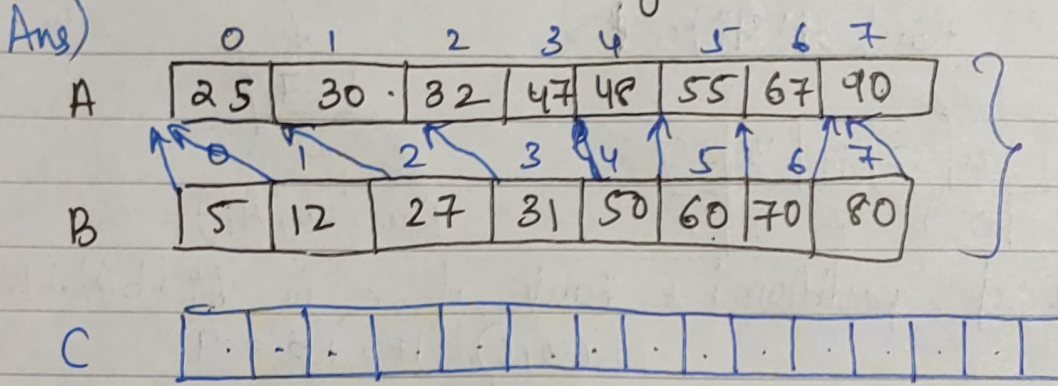
Q1) Show a run of the parallel merge algo on 2 arrays of 8 elem each where ranks are found for all elements directly.

Ans)



We need to fill C

Divide B into $\sqrt{m}$ element groups.

Let us take $\sqrt{8} = 3$ (ceiling value)

Search for B[2], B[5] in A in step 1

B[2] = 27 B[5] = 60

Rank(27, A) = 1 Rank(60, A) = 6 } parallel search in A

→ step 1

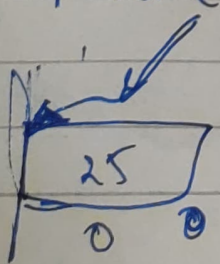
Divide the $\sqrt{m}$ element groups even further by another root = $m^{1/4}$ = 2 elements. parallel search in their respective groups.

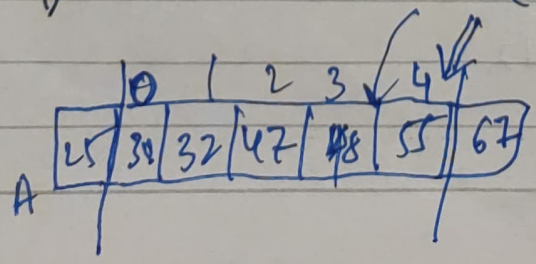
search for B[1], B[4] & B[7] in A_i's in step 2 (A_i's → partitions)

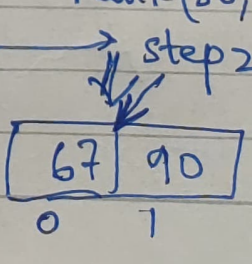
B[1] = 12 B[4] = 50 B[7] = 80

Rank(12, A_i) = 0 Rank(50, A_i) = 4 Rank(80, A_i) = 8

→ step 2





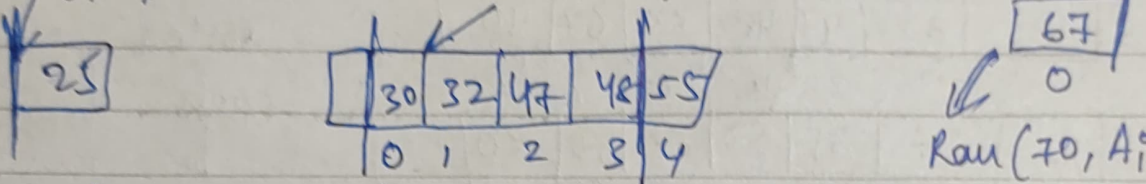


A_i^o → partition

Each sub group of $\sqrt{m}$ elements have only 1 element left

B[0] = 1 B[3] = 31 B[6] = 70

Rank(5, A_i^o) = 0 Rank(31, A_i^o) = 1 Rank(70, A_i^o) = 0



Rank (x, C) = Rank of partition i + Rank of x in A_i^o + R(x) in B

R(5, C) = 0

R(12, C) = 1 (12 in B = rank 1)

R(27, C) = 1+2 = 3

R(31, C) = 1+4+0 = 5

R(50, C) = 1+3+5 = 9

R(60, C) = 5+6+0 = 11

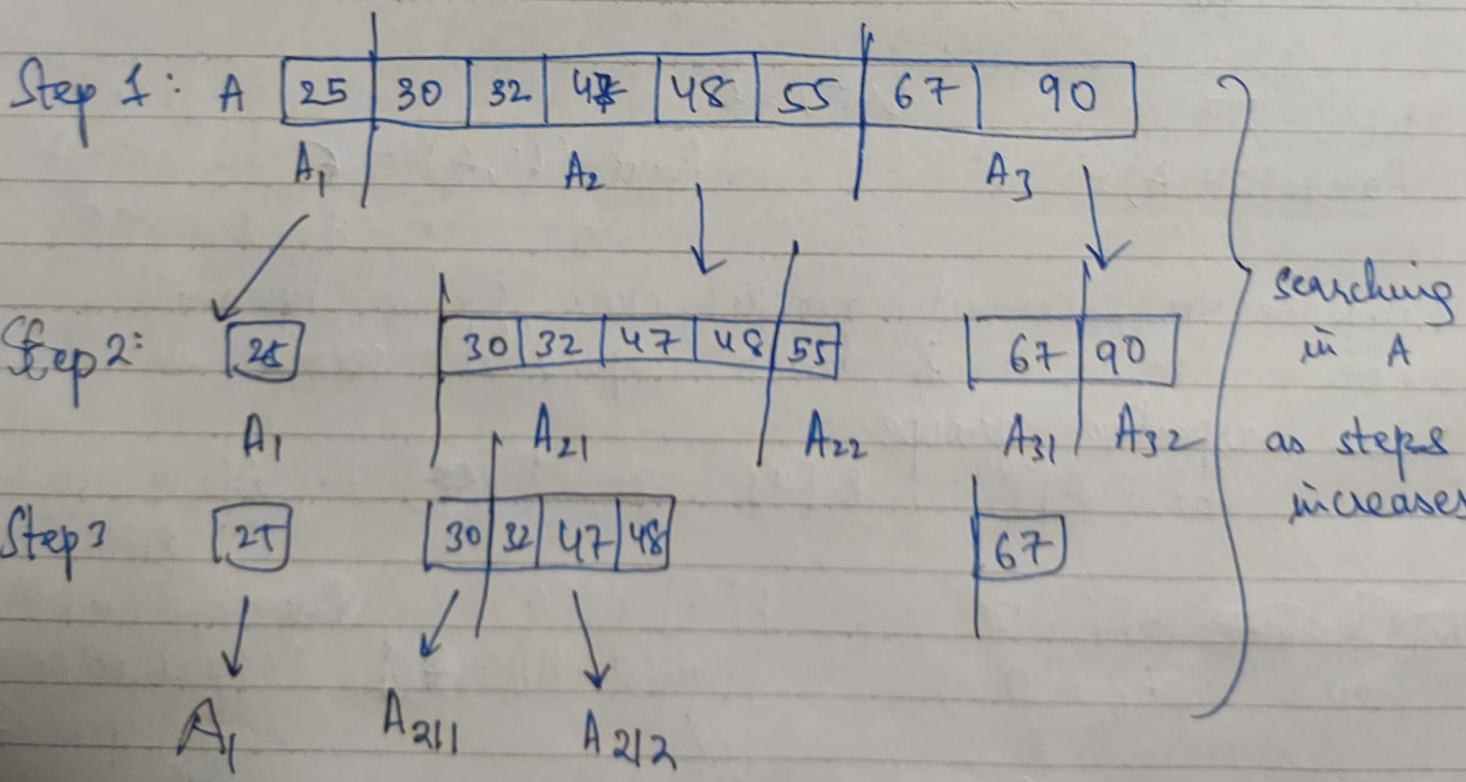
R(70, C) = 6+6+0+1 = 13

R(80, C) = 6+7+0+1 = 14

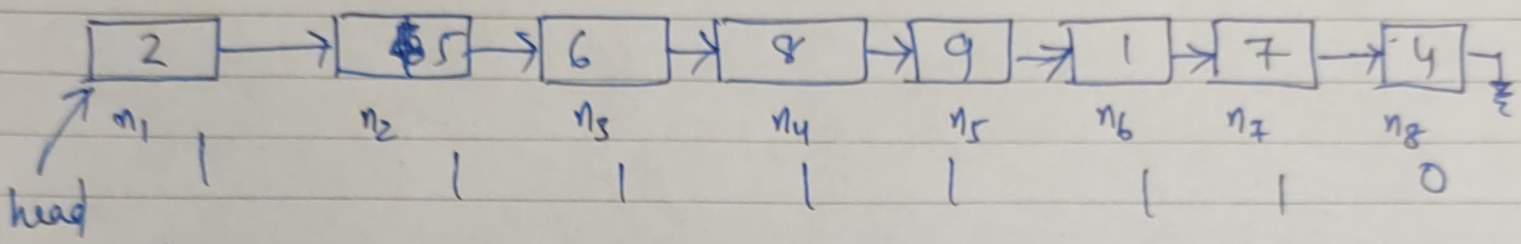
→ Plug A in the remaining gaps in order.

C

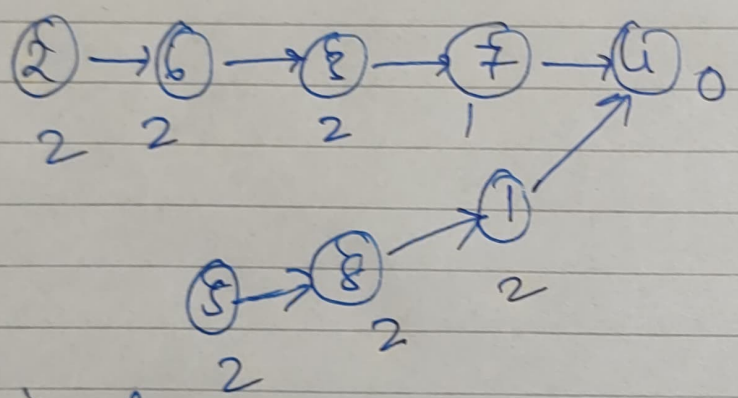
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
5	12	25	27	30	31	32	47	48	50	55	60	67	70	80	90



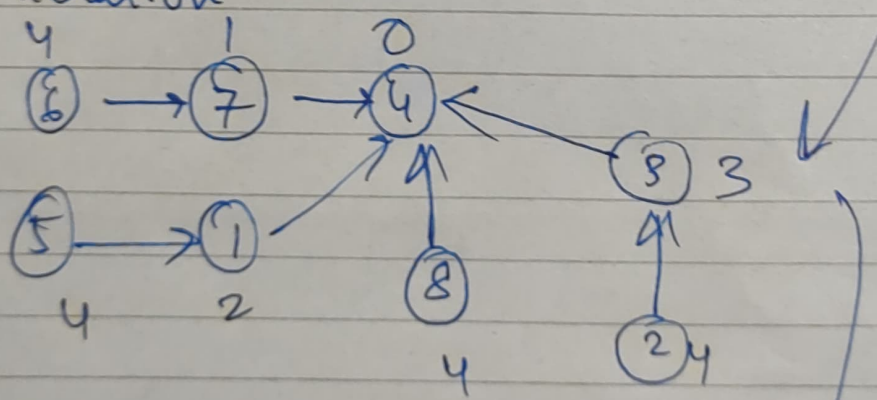
Q2) Linked list: 8 nodes, show progress of the simple list ranking algo across iterations to find the rank



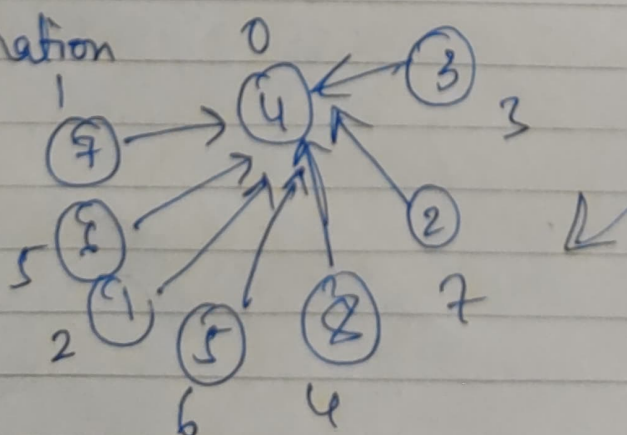
1st iteration:



2nd iteration:



3rd iteration



Q3)

$\phi = C_1 \wedge C_2 \wedge C_3 \wedge C_4$

$C_1 = x_1 \vee x_2 \vee \neg x_4$

$C_2 = x_2 \vee \neg x_3$

$C_3 = x_2 \vee x_3 \vee \neg x_4$

$C_4 = x_1 \vee x_3$

Equivalent integer linear program

C_1: $x_1 + x_2 + 1 - x_4 \geq z_1$
C_2: $x_2 + 1 - x_3 \geq z_2$
C_3: $x_2 + x_3 + 1 - x_4 \geq z_3$
C_4: $x_1 + x_3 \geq z_4$

x_i used here, $z_i \in \{0,1\}$ → only integer values

ϕ: $(x_1 + x_2 - x_4 + 1) * (x_2 - x_3 + 1) * (x_2 + x_3 - x_4 + 1) * (x_1 + x_3) \geq z_1 \cdot z_2 \cdot z_3 \cdot z_4$

Relaxed linear program:

C_1: $u_1 + u_2 + 1 - u_4 \geq v_1$ ⇒ $Pr(C_1) = 1 - (1-u_1)(1-u_2)(u_4)$
C_2: $u_2 + 1 - u_3 \geq v_2$ ⇒ $Pr(C_2) = 1 - (1-u_2)\cdot(u_4)$
C_3: $u_2 + u_3 + 1 - u_4 \geq v_3$ ⇒ $Pr(C_3) = 1 - (1-u_2)(1-u_3)(u_4)$
C_4: $u_1 + u_3 \geq v_4$ ⇒ $Pr(C_4) = 1 - (1-u_1)(1-u_3)$

$u_i \in [0,1]$ → Not just integer values

$Pr(C_i)$ is satisfied.

$\sum v_i^*$ = upper bound on the no of clauses to be satisfied