Assignment 4

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment4/codes

Download all latex codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment4

Problem

(Linear Forms Q2.20) Find the equation of the line through the point $\begin{pmatrix}0\\2\end{pmatrix}$ making an angle $\frac{2\pi}{3}$ with the positive x-axis. Also, find the equation of the line parallel to it and crossing the y-axis at a distance of 2 units below the origin.

Solution

The direction vector of the line is $\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix}$. The normal vector $\mathbf{n}$

$$\mathbf{n} = \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix}\begin{pmatrix}1\\-\sqrt{3}\end{pmatrix} \tag{0.0.1}$$

$$= \begin{pmatrix}\sqrt{3}\\1\end{pmatrix} \tag{0.0.2}$$

Let $\mathbf{P}$ be $\begin{pmatrix}0\\2\end{pmatrix}$. The equation of line in terms of normal vector

$$\mathbf{n}^T(\mathbf{x}-\mathbf{P}) = 0 \tag{0.0.3}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{P} \tag{0.0.4}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}0\\2\end{pmatrix} \tag{0.0.5}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = 2 \tag{0.0.6}$$

The standard basis vectors in 2D plane are

$$\mathbf{e_1} = \begin{pmatrix}1\\0\end{pmatrix} \tag{0.0.7}$$

$$\mathbf{e_2} = \begin{pmatrix}0\\1\end{pmatrix} \tag{0.0.8}$$

The point which crosses the y-axis at a distance of 2 units below the origin

$$\mathbf{Q} = \frac{-2\mathbf{e_2}}{\mathbf{n}^T\mathbf{e_2}} \tag{0.0.9}$$

$$= \frac{-2}{1}\mathbf{e_2} \tag{0.0.10}$$

$$= \begin{pmatrix}0\\-2\end{pmatrix} \tag{0.0.11}$$

The equation of line which passes through $\mathbf{Q}$

$$\mathbf{n}^T(\mathbf{x}-\mathbf{Q}) = 0 \tag{0.0.12}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{Q} \tag{0.0.13}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = \begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\begin{pmatrix}0\\-2\end{pmatrix} \tag{0.0.14}$$

$$\begin{pmatrix}\sqrt{3} & 1\end{pmatrix}\mathbf{x} = -2 \tag{0.0.15}$$

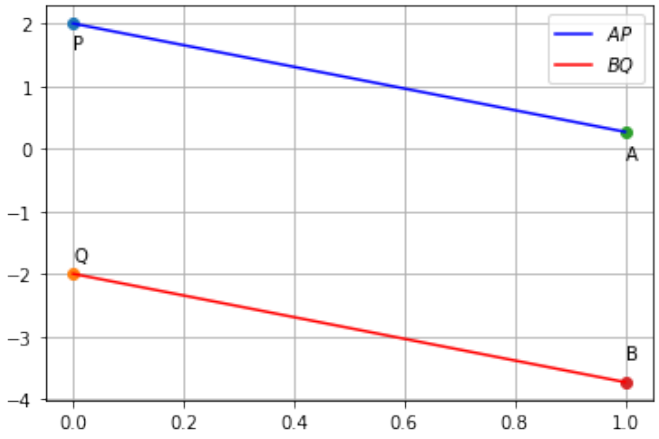

Fig. 0: Plot of the given points and lines