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Foundations of Computer Science
Homework #1 - Basics + Using Latex
Problems: 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9
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0.1 Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.

a. $\{1, 3, 5, 7, \dots\}$

This is a set of all real odd numbers.

b. $\{\dots, -4, -2, 0, 2, 4, \dots\}$

This is a set of all positive and negative even numbers.

c. $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}\}$

This is a set of all even positive numbers.

d. $\{n \mid n = 2m \text{ for some } m \text{ in } \mathbb{N}, \text{ and } n = 3k \text{ for some } k \text{ in } \mathbb{N}\}$

This is a set of all positive numbers which are also a multiple of six.

e. $\{w \mid w \text{ is a string of 0s and 1s and } w \text{ equals the reverse of } w\}$

This is a set of all binary palindrome strings, which are strings that are the same when read forward or backwards.

f. $\{n \mid n \text{ is an integer and } n = n + 1\}$

This is an empty set: \emptyset

0.2 Write formal descriptions of the following sets.

a. The set containing the numbers 1, 10, and 100

$$\{1, 10, 100\}$$

b. The set containing all integers that are greater than 5

$$\{n \in \mathbb{Z} \mid n > 5\}$$

c. The set containing all natural numbers that are less than 5

$$\{n \in \mathbb{N} \mid n < 5\}$$

- d. The set containing the string aba
 $\{aba\}$
- e. The set containing the empty string
 $\{\epsilon\}$
- f. The set containing nothing at all
 $\{\emptyset\}$

0.3 Let A be the set $\{x, y, z\}$ and B be the set $\{x, y\}$

a. Is A a subset of B?

No, A is not a subset of B because "z" is not in B, and to be a subset of another set all elements of A must be contained in B.

b. Is B a subset of A?

Yes, B is a subset of A because all elements in B are contained in A.

c. What is $A \cup B$?

$$A \cup B = \{x, y, z\}$$

d. What is $A \cap B$?

$$A \cap B = \{x, y\}$$

e. What is $A \times B$?

$$A \times B = \{(x, x), (x, y), (y, x), (y, y), (z, x), (z, y)\}$$

f. What is the power set of B?

$$P(B) = \{\emptyset, \{x\}, \{y\}, \{x, y\}\}$$

0.4 If A has a elements and B has b elements, how many elements are in $A \times B$? Explain your answer

For every element in A, there are B ordered pairs. This means that there will be $A * B$ elements

- 0.5 If C is a set with c elements, how many elements are in the power set of C ? Explain your answer.

If $|C| = c$, then $P(C) = 2^C$. This is because if a set has n members, then the power set will have 2^n members.

As an example, consider set $A: \{1, 2, 3\}$:

subsets: $\{1\}$, $\{2\}$, $\{3\}$, $\{1,2\}$, $\{1,3\}$, $\{2,3\}$ as well as $\{1,2,3\}$ and the empty set of $\{\}$

When we combine all of these sets, we get the powerset:

$$P(A) = \{\{\}, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\}$$

There are 3 elements in set A with 8, and $2^3 = 8$, which shows that $P(C) = 2^C$.

0.6

- a. The value of $f(2)$ is 7.
- b. The domain of f is X and the range is Y .
- c. The value of $g(2, 10)$ is 6.
- d. The domain of g is $X \times Y$ and the range is Y .
- e. $g(4, f(4))$ is equal to $g(4, 7)$ which equals 8.

0.7 For each part, give a relation that satisfies the condition.

- a. Reflexive and symmetric but not transitive

Set A contains $\{1, 2, 3\}$

$$R = \{(1, 1), (2, 2), (3, 3), (2, 1), (1, 2)\}$$

- b. Reflexive and transitive but not symmetric

Set B contains $\{1, 2, 3\}$

$$R = \{(1, 2), (2, 1), (2, 2), (1, 1)\}$$

- c. Symmetric and transitive but not reflexive

Set C contains 1, 2, 3

$$R = \{(1, 2), (2, 1), (1, 1), (2, 2)\}$$

0.8

Node 1 has a degree of 3

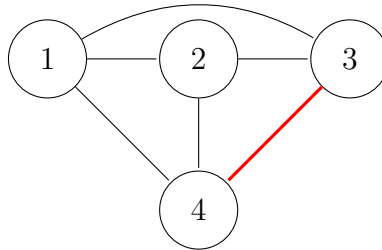
Node 2 has a degree of 3

Node 3 has a degree of 3

Node 4 has a degree of 3

Note: before we $3 \rightarrow 4$ was added, node 3 had a degree of 2 and node 4 has a degree of 2.

The graph looks like:



0.9 Write a formal description of the following graph.

$G = (V, E)$ where $V = \{1, 2, 3, 4, 5, 6\}$ and

$E = \{(1,4), (1,5), (1,6), (2,4), (2,5), (2,6), (3,4), (3,5), (3,6)\}$

Drawn in Latex this would be:

