Assignment 2

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PROBLEM

(Gate-EC 17) The input X to the binary Symmetric Channel (BSC) shown in Fig. 0 is '1' with probability 0.8. The cross-over probability is $\frac{1}{7}$. If the received bit $Y = 0$, the condition probability that '1' was transmitted is ...
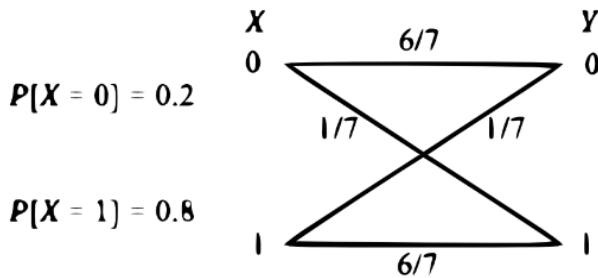

Fig. 0: Binary Symmetric Channel

SOLUTION

$$\Pr(X = 1|Y = 0) = \frac{\Pr(\{X = 1\}\{Y = 0\})}{\Pr(Y = 0)} \quad (0.0.1)$$

$$\Pr(Y = 0|X = 1) = \frac{\Pr(\{X = 1\}\{Y = 0\})}{\Pr(X = 1)} \quad (0.0.2)$$

From eq. (0.0.2),

$$\Pr(\{X = 1\}\{Y = 0\}) = \Pr(Y = 0|X = 1)\Pr(X = 1) \quad (0.0.3)$$

Substituting eq.(0.0.3) in eq. (0.0.1),

$$\Pr(X = 1|Y = 0) = \frac{\Pr(Y = 0|X = 1)\Pr(X = 1)}{\Pr(Y = 0)} \quad (0.0.4)$$

Given data,

$$\Pr(Y = 0|X = 1) = \frac{1}{7}, \Pr(Y = 0|X = 0) = \frac{6}{7} \quad (0.0.5)$$

$$\Pr(Y = 0) = \Pr(Y = 0|X = 1)\Pr(X = 1) + \Pr(Y = 0|X = 0)\Pr(X = 0) \quad (0.0.6)$$

Substituting the values from eq. (0.0.5) and the data given in the question in eq. (0.0.6),

$$\Pr(Y = 0) = \frac{2}{7} \quad (0.0.7)$$

Substituting eqs. (0.0.5), (0.0.7) and the data given in the question in eq. (0.0.4),

$$\Pr(X = 1|Y = 0) = 0.4 \quad (0.0.8)$$

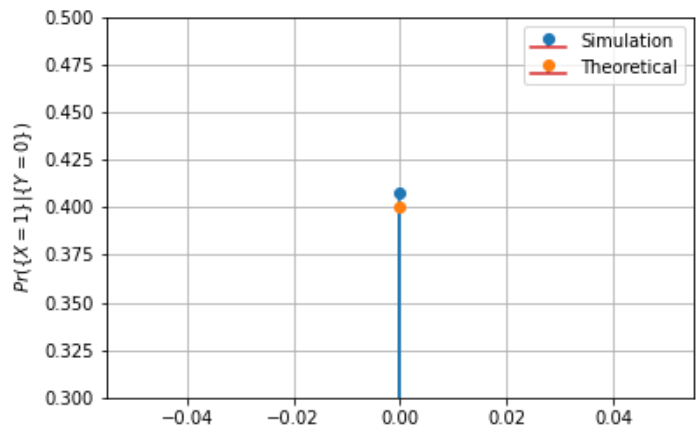If the received bit is $Y = 0$, the conditional probability that '1' was transmitted is 0.4.

Fig. 0: Plot for Simulation v/s Theoretical