End Sem LPDE

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1) $H(S,P) = U + PV$

$\partial H = \partial U + \partial(PV)$

$= Tds - P\cancel{dV} + P\cancel{dV} + vdp$

$\partial H = Tds + vdp$

$\left(\dfrac{\partial H(S,P)}{\partial s}\right)_P = T \quad P = \text{const},\ dp = 0$

$\left(\dfrac{\partial H(S,P)}{\partial p}\right)_S = V \quad s = \text{const},\ ds = 0.$

$\therefore \left(\left(\dfrac{\partial H}{\partial s}\right)_P, \left(\dfrac{\partial H}{\partial p}\right)_S\right) = (T, V)$ option I

2) $F = U - TS$

$\partial f = \partial U - \partial(TS)$

$= T\cancel{ds} - p\partial V - T\cancel{ds} - S\partial T$

$= -p\partial V - s\partial T$

$\partial G = \partial f + \partial(PV)$

$= -p\cancel{\partial V} - s\partial T + p\cancel{\partial V} + v\partial p$

$= -s\partial T + v\partial p$

If $t = \text{const},\ dT = 0$

$\Rightarrow \left(\dfrac{\partial G}{\partial p}\right)_T = V$ } Not option I

If $p = $ const, $\partial P = 0$ then,

$$\left(\frac{\partial G}{\partial T}\right)_P = -S \quad \Big\}\ \text{Not option 2}$$

$G = F + PV$, $F = U - TS$

$$\partial\left(\frac{F}{V}\right) = \partial F \times \frac{1}{V} + F\,\partial\left(\frac{1}{V}\right)$$

$$= \partial F \cdot \frac{1}{V} + F\left(-\frac{1}{V^2}\right)\partial V$$

$$\partial F = dU - \partial(TS)$$

$$= T\cancel{dS} - p\partial V - T\cancel{dS} - S\partial T$$

$$= -p\partial V - S\partial T$$

If $T = $ const, $\partial T = 0$.

$$\partial F = -p\partial V$$

Also $F = G - PV$

$$\partial\left(\frac{F}{V}\right)_T = \frac{\partial F}{V} - \frac{F}{V^2}\partial V$$

$$= -\frac{P}{V}\partial V - \left(\frac{G - PV}{V^2}\right)\partial V$$

$$= \cancel{-\frac{P}{V}\partial V} - \frac{G}{V^2}\partial V + \cancel{\frac{P}{V}\partial V}$$

$$\partial\left(F/V\right)_T = -G\left(\frac{1}{V^2}\partial V\right) = G\,\partial\left(1/V\right)_T$$

$$\Rightarrow G = \frac{\partial\left(F/V\right)_T}{\partial\left(1/V\right)_T}$$ option 3

3) $(x+2z)z_x + (4zx-y)z_y = 2x^2+y$

$$\frac{dx}{x+2z} = \frac{dy}{4xz-y} = \frac{dz}{2x^2+y}$$

$$y\,dx + x\,dy = \cancel{xy} + 2yz + 4x^2z - \cancel{xy}$$

$$= (2x^2+y)(2z)$$

$$\Rightarrow \underset{P_1}{y\,dx} + \underset{Q_1}{x\,dy} - \underset{R_1}{2z\,dz} = 0 \quad (P_1, Q_1, R_1) = (y, x, -2z)$$

$$\Rightarrow d(xy) - 2z\,dz = 0$$

$$\Rightarrow xy - z^2 = c_1$$

$$2x\,dx - dy - dz = \cancel{2x^2} + \cancel{4xz} - \cancel{4xz} + \cancel{y} - \cancel{2x^2} - \cancel{y}$$

$$P_1 \quad Q_1 \quad R_1 = 0$$

$$\Rightarrow 2x\,dx - dy - dz = 0 \quad (P_1, Q_1, R_1) = (2x, -1, -1)$$

$$x^2 - y - z = c_2$$

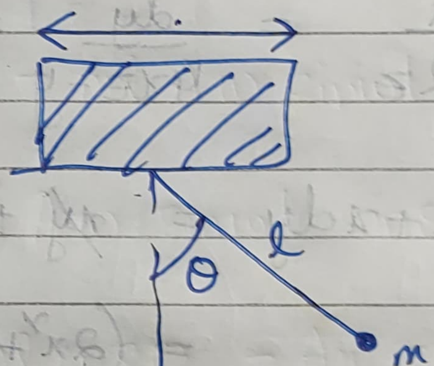
$f(xy - z^2,\ x^2 - y - z) = 0$. Option D

Q5) Pendulum mass = m
length = l

Pendulum support oscillates horizontally with a position given by $x(t) = A\cos\omega t$

General solu for the angle of the pendulum as a fn of time?

Figure :



Angle = θ between the vertical & pendulum

The position vector of the bob of mass 'm' is :

$$r = x(t)\,\hat{i} + l\sin\theta\,\hat{i} + l\cos\theta\,\hat{j}$$

$$= (A\cos\omega t + l\sin\theta)\,\hat{i} + l\cos\theta\,\hat{j}$$

$$\text{Velocity} = \frac{\partial r}{dt} = \dot{r}$$

$$= \left(-A\sin\omega t \times \omega + l\cos\theta \cdot \frac{d\theta}{dt}\right)\hat{i}$$

$$- l \sin\theta \cdot \frac{d\theta}{dt}\,\hat{j}$$

$$v = \left(-A\omega \sin\omega t + l\dot{\theta}\cos\theta\right)\hat{i} - l\dot{\theta}\sin\theta\,\hat{j}$$

Kinetic energy $= \frac{1}{2}mv^2$

$$= \frac{1}{2}m\left(\left(l\dot{\theta}\cos\theta - A\omega\sin\omega t\right)^2 + \left(l\dot{\theta}\sin\theta\right)^2\right)$$

$$= \frac{1}{2}m\left(l^2\dot{\theta}^2 \sin^2\theta + l^2\dot{\theta}^2\cos^2\theta + A^2\omega^2\sin^2\omega t - 2Al\omega\dot{\theta}\sin\omega t\cos\theta\right)$$

$$= \frac{1}{2}m\left(l^2\dot{\theta}^2 + A^2\omega^2\sin^2\omega t - 2Al\omega\dot{\theta}\sin\omega t\cos\theta\right)$$

Potential Energy $= -mgr$ P.E only in Vertical direction.

$$= -mgl\cos\theta$$

Lagrangian $L = T - V$

$$L = \frac{1}{2}ml^2\dot{\theta}^2 + ml\dot{\theta}\left(-A\omega\sin\omega t\cos\theta\right) + \frac{1}{2}mA^2\omega^2\sin^2\omega t + mgl\cos\theta$$

Lagrangian Eq is:

$$0 = \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta}$$

$$0 = \frac{\partial}{\partial t}\left(ml^2\dot{\theta} + ml\left(-A\omega \sin\omega t \cos\theta\right)\right)$$
$$- ml\dot{\theta}\left(-A\omega \sin\omega t \cdot -\sin\theta\right)$$
$$+ mgl \sin\theta$$

$$0 = \frac{\partial}{dt}\left(ml^2\dot{\theta} - mlA\omega \sin\omega t \cos\theta\right)$$
$$- ml\dot{\theta}A\omega \cdot \sin\omega t \sin\theta + mgl \sin\theta$$

$$= ml^2\ddot{\theta} + ml\cos\theta - A\omega\cos\omega t \cdot \omega - mlA\omega\sin\omega t \cdot -\sin\theta \cdot \dot{\theta}$$
$$- ml\dot{\theta}A\omega \sin\omega t \sin\theta + mgl\sin\theta$$

$$0 = ml^2\ddot{\theta} - ml\omega^2 A\cos\theta\cos\omega t + \cancel{mlA\omega\dot{\theta}\sin\omega t\sin\theta}$$
$$\cancel{- ml\dot{\theta}A\omega\sin\omega t\sin\theta} + mgl\sin\theta$$

$$= \cancel{l\ddot{\theta} - A\omega^2\cos\theta\cos\omega t - A\omega\dot{\theta}\sin\omega t\sin\theta + g\sin\theta}$$

$$0 = l\ddot{\theta} - A\omega^2\cos\theta\cos\omega t + g\sin\theta$$

option A

4) $(y+z)z_x + (x+z)z_y = x+y$

$$\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y} \longrightarrow 1, 2, 3$$

$$\frac{dx}{y+z} = \frac{dx}{x+z} = \frac{dz}{x+y} = \frac{dx+dy+dz}{2(x+y+z)}$$

$$dx+dy+dz = 2(x+y+z) \longrightarrow 4$$

$$dx - dy = y+z-x-z = y-x$$
$$= -(x-y)$$

$$\frac{dx-dy}{-(x-y)} = k \longrightarrow 5$$

$$dy - dz = x+z-x-y = z-y$$

$$\frac{dy-dz}{-(y-z)} = k$$

$$dz - dx = x+y-y-z = x-z$$

$$\frac{dz-dx}{-(z-x)} = k \longrightarrow 6$$

$$\frac{dx-dy}{-(x-y)} = \frac{dy-dz}{-(y-z)}$$

$$\log(x-y) = \log(y-z) + c_1$$

$$c_1 = \log\left(\frac{x-y}{y-z}\right)$$

$$\Rightarrow c_1 = \frac{x-y}{y-z} \longrightarrow (7)$$

$$c_5 = \frac{x-y}{\cancel{z}-\cancel{x}} \longrightarrow (9)$$

Similaly, $c_2 = \dfrac{y-z}{z-x} \longrightarrow (8)$

$$\frac{dx+dy+dz}{2(x+y+z)} = \frac{dx-dy}{-(x-y)}$$

$$\frac{1}{2}\log(x+y+z) = -\log(x-y) + c_3$$

$$c_3 = \log\left(\sqrt{x+y+z}\right) + \log(x-y)$$

$$= \log\left((x-y)\cdot\sqrt{x+y+z}\right)$$

$$c_3 = (x-y)\sqrt{x+y+z} \longrightarrow (10)$$

$$c_4 = (y-z)\left(\sqrt{x+y+z}\right) \longrightarrow (11)$$

$$c_5 = (z-x)\left(\sqrt{x+y+z}\right) \longrightarrow (12)$$

$F\left(\frac{x-y}{y-z}, (x-y)\sqrt{x+y+z}\right) \longrightarrow$ One of the many solutions of $F(u, v) = 0$.

Option: None of the above