Advanced Algorithms
Spring 2021
IIIT Hyderabad

Homework 1, Due: January 18, 2021

1. Consider the random experiment of throwing an unbiased coin n times independently. What is the probability of the following events:

   (a) The event that every 10th throw lands on Heads. Assume that n is divisible by 10, and also that we are not

   (b) The event that there is no head in any consecutive $10 \log n$ throws. For this question, you may want to use some assumptions and justify the same.

   (4+6=10 Points)

2. Recall the axioms of probability. Use the axioms to show that the way conditional probability as defined satisifes the axioms of probability. **(5 Points)**

3. Consider the experiment of giving away m T-shirts at an event to n participants. Each participant can receive any number of T-shirts including 0. All the shirts were given away at the end of the experiment. Compute/estimate the following.

   (a) The expected number of participants who do not get any T-shirt.

   (b) The expected number of people who get exactly one T-shirt.

   (c) The probability that some participant gets more than $10m \log n/n$ shirts. You can use any base of the logarithm you want to use.

   (3+2+5=10 Points)

4. Recall that you random variables are said to be identical if they have the same distribution. Consider two discrete random variables X and Y. You are told that X and Y have the same expectation. Are X and Y identical? Suppose now you are told that X and Y also have the same variance. Are X and Y idencial? State any assumptions you need to make to solve this question. **(5 Points)**