 1 1.1 Mason's Gain Formula 3.2 Example 1.2 Matrix Formula 2 Bode Plot 4 **Routh Hurwitz Criterion** 1 4.1 1 Routh Array 2.1 Introduction 4.2 Marginal Stability 2.2 Example Stability 4.3 1 3 Second order System 4.4 Example 1 3.1 Damping 5 State-Space Model 3.2 Example 5.1 Controllability and Observ-4 ROUTH HURWITZ CRITERION ability 1 4.1 Routh Array 5.2 Second Order System 1 4.2 Marginal Stability 5.3 1 Example 5.4 Example 2 4.3 Stability 4.4 Example 2 6 **Nyquist Plot** 5 STATE-SPACE MODEL 7 **Compensators** 5.1 Controllability and Observability 7.1 Phase Lead 2 5.2 Second Order System 7.2 Example 2 5.3 Example 5.1. The state equation and the output equation of 8 2 Gain Margin a control system are given below: 2 8.1 Introduction 8.2 2 Example $\dot{\mathbf{X}} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{U}$ (5.1.1)9 2 **Phase Margin** $Y = (1.5 \ 0.625) X$ (5.1.2)

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 $\mathbf{Y} = (1.5 \ 0.025) \mathbf{X}$ (5.1.2)

Then transfer function representation of the system is

5.2. **Solution:** when

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \tag{5.2.1}$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \tag{5.2.2}$$

where A,B,C,D are matrices. Then the transfer function can be find using

$$T(s) = \mathbf{C} \left[(sI - \mathbf{A})^{-1} \right] .\mathbf{B} + \mathbf{D}$$
 (5.2.3)

From the given state space representation of the system, we can find matrices as

$$\mathbf{A} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.4}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.5}$$

$$\mathbf{C} = (1.5 \quad 0.625) \tag{5.2.6}$$

We can find the transfer function using

$$T(s) = \mathbf{C} \left[(sI - \mathbf{A})^{-1} \right] .\mathbf{B} + \mathbf{D}$$
 (5.2.7)

$$(sI - \mathbf{A}) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix}$$
 (5.2.8)

$$(sI - \mathbf{A}) = \begin{pmatrix} s - 4 & -1.5 \\ 4 & s \end{pmatrix} \tag{5.2.9}$$

$$|sI - \mathbf{A}| = s(s+4) - (-4) \times (-1.5)$$
 (5.2.10)

$$|sI - \mathbf{A}| = s^2 + 4s + 6$$
 (5.2.11)

and from (5.2.9)

$$Adj[sI - \mathbf{A}] = \begin{pmatrix} s & -1.5 \\ 4 & s+4 \end{pmatrix}$$
 (5.2.12)

$$[sI - \mathbf{A}]^{-1} = \frac{Adj[sI - \mathbf{A}]}{|sI - \mathbf{A}|}$$

$$= \begin{pmatrix} \frac{s}{(s^2 + 4s + 6)} & \frac{-1.5}{(s^2 + 4s + 6)} \\ \frac{(s^2 + 4s + 6)}{(s^2 + 4s + 6)} & \frac{s + 4}{(s^2 + 4s + 6)} \end{pmatrix}$$
(5.2.13)

$$[sI - \mathbf{A}]^{-1} \cdot \mathbf{B} = \begin{pmatrix} \frac{s}{(s^2 + 4s + 6)} & \frac{-1.5}{(s^2 + 4s + 6)} \\ \frac{4}{(s^2 + 4s + 6)} & \frac{s + 4}{(s^2 + 4s + 6)} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$
(5.2.14)

$$. \dot{} . \qquad [sI - \mathbf{A}]^{-1} .\mathbf{B} = \begin{pmatrix} \frac{4s}{(s^2 + 4s + 6)} \\ \frac{16}{(s^2 + 4s + 6)} \end{pmatrix} (5.2.15)$$

Substituting the values of $[sI - A]^{-1}$.**B** and **C** in equation()

$$T(s) = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{1}{(s^2+4s+6)} \end{pmatrix}$$
 (5.2.16)

$$T(s) = \left(\frac{6s}{(s^2 + 4s + 6)} + \frac{10}{(s^2 + 4s + 6)}\right)$$
 (5.2.17)

the transfer function representation of the system is

. :
$$\mathbf{T}(\mathbf{s}) = \left(\frac{6s+10}{(s^2+4s+6)}\right)$$
 (5.2.18)

verify the answer with python code https://github.com/srikanth2001/EE2227control-systems/tree/master/codes

5.4 Example

6 Nyouist Plot

7 Compensators

7.2 Example

8 Gain Margin

8.2 Example

9 Phase Margin

10 OSCILLATOR