University of Zurich[UZH] | ETH zürich

Institute of Informatics – Institute of Neuroinformatics

ROBOTICS & PERCEPTION GROUP

Lecture 04
Image Filtering

Davide Scaramuzza

http://rpg.ifi.uzh.ch

No exercise this afternoon

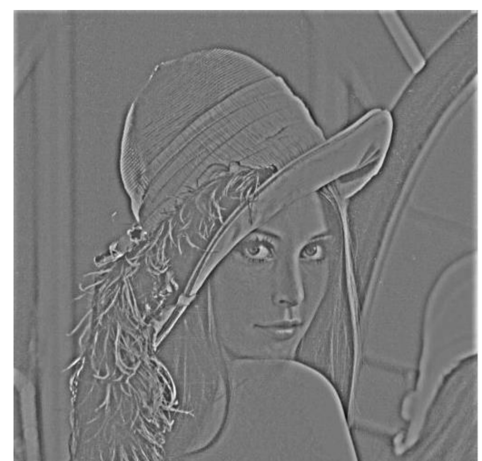
Date	Time	Description of the lecture/exercise	Lecturer
20.09.2018	10:15 - 12:00	01 – Introduction	Davide Scaramuzza
27.09.2018	10:15 - 12:00	02 - Image Formation 1: perspective projection and camera models	Davide Scaramuzza
	13:15 – 15:00	Exercise 1: Augmented reality wireframe cube	Titus Cieslewski & Mathias Gehrig
04.10.2018	10:15 - 12:00	03 - Image Formation 2: camera calibration algorithms	Guillermo Gallego
	13:15 – 15:00	Exercise 2: PnP problem	Antonio Loquercio & Mathias Gehrig
11.10.2018	10:15 - 12:00	04 - Filtering & Edge detection	Davide Scaramuzza
18.10.2018	10:15 - 12:00	05 - Point Feature Detectors 1: Harris detector	Guillermo Gallego
	13:15 – 15:00	Exercise 3: Harris detector + descriptor + matching	Antonio Loquercio & Mathias Gehrig
25.10.2018	10:15 - 12:00	06 - Point Feature Detectors 2: SIFT, BRIEF, BRISK	Davide Scaramuzza
01.11.2018	10:15 - 12:00	07 - Multiple-view geometry 1	Guillermo Gallego
	13:15 – 15:00	Exercise 4: Stereo vision: rectification, epipolar matching, disparity, triangulation	Antonio Loquercio & Mathias Gehrig
08.11.2018	10:15 - 12:00	08 - Multiple-view geometry 2	Davide Scaramuzza
	13:15 – 15:00	Exercise 5: Eight-Point algorithm	Antonio Loquercio & Mathias Gehrig
15.11.2018	10:15 - 12:00	09 - Multiple-view geometry 3	Davide Scaramuzza
	13:15 – 15:00	Exercise 6: P3P algorithm and RANSAC	Antonio Loquercio & Mathias Gehrig
22.11.2018	10:15 - 12:00	10 - Dense 3D Reconstruction (Multi-view Stereo)	Davide Scaramuzza
	13:15 – 15:00	Exercise session: Intermediate VO Integration	Antonio Loquercio & Mathias Gehrig
29.11.2018	10:15 - 12:00	11 - Optical Flow and Tracking (Lucas-Kanade)	Davide Scaramuzza
	13:15 – 15:00	Exercise 7: Lucas-Kanade tracker	Antonio Loquercio & Mathias Gehrig
06.12.2018	10:15 - 12:00	12 – Place recognition	Davide Scaramuzza
	13:15 – 15:00	Exercise session: Deep Learning Tutorial	Antonio Loquercio
	10:15 - 12:00	13 – Visual inertial fusion	Davide Scaramuzza
13.12.2018	13:15 – 15:00	Exercise 8: Bundle adjustment	Antonio Loquercio & Mathias Gehrig
20.12.2018	10:15 - 12:00	14 - Event based vision	Davide Scaramuzza
	12:30 – 13:30	**Scaramuzza's lab visit and live demonstrations**: Andreasstrasse 15, 2.11, 8050	Davide Scaramuzza & his lab
	14:00 – 16:00	Exercise session: final VO integration (it will take place close to Scaramuzza's lab)	Antonio Loquercio & Mathias Gehrig

Image filtering

- The word *filter* comes from frequency-domain processing, where “filtering” refers to the process of accepting or rejecting certain frequency components
- We distinguish between low-pass and high-pass filtering
  - A **low-pass filter** smooths an image (retains low-frequency components)
  - A **high-pass filter** retains the contours (also called edges) of an image (high frequency)

Low-pass filtered image

High-pass filtered image



Low-pass filtering

Low-pass filtering
Motivation: noise reduction

- **Salt and pepper noise**: random occurrences of black and white pixels
- **Impulse noise:** random occurrences of white pixels
- **Gaussian noise**: variations in intensity drawn from a Gaussian distribution

Original

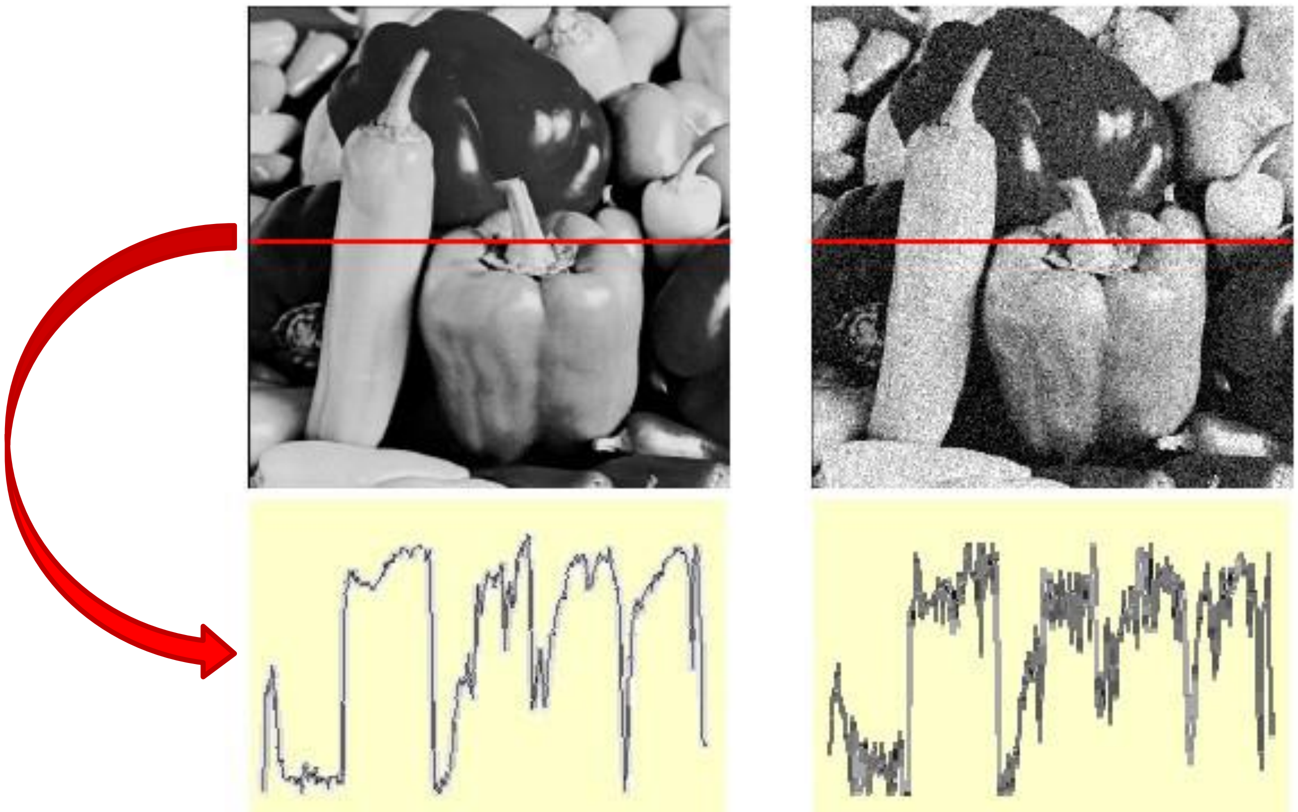
Salt and pepper noise

Impulse noise

Gaussian noise

Source: S. Seitz

Gaussian noise



$$f(x,y) = \overbrace{\tilde{f}(x,y)}^{\text{Ideal Image}} + \overbrace{\eta(x,y)}^{\text{Noise process}}$$

Gaussian i.i.d. (“white”) noise:
$\eta(x,y) \sim \mathcal{N}(\mu,\sigma)$

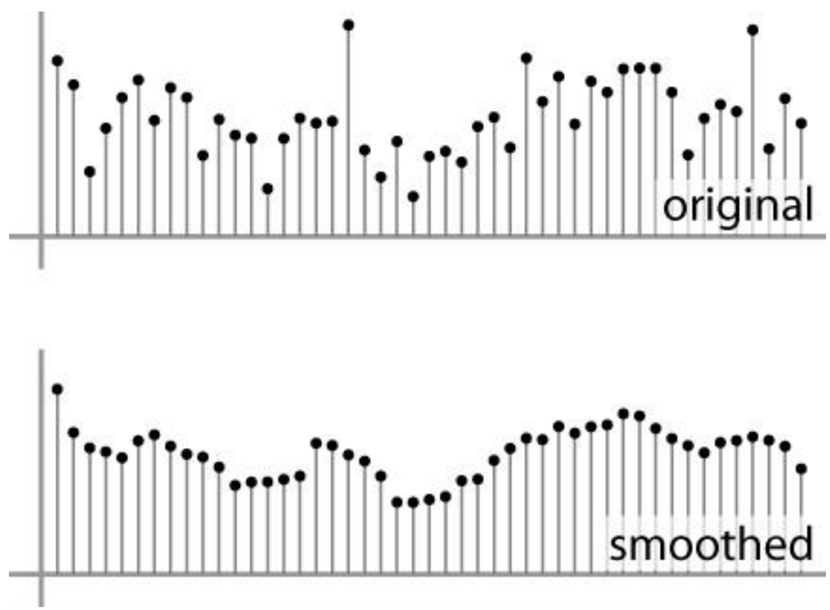
How could we reduce the noise to try to recover the “ideal image”?

Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Assumptions:
  - Expect pixels to be like their neighbors
  - Expect noise process to be independent from pixel to pixel

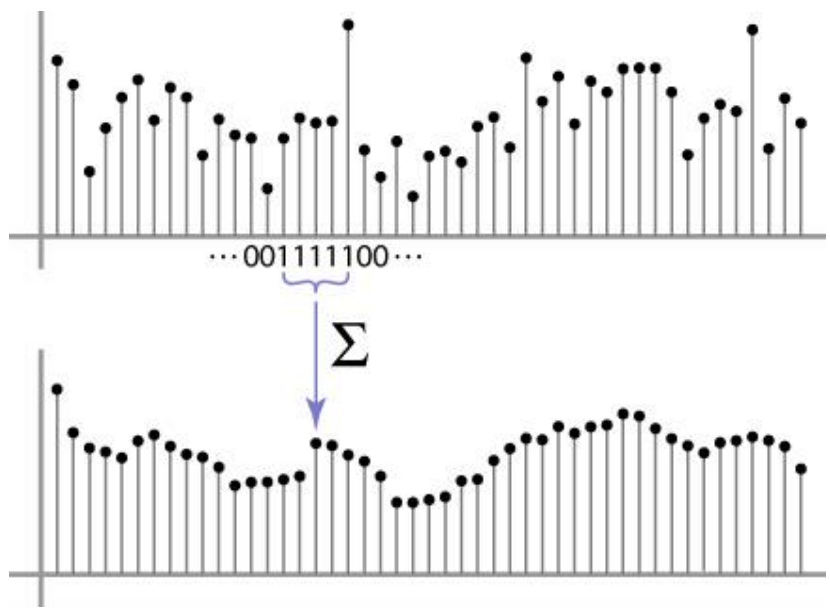
Moving average

- Replaces each pixel with an average of all the values in its neighborhood
- Moving average in 1D:



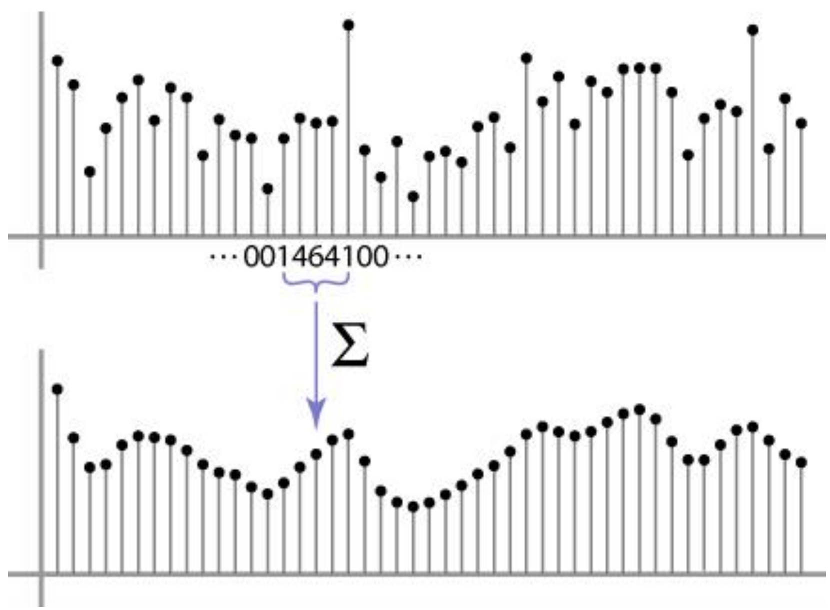
Weighted Moving Average

- Can add weights to our moving average
- *Weights* [1, 1, 1, 1, 1] / 5



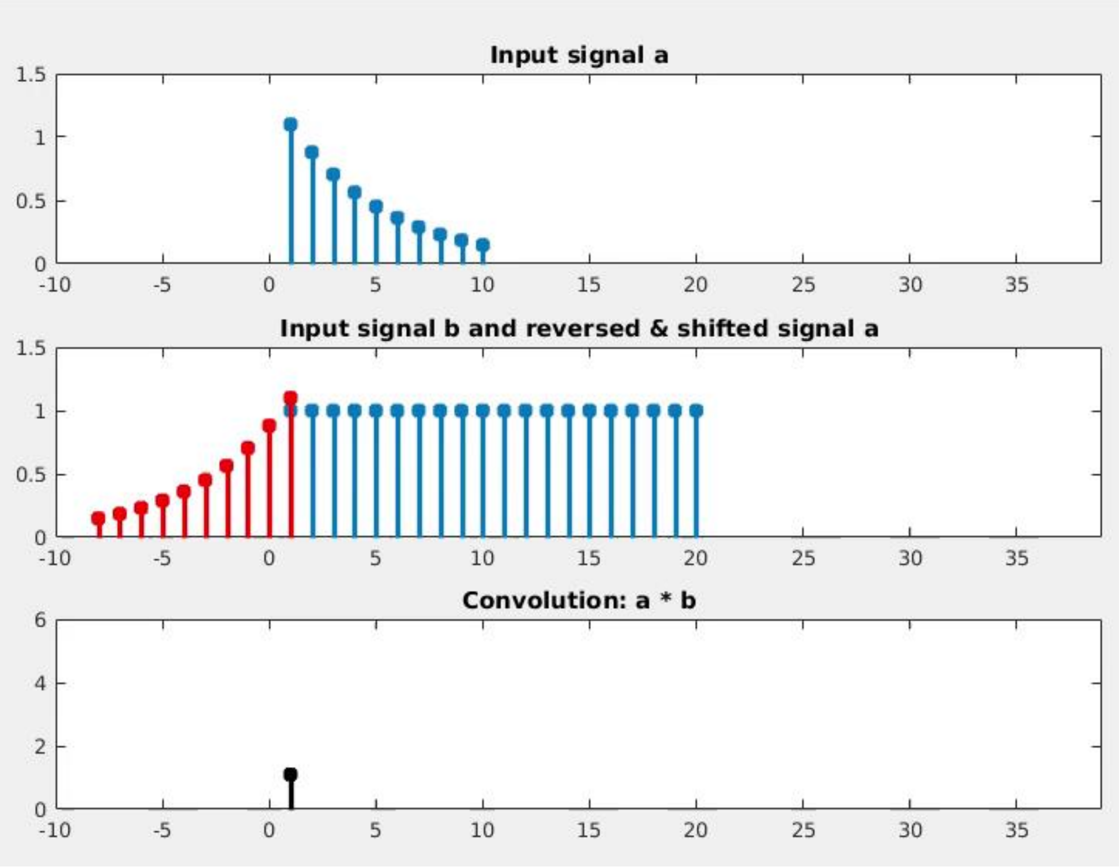
Weighted Moving Average

- Non-uniform weights [1, 4, 6, 4, 1] / 16



This operation is called *convolution*

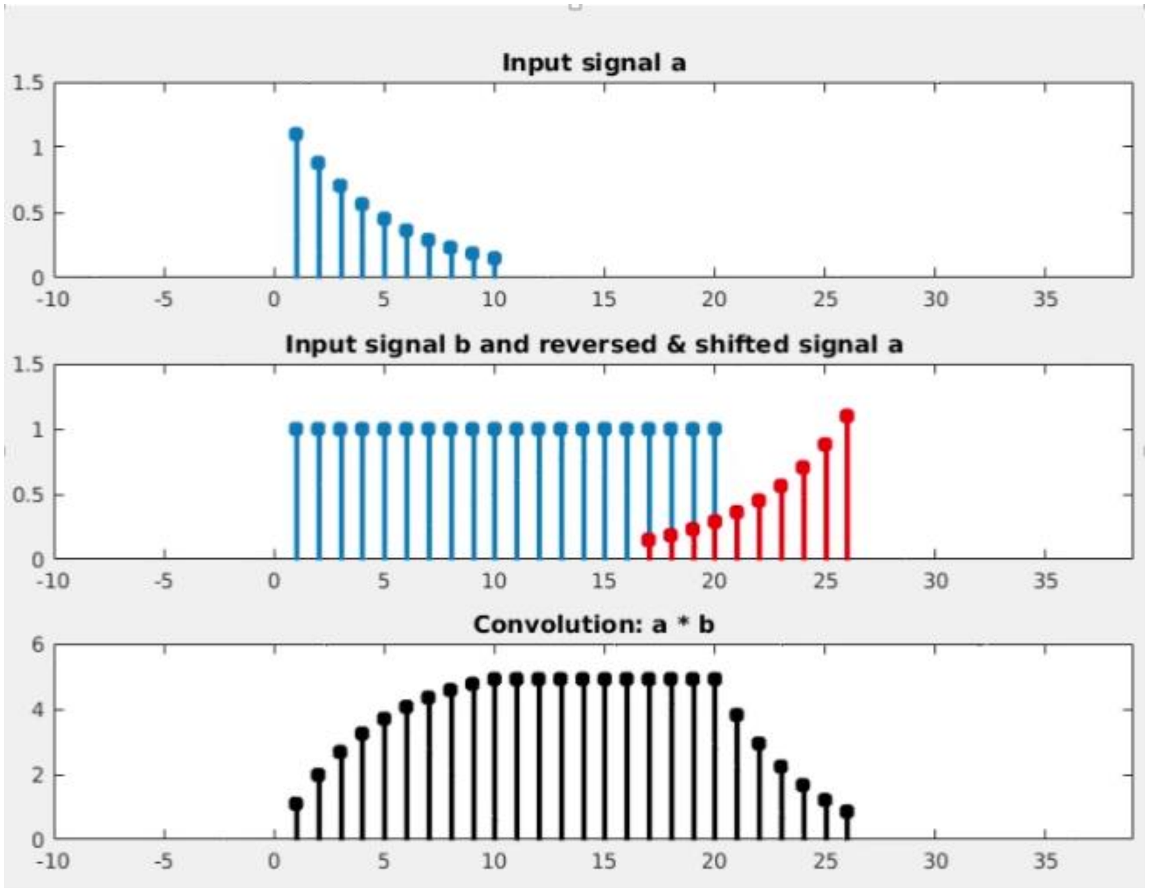
Example of convolution of two sequences (or “signals”)
- One of the sequences is flipped (right to left) before sliding over the other
- Notation: $a \star b$
- Nice properties: linearity, associativity, commutativity, etc.



This operation is called *convolution*

Example of convolution of two sequences (or “signals”)

- One of the sequences is flipped (right to left) before sliding over the other
- Notation: $a * b$
- Nice properties: linearity, associativity, commutativity, etc.

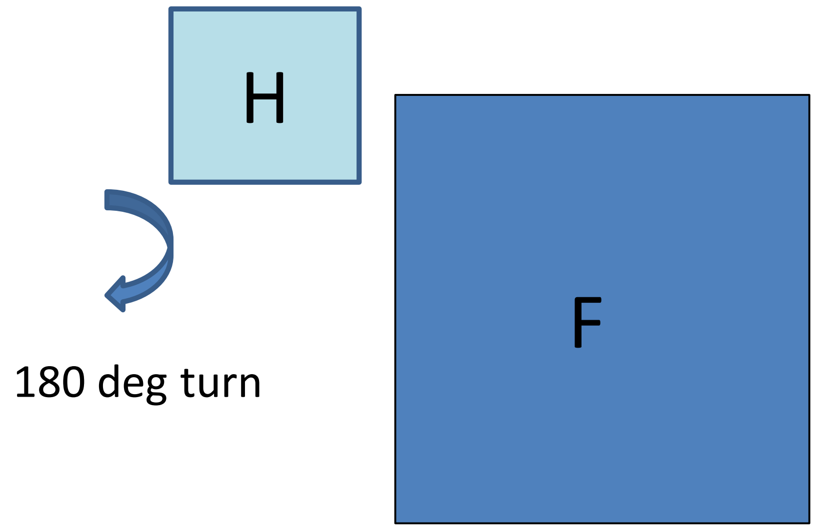


2D Filtering

- Convolution:
  - Flip the filter in both dimensions (bottom to top, right to left) (=180 deg turn)
  - Then slide the filter over the image and compute sum of products

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x-u, y-v]H[u,v]$$

$$G = F * H$$



Filtering an image: replace each pixel with a linear combination of its neighbors.

The **filter** $\boldsymbol{H}$ is also called "**kernel**" or "**mask**".

Review: Convolution vs. Cross-correlation

Convolution

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x-u, y-v]H[u,v]$$

$$G = F * H$$

Properties: linearity, associativity, commutativity

Cross-correlation

$$G[x,y] = \sum_{u=-k}^{k} \sum_{v=-k}^{k} F[x+u, y+v]H[u,v]$$

$$G = F \otimes H$$

Properties: linearity, but not associativity and commutativity

For a Gaussian or box filter, how will the outputs differ?

Example: Moving Average in 2D

"box filter"

$$\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

Input image

$F[x, y]$

1	1	1	0	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0
1	1	1	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$G[x, y]$

	0								

Example: Moving Average in 2D

Input image

$$F[x, y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$$G[x, y]$$

	0	10							

Example: Moving Average in 2D

Input image

$$F[x,y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$$G[x,y]$$

	0	10	20						

Example: Moving Average in 2D

Input image

$$F[x,y]$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$$G[x,y]$$

	0	10	20	30					

Example: Moving Average in 2D

Input image

$F[x, y]$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$G[x, y]$

	0	10	20	30	30				

Example: Moving Average in 2D

Input image

$$F[x,y]$$

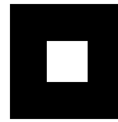
0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

Filtered image

$$G[x,y]$$

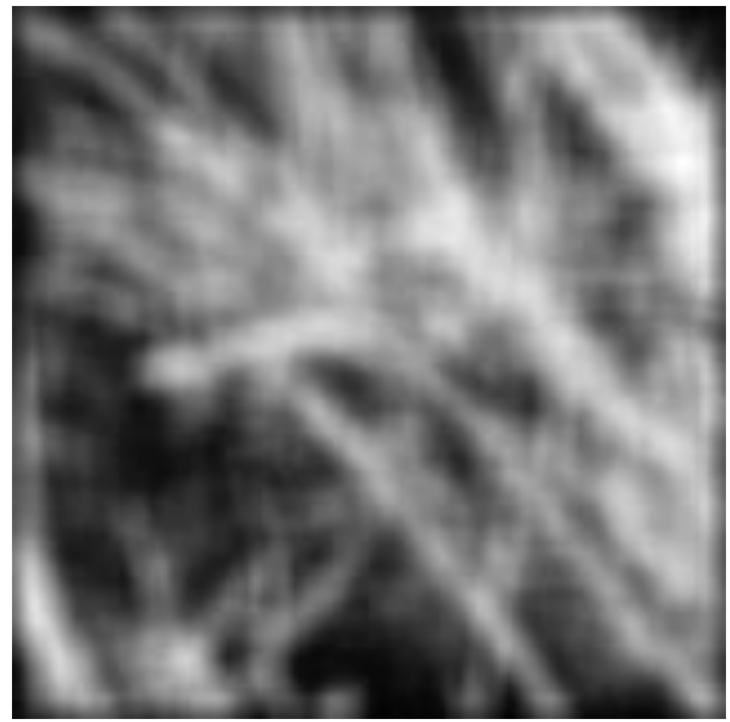
	0	10	20	30	30	30	20	10	
	0	20	40	60	60	60	40	20	
	0	30	60	90	90	90	60	30	
	0	30	50	80	80	90	60	30	
	0	30	50	80	80	90	60	30	
	0	20	30	50	50	60	40	20	
	10	20	30	30	30	30	20	10	
	10	10	10	0	0	0	0	0	

Example: Moving Average in 2D



Box filter:
white = high value, black = low value

original



filtered

Gaussian filter

- What if we want the closest pixels to have higher influence on the output?

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$F[x,y]$

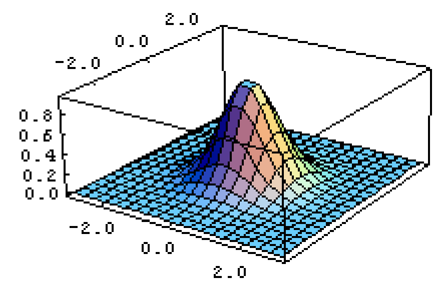
$\frac{1}{16}$

1	2	1
2	4	2
1	2	1

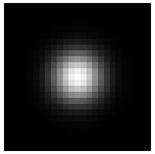
$H[u,v]$

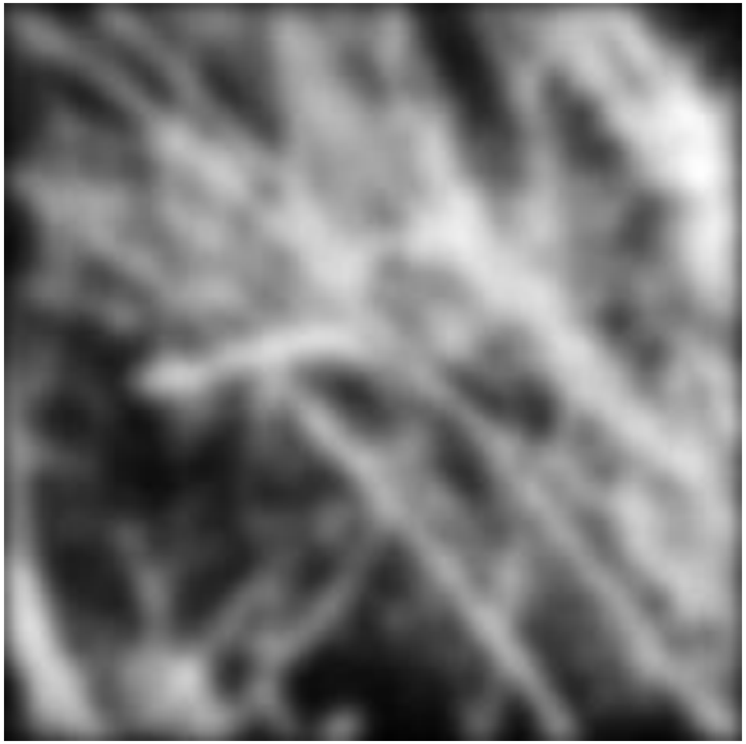
This kernel is the approximation of a Gaussian function:

$$H[u,v] = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{2\sigma^2}}$$

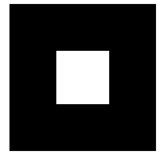


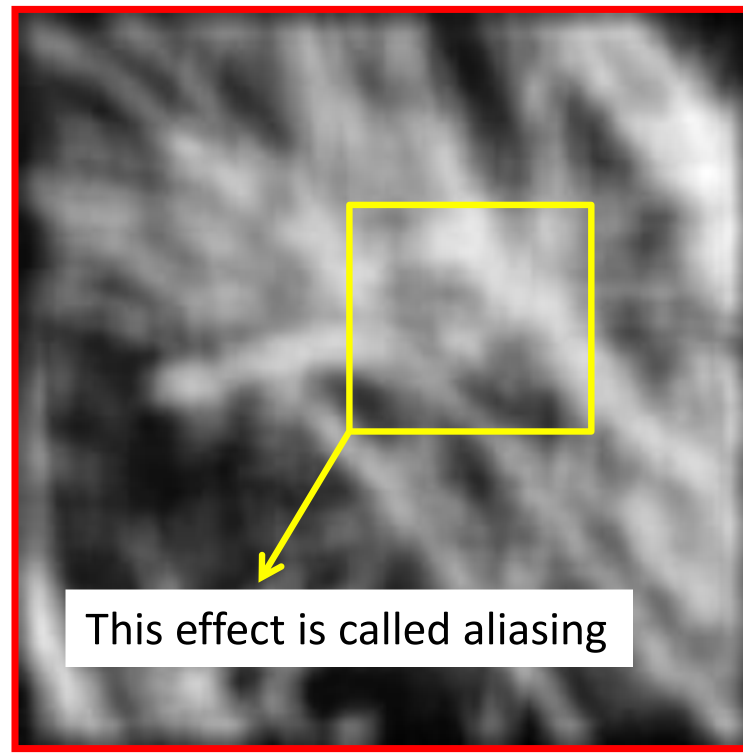
Smoothing with a Gaussian





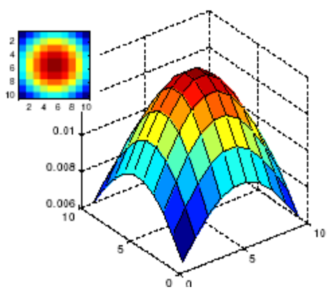
Compare the result with a box filter



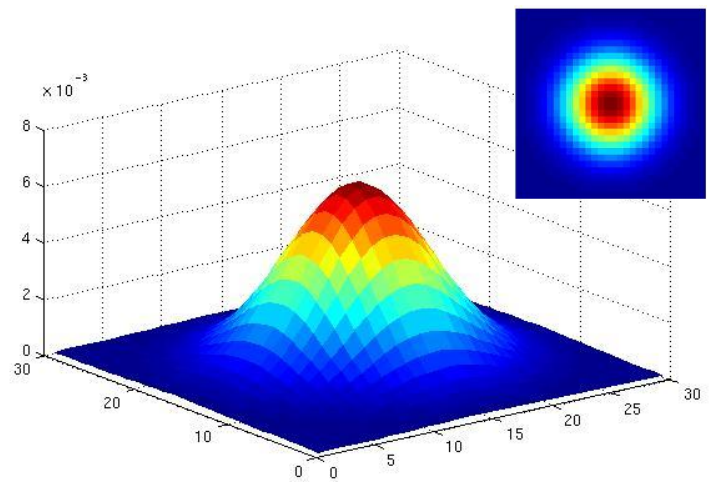


Gaussian filters

- What parameters matter?
- **Size** of the kernel
  - NB: a Gaussian function has **infinite support**, but discrete filters use finite kernels



σ = 5 pixels
with 10 x 10 pixel kernel

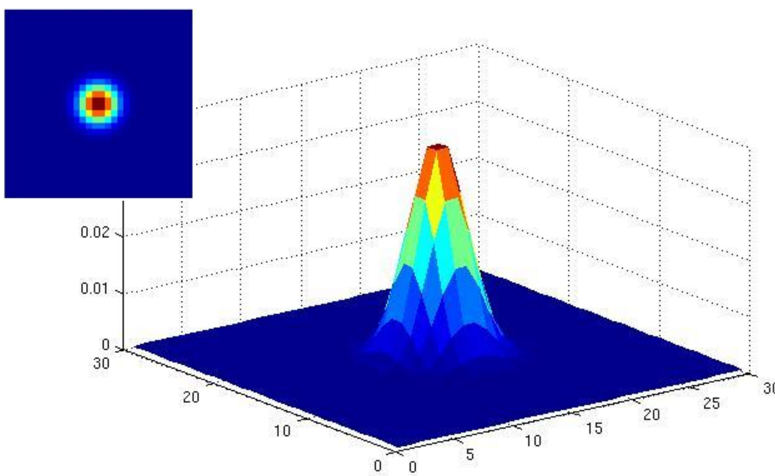


σ = 5 pixels
with 30 x 30 pixel kernel

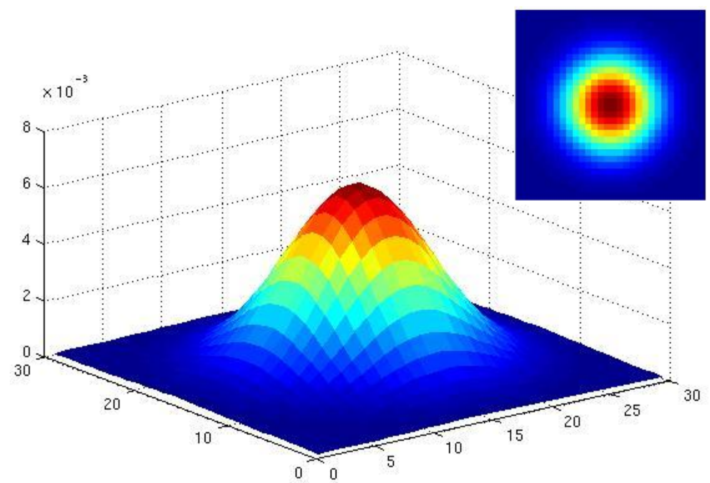
Which one is better (left or right)?

Gaussian filters

- What parameters matter?
- **Variance** of Gaussian: control the amount of smoothing



σ = 2 pixels
with 30 x 30 pixel kernel

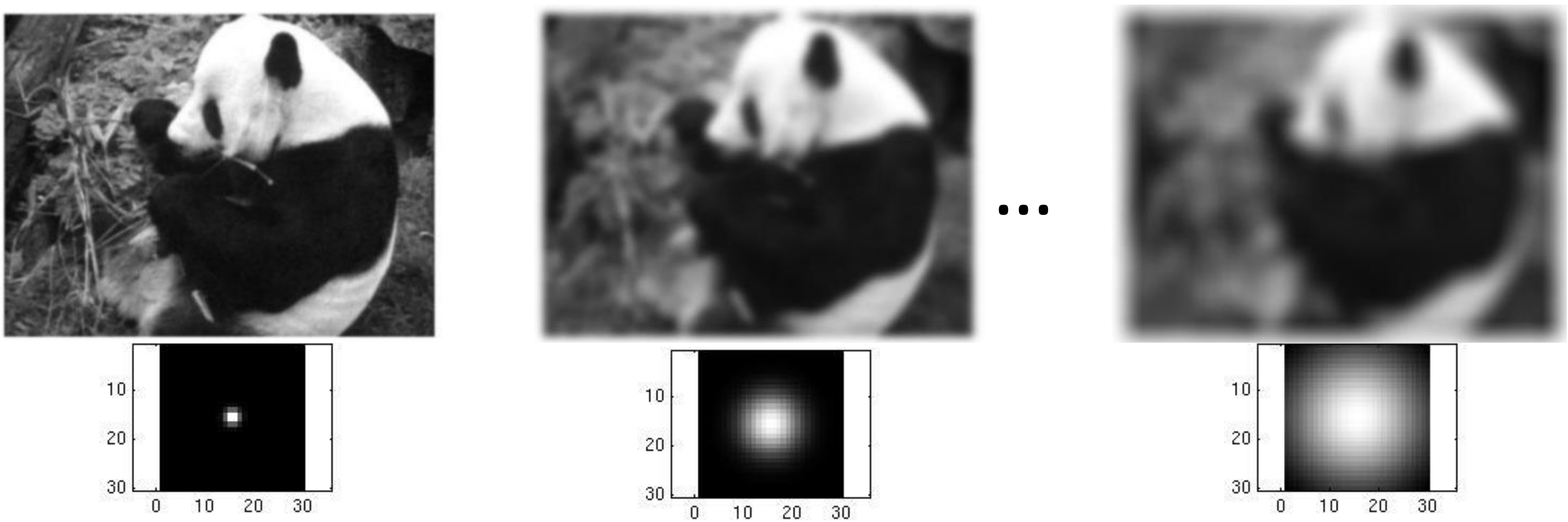


σ = 5 pixels
with 30 x 30 pixel kernel

Recall: standard deviation = σ [pixels], variance = σ^2 [pixels^2]

Smoothing with a Gaussian

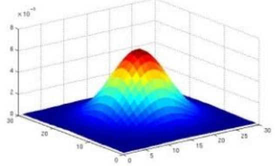
σ is called “scale” or “width” or “spread” of the Gaussian kernel, and controls the amount of smoothing.



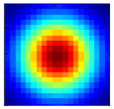
Sample Matlab code

```
>> hsize = 20;
>> sigma = 5;
>> h = fspecial('gaussian', hsize, sigma);
```

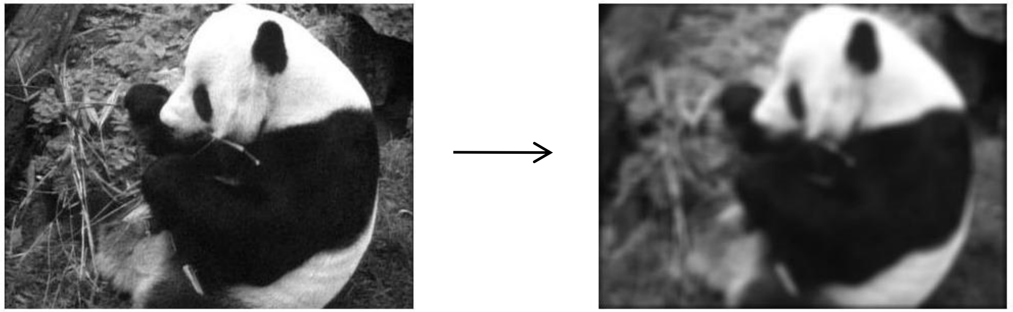
```
>> mesh(h);
```



```
>> imagesc(h);
```



```
>> im = imread('panda.jpg');
>> outim = imfilter(im, h);
>> imshow(outim);
```



outim

Boundary issues

- What about near the image edges?
  - the filter window falls off the edges of the image
  - need to pad the image borders
  - methods:
    - zero padding (black)
    - wrap around
    - copy edge
    - reflect across edge

Summary on (linear) smoothing filters

- <u>Smoothing filter</u>
  - has positive values (also called coefficients)
  - sums to 1 → preserve brightness of constant regions
  - removes “high-frequency” components; “low-pass” filter

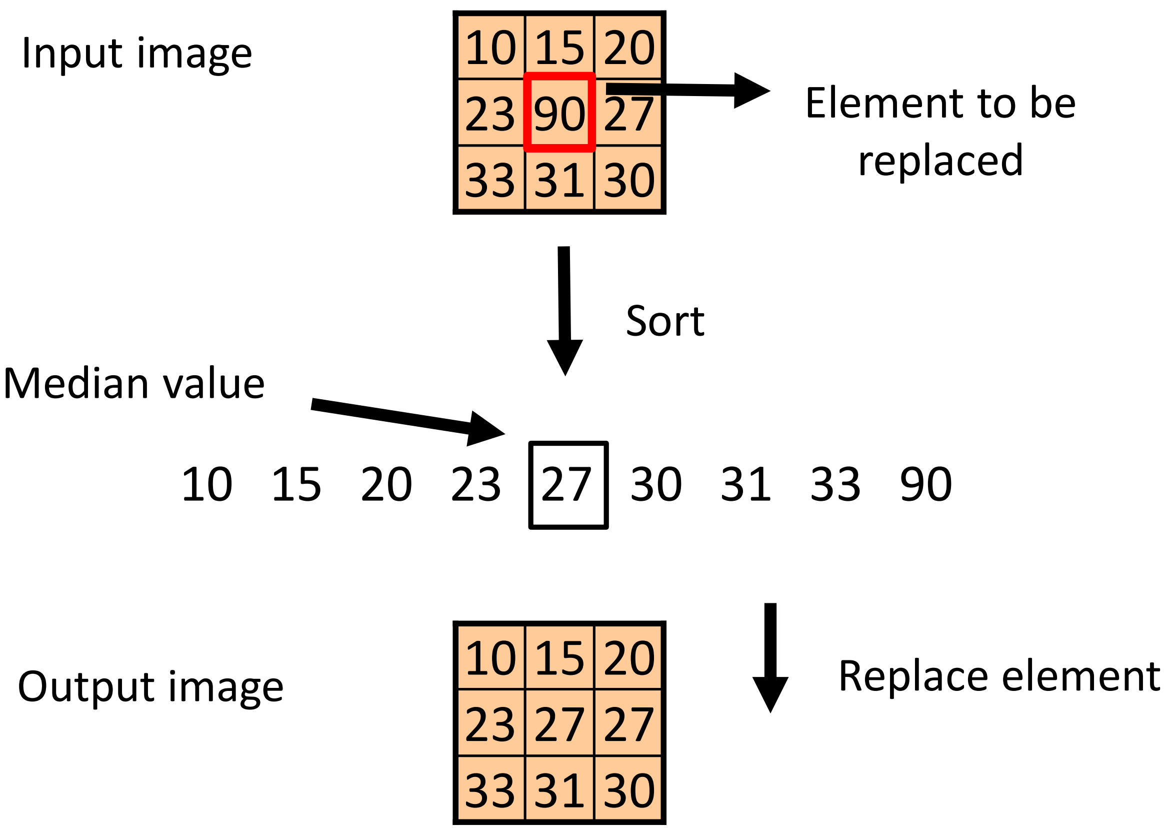
Non-linear filtering

Effect of smoothing filters

Linear smoothing filters do not alleviate salt and pepper noise!

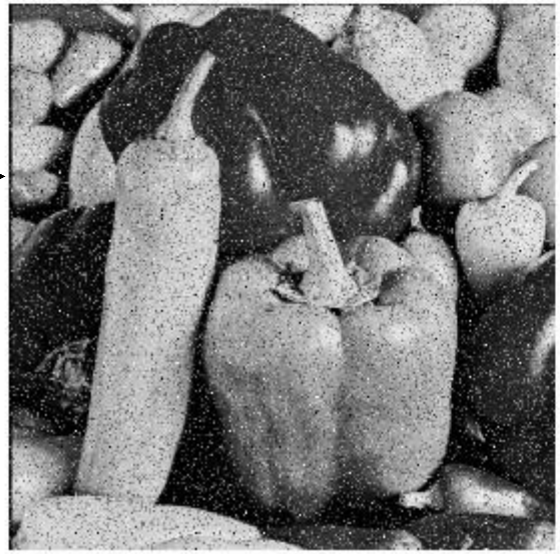
Median filter

- It is a non-linear filter
- Removes spikes: good for “*impulse noise*” and “*salt & pepper noise*”

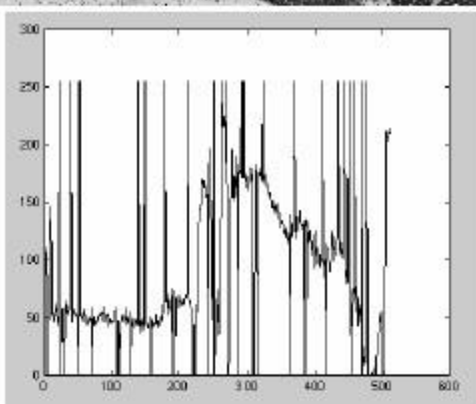


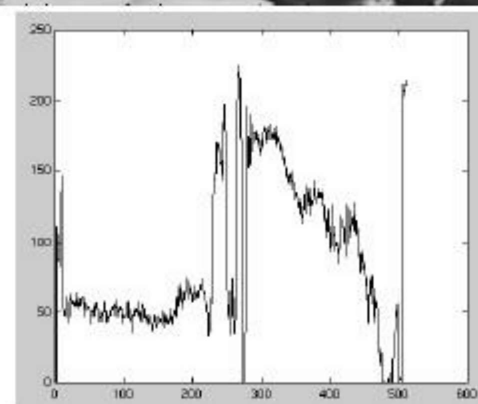
Median filter

Salt and pepper noise



Median filtered

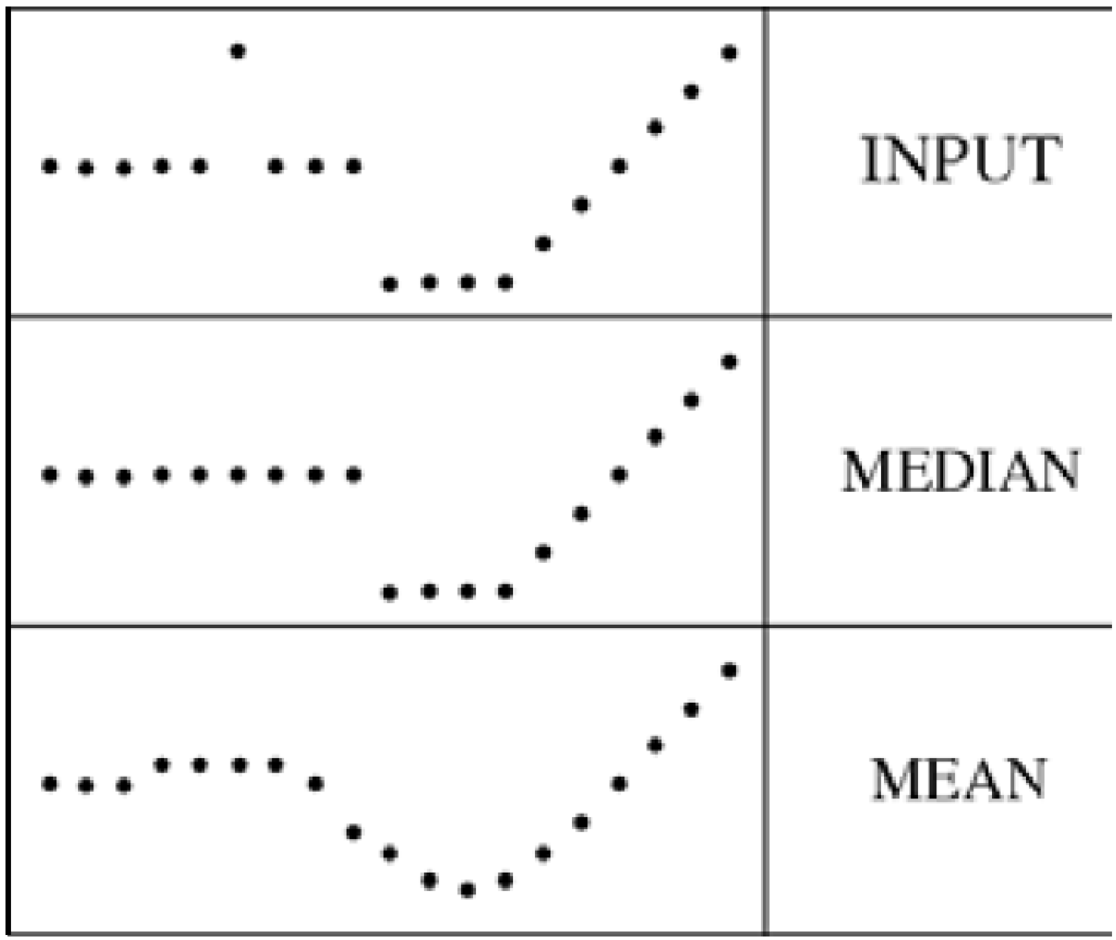




Plots of a row of the image

Median filter

- Median filter preserves sharp transitions (i.e., edges),

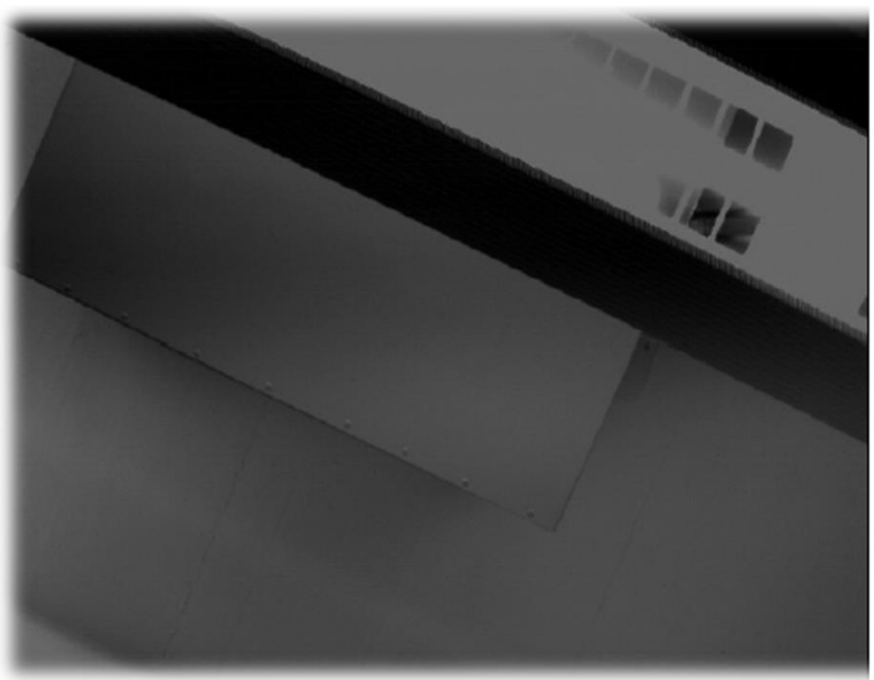
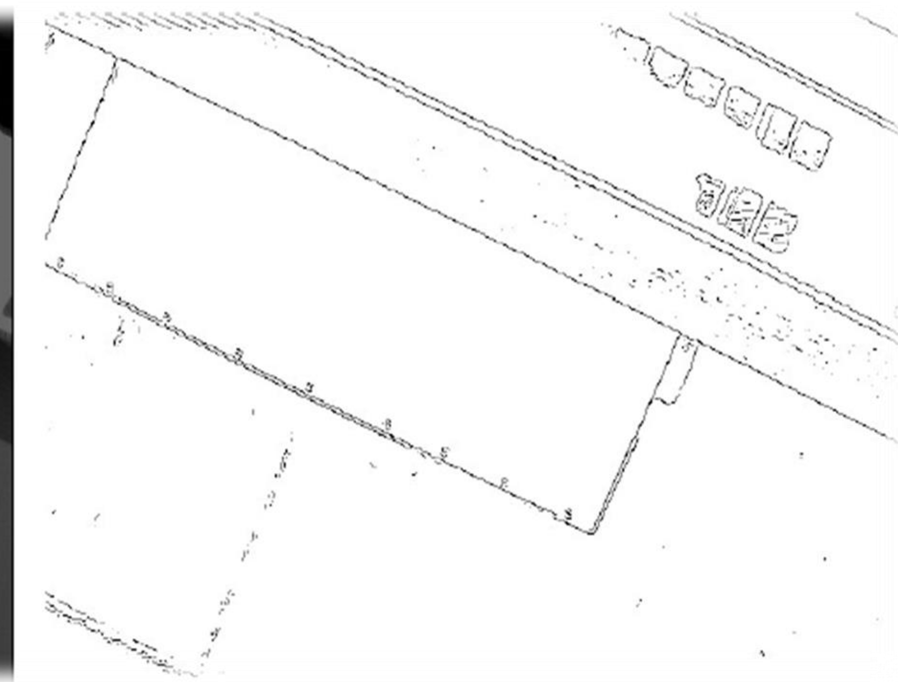


… but it removes small brightness variations.

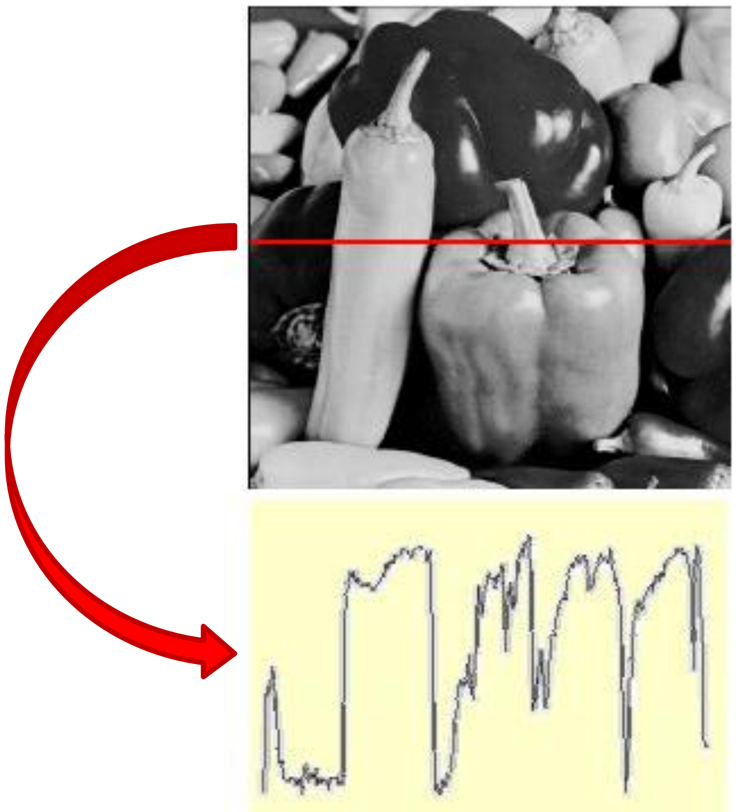
High-pass filtering
(edge detection)

Edge detection

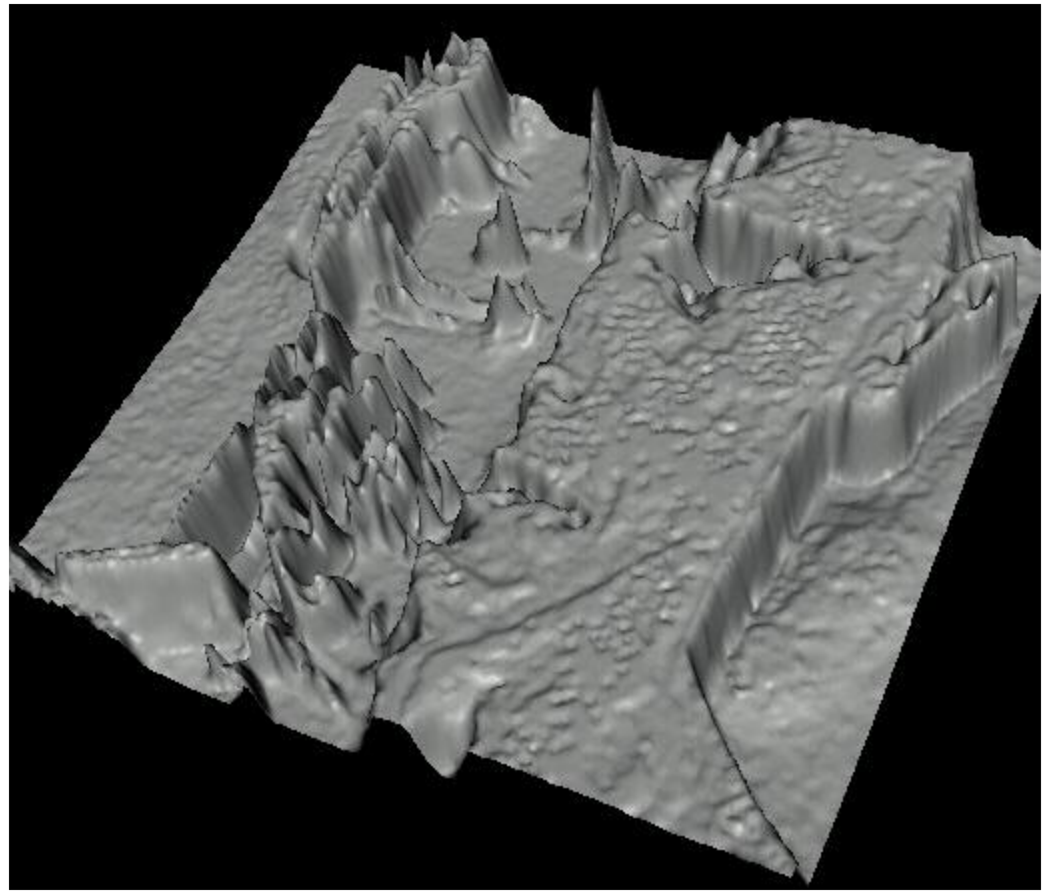
- Ultimate goal of edge detection: an idealized line drawing.
- Edge contours in the image correspond to important scene contours.

Edges are sharp intensity changes



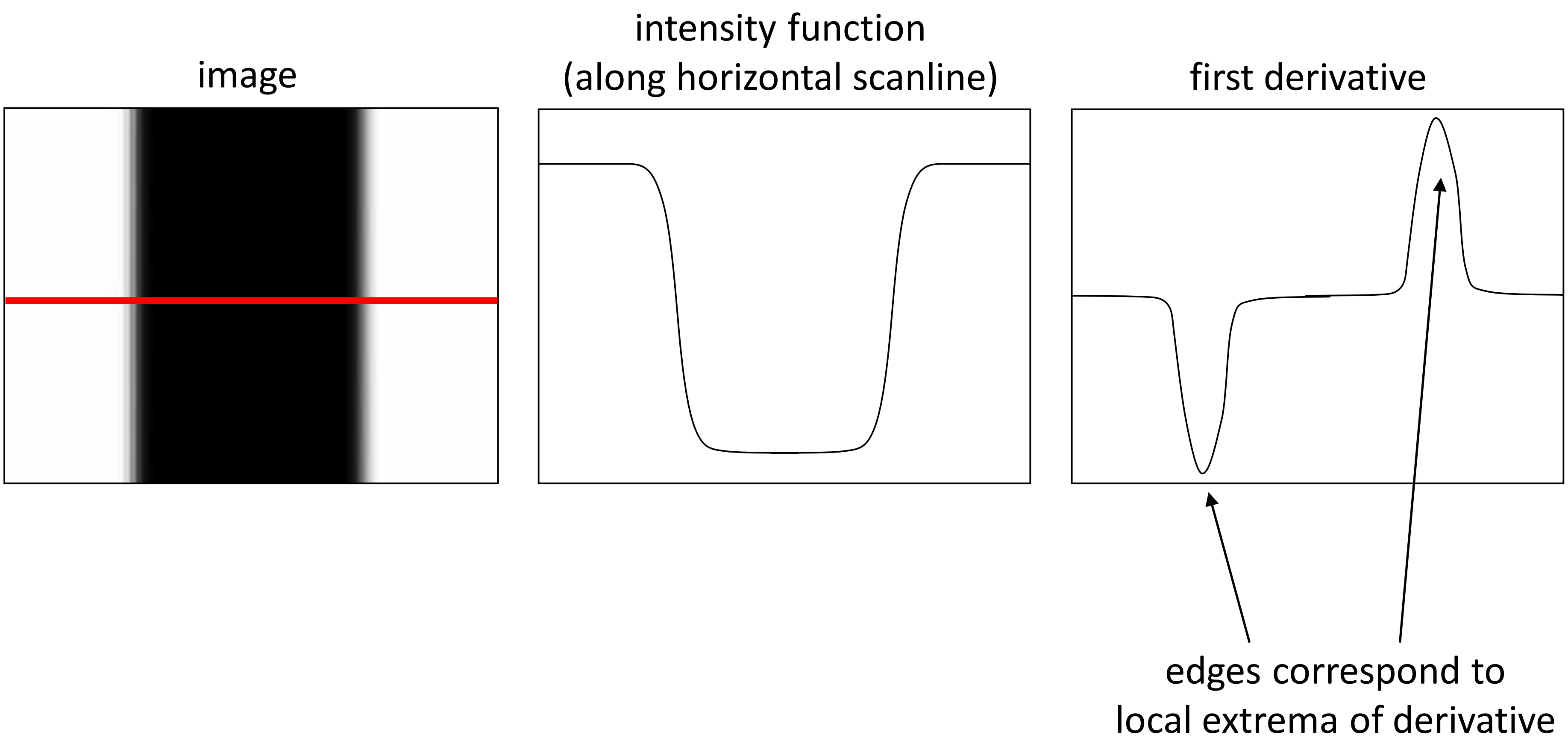
Images as functions $\mathrm{F}(x, y)$



Edges look like steep cliffs

Derivatives and edges

An edge is a place of rapid change in the image intensity function.



Differentiation and convolution

For a 2D function $F(x, y)$ the partial derivative along x is:

$$\frac{\partial F(x,y)}{\partial x} = \lim_{\varepsilon \to 0} \frac{F(x+\varepsilon, y) - F(x,y)}{\varepsilon}$$

For discrete data, we can approximate using finite differences:

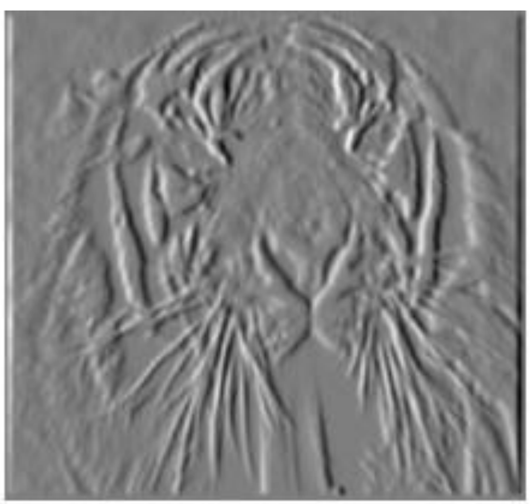
$$\frac{\partial F(x,y)}{\partial x} \approx \frac{F(x+1, y) - F(x,y)}{1}$$

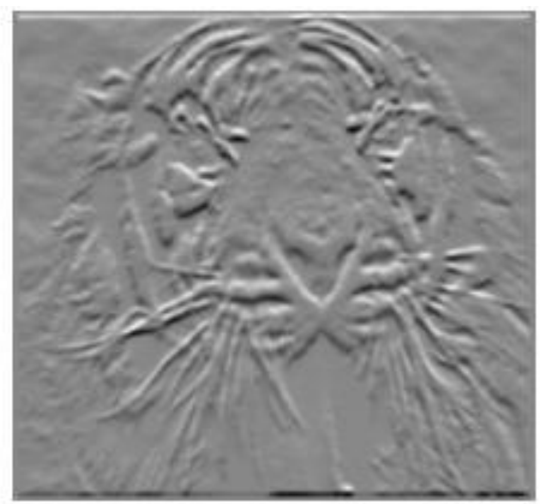
What would be the respective filters along x and y to implement the partial derivatives as a convolution?

Partial derivatives of an image

$$\frac{\partial F(x,y)}{\partial x}$$

$$\frac{\partial F(x,y)}{\partial y}$$





-1	1

-1
1

Alternative Finite-difference filters

Prewitt filter $G_x = \begin{bmatrix} -1 & 0 & +1 \\ -1 & 0 & +1 \\ -1 & 0 & +1 \end{bmatrix}$ and $G_y = \begin{bmatrix} -1 & -1 & -1 \\ 0 & 0 & 0 \\ +1 & +1 & +1 \end{bmatrix}$

Sobel filter $G_x = \begin{bmatrix} -1 & 0 & +1 \\ -2 & 0 & +2 \\ -1 & 0 & +1 \end{bmatrix}$ and $G_y = \begin{bmatrix} -1 & -2 & -1 \\ 0 & 0 & 0 \\ +1 & +2 & +1 \end{bmatrix}$

```
Sample Matlab code
>> im = imread('lion.jpg');
>> h = fspecial('sobel');
>> outim = imfilter(double(im), h);
>> imagesc(outim);
>> colormap gray;
```

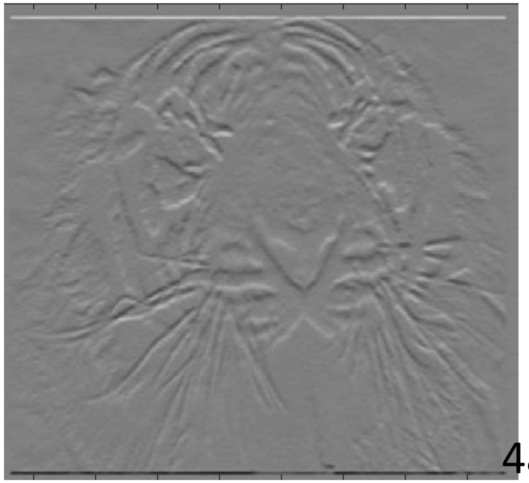
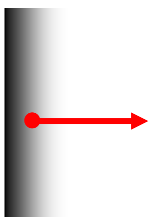


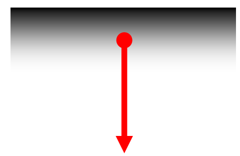
Image gradient

The gradient of an image: $$\nabla F = \left[\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}\right]$$

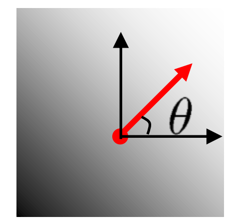
The gradient points in the direction of fastest intensity change



$$\nabla F = \left[\frac{\partial F}{\partial x}, 0\right]$$



$$\nabla F = \left[0, \frac{\partial F}{\partial y}\right]$$



$$\nabla F = \left[\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}\right]$$

The gradient direction (orientation of edge normal) is given by:

$$\theta = tan^{-1}\left(\frac{\partial F}{\partial y} \Big/ \frac{\partial F}{\partial x}\right)$$

The *edge strength* is given by the gradient magnitude

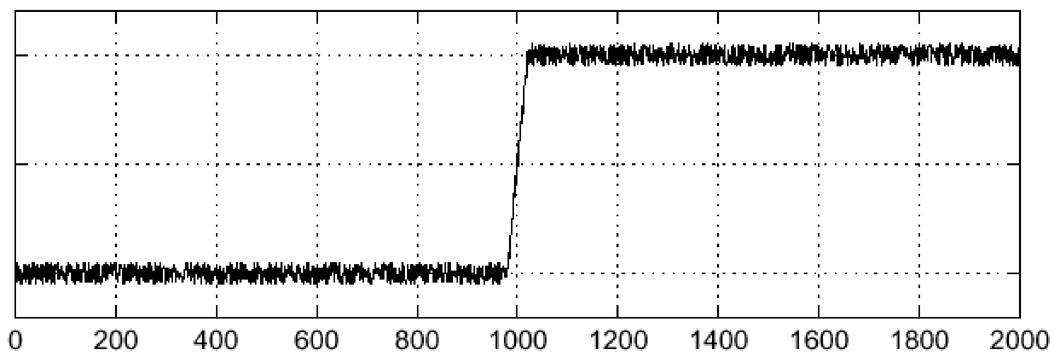
$$\|\nabla F\| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}$$

Effects of noise

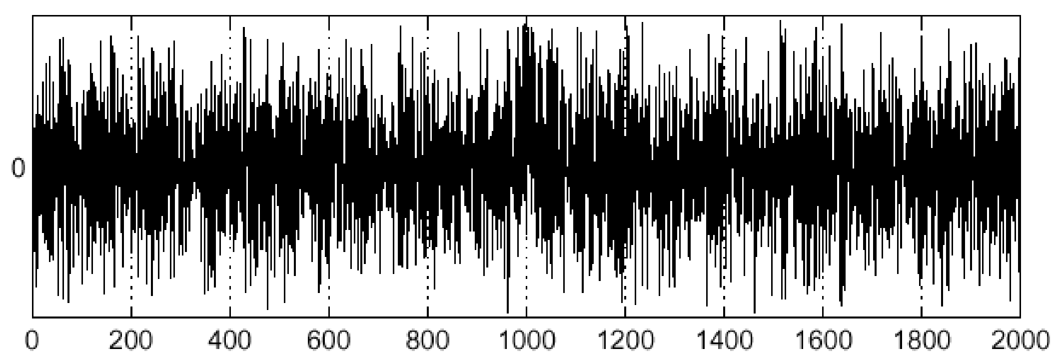
Consider a single row or column of the image

– Plotting intensity as a function of position gives a signal

$F(x)$



$\dfrac{\partial F(x)}{\partial x}$



Where is the edge?

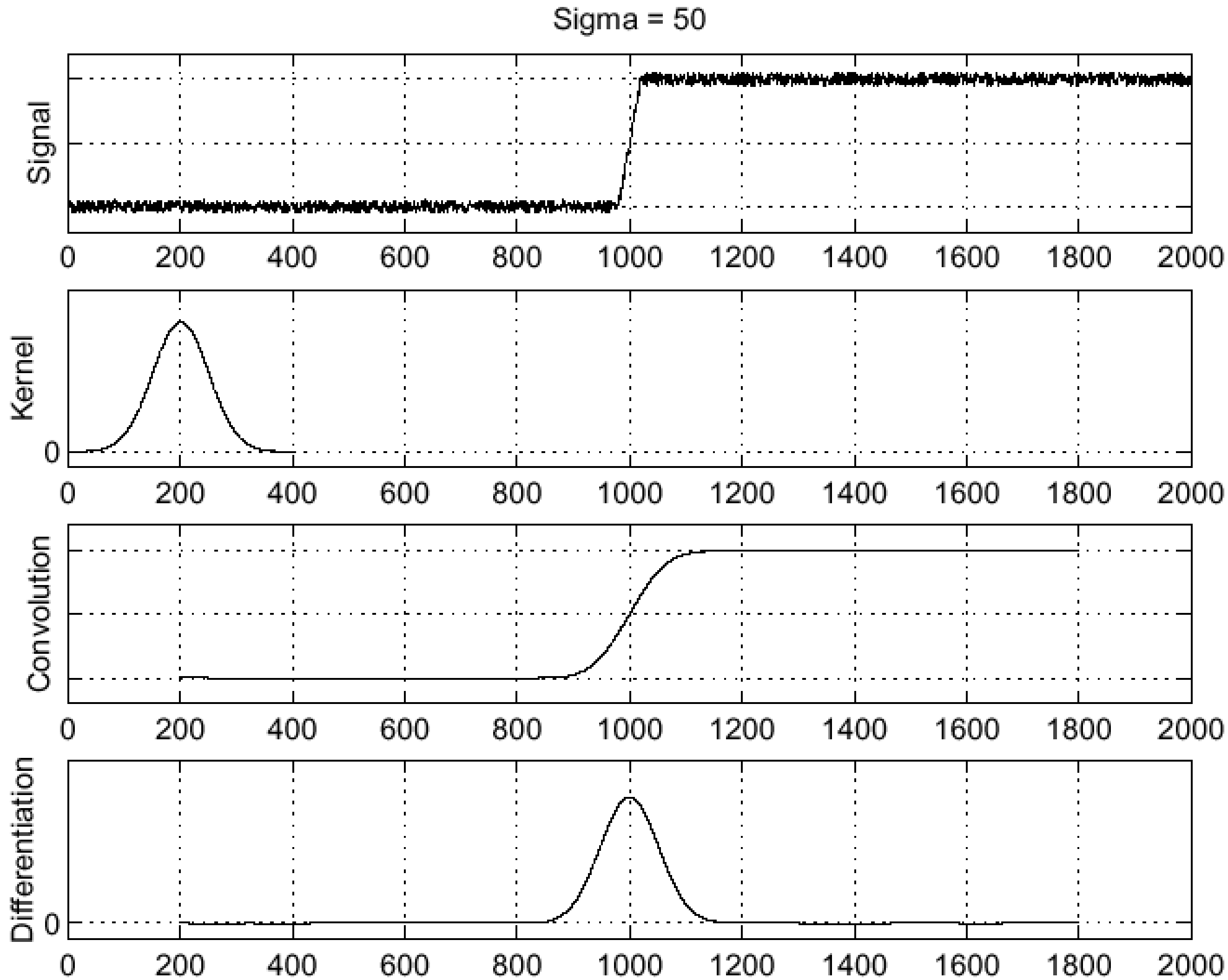
Solution: smooth first

F

H

$F * H$

$\frac{\partial}{\partial x}(F * H)$



Where is the edge?

Look for peaks in $\frac{\partial}{\partial x}(F * H)$

Alternative: combine derivative and smoothing filter

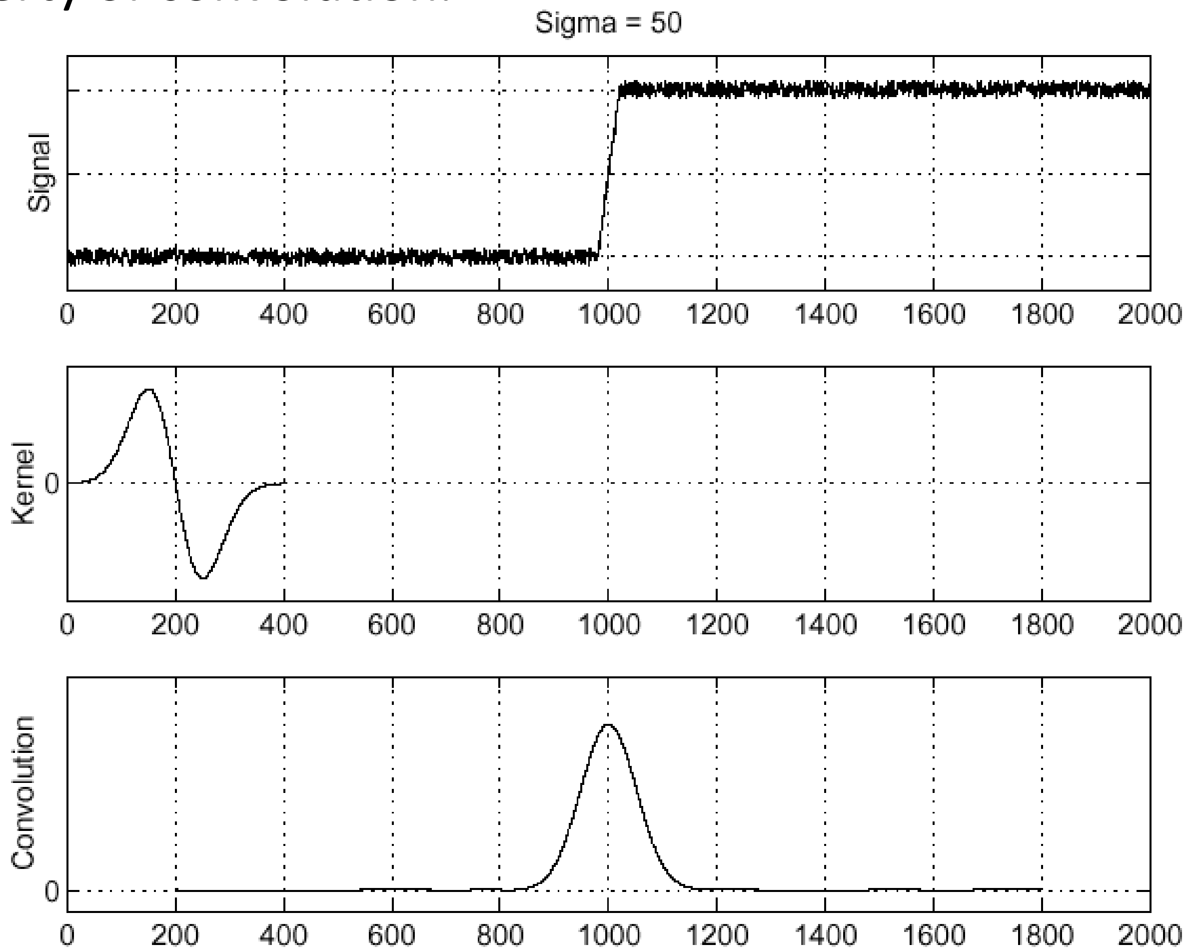
$$\frac{\partial}{\partial x}(F * H) = F * \frac{\partial H}{\partial x}$$

Differentiation property of convolution.

F

$\frac{\partial H}{\partial x}$

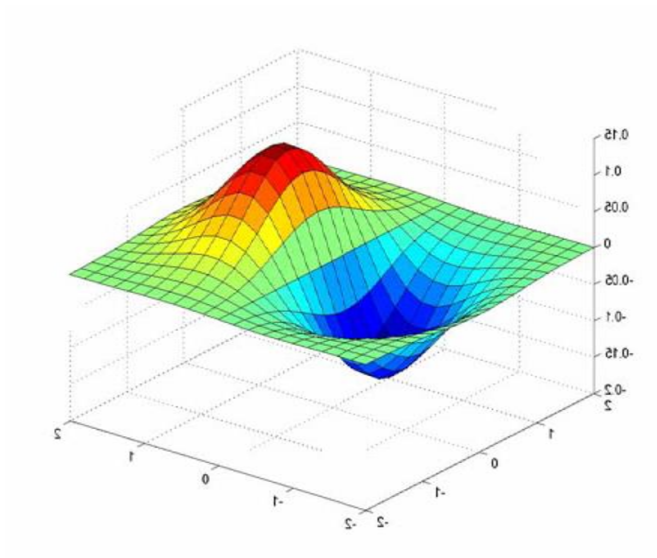
$F * \frac{\partial H}{\partial x}$



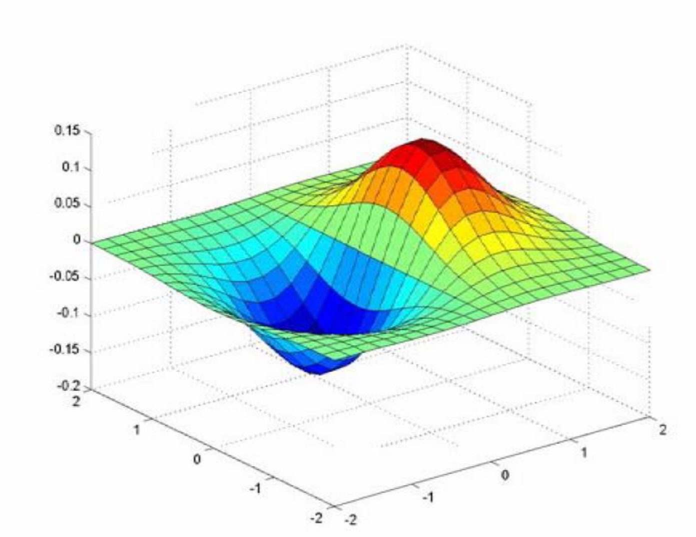
Derivative of Gaussian filter (along x)

$$(I * G) * H = I * (G * H)$$

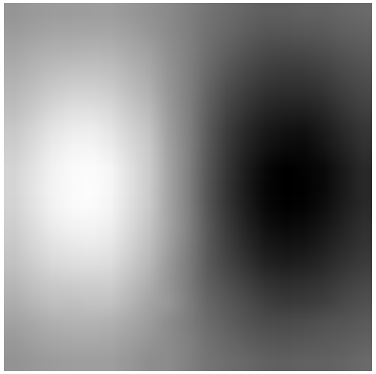
$$\begin{bmatrix} 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0219 & 0.0983 & 0.1621 & 0.0983 & 0.0219 \\ 0.0133 & 0.0596 & 0.0983 & 0.0596 & 0.0133 \\ 0.0030 & 0.0133 & 0.0219 & 0.0133 & 0.0030 \end{bmatrix} * \begin{bmatrix} 1 & -1 \end{bmatrix}$$

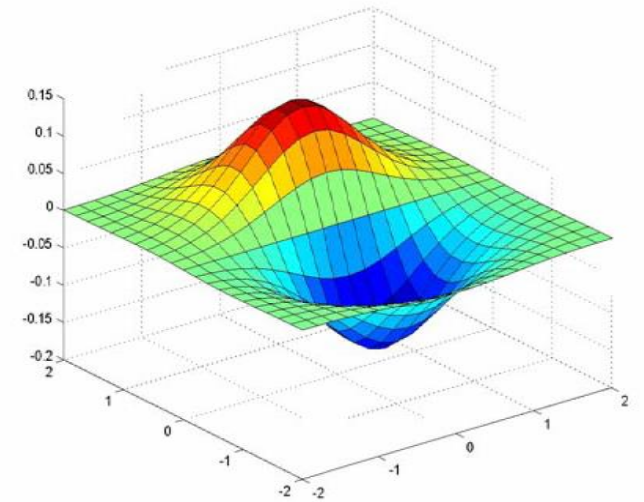


Derivative of Gaussian filters

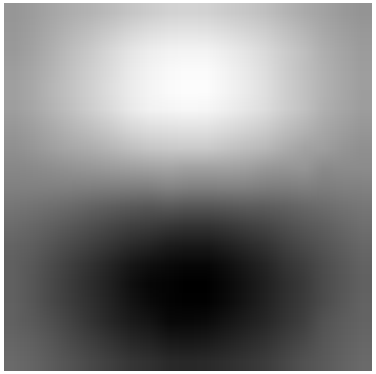


x-direction





y-direction



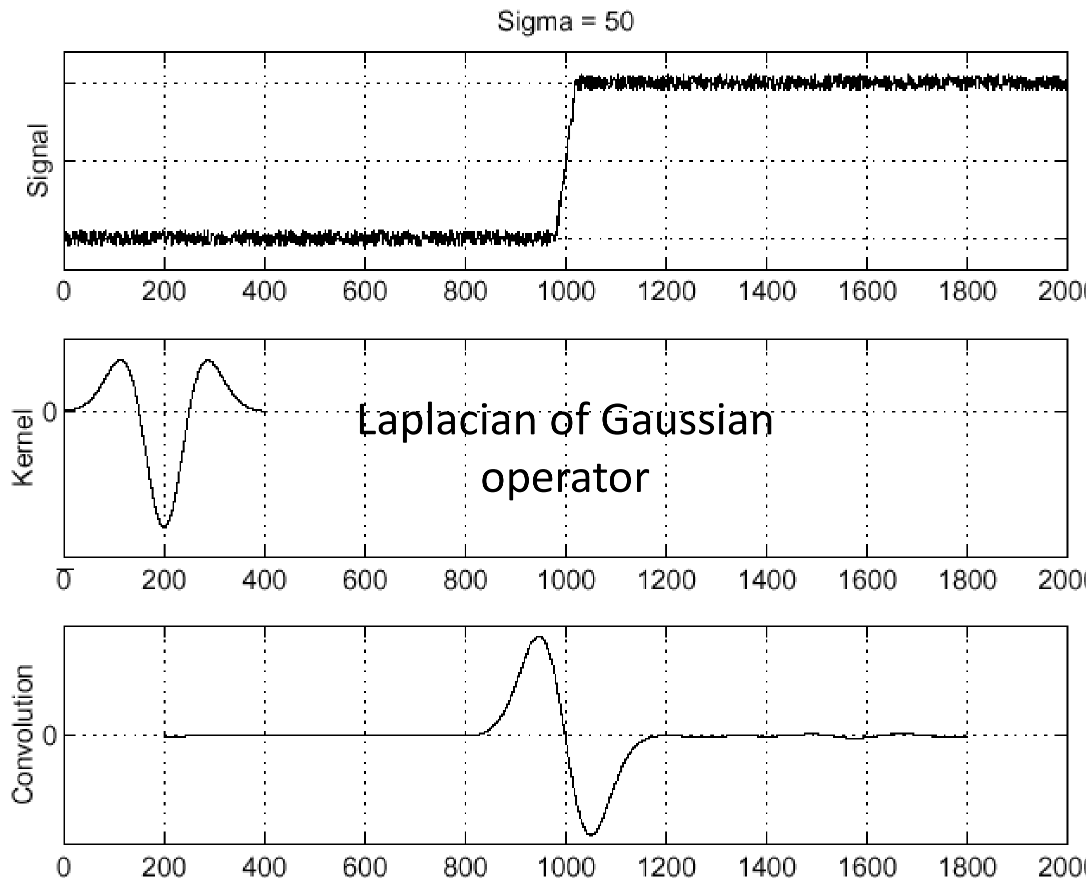
Laplacian of Gaussian

Consider $\frac{\partial^2}{\partial x^2}(F * H) = F * \frac{\partial^2 H}{\partial x^2}$

$F(x)$

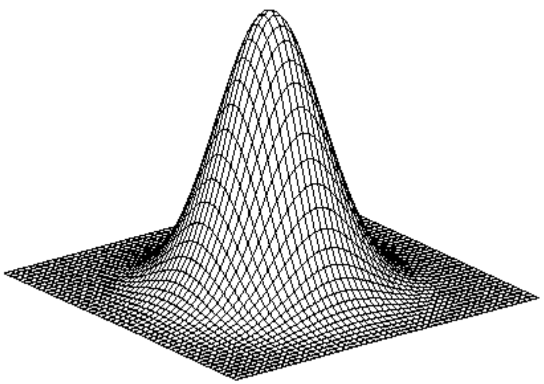
$\frac{\partial^2 H}{\partial x^2}$

$F * \frac{\partial^2 H}{\partial x^2}$



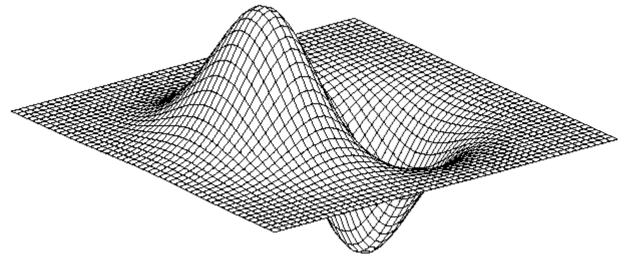
Where is the edge? Zero-crossings of bottom graph

2D edge detection filters



Gaussian

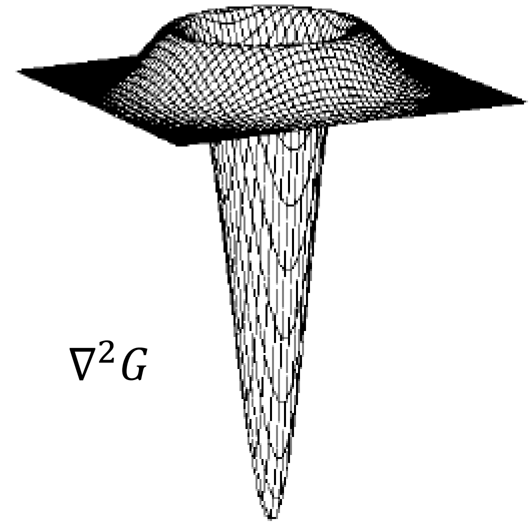
$$G = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$$



derivative of Gaussian

$$\frac{\partial G}{\partial x}$$

Laplacian of Gaussian



$$\nabla^2 G$$

- ∇^2 is the Laplacian operator: $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$

Summary on (linear) filters

- Smoothing filter:
  - **has positive values**
  - **sums to 1** → preserve brightness of constant regions
  - removes “high-frequency” components: “low-pass” filter
- Derivative filter:
  - **has opposite signs** used to get high response in regions of high contrast
  - **sums to 0** → no response in constant regions
  - **highlights “high-frequency”** components: “high-pass” filter

The Canny edge-detection algorithm (1986)

- Compute gradient of smoothed image in both directions
- Discard pixels whose gradient magnitude is below a certain threshold
- **Non-maximal suppression**: identify local maxima along gradient direction

The Canny edge-detection algorithm (1986)

Take a grayscale image. If not grayscale (i.g., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

Original image (Lenna image: https://en.wikipedia.org/wiki/Lenna)

The Canny edge-detection algorithm (1986)

Take a grayscale image. If not grayscale (i.g., RGB), convert it into a grayscale by replacing each pixel by the mean value of its R, G, B components.

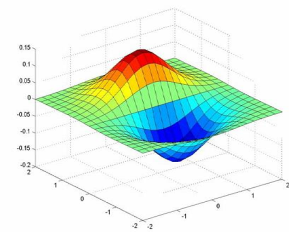
Original image (Lenna image: https://en.wikipedia.org/wiki/Lenna)

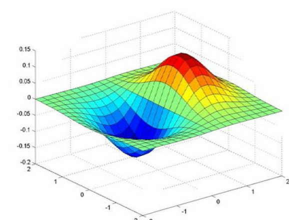
The Canny edge-detection algorithm (1986)

Convolve the image F with x and y derivatives of Gaussian filter

$$\frac{\partial F}{\partial x} = F * \frac{\partial G}{\partial x}$$

$$\frac{\partial F}{\partial y} = F * \frac{\partial G}{\partial y}$$

$\frac{\partial G}{\partial x}$ 

$\frac{\partial G}{\partial y}$ 

$$\|\nabla F\| = \sqrt{\left(\frac{\partial F}{\partial x}\right)^2 + \left(\frac{\partial F}{\partial y}\right)^2}$$: Edge strength

The Canny edge-detection algorithm (1986)

Thresholded $||\nabla F||$

Threshold it (i.e., set to 0 all pixels whose value is below a given threshold)

The Canny edge-detection algorithm (1986)

Take local maximum along gradient direction

Thinning: non-maxima suppression (local-maxima detection) along edge direction

Summary (things to remember)

- Image filtering (definition, motivation, applications)
- Moving average
- Linear filters and formulation: box filter, Gaussian filter
- Boundary issues
- Non-linear filters
  – Median filter and its applications
- Edge detection
  – Derivating filters (Prewitt, Sobel)
  – Combined derivative and smoothing filters (deriv. of Gaussian)
  – Laplacian of Gaussian
  – Canny edge detector
- Readings: Ch. 3.2, 4.2.1 of Szeliski book

Understanding Check

Are you able to:

- Explain the differences between convolution and correlation?
- Explain the differences between a box filter and a Gaussian filter?
- Explain why should one increase the size of the kernel of a Gaussian filter if is large (i.e. close to the size of the filter kernel?
- Explain when would we need a median filter?
- Explain how to handle boundary issues?
- Explain the working principle of edge detection with a $1D$ signal?
- Explain how noise does affect this procedure?
- Explain the differential property of convolution?
- Show how to compute the first derivative of an image intensity function along x and y?
- Explain why the Laplacian of Gaussian operator is useful?
- List the properties of smoothing and derivative filters?
- Illustrate the Canny edge detection algorithm?
- Explain what non-maxima suppression is and how it is implemented?