

Advanced Algorithms  
Spring 2021  
IIIT Hyderabad

Homework 1, Due: January 18, 2021

1. Consider the random experiment of throwing an unbiased coin  $n$  times independently. What is the probability of the following events:

- (a) The event that every 10th throw lands on Heads. Assume that  $n$  is divisible by 10, and also that we are not
- (b) The event that there is no head in any consecutive  $10 \log n$  throws. For this question, you may want to use some assumptions and justify the same.

**(4+6=10 Points)**

2. Recall the axioms of probability. Use the axioms to show that the way conditional probability as defined satisfies the axioms of probability. **(5 Points)**
3. Consider the experiment of giving away  $m$  T-shirts at an event to  $n$  participants. Each participant can receive any number of T-shirts including 0. All the shirts were given away at the end of the experiment. Compute/estimate the following.

- (a) The expected number of participants who do not get any T-shirt.
- (b) The expected number of people who get exactly one T-shirt.
- (c) The probability that some participant gets more than  $10m \log n/n$  shirts. You can use any base of the logarithm you want to use.

**(3+2+5=10 Points)**

4. Recall that two random variables are said to be identical if they have the same distribution. Consider two discrete random variables  $X$  and  $Y$ . You are told that  $X$  and  $Y$  have the same expectation. Are  $X$  and  $Y$  identical? Suppose now you are told that  $X$  and  $Y$  also have the same variance. Are  $X$  and  $Y$  identical? State any assumptions you need to make to solve this question. **(5 Points)**