Assignment 5

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

```
https://github.com/srikaran-p/EE3900/tree/main/
    Assignment5/codes
```

Download all latex codes from

```
https://github.com/srikaran-p/EE3900/tree/main/
    Assignment5
```

Problem

(Quadratic Forms Q2.19) Find the roots of $4x^2 + 3x + 5 = 0$.

Solution

The given equation can be written as,

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} + 5 = 0 \tag{0.0.1}$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \tag{0.0.2}$$

Substituting (0.0.2) in (0.0.1),

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + 5 = 0 \tag{0.0.3}$$

$$\implies 4x^2 + 3x + 5 = 0 \tag{0.0.4}$$

$$\implies \left(2x + \frac{3}{4}\right)^2 = -\frac{71}{16} \tag{0.0.5}$$

The square of a real number is always non-negative. In (0.0.5), we can say that $2x+\frac{3}{4}$ is not a real number. So, the roots are not real. From the figure, we can see that the function does not cross the x-axis, so, the quadratic equation has no real roots.

Alternate Solution

The equation of line is,

$$L : \mathbf{x} = \mathbf{q} + \mu\mathbf{m}, \mu \in \mathbb{R} \tag{0.0.6}$$

The vector form of the quadratic equation $y = 4x^2 + 3x + 5$ is,

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + 2 \begin{pmatrix} \frac{3}{2} & \frac{-3}{2} \end{pmatrix} \mathbf{x} + 5 = 0 \tag{0.0.7}$$

The line L is the x-axis,

$$\mathbf{q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tag{0.0.8}$$

$$\mathbf{m} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.9}$$

Comparing the quadratic equation with general equation of conic,

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.0.10}$$

$$\mathbf{u} = \begin{pmatrix} \frac{3}{2} \\ \frac{-3}{2} \end{pmatrix} \tag{0.0.11}$$

$$f = 5 \tag{0.0.12}$$

Here, the line L intersects the conic. The condition for real roots is,

$$\left(\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + u\right)\right)^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right) \geq 0 \tag{0.0.13}$$

Substituting the values in the LHS,

$$\left(\mathbf{m}^T \left(\mathbf{V}\mathbf{q} + u\right)\right)^2 - \left(\mathbf{q}^T\mathbf{V}\mathbf{q} + 2\mathbf{u}^T\mathbf{q} + f\right)\left(\mathbf{m}^T\mathbf{V}\mathbf{m}\right) = \frac{-71}{4} \tag{0.0.14}$$

Since the value is negative, there are no real roots. The equation used for affine transformation

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \tag{0.0.15}$$

The eigenvalues of V are

$$\lambda_1 = 4 \tag{0.0.16}$$

$$\lambda_2 = 0 \tag{0.0.17}$$

$$\mathbf{D} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \tag{0.0.18}$$

The eigenvectors of V are

$$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{0.0.19}$$

$$\mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{0.0.20}$$

$$\mathbf{P} = \begin{pmatrix} \mathbf{p}_1 & \mathbf{p}_2 \end{pmatrix} \tag{0.0.21}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{0.0.22}$$

Since $|\mathbf{V}| = 0$,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \tag{0.0.23}$$

$$\eta = \mathbf{u}^\top \mathbf{p}_1 \tag{0.0.24}$$

$$\implies \eta = -\frac{3}{2} \tag{0.0.25}$$

$$\implies \begin{pmatrix} \frac{3}{2} & -3 \\ 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ -\frac{3}{2} \\ 0 \end{pmatrix} \tag{0.0.26}$$

$$\implies \mathbf{c} = \begin{pmatrix} \frac{-3}{8} \\ \frac{71}{48} \end{pmatrix} \tag{0.0.27}$$

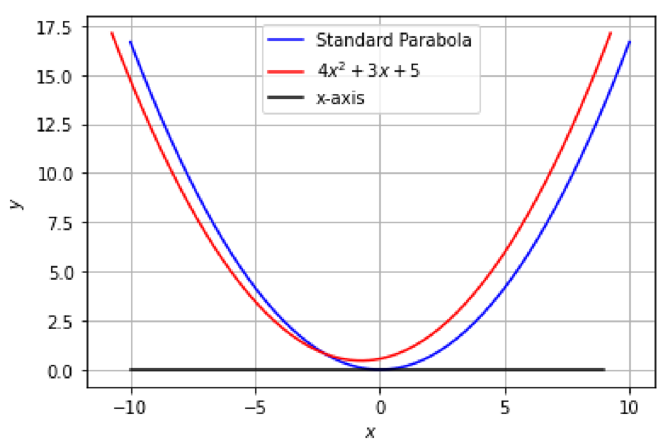

Fig. 0: Plot of the function