

Assignment 4

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Download all python codes from

<https://github.com/srikan-p/AI1103/tree/main/Assignment4/codes>

and latex codes from

<https://github.com/srikan-p/AI1103/tree/main/Assignment4>

PROBLEM

(STATS P1 IES ISS 2019 Q16) Let X be a Poisson random variable with p.m.f

$$P(X = k) = \begin{cases} \frac{e^{-\lambda} \lambda^k}{k!}, & k = 0, 1, 2, \dots; \lambda > 0 \\ 0 & \text{otherwise} \end{cases} \quad (0.0.1)$$

If $Y = X^2 + 3$, then what is $P(Y = y)$ equal to?

- (A) $\frac{e^{-\lambda} \lambda^{\sqrt{y-3}}}{\sqrt{(y-3)!}}$, for $y = \{3, 4, 7, 12, \dots\}$
- (B) $\frac{e^{-\lambda} \lambda^{-\sqrt{y-3}}}{\sqrt{(3-y)!}}$, for $y = \{3, 4, 7, 12, \dots\}$
- (C) $\frac{e^{-\lambda} \lambda^{\sqrt{3-y}}}{\sqrt{(3-y)!}}$, for $y = \{4, 7, 12, \dots\}$
- (D) $\frac{e^{-\lambda} \lambda^{-\sqrt{3-y}}}{\sqrt{(3-y)!}}$, for $y = \{4, 7, 12, \dots\}$

SOLUTION

$$Y = X^2 + 3 \quad (0.0.2)$$

$$X = \sqrt{Y - 3} \quad (0.0.3)$$

We can substitute $k = \sqrt{y - 3}$ in (0.0.1)

$$p_Y(y) = \begin{cases} \frac{e^{-\lambda} \lambda^{\sqrt{y-3}}}{\sqrt{(y-3)!}}, & y = 3, 4, 7, 12, \dots \\ 0 & \text{otherwise} \end{cases} \quad (0.0.4)$$

Hence, the correct option is (A).

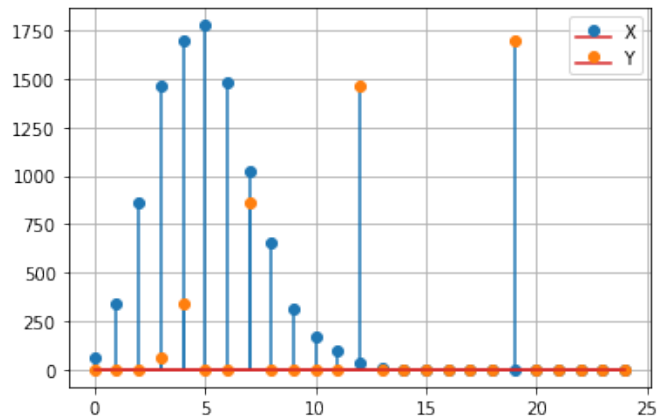


Fig. 4: Poisson stem plot for X and Y