Project problems

1. Using spherical polar co-ordinates (r, θ, ϕ) defined by $x = r\ sin\theta cos\phi,\ y = r\ sin\theta sin\phi, z = r\ cos\theta$, write down the explicit Lagrange's equation of motion for a particle of mass m moving in a central potential $V(r)$. Further take $V(r) = \frac{k}{r}$ and describe the nature of the equation of motion and propose a way to solve that equation of motion (If you can explicitly solve it completely, you will be given additional credit).
2. a) Write down the equations of motion for the Lagrangian $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(r) + \frac{eB}{2c}(x\dot{y} - y\dot{x}),$ with $r = \sqrt{x^2 + y^2 + z^2}$. b) Using the rotated co-ordinate system $x' = xcos\omega t + ysin\omega t,\ y' = -xsin\omega t + ycos\omega t, z' = z$ write down the Lagrangian and the equations of motion. (Additional credits will be given if you can discuss the physical system corresponding to these equations of motion)
3. Find the temperature in a thin metal rod of length L, with both the ends insulated (so that there is no passage of heat through the ends) and with initial temperature in the rod sin(πx/L).
4. A) One mole of gas obeys Van der Waals equation of state. If the molar internal energy is given by $cT - \frac{a}{V}$ with a, c constants. Calculate the molar heat capacity $C_p,\ C_v$.
   B) A solid object has a density ρ, mass M and coefficient of linear expansion α. Show that at pressure p, the heat capacities are related by $C_p - C_v = 3\alpha Mp/\rho$.
   C) One mole of a monatomic perfect gas initially at temperature T_0 expands from volume V_0 to $2V_0$ (a) at constant temperature, (b) at constant pressure. Calculate the work of expansion and the heat absorbed by the gas in each case.
5. A) Derive the equation of motion of a projectile, using the principle of least action.

   B) A bead slides on a wire in the shape of a cycloid described by equations $x = a(\theta - sin\theta)$, $y = a(1 + cos\theta)$ with $0 \le \theta \le 2\pi$. Find the Lagrangian and then the equation of motion .

6. The equation of state of a system is given by $pV = \alpha U(T, V)$, where α is a constant and $U(T, V)$ is the specific internal energy. Show that the specific internal energy and specific entropy can be expressed in the form $U = V^{-\alpha}\phi(TV^{\alpha}); S = \psi(TV^{\alpha})$, where it is given that $\phi'(x) = x\psi'(x)$.

[Hint: use $\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p$ and then finally you have to use Lagrange's method of solving 1st order PDE]

Instruction: You have to submit a report on the problem you have been given. Any independent insights will also be highly appreciated and be given extra credits.