Assignment 3

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

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https://github.com/srikaran−p/AI1103/tree/main/Assignment3/codes
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and latex codes from

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https://github.com/srikaran−p/AI1103/tree/main/Assignment3
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Problem

(GATE-MA 2015 Q17) Let X be a random variable having the distribution function

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \le x < 1 \\ \frac{1}{3} & \text{if } 1 \le x < 2 \\ \frac{1}{2} & \text{if } 2 \le x < \frac{11}{3} \\ 1 & \text{if } x \ge \frac{11}{3} \end{cases} \tag{0.0.1}$$

Then $E(X)$ is equal to ...

Solution

Consider a unit step function $u(x)$,

$$u(x) = \begin{cases} 1 & x \ge 0 \\ 0 & \text{otherwise} \end{cases} \tag{0.0.2}$$

We will remove the discontinuity at $x = 0$ by defining $u_\alpha(x)$ for any $\alpha > 0$,

$$u_\alpha(x) = \begin{cases} 1 & x > \frac{\alpha}{2} \\ \frac{1}{\alpha}(x + \frac{\alpha}{2}) & -\frac{\alpha}{2} \le x \le \frac{\alpha}{2} \\ 0 & x < -\frac{\alpha}{2} \end{cases} \tag{0.0.3}$$

$$\delta_\alpha(x) = \frac{du_\alpha(x)}{dx} \tag{0.0.4}$$

$$\delta_\alpha(x) = \begin{cases} \frac{1}{\alpha} & -\frac{\alpha}{2} \le x \le \frac{\alpha}{2} \\ 0 & |x| > \frac{\alpha}{2} \end{cases} \tag{0.0.5}$$

$$u(x) = \lim_{\alpha \to 0} u_\alpha(x) \tag{0.0.6}$$

$$\delta(x) = \lim_{\alpha \to 0} \delta_\alpha(x) \tag{0.0.7}$$

$$\int_{-\infty}^{\infty} g(x)\delta(x - x_0)dx = g(x_0) \tag{0.0.8}$$

We will add dirac δ functions to remove discontinuities in $F(x)$.

$$f_X(x) = \frac{dF(x)}{dx} \tag{0.0.9}$$

$$f_X(x) = \frac{1}{4}\delta(x) + \frac{1}{12}\delta(x-1) + \frac{1}{6}\delta(x-2) + \frac{1}{2}\delta\left(x - \frac{11}{3}\right) + 0 \tag{0.0.10}$$

We will use (0.0.8) to compute E(X)

$$E(X) = \int_{-\infty}^{\infty} x f_X(x) dx \tag{0.0.11}$$

$$E(X) = \int_{-\infty}^{\infty} \frac{1}{4}x\delta(x) + \frac{1}{12}x\delta(x-1) + \frac{1}{6}x\delta(x-2) + \frac{1}{2}x\delta\left(x - \frac{11}{3}\right) dx \tag{0.0.12}$$

$$E(X) = \frac{1}{4} \times 0 + \frac{1}{12} \times 1 + \frac{1}{6} \times 2 + \frac{1}{2} \times \frac{11}{3} \tag{0.0.13}$$

$$E(X) = 2.25 \tag{0.0.14}$$

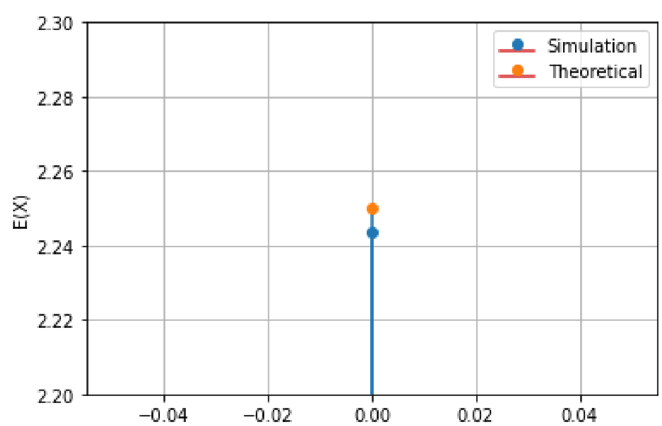

Fig. 0: Plot for Simulation v/s Theoretical