Control Systems

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Contents

***Abstract*—This manual is an introduction to control systems in feedback circuits. Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/feedback/codes
```

1 Feedback Voltage Amplifier: Series-Shunt

2 Feedback Current Amplifier: Shunt-Series

2.1 Ideal Case

2.1. The feedback current amplifier in fig.2.1 can be thought of as a “super” CG transistor. Note that rather than connecting the gate of Q_2 to signal ground, an amplifier is placed between source and gate.
for the fig.2.1 , the parameter's table is TABLE.2.1

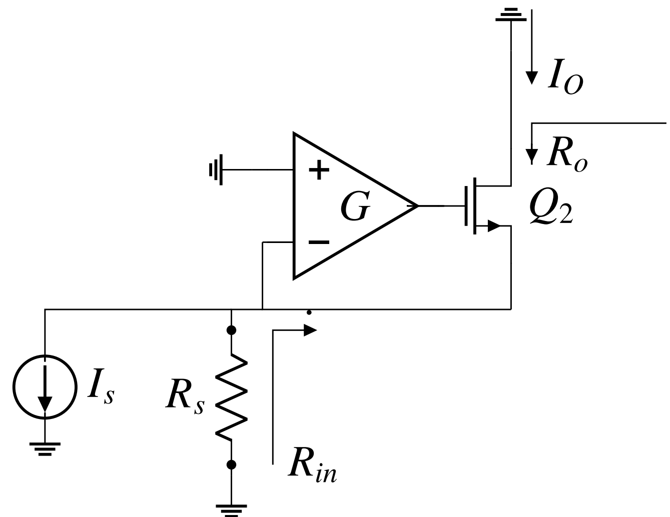

Fig. 2.1

Parameter	Value
input resistance(large μ)	0
output resistance(large μ)	∞
Input voltage	$-I_sR_s$
input resistance(finite μ)	R_s
output resistance(finite μ)	r_o
source resistance	R_s
feedback factor H	**1**
Open Loop Gain, G	$\mu g_m R_s\ A/A$
Closed Loop Gain, T	**1 A/A**

TABLE 2.1

2.2. (a) If μ is very large, what is the signal voltage at the input terminal?What is the input resistance?What is the current gain Io/Is ?

Solution:
Refer to the fig. 2.1 for the feedback current amplifier circuit, in this super common gate transistor is connected between the gate and source terminals of the MOSFET.
Replace the op-amp with its equivalent modal and replace the MOSFET with its small signal equivalent circuit.
with reference to the fig,2.3. For ideal op-amp,the input resistance(R_{id}) is very high (infinite) . And the drain current is approximately equal to source current

$$I_D \cong I_S \tag{2.2.1}$$

The closed loop gain of op-amp is

$$T = \frac{\mu}{1 + \mu H} \tag{2.2.2}$$

for larger value of closed loop gain , open loop gain 'μ' will be large. from fig.2.3 we

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can observe that input voltage is,

$$V_{in} = -R_s I_s \tag{2.2.3}$$

Since the drain current is approximately is equal to source current, And the current flowing through resistor R_S is I_s since there is no current flowing through the negative terminal of op-amp. Therefore, it is connected that the output current I_O is flowing through the resister R_s, then

$$I_o = I_S \tag{2.2.4}$$

$$\frac{I_o}{I_s} \equiv 1$$

therefore the current gain is,

$$\frac{I_o}{I_s} = 1\ A/A \tag{2.2.5}$$

and from fig.2.3 the input Resistance is equal to R_{id}

$$R_i = R_{id} \tag{2.2.6}$$

2.3. (b) For finite μ but assuming that the input resistance of the amplifier g is very large, find the 'G' circuit and derive expressions for G, Ri, and R_o.?
Solution:
For the finite value of μ and the input resistance of a ideal op-amp is very high(infinite). the open loop amplifier circuit ('G' circuit) fig.2.3

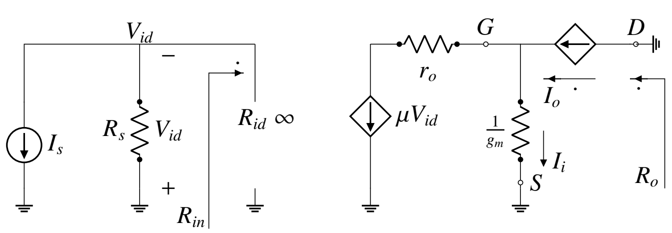

Fig. 2.3

For a ideal op-amp the output Resistance is very small, so we can neglect the resistance r_o. from fig2.3

$$V_{id} = I_s R_s \tag{2.3.1}$$

current through resistance R_s is

$$since \ \ I_o \equiv I_s$$

$$I_o = \frac{\mu V_{id}}{\frac{1}{g_m}} \tag{2.3.2}$$

$$I_o = \mu g_m I_s R_S \tag{2.3.3}$$

Expression for current gain G is:

$$\frac{I_o}{I_s} = \mu g_m R_s \tag{2.3.4}$$

$$G = \mu g_m R_s \tag{2.3.5}$$

from the fig.2.3

$$since \ \ R_{id} = \infty$$

the input resistance:

$$R_I = R_s \tag{2.3.6}$$

the output Resistance:

$$R_o = r_o \tag{2.3.7}$$

2.4. (c) What is the value of H?
Solution:
closed loop gain of the op-amp is:

$$T = \frac{\mu}{1 + \mu H} \tag{2.4.1}$$

$$since \ \ \mu H >> 1$$

then

$$T = \frac{\mu}{\mu H} \tag{2.4.2}$$

for the larger value of μ

$$T \implies 1$$

the value of 'H' will be:

$$H = 1 \tag{2.4.3}$$

2.5. (d) Find GH and T . If μ is large, what is the value of T ?
Solution:
from eq.2.3.4 and from eq.2.4.3

$$G = \mu g_m R_s \ \ , \quad H = 1$$

value of GH:

$$GH = \mu g_m R_s \tag{2.5.1}$$

closed loop gain of op-amp is:

$$T = \frac{G}{1+GH} \tag{2.5.2}$$

substitute values of G and GH in eq.2.5.2

$$T = \frac{\mu g_m R_s}{1+\mu g_m R_s} \tag{2.5.3}$$

for the larger valve of μ

$$T \implies 1 \tag{2.5.4}$$

2.6. (e) Find R_{in} and R_{out} assuming the loop gain is large
Solution:
for a Feedback Amplifier

$$R_{if} = \frac{R_i}{1+GH} \tag{2.6.1}$$

Here, $R_i = R_s$
substituting the values of GH and R_i in eq.2.6.1 we get:

$$R_{if} = \frac{R_s}{1+\mu g_m R_s} \tag{2.6.2}$$

dividing the eq.2.6.2 with R_s

$$R_{if} = \frac{1}{\frac{1}{R_s}+\mu g_m} \tag{2.6.3}$$

the eq.2.6.3 can also written like

$$R_{in} = R_s \parallel \frac{1}{\mu g_m} \tag{2.6.4}$$

$$R_{if} = R_s \parallel R_{in} \tag{2.6.5}$$

from the equations 2.6.4 and 2.6.5 R_{in} ca be written as:

$$R_{in} = \frac{1}{\mu g_m} \tag{2.6.6}$$

for the lager value of μ then R will be small

$$for \quad \mu \implies \infty$$

then R_{in} becomes:

$$R_{in} = 0 \tag{2.6.7}$$

for a Feedback Amplifier output Resistance is:

$$R_{out} = (1+GH)R_o \tag{2.6.8}$$

From the figure Fig.2.3

$$R_o = \frac{1}{g_m} \tag{2.6.9}$$

substitute the values of GH and R_o in eq.2.6.8 we get:

$$R_{out} = (1+\mu g_m R_s)\frac{1}{g_m} \tag{2.6.10}$$

$$R_{out} = \frac{1}{g_m} + \mu R_s \tag{2.6.11}$$

By observing the eq.2.6.11 , for the lager value for μ we will have larger value for R_{out}

$$\mu \implies \infty$$

$$R_{out} \implies \infty \tag{2.6.12}$$

2.7. (f) The “super” CG transistor can be utilized in the cascode configuration shown in Fig.2.7, where V_G is a dc bias voltage. Replacing Q1 by its small-signal model, use the analogy of the resulting circuit to that in Fig.2.1 to find Io and Rout.

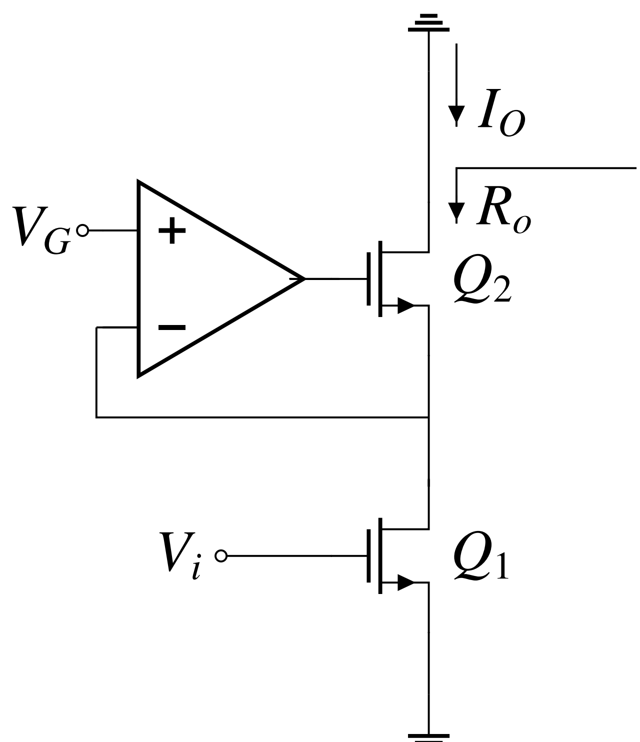

Fig. 2.7

parameter table for fig.2.7 is TABLE.2.7

Parameter	Value
output resistance(large μ)	$\mu g_{m2} R_s R_o$
Input voltage	I_s/g_{m1}
output current I_o	$g_{m1} V_i$
feedback factor H	**1**
Open Loop Gain, G	$\mu g_{m2} R_s \ A/A$
Closed Loop Gain, T	**1 A/A**

TABLE 2.7

2.8. **Solution:**
Refer to the fig.2.7 for the cascode configuration in which the super CG transistor is used. the small signal equivalent circuit is shown in fig.2.8,

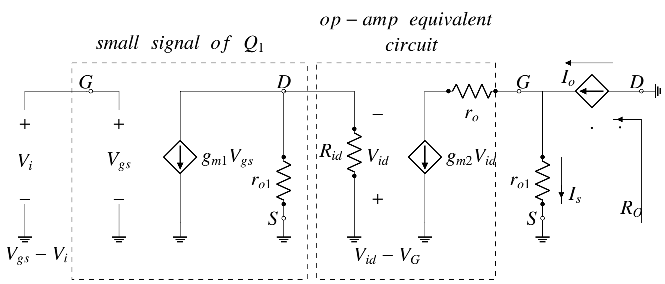

Fig. 2.8

from the fig.2.8 clearly observed that the current I_s

$$I_s = g_{m1} V_{gs} \tag{2.8.1}$$

the voltage V_i is applied at the gate terminal of the transistor Q_1 , therefore the gate source voltage becomes V_i :

$$V_i = V_{gs} \tag{2.8.2}$$

form the eq.2.8.2 and eq.2.8.1

$$I_s = g_{m1} V_i \tag{2.8.3}$$

the closed loop gain of the op-amp is:

$$T = \frac{G}{1 + GH} \tag{2.8.4}$$

from the questions Q.2.3 and Q.2.4 we will get:

$$G = \mu g_{m2} R_s \tag{2.8.5}$$

$$H \implies 1 \tag{2.8.6}$$

substitute the values of H and G in eq.2.8.4

$$T = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s} \tag{2.8.7}$$

$$since \quad T = \frac{I_o}{I_s}$$

$$\frac{I_o}{I_s} = \frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s} \tag{2.8.8}$$

$$I_o = I_s \left[\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s} \right] \tag{2.8.9}$$

from eq.2.8.3 substitute the value of I_s in eq.2.8.9

$$I_o = g_{m1} V_i \left[\frac{\mu g_{m2} R_s}{1 + \mu g_{m2} R_s} \right] \tag{2.8.10}$$

$$since \ \ \mu g_{m2} R_s >> 1$$

$$I_o = g_{m1} V_i \left[\frac{\mu g_{m2} R_s}{\mu g_{m2} R_s} \right] \tag{2.8.11}$$

$$I_o = g_{m1} V_i \tag{2.8.12}$$

the expression for the output current is:

$$I_o = g_{m1} V_i \tag{2.8.13}$$

the output amplifier is:

$$R_{out} = (1 + GH) R_o \tag{2.8.14}$$

from the question Q.2.5 we will get:

$$GH = \mu g_{m2} R_s \tag{2.8.15}$$

$$R_{out} = (1 + \mu g_{m2} R_s) R_o \tag{2.8.16}$$

$$R_{out} \equiv \mu g_{m2} R_s R_o \tag{2.8.17}$$

therefore the expression for output resistance is :

$$R_{out} = \mu g_{m2} R_s R_o \tag{2.8.18}$$

2.2 Practical Case