EE3025 Presentation

K Srikanth - EE18BTECH11023

Indian Institute of Technology Hyderabad

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Outline

Question

Assignment-01

- Let

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\}$$

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

  - Compute X(k), H(k) and y(n) using FFT and IFFT
  - Wherever possible, express all the above equations as matrix equations.

Assignment-02

- Filter Design
  - Filter Design Specifications
  - The Bilinear Transform: I
  - The FIR Filter

Computing X(k), H(k) Using FFT

- input signal x(n)

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\}$$

- Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

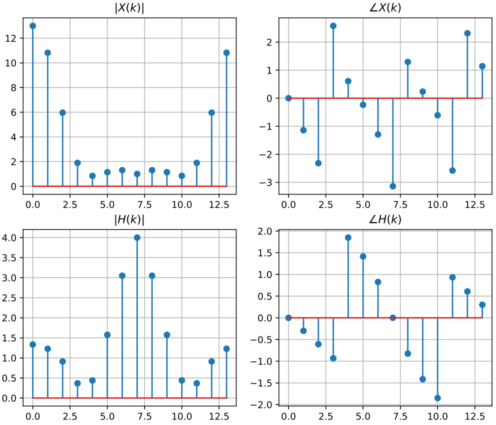
- FFT of a Input Signal $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1 \ldots N-1$$

- FFT of a Impulse Response $h(n)$ is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

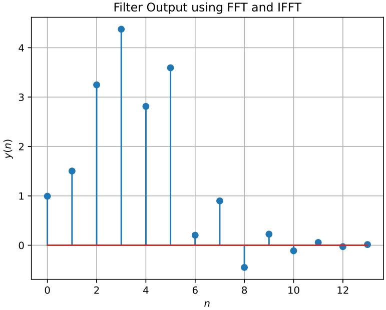
Plots of Magnitude and Phase of X(k) and H(k)



Filter Output Using FFT and IFFT

- $y(n)$ can be computed by doing IFFT for $Y(k)$

$$y(n) = \frac{1}{N} \sum_{n=0}^{N-1} Y(k) e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1$$



expressing all the above equations as matrix equations

- FFT of signal X(n)

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

- Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be $\mathrm{ex}-$ pressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

Computing X(K) from Matrix Equations

- On solving we get,

$$\implies X(0) = 13 + 0j,$$

$$X(1) = -4 - 1.732j,$$

$$X(2) = 1 + 0j$$

$$X(3) = -1 + 0j,$$

$$X(4) = 1 + 0j,$$

$$X(5) = -4 + 1.732j$$

- the impluse

$$h(n) = \frac{-1}{2}^{n} u(n) + \frac{-1}{2}^{n-2} u(n-2)$$

assuming that length of h(n) is same as length of x(n) i.e., N = 6. Similarly like x(n), solving h(n) using matrix method for each value of k we get,

Computing H(K) from Matrix Equations

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5) \\ h(0)+h(1)e^{-j\pi/3}+...+h(5)e^{-j5\pi/3} \\ h(0)+h(1)e^{-2j\pi/3}+...+h(5)e^{-2j5\pi/3} \\ h(0)+h(1)e^{-3j\pi/3}+...+h(5)e^{-3j5\pi/3} \\ h(0)+h(1)e^{-4j\pi/3}+...+h(5)e^{-4j5\pi/3} \\ h(0)+h(1)e^{-5j\pi/3}+...+h(5)e^{-5j5\pi/3} \end{bmatrix}$$

On solving we get

H(0) = 1.28125 + 0j , H(1) = 0.51625 - 0.5141875j ,
H(2) = -0.078125 + 1.1095625j , H(3) = 3.84375 + 0j ,
H(4) = -0.071825 - 1.1095625j , H(5) = 0.515625 + 0.5141875j

Computing Y(K) from Matrix Equations

We can now compute Y(k) using below equation

$$Y(k) = X(k)H(k)$$

So, Y(k) is obtained element wise multiplication of X(k) and H(k)

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix}$$

Computinh y(n) from IFFT of Y matrix

y(n) is given by IFFT of Y matrix. this y(n) can be calculated by a python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 + 0j \\ 2.28125071 + 0j \\ 2.6250019 - 1.11022302 \times 10^{-16} j \\ 4.37499667 - 1.47104551 \times 10^{-15} j \\ 2.6562481 + 6.10622664 \times 10^{-16} j \\ 3.59375262 - 1.60982339 \times 10^{-15} j \end{bmatrix}$$

Filter Design Specifications

- **Tolerances:**
  The magnitude of resonance in Pass-band is called Pass-band Tolerance and similarly magnitude of resonance in Stop-band is called Stop-band Tolerance.
- **Passband:**
  The Frequency band within which signals are transmitted by filter without attenuation.
- **Stopband:**
  The Frequency band within which signals are not transmitted by filter or with a large attenuation.

The Bilinear Transform

It is a technique in Signal Processing that is used to Continuous-Time System Representations to Discrete-Time System Representations and vice-versa. The bilinear transform is a first-order approximation of the natural logarithm function that is an exact mapping of the z-plane to the s-plane. When the Laplace transform is performed on a discrete-time signal (with each element of the discrete-time sequence attached to a correspondingly delayed unit impulse), the result is precisely the Z transform of the discrete-time sequence with the substitution of $z = e^{st}$

Small Proof:

$z = e^{sT}$ $s = \frac{1}{T}\ln(z)$

$s = \frac{2}{T}\left[\frac{z-1}{z+1} + \frac{1}{3}\left(\frac{z-1}{z+1}\right)^3 + \frac{1}{5}\left(\frac{z-1}{z+1}\right)^5 + \frac{1}{7}\left(\frac{z-1}{z+1}\right)^7 + \cdots\right]$

$s = \frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}$

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The Bilinear Transform

$$H_d(z) = H_a(s)\big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = H_a\left(\frac{2}{T}\frac{z-1}{z+1}\right)$$

$$H_d\left(e^{j\omega_d T}\right) = H_a\left(\frac{2}{T}\frac{e^{j\omega_d T}-1}{e^{j\omega_d T+1}}\right)$$

$$H_d\left(e^{j\omega_d T}\right) = H_a\left(\frac{2}{T}\cdot\frac{\left(e^{j\omega_d T/2}-e^{-j\omega_d T/2}\right)}{\left(e^{j\omega_d T/2}+e^{-j\omega_d T/2}\right)}\right)$$

$$H_d\left(e^{j\omega_d T}\right) = H_a\left(j\frac{2}{T}\cdot\frac{\sin(\omega_d T/2)}{\cos(\omega_d T/2)}\right)$$

$$H_d\left(e^{j\omega_d T}\right) = H_a\left(j\frac{2}{T}\cdot\tan\left(\omega_d T/2\right)\right)$$

$$\omega_a = \frac{2}{T}\tan\left(\omega_d\frac{T}{2}\right)$$

The FIR Filter

The lowpass filter has a pass frequency ω_l and transition band $\Delta\omega$ with Stop Band Tolerance δ is $h_{lp}(n) = \frac{\sin(n\omega_l)}{n\pi} w(n)$

$$w(n) = \begin{cases} \frac{I\left[\beta N \sqrt{1-\left(\frac{n}{N}\right)^2}\right]}{I_0(\beta N)}, & -N \leq n \leq N, \quad \beta > 0 \\ 0 \text{ otherwise} \end{cases}$$

where $w(n)$ is Kaiser Window and $I_0(x)$ is the modified Bessel function of the first kind of order zero in x, and N are the window shaping factors and they are selected as follows

$$N \geq \frac{A-8}{4.57\Delta\omega}$$

$$A = -20\log_{10}\delta$$

$$\beta N = \begin{cases} 0.1102(A-8.7) & A > 50 \\ 0.5849(A-21)^{0.4} + 0.07886(A-21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases}$$

The FIR Filter

The FIR Bandpass Filter: The centre of the passband of the desired bandpass filter was found to be $\omega_c = 0.275\pi$ The impulse response of the desired bandpass filter is obtained from the impulse response of the corresponding lowpass filter as

$$h_{bp}(n) = 2h_{lp}(n)\cos(n\omega_c)$$

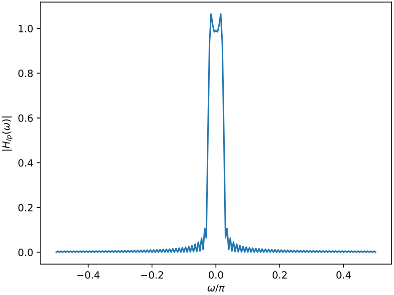

Figure 1: FIR Low Pass Filter

The FIR Filter

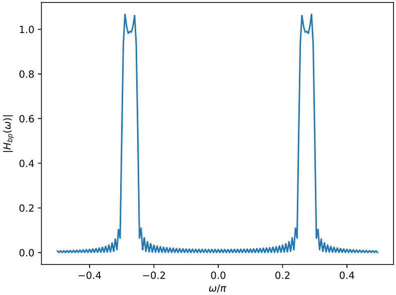

Figure 2: FIR Low Pass Filter

Thank You