Assignment 1

Perambuduri Srikaran - AI20BTECH11018

Download all python codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment1/codes

Download all latex codes from

https://github.com/srikaran-p/EE3900/tree/main/Assignment1

Problem

(Ramsey 1.1 Q6) Show that $\mathbf{B} = \begin{pmatrix}-2\\-2\end{pmatrix}$, $\mathbf{A} = \begin{pmatrix}-1\\2\end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix}3\\1\end{pmatrix}$ are the vertices of an isosceles triangle.

Solution

Define a matrix $\mathbf{M}$ such that,

$$\mathbf{M} = \begin{pmatrix}\mathbf{B}-\mathbf{A} & \mathbf{C}-\mathbf{A}\end{pmatrix}^T \tag{0.0.1}$$

$$\mathbf{M} = \begin{pmatrix}-1 & -4\\ 4 & -1\end{pmatrix} \tag{0.0.2}$$

Using matrix transformation,

$$\mathbf{M} = \begin{pmatrix}-1 & -4\\ 4 & -1\end{pmatrix} \xleftrightarrow{R_1 \leftarrow -R_1 - \frac{R_2}{4}} \begin{pmatrix}0 & \frac{17}{4}\\ 4 & -1\end{pmatrix} \tag{0.0.3}$$

$$\implies rank(\mathbf{M}) = 2 \tag{0.0.4}$$

Since the rank of matrix $\mathbf{M}$ is 2, the points form a triangle.

$$AB^2 = (\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{B}) \tag{0.0.5}$$

$$= \begin{pmatrix}1 & 4\end{pmatrix}\begin{pmatrix}1\\4\end{pmatrix} \tag{0.0.6}$$

$$= (1)^2 + (4)^2 \tag{0.0.7}$$

$$= 17 \tag{0.0.8}$$

$$BC^2 = (\mathbf{B}-\mathbf{C})^T(\mathbf{B}-\mathbf{C}) \tag{0.0.9}$$

$$= \begin{pmatrix}-5 & -3\end{pmatrix}\begin{pmatrix}-5\\-3\end{pmatrix} \tag{0.0.10}$$

$$= (-5)^2 + (-3)^2 \tag{0.0.11}$$

$$= 34 \tag{0.0.12}$$

$$CA^2 = (\mathbf{C}-\mathbf{A})^T(\mathbf{C}-\mathbf{A}) \tag{0.0.13}$$

$$= \begin{pmatrix}4 & -1\end{pmatrix}\begin{pmatrix}4\\-1\end{pmatrix} \tag{0.0.14}$$

$$= (4)^2 + (-1)^2 \tag{0.0.15}$$

$$= 17 \tag{0.0.16}$$

So, AB and CA are equal. Hence, the triangle is an isosceles triangle.

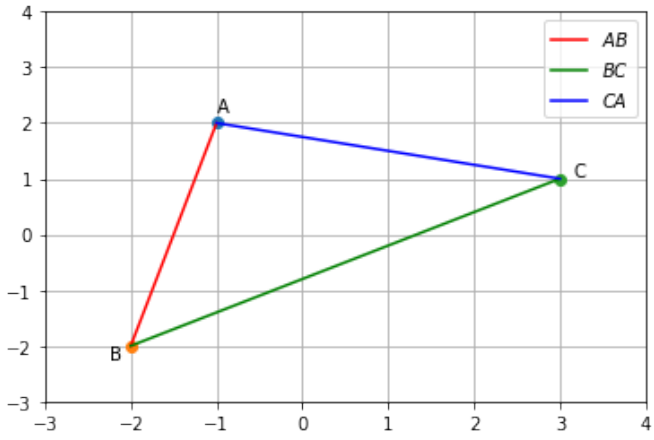

Fig. 0: Plot of the given points