Preparation for the Final

18.303 Linear Partial Differential Equations

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Final Exam: **Wednesday, December 21, 1:30PM - 4:30PM**

This document contains an outline of material I expect you to know for the final. It also contains extra practice problems for the final. Review all notes, except where noted below. Unless I say below that you are NOT responsible for something, you should assume you are responsible for that topic, even if it is not listed. However, concentrate on the topics below. Be able to do all Problem Set problems, except when they involve things I've said you are NOT responsible for. Be able to do all problems on last year's problem sets, on the OCW website. Be able to do all problems on the practice Exams and the Exams 1, 2 you wrote this year.

The final will be a mix of problems like the ones on your assignments, exams, and last years assignments and exams (see OCW). 70% of the problems on your final will be basic, and 30% more difficult. To get a B in the course you should be able to solve the basic problems, and show some effort on the more advanced problems. To get an A you should be able to do well on the advanced type problems.

(NEW) I suggest you work through the old pset q's, last year's pset q's, and the practice problems - you should be able to do all of these, and they will form the "basic problems", i.e. basic q's on the final will be similar to these. You'll notice some of these are a little different. Anything that's different than the norm (like the annulus practice problem) would be of advanced nature, but of course i can adjust the difficulty by giving hints. For example, 1(c) on PSet 6, and the extra part I added to the solution of Q4 on PSet 5, and the last line of Q5 on PSet 5, would be of advanced nature, unless I gave hints.

(NEW) You are NOT responsible for Green's Functions.

1 Review

1.1 Heat and Wave Equations (1D, 2D, 3D)

- be able to work with dimensional and non-dimensional versions of problems

1.1.1 Separation of variables

- solve homogeneous problem with initial condition, BCs

- Fourier Series, orthogonality - I'll give you the orthogonality relations; be sure you know how to use them

- (UPDATED) don't worry about convergence definitions and proofs in HeatEqI.pdf (i.e. sections 4.1, 4.2, 5.4, 5.5, section 9)

- linearity, superposition, types of BCs

- First term approx, smallest eigenvalue, and how this relates to cooling rate in the Heat Equation and frequencies of free oscillation for the wave equation

1.1.2 Inhomogeneous BCs

- (time independent) find steady-state solution (separation of variables, etc)

- (time dependent) find quasi-steady state (complexify - only simple versions of this problem)

- multiple inhomogeneous BCs - use superposition of solutions for problems with a single inhomogeneous BC (e.g. Haberman §2.5 and §7.9)

1.1.3 Inhomogeneous PDEs

- sources, convection terms - transform (problem 5, PSet 2)

1.1.4 Sturm Liouville Problem

- (UPDATED) on rectangle, 3D box, circle, cylinder, sphere, triangle (see also last year's PSet 5 on OCW)

- I'll give you the Laplacian in polar, cylindrical, spherical coords - so know how to use it

- I'll also give you the relevant Bessel Equation and solution, and any necessary info about the solution - I expect you to be able to get the Bessel Equation using the Laplacian I give you with separation of variables

- (NEW) Mean Value Property (statement and proof) - see last year's exam for an example problem

- (NEW) Maximum Principle (statement, be able to use, not responsible for proof)

  - e.g. what is the value of the solution u(x,y) to Laplace's equation on the interior of a domain if u(x,y)=10 everywhere on the boundary?

- (UPDATED) remember the facts: positive (for Type I BCs), real eigenvalues, orthogonal eigenfunctions (don't have to prove, which is what takes up most of sections 6.1, 6.2, 6.3)

- don't worry about Faber Kahn inequality

- Section 13, pde3d.pdf - just remember the following results, and how to use:

  - Rayleigh Quotient - I'll give you the definition, but remember what it says about the smallest eigenvalue - look at the solution at the end of problem 4, PSet 5, for an example of the use of the Rayleigh Quotient to estimate the upper bound on the smallest eigenvalue.

  - If one domain A is contained in B, the smallest eigenvalue on B is smaller than the smallest eigenvalue on A, i.e. A cools faster than B.

- Nodal Lines and combining eigenfunctions of the same eigenvalue to obtain an eigenfunction on another domain - know the problem on assignment 5, and in addition how to set up the table to find the m,n's (e.g. (vii) on P 36-37 in pde3d.pdf))

1.1.5 Heat Equation only

- uniqueness proof

- spatial temperature profiles, time temperature profiles, level curves and heat flow lines

- symmetry arguments

- equilibrium (steady-state) temperature profile

1.1.6 Wave Equation only

- space time plane, characteristics

- displacement vs. x plot

- displacement vs. time plot

- normal modes: frequencies of free oscillations, period, amplitude

- energy conservation

  - I will give you the formula, but know how to use it
  - know how to show dE/dt=0, using the wave Eq PDE

- D'Alembert solution

  - be able to derive via change of variable (§5.2-5.5 of the wave equation notes), and use to plot u(x,t) at various t
  - you may assume the initial velocity g(x)=0, hence problem reduces to shifting the IC

- finite and semi-infinite string - how to generalize D'Alembert solution by extending f(x)

1.2 Quasi-Linear PDEs

- given a PDE, find parametric solution, characteristics

- you don't have to memorize any particular PDE, like the traffic flow PDE, just know how to work with them

- validity, time and location of breakdown of validity (shocks)

- how to plot u(x,t) at various t (table method)

- space time plot of characteristics

1.3   Infinite domain Heat Problems and Fourier Transform

- Solve Heat Equation on infinite and semi-infinite domains

- Fourier Transform (I'll give you definition, know how to use it)

- know how to get FT of derivatives (of x, of t or y)

- Inverse FT (IFT) (Haberman p 468, etc) - I will give you the definition, and any IFTs you need; know how to use it

- Error Function erf(x) - I'll give you the definition; know how to use and differentiate

- Convolution Thm (statement only, and how to use, not proof)

- Shifting Thm (Haberman p 468) - I'll give you that, but know how to use it

- semi-infinite domains and how to deal with boundary conditions

- Laplace's equation on infinite and semi-infinite domains

  - extend dimension with inhomogeneous BC, and apply FT along this dimension

2   Practice Problems

1.4.7, 1.4.11

2.2.3, 2.4.1(b), 2.5.8(a), 2.5.9

7.3.1, 7.3.2, 7.3.3, 7.3.5, 7.3.7, 7.5.1, 7.7.4, 7.7.9

10.5.16, 10.6.1, 10.6.3

12.4.1, 12.4.2, 12.6.8(c),(e)