

GATE Assignment 1

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Download all python codes from

<https://github.com/srikan-p/AI1103/tree/main/GATEAssignment1/codes>

Download all latex codes from

<https://github.com/srikan-p/EE3900/tree/main/GATEAssignment1>

To find the limits,

$$-1 \leq \sin^2 x \leq 0 \quad (0.0.6)$$

$$\frac{-1}{x} \leq -\frac{\sin^2 x}{x} \leq 0 \quad (0.0.7)$$

$$\lim_{x \rightarrow \infty} \frac{-1}{x} \leq \lim_{x \rightarrow \infty} -\frac{\sin^2 x}{x} \leq \lim_{x \rightarrow \infty} 0 \quad (0.0.8)$$

Using Sandwich theorem,

$$\lim_{x \rightarrow \infty} -\frac{\sin^2 x}{x} = 0 \quad (0.0.9)$$

Similarly,

$$\lim_{x \rightarrow -\infty} -\frac{\sin^2 x}{x} = 0 \quad (0.0.10)$$

Using (0.0.9) and (0.0.10) in (0.0.5),

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin(2x)}{x} dx \quad (0.0.11)$$

Set, $u = 2x$,

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin(u)}{u} du \quad (0.0.12)$$

Let,

$$f(x) = \begin{cases} 1 & \text{if } |x| \leq 1 \\ 0 & \text{if } |x| > 1 \end{cases} \quad (0.0.13)$$

$$f(x) \xrightarrow{\mathcal{F}} F(u) \quad (0.0.14)$$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-iut} dt \quad (0.0.15)$$

$$= \int_{-1}^1 e^{-iut} dt \quad (0.0.16)$$

$$= \frac{e^{iu} - e^{-iu}}{iu} \quad (0.0.17)$$

$$= \frac{2 \sin u}{u} \quad (0.0.18)$$

PROBLEM

(GATE EC-2018 Q.39) The input $4 \operatorname{sinc}(2t)$ is fed to a Hilbert transformer to obtain $y(t)$, as shown in the figure below:



Here, $\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2 dt$ is

SOLUTION

Since, Hilbert transform performs a phase shift operation, the amplitude of the signal remains the same. So, the energy remains the same.

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |4 \operatorname{sinc} 2t|^2 dt \quad (0.0.1)$$

$$= 16 \int_{-\infty}^{\infty} \frac{\sin^2(2\pi t)}{(2\pi t)^2} dt \quad (0.0.2)$$

$$(0.0.3)$$

Set, $2\pi t = x$

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin^2 x}{x} dx \quad (0.0.4)$$

$$= \frac{8}{\pi} \left[-\frac{\sin^2 x}{x} \Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{\sin(2x)}{x} dx \right] \quad (0.0.5)$$

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(u) e^{iut} du \quad (0.0.19)$$

$$= \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin u}{u} e^{iut} du \quad (0.0.20)$$

$$f(0) = 1 = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\sin u}{u} dt \quad (0.0.21)$$

$$\int_{-\infty}^{\infty} \frac{\sin u}{u} du = \pi \quad (0.0.22)$$

Using (0.0.22) in (0.0.12),

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = 8 \quad (0.0.23)$$

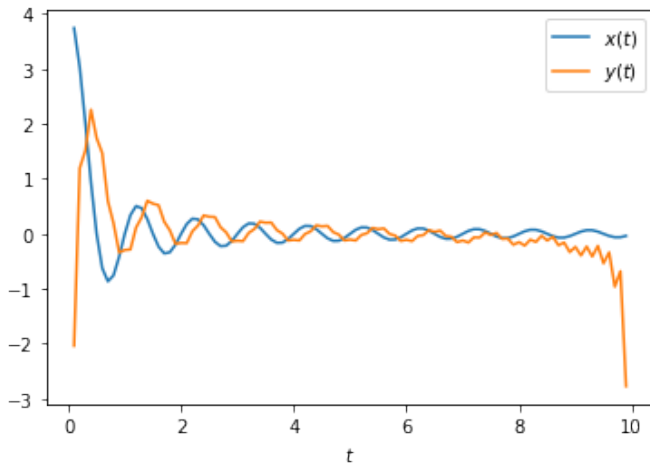


Fig. 0: Input and Output Signals