Assignment 4

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Download all python codes from

https://github.com/srikaran-p/AI1103/tree/main/Assignment4/codes

and latex codes from

https://github.com/srikaran-p/AI1103/tree/main/Assignment4

Problem

(STATS P1 IES ISS 2019 Q16) Let X be a Poisson random variable with p.m.f

$$P(X=k)=\begin{cases}\frac{e^{-\lambda}\lambda^{k}}{k!}, & k=0,1,2,...;\lambda>0\\ 0 & \text{otherwise}\end{cases} \tag{0.0.1}$$

If $Y=X^2+3$, then what is $P(Y=y)$ equal to?

(A) $\frac{e^{-\lambda}\lambda^{\sqrt{y-3}}}{\sqrt{(y-3)!}}$, for $y=\{3,4,7,12,...\}$

(B) $\frac{e^{-\lambda}\lambda^{-\sqrt{y-3}}}{\sqrt{(3-y)!}}$, for $y=\{3,4,7,12,...\}$

(C) $\frac{e^{-\lambda}\lambda^{\sqrt{3-y}}}{\sqrt{(3-y)!}}$, for $y=\{4,7,12,...\}$

(D) $\frac{e^{-\lambda}\lambda^{-\sqrt{3-y}}}{\sqrt{(3-y)!}}$, for $y=\{4,7,12,...\}$

Solution

$$Y=X^2+3 \tag{0.0.2}$$

$$X=\sqrt{Y-3} \tag{0.0.3}$$

We can substitute $k=\sqrt{y-3}$ in (0.0.1)

$$p_Y(y)=\begin{cases}\frac{e^{-\lambda}\lambda^{\sqrt{y-3}}}{\sqrt{(y-3)!}}, & y=3,4,7,12,...\\ 0 & \text{otherwise}\end{cases} \tag{0.0.4}$$

Hence, the correct option is (A).

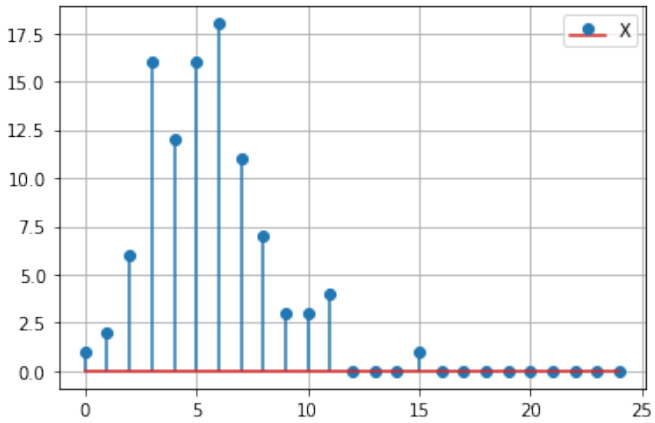

Fig. 4: Poisson stem plot for X ($\lambda=5$)

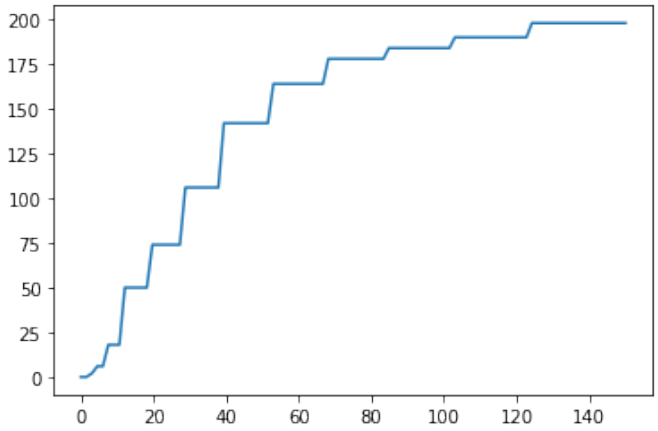

Fig. 4: CDF for Y ($\lambda=5$)

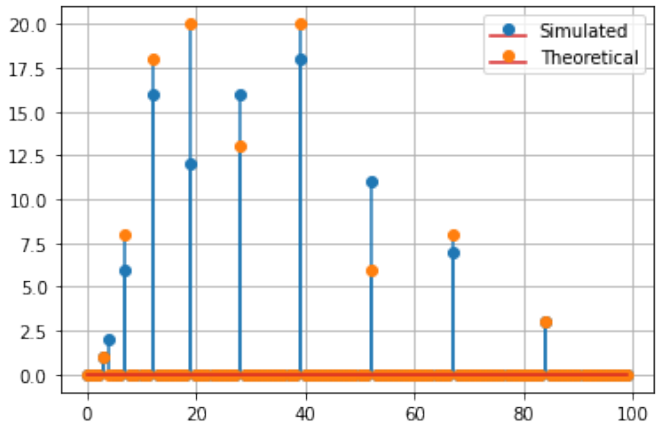

Fig. 4: Poisson stem plot for Y (Simulated and Theoretical) ($\lambda = 5$)