

# Assignment 5

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Download all python codes from

<https://github.com/srikan-p/EE3900/tree/main/Assignment5/codes>

Download all latex codes from

<https://github.com/srikan-p/EE3900/tree/main/Assignment5>

The equation used for affine transformation

$$\mathbf{x} = \mathbf{P}\mathbf{y} + \mathbf{c} \quad (0.0.9)$$

The eigenvalues of  $\mathbf{V}$  are

$$\lambda_1 = 4 \quad (0.0.10)$$

$$\lambda_2 = 0 \quad (0.0.11)$$

$$\mathbf{D} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.0.12)$$

The eigenvectors of  $\mathbf{V}$  are

$$\mathbf{p}_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad (0.0.13)$$

$$\mathbf{p}_2 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (0.0.14)$$

$$\mathbf{P} = (\mathbf{p}_1 \quad \mathbf{p}_2) \quad (0.0.15)$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad (0.0.16)$$

## PROBLEM

(Quadratic Forms Q2.19) Find the roots of  $4x^2 + 3x + 5 = 0$ .

## SOLUTION

The given equation can be written as,

$$\mathbf{x}^T \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{x} + \begin{pmatrix} 3 & 0 \end{pmatrix} \mathbf{x} + 5 = 0 \quad (0.0.1)$$

where,

$$\mathbf{x} = \begin{pmatrix} x \\ 0 \end{pmatrix} \quad (0.0.2)$$

Substituting (0.0.2) in (0.0.1),

$$\begin{pmatrix} x & 0 \end{pmatrix} \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + \begin{pmatrix} 3 & 0 \end{pmatrix} \begin{pmatrix} x \\ 0 \end{pmatrix} + 5 = 0 \quad (0.0.3)$$

$$\Rightarrow 4x^2 + 3x + 5 = 0 \quad (0.0.4)$$

$$\Rightarrow \left(2x + \frac{3}{4}\right)^2 = -\frac{71}{16} \quad (0.0.5)$$

The square of a real number is always non-negative. In (0.0.5), we can say that  $2x + \frac{3}{4}$  is not a real number. So, the roots are not real. From the figure, we can see that the function does not cross the x-axis, so, the quadratic equation has no real roots. Obtaining the affine transformation,

$$\mathbf{V} = \begin{pmatrix} 4 & 0 \\ 0 & 0 \end{pmatrix} \quad (0.0.6)$$

$$\mathbf{u} = \begin{pmatrix} 3 \\ -3 \end{pmatrix} \quad (0.0.7)$$

$$f = 5 \quad (0.0.8)$$

Since  $|\mathbf{V}| = 0$ ,

$$\begin{pmatrix} \mathbf{u}^T + \eta \mathbf{p}_1^T \\ \mathbf{V} \end{pmatrix} \mathbf{c} = \begin{pmatrix} -f \\ \eta \mathbf{p}_1 - \mathbf{u} \end{pmatrix} \quad (0.0.17)$$

$$\eta = \mathbf{u}^T \mathbf{p}_1 \quad (0.0.18)$$

$$\Rightarrow \eta = -3 \quad (0.0.19)$$

$$\Rightarrow \begin{pmatrix} 3 & -6 \\ 4 & 0 \\ 0 & 0 \end{pmatrix} \mathbf{c} = \begin{pmatrix} -5 \\ -3 \\ 0 \end{pmatrix} \quad (0.0.20)$$

$$\Rightarrow \mathbf{c} = \begin{pmatrix} -\frac{3}{4} \\ \frac{11}{24} \\ \frac{5}{24} \end{pmatrix} \quad (0.0.21)$$

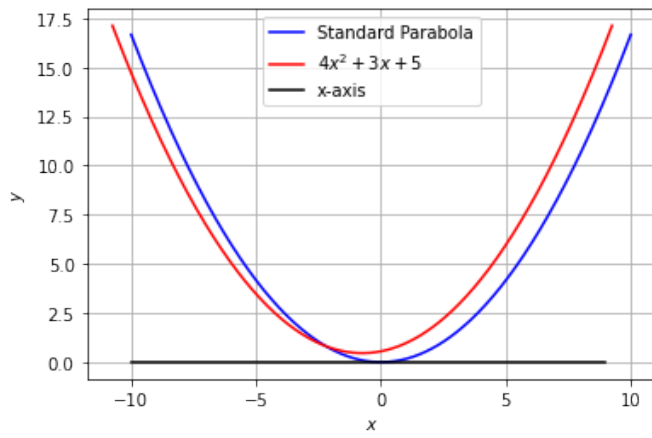


Fig. 0: Plot of the function