Quiz 2

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

```
https://github.com/srikaran-p/EE3900/tree/main/
    Quiz2/codes
```

Download all latex codes from

```
https://github.com/srikaran-p/EE3900/tree/main/
    Quiz2
```

Problem

(3.14) If $H(z) = \frac{1}{1-\frac{1}{4}z^{-2}}$ and $h[n] = A_1\alpha_1^n u[n] + A_2\alpha_2^n u[n]$, determine the values of A_1, A_2, α_1 and α_2.

Solution

$$h[n] = A_1\alpha_1^n u[n] + A_2\alpha_2^n u[n] \tag{0.0.1}$$

$$h[n] \overset{\mathcal{Z}}{\rightleftharpoons} H(z) \tag{0.0.2}$$

$$H(z) = \frac{1}{1-\frac{1}{4}z^{-2}} \tag{0.0.3}$$

$$= \frac{1}{2}\frac{2}{1-\frac{1}{4}z^{-2}} \tag{0.0.4}$$

$$= \frac{1}{2}\frac{1+\frac{1}{2}z^{-1}+1-\frac{1}{2}z^{-1}}{1-\frac{1}{4}z^{-2}} \tag{0.0.5}$$

$$= \frac{1}{2}\frac{\left(1+\frac{1}{2}z^{-1}\right)+\left(1-\frac{1}{2}z^{-1}\right)}{\left(1+\frac{1}{2}z^{-1}\right)\left(1-\frac{1}{2}z^{-1}\right)} \tag{0.0.6}$$

$$= \frac{1}{2}\left(\frac{1}{1-\frac{1}{2}z^{-1}}+\frac{1}{1+\frac{1}{2}z^{-1}}\right) \tag{0.0.7}$$

We know that,

$$a^n u[n] \overset{\mathcal{Z}}{\rightleftharpoons} \frac{1}{1-az^{-1}} \tag{0.0.8}$$

Using (0.0.8) in (0.0.7) and the linearity property of Z-transform,

$$h[n] = \frac{1}{2}\left(\left(\frac{1}{2}\right)^n u[n] + \left(\frac{-1}{2}\right)^n u[n]\right) \tag{0.0.9}$$

$$= \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{-1}{2}\right)^n u[n] \tag{0.0.10}$$

Comparing (0.0.1) and (0.0.10),

$$A_1 = \frac{1}{2} \tag{0.0.11}$$

$$A_2 = \frac{1}{2} \tag{0.0.12}$$

$$\alpha_1 = \frac{1}{2} \tag{0.0.13}$$

$$\alpha_2 = \frac{-1}{2} \tag{0.0.14}$$

Alternate Solution

$$H(z) = \frac{4z^2}{(2z-1)(2z+1)} \tag{0.0.15}$$

$$h[n] = \sum \text{Residues of } H(z)z^{n-1} \text{ at the poles inside C} \tag{0.0.16}$$

The ROC of $H(z)$ is $|z| > \frac{1}{2}$. The poles are $z = \frac{1}{2}$ and $\frac{-1}{2}$.

$$h[n] = \left(z-\frac{1}{2}\right)H(z)\Big|_{z=\frac{1}{2}} z^{n-1} + \left(z+\frac{1}{2}\right)H(z)\Big|_{z=\frac{-1}{2}} z^{n-1} \tag{0.0.17}$$

$$h[n] = \frac{1}{2}\left(\frac{1}{2}\right)^n u[n] + \frac{1}{2}\left(\frac{-1}{2}\right)^n u[n] \tag{0.0.18}$$

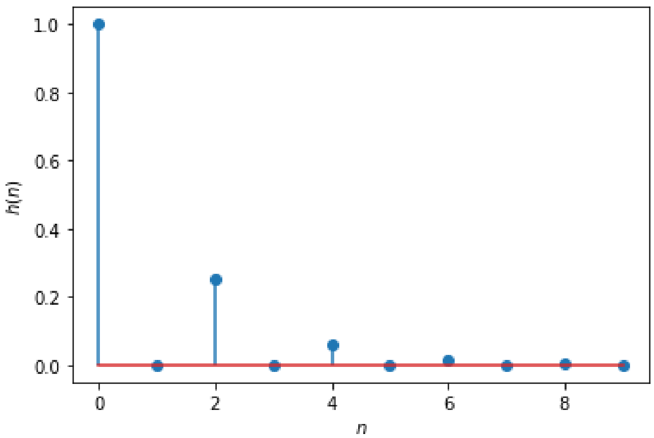

Fig. 0: Plot of h[n]

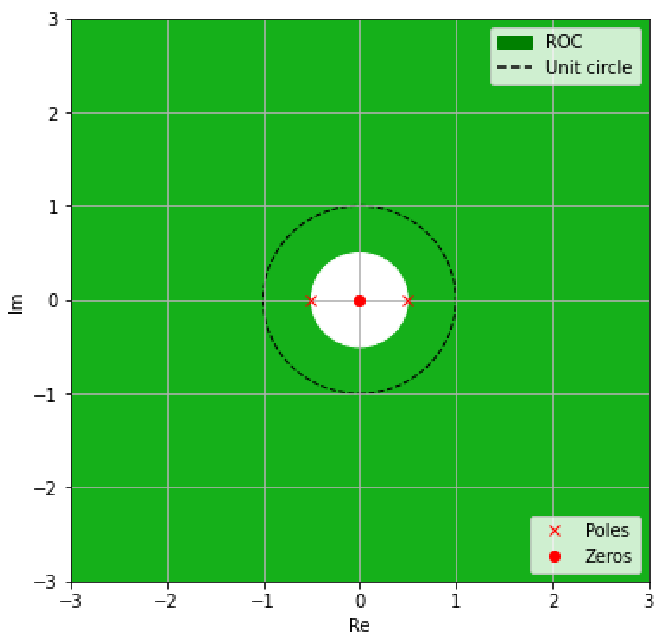

Fig. 0: Pole-zero plot of H(z)