Recipe to derive Kraus operators:

A dynamical map Λ_d is given, which acts on a d ~~ma~~ dimensional quantum system

Find the Choi matrix for this map:

$$\mathbb{I}_d \otimes \Lambda_d(|\psi\rangle\langle\psi|) = C_d$$

$|\psi\rangle = \frac{1}{\sqrt{d}} \sum_{i=1}^{d} |ii\rangle \rightarrow$ maximally entangled state in d dimension.

Diagonalize C_d : $C_d = \sum_{\alpha} \lambda_\alpha |\alpha\rangle\langle\alpha|$

$|\alpha\rangle \rightarrow$ d×d dimensional vector.

Now: $|\alpha\rangle = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_{d^2} \end{pmatrix}$

Build a matrix $A_\alpha = \begin{pmatrix} a_1 & a_{d+1} & \cdots & \vdots \\ a_2 & a_{d+2} & \cdots & \vdots \\ \vdots & \vdots & & \vdots \\ a_d & a_{2d} & & a_{d^2} \end{pmatrix}$

each Kraus operator

$$K_\alpha = \sqrt{\lambda_\alpha}\, A_\alpha .$$

Dephasing channel: (qubit).

The dynamical map:

$$\Lambda(\rho) = \begin{pmatrix} \rho_{11} & \rho_{12} e^{-\gamma t} \\ \rho_{21} e^{-\gamma t} & \rho_{22} \end{pmatrix}$$

Maximally entangled state:

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$$

$$\rho_\psi = |\psi\rangle\langle\psi| = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}.$$

$$\Lambda(\rho_\psi) = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{e^{-\gamma t}}{2} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ e^{-\gamma t}/2 & 0 & 0 & 1/2 \end{pmatrix}$$

Eigen values $\lambda_1 = \lambda_2 = 0$; $\lambda_3 = \frac{1 - e^{-\gamma t}}{2}$; $\lambda_4 = \frac{1 + e^{-\lambda t}}{2}$

Eigenvectors $|1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}$; $|2\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

$|3\rangle = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$; $|4\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 1 \end{pmatrix}$.

$$\therefore \quad K_1 = 0 \; ; \quad K_2 = 0$$

$$K_3 = \frac{\sqrt{1-e^{-\gamma t}}}{\sqrt{2}} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$K_4 = \frac{\sqrt{1+e^{-\gamma t}}}{\sqrt{2}} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$K_3^{\dagger} K_3 + K_4^{\dagger} K_4 = \frac{1-e^{-\gamma t}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \frac{1+e^{-\gamma t}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Lambda(\rho) = K_3 \rho K_3^{\dagger} + K_4 \rho K_4^{\dagger}$$

$$= \frac{1-e^{-\gamma t}}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$+ \frac{1+e^{-\gamma t}}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$= \frac{1-e^{-\gamma t}}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -\rho_{11} & \rho_{12} \\ -\rho_{21} & \rho_{22} \end{pmatrix} + \frac{1+e^{-\gamma t}}{2} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

$$= \frac{1-e^{-\gamma t}}{2} \begin{pmatrix} \rho_{11} & -\rho_{12} \\ -\rho_{21} & \rho_{22} \end{pmatrix} + \frac{1+e^{-\gamma t}}{2} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$$

$$= \begin{pmatrix} \rho_{11} & \rho_{12} e^{-\gamma t} \\ \rho_{21} e^{-\gamma t} & \rho_{22} \end{pmatrix}$$