IC Tutorial Class 2:

1. A fair die is thrown. If the outcome is either 1, 2, 3 or 4, then one coin is tossed. If the outcome is 5 or 6, then two coins are tossed. Find the joint entropy of the outcome of the die throw & the number of heads obtained. What is the average uncertainty in the number of heads if the outcome of the die throw is known? Is it less or more than the avg uncertainty of no. of heads? Verify by calculation & connect with class.

1.4 Establish the following:

a. $$H(Y, Z \mid X) \leq H(Y \mid X) + H(Z \mid X)$$

with equality if and only if $p(y_j, z_k \mid x_i) = p(y_j \mid x_i)p(z_k \mid x_i)$ for all i, j, k.

b. $$H(Y, Z \mid X) = H(Y \mid X) + H(Z \mid X, Y).$$

c. $$H(Z \mid X, Y) \leq H(Z \mid X)$$

with equality if and only if $p(y_j, z_k \mid x_i) = p(y_j \mid x_i)p(z_k \mid x_i)$ for all i, j, k.

2.

Solve the above problem. After that give a means to generalize the same statement, by comparing it to a result showed in class.

3.

1.6 A change that tends in the following sense to equalize a set of probabilities $p_1, \ldots, p_M$ always results in an increase in uncertainty:

Suppose $p_1 > p_2$. Define

$$p_1' = p_1 - \Delta p$$
$$p_2' = p_2 + \Delta p$$
$$p_i' = p_i, \; i = 3, \ldots, M$$

where $\Delta p > 0$ and $p_1 - \Delta p \geq p_2 + \Delta p$. Show that $H(p_1', \ldots, p_M') > H(p_1, \ldots, p_M)$.

Hint: For both prob 2 & prob 3, you can use relative entropy ≥ 0.