

Advanced Algorithms  
Spring 2021  
IIIT Hyderabad

Homework 3, Due: Saturday February 20, 2021

1. Using the probabilistic method, show that the following bipartite graph  $G = (L \cup R, E)$  exists on  $2n$  vertices.

- $|L| = |R| = n$ ,
- Every vertex  $v$  in  $L$  has a degree  $n^{3/4}$  and every vertex  $u$  in  $R$  has degree at most  $3n^{3/4}$ .
- Every subset of  $n^{3/4}$  vertices in  $L$  has at least  $n - n^{3/4}$  neighbors in  $R$ .

**(5 Points)**

2. Show that the function  $f(x) = 1 - (1 - x/k)^k$  is concave for any  $k > 0$ . **(5 Points)**
3. Show a complete example of the two level hashing scheme with your choice of  $p$ ,  $m$ , and  $n$ . **(5 Points)**
4. Write a Boolean formula on  $n$  variables such that the maximum number of satisfiable clauses is exactly half the total number of clauses. Repeat for the case when the maximum number of satisfiable clauses is exactly  $3/4$  of the total number of clauses.