Q1. (a) Let X denote the number of flips of a fair coin until the first head appears.

Find the entropy of X in bits.

The following expressions may be useful

$$\sum_{n=1}^{\infty} r^n = \frac{r}{1-r} \quad ; \quad \sum_{n=1}^{\infty} n r^n = \frac{r}{(1-r)^2}$$

(b) Let Y denote the no. of flips until the second head appears.

Show that $H(Y) \leq 2H(X)$

2. Let the joint distribution of Random Variables be given as follows

$P(X=x, Y=y)$ (which we write as $p(x,y)$)

is given by the following table

X \ Y	0	1
0	1/3	1/3
1	0	1/3

(a) Find $H(X)$, $H(Y)$

(b) $H(X,Y)$

(c) $H(X|Y)$, $H(Y|X)$

(d) $H(Y) - H(Y|X)$

(e) $H(X) - H(X|Y)$

(f) $I(X;Y)$