# Class no 3 :-

Signals → Carry Information

Comm → Reconstructing info at Rx end.

But what is information?

↳ Possible answers:

→ Anything interesting

→ Part of Data collected which is "useful"

→ What is interesting is something 'unexpected'

— Information of any outcome has to do with 'uncertainty' of that outcome.

'new' 'unexpected' → Indicate
'novel' 'high level of information.

Information deals with things/signals we don't know precisely but we are interested to find out

Say that $X$ is a random quantity representing the signal of interest (at some pt of 'time' or 'space')

Example: $X \in \{0, 1\}$ & can take value 0 with probability $p$, & value 1 with prob $1-p$.

↳ General abstraction of many real world scenarios

$X$ is also technically called a 'Random Variable'.

What is the information content in X ?

" Average uncertainty or surprise " in X ?

Information content in a particular event (here there are 2 possible outcomes $\{0,1\}$ & 2 corresponding events $X=0$ & $X=1$)

$\propto$ Inversely proportional to $P(\text{event})$

For ex:

Information content in event " $X=0$ "

$$\propto \frac{1}{P(X=0)}$$

ditto " $X=1$ " $\propto \frac{1}{P(X=1)}$

What fn can we assign to "Information content"?

→ Suppose $X_1, X_2$ are independent random variables

$$\boxed{\begin{aligned} &P(\underbrace{X_1 = x_1}_{\text{some value}}, \underbrace{X_2 = x_2}_{\text{some value}}) \\ &= P(X_1 = x_1)P(X_2 = x_2) \\ &\forall \text{ outcomes } x_1, x_2. \end{aligned}}$$

We expect the 'Joint information content in $X_1$ & $X_2$'

= Sum of Individual Info content

→ Thus product of probabilities has to convert into sum of Information content

So for $\boxed{\text{Info Content in event } X=x}$, the $\log_2\left(\frac{1}{P(X=x)}\right)$ is a good choice of fn.

Base 2

Note: $\log\left(\frac{1}{P(X_1=x_1, X_2=x_2)}\right) = \log_2 \frac{1}{P(X_1=x_1)} + \log_2 \frac{1}{P(X_2=x_2)}$

if $X_1, X_2$ are independent

The 'Info content' in event $X = x$ ← same value which $X$ can take

is assumed to be $\log \frac{1}{P(X=x)}$

So the 'average' info content ('expected' info content) 'mean'

Entropy in Random variable $X$ denoted by $H(X)$ $\triangleq \sum_{x \in \text{all possible values taken by } X} p(X=x) \left[ \log_2 \frac{1}{P(X=x)} \right]$

(Notation " $\triangleq$ " denotes that LHS is "defined as" RHS

'belongs to' (takes values in)

Lemma: Suppose $X_1 \in \mathcal{X}_1$ & $X_2 \in \mathcal{X}_2$ are independent random variables.

Then $H(X_1, X_2) = H(X_1) + H(X_2)$

where

$$H(X_1, X_2) \triangleq \sum_{\substack{x_1 \in \mathcal{X}_1 \\ x_2 \in \mathcal{X}_2}} P(X_1 = x_1, X_2 = x_2) \log \frac{1}{P(X_1 = x_1, X_2 = x_2)}$$

"Joint entropy of $X_1$ & $X_2$"

Proof: Use the fact that $X_1, X_2$ are indep & the fact that $\sum_{x_1 \in \mathcal{X}_1} P(X_1 = x_1) = 1 = \sum_{x_2 \in \mathcal{X}_2} P(X_2 = x_2)$ in defn of $H(X_1, X_2)$ and prove the result. [Exercise for Monday, 31st May]