Homework - 1

1. Calculate the efficiency of sequential search for average case. Assume input size is n.

We need to find the efficiency / no. of comparisons for average case which is the expected number of comparisons. To find the average case efficiency, we can assume that probability of finding the target at any position is equally likely.

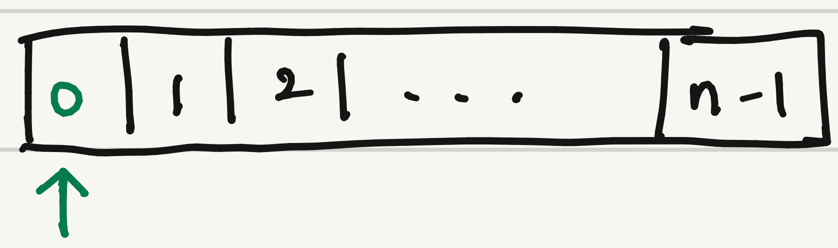
For an input size n, let $p[i] = \frac{1}{n}$ $\forall\ i = 0 : n-1$. Here i is the index of array of size n. Let $C[i]$ be the number of comparisons when target element is at i^{th} index.

$i = 0 \rightarrow C[0] = 1$

$i = 1 \rightarrow C[1] = 2$

$\vdots$

$i = n-1 \rightarrow C[n-1] = n$



In sequential search we iterate over each element in array.

$$\text{average efficiency} = \sum_{i=0}^{n-1} C[i]\, P[i]$$

$$\Rightarrow \frac{1}{n} \sum_{i=1}^{n-1} C[i] = \frac{1}{n}[1+2+\cdots+n]$$

$$= \frac{1}{n} \frac{(n)(n+1)}{2}$$

$$= \frac{n+1}{2}$$

$\therefore$ Average case efficiency $= \frac{n+1}{2}$ comparisons $= O(n)$.

3. Under what circumstances, when a searching operation is needed, would sequential search not be appropriate?

It is not appropriate to use sequential search when

- There are millions of elements in input array because searching is inefficient when input size increases since its time complexity is $O(n)$.

- If data is sorted, then 'Binary search' is more efficient than sequential search because of its $O(\log n)$ time complexity.

4. $f(n) = n^2 + 3n^3 \in \Theta(n^3)$.

To show that $f(n) \in \Theta(n^3)$, we need to prove

$$f(n) \in O(n^3) \quad \text{and} \quad f(n) \in \Omega(n^3)$$

(i) $f(n) = n^2 + 3n^3 \in O(n^3)$

We need to show that $\exists\, n_0, c \;\; \forall n \geq n_0 \;\; n^2 + 3n^3 \leq c \cdot n^3$

$$n^2 + 3n^3 \leq cn^3$$
$$(c-3)n^3 \geq n^2$$

For $c = 4$, $n = 1$ $\rightarrow$ $(4-3)\,1^3 \geq 1^2$

$1 \geq 1$ (True)

$\therefore f(n) \in O(n^3)$

(ii) $f(n) = n^2 + 3n^3 \in \Omega(n^3)$

We need to show that $\exists\, n_0, c \;\; \forall n \geq n_0 \;\; n^2 + 3n^3 \geq c \cdot n^3$

$$n^2 + 3n^3 \geq cn^3$$

$$n^2 \geq (c-3)n^3$$

$c = 0,\ n = 1$

$$1^2 \geq -3(1)^3$$

$$1 \geq -3 \quad \text{(True)}$$

$\therefore f(n) \in \Omega(n^3)$

Since, $f(n) \in O(n^3)$ and $f(n) \in \Omega(n^3)$ we can say that $f(n) \in \Theta(n^3) \rightarrow$ average case.

2. For the second question, I am attaching the output photos in this pdf and uploaded the .py on canvas.

```
…/courses/spring_2026/infsci_2591_ad                                   (infsci_2591_ad)
→ python hw1/hw1.py
For Test Case [1, 2, 3, 4]
[1, 2, 3]
[1, 2, 4]
[1, 3, 4]
[2, 3, 4]
Total subsets found = 4


For Test Case [7, 3]
Cannot print subsets of 3 if input data size less than 3
Total subsets = 0
```

```
For Test Case [4, 1, 7, 4, 3, 9, 1, 5]
[4, 1, 7]
[4, 1, 4]
[4, 1, 3]
[4, 1, 9]
[4, 1, 1]
[4, 1, 5]
[4, 7, 4]
[4, 7, 3]
[4, 7, 9]
[4, 7, 1]
[4, 7, 5]
[4, 4, 3]
[4, 4, 9]
[4, 4, 1]
[4, 4, 5]
[4, 3, 9]
[4, 3, 1]
[4, 3, 5]
[4, 9, 1]
[4, 9, 5]
[4, 1, 5]
[1, 7, 4]
[1, 7, 3]
[1, 7, 9]
[1, 7, 1]
[1, 7, 5]
[1, 4, 3]
[1, 4, 9]
[1, 4, 1]
[1, 4, 5]
[1, 3, 9]
[1, 3, 1]
[1, 3, 5]
[1, 9, 1]
[1, 9, 5]
[1, 1, 5]
[7, 4, 3]
[7, 4, 9]
[7, 4, 1]
[7, 4, 5]
[7, 3, 9]
[7, 3, 1]
[7, 3, 5]
[7, 9, 1]
[7, 9, 5]
[7, 1, 5]
[4, 3, 9]
[4, 3, 1]
[4, 3, 5]
[4, 9, 1]
[4, 9, 5]
[4, 1, 5]
[3, 9, 1]
[3, 9, 5]
[3, 1, 5]
[9, 1, 5]
Total subsets found = 56
```