

Due: 16.09.19

Instructor: Dr. P. Kumar

INSTRUCTIONS:

Problems to be discussed in Tutorial in the week of Monday 16th Sep 2019.

1. Let N be the set of natural numbers including zero. Determine which of the following functions are one-to-one, which are onto, and which are one-to-one and onto.

1. $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(j) = j^2 + 2$
2. $f : \mathbb{N} \rightarrow \mathbb{N}$, $f(j) = j \pmod{3}$
3. $f : \mathbb{N} \rightarrow \mathbb{N}$, where

$$f(j) = \begin{cases} 1, & j \text{ is odd} \\ 0, & j \text{ is even} \end{cases}$$

4. $f : \mathbb{N} \rightarrow \mathbb{N}$

$$f(j) = \begin{cases} 0, & j \text{ is odd} \\ 1, & j \text{ is even} \end{cases}$$

2. List all functions from the set $X = \{a, b, c\}$ to $Y = \{0, 1\}$ and indicate in which case whether the function is one-to-one, and is onto, and is one-to-one onto.
3. If $A = \{1, 2, \dots, n\}$ show that any function from A to A which is one-to-one must be also onto, and conversely.
4. Show that the functions f and g which are both from $\mathbb{N} \times \mathbb{N}$ to \mathbb{N} given by $f(x, y) = x + y$ and $g(x, y) = xy$ are onto but not one-to-one.
5. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$, where \mathbb{R} is the set of real numbers. Find $f \circ g$ and $g \circ f$, where $f(x) = x^2 - 2$ and $g(x) = x + 4$. State whether these functions are injective, surjective, and bijective.
6. If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ and both f and g are onto, show that $g \circ f$ is also onto. Is $g \circ f$ one-to-one if both g and f are one-to-one?
7. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be given by $f(x) = x^3 - 2$. Find f^{-1} .