19th order Andred Haysis Show that an inner product space of finite dimension of has a hasis {b,,b, -. boy of orthonormal vectors. Bessel Inner Passenal's egr ルンスセーナスピンナスピる [27,e]= x オーナルナイン く オーナーナー と オーナー と オーナー と 3 Let  $\{V_1, V_2, -..., V_n\}$  is a baris in this T.P.S.(V)This is L.T. and Spans the space Apply the Gram Schwidt  $\{e_1, e_2, -..., e_n\}$ Grie an warper of an n & l', buch that the Bessel Inequality EKnew strict enea. Let EE 30 hea standard rothonorval banisty l.

E= (0,0,0---1,0---)

implace dits define  $\{e_{k}\}_{k=1}^{\infty}$  an orthogramal fit defid length  $e_{k}=\{e_{k+1}\}_{k=1}^{\infty}$ CK= CK+1 - is equivalent to thomeway want relevent  $T = C_1 = (1, 0 - 1)$  (1) = 1 (1) = 1 (1) = 1¿é, eu = ((1,0 - - ·), (0,0, -1,0·-)) ≥0 5/27, ep) - 0 2/ - 117/11 Meninum property of Forrige Carpticients det {e,,ez,-. h} (2,3) in R<sup>2</sup>, i, i = 2i+3i

N= x, e, + de, + de, t - - -(zen) ære the So-called Former Cresticients Let e, ez. . En ble an orthonored set in an t.P.S. X. where of is fined. Let abx be any fixed clement and y = B, e, + Bzez + - - . B, en . Then 1/2 yll depends on B. - Br. Show by direct Calculation that Ma-y" is minimum iff Bi = {7,ei} where i=1, - . h Best Appointen!  $||x-y|| = \langle x-y, x-y \rangle = \langle x-\frac{\pi}{\kappa}, \beta_{\kappa}e_{\kappa} \rangle x - \frac{\pi}{\kappa} \beta_{\kappa}e_{\kappa} \rangle$  $= \langle \mathcal{A}, \mathcal{A} \rangle - \langle \mathcal{A}, \stackrel{\leftarrow}{\xi} \beta_{k} e_{k} \rangle - \langle \mathcal{E} \beta_{k} e_{k}, \mathcal{A} \rangle + \langle \mathcal{E} \beta_{k} e_{k} \mathcal{E} \beta_{k} e_{k} \rangle$ 1711 - EBKARN - EBKERN T. ZBBRA Mail + Z (-By Lnjew) - BK ajew) + Bu fu) (Add ad Substant The Ka) (Add ad Substant The Ka) (Alexander) - Chew Knjew) L- (1, BP, +B227 - Bren) = La, fe)+ - - La, fe)+ - - La, fe) = - La, fe)+ - - Br La, en for fewen 1/2-41 = 1/211 + Z (Bu & - (2) ew) + Bu} - (2) ew) { - (2) ew) { - (2) ew} { - (2) ew} { - (2) ew} =  $\|x\|^2 + \sum_{k=1}^{\infty} (b_k - (x_i e_k))(b_k - (x_i e_k)) - (x_i e_k)$ = 117117 + 2 1 BL - (2, PW) 2, e,,ez - en are fixe 112-41 is nin 18 Bi = <7,000 who i=1,-. h 7; (t) 一七 m orthonorwalize the 18 3 terms of the sequence intional [-1,1], <2,4)= [26) y(4) df  $Z_0 = t^0$ 11201= (1.1.dr = t.] = 2 117/211 = 52 V,= ス,- 〈れ,ら〉 と。 othonormal Sequences and Sets. Formin Seres Trisonemetre serus is of the form

a t San WKt + be Smith Let a he a periodic function of period ?, a(t+P)= a(t) + t + R  $f(t) = a_0 + \sum_{k=1}^{\infty} a_k b s k t + b_k s r k t$   $(f(t)) = \int_{0}^{\infty} a_0 dh + 0 + 0$   $= \int_{0}^{\infty} a_0 dh + a_0 = \int_{0}^{\infty} \int_{0}^{\infty} f(t) dh$ IT (f(t) coskt dh = Sab coskdy +ans coskt-dr + D an = 1(fr) But M Def: A total Othornomal Set, Efundamental Set) in a normal Space is a somet MCX whose span is "dense in X: Accordingly an oriverwal set in an inner product space X which is total in X is called a total orivered St. Dense Set: A subset II of a metern Space X is said to be densitive if M = XXis said be Separable if it has a contable subject which is denticle D=R The Set of Ratural Nas 12 dense in R.
The Set of Real Nas is Deparable of Sirce it has a Countable fit - 4 Ratural
Nos which is dense in it The Coneplex pearl is Separable Lording = linn+ ling. His total in X Span M = X In every Helpert space of \$ 503 F a total or turn and fit If Mis denseinx, Then surey Nbd inx will contour. Annot Min W. Three is no pt x ex which has all that tannt boutern pri of.