

Q2 The majority logic decoding algorithm is as follows -

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1  FOR l = r to 0 DO
2      FOR each A ∈ {1, ..., m}^l, |A| = l DO:
3          FOR each b ∈ F_2^{m-l}
4              m_A(b) = ∑_{\substack{x_A = b \\ x_{\{1, \dots, m\} \setminus A} = b}} y
5              cnt[m_A(b)] += 1.
6              m_A = argmax cnt
7      let M_r = ∑_{\substack{A \in \{1, \dots, m\} \\ |A| = l}} m_A x_A
8      For c ∈ F_2^m DO
9          y_c = M_r | c
10 Output ∑ M_A x_A
```

The outer loops in lines 1, 2 run  $\sum_{l=0}^r \binom{m}{l} = k$  times.

The loop in line 3 runs  $2^{m-l}$  times.

The summation in line 4 takes  $2^l$  operations.

Thus these require  $O(2^m)$  operations.

Line 7 requires  $\binom{m}{l}$  operations.

∴ total of  $\sum_{l=0}^r \binom{m}{l} = k$  operations.

line 8 iterates  $O(2^m)$  times. Evaluating  $M_r$  at each position requires  $\binom{m}{2}$  operations.

$$\Rightarrow \sum_{r=0}^r 2^m \cdot \binom{m}{2} = k \cdot 2^m \text{ in total.}$$

$$\begin{aligned} \text{Thus total time} &= O(k \cdot 2^m) + O(k) + O(k \cdot 2^m) \\ &= O(k \cdot 2^m). \end{aligned}$$