GATE Assignment 1

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

https://github.com/srikaran−p/AI1103/tree/main/GATE_Assignment1/codes

Download all latex codes from

https://github.com/srikaran−p/EE3900/tree/main/GATE_Assignment1

Problem

(GATE EC-2018 Q.39) The input $4\,\text{sinc}\,(2t)$ is fed to a Hilbert transformer to obtain $y\,(t)$, as shown in the figure below:

$$4\,\text{sinc}\,(2t) \longrightarrow \boxed{\text{Hilbert Transform}} \longrightarrow y\,(t)$$

Here, $\text{sinc}\,(x) = \frac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y\,(t)\,|^2 dt$ is

Solution

Lemma 0.1. *Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.*

$$\int_{-\infty}^{\infty} |x\,(t)\,|^2 dt = \int_{-\infty}^{\infty} |X\,(f)\,|^2 df \tag{0.0.1}$$

$$x(t) = 4\,\text{sinc}(2t) \tag{0.0.2}$$

$$h(t) = \frac{1}{\pi t} \tag{0.0.3}$$

$$y(t) = x(t) * h(t) \tag{0.0.4}$$

$$x(t) \overset{\mathcal{F}}{\rightleftharpoons} X(f) \tag{0.0.5}$$

$$h(t) \overset{\mathcal{F}}{\rightleftharpoons} H(f) \tag{0.0.6}$$

$$y(t) \overset{\mathcal{F}}{\rightleftharpoons} Y(f) \tag{0.0.7}$$

Define a rectangular function,

$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \le 1 \\ 0 & \text{if } |t| > 1 \end{cases} \tag{0.0.8}$$

Define a signum function,

$$\text{sgn}(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0 \end{cases} \tag{0.0.9}$$

The Fourier transforms are

$$X(f) = 2\,\text{rect}(f) \tag{0.0.10}$$

$$H(f) = -j\,\text{sgn}(f) \tag{0.0.11}$$

Applying Convolution theorem in (0.0.4),

$$Y(f) = X(f)H(f) \tag{0.0.12}$$

$$= -2j\,\text{rect}(f)\,\text{sgn}(f) \tag{0.0.13}$$

Applying Inverse Fourier Transform on $Y(f)$,

$$y(t) = \int_{-\infty}^{\infty} Y(f) e^{j2\pi f t} df \tag{0.0.14}$$

$$= \int_{-1}^{0} 2je^{j2\pi f t} df + \int_{0}^{1} -2je^{j2\pi f t} df \tag{0.0.15}$$

$$= \frac{2j}{j2\pi t} e^{j2\pi f t}\Big|_{-1}^{0} - \frac{2j}{j2\pi t} e^{j2\pi f t}\Big|_{0}^{1} \tag{0.0.16}$$

$$= \frac{2}{\pi t} - \frac{1}{\pi t} e^{-j2\pi t} - \frac{1}{\pi t} e^{j2\pi t} \tag{0.0.17}$$

$$= \frac{2}{\pi t} - \frac{2cos(\pi t)}{\pi t} \tag{0.0.18}$$

$$= \frac{2}{\pi t}\,(1 - cos(\pi t)) \tag{0.0.19}$$

By the Parseval's theorem,

$$\int_{-\infty}^{\infty} |y\,(t)\,|^2 dt = \int_{-\infty}^{\infty} |Y\,(f)\,|^2 df \tag{0.0.20}$$

$$= \int_{-1}^{1} |2rect(f)|^2 df \tag{0.0.21}$$

$$= 8 \tag{0.0.22}$$

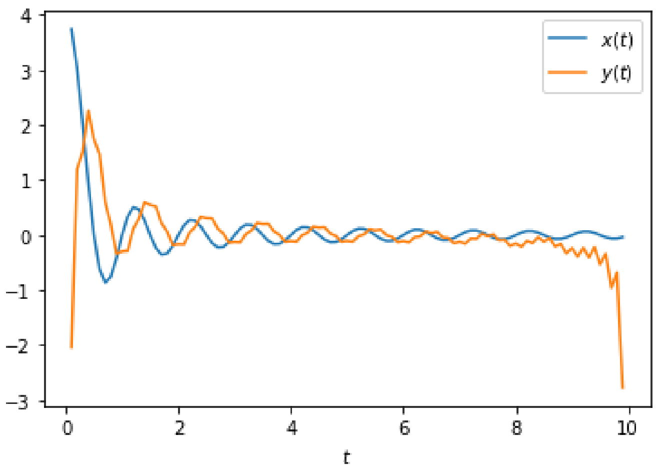

Fig. 0: Input and Output Signals