An Introduction to Financial Engineering

P. Srikant

Visiting Faculty, IIM Calcutta

September 2, 2024

What is Financial Engineering?

- Financial engineering refers to the process of using existing financial instruments to manufacture new financial instruments to cater to the needs of different clienteles
- My goals today:
  - interest you in financial economics
    - time, chance, financial claims
  - introduce some simple static examples

Arbitrage

- (chance of) Risk-free profit
- chocolate bar costs 15 rupees inside campus and 10 rupees outside– buy chocolate bar outside and sell inside pocketing the difference
- As you do this :
  - demand for chocolates increases outside campus, pushing up the prices of chocolates outside campus
  - and supply of chocolates increases inside campus, pushing down prices.
- Your profit seeking forces the prices to converge.
- Arbitrage can be across space, time or sum of parts

Value Additivity

- Pricing a phone

Value Additivity

- Pricing a phone
- C = A + B. If A is priced at 30 and B is priced at 70, and its is costless to combine A and B to make C; or to break down C into A and B. Then C should be priced at 100
- Note that we have assumed prices for A and B. Arbitrage pricing does NOT explain why A is priced at 30 (though you could argue, tautologically, that A is 30 because C is 100 and B is 70) . Equilibrium is a standalone theory of pricing, while arbitrage is not - it does not seeks to explain how the battery of your phone gets its price.
- Nonetheless, given prices of any two, the arbitrage approach to pricing lets you determine the price of the third
- Equilibrium and arbitrage pricing are both useful , and should be viewed as complementary approaches.

The Law of One Price

- In equilibrium things which **always** have the same payoffs have the the same price. This is called the law of one price.
- If this did not hold : Arbitrages would exist. Arbitrage is the glue that binds different markets together.
- In a Type 1 arbitrage, you make a sure profit. In a Type 2 arbitrage, you have some chance of making a profit, but no choice of making a loss (free lottery ticket)

FE Approach

- Find a combination of existing products that give you the desired payoffs
- You have replicated the desired product
- The price of the product is the sum of the prices of the parts
  - what it costs to make
- Let's look at some toy non-financial examples first ..

LEGO Bricks

Numberblocks

Repackaging

- Suppose you work at a grocery, where beer is sold in 6 packs and cigarettes in packs of 10.
  - Customer 1 comes along, and wants 4 beers and 2 cigarettes.
  - Customer 2 wants 8 beers and 8 cigarettes.
- How can you meet their needs?

Fruit Baskets

- A grocery store sells only the following combinations of fruit (baskets) : 2 apples and 6 bananas ; 1 apple and 2 bananas
  - The 2 apples and 6 banana combo is priced at 70 Rs.
  - The 1 apple and 2 bananas combo costs 30.
- Assume that apples and bananas are not available for sale individually, so you cannot just look up their prices. If it is costless to break up and put together combinations, can you determine what the price of an apple should be ? And the price of a banana? What will 5 apples and 3 bananas cost ?

Fruit Baskets

- A grocery store sells only the following combinations of fruit (baskets) : 2 apples and 6 bananas ; 1 apple and 2 bananas
  - The 2 apples and 6 banana combo is priced at 70 Rs.
  - The 1 apple and 2 bananas combo costs 30.
- Assume that apples and bananas are not available for sale individually, so you cannot just look up their prices. If it is costless to break up and put together combinations, can you determine what the price of an apple should be ? And the price of a banana? What will 5 apples and 3 bananas cost ?
- Building blocks– Apple : 20– Banana : 5
  Combine to desired basket– 115

Direct Replication

- Here, we broke existing products down into building blocks, and assembled them to give us whatever payoffs we wanted.
- We could also proceed directly - replicate 5 apples and 3 bananas in terms of the available baskets.

Direct Replication

- Here, we broke existing products down into building blocks, and assembled them to give us whatever payoffs we wanted.
- We could also proceed directly - replicate 5 apples and 3 bananas in terms of the available baskets.
- Let x of (2,6) and y of (1,2) give (5,3)
- Apple Equation: $2x + y = 5$
- Banana Equation: $6x + 2y = 3$

Direct Replication

- Here, we broke existing products down into building blocks, and assembled them to give us whatever payoffs we wanted.
- We could also proceed directly - replicate 5 apples and 3 bananas in terms of the available baskets.
- Let x of (2,6) and y of (1,2) give (5,3)
- Apple Equation: $2x + y = 5$
- Banana Equation: $6x + 2y = 3$
- (x,y) = (-3.5,12) ; Cost: 12*30-3.5*70=360-245=115
- Directly construct from tradeables; may involve fractional units, shortselling
- Analysis/synthesis versus replication; choice of building blocks isn't unique but some appear "obvious"

Government Bonds

- Default free, annual coupons on the same calendar date
  - 4% bond maturing next year
    cash flows = (104, 0)
    price today = 100
  - 6% bond maturing in 2 years
    cash flows = (6, 106)
    price today = 101
- What can we use for as building blocks?
- Can we use these to make a 5% bond, and what would that cost ?

Zeroes

- choose securities that pay off \$1 at time 1, 2.
  We call these zero coupon bonds Z1, Z2

time:	1	2	prices (time 0)
A	104	0	100
B	6	106	101
Z1	1	0	d1=?
Z2	0	1	d2=?

Zeroes

- choose securities that pay off \$1 at time 1, 2.
  We call these zero coupon bonds Z1, Z2

time:	1	2	prices (time 0)
A	104	0	100
B	6	106	101
Z1	1	0	d1=?
Z2	0	1	d2=?

- Prices: Z1 = 0.9615 (4.0000%); Z2 = 0.8984 (5.5029%)

Replicating a 5% bond

- Method 1: 5*0.9615 + 105*0.8984 = 99.14
- Method 2: x A, y B
  - time 0 : 104x + 6y = 5
  - time 1: 0 + 106y = 105

Replicating a 5% bond

- Method 1: 5*0.9615 + 105*0.8984 = 99.14
- Method 2: x A, y B
  - time 0 : 104x + 6y = 5
  - time 1: 0 + 106y = 105
  - (x,y) = (-0.0091, 0.9906)
    which costs (-0.0091*100 + 0.9906 * 101 = 99.14)
- Suppose a 5% bond were trading in the market at 100 / 98 .
  What could you do ?

Risky and Safe Assets

- 2 financial assets - a stock and a safe (government) bond; and two states - Growth and Recession.
  - An investment of 100 Rs in the bond yields 110 in both Growth and Recession.
  - an investment of 100 in the stock yields 200 in Growth, and 80 in recession
- What can we use for as building blocks?
- Can we use these to make a security that pays 80 in Growth and 0 in recession, and what would it cost ?
  - this is the payoff off a call option with a strike of 120

AD securities

- choose securities that pay off \$1 in Growth or Recesssion at time 1. We call these securities G and R
  These are known as Arrow-Debreu / pure securities

state:	Growth	Recession	prices (time 0)
Stock	200	80	100
Bond	110	110	100
G	1	0	d1=?
R	0	1	d2=?

AD securities

- choose securities that pay off \$1 in Growth or Recesssion at time 1. We call these securities G and R
  These are known as Arrow-Debreu / pure securities

state:	Growth	Recession	prices (time 0)
Stock	200	80	100
Bond	110	110	100
G	1	0	d1=?
R	0	1	d2=?

- Prices: G = 0.2273, R = 0.6818

Replicating a call

- Method 1: 80*0.2273 = 18.18
- Method 2: x Stock, y Bond
  - growth : 200x + 110y = 80
  - recession: 80x+110y = 0

Replicating a call

- Method 1: 80*0.2273 = 18.18
- Method 2: x Stock, y Bond
  - growth : 200x + 110y = 80
  - recession: 80x+110y = 0
  - (x,y) = (0.6667, -0.4848)
    which costs (0.6667*100 -0.4848* 100 = 18.18)
- Real world asset dynamics are more complex ..
- Nobels: Arrow, Debreu, Scholes+Merton

Financial Innovation: Good or Bad?

- Financial innovations have generally increased welfare (Ascent of Money, Against the Gods)
- Derivatives have been at the forefront of financial innovation
  - Allow for risk transfer and management , make trading cheaper, allocate capital efficiently
  - however increase risks : "financial weapons of mass destruction, carrying dangers that, while now latent, are potentially lethal"
- Financial innovations often triggered by investor needs, or circumvent taxes or regulations

Markets will find a way