INTERNATIONAL INSTITUTE OF INFORMATION TECHNOLOGY HYDERABAD

Monsoon Semester (Aug-Nov), 2019
Discrete Structures (DS)

Tutorial XIII
October 8, 2019

Due: 13.10.19 **Instructor: Dr. P. Kumar**

INSTRUCTIONS:

Problems to be discussed in Tutorial on Monday 14th Oct 2019.

1. **(Recurrence Relations)** Find all the solutions of the following recurrence relations

   (a) $a_n = a_{n-1} + 8a_{n-2} - 12a_{n-3}, \quad n \geq 3, \quad a_0 = 0,\ a_1 = 1,\ a_2 = 3$

   (b) $a_n = 2a_{n-1} - a_{n-2}, \quad a_0 = 0,\ a_1 = 3$

   (c) $a_n = 4a_{n-1} - 4a_{n-2} + n2^n + 3^n + 4, \quad n \geq 2, \quad a_0 = 0,\ a_1 = 1$
   [Remark: See Malik and Sen book for more examples on non-homogeneous recurrences]

   (d) $f_n = f_{n-1} + f_{n-2}, \quad f_0 = 1, f_1 = 1$

   (e) There are three posts. In one of the posts, there are n disks ordered from largest at the bottom to the smallest on top. The rules of the game are:

      (a) In one step, one can remove the top disk from one post and put it on another

      (b) Larger disk can never lie above a smaller disk

   How many steps are needed to move the disks to another post such that they are ordered from largest disk at bottom to smallest on top if $n = 50$? Derive recurrence relation, and solve the recurrence for $n = 50$.

   (f) There are three letters α, β, γ in a language. In this language, any word is not allowed to have two consecutive γ. How many words of length n are possible in this langiage? Derive (and explain) a recurrence relation, and find solution.

   (g) In a fresher's meeting, all students of UG1 shook hands exactly once with other. How many handshakes they did? Assume that no one does handshake with himself or herself. Also, no one did more than one handshake with same person. Assume that there are n students in UG1. Derive (and explain) a recurrence relation first and solve it.

   (h) There is a $m \times n$ grid. The rules of the game are:

      (a) At each step one can move either one step to the right (horizontaly) or up (vertically).

   In how many ways one can go from bottom left grid point say $(1, 1)$ to the top right corner grid (m, n)? Derive a recurrence relation.
