Due: 09.09.19   **Instructor: Dr. P. Kumar**

INSTRUCTIONS:

Problems to be discussed in Tutorial in the week of Monday 9th Sep 2019.

1. (**Free or bound**) Indicate the variables that are free and bound. Also show the scope of the quantifiers.

   (a) $(x)(P(x) \land R(x)) \to (x)P(x) \land Q(x)$

   (b) $(x)(P(x) \land (\exists x)Q(x)) \lor ((x)P(x) \to Q(x))$

   (c) $(x)(P(x) \leftrightarrow Q(x) \land (\exists x)R(x)) \land S(x)$

2. (**Incorrect Derivations**) What is wrong in the following derivations?

   1. (a) $P(x) \to Q(x)$ P
      (b) $(\exists x)(P(x) \to Q(x))$ (1), EG

   2. (a) $P(a) \to Q(b)$ P
      (b) $(\exists x)(P(x) \to Q(x))$ (1), EG

   3. (a) $P(a) \land (\exists x)(P(a) \land Q(x))$ P
      (b) $(\exists x)(P(x) \land (\exists x)(P(x) \land Q(x)))$ (1), EG

3. (**Implications**) Demonstrate the following implications.

   1. $\neg((\exists x)P(x) \land Q(a)) \implies (\exists x)P(x) \to \neg Q(a)$

   2. $(x)(\neg P(x) \to Q(x)),\ (x)\neg Q(x) \implies P(a)$

   3. $(x)(P(x) \to Q(x)),\ (x)(Q(x) \to R(x)) \implies P(x) \to R(x)$

4. (**Implication using CP rule**) Use CP rule if possible to show the following implications.

   1. $(x)(P(x) \to Q(x)),\ (x)(R(x) \to \neg Q(x)) \implies (x)(R(x) \to \neg P(x))$

5. (**Mistake in derivation**) There is a mistake in the following derivation. Find it.

   1. (1) $(x)(P(x) \to Q(x))$ P
   2. (2) $P(y) \to Q(y)$ US, (1)
   3. (3) $(\exists x)P(x)$ P
   4. (4) $P(y)$ ES, (3)
   5. (5) $Q(y)$ T,(2),(4), I_{11}
   6. (6) $(\exists x)Q(x)$ EG, (5)