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Basis set: $\{G_a\} = \frac{\sigma_a}{\sqrt{2}}$

$L_{k\ell}(t) = \mathrm{Tr}\left[G_k \Lambda(G_\ell)\right]$

$$\Lambda(\rho) = \frac{d\rho}{dt} = \gamma(n+1)\left(\sigma_- \rho \sigma_+ - \frac{1}{2}\{\sigma_+\sigma_-, \rho\}\right) + \gamma(n)\left(\sigma_+ \rho \sigma_- - \frac{1}{2}\{\sigma_-\sigma_+, \rho\}\right) + \lambda\left(\sigma_z \rho \sigma_z - \rho\right)$$

Let $\rho = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$

substituting ρ in the master Equation above, substituting the values of $\sigma_z, \sigma_+, \sigma_-$ and so on,

$$\frac{\partial \rho}{\partial t} = \begin{bmatrix} \gamma(n+1)d - \gamma(n)a & \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)b \\ \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)c & -\gamma(n+1)d + \gamma(n)a \end{bmatrix}$$

$\downarrow$

$\Lambda(\rho)$

We need to make the L matrix, we need values:

$$\Lambda(G_0) = \Lambda\left(\frac{\sigma_0}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\Lambda(\sigma_0)$$

$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \Rightarrow a = d = 1, \ b = c = 0$

$$\Lambda(G_0) = \frac{1}{\sqrt{2}}\begin{bmatrix} \gamma(n+1)1 - \gamma(n)1 & 0 \\ 0 & -\gamma(n+1) + \gamma(n) \end{bmatrix}$$

$$= \frac{1}{\sqrt{2}}\begin{bmatrix} \gamma & 0 \\ 0 & -\gamma \end{bmatrix} = \frac{\gamma}{\sqrt{2}}\sigma_z$$

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$$\Lambda(G_1) = \Lambda\left(\frac{\sigma_x}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\Lambda(\sigma_x) \quad , \sigma_x \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad a=d=0, \; b=c=1$$

$$\therefore \Lambda(G_1) = \frac{1}{\sqrt{2}}\Lambda(\sigma_x) = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 & \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)1 \\ \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)1 & 0 \end{bmatrix}$$

$$= \frac{\left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)}{\sqrt{2}} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$= \frac{\left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)}{\sqrt{2}} \sigma_x$$

$$\Lambda(G_2) = \Lambda\left(\frac{\sigma_y}{\sqrt{2}}\right) = \frac{-\gamma n - \frac{\gamma}{2} - 2\lambda}{\sqrt{2}} \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$= \frac{-\gamma n - \frac{\gamma}{2} - 2\lambda}{\sqrt{2}} \sigma_y$$

$$\Lambda(G_3) = \Lambda\left(\frac{\sigma_z}{\sqrt{2}}\right) = \frac{1}{\sqrt{2}}\Lambda(\sigma_z) \qquad a=1, \; d=-1, \; b=0, \; c=0)$$

$$= \frac{1}{\sqrt{2}} \begin{bmatrix} -\gamma(n+1) - \gamma n & 0 \\ 0 & \gamma(n+1) + \gamma(n) \end{bmatrix}$$

$$= \frac{\gamma(n+1) + \gamma(n)}{\sqrt{2}} \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= -\frac{(\gamma(n+1) + \gamma(n))}{\sqrt{2}} \sigma_z$$

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$$L_{00} = Tr\left[G_0 \cdot \Lambda(G_0)\right] = Tr\left[\frac{\sigma_0}{\sqrt{2}} \frac{\gamma}{\sqrt{2}} \sigma_z\right]$$

$$= \frac{\gamma}{2} Tr(\sigma_z) = 0$$

$$L_{01} = Tr\left[G_0 \cdot \Lambda(G_1)\right] = Tr\left[\frac{\sigma_0}{\sqrt{2}} \cdot \frac{\left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right)\sigma_x}{\sqrt{2}}\right]$$

$$= 0$$

$$\left.\begin{aligned} L_{02} &= 0 \\ L_{03} &= 0. \end{aligned}\right\} \longrightarrow \text{property: } Tr(\sigma_k \cdot \sigma_l) = 2\delta_{kl}$$

$$L_{10} = Tr\left(G_1 \Lambda(G_0)\right) = Tr\left(\frac{\sigma_x}{\sqrt{2}} \cdot \frac{\gamma}{\sqrt{2}} \sigma_z\right)$$

$$= \frac{\gamma}{2} Tr(\sigma_x \sigma_z) \rightarrow 0$$

$$= 0$$

$$L_{11} = Tr\left(G_1 \Lambda(G_1)\right) = Tr\left(\frac{\sigma_x}{\sqrt{2}} \cdot \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right) \frac{\sigma_x}{\sqrt{2}}\right)$$

$$= \frac{1}{\cancel{2}}\left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right) \cdot Tr(\sigma_x \cdot \sigma_x) \rightarrow \cancel{2}$$

$$= -\gamma n - \frac{\gamma}{2} - 2\lambda$$

$$L_{12} = Tr\left(G_1 \Lambda(G_2)\right) = Tr\left(\frac{\sigma_x}{\sqrt{2}} \cdot \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right) \frac{\sigma_y}{\sqrt{2}}\right)$$

$$= 0$$

$$L_{13} = 0 \longrightarrow Tr(\sigma_x \sigma_z)$$

$$L_{20} = 0 \longrightarrow Tr(\sigma_y \sigma_z), \quad L_{21} = 0 \quad Tr(\sigma_y \sigma_x)$$

$$L_{22} = Tr\left(\frac{\sigma_y}{\sqrt{2}} \cdot \left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right) \frac{\sigma_y}{\sqrt{2}}\right)$$

$$= \frac{1}{2}\left(-\gamma n - \frac{\gamma}{2} - 2\lambda\right) \cdot 2 \longrightarrow Tr(\sigma_y \sigma_y) = 2$$

$$= -\gamma n - \frac{\gamma}{2} - 2\lambda$$

$L_{23} = 0 \quad \mathrm{Tr}(\sigma_y \cdot \sigma_z)$

$L_{30} = \mathrm{Tr}\left(\frac{\sigma_z}{\sqrt{2}} \cdot \frac{\gamma}{\sqrt{2}} \sigma_z\right) = \frac{\gamma}{\cancel{2}} \cdot \underbrace{\mathrm{Tr}(\sigma_z \cdot \sigma_z)}_{\to \cancel{2}}$

$= \gamma$

$L_{31} = 0 \to \mathrm{Tr}(\sigma_z \cdot \sigma_x)$

$L_{32} = 0 \to \mathrm{Tr}(\sigma_z \cdot \sigma_y)$

$L_{33} = \mathrm{Tr}\left(\frac{\sigma_z}{\sqrt{2}} \cdot \frac{-\gamma(n+1) - \gamma(n)}{\sqrt{2}} \sigma_z\right)$

$= \frac{1}{\cancel{2}}(-2\gamma n - \gamma) \cdot \cancel{2} \longrightarrow \mathrm{Tr}(\sigma_z \cdot \sigma_z)$

$= -2\gamma_n - \gamma$

$$L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\gamma_n - \frac{\gamma}{2} - 2\lambda & 0 & 0 \\ 0 & 0 & -\gamma_n - \frac{\gamma}{2} - 2\lambda & 0 \\ \gamma & 0 & 0 & -2\gamma_n - \gamma \end{bmatrix}$$

Since L has no terms in 't', L is time independent

We know $\frac{\partial f}{dt} = L \cdot f$

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We need to find $\phi(\rho)$

$$\phi(\rho) = (F \cdot r)^T \cdot G.$$

$$r_\ell = tr\left[G_\ell \ \rho\right]$$

Let $\rho = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}$

$$r_0 = tr\left[G_0 \ \rho\right] = \frac{1}{\sqrt{2}} tr[\rho] = \frac{\rho_{11} + \rho_{22}}{\sqrt{2}}$$

$$r_1 = tr\left[G_1 \ \rho\right] = \frac{1}{\sqrt{2}} tr\left[\sigma_x \cdot \rho\right]$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} = \begin{pmatrix} \rho_{21} & \rho_{22} \\ \rho_{11} & \rho_{12} \end{pmatrix}$$

$$r_1 = \frac{1}{\sqrt{2}} \rho_{21} + \rho_{12}$$

$$r_2 = tr\left[G_2 \rho\right] = \frac{1}{\sqrt{2}} tr\left[\sigma_y \rho\right] = \frac{1}{\sqrt{2}} \left(-i \rho_{21} + i \rho_{12}\right)$$

$$r_3 = tr\left[G_3 \cdot \rho\right] = \frac{1}{\sqrt{2}} tr\left(\sigma_z \cdot \rho\right) = \frac{1}{\sqrt{2}} \left(\rho_{11} - \rho_{22}\right)$$

$$r = \frac{1}{\sqrt{2}} \begin{bmatrix} \rho_{11} + \rho_{12} \\ \rho_{21} + \rho_{12} \\ i\rho_{12} - i\rho_{21} \\ \rho_{11} - \rho_{22} \end{bmatrix}$$