1 Single dimension

Given robot position r and point on wall "x" along with frame width 2b. The step function looks like:

$$S(x,r) = \frac{1}{1+e^{-a(-r+x+b)}} + \frac{1}{1+e^{a(-r+x-b)}} - 1 \tag{1}$$

$$S(r,x) = \frac{2 + e^{-ab}(e^{-a(-r+x)} + e^{a(-r+x)})}{(e^{-ab}(e^{-a(-r+x)} + e^{a(-r+x)}) + e^{-2ab} + 1)} - 1 \tag{2}$$

$$e^{-a(-r+x)} + e^{a(-r+x)} = 2cosh(a(-r+x)) \tag{3}$$

$$e^{-ab} = k \tag{4}$$

by solving it further, and substituting eq 3,4 in eq 2

$$S(r,x) = \frac{2(1 + kcosh(a(-r+x)))}{k(2cosh(a(-r+x)) + k) + 1} - 1 \tag{5}$$

Expection of point x is covered on wall is

$$E(x) = \int_{-inf}^{+inf} P(r)S(r,x)dr \tag{6}$$

Consider one dimension space x and position distribution to be gaussian

$$P(r) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}(\frac{x-r}{\sigma})^2} \tag{7}$$

Then expectation becomes

$$E(x) = \tag{8}$$