

GATE Assignment 1

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Download all python codes from

https://github.com/srikan-p/AI1103/tree/main/GATE_Assignment1/codes

Download all latex codes from

https://github.com/srikan-p/EE3900/tree/main/GATE_Assignment1

PROBLEM

(GATE EC-2018 Q.39) The input $4 \text{ sinc}(2t)$ is fed to a Hilbert transformer to obtain $y(t)$, as shown in the figure below:



Here, $\text{sinc}(x) = \frac{\sin(\pi x)}{\pi x}$. The value (accurate to two decimal places) of $\int_{-\infty}^{\infty} |y(t)|^2 dt$ is

SOLUTION

Lemma 0.1. Parseval's theorem states that there is no loss of information in Fourier transform and the amount of energy remains the same in time and frequency domains.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df \quad (0.0.1)$$

$$x(t) = 4 \text{ sinc}(2t) \quad (0.0.2)$$

$$h(t) = \frac{1}{\pi t} \quad (0.0.3)$$

$$y(t) = x(t) * h(t) \quad (0.0.4)$$

$$x(t) \xrightarrow{\mathcal{F}} X(f) \quad (0.0.5)$$

$$h(t) \xrightarrow{\mathcal{F}} H(f) \quad (0.0.6)$$

$$y(t) \xrightarrow{\mathcal{F}} Y(f) \quad (0.0.7)$$

Define a rectangular function,

$$\text{rect}(t) = \begin{cases} 1 & \text{if } |t| \leq \frac{1}{2} \\ 0 & \text{if } |t| > \frac{1}{2} \end{cases} \quad (0.0.8)$$

Define a signum function,

$$\text{sgn}(t) = \begin{cases} 1 & \text{if } t > 0 \\ 0 & \text{if } t = 0 \\ -1 & \text{if } t < 0 \end{cases} \quad (0.0.9)$$

The Fourier transforms are

$$X(f) = 2 \text{ rect}\left(\frac{f}{2}\right) \quad (0.0.10)$$

$$H(f) = -j \text{sgn}(f) \quad (0.0.11)$$

Applying Convolution theorem in (0.0.4),

$$Y(f) = X(f)H(f) \quad (0.0.12)$$

$$= -2j \text{rect}\left(\frac{f}{2}\right) \text{sgn}(f) \quad (0.0.13)$$

$$= 2j \text{rect}\left(f + \frac{1}{2}\right) - 2j \text{rect}\left(f - \frac{1}{2}\right) \quad (0.0.14)$$

Applying Inverse Fourier Transform on $Y(f)$,

$$y(t) = 2j \text{sinc}(t)e^{-j\pi t} - 2j \text{sinc}(t)e^{j\pi t} \quad (0.0.15)$$

$$= -2j \text{sinc}(t) (e^{j\pi t} - e^{-j\pi t}) \quad (0.0.16)$$

$$= 2 \text{sinc}(t) \left(\frac{e^{j\pi t} - e^{-j\pi t}}{j} \right) \quad (0.0.17)$$

$$= 4 \text{sinc}(t) \sin(\pi t) \quad (0.0.18)$$

$$= 4\pi t \text{sinc}^2(t) \quad (0.0.19)$$

By the Parseval's theorem,

$$\int_{-\infty}^{\infty} |y(t)|^2 dt = \int_{-\infty}^{\infty} |Y(f)|^2 df \quad (0.0.20)$$

$$= \int_{-1}^1 |2 \text{rect}(f)|^2 df \quad (0.0.21)$$

$$= 8 \quad (0.0.22)$$

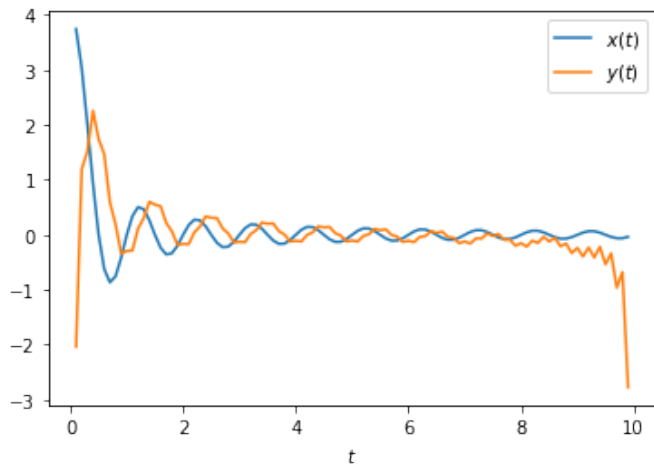


Fig. 0: Input and Output Signals