Homework 4

August 10, 2025

1. L&M 3.6.2,3.6.5,3.6.11,3.6.13,3.6.16,3.6.22,3.6.24.

2. Compute the mean and variance of Bernoulli, Binomial, Geometric, Hypergeometric, Poisson, Uniform, Exponential distributions. You are supposed to memorize the formulae as well.

3. We toss a fair coin, if it is heads we roll a pair of fair dice and record the sum of the face values as X. otherwise we draw a coupon from a box with coupons from the set $\{1, 2, 3, 4, \cdots, 12\}$ and record the coupon value as X. Compute the mean and variance of X.

4. Suppose that two teams are playing a series of games, each of which is independently won by team A with probability p and by team B with probability 1− p. The winner of the series is the first team to win 3 games. Let X be the number of games won by the loser when the series ends. Compute the mean and variance of X if $6p(1-p) = 1$.

5. A drunk man has n keys, exactly one of which opens his door. He tries keys at random until he finds the correct one.

   Case 1: He doesn't reuse keys that don't work (sampling without replacement).

   Case 2: He forgets which keys he's tried and might try the same key multiple times (sampling with replacement).

   For both cases, let **X** be the number of trials needed to open the door.

   Find:

   (a) The probability mass function $P(X = k)$ for each case.

   (b) The mean $E[X]$ for each case.

   (c) The variance $Var(X)$ for each case.

   (d) Compare the efficiency of the two approaches.