Joint and Marginal Probability Analysis of Markov Random Field Networks for Digital Logic Circuits

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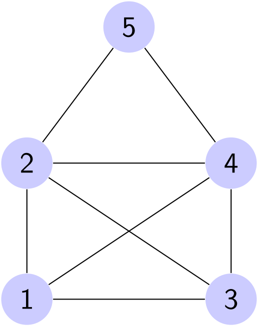
Introduction

- Nanoscale electronic circuits are suffering from both manufacturing defects and transient faults.
- Since, the integrated circuits are scaling to nano-level, they are expected to face high computing errors.
- Markov Random Field modelling is one approach to achieve noise-tolerance.

Cliques

- A clique is a complete subgraph of a given graph.
- A maximal clique is a clique that cannot be extended by including one more adjacent vertex.



Markov network

Joint Probability of MRF

Gibbs distribution

A Gibbs distribution on the graph G takes the form:

$$\Pr(X) = \frac{1}{Z} \prod_{c \in C} \phi_c(x_c) \tag{1}$$

$$\phi_c(x_c) = e^{-\frac{U_c(x_c)}{kT}} \tag{2}$$

- The Hammersley-Clifford theorem states that the joint probability distribution of any MRF can be written as a Gibbs distribution, and furthermore that for any Gibbs distribution there exists an MRF for which it is the joint.

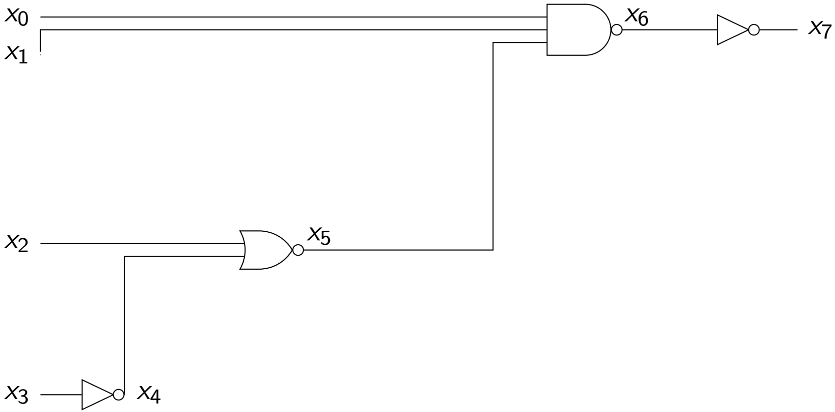
Marginal Probability

- We fix the value of one or more variables and sum it over non-fixed variables.

$$\Pr(X = x) = \sum_{y} \Pr(X = x, Y = y) \tag{3}$$

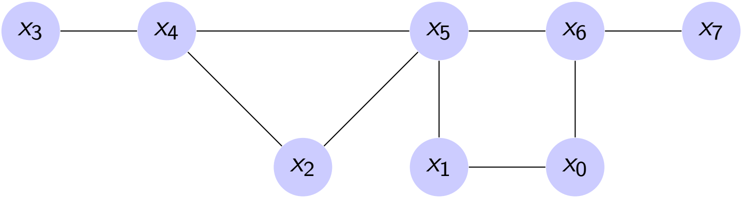
$$= \sum_{y} \Pr(X = x | Y = y) \Pr(Y = y) \tag{4}$$

Computing Joint Probability



M3 module of C432 Interrupt Controller

Computing Joint Probability (contd...)



Dependence graph of the test circuit

Cliques: $\{x_3, x_4\}, \{x_2, x_4, x_5\}, \{x_0, x_1, x_5, x_6\}, \{x_6, x_7\}$

Computing Joint Probability (contd...)

x_0	x_1	x_5	x_6	f
0	0	0	1	1
0	0	0	0	0
0	0	1	1	1
0	0	1	0	0
0	1	0	1	1
0	1	0	0	0
0	1	1	1	1
0	1	1	0	0

x_0	x_1	x_5	x_6	f
1	0	0	1	1
1	0	0	0	0
1	0	1	1	1
1	0	1	0	0
1	1	0	1	1
1	1	0	0	0
1	1	1	0	1
1	1	1	1	0

Logic Compatibility Function for NAND

Computing Joint Probability (contd...)

We will compute the clique energy function for $\{x_0, x_1, x_5, x_6\}$

$$U_c = -\sum (\text{Valid minterms } (f = 1) \text{ in the Logic Compatibility Function}) \quad (5)$$

$$= -\left[x_0'x_1'x_5'x_6 + x_0'x_1'x_5x_6 + x_0'x_1x_5'x_6 + x_0'x_1x_5x_6 + x_0x_1'x_5'x_6 + x_0x_1'x_5x_6 + x_0x_1x_5x_6'\right] \quad (6)$$

$$= 2x_0x_1x_5x_6 - x_0x_1x_5 - x_6 \quad (7)$$

Computing Joint Probability (contd...)

Clique energy functions for NOT and NOR gates

NOT 1	$2x_3x_4 - x_3 - x_4$
NOR	$x_2x_4 + 2x_4x_5 + 2x_2x_5 - 2x_2x_4x_5 - x_2 - x_4 - x_5$
NOT 2	$2x_6x_7 - x_6 - x_7$

Now, we will compute the joint probability

$$\Pr\left(x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7\right) = \frac{1}{Z}\left(e^{\frac{-U_c(\text{NOT 1})}{kT}} e^{\frac{-U_c(\text{NAND})}{kT}} e^{\frac{-U_c(\text{NOR})}{kT}} e^{\frac{-U_c(\text{NOT 2})}{kT}}\right) \tag{8}$$

$$= \frac{1}{Z} exp\left[\frac{\begin{array}{r} x_2 + x3 + 2x_4 + x_5 + 2x_6 + x_7 - x_2x_4 - 2x_2x_5 - 2x_3x_4 \\ - 2x_4x_5 - 2x_6x_7 + x_0x_1x_5 + 2x_2x_4x_5 - 2x_0x_1x_5x_6 \end{array}}{kT}\right] \tag{9}$$

Computing Joint Probability (contd...)

- Following the joint probability calculation, we need to find the node label combinations that maximize its value.
- The maximum value is $\frac{1}{Z}e^{\frac{4}{kT}}$. There are 16 node label combinations.
- These 16 combinations are same as the circuit's truth table, which shows that joint probability is maximum only for correct logic combinations.
- For the rest of the combinations, the joint probability is lower.

Design Principle of Joint Probability

For perfect logic operation of a circuit, we need to design our circuit such that joint probability of the circuit remains maximum at all times.

Computing Marginal Probability

- For computing the probability of the (intermediate and output) nodes, we will use the belief propagation algorithm.
- We will assign PDFs to all inputs and cliques.

$$\Pr(x_7) = f_0 f_1 f_2 f_3 f_4 f_5 f_6 f_7 \tag{10}$$

PDFs for Inputs and Cliques

Input	PDF	Clique	PDF
x_0	$f_0(s_0)$	$\{x_3, x_4\}$	$f_4(x_3, x_4)$
x_1	$f_1(s_1)$	$\{x_2, x_4, x_5\}$	$f_5(x_2, x_4, x_5)$
x_2	$f_2(s_2)$	$\{x_0, x_1, x_5, x_6\}$	$f_6(x_0, x_1, x_5, x_6)$
x_3	$f_3(s_3)$	$\{x_6, x_7\}$	$f_7(x_6, x_7)$

Computing Marginal Probability (contd...)

Step 1: (Eliminating x_3)

$$\Pr\left(x_4\right) = \sum_{x_3 \in \{0,1\}} \frac{1}{Z_1} e^{-\frac{U_c(\text{NOT } 1)}{kT}} \tag{11}$$

$$= \frac{1}{Z_1}\left(e^{\frac{x_4}{kT}} + e^{\frac{1-x_4}{kT}}\right) \tag{12}$$

$$= f_8\left(x_4\right) \tag{13}$$

$$\Pr\left(x_7\right) = f_0 f_1 f_2 f_5 f_6 f_7 f_8 \tag{14}$$

Computing Marginal Probability (contd...)

Step 2: (Eliminating x_2)

$$\Pr\left(x_5|x_4\right) = \sum_{x_2\in\{0,1\}} \frac{1}{Z_2} e^{-\frac{U_c(\text{NOR})}{kT}} \tag{15}$$

$$= \frac{1}{Z_2}\left(e^{\frac{x_4+x_5-2x_4x_5}{kT}} + e^{\frac{1-x_5}{kT}}\right) \tag{16}$$

$$= f_9\left(x_4, x_5\right) \tag{17}$$

$$\Pr\left(x_7\right) = f_0 f_1 f_6 f_7 f_8 f_9 \tag{18}$$

Computing Marginal Probability (contd...)

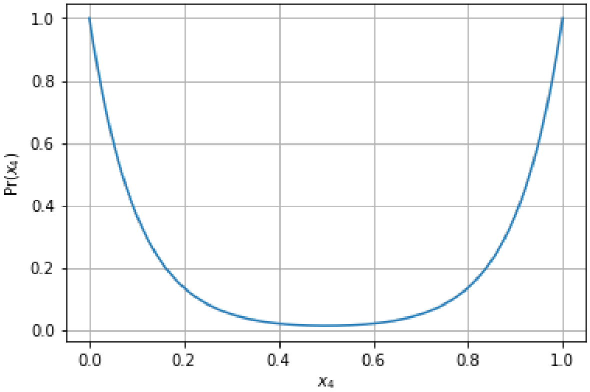


Figure: Plot of probability of x_4 against x_4

Computing Marginal Probability (contd...)

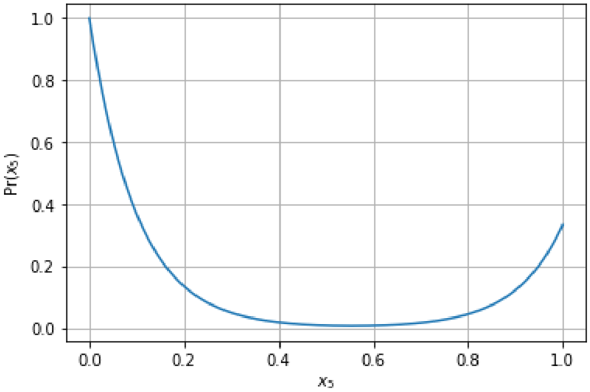


Figure: Plot of probability of x_5 against x_5

Computing Marginal Probability (contd...)

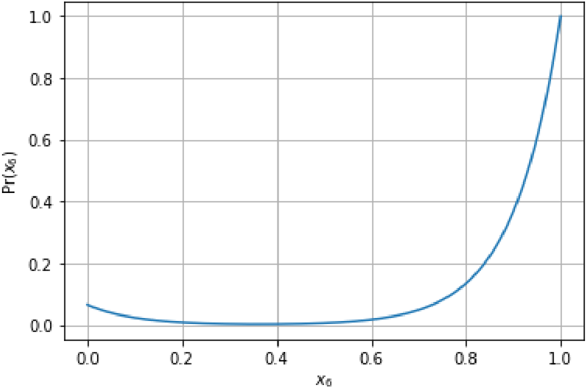


Figure: Plot of probability of x_6 against x_6

Computing Marginal Probability (contd...)

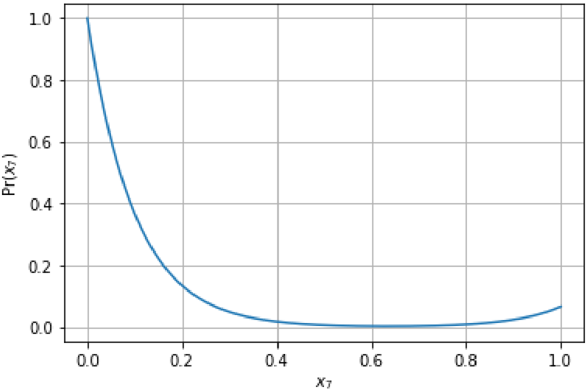


Figure: Plot of probability of x_7 against x_7

Computing Marginal Probability (contd...)

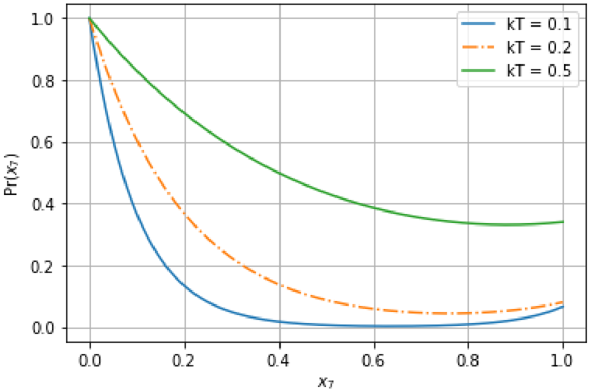


Figure: Plot of probability of x_7 for different values of T

Computing Marginal Probability (contd...)

Design Principle of Marginal Probability

The key to design a fault-tolerant circuit is to ensure a good heat removal system for the integrated circuit which would make the probability of intermediate states between '0' and '1' close to zero and maintain sufficient noise margin as well.