Assignment 3

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Download all python codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment3/codes

Download all latex codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment3

Problem

(Ramsey 4.2 Q18) Find the equation of the chord of the circle $\mathbf{x}^T\mathbf{x} - \begin{pmatrix}6 & 4\end{pmatrix}\mathbf{x} - 23 = 0$ which has the point $\begin{pmatrix}4\\1\end{pmatrix}$ as its middle point.

Solution

The general equation of circle is

$$\mathbf{x}^T\mathbf{x} - 2\mathbf{c}\mathbf{x} + f = 0 \tag{0.0.1}$$

where $\mathbf{c}$ is the centre of the circle.

$$\mathbf{c} = \begin{pmatrix}3 & 2\end{pmatrix} \tag{0.0.2}$$

$$\mathbf{O} = \mathbf{c}^T \tag{0.0.3}$$

$$\mathbf{M} = \begin{pmatrix}4\\1\end{pmatrix} \tag{0.0.4}$$

The line passing through the centre bisects any chord perpendicularly. The direction vector of $\mathbf{OM}$ is

$$\mathbf{OM} = \mathbf{M} - \mathbf{O} \tag{0.0.5}$$

$$= \begin{pmatrix}4\\1\end{pmatrix} - \begin{pmatrix}3\\2\end{pmatrix} \tag{0.0.6}$$

$$= \begin{pmatrix}1\\-1\end{pmatrix} \tag{0.0.7}$$

$$= \begin{pmatrix}1\\m\end{pmatrix} \tag{0.0.8}$$

The normal vector $\mathbf{n}$ is

$$\mathbf{n} = \mathbf{OM} \tag{0.0.9}$$

The equation of line in terms of normal vector

$$\mathbf{n}^T(\mathbf{x} - \mathbf{M}) = 0 \tag{0.0.10}$$

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & -1\end{pmatrix}\mathbf{M} \tag{0.0.11}$$

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = \begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}4\\1\end{pmatrix} \tag{0.0.12}$$

$$\begin{pmatrix}1 & -1\end{pmatrix}\mathbf{x} = 3 \tag{0.0.13}$$

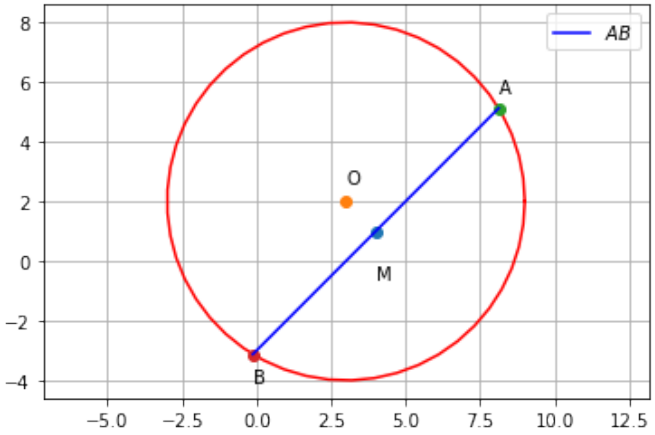

Fig. 0: Plot of the given points and circle