

Linear partial differential equations: End Set exam (Full marks 10)
Time 1.5 hours [Answer sheet should be submitted before 5 pm]

- If $H(S, p)$ is the enthalpy of a thermodynamic (p, V, T, S) system, the values of $(\frac{\partial H}{\partial S})_p, (\frac{\partial H}{\partial p})_S$ are respectively (a) (T, V) (b) (V, T) (c) (U, V) (d) (T, U) . [Marks 1]
- Helmoltz free energy $F = U - TS$ and Gibbs free energy $G = F + pV$. Which of the followings is true ? (a) $(\frac{\partial G}{\partial T})_p = V$ (b) $(\frac{\partial G}{\partial p})_T = -S$ (c) $G = (\frac{\partial(F/V)}{\partial(1/V)})_T$ (d) $F = (\frac{\partial(G/V)}{\partial(1/V)})_T$ [Marks 2]
- Solve $(x + 2z)z_x + (4zx - y)z_y = 2x^2 + y$. (a) $F(x^2 + y^2 + z^2, x - y - z) = 0$ (b) $F(xy - z, z^2x + y) = 0$ (c) $F(x - yz, x + y + z) = 0$ (d) $F(xy - z^2, x^2 - y - z) = 0$ [Marks 2]
- Solve $(y + z)z_x + (z + x)z_y = x + y$ (a) $F(\frac{x - y}{y - z}, \frac{y - z}{\sqrt{x + y + z}}) = 0$ (b) $F(\frac{x - z}{y - z}, \frac{z - x}{\sqrt{x + y + z}}) = 0$ (c) $F(\frac{y - z}{z - x}, \frac{x - y}{\sqrt{x + y + z}}) = 0$ (d) $F(x - y, x - y + z) = 0$ [Marks 2]
- A pendulum consists of a mass m and a massless stick of length l . The pendulum support oscillates horizontally with a position given by $x(t) = A \cos \omega t$ [Figure attached]. What is the general solution for the angle of the pendulum as a function of time?
(a) $l\ddot{\theta} - A\omega^2 \cos \omega t \cos \theta + g \sin \theta = 0$ (b) $l\ddot{\theta} - A \cos \omega t \sin \theta + g \cos \theta = 0$
(c) $l\ddot{\theta} + A\omega^2 \sin \omega t \sin \theta + g \cos \theta = 0$ (d) $l\ddot{\theta} + A\omega^2 \cos \omega t \cos \theta + g \cos \theta = 0$ [Marks 3]