GATE Assignment 1

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Download all python codes from

```
https://github.com/srikaran−p/AI1103/tree/main/
    GATE_Assignment1/codes
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Download all latex codes from

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https://github.com/srikaran−p/EE3900/tree/main/
    GATE_Assignment1
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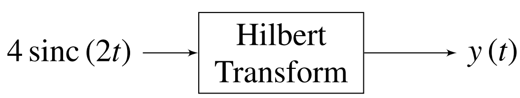
Problem

(GATE EC-2018 Q.39) The input $4\,\mathrm{sinc}\,(2t)$ is fed to a Hilbert transformer to obtain $y(t)$, as shown in the figure below:



Here, $\mathrm{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2 dt$ is

Solution

Since, Hilbert transform performs a phase shift operation, the amplitude of the signal remains the same. So, the energy remains the same.

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |4\,\mathrm{sinc}\,2t|^2 dt \tag{0.0.1}$$

$$= 16 \int_{-\infty}^{\infty} \frac{\sin^2(2\pi t)}{(2\pi t)^2} dt \tag{0.0.2}$$

$$\tag{0.0.3}$$

Set, $2\pi t = x$

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin^2 x}{x} dx \tag{0.0.4}$$

$$= \frac{8}{\pi}\left[-\frac{\sin^2 x}{x}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty} \frac{\sin(2x)}{x} dx \right] \tag{0.0.5}$$

To find the limits,

$$-1 \le \sin^2 x \le 0 \tag{0.0.6}$$

$$\frac{-1}{x} \le -\frac{\sin^2 x}{x} \le 0 \tag{0.0.7}$$

$$\lim_{x\to\infty} \frac{-1}{x} \le \lim_{x\to\infty} -\frac{\sin^2 x}{x} \le \lim_{x\to\infty} 0 \tag{0.0.8}$$

Using Sandwich theorem,

$$\lim_{x\to\infty} -\frac{\sin^2 x}{x} = 0 \tag{0.0.9}$$

Similarly,

$$\lim_{x\to-\infty} -\frac{\sin^2 x}{x} = 0 \tag{0.0.10}$$

Using (0.0.9) and (0.0.10) in (0.0.5),

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin(2x)}{x} dx \tag{0.0.11}$$

Set, $u = 2x$,

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \frac{8}{\pi} \int_{-\infty}^{\infty} \frac{\sin(u)}{u} du \tag{0.0.12}$$

Let,

$$f(x) = \begin{cases} 1 & \text{if } |x| \le 1 \\ 0 & \text{if } |x| > 1 \end{cases} \tag{0.0.13}$$

$$f(x) \overset{\mathcal{F}}{\rightleftharpoons} F(u) \tag{0.0.14}$$

$$F(u) = \int_{-\infty}^{\infty} f(x) e^{-iut} dt \tag{0.0.15}$$

$$= \int_{-1}^{1} e^{-iut} dt \tag{0.0.16}$$

$$= \frac{e^{iu} - e^{-iu}}{iu} \tag{0.0.17}$$

$$= \frac{2\sin u}{u} \tag{0.0.18}$$

$$f(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(u)e^{iut}du \tag{0.0.19}$$
$$= \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u} e^{iut}du \tag{0.0.20}$$

$$f(0) = 1 = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\sin u}{u} dt \tag{0.0.21}$$
$$\int_{-\infty}^{\infty} \frac{\sin u}{u} du = \pi \tag{0.0.22}$$

Using (0.0.22) in (0.0.12),

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = 8 \tag{0.0.23}$$

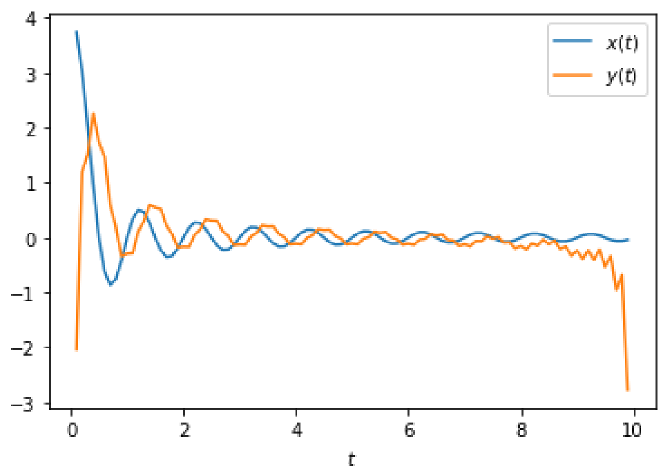


Fig. 0: Input and Output Signals