ESTIMATING MARKET CRASH RISK IN THE INDIAN STOCK MARKET THROUGH COPULAS

AMRITHA SATISH KUMAR

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Name of the Student	**:**	**AMRITHA SATISH KUMAR**
Registration Number	**:**	**QF/2023-25/003**
Name and Designation of the Supervisor	**:**	**Dr. EKTA SELARKA** **Associate Professor** MADRAS SCHOOL OF ECONOMICS Chennai- 600025

BONAFIDE CERTIFICATE

Certified that this post-graduate program dissertation Project Report titled *"Estimating Market Crash Risk through Copulas"* is the bonafide work of Ms. Amritha Satish Kumar who carried out the project under my supervision. Certified further, that to the best of my knowledge the work reported herein does not form part of any other Project Report of the basis of which a degree or award was conferred on an earlier occasion on this or any other candidate.

(Signature with date and Seal)	(Signature with date and Seal)
Dr. K.S. KAVI KUMAR	Dr. EKTA SELARKA
Dean (Academics)	Associate Professor
Madras School of Economics	Madras School of Economics
Chennai - 600025	Chennai - 600025

ABSTRACT

This thesis investigates the relationship between tail dependence and stock returns in the Indian stock market, with a focus on the Nifty 50 index. This study employs copula models to better estimate the dependence structure between individual stock returns and market returns. Lower tail dependence (LTD) is the measure of crash risk in this study since it measures the conditional probability of stocks crashing given the market return crashes whereas upper tail dependence (UTD) measures the probability that the stock returns take high positive values given the market returns are large and positive. LTD is used to analyse how stock returns are affected based on the level of crash risk. Using a combination of lower-tail, upper-tail, and no-tail dependence copulas, parameters are estimated for each company and year from 1995 to 2025.

The empirical results show that prior to 2008, stocks with stronger lower-tail dependence yielded higher excess returns, suggesting compensation for downside risk exposure. However, post-2008, this pattern reverses, with stronger downside dependence associated with lower returns, potentially signifying a change in the perception of risk and return relationship after the Global Financial Crisis of 2008. Additionally, stocks with higher upper-tail dependence consistently exhibit lower excess returns across the sample period proving the theoretical claim that only downside risk or lower tail risk concerns the investors more and the higher upper tail dependence need not be compensated with higher returns since there is no crash or extreme negative returns associated with upper tail dependence. The study aids in understanding lower tail risk in a developing economy's financial market, and highlight the role of copulas in modelling dependence beyond linear correlation that overcome distribution dependent assumptions.

Keywords: Copula Models, Tail Dependence, Nifty50

JEL Classification: D53

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CHAPTER 1

INTRODUCTION

1.1.Financial Markets and Return Distribution

Financial markets are the backbone of economies, and a well-functioning market paves way for long-term economic growth and financial stability. However, they also carry significant risks that can trigger far-reaching global consequences, potentially leading to financial contagion – the spread of financial shocks across borders or markets. Understanding this risk requires analysing how a stock responds to changes in overall market behaviour, including the potential impact of a market crash. More importantly, it involves estimating the probability of the stock crashing in the event of a market downturn. It is vital to estimate this relation between market returns and individual stock returns which inherently help in understanding the crash sensitivity of that particular stock.

The traditional approach to estimating the relationship between market returns and the individual stock returns is to run the market model regression as given below by Equation(1) and estimate the beta factor, where beta will give the expected movement of the stock returns with respect to the market returns.

$$R_i = \alpha_i + \beta_i(R_m) + \varepsilon_i \tag{1}$$

This model is said to work under assumptions such as the market returns being normally distributed. It further assumed that the error term ε_i is homoscedastic in nature, normally distributed and uncorrelated with the market returns.

Beta factor is derived from Pearson's correlation which theoretically works on the assumption of bivariate normality. Chok (2010) shows using this for distributions with significant deviations from normality compromises the reliability and the correlation coefficient loses its statistical power. Moreover, these assumptions, specifically that of normality and homoscedasticity undermine the model's validity because in reality, financial data such as stock returns exhibit volatility clustering and fat tails, meaning they are rarely normally distributed. Presence of fat tails imply a higher chance of extreme events happening in comparison to what is normally expected. Therefore, assuming normal distribution underestimates the occurrence of extreme events or the existence of abnormal market conditions.

Such non-normal market conditions include extreme fluctuation of prices, returns being skewed and exhibiting fat tails, sharp decline in returns and unexpected losses due to higher volatility clustering subsequently resulting in market crashes. By 1960s, various economists such as Markovitz(1959), Eugene Fama(1965) and Benoit Mandelbrot(1965) challenged the assumption of normal distribution of returns. Their papers suggested using a stable distribution called the ‘Levy Distribution’ which is better suited for modelling fat tails but for univariate analysis.

Market crash is a situation where there is a sudden and significant drop in the stock prices and subsequently the returns. The 2008 Global Financial crisis was one such event which had a far-reaching negative impact on the global economy. Moreover, in a world, where the economies are integrated, the spread of this crisis is inevitable. Vishwanathan (2010) showed India’s BSE Index lost 37.9% of its value in May 2008 due to the crisis and it unfavourably impacted wealth creation and businesses’ ability to raise funds from the capital market. Given that India is one of the most important players in the global stock markets, studying its crash risk is essential. With a market capitalisation of $4.33 trillion as of 2024, India is now the fourth largest stock market, making it a key economy for understanding market crash and estimating a measure to quantify it.

Considering the disadvantage of the market model, there is a need to estimate crash sensitivity, measured as the value of lower tail dependence between the returns, through a revised method that does not make assumptions about the underlying distribution. One such solution is to implement a different statistical tool, called copula to measure crash risk or crash sensitivity of a stock.

Copulas are functions that capture the dependence of two or more random variables, the stock returns and the market return in this study, not concerning itself with the distribution of the said returns. This is further discussed in detail in the following next section

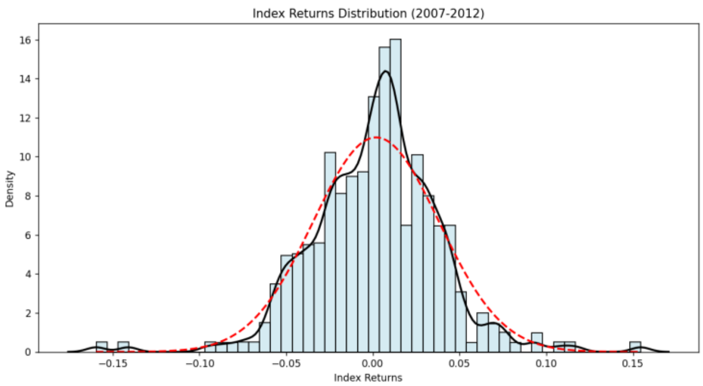
1.2. Introduction to Copulas

Consider two random variables X and Y following any two distributions. To study the dependence structure between the two random variables most statistical methods’ accuracy is subject to the marginal distributions of the two variables which in certain cases complicates the modelling process or limits the applicability or accuracy of the said models. Copulas on the other hand, offer an alternative solution to the assumptions related to the marginal distributions of the two variables under study. Copulas are a statistical model which models the dependence structure between two or more random variables making no assumptions about the individual marginal distributions. For instance, as mentioned earlier, the market model regression to understand the dependence structure between market returns and individual stock returns assumes normal distribution of the returns which leads to flawed assumptions about the reality of the said returns.

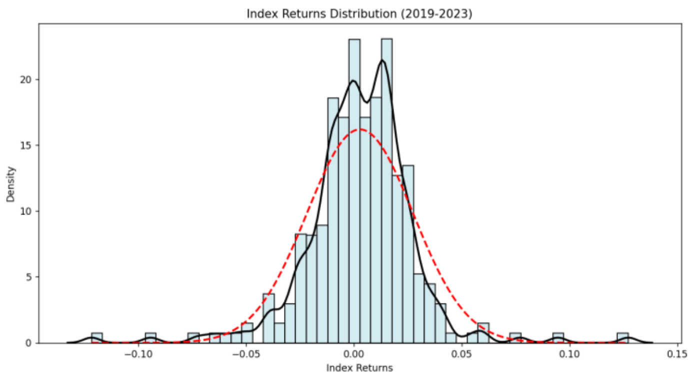
For instance, when one plots a normal distribution curve and simultaneously the market returns as well, it can be visually observed that the returns in fact do not follow a normal distribution. This is observed in Figure (1). Graph A shows the difference in distribution for the years 2007 to 2012 while Graph B does for the years 2019 to 2023.

Figure 1: Normal distribution vs Actual Returns

Graph A: 2007 to 2012



Graph B: 2019 to 2023



The graphs compare a normal distribution curve drawn from the market returns mean and variance versus the actual distribution of the returns. As seen, the dotted red curve representing the normal distribution curve does not fit the actual distribution curve of the market returns(black smooth line). Graph A plots the market returns during the period of global financial crisis 2008- 2012 and Graph B plots the same for the years 2019-2023, the years of peak Covid-19 pandemic.

This implies it is difficult to arrive at a conclusion about the actual distribution that the market follows and then fit a model that is distribution dependent. The alternative would be to understand the dependence structure between the market returns and the stock returns using a model that makes no assumptions about the underlying marginal distributions of the returns. This is offered by copulas as developed by Sklar.

Abe Sklar introduced the theory of copulas in 1959. According to this theory any pair of random variables X_1 and X_2 , and the joint cumulative distribution function (CDF) i.e., [F(x_1, x_2)=P($X_1 \leq x_1$, $X_2 \leq x_2$)] can be expressed in terms of their marginal CDFs and a function called the copula function. This can be extended to 'd' such variables X_1, X_2, X_3, … X_d and to arrive at

$$F_X\ (x_1, \dots\ x_d) =\ C_X(F_{X1}(x_1), \dots F_{Xd}(x_d)) \qquad (2)$$

Here, $F_X\ (x_1, \dots\ x_d)$ is the joint CDF of 'd' random variables, $C_X(\dots)$ is the copula function and $F_{X1}(x_1), \dots F_{Xd}(x_d)$ is the marginal CDFs i.e., individual CDFs of the variables we are trying to capture the dependence structure of.

The cumulative distribution function of a random variable X is defined as the probability that X takes up a values lesser than equal to a particular number. CDF is written as $F_X(x) = P(X\ \leq\ x)$.

Given the copula function, the advantage is that when the variables are input into the copula function, they are written as marginal CDFs of the random variables evaluated at the variable itself i.e., $(F_{X_1}(X_1), F_{X_2}(X_2))$, usually denoted as u and v to represent the copula as $C(u, v)$. These input variables u and v are always uniformly distributed no matter what the original distribution of the variables X_1 or X_2.

This can be illustrated with a simulation of 2000 data points that are normally distributed and input into different copulas to understand their dependence structure. Moreover, when the transformed variables u and v are plotted, it is visually noticed to be uniformly distributed.

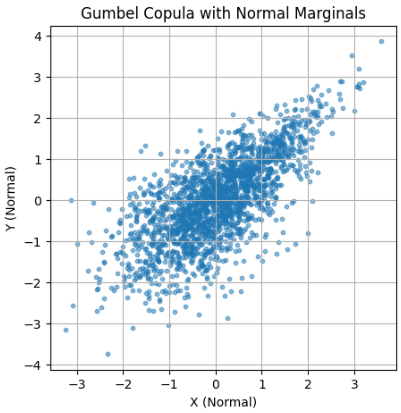
For this simulation, let us consider three different types of most commonly used copulas, namely Clayton, Gaussian and Gumbel. These copulas are used to model lower tail dependence, no tail dependence and upper tail dependence respectively. Lower tail dependence in essence refers to the probability of one variable reaching extreme low or negative values given that the other variable has reached an extreme negative or low value.

Considering the inverse of lower tail dependence, upper tail dependence refers to probability of one variable reaching extremely high values given that the tother variable has an extremely high value.

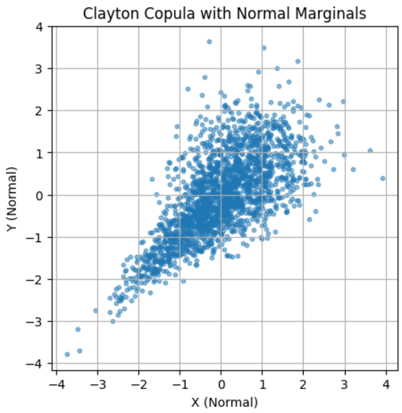
In Figure (2), Graphs A – E visualise the dependence structure modelled by the respective copulas despite having the same normal marginal distribution. Graph A shows strong clustering in the upper right quadrant indicating upper tail dependence in the Gumbel Copula. In extreme positive values of X, Y also takes on large values. The lower tail dependence is weak. Graph B shows clustering along the lower left side depicting lower tail dependence. In extreme negative outcomes of X, Y is also likely to be low. Consequently, the upper tail dependence is low. Graph C and Graph D seem to be identical since there is no patterned scatter plot due to the lack of strong association in either tails. The difference in Graph D is that, there is no association between the variables since we plot two purely independent variables X and Y. Lastly, Graph E has no scatter since with perfect dependence, every point would like along a straight line indicating deterministic relationship between X and Y

Figure 2: Dependence Structure under Different Copula

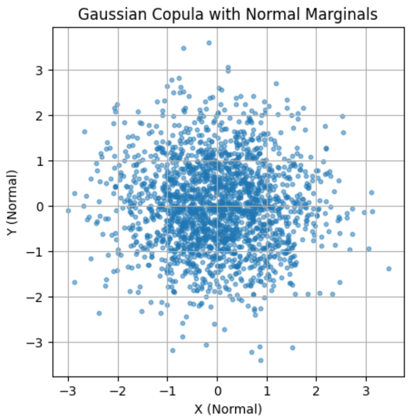
Graph A



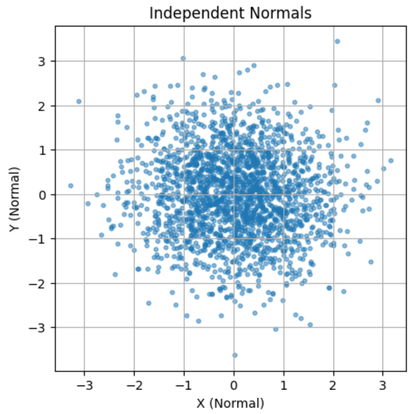
Graph B



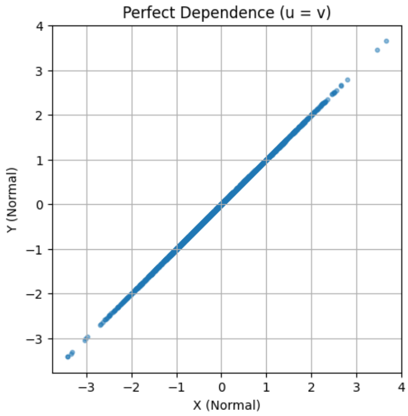
Graph C



Graph D



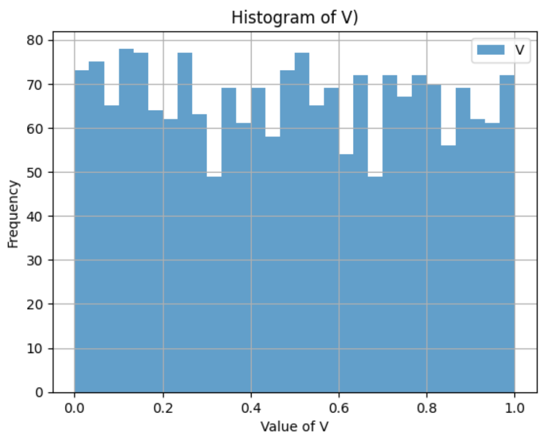
Graph E



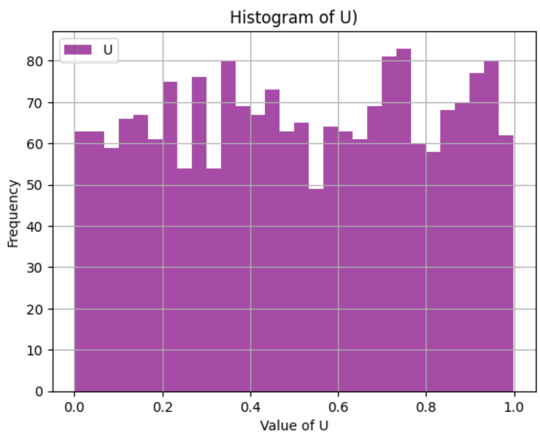
Moreover, no matter what the marginals are, when the inverse CDFs 'u' and 'v' are plotted to see their distribution though a histogram, it is observed to be uniformly distributed as demonstrated in Figure (3).

Figure 3: Histogram of Inverse Cumulative Distribution Function

Graph A



Graph B



The above histograms show very slight variations in the values of u and v indicating a uniform distribution followed by the transformed variables. This implies that no matter what the marginal distribution of the variables in the study are they are transformed into uniform distribution

Therefore, copulas are a universal statistical tool to study the dependence structure of any set of variables without making any assumptions about the marginal distributions.

This dependence structure as already mentioned is primarily of two types, namely lower tail dependence (LTD) and upper tail dependence (UTD). Alternatively, when neither apply, it is said to have no tail dependence. Lower tail dependence is defined in Equation (3).

$$\mathrm{P(q)} = \Pr[X_1 < F_{X1}^{-1}(q) | X_2 < F_{X2}^{-1}(q)] \ ; \ LTD \equiv \lim_{q \to 0} P_l\,(q) \quad (3)$$

Equation (3) , in simple , is a conditional probability where, given an event A (Market returns reaching a low negative value) has occurred, we try to estimate the probability that event B(Individual stock returns reaching a low negative value) will also occur.

In essence, $P(q) = Pr[A \mid B]$. Estimation of this, being defined as a measure of market crash risk, is the main objective of this study. The derivation of lower tail dependence is given by Equation (4.1) to Equation (4.4).

Equation (3) when further simplified can be derived as follows,

$$C(u, v) = \Pr[F_{X1}(X_1) < (u) | F_{X2}(X_2) < (v)] \quad (4.1)$$

$$And, \Pr[X_1 < F_{X1}^{-1}(q) | X_2 < F_{X2}^{-1}(q)] = \frac{\Pr[X_1 < F_{X1}^{-1}(q)\,, X_2 < F_{X2}^{-1}(q)]}{\Pr[X_1 < F_{X1}^{-1}(q)]} \quad (4.2)$$

$$From\ the\ definition\ of\ C(u, v), we\ can\ say \quad (4.3)$$

$$\Pr[X_1 < F_{X1}^{-1}(q)\,, X_2 < F_{X2}^{-1}(q)] = C(q, q)$$

$$then\ for\ a\ very\ negative\ value\ of\ the\ inverse\ CDF, \quad (4.4)$$

$$LTD\ can\ be\ written\ in\ terms\ of\ copulas\ as\ LTD = \lim_{q \to 0} \frac{C(q,q)}{q}$$

Therefore, to estimate lower tail dependence as defined above, this study uses copulas that gets rid of the individuality of the marginal distributions. Copulas are better fit for understanding real financial data and can predict the dependence structure. Through this predicted dependence structure, we arrive at the value of lower tail dependence and can conclude what the probability of a particular stock crashing is conditioned on the probability that the market crashes.

The objective of this thesis is to estimate the lower tail dependence to understand the crash sensitivity of the stock to check if there exists a relationship between the probability of stock price crashing (LTD) and the expected portfolio return based on univariate portfolio sorting.

The thesis is organised as follows: The next section presents an overview of the existing literature relevant to estimating tail risk, understanding crash risk, and the use of copulas to capture dependence structures between two variables. The ‘Methodology’ section provides a detailed description of the data collected and used in the study, followed by an explanation of the model implemented. The ‘Results and Discussion’ section examines the empirical findings, and finally, the ‘Conclusion’ summarizes the main insights and implications drawn from the study.

CHAPTER 2

LITERATURE REVIEW

The multiple financial crises motivated researchers around the world to study the impact of such an event, to understand if it is possible to estimate the risk of a crisis happening and the likelihood that individual stock returns will exhibit extreme downward movement.

2.1. Theoretical Literature on Risk

Nawrocki(2000) writes about how one of the pioneers of downside risk measurement was Markovitz (1959) where he emphasised on the idea that downside risk was more relevant to investors than upside potential and that the distributions of the securities or the returns may not necessarily be normally distributed. He proved that variance could be a valid measure only when returns are normally distributed but when that is not the case, there is a need for downside risk measurement. Therefore, variance would be insufficient to capture downside risk of financial data. Richardson & Smith(1993) show while analysing stock returns, many authors including Mandelbrot (1963) and Fama (1965) argued about the assumption of normal distribution of the stock returns have proved that returns are not normally distributed questioning the results that are dependent on this assumption. This implies that returns in reality exhibit skewness and fat tails. There is a need to measure the risk of extreme event occurring, called the tail risk.

One prominent measure of tail risk is Value at Risk (VaR). McNeil & Frey(2000) studied how historical simulation of VaR combined with GARCH and pseudo-maximum likelihood is another approach to modelling tail risk measures given the heteroscedasticity nature of financial data. But, Artzner, Delbaen, Heath, & Marc (1999) have proved shortages in the use of VaR as a risk measure of the market. VaR fails to capture joint tail risk, as it only considers the maximum potential loss for an individual stock or portfolio at a given confidence level, ignoring the possibility of simultaneous market-wide crashes since it cannot capture dependence.

2.2. Empirical Literature on Risk

Authors including Linmeier & Pearson (1996) and Huisman, Koedijk, & Pownall (1998) have implemented value at risk and modified VaR in measuring financial risk. Karmakar (2012) used daily BSE SENSEX for a period of 10 years (2000-2009) and implement AR-GARCH model to capture the heteroscedasticity of returns. Additional to VaR and conditional VaR they also use a third measure, expected shortfall to understand tail risk. Chen, Hong, & Stein (2001) forecasted crashes in stock returns by estimating negative skewness and down-to-up volatility measured as the log of the ratio between a stock's volatility on days the returns fall below the mean of that period and the volatility on the days its returns exceed the mean. Similarly, Chen, Huang, & Zhang (2015) calculate the same two variables as a measure of crash risk and additionally include a third measure, 'Count'. This is measured by the difference between the number of firm-specific weekly returns exceeding 3.09 times standard deviation below the mean weekly return and the number of firm-specific weekly returns exceeding 3.09 times standard deviations above the mean weekly return. Chauhan, Kumar, & Pathak (2017) calculate negative skewness and log of down-to-up volatility of daily returns as proxies for crash risk. Further, they also estimate the daily residual returns from an extended version of the market model regression to include lags and leads of the market return. The residuals from this are used to calculate the crash risk proxies instead of the returns directly. All these measures only capture individual crash risk of the stocks but does not focus on the dependence in estimating the crash risk, given the market returns crash. However, while individual crash risk measures have been extensively explored, few studies address the joint dependence of stock returns and market crashes.

2.3. Literature on Copulas

The alternative to using measures that were dependent on normal distribution or ones that could not capture co-movement and dependence of returns was the theory of copulas as proposed by Abe Sklar(1959). Burney (2020) explained how the theory of copulas was implemented in various fields and this theory grew importance towards the end of the 1990s due to its application in finance, insurance and risk analysis. Dependence structure is measured through tail dependence coefficients since simple linear correlation could not capture non-linear dependence relationship often seen in financial data. Furthermore, several authors including Wang, Chen, & Huang (2011), Peng & Ng (2012), and Jayech & Zina (2012) have employed time-varying copulas combined with time-series models including ARMA-GARCH, AR-GJR-GARCH to effectively capture volatility clustering and heteroscedasticity, especially to measure financial contagion effects. Additionally, Shirvani (2020) used a t-copula to study extreme variations in the time-series of log-return and log-roughness of the S&P 500 index during both the 2008 and 2015 flash crash. Given the break in data points, they used FARIMA and FIGARCH for fractionally integrated datasets to account for the crashes captured by two indices, tail dependence index and degree of freedom index. Srilakshminarayana (2021) studied the tail index value of Nifty50 stocks especially during the crises periods by dividing the entire study(2007-2020) into 6 sub-periods. Their results showed that these stocks exhibit heavy tailed behaviour during 2007-09 and never returned to thin tailed behaviour. Further their research showed that certain large-caps repeatedly exhibit fat-tailed behaviour and this helps identify crash-prone and crash-resilient companies.

Hu (2006) modelled and estimated the dependence pattern across financial markets by applying mixed copula by using Gumbel, Gaussian and Survival (Rotated) Gumbel in a weighted combination. But to estimate this they filter data through GARCH(1,1) to remove heteroskedasticity and get residuals closer to independent and identically distributed returns and then use the filtered data to find the Cumulative Distribution Function that gets input in the copula. Their results showed that US and UK correlation was the strongest at 0.6365. Further, in the indices they analysed, no combination was taking a positive weight, showing the inequality constraint of weight not compatible for the data on the upper tail modelling copula Gumbel. Therefore, they consider only Gaussian and Survival Gumbel to analyse no tail dependence and lower tail dependence respectively. Their findings show that markets with lower correlation have almost same probability to crash together as pairs with higher correlation coefficient.

Recent literature has increasingly applied copula models to analyse market crash risk in various global financial markets especially by Chabi-Yo, Ruenzi, & Weigert (2018) in the United States and the same inspired other authors such as Fjærvik (2023) to analyse the Nordic Stock Market. Chabi-Yo, Ruenzi, & Weigert (2018) implemented a cross-sectional analysis on the returns to analyse the tail dependence through multiple convex combinations of copulas to understand the entire dependence structure including lower tail dependence, upper tail dependence and no tail dependence. They found that stocks with strong LTD have higher average excess returns compared to weak LTD stock.

Although authors have researched the tail-index of stocks in India, especially Nifty50, it focuses on each stock in isolation. Studies on the dependence and extreme co-movement of stock returns and the market returns still remain scarce. Inspired by the methodology and driven by the need to understand market crash risk and its measurement in a developing country such as India, which is an active player and a key competitor in the growing global stock market, this thesis applies the methodology of Chabi-Yo, Ruenzi, & Weigert (2018) to analyse the impact of tail risk on returns.

CHAPTER 3

METHODOLOGY

3.1. Data

The analysis is focused on the firms that are constituent of the current Nifty50 index of the National Stock Exchange. The final sample consists of 50 firms for the years 1995-2025. The weekly stock prices data is collected from the Prowess, a database maintained by the Centre for Monitoring Indian Economy (CMIE). The risk-free rate is measured by the annual 10-year G-Sec rate. Table (1) presents the summary statistics of the data collected.

Table 1: Summary Statistics of Input Variables

Table 1 presents the summary statistics of the main input variables pooled over 30 years. The columns show the mean value of each of the variables, the minimum value, the maximum value and the standard deviation of the variables respectively. Standard deviation is not included for companies per year since it is a count variable.

Variable	Mean	Min	Max	Std Dev
Companies per Year	43	30	50	
Index Return	0.0024	-0.159	0.154	0.03
Risk-Free Rate	0.08	0.05	0.12	0.02

3.2 Overview of the Methodology

Figure 4: Copula Based Returns and Tail Dependence Analysis

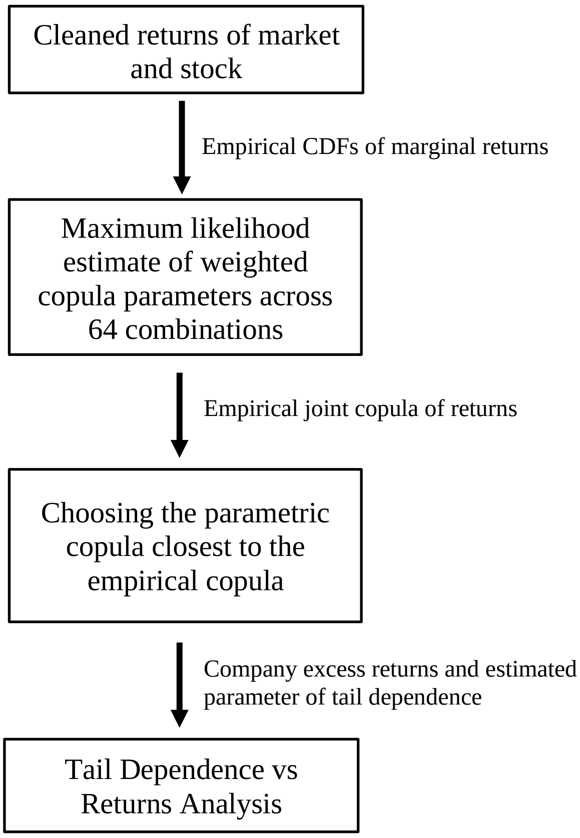


Figure (4) illustrates the methodology used in this thesis: estimating the copula and its parameters, selecting the best model and applying the observed parameters to draw conclusions about the relationship between tail dependence and the excess return realized.

3.3. Copula Estimation:

Since this study is attempting to estimate the lower tail dependence as well the to understand the idea of the whole dependence structure, i.e., upper tail, lower tail as well as to check for no tail dependence (UTD,LTD,NTD), the copula function used incorporates three sets of copulas specifically catering to capture all three dependence structures.

Table 2: Copulas for Different Dependence Structures

Table 2 represents the different copula types that capture different dependence structures. The first column is the set of copulas that capture the lower tail dependence, the second column is for no tail dependence and the third column is for upper tail dependence. The parametric form of these copulas are available in Table (A1) in the Appendix.

LTD	***NTD***	***UTD***
Clayton	Gaussian	Gumbel
Rotated Gumbel	Frank	Joe
Rotated Joe	FGM	Galambos
Rotated Galambos	Plackett	Rotated Clayton

The copula sets mentioned above refer to the use of different models of copula together to capture LTD and UTD simultaneously since using them independently does not allow to do so. Therefore, convex combinations of parametric copulas that either exhibit no tail dependence, lower tail dependence or upper tail dependence are used. Consequently, this involves 64 models for 4 * 4 * 4 combinations.

$$C(u_1, u_2, \Theta) = w_1 \times C_{LTD}(u_1, u_2, \theta_1) + w_2 \times C_{NTD}(u_1, u_2, \theta_2) + (1 - w_1 - w_2) \times C_{UTD}(u_1, u_2, \theta_3) \quad (5)$$

In the combined copula, as given by equation (5), w_1 and w_2 are the weights attached to each copula set, θ_1 θ_2 and θ_3 are the parameters of the said copula sets and $\Theta_j = w_1, w_2, \theta_1, \theta_2, \theta_3$ is the set of all the parameters for j $models$ $where$ $j = 1,2,3 \ldots 64$.

In order to estimate the marginal distribution in the copula and to arrive at parameters, the marginal distributions are estimated by using empirical cumulative distribution function as given by equation(6).

$$\widehat{F_i}(x) = \frac{1}{n+1}\sum_{k=1}^{n} 1_{r_{i,k} \le x} \quad and\; \widehat{F_m}(x) = \frac{1}{n+1}\sum_{k=1}^{n} 1_{r_{m,k} \le x} \tag{6}$$

Here, $\widehat{F_i}(x)$ and $\widehat{F_m}(x)$ are the marginal distribution of individual stock returns r_i and market return r_m respectively.This is used to input in the copula function $\mathrm{C}(\widehat{F_i}(r_i), \widehat{F_m}(r_m))$. $1_{r_{m,k} \le x}$ is an indicator function which will take the value 1 if $r_{m,k} \le x$ and 0 otherwise and n is the number of weekly returns observed in a given period. These marginal distributions are the u and v input into the copula that are always uniformly distributed.

The next step is to input the estimated marginal distributions in the density function of the copula and then further arrive at the loglikelihood of the density function of the combined copula function that captures all three dependence structures as given in Equation (5). This log likelihood is defined in Equation (7)

$$\widehat{\Theta_j} = \underset{\Theta_j}{\operatorname{argmax}}\, \mathrm{L}_j(\Theta_j) \; with\; \mathrm{L}_j(\Theta_j) = \sum_{k=1}^{n} \ln\left(c_j(\widehat{F_i}_{r_{i,k}}, \widehat{F_m}_{r_{i,k}};\ \Theta_j)\right) \tag{7}$$

$$c(u_1, u_2, \dots u_n) = \frac{\partial^n C(u_1, u_2, \dots u_n)}{\partial u_1 \partial u_2 \dots \partial u_n} \tag{8.1}$$

$$Given\ that\ there\ are\ only\ 2\ variables\ in\ this\ study,$$

$$c(u_1, u_2, \Theta) = \frac{\partial^2 C(u_1, u_2, \Theta)}{\partial u_1 \partial u_2} \tag{8.2}$$

Equations (8.1) and (8.2) define the density function of the copula. This density function is the input function for the log-likelihood equation where maximum likelihood estimate approach is implemented to arrive at best parameters.

3.4. Model Selection

In order to select the right copula, an empirical copula is to be estimated on the ranked returns. Further, the best copula combination is one that minimises the distance between the parametric copula $C_{(j)}(.,.;\ \widehat{\Theta}_{(j)})$ and the estimated empirical copula $\hat{C}_{(n)}$. This is necessary because one cannot make a judgement call on which exact copula combination to use to fit their data and the tail dependency of the said data without testing for it since it affects the resultant LTD values.

Suppose there are n observations of returns, these n returns are to be ranked such that $r_{i,k} = 1$ for being the smallest return and $r_{i,k} = \mathrm{n}$ for being the largest return. The empirical copula is defined as given in equation (9).

$$\hat{C}_{(n)}\left(\frac{t_i}{n}, \frac{t_m}{n}\right) = \frac{1}{n}\sum_{k=1}^{n} 1_{r_{i,k} \le t_i} \times 1_{r_{m,k} \le t_i} \; ; t_i = 0,1, \dots n \;\; and \; t_m = 0, 1, \dots n \tag{9}$$

For the purpose of calculating distance, the Integrated Anderson-Darling distance$\mathrm{D}_{\mathrm{j,IAD}}$ is measured and the copula combination that produces minimised distance is chosen.

$$\mathrm{D}_{\mathrm{j,IAD}} = \sum_{t_i=1}^{n}\sum_{t_m=1}^{n} \frac{\left(\hat{C}_{(n)}\left(\frac{t_i}{n}, \frac{t_m}{n}\right) - C_{(j)}\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_J\right)\right)^2}{C_{(j)}\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_J\right) \times \left(1 - C_{(j)}\left(\frac{t_i}{n}, \frac{t_m}{n}; \widehat{\Theta}_J\right)\right)} \tag{10}$$

In this distance equation, the distance is calculated between the predicted value of the parametric copula and the empirical copula for every grid point on the lattice 'L'.

$$L = \left(\frac{t_i}{n}, \frac{t_m}{n}\right) \; ; \; t_i = 0,1, \dots n \;\; and \; t_m = 0, 1, \dots n \tag{11}$$

Once the appropriate combination that minimises the distance is selected and the optimised parameter for the selected model is arrived at, we compute the LTD measures based on the formula appropriate for each copula type. The calculated LTD values are then used to understand the relation between tail dependence and the expected future returns.

3.5. Univariate Portfolio Sorting

The last part of this study is to test the relation between the lower tail dependence values the excess returns through univariate portfolio sorting where portfolios are sorted based on LTD values. For this purpose, portfolios are sorted through quintiles of the LTD values ranging from Weak LTD as the first Quintile and Strong LTD as the fifth Quintile. For each year, we sort companies into 5 quintile portfolios using the yearly LTD values arrived at for each company in that year. The values of LTD of these portfolios is studied against the average expected returns of the portfolio calculated as risk-free return subtracted from portfolio return. Portfolio return is the sum of the weighted return of each company in the portfolio and in this paper, they are considered to be equally weighted portfolios. This process is further repeated for UTD based portfolio sorting.

CHAPTER 4

RESULTS AND DISCUSSION

4.1. Summary statistics of company and index excess returns

Table 3: Summary Statistics of Excess Returns

Table 3 shows the summary statistics of Excess Returns of the company as well as the market pooled over all stocks and years. The returns presented in this chapter is the return excess of the risk free rate. The first five columns show the mean, 25% quantile, 50% quantile, 75% quantile and the standard deviation of each variable. This is presented as 3 separate tables to analyse separate sub-periods as well as the full sample.

Table 3.1: Summary Statistics for the period 1995 - 2008

Variable	Mean	25% Quantile	MEDIAN	75% Quantile	Std Dev
Return (Market)	0.009	-0.232	0.028	0.272	0.341
Return (Company)	0.289	-0.282	0.032	0.481	1.176

Table 3.2: Summary Statistics for the period 2009 - 2025

Variable	Mean	25% Quantile	MEDIAN	75% Quantile	Std Dev
Return (Market)	0.056	-0.014	0.053	0.149	0.187
Return (Company)	0.192	-0.105	0.058	0.327	0.558

Table 3.3: Summary Statistics for the full sample 1995 -2025

Variable	Mean	25% Quantile	MEDIAN	75% Quantile	Std Dev
Return (Market)	0.035	-0.093	0.046	0.176	0.264
Return (Company)	0.230	-0.152	0.047	0.381	0.854

From Table(3.3), it is observed that the market offered an average excess return of 3.5% per annum over the risk-free rate over the full sample period. Comparing Table(3.1) and Table(3.2), it can be seen that the volatilities have noticeably reduced before and after 2008 while the mean returns of the companies have reduced.

4.2. Relationship between LTD and Excess Returns

The main part of the empirical analysis is to understand the relationship between lower-tail dependence and returns after the estimation of lower tail dependence values. For each year, each company, the LTD values are estimated based on the best model selected. The copulas that fit most of the companies is Clayton for lower tail dependence and Joe for upper tail dependence.

Table 4: Summary Statistics of LTD vs Excess Return

Table 4 shows the summary statistics of LTD and Excess Return pooled over all stocks and years. The first five columns show the mean, 25% quantile, 50% quantile, 75% quantile and the standard deviation of each variable. The last three columns show the mean values of the variables conditional on the lower tail dependence being above the 50% quantile and below the 50% quantile as well as the difference and the respective statistical significance. This is presented as 3 separate tables to analyse separate sub-periods as well as the full sample.

Table 4.1: Summary Statistics for the period 1995 - 2008

Variable	Mean	25% Quantile	Median	75% Quantile	Std Dev	Above LTD median	Below LTD median	Above - Below
LTD	0.527	0.384	0.549	0.693	0.232	0.713	0.341	0.3712***
Excess Return	0.289	-0.282	0.032	0.481	1.176	0.404	0.174	0.2307**

Table 4.2: Summary Statistics for the period 2009 - 2025

Variable	Mean	25% Quantile	Median	75% Quantile	Std Dev	Above LTD median	Below LTD median	Above - Below
LTD	0.641	0.543	0.679	0.774	0.195	0.789	0.494	0.2945***
Excess Return	0.192	-0.105	0.058	0.327	0.558	0.132	0.252	(-0.1207)***

Table 4.3: Summary Statistics for the full sample 1995 -2025

Variable	Mean	25% Quantile	Median	75% Quantile	Std Dev	Above LTD median	Below LTD median	Above - Below
LTD	0.597	0.471	0.637	0.754	0.217	0.767	0.427	0.3398***
Excess Return	0.230	-0.152	0.047	0.381	0.854	0.232	0.228	0.0043

In Table(4.1), the sub-period 1995-2008, the mean LTD is 0.527 and the mean excess return when conditioned on the LTD value being higher than the median is 40.4% per annum whereas, for LTD below mean the average return is 17.4% per annum. This suggests that on an average prior to 2009, stocks with higher lower tail dependence tended to earn higher excess return. This result is statistically significant at 5% level of significance. Conversely, in Table (4.2) in the sub-period 2009-2025, the mean LTD is higher at 0.641. In this period, the excess returns are lower when LTD is above median compared to when LTD is below median and this difference is statistically significant at 1% level of significance.

In Table(4.3), for the full sample 1995-2025, the mean LTD is 0.597 and the excess return is slightly higher when LTD is above median compared to the excess return for an LTD below median. But this difference is not statistically significant.

Overall, there is a structural change in the relationship between LTD and excess return after 2008. Before 2008, higher lower tail dependence was compensated with higher excess return whereas this patter reverses post 2008 where investors received lower return excess of the risk-free rate for a higher lower tail dependence and higher excess return for smaller lower tail dependence.

4.3. Relationship between UTD and Excess Returns

To further understand the relationship between returns and tail dependence, this thesis— while primarily focused on lower tail dependence (LTD)— also studies the upper tail dependence to identify potential patters between UTD and returns. Table (5) gives the summary statistics of the UTD and the returns excess of the risk-free rate. Similarly, this part of analysis is also divided into 2 sub-periods aside from observing the full sample.

Table 5: Summary Statistics of UTD vs Excess Return

Table 5 shows the summary statistics of UTD and Excess Return pooled over all stocks and years. The first five columns show the mean, 25% quantile, 50% quantile, 75% quantile and the standard deviation of each variable. The last three columns show the mean values of the variables conditional on the upper tail dependence being above the 50% quantile and below the 50% quantile as well as the difference and the respective statistical significance. This is presented as 3 separate tables to analyse separate sub-periods as well as the full sample

Table 5.1: Summary Statistics for the period 1995 - 2008

Variable	**Mean**	**25% Quantile**	**Median**	**75% Quantile**	**Std Dev**	**Above UTD median**	**Below UTD median**	**Above-Below**
UTD	0.536	0.360	0.582	0.740	0.270	0.754	0.318	0.4354***
Excess Return	0.289	-0.282	0.032	0.481	1.176	0.116	0.462	(-0.3461)***

Table 5.2: Summary Statistics for the period 2009 – 2025

Variable	**Mean**	**25% Quantile**	**Median**	**75% Quantile**	**Std Dev**	**Above UTD median**	**Below UTD median**	**Above-Below**
UTD	0.621	0.508	0.663	0.784	0.229	0.797	0.445	0.3516***
Excess Return	0.192	-0.105	0.058	0.327	0.558	0.152	0.232	(-0.0796)**

Table 5.3: Summary Statistics for the period 1995 – 2025

Variable	Mean	25% Quantile	Median	75% Quantile	Std Dev	Above UTD median	Below UTD median	Above-Below
UTD	0.588	0.451	0.628	0.774	0.249	0.783	0.393	0.3898***
Excess Return	0.230	-0.152	0.047	0.381	0.854	0.146	0.313	(-0.1671)***

In Table(5.1), for the period 1995-2008, as UTD increases, lower excess returns are realised. This implies that for stocks with lower downside risk tend to have lesser excess returns. Similar pattern is followed post-2008, in Table(5.2) where with higher UTD, lower average excess returns are observed, though the gap narrows relative to pre-2008 significant at 5% level of significance. This pattern is noticed to follow through when the whole sample is analysed as well. This result is statistically significant at 1% level. This is in line with the theoretical understanding the investors are more concerned about downward risk and that upward movement of returns does not carry a risk that needs to be compensated with excess returns.

4.4. Univariate Portfolio Sorting

Portfolio sorting includes sorting based on UTD and LTD to identify any potential patterns. Equally weighted portfolios were constructed annually, with stocks sorted into quintiles based on their LTD and UTD estimates, ranging from weakest to strongest dependence. This approach enables an examination of whether portfolios with higher dependence on market movements yield higher returns in excess of the risk-free rate. The portfolio sorting results, aggregated over 30 years using a weighted average, are presented in Table (6).

Table 6 : Portfolio Sorting Based on LTD and UTD values

Table 6.1: Portfolio sorted based on LTD

Portfolio	Portfolio LTD	Excess Return
1 Weak LTD	0.3101	0.2846
2	0.5182	0.2519
3	0.6203	0.1745
4	0.7078	0.2143
5 Strong LTD	0.8404	0.2255
Strong – Weak	0.5303	-0.0591

Table 6.2: Portfolio sorted based on UTD

Portfolio	Portfolio UTD	Excess Return
1 Weak UTD	0.2333	0.3045
2	0.4984	0.2416
3	0.6258	0.2116
4	0.7281	0.2225
5 Strong UTD	0.8684	0.1571
Strong – Weak	0.6351	-0.1475

In Table (6.1), as LTD increases from quintile 1 to quintile 5, from 0.3101 to 0.8404, the excess return is lower for the strongest LTD compared to the weakest ltd. Excess return is seen to be the highest for the weakest LTD portfolio. The Strong – Weak difference shows a return difference of -0.0591, shows that portfolios more exposed to downside risk do not deliver higher returns.

In Table (6.2), as UTD increases steadily from 0.2333 to 0.8684 across quintiles, excess returns decline from 0.3045 in the weakest UTD portfolio to 1.571 in the strongest UTD portfolio. The return difference is -0.1475, relatively larger than LTD showing that portfolios more exposed to upper tail dependence tend to have steadily declining returns.

In summary, data shows that while upper tail dependence provides lower excess return, investors in stocks and portfolios that exhibit lower tail dependence are also not compensated for the increasing returns, especially post 2008, the year of the outbreak of the global financial crisis. This challenges the theoretical expectation that bearing higher downside risk should be compensated with higher returns showing evidence that there might exist a factor of uncompensated risk in the Indian market scenario.

CHAPTER 5

CONCLUSION

This thesis investigated the relationship between stock returns and their dependence on the movement of market returns by applying copulas to capture this dependence. By applying various copula functions to estimate lower tail dependence and upper tail dependence for the stocks in Nifty50 index over the time horizon of 1995 to 2025, the study sought to examine whether stocks with higher lower tail dependence compensated investors with higher excess return over the risk-free rate.

The empirical results reveal that stocks exhibiting higher lower tail dependence do not always offer higher excess returns. Through sorting portfolios based on LTD, in most cases the strongest LTD portfolio did not have the highest excess return while the 4^{th} quintile or 3^{rd} quintile LTD offered the better returns. Furthermore, the analysis of upper tail dependence showed that for greater UTD, the excess returns were declining.

These finding challenge the theoretical notion that higher downward risk is not always compensated with higher returns. This shows that there might be a need to consider separate factor of lower tail risk that is left uncaptured by traditional beta. Furthermore, the structural change in the relation between LTD and returns post the global financial crisis of 2008 could signify a profound impact of the crisis on the risk perception of returns realised. This implies that rather than being compensated with higher return for higher market crash exposure, investors realise lower returns possibly signifying that stocks with higher lower tail dependence hold an uncompensated tail risk factor.

Limitation of this study is the implementation of an equally weighted portfolio sorting method which may not accurately reflect the weighting schemes of real-world investment portfolios.

Further research could be extended to include a broader set of companies across different industries or a more specific sector-wise analysis which would help understand how dependency changes based on the sector in which a particular firm is. Additionally, integrating dynamic or time varying copula models could allow for a more elaborate understanding of dependence that would capture the evolving market conditions through a time-series analysis. Examining the interaction between firm specific factors such as liquidity, size and book to market equity through bivariate or multivariate portfolio analysis could help further understand the contributing factors to tail risk.

APPENDIX

Table A1 : Bivariate Copula Functions with Tail Dependence Coefficients

This table reports the parametric forms of the bivariate copula functions considered in this study in the second column and the corresponding lower and upper tail dependence coefficients, LTD and UTD, in the last two columns. In this thesis, we define $\bar{u}_1 = 1 - u_1$ and $\bar{u}_2 = 1 - u_2$. Φ denotes the standard normal N(0, 1) distribution function, Φ^{-1} is the functional inverse of Φ and Φ_θ is the bivariate normal distribution function with correlation θ. standard normal distribution function with correlation θ.

Copula	Parametric Form	LTD	UTD
Clayton	$C_{Cla}(u_1, u_2; \theta) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$2^{-1/\theta}$	—
Rotated-Gumbel	$C_{RGum}(u_1, u_2) = u_1 + u_2 - 1 + \exp(-((-\log(\bar{u}_1))^\theta + (-\log(\bar{u}_2))^\theta)^{1/\theta})$	$2 - 2^{1/\theta}$	—
Rotated-Joe	$C_{RJoe}(u_1, u_2) = u_1 + u_2 - (u_1^\theta + u_2^\theta - u_1^\theta \cdot u_2^\theta)^{1/\theta}$	$2 - 2^{1/\theta}$	—
Rotated-Galambos	$C_{RGal}(u_1, u_2) = u_1 + u_2 - 1 + (\bar{u}_1) \cdot (\bar{u}_2) \cdot \exp(((-\log(\bar{u}_1))^{-\theta} + (-\log(\bar{u}_2))^{-\theta})^{-1/\theta})$	$2^{-1/\theta}$	—
Gauss	$C_{Gau}(u_1, u_2; \theta) = \Phi_\theta(\Phi^{-1}(u_1), \Phi^{-1}(u_2))$	—	—
Frank	$C_{Fra}(u_1, u_2; \theta) = -\theta^{-1} \log((1-\exp(-\theta) - ((1-\exp(-\theta u_1))(1-\exp(-\theta u_2))) / (1-\exp(-\theta)))$	—	—
Plackett	$C_{Pla}(u_1, u_2; \theta) = \frac{1}{2}((\theta-1)^{-1}\{1+(\theta-1)(u_1+u_2) - [(1+(\theta-1)(u_1+u_2))^2 - 4\theta u_1 u_2]^{1/2}\}$	—	—
F-G-M	$C_{Fgm}(u_1, u_2; \theta) = u_1 u_2 (1 + \theta(1 - u_1)(\bar{u}_2))$	—	—
Joe	$C_{Joe}(u_1, u_2; \theta) = 1 - ((\bar{u}_1)^\theta + (\bar{u}_2)^\theta - (\bar{u}_1)^\theta \cdot (\bar{u}_2)^\theta)^{1/\theta}$	—	$2 - 2^{1/\theta}$
Gumbel	$C_{Gum}(u_1, u_2; \theta) = \exp(-((-\log(u_1))^\theta + (-\log(u_2))^\theta)^{1/\theta})$	—	$2 - 2^{1/\theta}$
Galambos	$C_{Gal}(u_1, u_2; \theta) = u_1 \cdot u_2 \cdot \exp(-((-\log u_1)^\theta + (-\log u_2))^\theta)^{1/\theta})$	—	$2^{-1/\theta}$
Rotated-Clayton	$C_{RCla}(u_1, u_2; \theta) = u_1 + u_2 - 1 + ((\bar{u}_1)^{-\theta} + (\bar{u}_2)^{-\theta} - 1)^{-1/\theta}$	—	$2^{-1/\theta}$

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