Open Quantum Systems and Quantum Thermodynamics

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Instructor: Samyadeb Bhattacharya (*Phone:* 4066531000, *E-mail:* samyadeb.b@iiit.ac.in)
TA: Utkarsh Azad (*Phone:* 9491750674 , *E-mail:* utkarsh.azad@research.iiit.ac.in)

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Problem Set - II

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Q1. A spin $1/2$ system is known to be in an eigenstate of $\vec{s}\cdot\hat{n}$ with eigenvalue $\hbar/2$, where $\hat{n}$ is a unit vector lying in the xz plane that makes an angle γ with the positive z axis.

(i) Suppose $\hat{S}_x$ is measured. What is the probability of getting $+\hbar/2$?

(ii) Evaluate the dispersion in $\hat{S}_x$, that is $\langle(\hat{S}_x-\langle\hat{S}_x\rangle)^2\rangle$.

Q2. Consider the spin-precession problem with Hamiltonian

$$H=-\left(\frac{eB}{mc}\right)S_z=\omega S_z$$

(i) Write down the Heisenberg equation of motion for S_x, S_y, S_z.

(ii) Solve those equations to find the $S_x(t), S_y(t), S_z(t)$ as a function of time.

Q3. Consider a particle in three dimensions whose Hamiltonian is given by:

$$H=\frac{\hat{p}^2}{2m}+V(\hat{x})$$

By calculating $[\vec{x}\cdot\vec{p},H]$, obtain

$$\frac{d}{dt}\langle\vec{x}\cdot\vec{p}\rangle=\left\langle\frac{\vec{p}^2}{2m}\right\rangle-\langle\vec{x}\cdot\vec{\nabla}V\rangle$$

Q4. Consider an one dimensional Harmonic oscillator. Then consider a normalized state $|\alpha\rangle=c_0|0\rangle+c_1|1\rangle$, where $|n\rangle$ is a number state.

(i) Show the expectation value $\langle x\rangle_\alpha$ to be:

$$\langle x\rangle_\alpha=2\sqrt{\frac{\hbar}{2m\omega}}\cos(\delta_1-\delta_0)|c_0|\sqrt{1-|c_0|^2}$$

where $c_n=|c_n|e^{i\delta_n}$.

(ii) Now find the dispersion $\langle\Delta x^2\rangle$.

Q5. Let $x(t)$ be the coordinate operator for a free particle with Hamiltonian $H=p^2/2m$.

Then find the commutator $[x(t),x(0)]$.

Q6. Consider an one-dimensional simple Harmonic oscillator. Then consider a coherent state $|\lambda\rangle$, which is an eigenstate of the annihilation operator a, i.e. $a|\lambda\rangle=\lambda|\lambda\rangle$.

(i) Prove $|\lambda\rangle=e^{-|\lambda|^2/2}e^{\lambda a^\dagger}|0\rangle$ is a normalized coherent state.

(ii) Prove that $|\lambda\rangle$ can be written as $|\lambda\rangle=\sum_{n=0}^{\infty}f(n)|n\rangle$. What kind of distribution does $|f(n)|^2$ follows with respect to n? Using this find the most probable value of E.