Quiz - 5

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Subject: (Date / / التاريخ) الموضوع:

1) Find an input for the greedy assignment algorithm that has an approximation ratio as close to 2 as possible

→ Conside $m(m-1)$ jobs of length '1', 1 length 'm' job. Total no of machines ~~are~~ m.

Greedy:

M_1	M_2		. . . . .	M_m
1	1	1	. . . . .	1
1	1	1	. . . . .	1
m				

$\Big\}$ $m-1$ times

$\therefore$ Total makespan time $= \max\{m+m-1, m-1, m-1, \ldots\}$
$= 2m-1$

OPT: sort by decreasing order of time taken

M_1	M_2		. . . . .	M_m
m	1	1	. . . . .	1
	1	1	. . . . .	1

$\Big\}$ m times

Total makespan time $= \max\{m, m, m \ldots\}$
$= m$

$\therefore$ Ratio $= \dfrac{2m-1}{m} = 2 - \dfrac{1}{m}$

as $m \to \infty$, $\lim\limits_{m\to\infty}$ Ratio $= 2$

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2) Sorted greedy assignment algorithm: See if there exists i/ps that push the approximation as close to $3/2$ as possible

→ Let the set of tasks be:

$$(\alpha_1, \alpha_2, \ldots \alpha_n) = \underbrace{(2m-1, 2m-1, 2m-2, \ldots\ldots m, m, m)}_{\text{Total } 2m+1 \text{ tasks}}$$

$$\alpha_k = 2m - \left\lceil \frac{k+1}{2} \right\rceil \qquad k = 1, 2, \ldots 2m \qquad \alpha_{2m+1} = \alpha_n = m$$

Greedy Ordered algo

M_1	M_2	M_3	$\ldots$		M_m
α_1	α_2	α_3			α_m
α_{2m}			$\ldots$	α_{m+2}	α_{m+1}
α_{2m+1}					

Makespan: $\alpha_1 + \alpha_{2m} + \alpha_{2m+1} = 2m-1 + m + m = 4m-1$

OPT algo

M_1	M_2	M_3		M_{m-1}	M_m
α_1	α_2	α_3	$\ldots$	α_{m-1}	α_{2m-1}
α_{2m-2}	α_{2m-3}		$\ldots$	α_m	α_{2m}
					α_{2m+1}

Makespan: $\alpha_{2m-1} + \alpha_{2m} + \alpha_{2m+1} = m + m + m$
$= 3m$

$\therefore$ Ratio $= \dfrac{4m-1}{3m} = \dfrac{4}{3} - \dfrac{1}{3m}$ → This is the tighest approx ratio of Ordered Greedy assigment algorithm.

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→ Getting close to a $3/2$ ratio using sorted algorithm is Tough. Incase of a normal greedy approach,

Take $m+1$ jobs of length m
$m(m-1)$ jobs of length 1

Greedy :

M_1	M_2	M_3	. . . .	M_m	
1	1	1	. . .	1	
1	1	1	. . .	1	$m-1$ times
m	m	n	. . .	m	
m					

$$\text{Timespan} = \max\{3m-1, 2m-1, 2m-1, \ldots\} = 3m-1$$

Opt :

M_1	M_2	M_3	. . .	M_m	
m	m	m		m	
m	1	1		1	
	1	1		1	m times
	⋮				

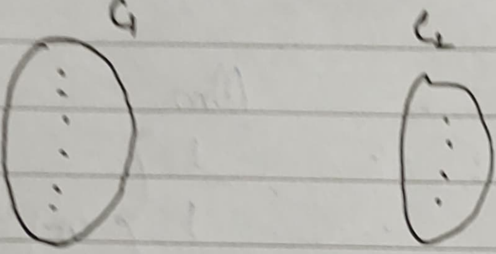
$$\text{Timespan} = \max\{2m, 2m, 2m, \ldots\} = 2m$$

$$\therefore \text{Ratio} = \frac{3m-1}{2m} = \frac{3}{2} - \frac{1}{2m}$$

as $m \to \infty$, $\lim_{m\to\infty} \text{Ratio} = 3/2$

3) Given a constant c, find a graph class that has a fractional independent set of size ~~at~~ least n/c
i) $c = 2$, ii) $c = 3$

Ex I: Let us take the class of bipartite graphs,



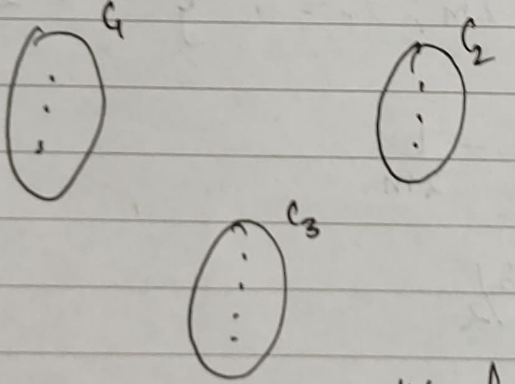
We have 2 ~~I~~ independent sets.

For any split of 'n' vertices into C_1 or C_2, one of the sets is bound to have a size of atleast $n/2$ $(c=2)$.

If one of the set has size atleast $n/2$, then it will also have atleast $n/3$ $(c=3)$ (As $n/3 < n/2$)

For both $c=2$, $c=3$ the bipartite graph class will have an FIS of size atleast $n/2$ & $n/3$

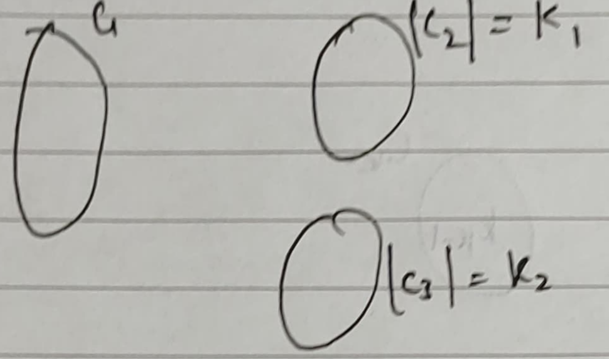
Ex2: Let us consider the class of tripartite graphs,



We have 3 independent sets

For any 3-way split of n vertices, one of the sets is bound to have a size of atleast $n/3$ $(c=3)$

For $c=2$ case,



$|C_1| = n-(k_1+k_2)$

We want $|C_1| \geq n/2$

$\Rightarrow n-(k_1+k_2) \geq n/2 \quad \Rightarrow k_1+k_2 \leq n/2$

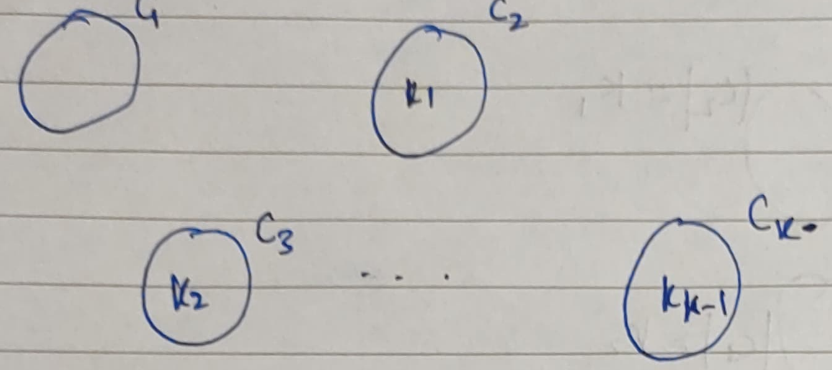
For every vertex in C_1, the max degree it can contain is k_1+k_2

↳ Which by the definition is the parameter 'd'

So we have a $\left(2, \frac{n}{2}\right)$ FIS of size atleast $\frac{n}{2}$

4) For any given constant c, will there exist graph classes such that they have an independent set of size atleast n/c.

Let us consider the graph class k-partite graphs



$|C_2| = k_1$, $|C_2| = k_2$ ... $|C_i| = k_{i-1}$

Let $|C_1| = \cancel{e}\, n - (k_1 + k_2 + \dots k_{k-1})$

Let c be a constant 'x' without loss of generality

We want one of these independent sets to be of size atleast n/x.

If any of $C_2, C_3, \dots C_k$ are of size $\geq n/x$, then It is satisfying the FIS definition.

Suppose none of $C_2, C_3, \dots C_k$ are satisfying the size $\geq n/x$ condition. Then C_1 has to satisfy it.

$$|C_1| = n - (k_1 + k_2 + \dots k_{k-1}) \geq \frac{n}{x}$$

$$\Rightarrow n\left(1 - \frac{1}{x}\right) \geq \underbrace{(k_1 + k_2 + \dots k_{k-1})}_{\text{parameter 'd' of FIS definition}}$$

So we have a $\left(x, n\left(1 - \frac{1}{x}\right)\right)$ FIS ~~graph~~ bipartite graph.

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For $x = 2$, $\left(2, \frac{n}{2}\right)$ FIS k-partite graph

$x = 3$, $\left(3, \frac{2n}{3}\right)$ FIS k-partite graph

$\vdots$

for $\left(2, \frac{n}{2}\right)$ FIS k partite graph, $x \geq 2$

as $\left|\frac{n}{2}\right| > \left|\frac{n}{3}\right| > \frac{n}{4} \ldots$

If it satisfies size constraint $\geq n/2$, then it satisfies size constraint $\geq \frac{n}{x}$ $x \geq 2$