

# EE3025 Presentation

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# Outline

## Problem

Assignment-01

Assignment-02

## Solution

Filter Output  $y(n)$

## Matrix Equations

## Filter Design

The Bilinear Transform

The FIR Filter

# Question

## Assignment-01

- ▶ Let

$$x(n) = \{ \underset{\uparrow}{1}, 2, 3, 4, 2, 1 \}$$

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

- ▶ Compute  $X(k)$ ,  $H(k)$  and  $y(n)$  using FFT and IFFT
- ▶ Wherever possible, express all the above equations as matrix equations.

## Assignment-02

- ▶ Filter Design
  - ▶ Filter Design Specifications
  - ▶ The Bilinear Transform: I
  - ▶ The FIR Filter

# Computing $X(k)$ , $H(k)$ Using FFT

- ▶ input signal  $x(n)$

$$x(n) = \{1, 2, 3, 4, 2, 1\}$$

$\uparrow$

- ▶ Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

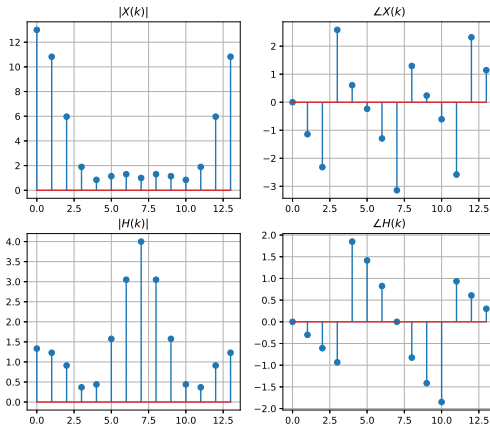
- ▶ FFT of a Input Signal  $x(n)$  is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

- ▶ FFT of a Impulse Response  $h(n)$  is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

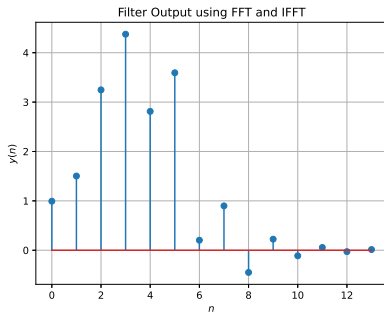
# Plots of Magnitude and Phase of $X(k)$ and $H(k)$



# Filter Output Using FFT and IFFT

- $y(n)$  can be computed by doing IFFT for  $Y(k)$

$$y(n) = \frac{1}{N} \sum_{k=0}^{N-1} Y(k) e^{j2\pi nk/N}, \quad k = 0, 1, \dots, N-1$$



## expressing all the above equations as matrix equations

- ▶ FFT of signal  $X(n)$

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1$$

- ▶ Let  $W_N^{nk} = e^{-j2\pi kn/N}$  then this can be expressed in terms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

## Computing $X(K)$ from Matrix Equations

- ▶ On solving we get,

$$\implies X(0) = 13 + 0j,$$

$$X(1) = -4 - 1.732j,$$

$$X(2) = 1 + 0j$$

$$X(3) = -1 + 0j,$$

$$X(4) = 1 + 0j,$$

$$X(5) = -4 + 1.732j$$

- ▶ the impulse

$$h(n) = \frac{-1^n}{2} u(n) + \frac{-1^{n-2}}{2} u(n-2)$$

assuming that length of  $h(n)$  is same as length of  $x(n)$  i.e.,  $N = 6$ . Similarly like  $x(n)$ , solving  $h(n)$  using matrix method for each value of  $k$  we get,



## Computing H(K) from Matrix Equations

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0) + h(1) + h(2) + h(3) + h(4) + h(5) \\ h(0) + h(1)e^{-j\pi/3} + \dots + h(5)e^{-j5\pi/3} \\ h(0) + h(1)e^{-2j\pi/3} + \dots + h(5)e^{-2j5\pi/3} \\ h(0) + h(1)e^{-3j\pi/3} + \dots + h(5)e^{-3j5\pi/3} \\ h(0) + h(1)e^{-4j\pi/3} + \dots + h(5)e^{-4j5\pi/3} \\ h(0) + h(1)e^{-5j\pi/3} + \dots + h(5)e^{-5j5\pi/3} \end{bmatrix}$$

On solving we get

$$H(0) = 1.28125 + 0j, \quad H(1) = 0.51625 - 0.5141875j,$$

$$H(2) = -0.078125 + 1.1095625j, \quad H(3) = 3.84375 + 0j,$$

$$H(4) = -0.071825 - 1.1095625j, \quad H(5) = 0.515625 + 0.5141875j$$

## Computing Y(K) from Matrix Equations

We can now compute Y(k) using below equation

$$Y(k) = X(k)H(k)$$

So, Y(k) is obtained element wise multiplication of X(k) and H(k)

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix}$$

## Computing $y(n)$ from IFFT of $Y$ matrix

$y(n)$  is given by IFFT of  $Y$  matrix. this  $y(n)$  can be calculated by a python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 + 0j \\ 2.28125071 + 0j \\ 2.6250019 - 1.11022302 \times 10^{-16}j \\ 4.37499667 - 1.47104551 \times 10^{-15}j \\ 2.6562481 + 6.10622664 \times 10^{-16}j \\ 3.59375262 - 1.60982339 \times 10^{-15}j \end{bmatrix}$$

# Filter Design Specifications

- ▶ **Tolerances:**

The magnitude of resonance in Pass-band is called Pass-band Tolerance and similarly magnitude of resonance in Stop-band is called Stop-band Tolerance.

- ▶ **Passband:**

The Frequency band within which signals are transmitted by filter without attenuation.

- ▶ **Stopband:**

The Frequency band within which signals are not transmitted by filter or with a large attenuation.

# The Bilinear Transform

It is a technique in Signal Processing that is used to Continuous-Time System Representations to Discrete-Time System Representations and vice-versa. The bilinear transform is a first-order approximation of the natural logarithm function that is an exact mapping of the  $z$ -plane to the  $s$ -plane. When the Laplace transform is performed on a discrete-time signal (with each element of the discrete-time sequence attached to a correspondingly delayed unit impulse), the result is precisely the  $Z$  transform of the discrete-time sequence with the substitution of  $z = e^{st}$

## Small Proof:

$$z = e^{sT} \quad s = \frac{1}{T} \ln(z)$$

$$s = \frac{2}{T} \left[ \frac{z-1}{z+1} + \frac{1}{3} \left( \frac{z-1}{z+1} \right)^3 + \frac{1}{5} \left( \frac{z-1}{z+1} \right)^5 + \frac{1}{7} \left( \frac{z-1}{z+1} \right)^7 + \dots \right]$$

$$s = \frac{2}{T} \frac{1-z^{-1}}{1+z^{-1}}$$

# The Bilinear Transform

$$H_d(z) = H_a(s)|_{s=\frac{2}{T} \frac{z-1}{z+1}} = H_a\left(\frac{2}{T} \frac{z-1}{z+1}\right)$$

$$H_d(e^{j\omega_d T}) = H_a\left(\frac{2}{T} \frac{e^{j\omega_d T}-1}{e^{j\omega_d T}+1}\right)$$

$$H_d(e^{j\omega_d T}) = H_a\left(\frac{2}{T} \cdot \frac{(e^{j\omega_d T/2}-e^{-j\omega_d T/2})}{(e^{j\omega_d T/2}+e^{-j\omega_d T/2})}\right)$$

$$H_d(e^{j\omega_d T}) = H_a\left(j \frac{2}{T} \cdot \frac{\sin(\omega_d T/2)}{\cos(\omega_d T/2)}\right)$$

$$H_d(e^{j\omega_d T}) = H_a\left(j \frac{2}{T} \cdot \tan(\omega_d T/2)\right)$$

$$\omega_a = \frac{2}{T} \tan\left(\omega_d \frac{T}{2}\right)$$

# The FIR Filter

The lowpass filter has a pass frequency  $\omega_l$  and transition band  $\Delta\omega$  with Stop Band Tolerance  $\delta$  is  $h_{lp}(n) = \frac{\sin(n\omega_l)}{n\pi} w(n)$

$$w(n) = \begin{cases} \frac{I\left[\beta N \sqrt{1 - \left(\frac{n}{N}\right)^2}\right]}{I_0(\beta N)}, & -N \leq n \leq N, \quad \beta > 0 \\ 0 & \text{otherwise} \end{cases}$$

where  $w(n)$  is Kaiser Window and  $I_0(x)$  is the modified Bessel function of the first kind of order zero in  $x$ , and  $N$  are the window shaping factors and they are selected as follows

$$N \geq \frac{A-8}{4.57\Delta\omega}$$

$$A = -20 \log_{10} \delta$$

$$\beta N = \begin{cases} 0.1102(A - 8.7) & A > 50 \\ 0.5849(A - 21)^{0.4} + 0.07886(A - 21) & 21 \leq A \leq 50 \\ 0 & A < 21 \end{cases}$$

# The FIR Filter

The FIR Bandpass Filter: The centre of the passband of the desired bandpass filter was found to be  $\omega_c = 0.275\pi$ . The impulse response of the desired bandpass filter is obtained from the impulse response of the corresponding lowpass filter as

$$h_{bp}(n) = 2h_{lp}(n) \cos(n\omega_c)$$

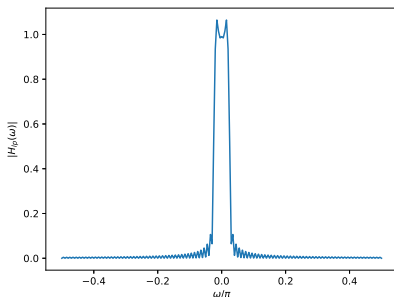


Figure 1: FIR Low Pass Filter



# The FIR Filter

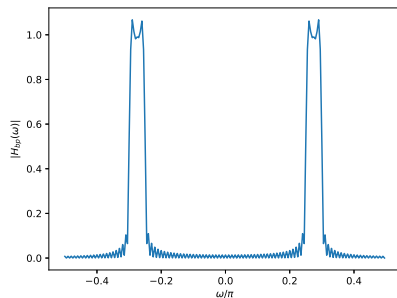


Figure 2: FIR Band Pass Filter

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Thank You