OPEN QUANTUM SYSTEMS AND QUANTUM THERMODYNAMICS

M20Temp16 Monsoon 2020

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Date: OCTOBER 31, 2020

Course Project Guidelines

Date of Submission: November 27, 2020 23:55 HRS

The course project is the capstone of the class. *It is preferred that you do this project in a group of two*. From the viewpoint of grading research-like problem solving is encouraged, but not necessary.

A list of assigned project topics is provided below. Additionally, for **BONUS**, you might want to extend your problem to showcase an interesting research oriented task or application. This will allow you to understand your results better and explore the topic in depth. *The grading of the bonus part will depend entirely on the relevance and quality of your submission.*

Importantly, you and project partner will have to submit the project deliverables by **27th November 2020**, and we will be available to guide you till **20th November 2020**.

1 Project Deliverables

Here is what is expected of you:

1. **Project Progress Update** ⇒ A less than one page written update on your project progress from each group. This is due on **15th November 2020**. It should describe the progress you have made against your originally assigned problem, including any changes of direction and results so far. You might wanna include the bonus tasks you're planning to do in it.

2. **Code Repository** ⇒ In case you are using a programming library to either solve numerical results or for simulations, then your group must upload all such project code by **27th November 2020**. This can be done to either (i) a publicly viewable repository (github, gitlab, bitbucket, etc.) or (ii) a private repository whose access credentials are provided in your report (see below). Your code should be documented and should include full instructions on how to install your project.

3. **Written Report** ⇒ Your group must upload a final report by no later than **27th November 2020** to **Moodle**. LATEXis preferred, and please use reasonable margins and font sizes. Apart from the results to the problem given, this report will describe the research background for your project and what extensions/constructions/proofs you have made in your project. It should also include a discussion section which explains your understanding of the obtained results.

4. **Project VIVA** ⇒ To further investigate your contributions and understanding about your project, we will take a project VIVA on **28th November 2020**.

2 Project Topics

Here are the potential course project topics. However, if you end up curious about an area and are looking for more references then please do ask on Moodle and we'll be happy to follow up. In general, we recommend you to stick with your assigned project (given below) unless you've some really interesting original idea that you might want to explore along with your partner. The latter case should be discussed with the TA by no later than **2nd November 2020**.

1. Consider the following dynamical map for a qubit:

$$\Lambda_t(\rho) = p(t) \sum_{i,j=0}^{1} |i\rangle \langle j| \rho |j\rangle \langle i| + (1-p(t))\rho \tag{1.1}$$

where $p(t) = e^{-\gamma t}$, and

$$|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{1.2}$$

   a. Find the Kraus operators corresponding to this evolution.
   b. Let us now consider the following two qubit maximally entangled state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big) \tag{1.3}$$

   and apply the previous dynamical map one side of this state: $\mathbb{I} \otimes \Lambda(|\psi\rangle \langle \psi|)$, using the Kraus operator representation.
   c. Now, find the instantaneous entanglement for $p(t) = e^{-\gamma t}$ and $p(t) = \cos \gamma t$ and plot the entanglement for the these two cases with time. By comparison, then conclude which channel between the two is better to preserve entanglement. The entanglement for this case can be calculated by the following recipe or any other measure of your choice.

   Entanglement as Concurrence $\Rightarrow$ If we have a two qubit state of the following form

$$\rho = \begin{pmatrix} \rho_{11} & 0 & 0 & \rho_{14} \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ \rho_{41} & 0 & 0 & \rho_{44} \end{pmatrix} \tag{1.4}$$

   then its entanglement (concurrence) can be measured as $C(\rho)$ with $0 \leq C(\rho) \leq 1$:

$$C(\rho) = 2\sqrt{|\rho_{14}|^2 - \rho_{22}\rho_{33}} \tag{1.5}$$

   or,

$$C(\rho) = 2\sqrt{|\rho_{23}|^2 - \rho_{11}\rho_{44}} \tag{1.6}$$

2. Consider the following positive map for a qutrit system:

$$\Lambda_c(\rho) = \begin{pmatrix} \rho_{11}+\rho_{22} & -\rho_{12} & -\rho_{13} \\ -\rho_{21} & \rho_{22}+\rho_{33} & -\rho_{23} \\ -\rho_{31} & -\rho_{32} & \rho_{33}+\rho_{11} \end{pmatrix} \tag{1.7}$$

where,

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix} \tag{1.8}$$

   a. Show that this map is not completely positive.
   b. Now consider the modified map

$$\Lambda_{mc}(\rho) = p\rho + (1-p)\Lambda_c(\rho) \tag{1.9}$$

   and find the threshold value of p, above which the modified map is completely positive ($p \geq p_{i\hbar}$).
   c. Find the Kraus operators for the modified map with $p = p_{i\hbar}$.

3. Consider the following master equation for qubits (two-level system)

$$\frac{d\rho}{dt} = \gamma(n+1)\big(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\big) + \gamma n\big(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\rho\}\big) + \lambda\left(\sigma_z\rho\sigma_z - \rho\right) \tag{1.10}$$

   a. Solve the master equation to find the corresponding dynamical map $\Lambda(\rho)$.
   b. Find the fixed state ρ_{fix}, for which $\Lambda(\rho_{fix}) = \rho_{fix}$
   c. Find the Kraus operators for the map $\Lambda(\rho)$.
   d. Find the values of n, for which $\rho_{fix} = \frac{\mathbb{I}}{2}$ and $\rho_{fix} = |0\rangle\langle 0|$ where

$$|0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1.11}$$

$$\sigma_+ = |0\rangle\langle 1| \qquad \sigma_- = |1\rangle\langle 0| \tag{1.12}$$

4. Consider the following map

$$\Lambda_T(\rho) = \begin{pmatrix} \rho_{33} & \rho_{23} & \rho_{13} \\ \rho_{32} & \rho_{22} & \rho_{12} \\ \rho_{31} & \rho_{21} & \rho_{11} \end{pmatrix} \tag{1.13}$$

   where,

$$\rho = \begin{pmatrix} \rho_{11} & \rho_{12} & \rho_{13} \\ \rho_{21} & \rho_{22} & \rho_{23} \\ \rho_{31} & \rho_{32} & \rho_{33} \end{pmatrix} \tag{1.14}$$

   a. Prove that this map is positive, but not completely positive.
      Hint: You might want to use Sylvestor's criteria here.
   b. Now consider the state:

$$\rho = p\frac{\mathbb{I}}{9} + (1-p)\,|\psi\rangle\langle\psi| \tag{1.15}$$

      where $|\psi\rangle$ is a maximally entangled state:

$$\psi = \frac{1}{\sqrt{3}}\big(|00\rangle + |11\rangle + |22\rangle\big) \tag{1.16}$$

$$|0\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \qquad |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \qquad |2\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \tag{1.17}$$

      by applying the map: $(\mathbb{I}\otimes\Lambda_T)$, find the threshold value of p, under which the state is entangled.

5. Consider the following master equation for qubits:

$$\frac{d\rho}{dt} = L_1(\rho) + L_2(\rho) \tag{1.18}$$

   where,

$$L_i(\rho) = \gamma_i(n_i+1)\big(\sigma_-\rho\sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-,\rho\}\big) + \gamma_i n_i\big(\sigma_+\rho\sigma_- - \frac{1}{2}\{\sigma_-\sigma_+,\rho\}\big) \tag{1.19}$$

   a. Solve the master equation to find the corresponding dynamical map Λ.
   b. Find the fixed state ρ_{fix}, for which $\Lambda(\rho_{fix}) = \rho_{fix}$
   c. Find the Kraus operators for the map $\Lambda(\rho)$.
   d. Find the following quantity called entropy production rate:

$$\sigma(t) = \frac{d}{dt}S(\rho(t)||\rho_{fix}) \tag{1.20}$$

      where,

$$S(\rho_1||\rho_2) = \mathrm{Tr}[\rho_1(\ln\rho_1 - \ln\rho_2)] \tag{1.21}$$

      and plot $\sigma(t)$ w.r.t time for different values of γ_1, γ_2, n_1 and n_2. From the plots interpret the nature of the time evolution of $\sigma(t)$.

3 Project Teams

Project Member 1		Project Member 2		Project Number
AMEY	KUDARI	Ganesh	Mylavarapu	2
Ankit	Vaishy	YASH	CHAURASIA	1
Sriven Reddy	Nookala	AadilMehdi	Sanchawala	3
ALOK	KAR	Samanvaya	Panda	5
Anushka	Wakankar	Suryansh	Srivastava	4
Rohan	Sharma	Nikhil	Tadigoppula	2
Mayank	Goyal	Swastik	Murawat	1
Abhinav	Vaishya	Aditya	Morolia	5
Srikar	Kale	Deepti	Mahesh	3
Athreya	Chandramouli	Aaron	Saju Augustine	4
Varun	Chhangani	Freya	Mehta	2
Jayadev	Naram	SAHIL	BHATT	5
Ritam	Basu	Debojit	Das	4
Sailendra	D S	SAI DINESH	BIJJAM	3
Atirek	Kumar	Kripa Anne	Tharakan	1
TANMAY KUMAR	SINHA	Mehul	Gupta	4
Fanish	Jain	Arpan	Dasgupta	5
Rutwik	Segireddy	YASHAS	SAMAGA	2
Anurag	pateriya	Akash	Verma	1
Vaibhav	Chimalgi	Krishna Mahesh teja	Nukala	3

Didn't get a partner of your choice? Read this **comic** to cheer up!

4 Important Dates

1. **15th November 2020** - One page project update due.
2. **27th November 2020** - Rest project deliverables due.
3. **28th November 2020** - Project VIVA due.