

Control Systems

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Abstract—This manual is an introduction to control systems based on GATE problems. Links to sample Python codes are available in the text.

Download python codes using

```
svn co https://github.com/gadepall/school/trunk/
control/codes
```

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1 MASON'S GAIN FORMULA

2 BODE PLOT

2.1 Introduction

2.2 Example

3 SECOND ORDER SYSTEM

3.1 Damping

3.2 Example

4 ROUTH HURWITZ CRITERION

4.1 Routh Array

4.2 Marginal Stability

4.3 Stability

5 STATE-SPACE MODEL

5.1 Controllability and Observability

5.1. The state equation and the output equation of a control system are given below :

$$\dot{\mathbf{X}} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \mathbf{X} + \begin{pmatrix} 4 \\ 0 \end{pmatrix} \mathbf{U} \quad (5.1.1)$$

$$\mathbf{Y} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \mathbf{X} \quad (5.1.2)$$

Then transfer function representation of the system is

5.2. **Solution:** when

$$\dot{\mathbf{X}} = \mathbf{A}\mathbf{X} + \mathbf{B}\mathbf{U} \quad (5.2.1)$$

$$\mathbf{Y} = \mathbf{C}\mathbf{X} + \mathbf{D}\mathbf{U} \quad (5.2.2)$$

where $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are matrices. Then the transfer function can be find using

$$T(s) = \mathbf{C} \left[(s\mathbf{I} - \mathbf{A})^{-1} \right] \mathbf{B} + \mathbf{D} \quad (5.2.3)$$

From the given state space representation of the system, we can find matrices as

$$\mathbf{A} = \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \quad (5.2.4)$$

$$\mathbf{B} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (5.2.5)$$

the transfer function representation of the system is

$$\mathbf{C} = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \quad (5.2.6)$$

We can find the transfer function using

$$T(s) = \mathbf{C}[(sI - \mathbf{A})^{-1}] \cdot \mathbf{B} + \mathbf{D} \quad (5.2.7)$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} -4 & -1.5 \\ 4 & 0 \end{pmatrix} \quad (5.2.8)$$

$$(sI - \mathbf{A}) = \begin{pmatrix} s - 4 & -1.5 \\ 4 & s \end{pmatrix} \quad (5.2.9)$$

$$|sI - \mathbf{A}| = s(s + 4) - (-4) \times (-1.5) \quad (5.2.10)$$

$$|sI - \mathbf{A}| = s^2 + 4s + 6 \quad (5.2.11)$$

and from (1.2.9)

$$\text{Adj}[sI - \mathbf{A}] = \begin{pmatrix} s & -1.5 \\ 4 & s + 4 \end{pmatrix} \quad (5.2.12)$$

$$[sI - \mathbf{A}]^{-1} = \frac{\text{Adj}[sI - \mathbf{A}]}{|sI - \mathbf{A}|} \quad (5.2.13)$$

$$= \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix}$$

$$[sI - \mathbf{A}]^{-1} \cdot \mathbf{B} = \begin{pmatrix} \frac{s}{(s^2+4s+6)} & \frac{-1.5}{(s^2+4s+6)} \\ \frac{4}{(s^2+4s+6)} & \frac{s+4}{(s^2+4s+6)} \end{pmatrix} \begin{pmatrix} 4 \\ 0 \end{pmatrix} \quad (5.2.14)$$

$$\therefore [sI - \mathbf{A}]^{-1} \cdot \mathbf{B} = \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \quad (5.2.15)$$

Substituting the values of $[sI - \mathbf{A}]^{-1} \cdot \mathbf{B}$ and \mathbf{C} in equation(1.2.7)

$$T(s) = \begin{pmatrix} 1.5 & 0.625 \end{pmatrix} \begin{pmatrix} \frac{4s}{(s^2+4s+6)} \\ \frac{16}{(s^2+4s+6)} \end{pmatrix} \quad (5.2.16)$$

$$T(s) = \left(\frac{6s}{(s^2+4s+6)} + \frac{10}{(s^2+4s+6)} \right) \quad (5.2.17)$$

$$\therefore \mathbf{T}(s) = \left(\frac{6s+10}{(s^2+4s+6)} \right) \quad (5.2.18)$$

5.3.

5.4. verify the answer with python code
<https://github.com/srikanth2001/EE2227-control-systems/tree/master/codes>

5.2 Second Order System

6 NYQUIST PLOT

7 PHASE MARGIN

8 GAIN MARGIN

9 COMPENSATORS

9.1 Phase Lead

10 OSCILLATOR