EE3025 Assignment-1

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Download all python codes from

> https://github.com/srikanth2001/EE3025−DSP/tree/main/Assignment−01/codes

and latex-tikz codes from

> https://github.com/srikanth2001/EE3025−DSP/blob/main/Assignment−01/ee18btech11023.tex

1 Problem

1.1. Let

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{1.1.1}$$

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{1.1.2}$$

1.2. Compute X(k), H(k) and y(n) using FFT and IFFT

2 Solution

2.1. input signal x(n)

$$x(n) = \left\{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\right\} \tag{2.1.1}$$

2.2. Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^n u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2) \tag{2.2.1}$$

2.3. FFT of a Input Signal $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1 \dots N-1 \tag{2.3.1}$$

2.4. FFT of a Impulse Response $h(n)$ is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \dots, N-1 \tag{2.4.1}$$

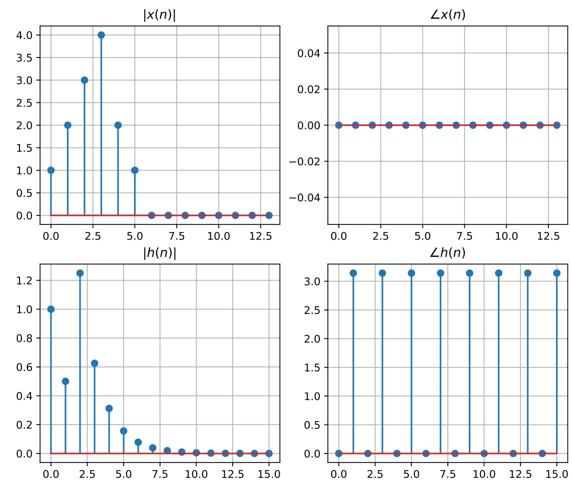

Fig. 2.3: input signal x(n) and Impulse responce h(n)

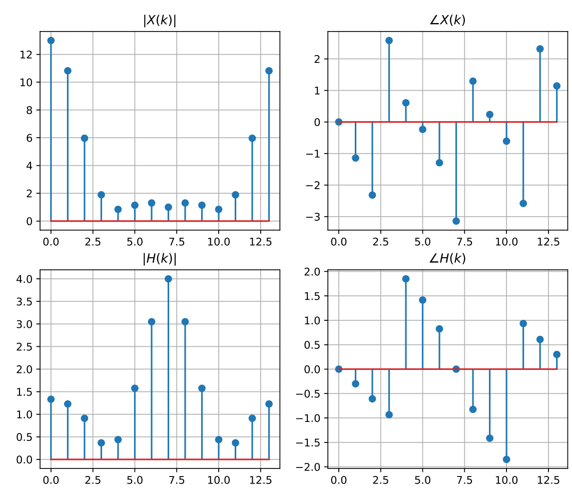

Fig. 2.4: FFT of x(n)- X(k) and FFT of h(n)- H(k)

2.5. then FFT of output Signal $y(n)$ can be computed by

$$Y(k) = X(k)H(k) \tag{2.5.1}$$

2.6. $y(n)$ can be computed by doing IFFT for $Y(k)$

$$y(n) = \frac{1}{N}\sum_{n=0}^{N-1} Y(k)e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1 \tag{2.6.1}$$

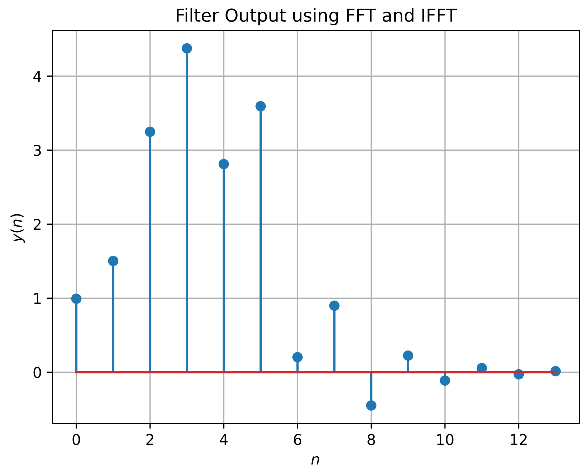

Fig. 2.6: output signal y(n)

2.7. plotting FFT of output signal

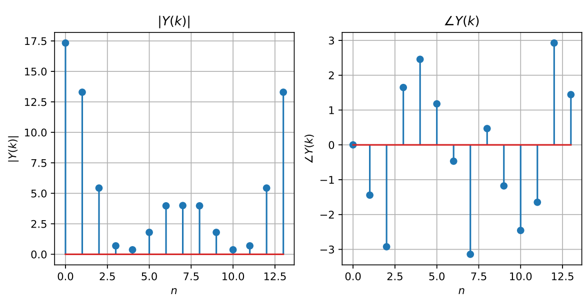

Fig. 2.7: FFT of output signal y(n)

3 Problem

3.1. Wherever possible, express all the above equations as matrix equations.

4 Solution

4.1. FFT of signal X(n)

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1 \tag{4.1.1}$$

4.2. Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be ex-pressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix} \tag{4.2.1}$$

4.3. Given that $x(n) = \{1, 2, 3, 4, 2, 1\}$ and As, N = 6 then above equation on multiplying matrices becomes

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1+2+3+4+2+1 \\ 1+(2)e^{-j\pi/3}+\ldots+(1)e^{-j5\pi/3} \\ 1+(2)e^{-2j\pi/3}+\ldots+(1)(e^{-2j5\pi/3} \\ 1+(2)e^{-3j\pi/3}+\ldots+(1)e^{-3j5\pi/3} \\ 1+(2)e^{-4j\pi/3}+\ldots+(1)e^{-4j5\pi/3} \\ 1+(2)e^{-5j\pi/3}+\ldots+(1)e^{-5j5\pi/3} \end{bmatrix} \tag{4.3.1}$$

4.4. On solving we get,

$$\implies X(0) = 13 + 0j, \tag{4.4.1}$$
$$X(1) = -4 - 1.732j, \tag{4.4.2}$$
$$X(2) = 1 + 0j, \tag{4.4.3}$$
$$X(3) = -1 + 0j, \tag{4.4.4}$$
$$X(4) = 1 + 0j, \tag{4.4.5}$$
$$X(5) = -4 + 1.732j \tag{4.4.6}$$

4.5. Now to find $H(k)$ we need to know h(n) first. So we will first calculate h(n). For that we need to first find the Y(z) by applying Z-transform on equation (**??**) i.e.,

$$Y(z) + \frac{1}{2}z^{-1}Y(z) = X(z) + z^{-2}X(z) \tag{4.5.1}$$

$$\implies Y(z) = \frac{2(z^2+1)}{z(2z+1)}X(z) \tag{4.5.2}$$

Now we can find H(z) using Y(z) i.e.,

$$H(z) = \frac{Y(z)}{X(z)} \tag{4.5.3}$$

$$H(z) = \frac{2(z^2+1)}{z(2z+1)} \tag{4.5.4}$$

$$H(z) = \frac{1 + z^{-2}}{1 + \frac{1}{2}z^{-1}} \tag{4.5.5}$$

From this we can say that h(n) is,

$$h(n) = Z^{-1}\left[\frac{1}{1 + \frac{1}{2}z^{-1}} + \frac{z^{-2}}{1 + \frac{1}{2}z^{-1}}\right] \tag{4.5.6}$$

$$h(n) = \left[\frac{-1}{2}\right]^{n} u(n) + \left[\frac{-1}{2}\right]^{n-2} u(n-2) \tag{4.5.7}$$

Now for the calculations we can assume that length of h(n) is same as length of x(n) i.e., N = 6. Similarly Now on solving equation (**??**) using matrix method for each value of k we get,

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0) + h(1) + h(2) + h(3) + h(4) + h(5) \\ h(0) + h(1)e^{-j\pi/3} + ... + h(5)e^{-j5\pi/3} \\ h(0) + h(1)e^{-2j\pi/3} + ... + h(5)e^{-2j5\pi/3} \\ h(0) + h(1)e^{-3j\pi/3} + ... + h(5)e^{-3j5\pi/3} \\ h(0) + h(1)e^{-4j\pi/3} + ... + h(5)e^{-4j5\pi/3} \\ h(0) + h(1)e^{-5j\pi/3} + ... + h(5)e^{-5j5\pi/3} \end{bmatrix} \tag{4.5.8}$$

4.6. On solving we get,

$$\implies H(0) = 1.28125 + 0j, \tag{4.6.1}$$
$$H(1) = 0.51625 - 0.5141875j, \tag{4.6.2}$$
$$H(2) = -0.078125 + 1.1095625j, \tag{4.6.3}$$
$$H(3) = 3.84375 + 0j, \tag{4.6.4}$$
$$H(4) = -0.071825 - 1.1095625j. \tag{4.6.5}$$
$$H(5) = 0.515625 + 0.5141875j \tag{4.6.6}$$

So,These values which we got are same as that of from the plots.

4.7. We can now compute $Y(k)$ using Eq (4.7.1)

$$Y(k) = X(k)H(k) \tag{4.7.1}$$

So, $Y(k)$ is obtained element wise multiplication of $X(k)$ and $H(k)$

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \end{bmatrix} \tag{4.7.2}$$

Computing the above expression we get,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix} \tag{4.7.3}$$

4.8. y(n) is given by IFFT of Y matrix. this y(n) can be calculated by a python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 + 0j \\ 2.28125071 + 0j \\ 2.6250019 - 1.11022302 \times 10^{-16}j \\ 4.37499667 - 1.47104551 \times 10^{-15}j \\ 2.6562481 + 6.10622664 \times 10^{-16}j \\ 3.59375262 - 1.60982339 \times 10^{-15}j \end{bmatrix} \tag{4.8.1}$$