DWQP: A large scale box-quadratic programming solver.

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Joint work with

Victor Bittorf, Christopher Ré and Stephen J. Wright

One slide summary

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  - How large is large scale?
  - Current state of commercial solvers.

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  - Review convergence rates (serial and parallel) versions of SCD.
- Constraints: Implementation on multi-core processors.
  - Non uniform memory access (NUMA).
  - Processor affinity.
- Optimal solution: Our BQP solver can be 100x faster than commercial solvers. (Optimistic preliminary results).

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Solving *large* optimization problems:

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Compelling applications: learning from data, *low-accuracy LP*.

What is large scale?

Table: Solve times for commercial solvers using 32 cores on a dense box-constrained quadratic programs.

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Box constrained quadratic programs

Applications and algorithms

Problem description

Box constrained quadratic program

$$\min_{x \in \mathcal{R}^n} \frac{1}{2} x^\top Q x + p^\top x$$
$$\text{s.t.} \quad l_i \leq x_i \leq u_i \quad \forall i \in \{1, 2 \dots n\}$$

Some applications...

- Support vector machines.

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- and many more...

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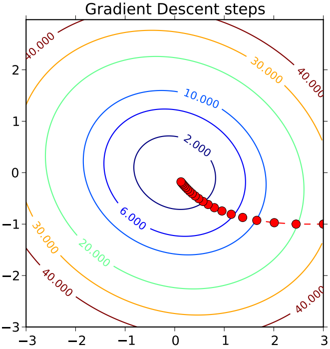
Full gradient based methods



Gradient descent

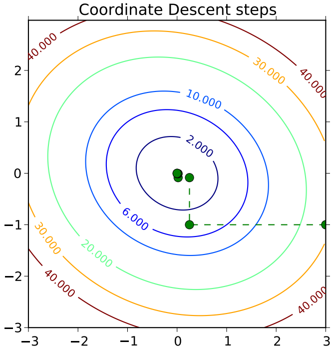
while not converged **do**
  for $i \in \{1, 2, \ldots, n\}$ **do**
    Compute $\nabla f = Qx + p$;
    $x \leftarrow \max(x - \alpha \nabla f, l)$;
    $x \leftarrow \min(x, u)$;
  end for
end while

Full gradients are expensive!

Stochastic coordinate descent (SCD)

SCD is ideal for large scale!

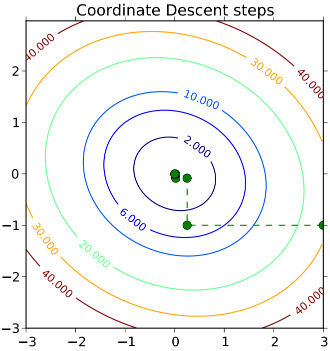
- Computing partial gradients are cheap.



Serial SCD

- Step 1: Compute the gradient ∇f_i along a single coordinate i.
- Step 2: Take a step along a single coordinate.
- Step 3: Projection to the feasible set of that coordinate $[l_i, u_i]$.

Stochastic coordinate descent (SCD)



SCD algorithm

while not converged **do**
  for $i \in \{1, 2, \ldots, n\}$ **do**
    Compute $\nabla f_i = Q_{i.}x + p_i$;
    $x_i \leftarrow \max(x_i - \nabla f_i / Q_{ii}, l_i)$;
    $x_i \leftarrow \min(x_i, u_i)$;
  end for
end while

- Nesterov (2012) showed that with high probability convergence of $f(\cdot)$ to within a specified threshold ϵ of $f(x^*)$ in about $O(1/k)$ iterations.
- Linear convergence, in expectation, when $f(\cdot)$ is strongly convex.

Parallel Stochastic co-ordinate descent (PSCD)

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Parallel SCD

while not converged **do**
  for $i \in \{1, 2, \ldots, n\}$ in parallel **do**
    Read the current state of x;
    Compute $\nabla f_i = Q_{i.}x + p_i$;
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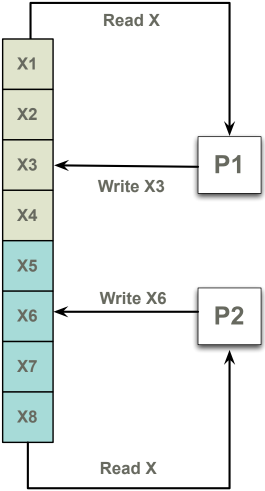

Asynchronous: HOGWILD! style

- Each core grabs the centrally-stored x and evaluates ∇f_i and then writes the updates back into x. (Niu, Ré, Recht, Wright, NIPS, 2011).

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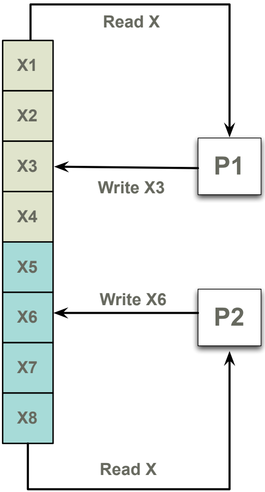

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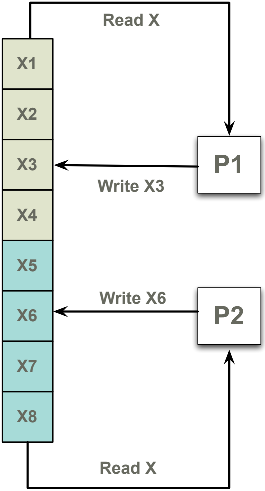

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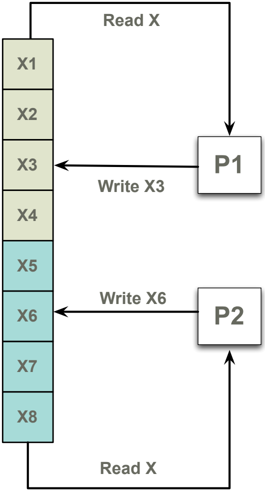

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- Processors don't overwrite each other's work!
- Asynchronous SCD analyzed by Richtarik and Takac (2012)

Implementation issues

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$Q = AA^T$: Lazy vs Eager

Dual SVM with linear kernel is a bound-constrained QP, with $Q = A^T A$. Each row of A is the feature vector for a single item of data.

- Eager: Q is precomputed.
- Lazy: Use A; don't compute Q explicitly.

In Lazy, the key operation at each SCD iteration is

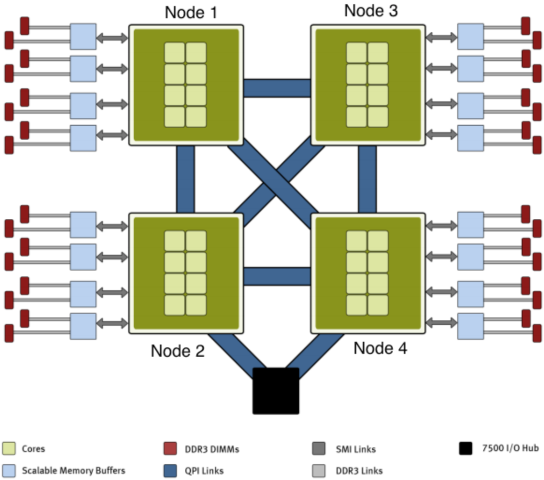
$$Q_{i\cdot} x = A_{\cdot i}^T A x.$$

With sparse A, implement this by

- compute $A_{j\cdot} x$ for those j for which $A_{ij} \neq 0$;

$$\sum_{j \,:\, A_{ij} \neq 0} A_{ij} (A_{j\cdot} x).$$

NUMA aware SCD: 4 socket, 40 cores



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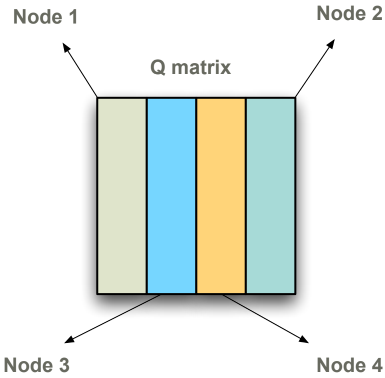

- Each SCD step requires access to only a single column of Q.
- Distribute columns of the Q matrix to separate cores.
- Each core accesses a pre-determined set of columns of Q.

Full-blown Eager

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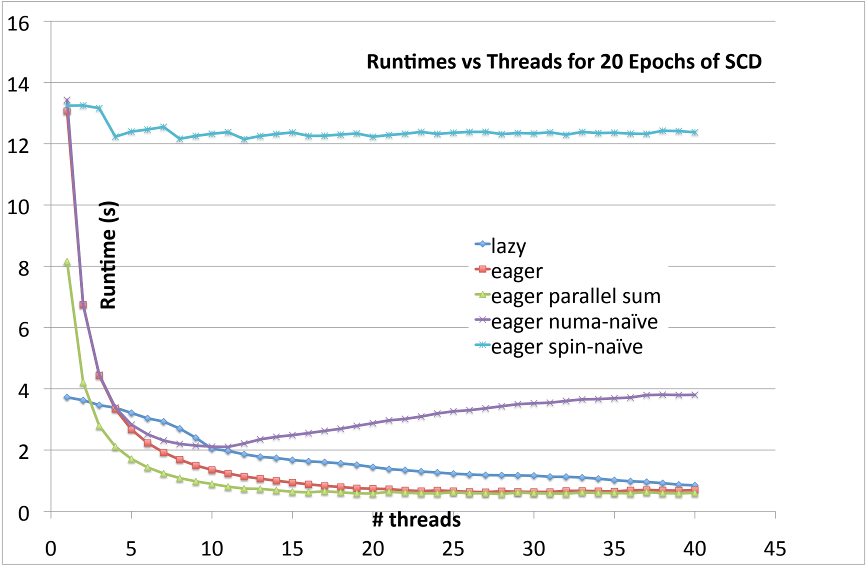
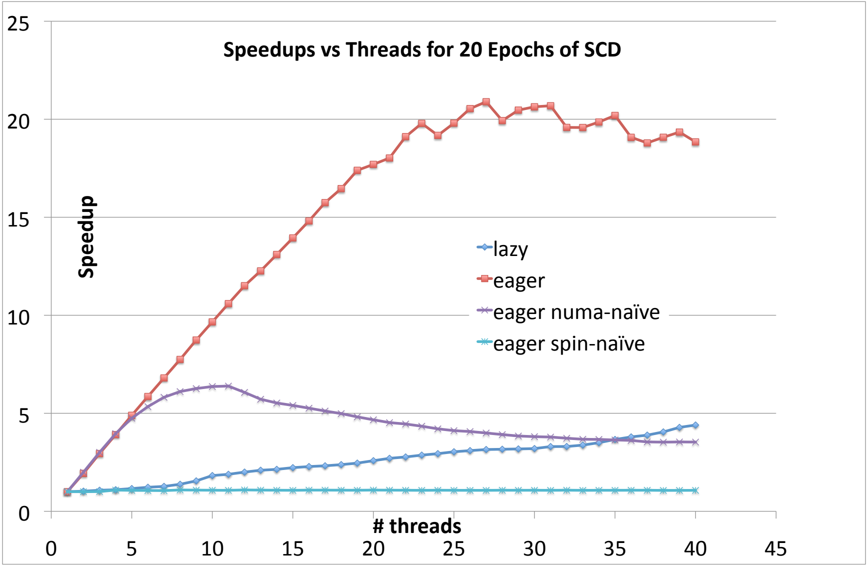
- When a core writes to x_i, the hardware ensures that this x_i is simultaneously removed from the cache of other cores.

Computational Experiments

Do 20 epochs of SCD on the problem with 1.2 GB of data ($n = 12596$).

- **Lazy**: Store A (on every socket), not Q.
- **Eager**: Algorithm described above, with Q precomputed (which takes approximately 6 seconds)
- **Eager: Spin-Naive**: Lock x while reading and writing.
- **Eager: NUMA-naive**: Cores select index i to update without regard to where $Q_{i\cdot}$ is stored — possibly need to fetch it from another socket.
- **Parallel Sum**: Speed limit: simply sum the elements of Q, 20 times. SCD "Eager" cannot be faster than this.


Runtimes vs Threads for 20 Epochs of SCD
Runtime (s)
threads
lazy
eager
eager parallel sum
eager numa-naïve
eager spin-naïve
0
2
4
6
8
10
12
14
16
0
5
10
15
20
25
30
35
40
45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Preliminary results: Comparison with commercial solvers

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Snow storm arriving. Time to go home.