Quantum Computing

20 October 2020 12:54

80' Feynman . . .

85' Deutsch QTM

94' Shor Factorization, Discrete Log.

Principle of Superposition

0, .. k − 1 states

$\alpha_0|0\rangle + \alpha_1|1\rangle + \dots + \alpha_{k-1}|k-1\rangle$

$\alpha_i \in \mathbb{C}$ amplitudes $\begin{bmatrix}\alpha_0\\ \vdots \\ \alpha_{k-1}\end{bmatrix} \in \mathbb{C}^k$ $\mathcal{H}$ $|0\rangle = \begin{bmatrix}1\\0\\0\\0\\ \vdots\\0\end{bmatrix}$

2 particles

$\alpha_0|0\rangle \dots \alpha_{n-1}|n-1\rangle \equiv \mathcal{H}_A$

$\beta_0|0\rangle \dots \beta_{m-1}|m-1\rangle \equiv \mathcal{H}_B$

$\mathcal{H}_{AB} = \mathcal{H}_A \otimes \mathcal{H}_B$ dim = n×m

$\sum_{i=1}^{n}\sum_{j=1}^{m} \gamma_{ij}\, |i\rangle \otimes |j\rangle \in \mathbb{C}^{nm}$

Measurements.

Bra-ket Dirac

$\alpha_0|0\rangle + \alpha_1|1\rangle$

$\{|0\rangle\langle 0|, |1\rangle\langle 1|\} \approx \left\{\begin{bmatrix}1&0\\0&0\end{bmatrix}, \begin{bmatrix}0&0\\0&1\end{bmatrix}\right\}$

$\alpha_0|0\rangle + \alpha_1|1\rangle = \varphi$

$|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ $|-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$

$\{|+\rangle\langle +|, |-\rangle\langle -|\}$ = Measurement

$|0\rangle = (|+\rangle + |-\rangle)\frac{1}{\sqrt{2}}$ $|1\rangle = (|+\rangle - |-\rangle)\frac{1}{\sqrt{2}}$

$\varphi = \left(\frac{\alpha_0 + \alpha_1}{\sqrt{2}}\right)|+\rangle + \left(\frac{\alpha_0 - \alpha_1}{\sqrt{2}}\right)|-\rangle$

Qubits.

$\alpha_0|0\rangle + \alpha_1|1\rangle \in \mathbb{C}^2$

$|00\rangle = |0\rangle \otimes |0\rangle$

$\alpha_{00}|00\rangle + \alpha_{01}|01\rangle + \alpha_{10}|10\rangle + \alpha_{11}|11\rangle$

unentangled.

$(\alpha_0|0\rangle + \alpha_1|1\rangle) \otimes (\beta_0|0\rangle + \beta_1|1\rangle)$

$= \alpha_0\beta_0|00\rangle + \alpha_0\beta_1|01\rangle + \alpha_1\beta_0|10\rangle + \beta_1\alpha_1|11\rangle$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ Can this be written as prod of indi. states?

entangled state

n qubits

unentangled → 2n

entangled → 2^n

$(|0\rangle\langle 0|)(\alpha_0|0\rangle + \alpha_1|1\rangle) = (\alpha_0|0\rangle\langle 0|0\rangle + \alpha_1|0\rangle\langle 0|1\rangle)$

$= \alpha_0|0\rangle$

w.p. $\|\alpha_0\|^2$ $|0\rangle$ and with pr $\|\alpha_1\|^2$ $|1\rangle$

a measurement is a set of matrices

after observing 0, 1

state $|0\rangle$ $|1\rangle$

$\|\alpha_0\|^2 + \|\alpha_1\|^2 = 1$

sum of abs. val. of amplitude

$= \sum_i |\alpha_i|^2 = 1$

$\Pr[\text{state } +] = \left|\frac{\alpha_0 + \alpha_1}{\sqrt{2}}\right|^2$

$\Pr[\text{state } -] = \left|\frac{\alpha_0 - \alpha_1}{\sqrt{2}}\right|^2$

$\alpha_{00}|00\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{11}|11\rangle$

$\{|0\rangle\langle 0| \otimes I_{2\times 2}, |1\rangle\langle 1| \otimes I_{2\times 2}\}$

$\left(\frac{\ \ }{\sqrt{2}}\right)|1\rangle + \left(\frac{\ \ }{\sqrt{2}}\right)|\ \rangle$

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ EPR / Bell state

Einstein Podolsky, Rosen

Mesurement according to

$$\left\{ |0\rangle\langle 0| \otimes I_{2\times 2},\ |1\rangle\langle 1| \otimes I_{2\times 2} \right\}$$

state: $\dfrac{\alpha_{00}|00\rangle + \alpha_{01}|01\rangle}{\sqrt{|\alpha_{00}|^2 + |\alpha_{01}|^2}}$ $\qquad$ $\dfrac{\alpha_{10}|10\rangle + \alpha_{11}|11\rangle}{\sqrt{|\alpha_{10}|^2 + |\alpha_{11}|^2}}$

pr $|\alpha_{00}|^2 + |\alpha_{01}|^2$ $\qquad$ $|\alpha_{10}|^2 + |\alpha_{11}|^2$

w.pr $\frac{1}{2}$ $\quad$ 0 $\quad$ $|00\rangle$

"spooky action at a distance"

Unitary Operation:

$$\begin{bmatrix} & U & \end{bmatrix} \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix} = \begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix}$$

Unitary:

$$U \times U^{\dagger} = I \qquad U^{\dagger} = \overline{U^T} = U^{-1}$$

invertible

"reversible computation"

$$U(\alpha_0|0\rangle + \alpha_1|1\rangle) = \alpha_0|1\rangle + \alpha_1|0\rangle$$

$$U = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \equiv |0\rangle\langle 1| + |1\rangle\langle 0|$$

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$Z(\alpha_0|0\rangle + \alpha_1|1\rangle) = \alpha_0|0\rangle - \alpha_1|1\rangle$$

H : Hadamard op.

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$$

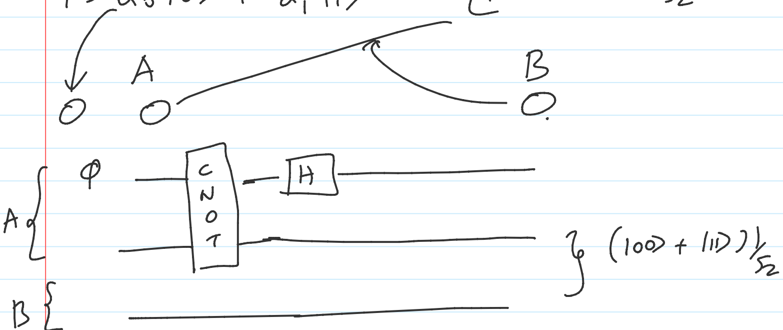
$$H|0\rangle = |+\rangle$$
$$H|1\rangle = |-\rangle$$

CNOT : 2 qubit operation

$$\text{CNOT } |0b\rangle = |0b\rangle$$
$$\text{CNOT } |1b\rangle = |1\bar{b}\rangle$$
$$\text{CNOT } (\alpha_{00}|00\rangle + \alpha_{10}|10\rangle + \alpha_{01}|01\rangle + \alpha_{11}|11\rangle)$$
$$= \alpha_{00}|00\rangle + \alpha_{10}|11\rangle + \alpha_{01}|01\rangle + \alpha_{11}|10\rangle$$

Quantum Teleportation.

$\varphi = \alpha_0|0\rangle + \alpha_1|1\rangle$ $\qquad$ $(|00\rangle + |11\rangle)\frac{1}{\sqrt{2}}$



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