Fast Fourier transform

EE3025 - DSP Lab

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Question

- Let

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\}$$

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

- Compute X(k), H(k) and y(n) using FFT and IFFT
- Wherever possible, express all the above equa-tions as matrix equations.

Computing X(k), H(k) Using FFT

- input signal x(n)

$$x(n) = \{\underset{\uparrow}{1}, 2, 3, 4, 2, 1\}$$

- Impulse Response of the System is

$$h(n) = \left(-\frac{1}{2}\right)^{n} u(n) + \left(-\frac{1}{2}\right)^{n-2} u(n-2)$$

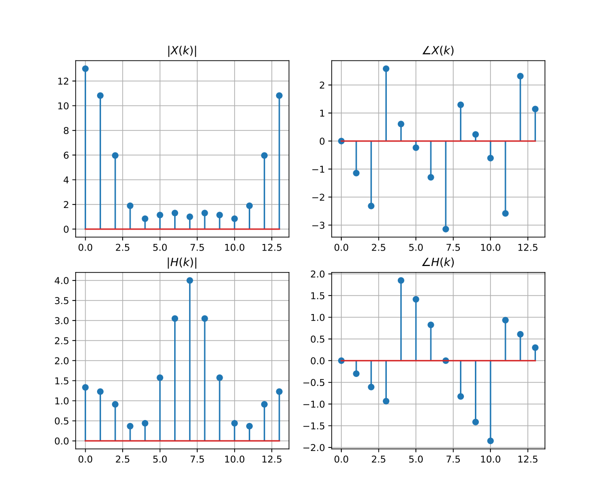
- FFT of a Input Signal $x(n)$ is

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j2\pi kn/N}, \quad k = 0, 1 \ldots N-1$$

- FFT of a Impulse Response $h(n)$ is

$$H(k) = \sum_{n=0}^{N-1} h(n) e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

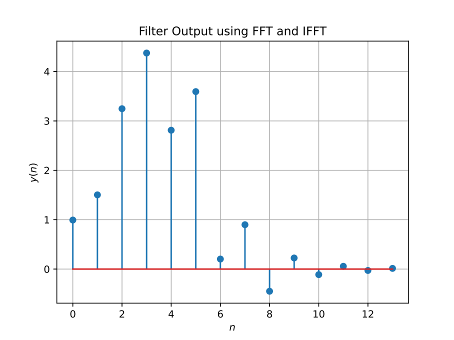
Plots of Magnitude and Phase of X(k) and H(k)



Filter Output Using FFT and IFFT

- $y(n)$ can be computed by doing IFFT for $Y(k)$

$$y(n) = \frac{1}{N}\sum_{n=0}^{N-1} Y(k)e^{j2\pi nk/N}, \quad k = 0, 1, \ldots, N-1$$



expressing all the above equations as matrix equations

- FFT of signal X(n)

$$X(k) \triangleq \sum_{n=0}^{N-1} x(n)e^{-j2\pi kn/N}, \quad k = 0, 1, \ldots, N-1$$

- Let $W_N^{nk} = e^{-j2\pi kn/N}$ then this can be **ex**− pressed interms of matrices as:

$$\begin{bmatrix} X(0) \\ X(1) \\ X(2) \\ X(3) \\ X(4) \\ X(5) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & W_N^1 & W_N^2 & W_N^3 & W_N^4 & W_N^5 \\ 1 & W_N^2 & W_N^4 & W_N^6 & W_N^8 & W_N^{10} \\ 1 & W_N^3 & W_N^6 & W_N^9 & W_N^{12} & W_N^{15} \\ 1 & W_N^4 & W_N^8 & W_N^{12} & W_N^{16} & W_N^{20} \\ 1 & W_N^5 & W_N^{10} & W_N^{15} & W_N^{20} & W_N^{25} \end{bmatrix} \begin{bmatrix} x(0) \\ x(1) \\ x(2) \\ x(3) \\ x(4) \\ x(5) \end{bmatrix}$$

Computing X(K) from Matrix Equations

- On solving we get,

$$\implies X(0) = 13 + 0j,$$
$$X(1) = -4 - 1.732j,$$
$$X(2) = 1 + 0j$$
$$X(3) = -1 + 0j,$$
$$X(4) = 1 + 0j,$$
$$X(5) = -4 + 1.732j$$

- the impluse

$$h(n) = \frac{-1}{2}^{n} u(n) + \frac{-1}{2}^{n-2} u(n-2)$$

assuming that length of h(n) is same as length of x(n) i.e., N = 6. Similarly like x(n), solving h(n) using matrix method for each value of k we get,

Computing H(K) from Matrix Equations

$$\begin{bmatrix} H(0) \\ H(1) \\ H(2) \\ H(3) \\ H(4) \\ H(5) \end{bmatrix} = \begin{bmatrix} h(0)+h(1)+h(2)+h(3)+h(4)+h(5) \\ h(0)+h(1)e^{-j\pi/3}+...+h(5)e^{-j5\pi/3} \\ h(0)+h(1)e^{-2j\pi/3}+...+h(5)e^{-2j5\pi/3} \\ h(0)+h(1)e^{-3j\pi/3}+...+h(5)e^{-3j5\pi/3} \\ h(0)+h(1)e^{-4j\pi/3}+...+h(5)e^{-4j5\pi/3} \\ h(0)+h(1)e^{-5j\pi/3}+...+h(5)e^{-5j5\pi/3} \end{bmatrix}$$

On solving we get
H(0) = 1.28125 + 0j , H(1) = 0.51625 - 0.5141875j ,
H(2) = -0.078125 + 1.1095625j , H(3) = 3.84375 + 0j ,
H(4) = -0.071825 - 1.1095625j , H(5) = 0.515625 + 0.5141875j

Computing Y(K) from Matrix Equations

We can now compute Y(k) using below equation

$$Y(k) = X(k)H(k)$$

So, Y(k) is obtained element wise multiplication of X(k) and H(k)

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} X(0) \cdot H(0) \\ X(1) \cdot H(1) \\ X(2) \cdot H(2) \\ X(3) \cdot H(3) \\ X(4) \cdot H(4) \\ X(5) \cdot H(5) \end{bmatrix}$$

Computing the above expression we get,

$$\begin{bmatrix} Y(0) \\ Y(1) \\ Y(2) \\ Y(3) \\ Y(4) \\ Y(5) \end{bmatrix} = \begin{bmatrix} 16.6562 + 0j \\ -2.95312 + 1.16372j \\ -0.07812 + 1.10959j \\ -3.84375 - 9.27556j \\ -0.07812 - 1.10959j \\ -2.95312 - 1.16372j \end{bmatrix}$$

Computinh y(n) from IFFT of Y matrix

y(n) is given by IFFT of Y matrix. this y(n) can be calculated by a python code

$$\begin{bmatrix} y(0) \\ y(1) \\ y(2) \\ y(3) \\ y(4) \\ y(5) \end{bmatrix} = \begin{bmatrix} 1.125 + 0j \\ 2.28125071 + 0j \\ 2.6250019 - 1.11022302 \times 10^{-16}j \\ 4.37499667 - 1.47104551 \times 10^{-15}j \\ 2.6562481 + 6.10622664 \times 10^{-16}j \\ 3.59375262 - 1.60982339 \times 10^{-15}j \end{bmatrix}$$