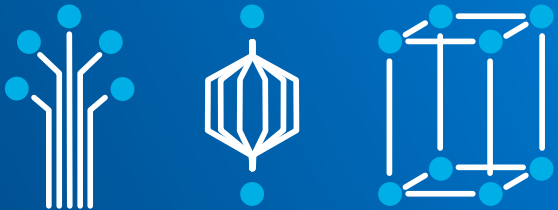
Fundamentals of Parallelism on Intel® Architecture

Week 2
Vectorization



COLFAX RESEARCH

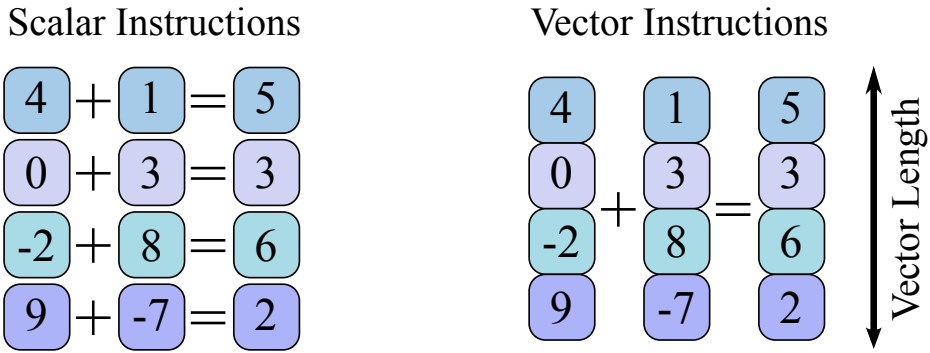
May 2017

intel

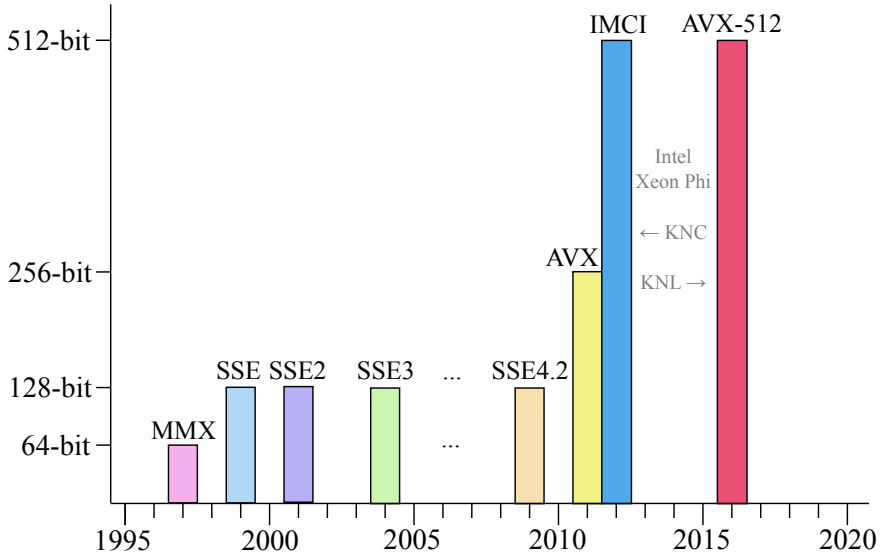
§1. Vector Operations

Short Vector Support

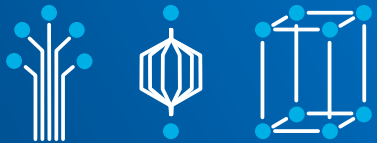
Vector instructions – one of the implementations of SIMD (Single Instruction Multiple Data) parallelism.



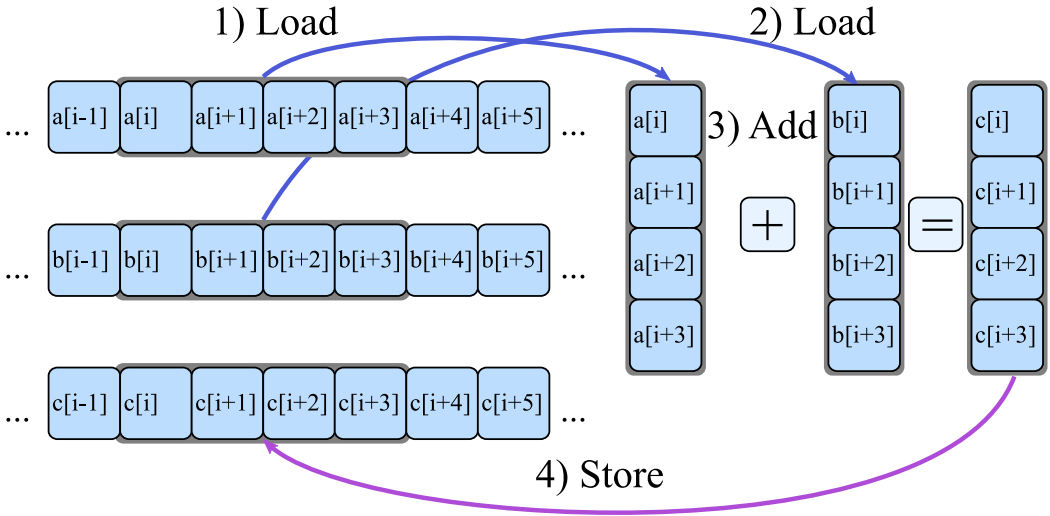
Instruction Sets in Intel Architecture



§2. Vectorizing your Code



Workflow of Vector Computation



Using Vector Instructions: Two Approaches

Automatic Vectorization →

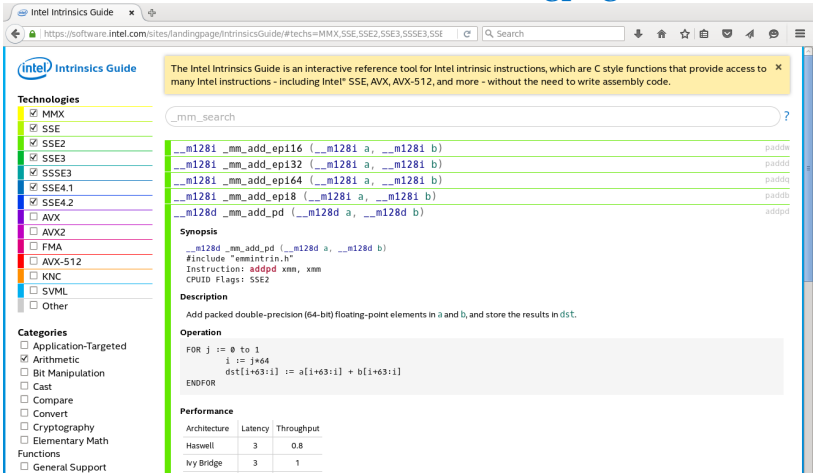
```
double A[vec_width], B[vec_width];
// ...
for(int i = 0; i < vec_width; i++)
  A[i]+=B[i];
```

```
double A[8], B[8];
__m512d A_v = _mm512_load_pd(A);
__m512d B_v = _mm512_load_pd(B);
A_v = _mm512_add_pd(A_v,B_v);
_mm512_store_pd(A, A_v);
```

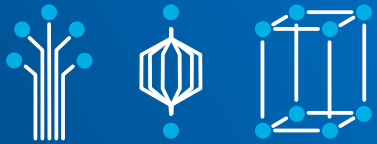
← Explicit Vectorization

Intel Intrinsics Guide

https://software.intel.com/sites/landingpage/IntrinsicsGuide



§3. Automatic Vectorization



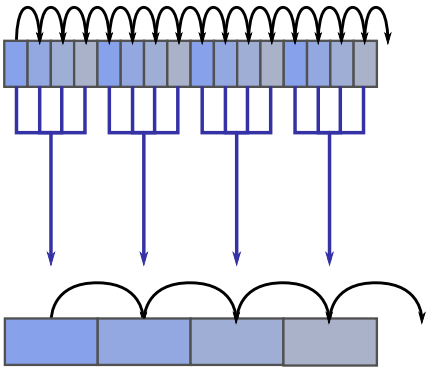
Automatic Vectorization of Loops

```
#include <cstdio>

int main(){
  const int n=1024;
  int A[n] __attribute__((aligned(64)));
  int B[n] __attribute__((aligned(64)));

  for (int i = 0; i < n; i++)
    A[i] = B[i] = i;

  // This loop will be auto-vectorized
  for (int i = 0; i < n; i++)
    A[i] = A[i] + B[i];

  for (int i = 0; i < n; i++)
    printf("%2d %2d %2d\n",i,A[i],B[i]);
}
```

```
vega@lyra% icpc autovec.cc -qopt-report
vega@lyra% cat autovec.optrpt
...
LOOP BEGIN at autovec.cc(12,3)
remark #15399: vectorization support:
unroll factor set to 2 [autovec.cc(12,3)]
remark #15300: LOOP WAS VECTORIZED
[autovec.cc(12,3)]
LOOP END
...
vega@lyra% ./a.out
0  0  0
1  2  1
2  4  2
3  6  3
4  8  4
...
```


Limitations on Automatic Vectorization

- Innermost loops*
- Known number of iterations
- No vector dependence
- Functions must be SIMD-enabled



* `#pragma omp simd` to override

Targeting a Specific Instruction Set

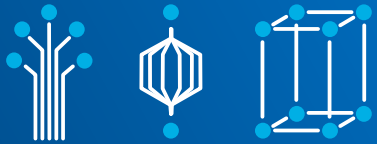
`-x[code]` to target specific processor architecture
`-ax[code]` for multi-architecture dispatch

`code`	Target architecture
MIC-AVX512	Intel Xeon Phi processors (KNL)
CORE-AVX512	Future Intel Xeon processors
CORE-AVX2	Intel Xeon processor E3/E5/E7 v3, v4 family
AVX	Intel Xeon processor E3/E5 and E3/E5/E7 v2 family
SSE4.2	Intel Xeon processor 55XX, 56XX, 75XX and E7 family
host	architecture on which the code is compiled

Auto-Vectorized Loops May Be Complex

```
1  for (int i = ii; i < ii + tileSize; i++) { // Auto-vectorized
2
3    // Newton's law of universal gravity
4    const float dx = particle.x[j] - particle.x[i]; // x[j] is a const
5    const float dy = particle.y[j] - particle.y[i]; // x[i] -> vector
6    const float dz = particle.z[j] - particle.z[i];
7    const float rr = 1.0f/sqrtf(dx*dx + dy*dy + dz*dz + softening);
8    const float drPowerN32  = rr*rr*rr;
9
10   // Calculate the net force
11   Fx[i-ii] += dx * drPowerN32;
12   Fy[i-ii] += dy * drPowerN32;
13   Fz[i-ii] += dz * drPowerN32;
14 }
```


§4. Guided Automatic Vectorization



Vectorize more loops: `#pragma omp simd`

Used to “enforce vectorization of loops”, which includes:

- Loops with SIMD-enabled functions
- Second innermost loops
- Failed vectorization due to compiler decision
- Where guidance is required (vector length, reduction, etc.)

See OpenMP reference for syntax; `#pragma simd`

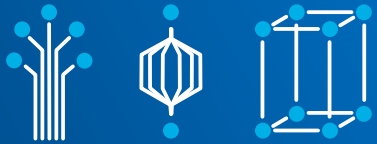
Example for #pragma omp simd

```
1  const int N=128, T=4;
2  float A[N*N], B[N*N], C[T*T];
3
4  for (int jj = 0; jj < N; jj+=T) // Tile in j
5    for (int ii = 0; ii < N; ii+=T) // and tile in i
6  #pragma omp simd   // Vectorize outer loop
7    for (int k = 0; k < N; ++k) // long loop, vectorize it
8      for (int i = 0; i < T; i++) { // Loop between ii and ii+T
9        // Instead of a loop between jj and jj+T, unrolling that loop:
10       C[0*T + i] += A[(jj+0)*N + k]*B[(ii+i)*N + k];
11       C[1*T + i] += A[(jj+1)*N + k]*B[(ii+i)*N + k];
12       C[2*T + i] += A[(jj+2)*N + k]*B[(ii+i)*N + k];
13       C[3*T + i] += A[(jj+3)*N + k]*B[(ii+i)*N + k];
14 }
```


Vectorization Directives

- `#pragma omp simd`
- `#pragma vector always`
- `#pragma vector aligned | unaligned`
- `__assume_aligned` keyword
- `#pragma vector nontemporal | temporal`
- `#pragma novector`
- `#pragma ivdep`
- `restrict` qualifier and `-restrict` command-line argument
- `#pragma loop count`

§5. SIMD-enabled Functions



SIMD-Enabled Functions

Define function in one file (e.g., library), use in another

```
// Compiler will produce 3 versions:
#pragma omp declare simd
float my_simple_add(float x1, float x2){
  return x1 + x2;
}
```

```
// May be in a separate file
#pragma omp simd
for (int i = 0; i < N, ++i) {
  output[i] = my_simple_add(inputa[i], inputb[i]);
}
```


SIMD-enabled Functions May Be Complex

```
1  #pragma omp declare simd
2  float MyErfElemental(const float inx){
3      const float x = fabsf(inx); // Absolute value (in each vector lane)
4      const float p = 0.3275911f; // Constant parameter across vector lanes
5      const float t = 1.0f/(1.0f+p*x); // Expression in each vector lanes
6      const float l2e = 1.442695040f; // log2f(expf(1.0f))
7      const float e = exp2f(-x*x*l2e); // Transcendental in each vector lane
8      float res = -1.453152027f + 1.061405429f*t; // Computing a polynomial
9      res = 1.421413741f + t*res;                 // in each vector lane
10     res =-0.284496736f + t*res;
11     res = 0.254829592f + t*res;
12     res *= e;
13     res = 1.0f - t*res; // Analytic approximation in each vector lane
14     return copysignf(res, inx); // Copy sign in each vector lane
15 }
```


§6. Vector Dependence

True Vector Dependence

- True vector dependence – vectorization impossible:

```
1 for (int i = 1; i < n; i++)
2   a[i] += a[i-1]; // dependence on the previous element
```

- Safe to vectorize:

```
1 for (int i = 0; i < n-1; i++)
2   a[i] += a[i+1]; // no dependence on the previous element
```

- May be safe to vectorize:

```
1 for (int i = 16; i < n; i++)
2   a[i] += a[i-16]; // no dependence if vector length <=16
```


Assumed Vector Dependence

Not enough information to confirm or rule out vector dependence:

```
1 void AmbiguousFunction(int n, int *a, int *b) {
2   for (int i = 0; i < n; i++)
3     a[i] = b[i];
4 }
```

- If `a`, `b` are not aliased or `b>a`, then safe to vectorize
- If `a`, `b` are aliased (e.g., `b==a-1`), requires scalar computation

Multiversioning

```
user@host% icpc -c code.cc -qopt-report
user@host% cat code.optrpt
...
LOOP BEGIN at code.cc(4,1)
<Multiversioned v1>
   remark #25228: LOOP WAS VECTORIZED
LOOP END
...
LOOP BEGIN at code.cc(4,1)
<Multiversioned v2>
   remark #15304: loop was not vectorized: non-vectorizable loop instance ..
LOOP END
```

Pointers checked for aliasing *runtime* to choose code path.

Pointer Disambiguation

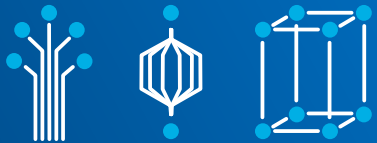
Prevent multiversioning or allow vectorization with a directive:

```
1 #pragma ivdep
2   for (int i = 0; i < n; i++)
3     // ...
```

```
user@host% icpc -c code.cc -qopt-report -qopt-report-phase:vec
user@host% cat vdep.optrpt
...
LOOP BEGIN at code.cc(4,1)
   remark #25228: LOOP WAS VECTORIZED
LOOP END
...
```

Alternative: keyword `restrict` – more fine-grained, weaker.

§7. Strip-Mining



Strip-Mining for Vectorization

- Programming technique that turns one loop into two nested loops.
- Used to expose vectorization opportunities.

Original:

```
1 for (int i = 0; i < n; i++) {
2   // ... do work
3 }
```

Strip-mined:

```
1 const int STRIP=1024;
2 const int nPrime = n - n%STRIP;
3 for (int ii=0; ii<nPrime; ii+=STRIP)
4   for (int i=ii; i<ii+STRIP; i++)
5     // ... do work
6
7 for (int i=nPrime; i<n; i++)
8   // ... do work
```


§8. Example: Stencil Code

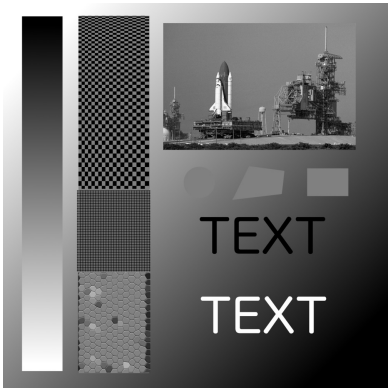
Stencil Operators

- Linear systems of equations
- Partial differential equations

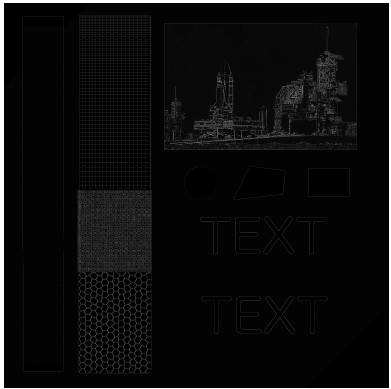
$$
\begin{aligned}
Q_{x,y} = \; & c_{00}P_{\mathrm{x-1,y-1}} + c_{01}P_{\mathrm{x,y-1}} + c_{02}P_{\mathrm{x+1,y-1}} + \\
& c_{10}P_{\mathrm{x-1,y}} + c_{11}P_{\mathrm{x,y}} + c_{12}P_{\mathrm{x+1,y}} + \\
& c_{20}P_{\mathrm{x-1,y+1}} + c_{21}P_{\mathrm{x,y+1}} + c_{22}P_{\mathrm{x+1,y+1}}
\end{aligned}
$$

Fluid dynamics, heat transfer, image processing (convolution matrix), cellular automata.

Edge Detection



$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix} \rightarrow$$



Basic Implementation

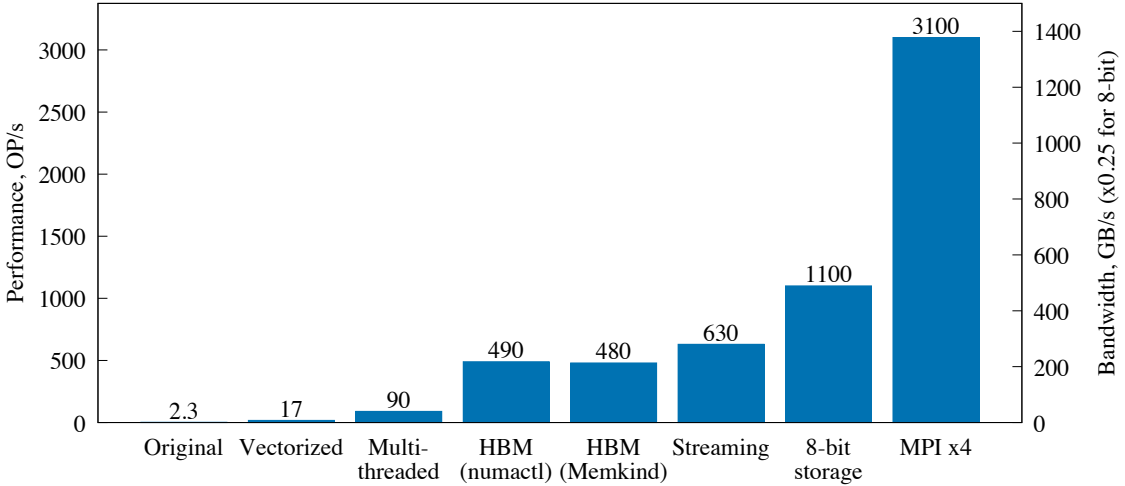
```
1  float *in, *out; // Input and output images
2  int width, height; // Dimensions of the input image
3
4  // Image convolution with the edge detection stencil kernel:
5  for (int i = 1; i < height-1; i++)
6   for (int j = 1; j < width-1; j++)
7    out[i*width + j] =
8    -in[(i-1)*width + j-1] -   in[(i-1)*width + j] - in[(i-1)*width + j+1]
9    -in[(i  )*width + j-1] + 8*in[(i  )*width + j] - in[(i  )*width + j+1]
10   -in[(i+1)*width + j-1] -   in[(i+1)*width + j] - in[(i+1)*width + j+1];
```


Vectorization

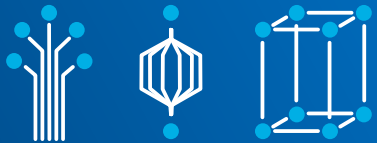
```
user@vega% icpc -c -qopt-report=5 -xMIC-AVX512 stencil.cc
```

```
1 for (int i = 1; i < height-1; i++)
2 #pragma omp simd
3  for (int j = 1; j < width-1; j++)
4   out[i*width + j] =
5   -in[(i-1)*width + j-1] -   in[(i-1)*width + j] - in[(i-1)*width + j+1]
6   -in[(i  )*width + j-1] + 8*in[(i  )*width + j] - in[(i  )*width + j+1]
7   -in[(i+1)*width + j-1] -   in[(i+1)*width + j] - in[(i+1)*width + j+1];
```

Performance



§9. Example: Numerical Integration



Midpoint Rectangle Method

$$I(a,b) = \int_0^a f(x)\mathrm{d}x \approx \sum_{i=0}^{n-1} f\left(x_{\mathrm{i}+\frac{1}{2}}\right)\Delta x,$$

where

$$\Delta x = \frac{a}{n}, \qquad x_{\mathrm{i}+\frac{1}{2}} = \left(i+\frac{1}{2}\right)\Delta x.$$

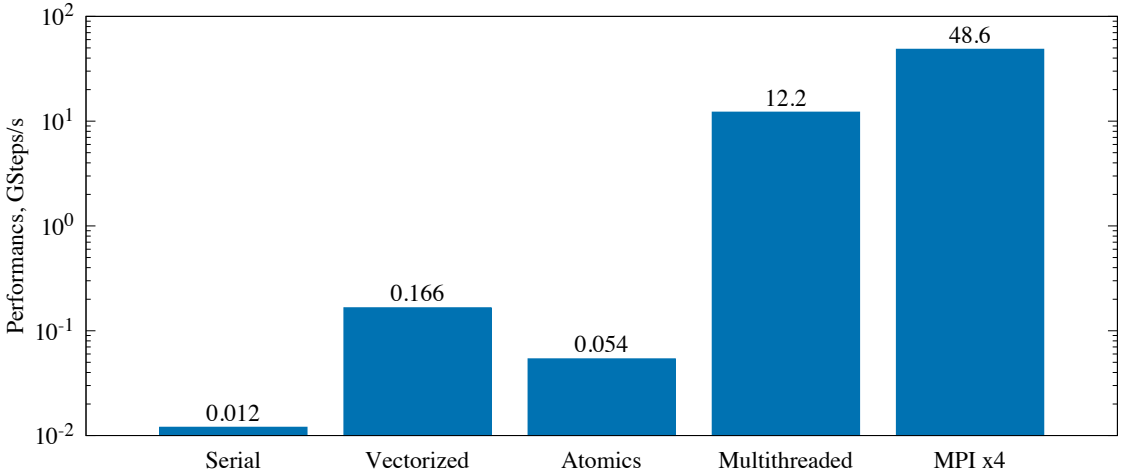
Scalar Implementation

```
1  double BlackBoxFunction(double x);
2  ...
3  const double dx = a/(double)n;
4  double integral = 0.0;
5
6  for (int i = 0; i < n; i++) {
7    const double xip12 = dx*((double)i + 0.5);
8    const double dI = BlackBoxFunction(xip12)*dx;
9    integral += dI;
10 }
```

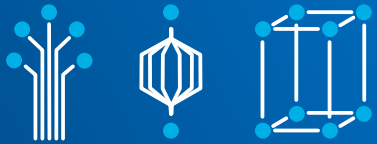

Vectorized Implementation

```
1  #pragma omp declare simd
2  double BlackBoxFunction(double x);
3  ...
4  const double dx = a/(double)n;
5  double integral = 0.0;
6
7  #pragma omp simd reduction(+: integral)
8  for (int i = 0; i < n; i++) {
9    const double xip12 = dx*((double)i + 0.5);
10   const double dI = BlackBoxFunction(xip12)*dx;
11   integral += dI;
12 }
```


Performance

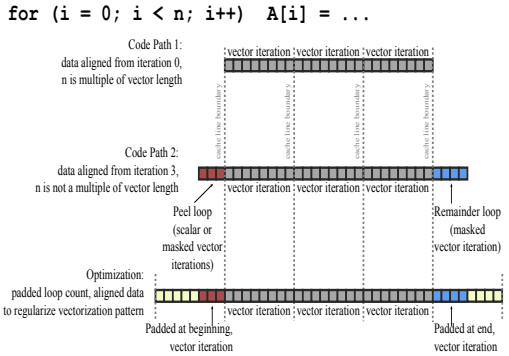


§10. Learn More



Loop Was Vectorized, Now What?

1. Unit-stride access
2. Data alignment
3. Container padding
4. Eliminate peel loops
5. Eliminate multiversioning
6. **Optimize data re-use in caches**



Loop Was Vectorized, Now What?

1. Unit-stride access
2. Data alignment
3. Container padding
4. Eliminate peel loops
5. Eliminate multiversioning
6. **Optimize data re-use in caches**

Vector Arithmetics is Cheap, Memory Access is Expensive

If you don't optimize cache usage, vectorization will not matter.
You will be bottlenecked by memory access.