Control Systems

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***Abstract*—This manual is an introduction to control systems based on GATE problems.Links to sample Python codes are available in the text.**

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
    control/codes
```

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5.1. The state equation and the output equation of a control system are given below :

$$\dot{\mathbf{X}} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{U} \tag{5.1.1}$$

$$\mathbf{Y} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \mathbf{X} \tag{5.1.2}$$

Then transfer function representation of the system is

5.2. **Solution:** when

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU} \tag{5.2.1}$$

$$\mathbf{Y} = \mathbf{CX} + \mathbf{DU} \tag{5.2.2}$$

where A,B,C,D are matrices. Then the transfer function can be find using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.3}$$

From the given state space representation of the system, we can find matrices as

$$\mathbf{A} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.4}$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.5}$$

$$\mathbf{C} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \tag{5.2.6}$$

We can find the transfer function using

$$T(s) = \mathbf{C}\left[(sI - \mathbf{A})^{-1}\right].\mathbf{B} + \mathbf{D} \tag{5.2.7}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \tag{5.2.8}$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s-4 & -1.5 \\ 4 & s \end{pmatrix} \tag{5.2.9}$$

$$|sI - \mathbf{A}| = s(s+4) - (-4) \times (-1.5) \tag{5.2.10}$$

$$|sI - \mathbf{A}| = s^2 + 4s + 6 \tag{5.2.11}$$

and from (5.2.9)

$$Adj\,[sI - \mathbf{A}] = \begin{pmatrix} s & -1.5 \\ 4 & s+4 \end{pmatrix} \tag{5.2.12}$$

$$[sI - \mathbf{A}]^{-1} = \frac{Adj[sI - \mathbf{A}]}{|sI - \mathbf{A}|} \tag{5.2.13}$$

$$= \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix}$$

$$[sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5.2.14}$$

$$\therefore \quad [sI - \mathbf{A}]^{-1}.\mathbf{B} = \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.15}$$

$$Substituting\, the\, values\, of\ [sI - \mathbf{A}]^{-1}.\mathbf{B}\ and\ \mathbf{C}\ in\ equation$$

$$T(s) = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \tag{5.2.16}$$

$$T(s) = \left(\frac{6s}{(s^2+4s+6)} + \frac{10}{(s^2+4s+6)} \right) \tag{5.2.17}$$

the transfer function representation of the system is

$$\therefore \quad \mathbf{T(s)} = \left(\frac{6s+10}{(s^2+4s+6)} \right) \tag{5.2.18}$$

verify the answer with python code https://github.com/srikanth2001/EE2227-control-systems/tree/master/codes

5.4 Example

6 Nyquist Plot

6.1. Using the Nyquist criterian , find out whether the system is stable or not

$$G(s) = \frac{20}{s(s+1)} \tag{6.1.1}$$

$$H(s) = \frac{s+3}{s+4} \tag{6.1.2}$$

6.2. **Solution:**

$$G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \tag{6.2.1}$$

$$= \frac{20s + 60}{s^3 + 5s^2 + 4s}$$

$$1 + G(s)H(s) = \frac{s^3 + 5s^2 + 24s + 60}{s^3 + 5s^2 + 4s} \tag{6.2.2}$$

6.3. Nyquist Stability Criterion can be expressed as:

$$Z = N + P \tag{6.3.1}$$

Where:

Z = number of roots of 1+G(s)H(s) in right-hand side (RHS) of s-plane (It is also called zeros of characteristics equation)

N = number of encirclement of critical point 1+j0 in the clockwise direction

P = number of poles of open loop transfer function (OLTF) [i.e. G(s)H(s)] in RHS of s-plane.

Z=N+P is valid for all the systems whether stable or unstable. For the stable system, Z=0, So for the stable system N = –P.

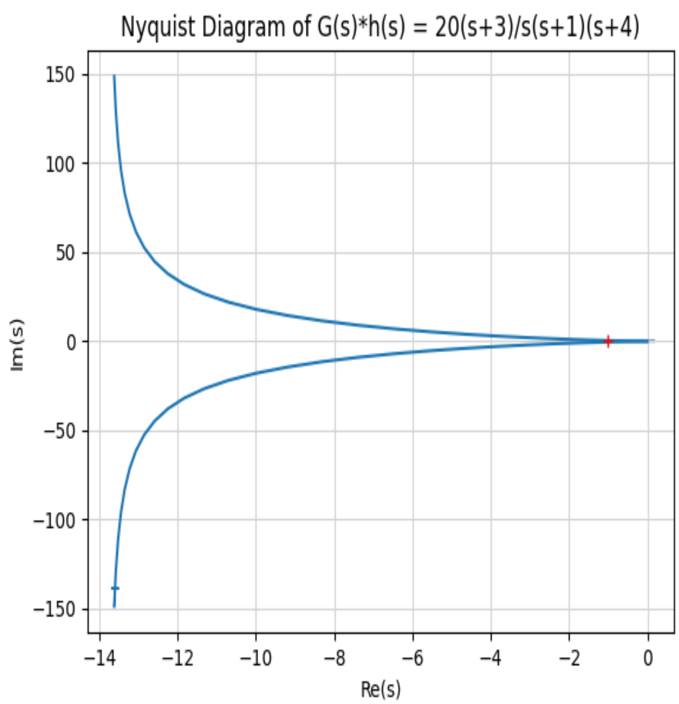
if p = 0

there will be no Encirclement of Nyquist plot and the system is stable

$$G(s)H(s) = \frac{20(s+3)}{s(s+1)(s+4)} \tag{6.3.2}$$

$$Here\, P = 0 \tag{6.3.3}$$

$$Then\, N = 0 \tag{6.3.4}$$



by seeing the we conclude that N = 0 and P = 0

$$hence\ the\ systen\ is\ stable \tag{6.3.5}$$

verify the answer with python code https://github.com/srikanth2001/EE2227-control-systems/tree/master/codes

7 Compensators

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