# Assignment 5

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### PROBLEM

(Quadratic Forms Q2.19) Find the roots of $4x^2 + 3x + 5 = 0$.

### SOLUTION

The given equation can be written as,

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} + 5 = 0 \qquad (0.0.1)$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \qquad (0.0.2)$$

Substituting (0.0.2) in (0.0.1),

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + 5 = 0 \qquad (0.0.3)$$

$$\implies 4x^2 + 3x + 5 = 0 \qquad (0.0.4)$$

$$\implies \left(2x + \frac{3}{4}\right)^2 = -\frac{71}{16} \qquad (0.0.5)$$

The square of a real number is always non-negative. In (0.0.5), we can say that $2x + \frac{3}{4}$ is not a real number. So, the roots are not real. From the figure, we can see that the function does not cross the x-axis, so, the quadratic equation has no real roots. Obtaining the affine transformation,

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \qquad (0.0.6)$$

$$\mathbf{u} = \begin{pmatrix} \frac{3}{2} \\ \frac{-3}{2} \end{pmatrix} \qquad (0.0.7)$$

$$f = 5 \qquad (0.0.8)$$

The equation used for affine transformation

$$\mathbf{x} = \mathbf{Py} + \mathbf{c} \qquad (0.0.9)$$

The eigenvalues of $V$ are

$$\lambda_1 = 4 \qquad (0.0.10)$$

$$\lambda_2 = 0 \qquad (0.0.11)$$

$$\mathbf{D} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \qquad (0.0.12)$$

The eigenvectors of $V$ are

$$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad (0.0.13)$$

$$\mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad (0.0.14)$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} \qquad (0.0.15)$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (0.0.16)$$

Since $|\mathbf{V}| = 0$,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \qquad (0.0.17)$$

$$\eta = \mathbf{u}^\top \mathbf{p}_1 \qquad (0.0.18)$$

$$\implies \eta = -\frac{3}{2} \qquad (0.0.19)$$

$$\implies \begin{pmatrix} \frac{3}{4} & -3 \\ 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ -\frac{3}{2} \\ 0 \end{pmatrix} \qquad (0.0.20)$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{-3}{8} \\ \frac{71}{48} \end{pmatrix} \qquad (0.0.21)$$

The quadratic equation will not have real roots if

$$\left(\mathbf{p_1}^T \mathbf{c}\right)\left(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}\right) > 0 \qquad (0.0.22)$$

Substituting the values in LHS,

$$\left(\mathbf{p_1}^T \mathbf{c}\right)\left(\mathbf{p_2}^T \mathbf{V} \mathbf{p_2}\right) = \left(\frac{71}{48}\right)(4) \qquad (0.0.23)$$

$$= \frac{71}{12} \qquad (0.0.24)$$
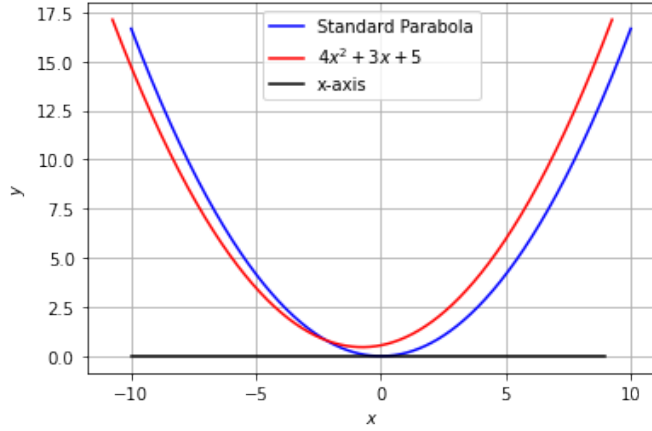
Since the value is positive, the quadratic equation has no real roots.



Fig. 0: Plot of the function