OPEN QUANTUM SYSTEMS PROJECT UPDATE

We have gone through all the class materials and read papers online for understanding how to proceed to solve the question.(Question-3)

$$|0\rangle = \begin{pmatrix}0\\1\end{pmatrix} \qquad |1\rangle = \begin{pmatrix}1\\0\end{pmatrix}$$

$$\sigma_x = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} \qquad \sigma_y = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix} \qquad \sigma_z = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$

$$\sigma_+ = |0\rangle\langle 1| = \begin{pmatrix}0\\1\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}$$

$$\sigma_- = |1\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}0 & 1\end{pmatrix} = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}$$

$$\sigma_+\sigma_- = \begin{pmatrix}0 & 0\\1 & 0\end{pmatrix}\begin{pmatrix}0 & 1\\0 & 0\end{pmatrix} = \begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}$$

$$\sigma_-\sigma_+ = \begin{pmatrix}0 & 1\\0 & 0\end{pmatrix}\begin{pmatrix}0 & 0\\1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 0\end{pmatrix}$$

3 (i)

$$\frac{d\rho}{dt} = \gamma(n+1)\Big(\overbrace{\sigma_-\rho\sigma_+}^{T_1} - \overbrace{\tfrac{1}{2}\{\sigma_+\sigma_-,\rho\}}^{T_2}\Big) + \gamma n\Big(\underbrace{\sigma_+\rho\sigma_-}_{T_3} - \overbrace{\tfrac{1}{2}\{\sigma_-\sigma_+,\rho\}}^{T_4}\Big) + \underbrace{\lambda(\sigma_z\rho\sigma_z - \rho)}_{T_5}$$

$\underline{\underline{T_1}}$ $\quad \sigma_- \rho \sigma_+ = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix}$

$\underline{\underline{T_2}}$ $\quad -\frac{1}{2}\left(\sigma_+ \sigma_- \rho + \rho \sigma_+ \sigma_-\right) = -\frac{1}{2}\left(\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right)$

$$= -\frac{1}{2}\left(\begin{pmatrix} 0 & 0 \\ c & d \end{pmatrix} + \begin{pmatrix} 0 & b \\ 0 & d \end{pmatrix}\right)$$

$$= -\frac{1}{2}\begin{pmatrix} 0 & b \\ c & 2d \end{pmatrix}$$

$\underline{\underline{T_3}}$ $\quad \sigma_+ \rho \sigma_- = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$

$$= \begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix}$$

$\underline{\underline{T_4}}$ $\quad -\frac{1}{2}\left(\sigma_- \sigma_+ \rho + \rho \sigma_- \sigma_+\right) = -\frac{1}{2}\left(\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} + \begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\right)$

$$= -\frac{1}{2}\begin{pmatrix} 2a & b \\ c & 0 \end{pmatrix}$$

$\underline{\underline{T_S}}$

$$\sigma_z \rho \sigma_z - \rho = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} a & -b \\ -c & d \end{pmatrix} - \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$= \begin{pmatrix} 0 & -2b \\ -2c & 0 \end{pmatrix}$$

$$\Rightarrow \underline{\underline{R.H.S}} = \gamma(n+1)\left[\begin{pmatrix} d & 0 \\ 0 & 0 \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 0 & b \\ c & 2d \end{pmatrix}\right]$$

$$+ \gamma n \left[\begin{pmatrix} 0 & 0 \\ 0 & a \end{pmatrix} - \frac{1}{2}\begin{pmatrix} 2a & b \\ c & 0 \end{pmatrix}\right]$$

$$+ \lambda \begin{pmatrix} 0 & -2b \\ -2c & 0 \end{pmatrix}$$

$$= \begin{pmatrix} \gamma(n+1)d - \gamma n a & -\frac{\gamma(n+1)}{2}b - \frac{\gamma n}{2}b - 2\lambda b = (-\gamma n - \frac{\gamma}{2} - 2\lambda)b \\ -\frac{\gamma(n+1)c}{2} - \frac{\gamma n}{2}c - 2\lambda c = (-\gamma n - \frac{\gamma}{2} - 2\lambda)c & -\gamma(n+1)d + \gamma n a \end{pmatrix}$$

as $\rho = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow a = \rho_{11}$, $b = \rho_{12}$, $c = \rho_{21}$, $d = \rho_{22}$

$$\boxed{\Rightarrow \frac{d}{dt}\begin{pmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{21} \\ \rho_{22} \end{pmatrix} = \begin{pmatrix} -\gamma n & 0 & 0 & \gamma(n+1) \\ 0 & -\gamma n - \frac{\gamma}{2} - 2\lambda & 0 & 0 \\ 0 & 0 & -\gamma n - \frac{\gamma}{2} - 2\lambda & 0 \\ \gamma n & 0 & 0 & -\gamma(n+1) \end{pmatrix}\begin{pmatrix} \rho_{11} \\ \rho_{12} \\ \rho_{21} \\ \rho_{22} \end{pmatrix}}$$

By solving the above differential equation, we can get the dynamical map Λ.

This completes the first part of question.

ii) once, we get the dynamical map Λ, we can solve for fixed point.

iii) We can solve for kraus operators as discussed in the class $\Rightarrow$ taking maximally entangled state, applying the dynamical map etc.

iv) As, we find ρ_{fix} in 2nd part of the question in terms of n, we can solve for particular ρ_{fix} needed in this part and find n.

We intend to proceed in the following way as explained above.

Team Members
1)Krishna Mahesh Teja Nukala
2)Vaibhav Chimalgi