Monsoon Semester (Aug-Nov), 2019
Discrete Structures (DS)

Tutorial XI
September 12, 2019

Due: 16.09.19

Instructor: Dr. P. Kumar

INSTRUCTIONS:

Problems to be discussed in Tutorial in the week of Monday 16th Sep 2019.

1. **(Parity)** Prove the following by identifying parity.
   (a) Can a knight in a chess start at $a1$ of a chessboard, and go to square $h8$, visiting each of the remaining squares exactly once on the way?
   (b) A grasshopper jumps along a line. His first jump takes him 1 cm, his second 2 cm, and so on. Each jump can take him to the right or left. Show that after 1985 jumps the grasshoper cannot return to the point at which he started.
   (c) Twenty-five boys and twenty-five girls are seated at a round table. Show that both neighbors of at least one student are boys.

2. **(Pigeon hole principle** The **pigeon-hole principle** states that:

   If we put $N + 1$ pigeons in N pigeon-holes, then there will be atleast one pigeon hole with at least two pigeons. Prove this statement using contrapositive proof.

   A **general pigeon-hole principle** is stated as follows:

   If we must put $Nk + 1$ or more pigeons into N pigeon holes, then some pigeon-hole must contain at least $k + 1$ pigeons. Prove this using contrapositive proof.

   Prove the following using pigeon-hole principle.

   1. Given 8 different natural numbers, none greater than 15, show that at least three pairs of them have the same positive difference (The pairs need not be disjoint as sets).
   2. What is the largest number of kings that can be placed on a chessboard so that no two of them put each other in check?

3. **(Induction Proofs)** Prove the following using induction.

   1. Show that
   $$\frac{1}{a(a+b)} + \frac{1}{(a+b)(a+2b)} + \cdots + \frac{1}{(a+(n-1)b)(a+nb)} = \frac{n}{a(a+nb)},$$
   where a and b are any natural numbers.
   2. Show that
   $$\frac{m!}{0!} + \frac{(m+1)!}{1!} + \cdots + \frac{(m+n)!}{n!} = \frac{(m+n+1)!}{n!(m+1)},$$
   where $m, n = 0, 1, 2$.

3. Prove that a square **can** be dissected into n squares for $n \geq 6$.

4. **(Proof by finding invariants)** Prove the following by finding an invariant property.

   1. There are Martian amoebae of three types A, B, and C in a test tube. Two amoebae of any two different types can merge into one amoebae of the third type. After several such merges only one amoebae remains in the test tube. What is its type, if initially there were 20 amoebae of type A, 21 amoebae of type B, and 22 amoebae of type C?