



4

Event Detection

Biomedical signals carry signatures of physiological events.

Epoch: part of a signal related to a specific event.

The analysis of a signal requires the identification of epochs and investigation of the corresponding events.



4.1 Problem Statement

Given a biomedical signal, identify discrete signal epochs and correlate them with events in the related physiological process.

Solution: Development of signal processing techniques to emphasize, detect, segment, and analyze epochs.



4.2 Illustration of the Problem with Case-studies

4.2.1 *The P, QRS, and T waves in the ECG*

Epochs in an ECG waveform:

- **The P wave:**

Contraction of the atria triggered by the SA node.

- **The PQ segment:**

AV node provides delay to facilitate completion of atrial contraction and transfer of blood to ventricles before ventricular contraction is initiated.

Normally flat or iso-electric.



- **The QRS wave:**

Purkinje fibers stimulate contraction of ventricles.

- **The ST segment:**

Normally flat or iso-electric – related to the plateau in action potential of left ventricular muscle cells.

Myocardial ischemia or infarction could change the action potentials of left ventricular musculature:

ST segment depressed or elevated.



- **The T wave:**

Related to the last phase of the action potential of ventricular muscle cells: ventricular repolarization or relaxation.



4.2.2 *The first and second heart sounds*

- **The first heart sound S1:**

S1 reflects a sequence of events related to ventricular contraction —

closure of the atrio-ventricular valves,

isovolumic contraction,

opening of the semilunar valves, and

ejection of the blood from the ventricles.



- **The second heart sound S2:**

S2 is related to the end of ventricular contraction — closure of the aortic and pulmonary valves (A2 and P2).



▪ Murmurs:

Systolic murmur of aortic stenosis —
turbulent ejection of blood from the left ventricle
through a restricted opening of the aortic valve.

Diastolic murmur in case of aortic insufficiency —
regurgitation of blood from the aorta back into the
left ventricle through a leaky aortic valve.



4.2.3 *The dicrotic notch in the carotid pulse*

Closure of the aortic valve causes a sudden drop in aortic pressure on a downward slope at end of systole.

Dicrotic notch in the carotid pulse is a delayed, upstream manifestation of the incisura in the aortic pressure wave.



4.2.4 EEG rhythms, waves, and transients

EEG rhythms: α , β , δ , and θ waves.

Additional waves and events:

- **K-complex:** transient waveform with slow waves, sometimes associated with sharp components, often followed by 14 Hz waves.

Occurs spontaneously or in response to a sudden stimulus during sleep; amplitude $\sim 200 \mu V$.

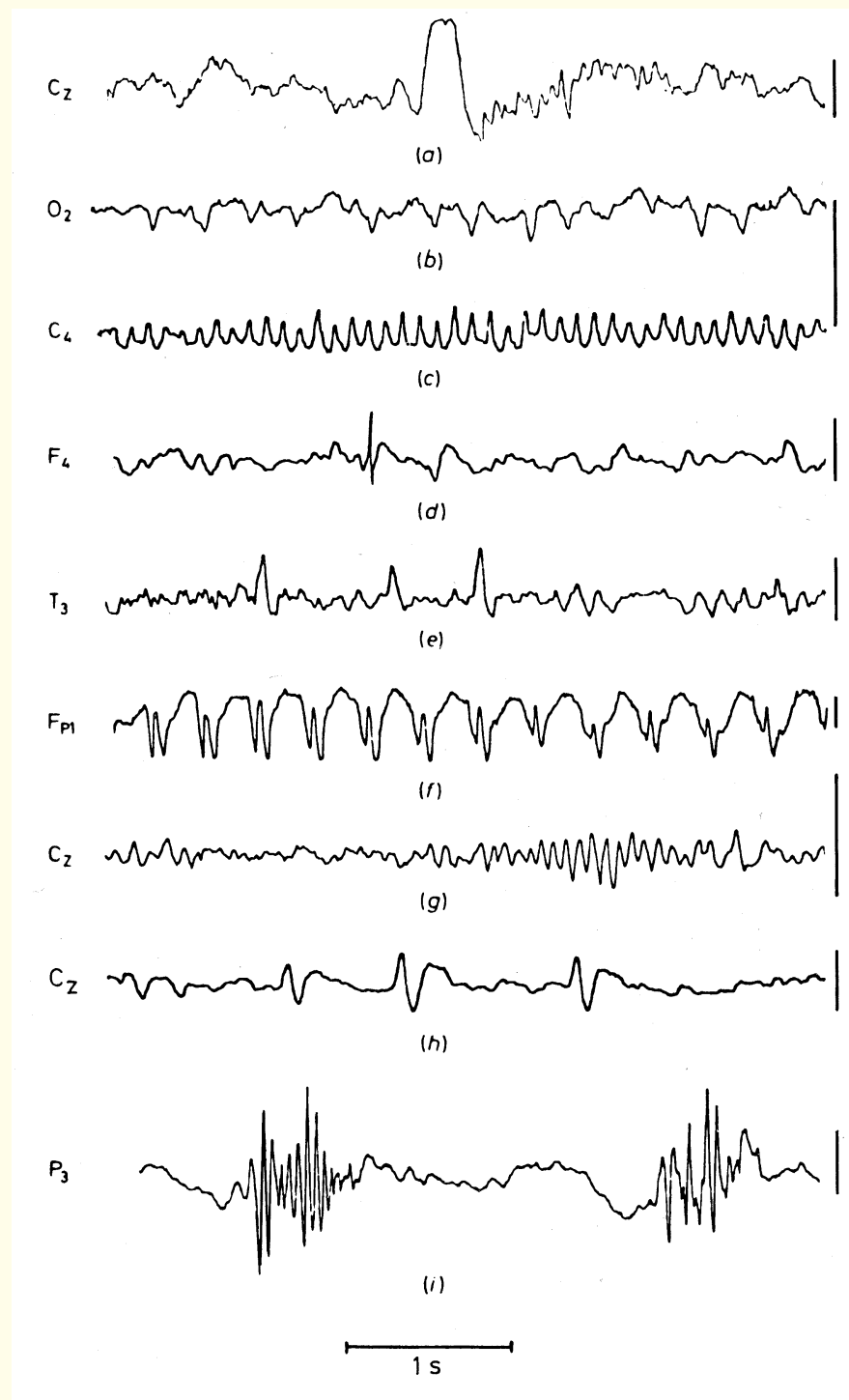




Figure 4.1: From top to bottom: (a) the K-complex; (b) the lambda wave; (c) the mu rhythm; (d) a spike; (e) sharp waves; (f) spike-and-wave complexes; (g) a sleep spindle; (h) vertex sharp waves; and (i) polyspike discharges. The horizontal bar at the bottom indicates a duration of 1 s; the vertical bars at the right indicate 100 μV . Reproduced with permission from R. Cooper, J.W. Osselton, and J.C. Shaw, EEG Technology, 3rd Edition, 1980. ©Butterworth Heinemann Publishers, a division of Reed Educational & Professional Publishing Ltd., Oxford, UK.



- **Lambda waves:** monophasic, positive, sharp waves;
occur in the occipital location with amplitude $< 50 \mu V$.
Related to eye movement;
associated with visual exploration.
- **Mu rhythm:** group of waves, $7 - 11 Hz$,
arcade or comb shape in the central location.
Amplitude $< 50 \mu V$.
Attenuated by contralateral movement,
thought of movement,
readiness to move, or tactile stimulation.



- **Spike:** transient with a pointed peak, duration $20 - 30 \text{ ms}$.
- **Sharp wave:** transient with a pointed peak, longer duration than a spike of $70 - 200 \text{ ms}$.
- **Spike-and-wave rhythm:** sequence of surface-negative slow waves, $2.5 - 3.5 \text{ Hz}$, spike associated with each wave.
Spike amplitude up to $1,000 \mu\text{V}$ in each complex.



- **Sleep spindle:** episodic rhythm, 14 Hz , $50\text{ }\mu\text{V}$, occurring maximally over the fronto-central regions during certain stages of sleep.
- **Vertex sharp transient** or **V-wave:**
sharp potential maximal at the vertex at about $300\text{ }\mu\text{V}$, negative in relation to the EEG in other areas.
Occurs spontaneously during sleep or in response to a sensory stimulus during sleep or wakefulness.



EEG record is described in terms of:

- most persistent rhythm (for example, α);
- presence of other rhythmic features, such as δ , θ , or β ;
- discrete features of relatively long duration, such as an episode of spike-and-wave activity;



- discrete features of relatively short duration, such as isolated spikes or sharp waves;
- activity remaining when all of the above have been described: background activity; and
- artifacts; ambiguity in interpretation.



Each EEG wave or activity is described

in chronological sequence

in terms of amplitude;

frequency, in the case of rhythmic features;

waveform, for both rhythmic and transient features;



location or spatial distribution;

incidence or temporal variability;

right – left symmetry in location of activity;

responsiveness to stimuli, e.g., eye opening and closure.



EEG record at rest is first described as above;

the effects of evocative techniques

are then specified in the same terms.

Behavioral changes, such as the subject

becoming drowsy or falling asleep, are also noted.



4.3 Detection of Events and Waves

4.3.1 Derivative-based methods for QRS detection

Problem: *Develop signal processing techniques*

to facilitate detection of the QRS complex,

given that it is the sharpest wave in an ECG cycle.



Solution 1: QRS complex has the largest slope

or rate of change of voltage in a cardiac cycle —

rapid conduction and depolarization of the ventricles.

Rate of change: derivative operator $\frac{d}{dt}$.

The derivative operator enhances the QRS;

the result has no resemblance to a typical QRS complex.

The slow P and T waves are suppressed.

Result is noisy: significant smoothing required.



Algorithm of Balda et al. for QRS detection:

Smoothed three-point first derivative $y_0(n)$

of the given signal $x(n)$ approximated as

$$y_0(n) = |x(n) - x(n - 2)|. \quad (4.1)$$

The second derivative approximated as

$$y_1(n) = |x(n) - 2x(n - 2) + x(n - 4)|. \quad (4.2)$$



The two results are weighted and combined to obtain

$$y_2(n) = 1.3 y_0(n) + 1.1 y_1(n). \quad (4.3)$$

The result $y_2(n)$ is scanned with a threshold of 1.0.

Whenever threshold is crossed, the subsequent eight samples are also tested against the same threshold.

If at least six of the eight points pass the threshold test, the segment of eight samples taken to be a part of a QRS.

Result: pulse with width proportional to QRS width.



Illustration of application: Figure 4.2.

Two cycles of a filtered version of the

ECG signal shown in Figure 3.5:

eighth-order Butterworth lowpass filter, $f_c = 90 \text{ Hz}$,

down-sampled by a factor of five,

notch filter with $f_o = 60 \text{ Hz}$.

Effective sampling rate is 200 Hz .

Signal normalized by dividing by its maximum value.

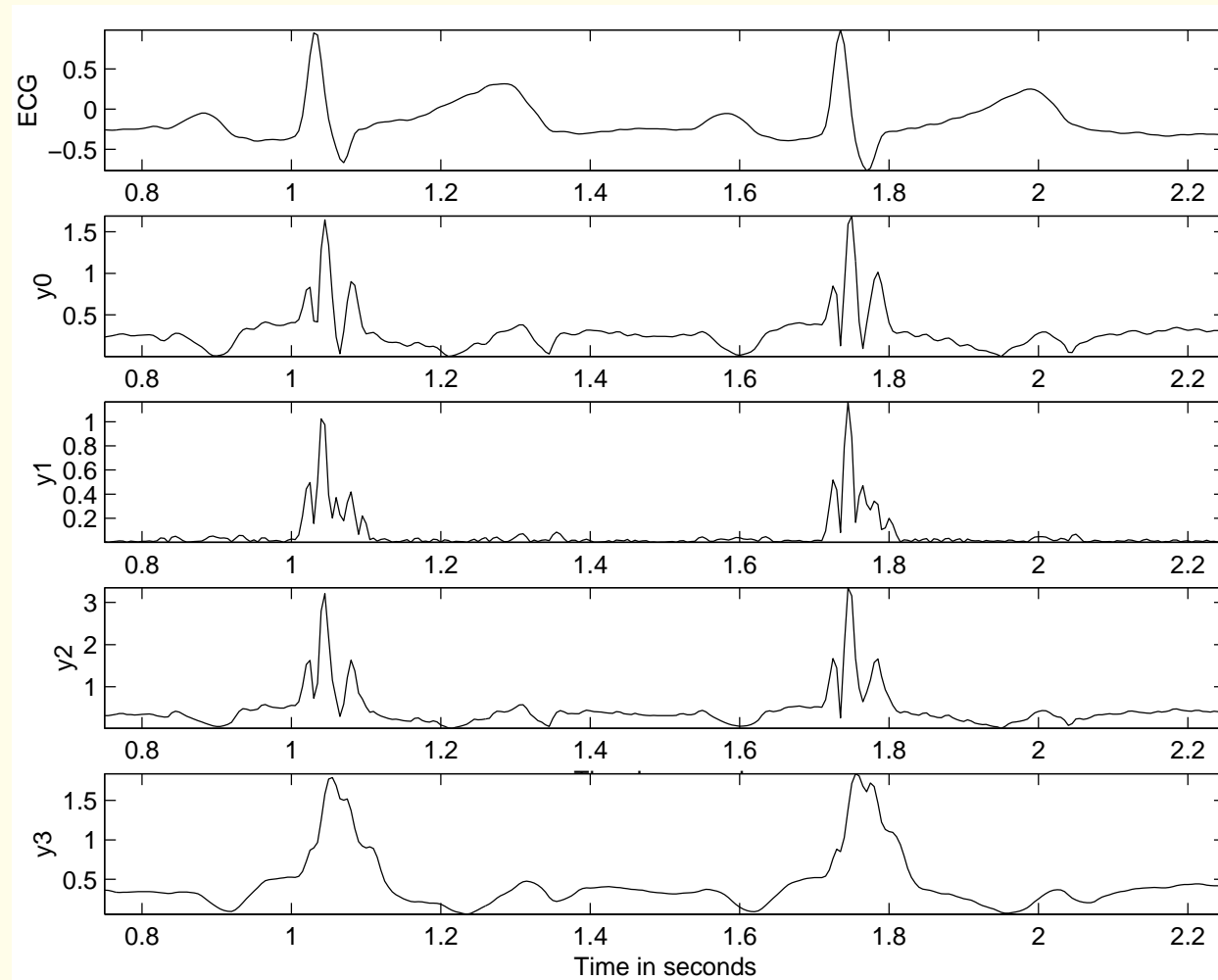


Figure 4.2: From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5; output $y_0(n)$ of the first-derivative-based operator in Equation 4.1; output $y_1(n)$ of the second-derivative-based operator in Equation 4.2; the combined result $y_2(n)$ from Equation 4.3; and the result $y_3(n)$ of passing $y_2(n)$ through the 8-point MA filter in Equation 3.27.



Solution 2:

Algorithm of Murthy and Rangaraj —

squared first-derivative and MA filter.

$$g_1(n) = \sum_{i=1}^N |x(n-i+1) - x(n-i)|^2 (N-i+1), \quad (4.4)$$

$$g(n) = \frac{1}{M} \sum_{j=0}^{M-1} g_1(n-j). \quad (4.5)$$

Sampling rate 100 Hz , $M = N = 8$.



Peak-searching algorithm:

1. Scan a portion of the signal $g(n)$ expected to contain a peak; determine maximum value g_{\max} .
2. Define a threshold as a fraction of the maximum, for example, $Th = 0.5 g_{\max}$.
3. For all $g(n) > Th$, select those samples for which the corresponding $g(n)$ values are greater than a predefined number M of preceding and succeeding samples of $g(n)$:

$$\begin{aligned} \{p\} = [n \mid g(n) > Th] \text{ AND} & \quad (4.6) \\ [g(n) > g(n-i), i = 1, 2, \dots, M] \text{ AND} & \\ [g(n) > g(n+i), i = 1, 2, \dots, M]. & \end{aligned}$$

The set $\{p\}$ contains the indices of the peaks in $g(n)$.



Illustration of application: Figure 4.3.

Two cycles of a filtered version of the

ECG signal shown in Figure 3.5:

eighth-order Butterworth lowpass filter, $f_c = 40 \text{ Hz}$,

down-sampled by a factor of ten.

Effective sampling rate is 100 Hz .

Signal normalized by dividing by its maximum value.

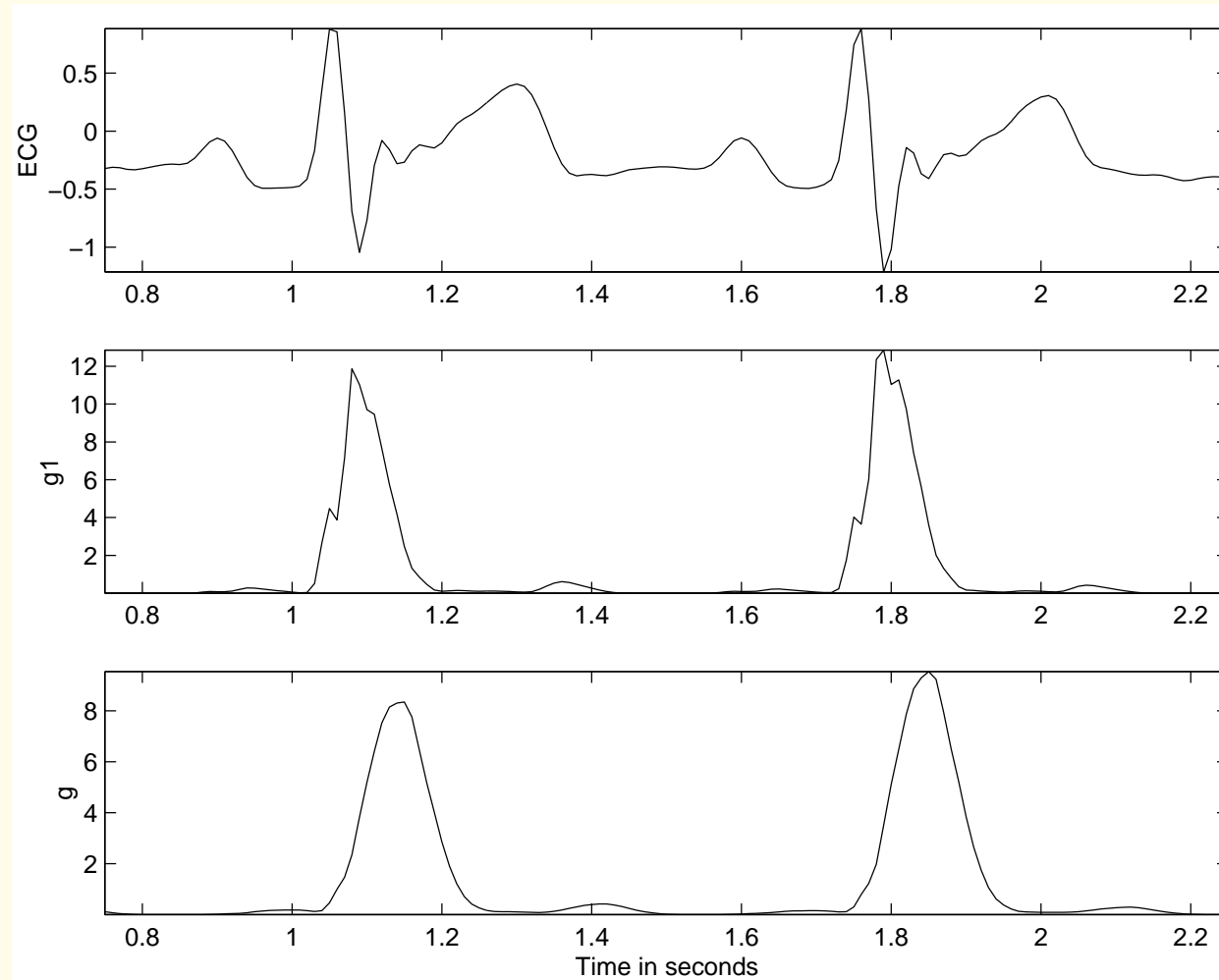


Figure 4.3: From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5; output $g_1(n)$ of the weighted and squared first-derivative operator in Equation 4.4; output $g(n)$ of the smoothing filter in Equation 4.5.



4.3.2 The Pan-Tompkins algorithm for QRS detection

Problem: *Propose an algorithm to detect QRS complexes in an ongoing ECG signal.*

Solution: Pan-Tompkins real-time QRS detector.

Analysis of slope, amplitude, and width of QRS complexes.

Algorithm includes a series of filters and methods:

lowpass, highpass, derivative, squaring, integration,

adaptive thresholding, and search procedures.

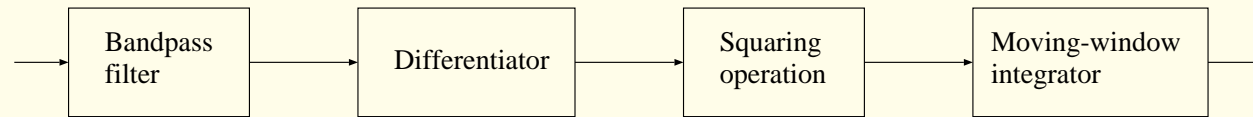


Figure 4.4: Block diagram of the Pan-Tompkins algorithm for QRS detection.



Lowpass filter: Recursive filter with integer coefficients.

$$H(z) = \frac{1}{32} \frac{(1 - z^{-6})^2}{(1 - z^{-1})^2}. \quad (4.7)$$

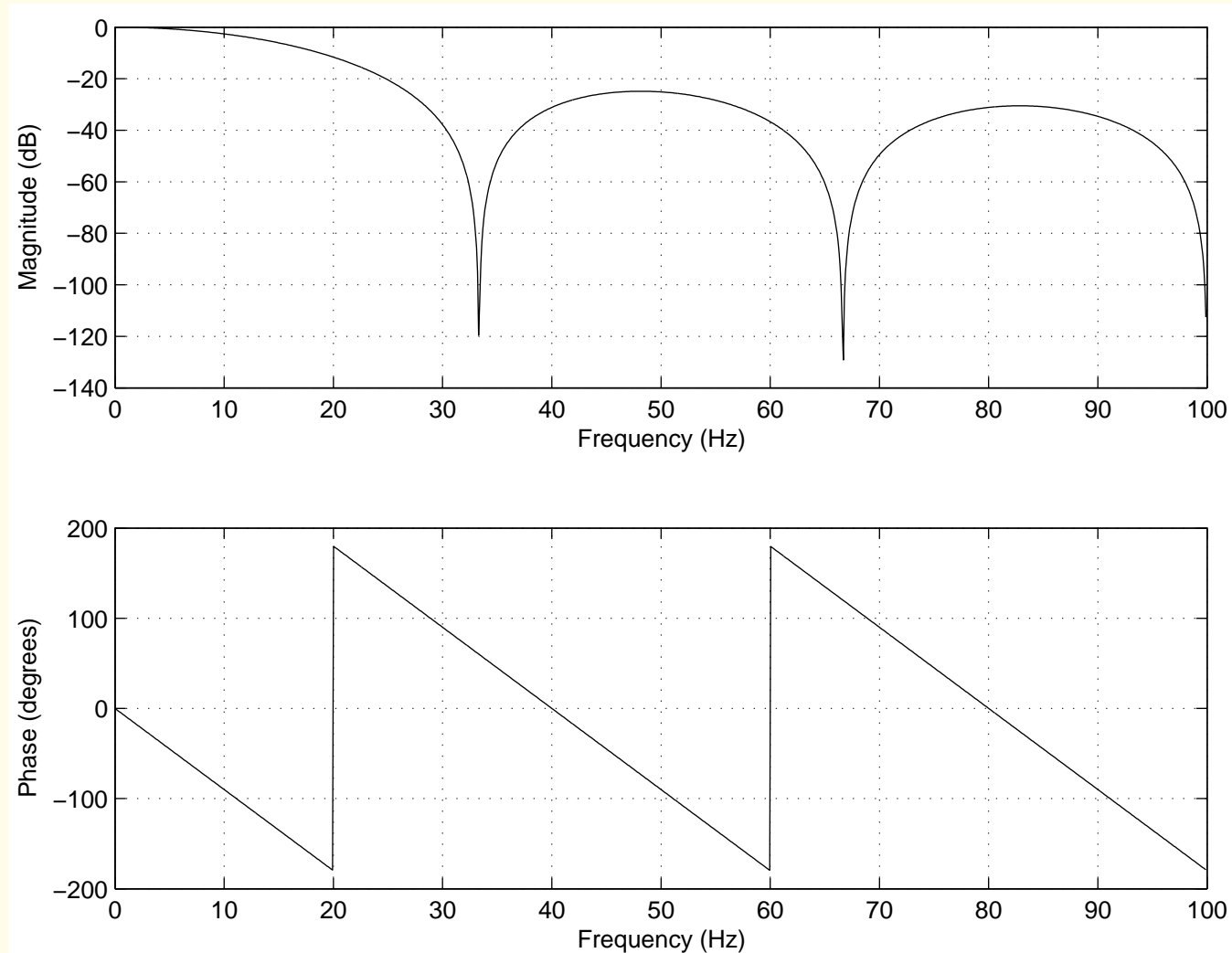
$$y(n) = 2y(n-1) - y(n-2) + \frac{1}{32} [x(n) - 2x(n-6) + x(n-12)]. \quad (4.8)$$

Sampling rate 200 Hz , $f_c = 11 \text{ Hz}$,

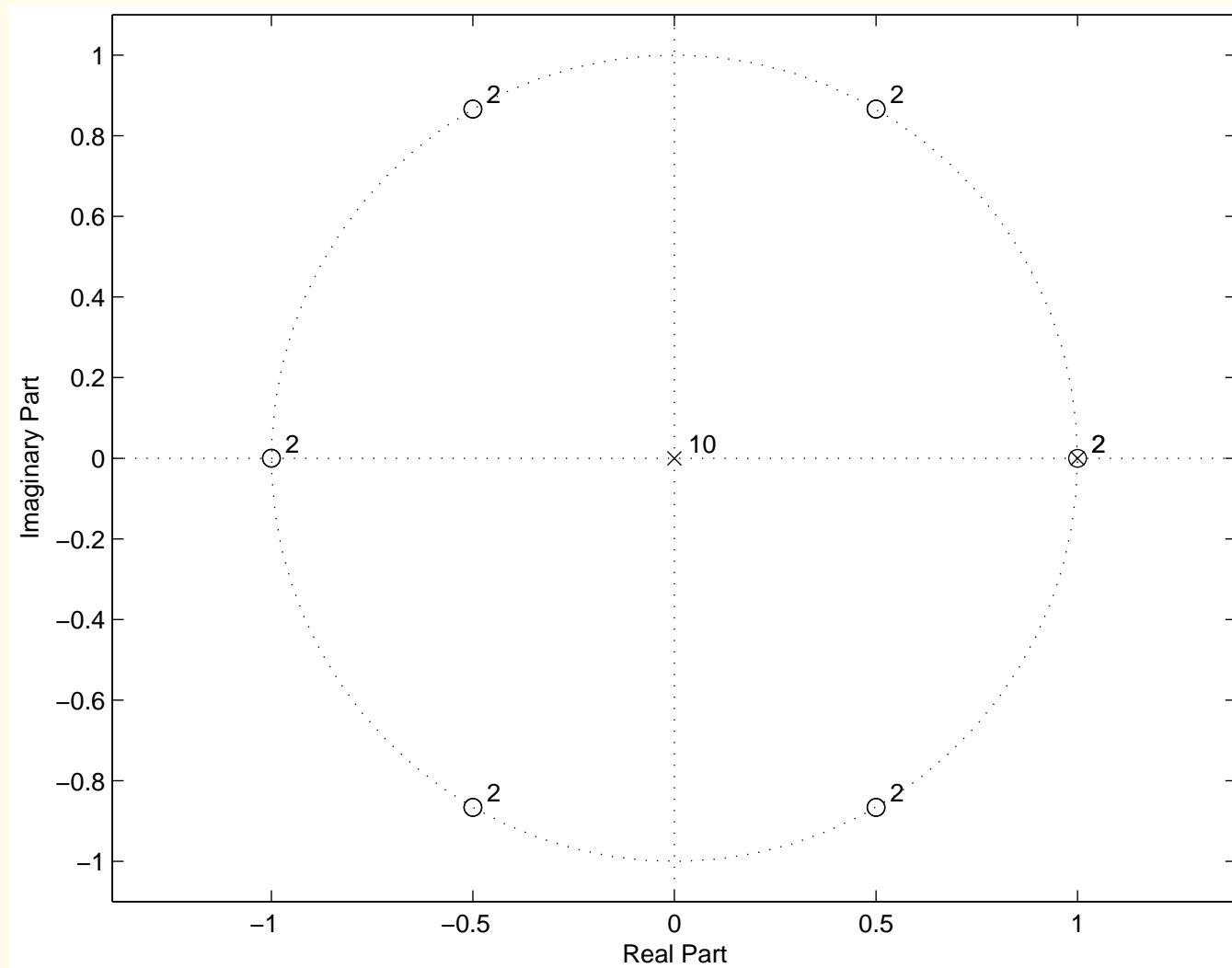
delay of 5 samples or 25 ms .



Attenuation greater than 35 dB at 60 Hz ,
effectively suppresses power-line interference.



Frequency response of the lowpass filter in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the lowpass filter in the Pan-Tompkins algorithm for QRS detection.



Highpass filter: allpass filter minus a lowpass filter.

Lowpass component:

$$H_{lp}(z) = \frac{(1 - z^{-32})}{(1 - z^{-1})}; \quad (4.9)$$

$$y(n) = y(n - 1) + x(n) - x(n - 32). \quad (4.10)$$



Transfer function $H_{hp}(z)$ of the highpass filter:

$$H_{hp}(z) = z^{-16} - \frac{1}{32} H_{lp}(z) . \quad (4.11)$$



Output $p(n)$ of the highpass filter:

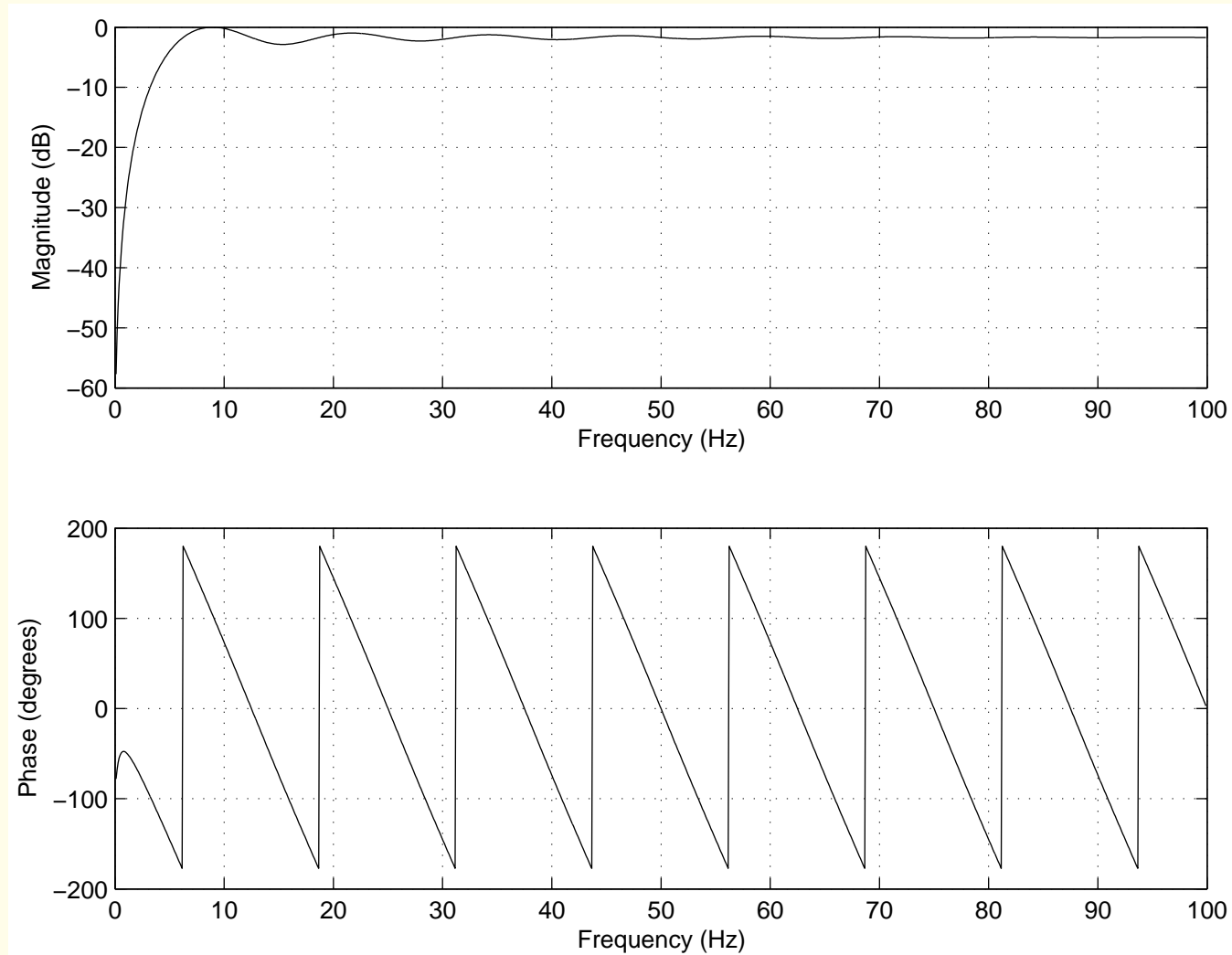
$$p(n) = x(n-16) - \frac{1}{32} [y(n-1) + x(n) - x(n-32)], \quad (4.12)$$

$x(n)$ and $y(n)$ related as in Equation 4.10.

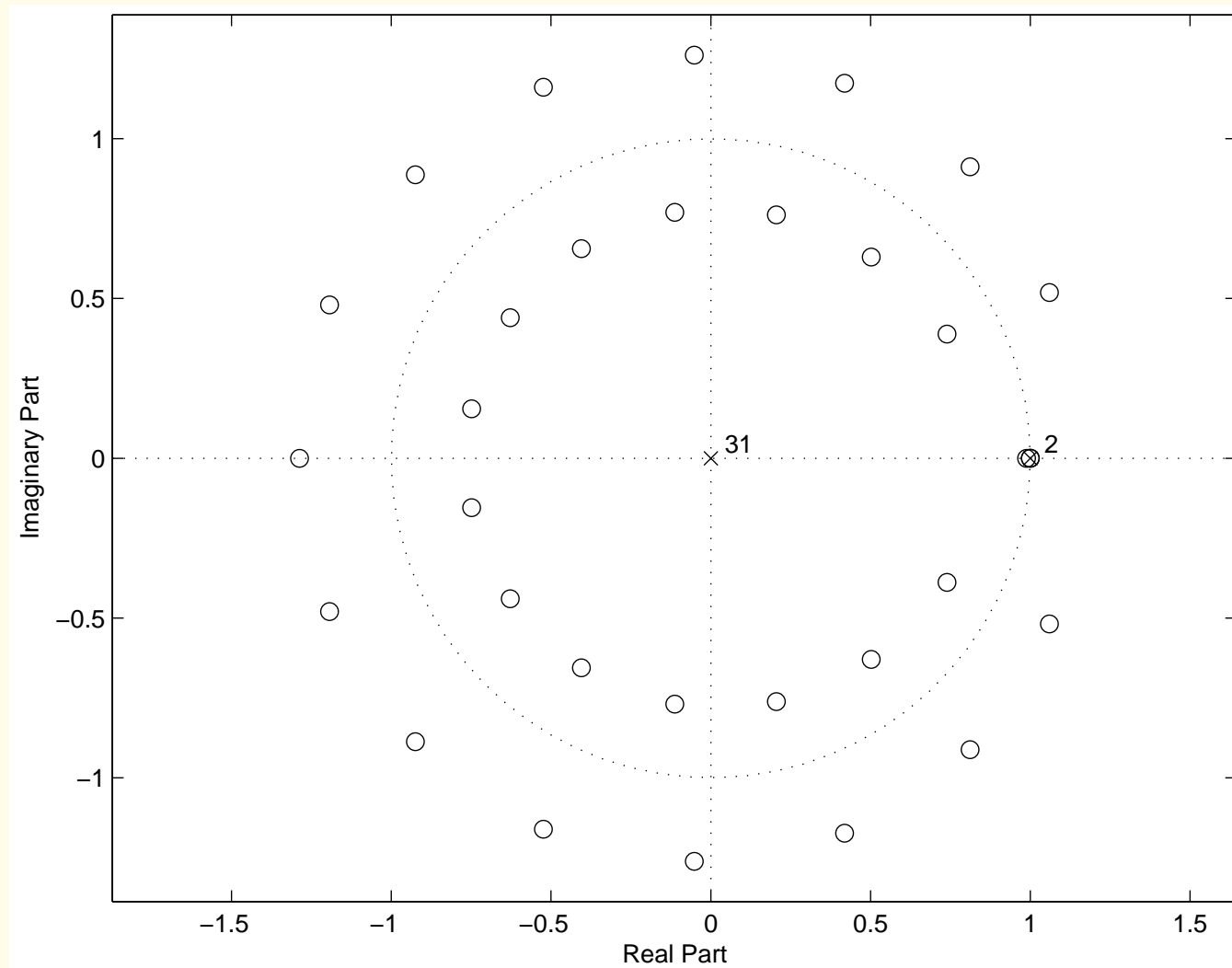
Input-output relationship of the overall highpass filter:

$$p(n) = p(n-1) - \frac{1}{32}x(n) + x(n-16) - x(n-17) + \frac{1}{32}x(n-32).$$

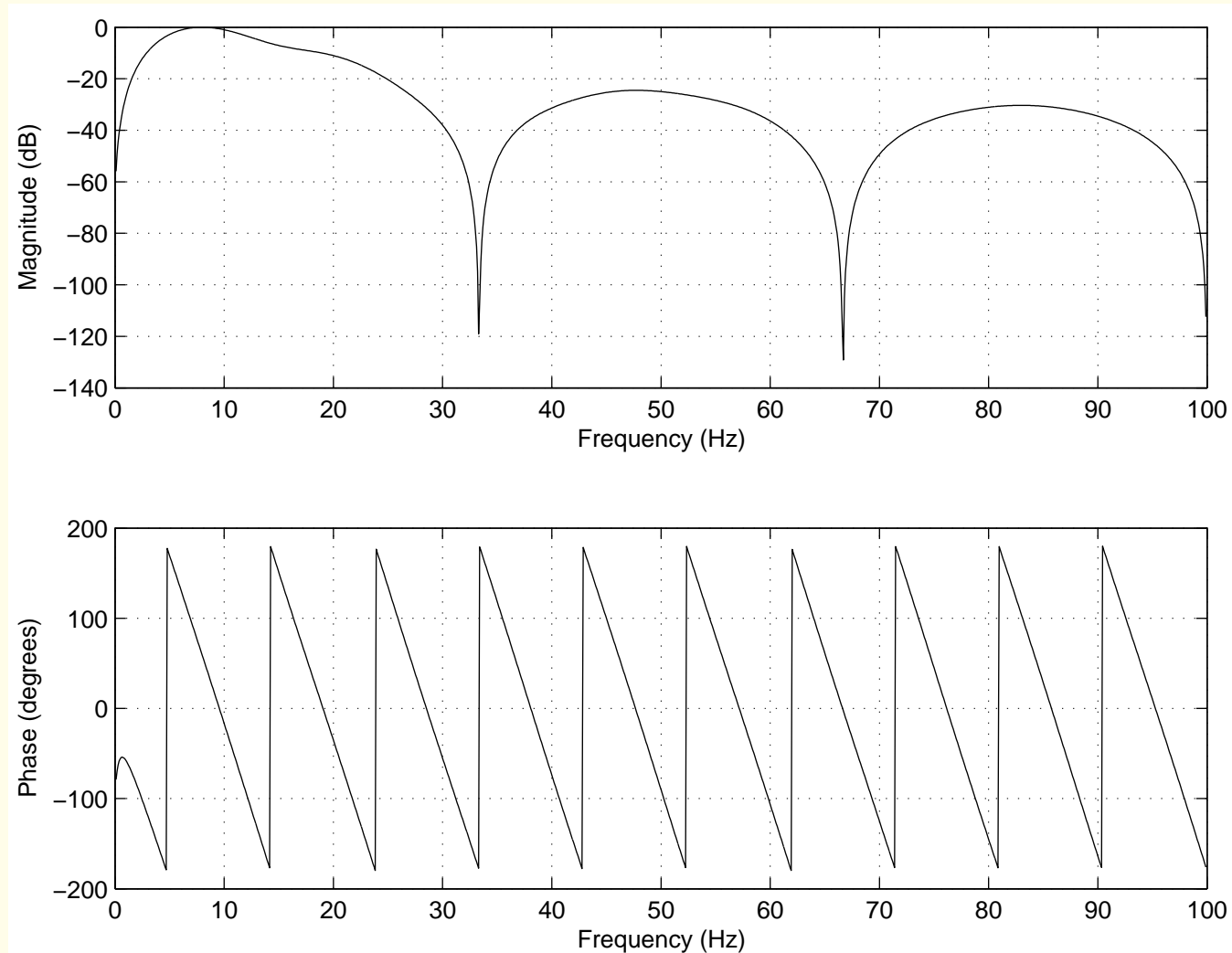
Highpass cutoff 5 Hz , delay of 80 ms .



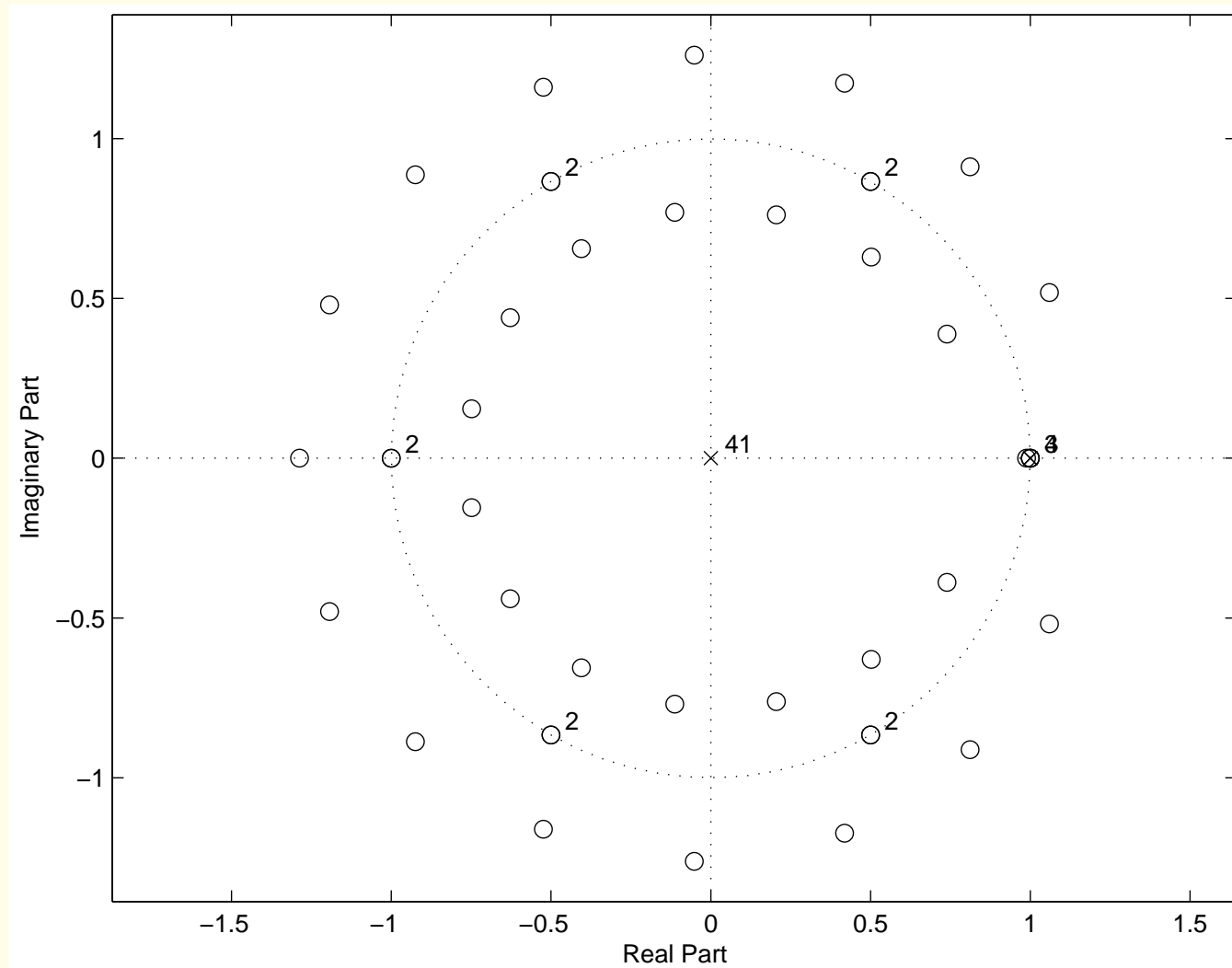
Frequency response of the highpass filter in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the highpass filter in the Pan-Tompkins algorithm for QRS detection.



Frequency response of the bandpass filter in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the bandpass filter in the Pan-Tompkins algorithm for QRS detection.

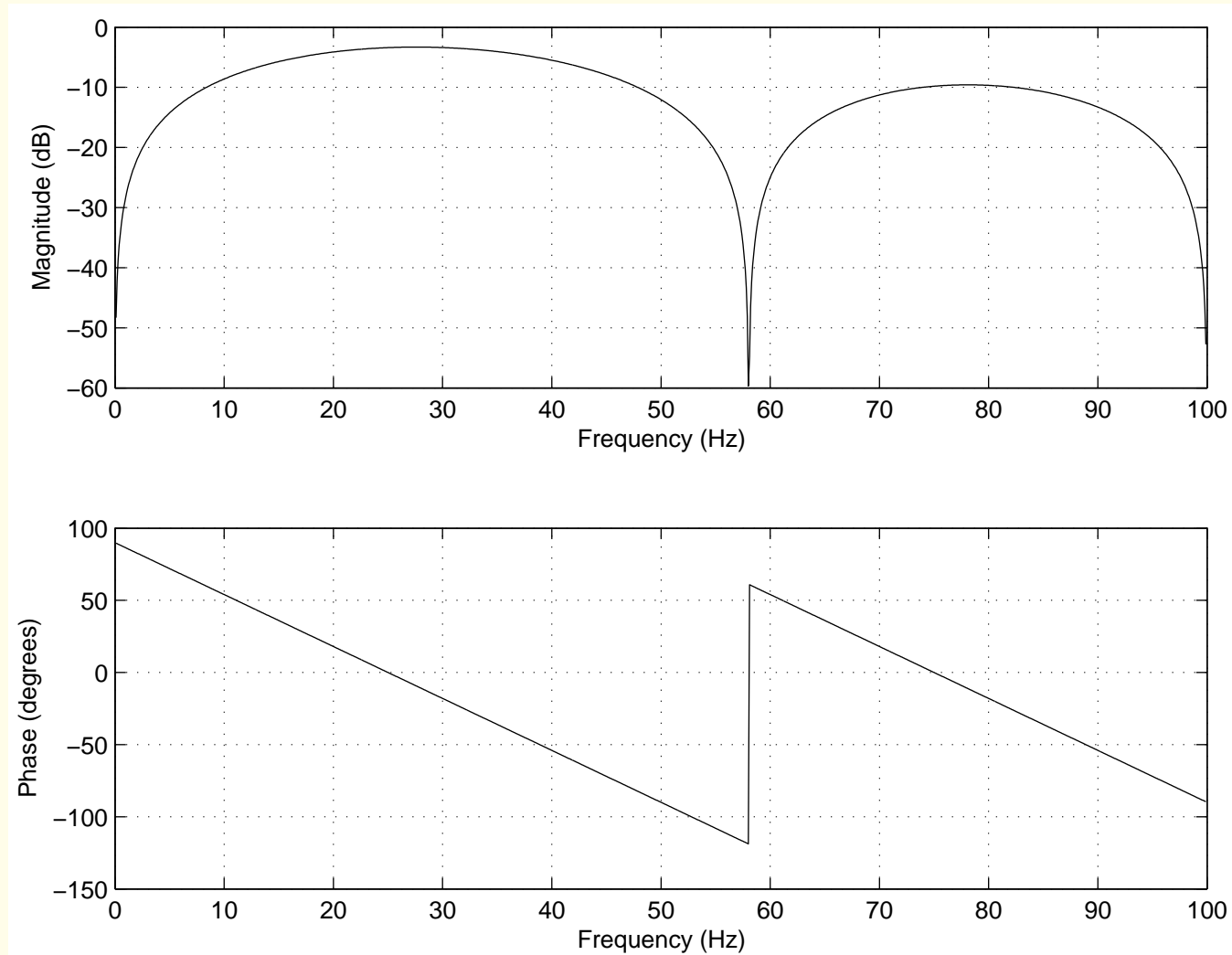


Derivative operator:

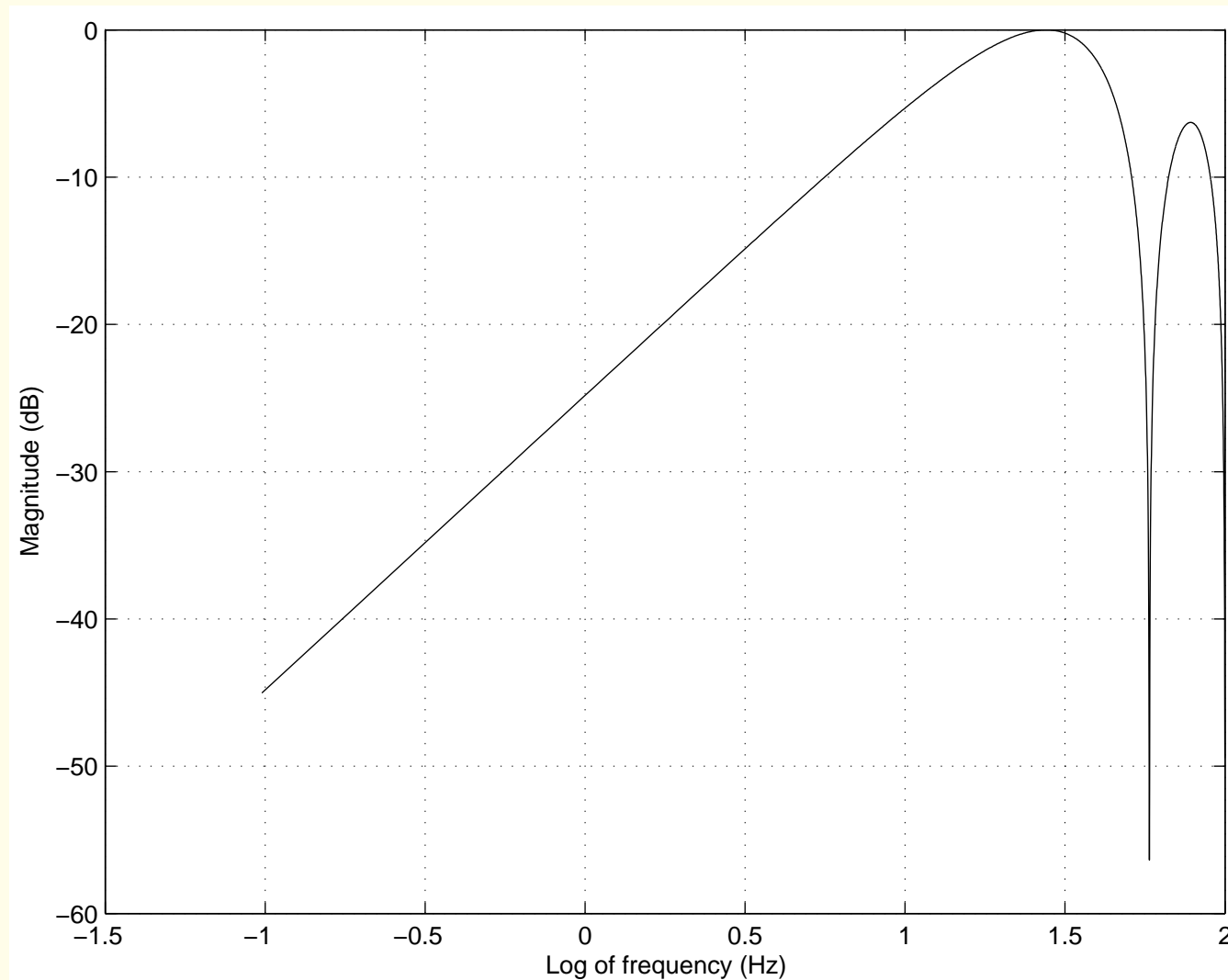
$$y(n) = \frac{1}{8} [2x(n) + x(n-1) - x(n-3) - 2x(n-4)]. \quad (4.13)$$

Approximates the ideal $\frac{d}{dt}$ operator up to 30 Hz .

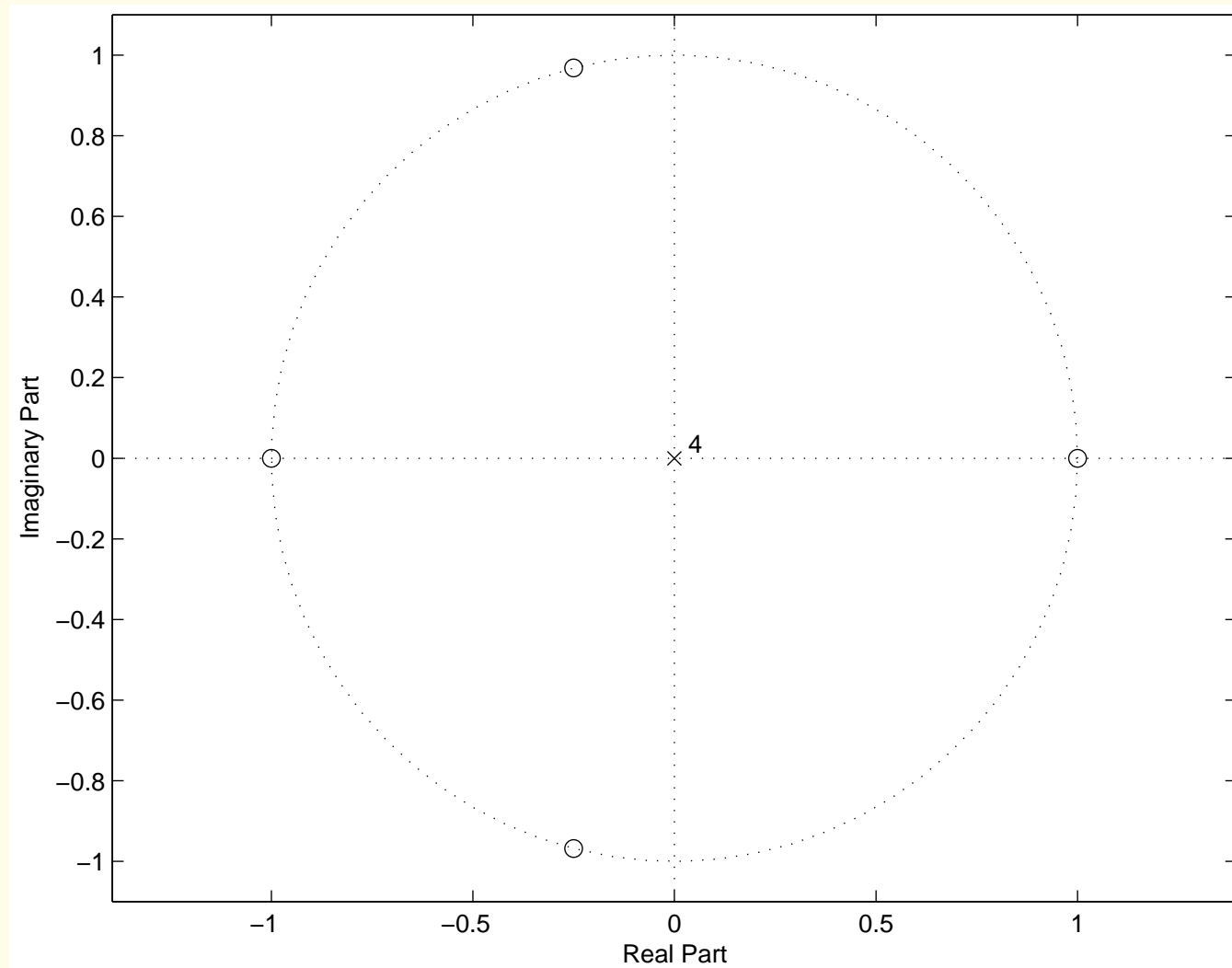
Suppresses the low-frequency components of P and T,
provides a large gain to the high-frequency components
arising from the high slopes of the QRS complex.



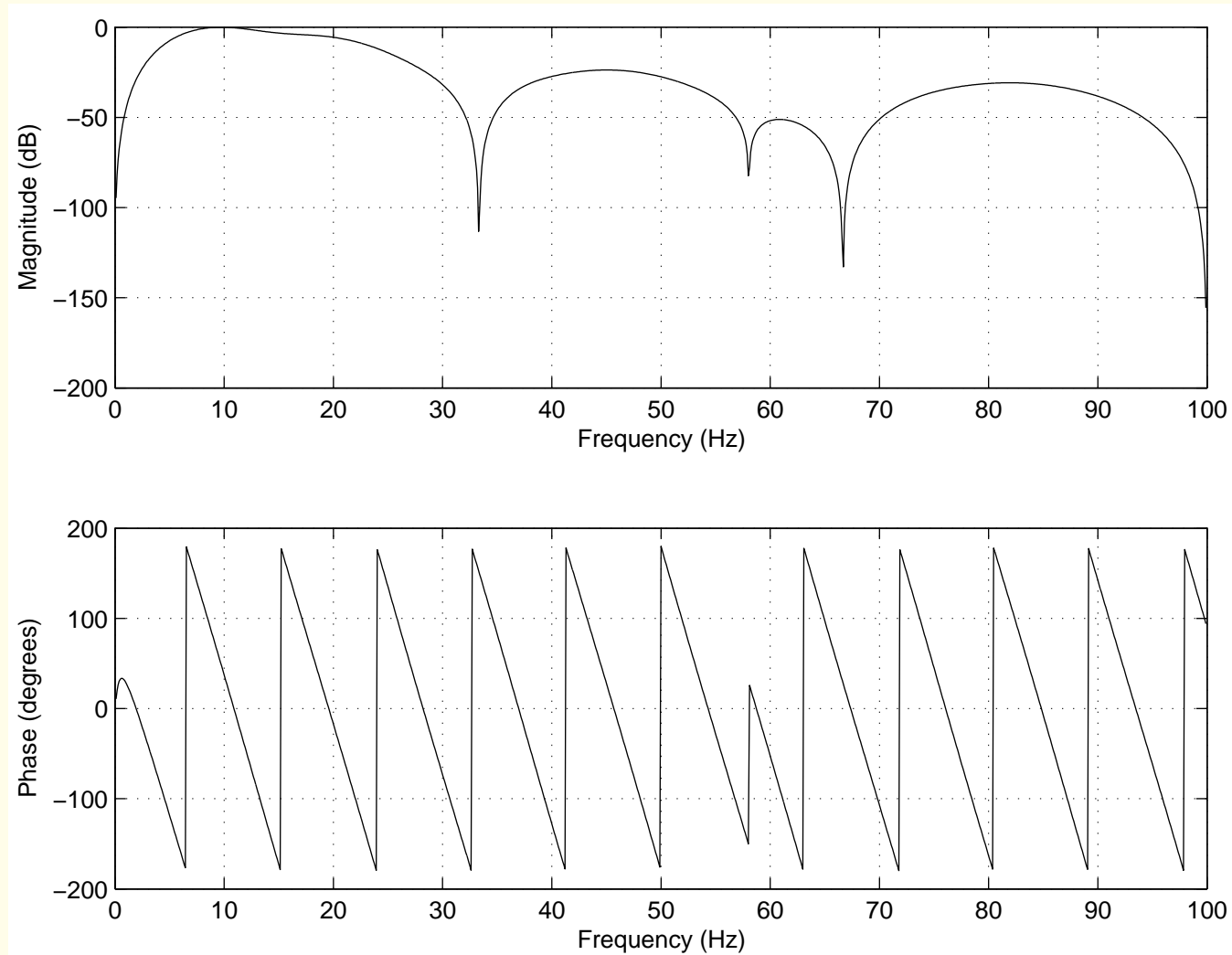
Frequency response of the derivative operator in the Pan-Tompkins algorithm for QRS detection.



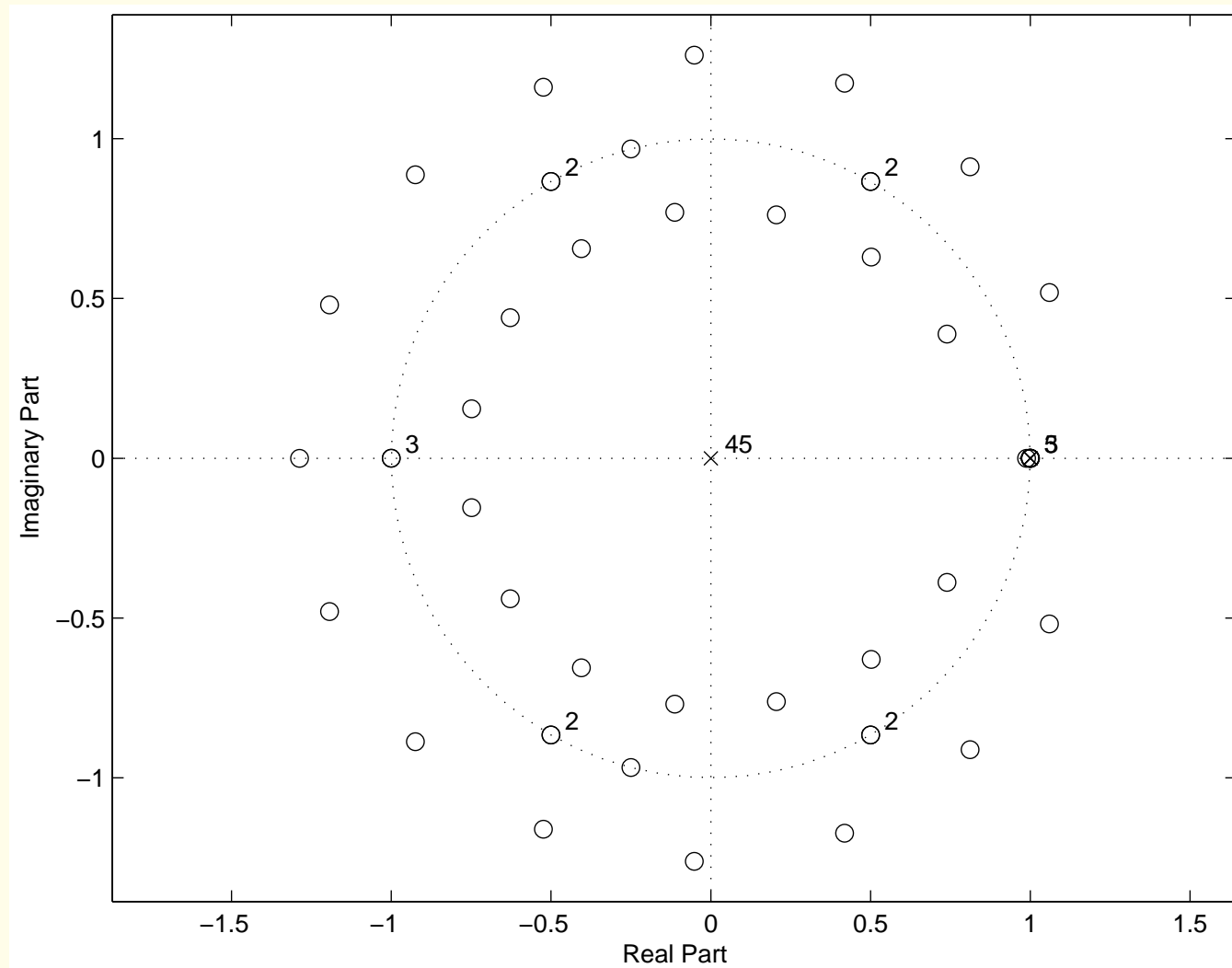
Frequency response (log-log) of the derivative operator in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the derivative operator in the Pan-Tompkins algorithm for QRS detection.



Combined frequency response of the bandpass filter and the derivative operator in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the bandpass filter and the derivative operator in the Pan-Tompkins algorithm for QRS detection.



Squaring: makes the result positive and emphasizes

large differences resulting from QRS complexes;

small differences from P and T waves are suppressed.

Nonlinear operation.



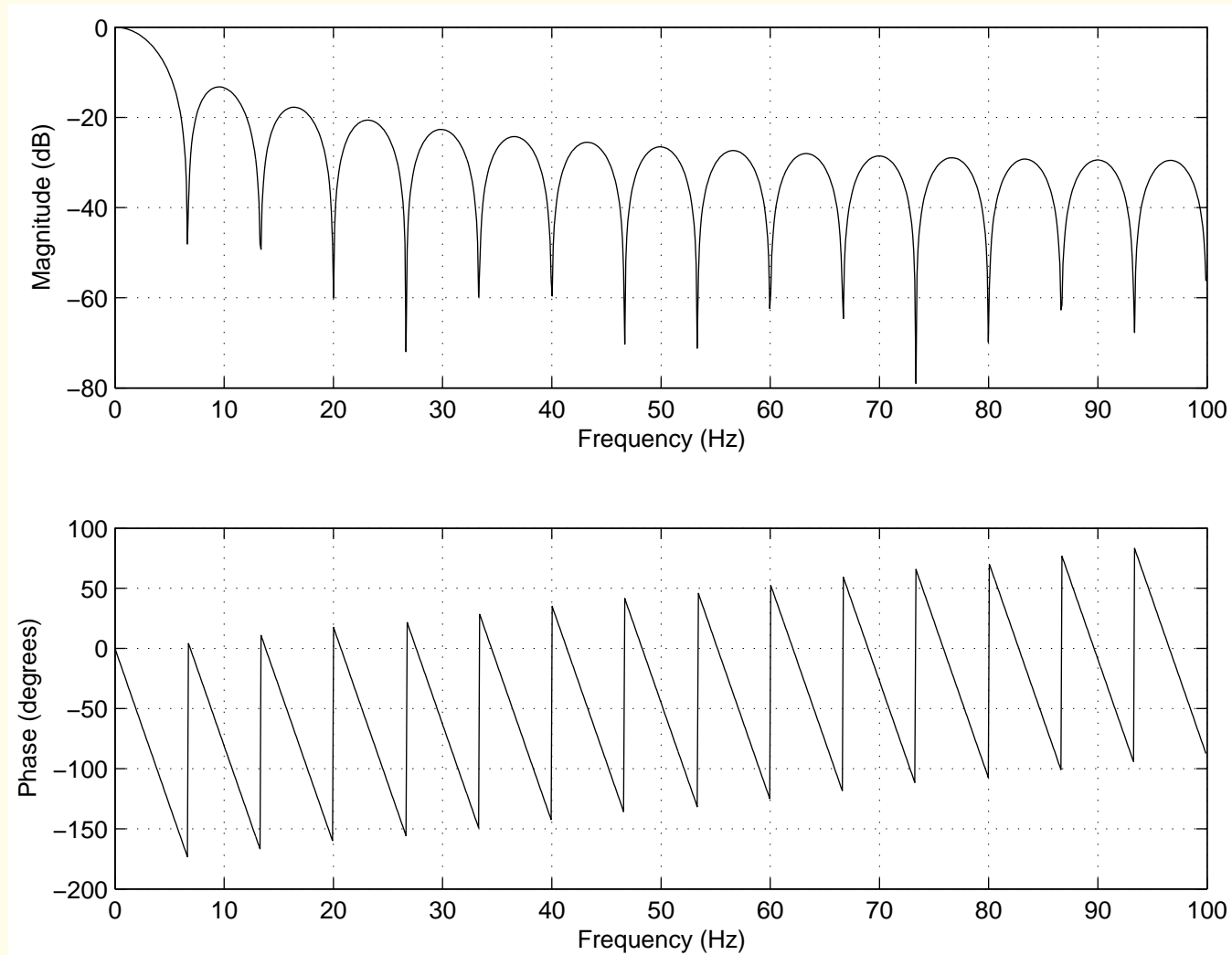
Integration:

Output of a derivative-based operation will exhibit multiple peaks within the duration of a single QRS complex.

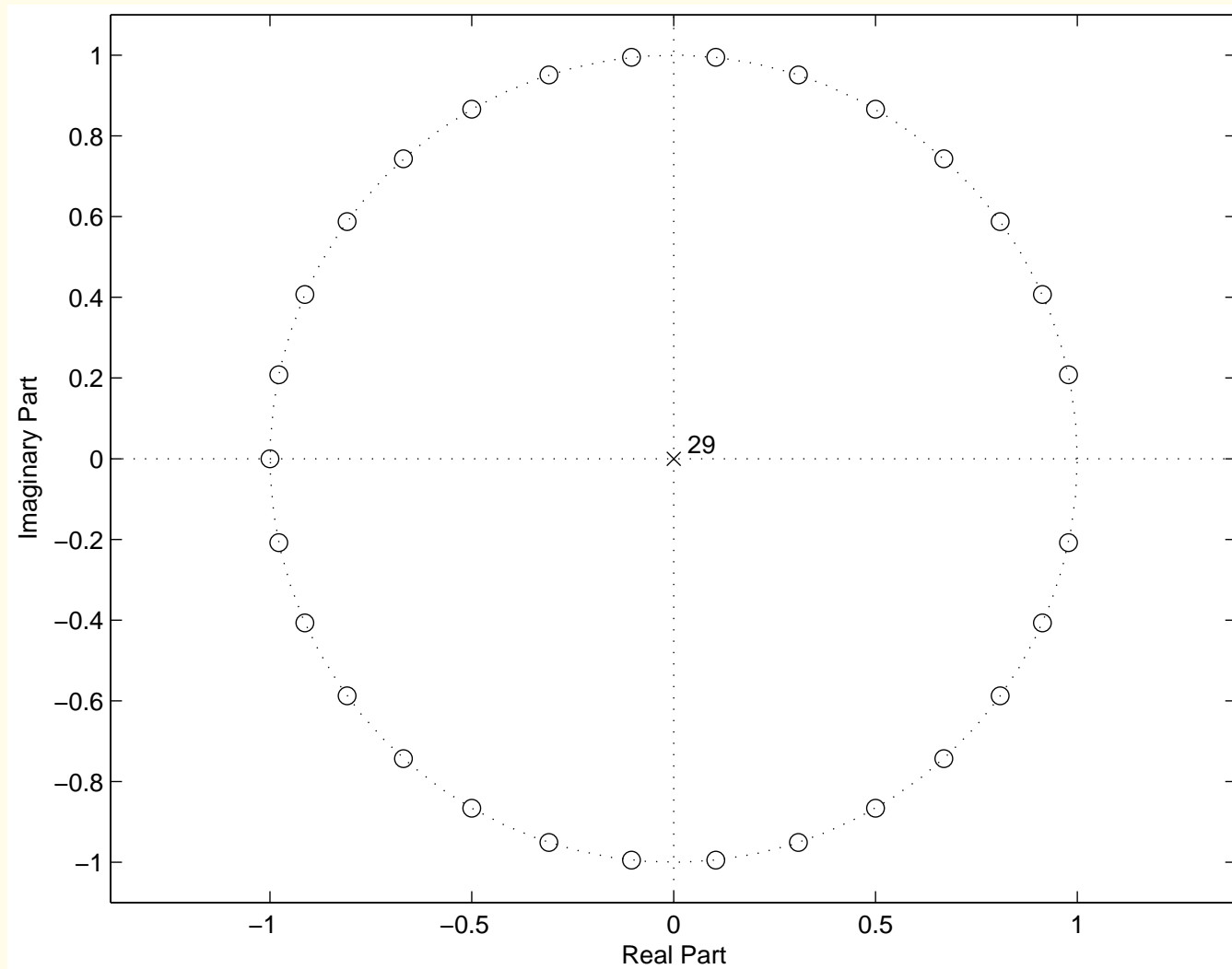
Smoothing with a moving-window integration filter:

$$y(n) = \frac{1}{N} [x(n - (N - 1)) + x(n - (N - 2)) + \cdots + x(n)]. \quad (4.14)$$

$N = 30$ for $f_s = 200 \text{ Hz}$.



Frequency response of the integrator in the Pan-Tompkins algorithm for QRS detection.



Pole-zero diagram of the integrator in the Pan-Tompkins algorithm for QRS detection.

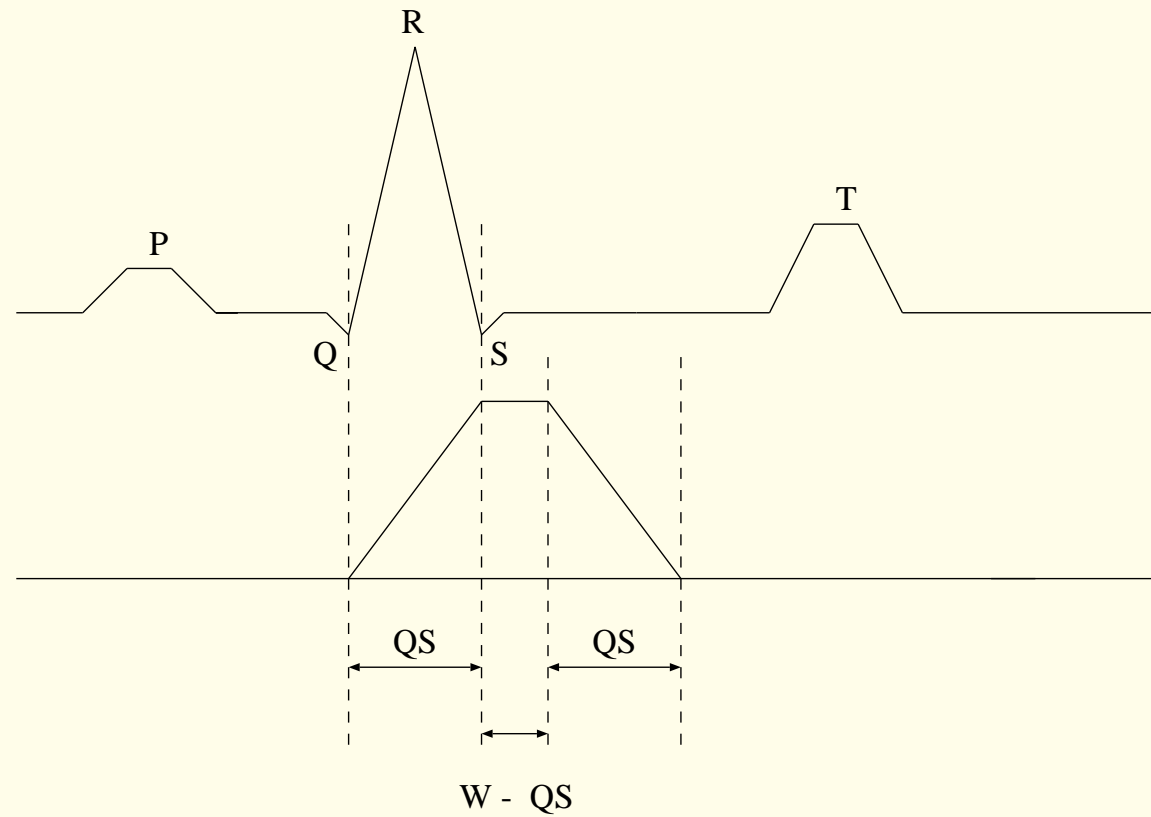


Figure 4.5: The relationship of a QRS complex to the moving-window integrator output. Upper plot: Schematic ECG signal. Lower plot: Output of the moving-window integrator. QS: QRS complex width. W: width of the integrator window, given as N/f_s s. Adapted from Tompkins.



Adaptive thresholding:

Running estimates of signal and noise peaks maintained.

SPKI: peak level the algorithm has learned for a QRS.

NPKI : peak level related to non-QRS events (noise).

THRESHOLD I1 and *THRESHOLD I2*:

to categorize peaks detected as signal (QRS) or noise.

Every new peak detected categorized as signal peak or noise.



If a peak exceeds *THRESHOLD I1* during the first step of analysis: classified as a QRS (signal) peak.

If the searchback technique is used,

the peak should be above *THRESHOLD I2*

to be called a QRS.

Peak levels and thresholds updated

after each peak is detected and classified.



$$SPKI = 0.125 PEAKI + 0.875 SPKI \quad (4.15)$$

if *PEAKI* is a signal peak;

$$NPKI = 0.125 PEAKI + 0.875 NPKI$$

if *PEAKI* is a noise peak;

$$THRESHOLD I1 = NPKI + 0.25(SPKI - NPKI);$$

$$THRESHOLD I2 = 0.5 THRESHOLD I1. \quad (4.16)$$



The updating formula for $SPKI$ is changed to

$$SPKI = 0.25 \text{ } PEAKI + 0.75 \text{ } SPKI \quad (4.17)$$

if a QRS is detected in the searchback procedure

using $THRESHOLD \text{ } I2$.



Searchback procedure:

Maintains two *RR*-interval averages.

RR AVERAGE1 : average of the eight most-recent beats;

RR AVERAGE2 : average of the eight most-recent beats

having *RR* intervals within the range specified by

$RR\ LOW\ LIMIT = 0.92 \times RR\ AVERAGE2$ and

$RR\ HIGH\ LIMIT = 1.16 \times RR\ AVERAGE2$.



If a QRS is not detected for the interval

$$RR\ MISSED\ LIMIT = 1.66 \times RR\ AVERAGE2,$$

the QRS is taken to be the peak between the established thresholds applied in the searchback procedure.

Low error rate of 0.68%, or 33 beats per hour

on a database of about 116,000 beats obtained from

24-hour records of the ECGs of 48 patients.

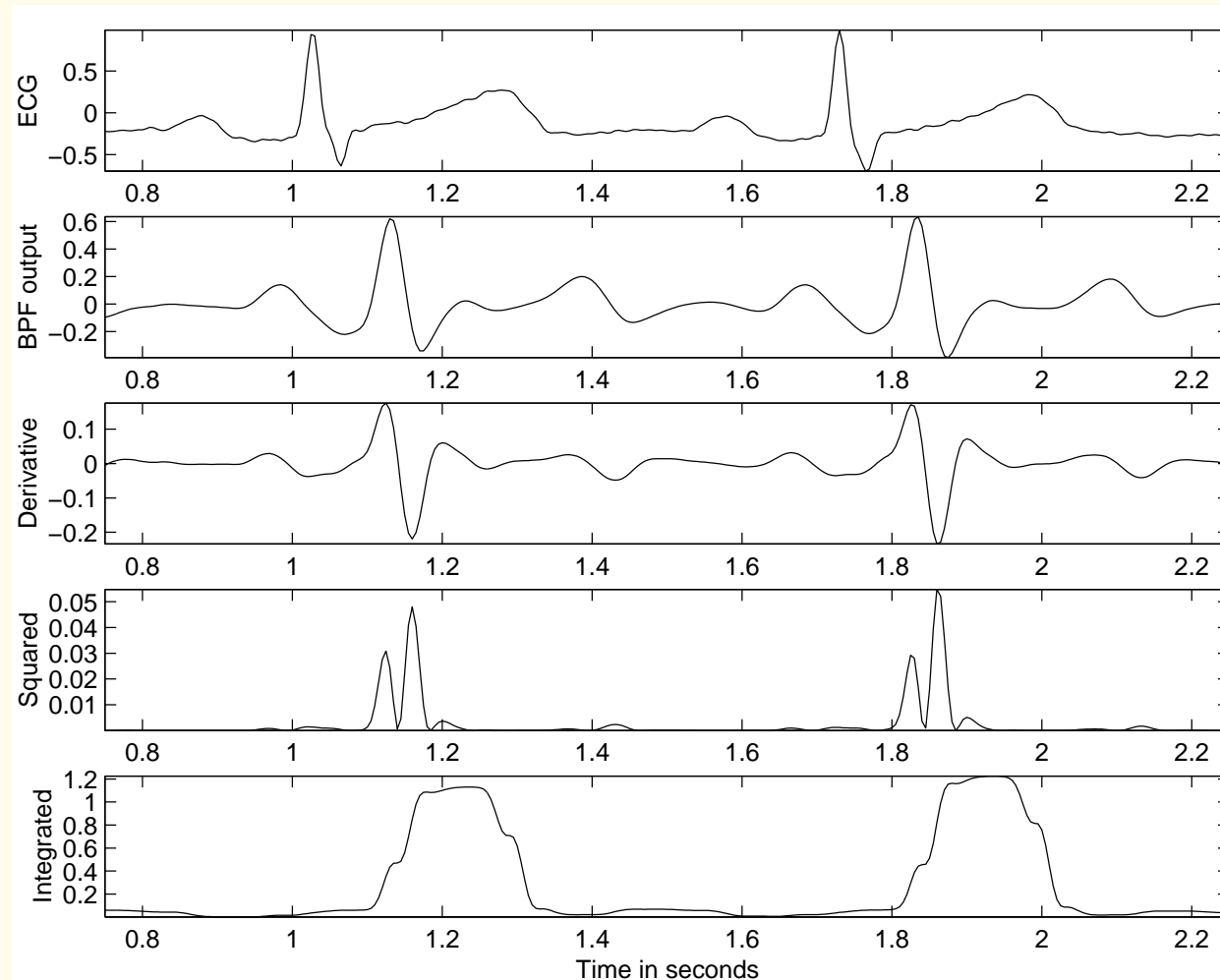


Figure 4.6: Results of the Pan-Tompkins algorithm. From top to bottom: two cycles of a filtered version of the ECG signal shown in Figure 3.5 (the same as that in Figure 4.2); output of the bandpass filter (BPF, a combination of lowpass and highpass filters); output of the derivative-based operator; the result of squaring; and $100\times$ the result of the final integrator.



4.3.3 Detection of the dicrotic notch

Problem: *Propose a method to detect the dicrotic notch in the carotid pulse signal.*

Solution: Lehner and Rangayyan method.

Dicrotic notch: short wave on downward slope of the carotid pulse.

Use the second derivative.



Least-squares estimate of the second derivative:

$$p(n) = 2y(n-2) - y(n-1) - 2y(n) - y(n+1) + 2y(n+2). \quad (4.18)$$

Noncausal; made causal by adding a delay of two samples.



Second derivative removes effect of downward slope;

enhances the notch itself.

Result squared and smoothed:

$$s(n) = \sum_{k=1}^M p^2(n - k + 1) w(k), \quad (4.19)$$

where $w(k) = (M - k + 1)$ is a linear weighting function,

$M = 16$ for $f_s = 256 \text{ Hz}$.



Two peaks for each period of the carotid pulse signal:

first peak represents the onset of the carotid upstroke;

second peak is due to the dicrotic notch.

To locate the dicrotic notch:

search for the local minimum in the carotid pulse

within a ± 20 *ms* interval of the second peak.

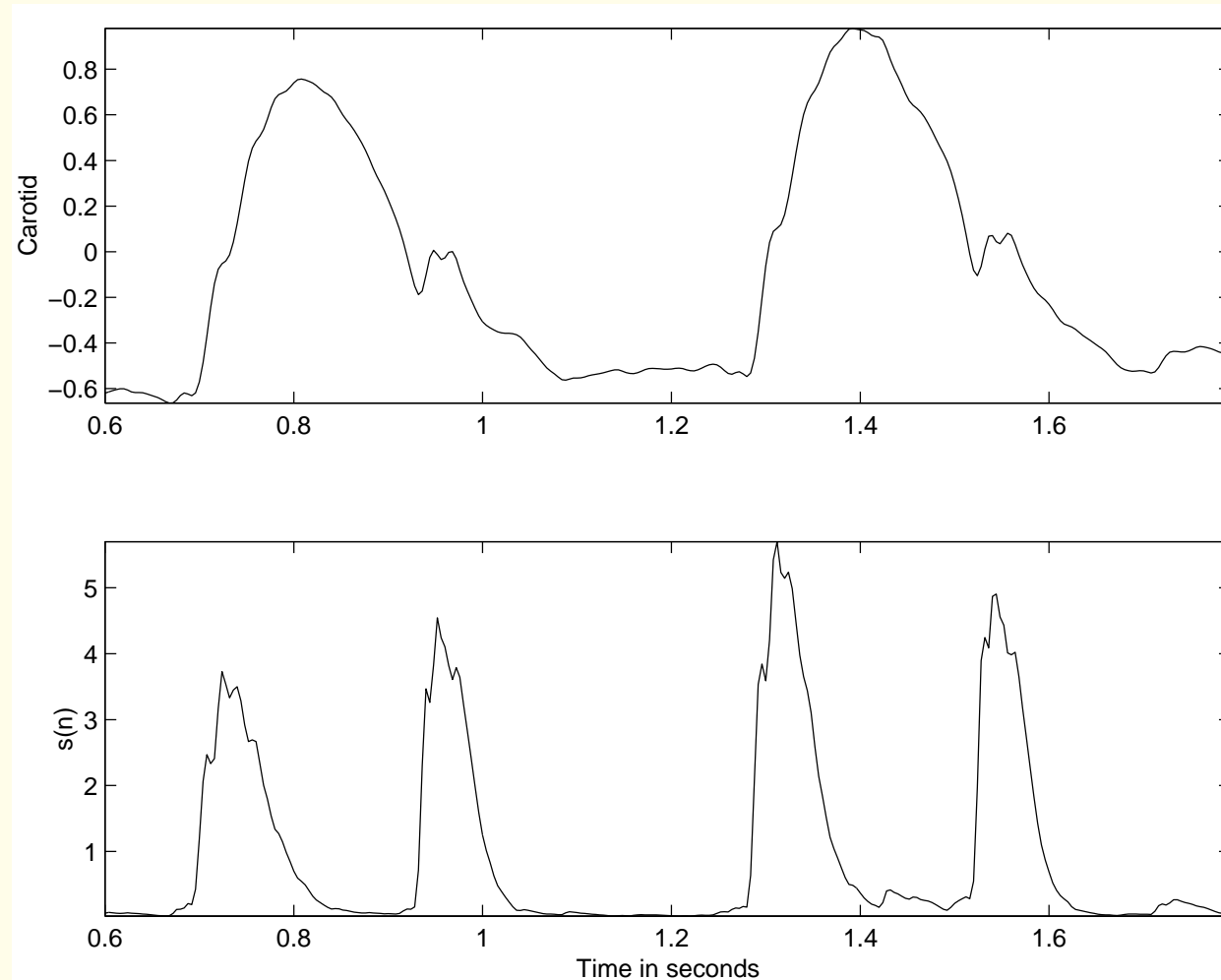


Figure 4.7: Two cycles of a carotid pulse signal and the result of the Lehner and Rangayyan method for detection of the dicrotic notch.

Signal lowpass filtered at 100 Hz and sampled at 250 Hz .



4.4 Correlation Analysis of EEG channels

4.4.1 Detection of EEG rhythms

Problem: *Propose a method to detect the*

presence of the α rhythm in an EEG channel.

How would you extend the method to detect the presence of

the same rhythm simultaneously in two EEG channels?



Solution: Two signals may be compared

to detect common characteristics between them via their dot product (inner or scalar product).

$$x \cdot y = \langle x, y \rangle = \sum_{n=0}^{N-1} x(n) y(n). \quad (4.20)$$

Dot product: projection of one signal onto the other,

with each signal being viewed as an N -dimensional vector.



Dot product normalized by the geometric mean of the energies of the two signals to get a correlation coefficient:

$$\gamma_{xy} = \frac{\sum_{n=0}^{N-1} x(n) y(n)}{\left[\sum_{n=0}^{N-1} x^2(n) \sum_{n=0}^{N-1} y^2(n) \right]^{1/2}} . \quad (4.21)$$

The means of the signals may be subtracted out, if desired.



In the case of two continuous-time signals $x(t)$ and $y(t)$,

the projection of one signal onto the other is defined as

$$\theta_{xy} = \int_{-\infty}^{\infty} x(t) y(t) dt. \quad (4.22)$$



When a shift or time delay may be present in the occurrence of the epoch of interest in the two signals, it becomes necessary to introduce a time-shift parameter.

Cross-correlation function (CCF) for a shift or delay of τ seconds or k samples:

$$\theta_{xy}(\tau) = \int_{-\infty}^{\infty} x(t) y(t + \tau) dt. \quad (4.23)$$



$$\theta_{xy}(k) = \sum_n x(n) y(n + k). \quad (4.24)$$

Range of summation limited to range of

available overlapped data.

Scale factor: depending upon the number of

data samples used.



In the case of random signals: need to take the expectation or sample average of the outer product of the vectors.

Let $\mathbf{x}(n) = [x(n), x(n-1), \dots, x(n-N+1)]^T$

and $\mathbf{y}(n) = [y(n), y(n-1), \dots, y(n-N+1)]^T$

represent the N -dimensional vectorial form of

$x(n)$ and $y(n)$ with the most-recent N samples

in each signal at the time instant n .



If $\mathbf{x}(n)$ and $\mathbf{y}(n)$ are sample observations of random processes, their CCF is defined as

$$\Theta_{xy} = E[\mathbf{x}(n) \mathbf{y}^T(n)]. \quad (4.25)$$

This outer product, which is an $N \times N$ matrix, provides cross-terms that include all possible delays or shifts within the duration of the signals.



The ACF displays peaks at intervals corresponding to the period and integral multiples thereof of any periodic or repetitive pattern present in the signal.

Facilitates detection of rhythms in signals such as the EEG:

α rhythm indicated by a peak near 0.1 s.



ACF of most signals decays and reaches negligible values after delays of a few milliseconds, except for periodic signals of infinite or indefinite duration for which the ACF will also exhibit periodic peaks. ACF will also exhibit multiple peaks when the same event repeats itself at regular or irregular intervals.



CCF displays peaks at the period of any periodic pattern

present in *both* of the signals being analyzed.

CCF may be used to detect common rhythms present

between two signals: α rhythm in two EEG channels.

When one of the functions is a template of an event,

the procedure is known as *template matching*.

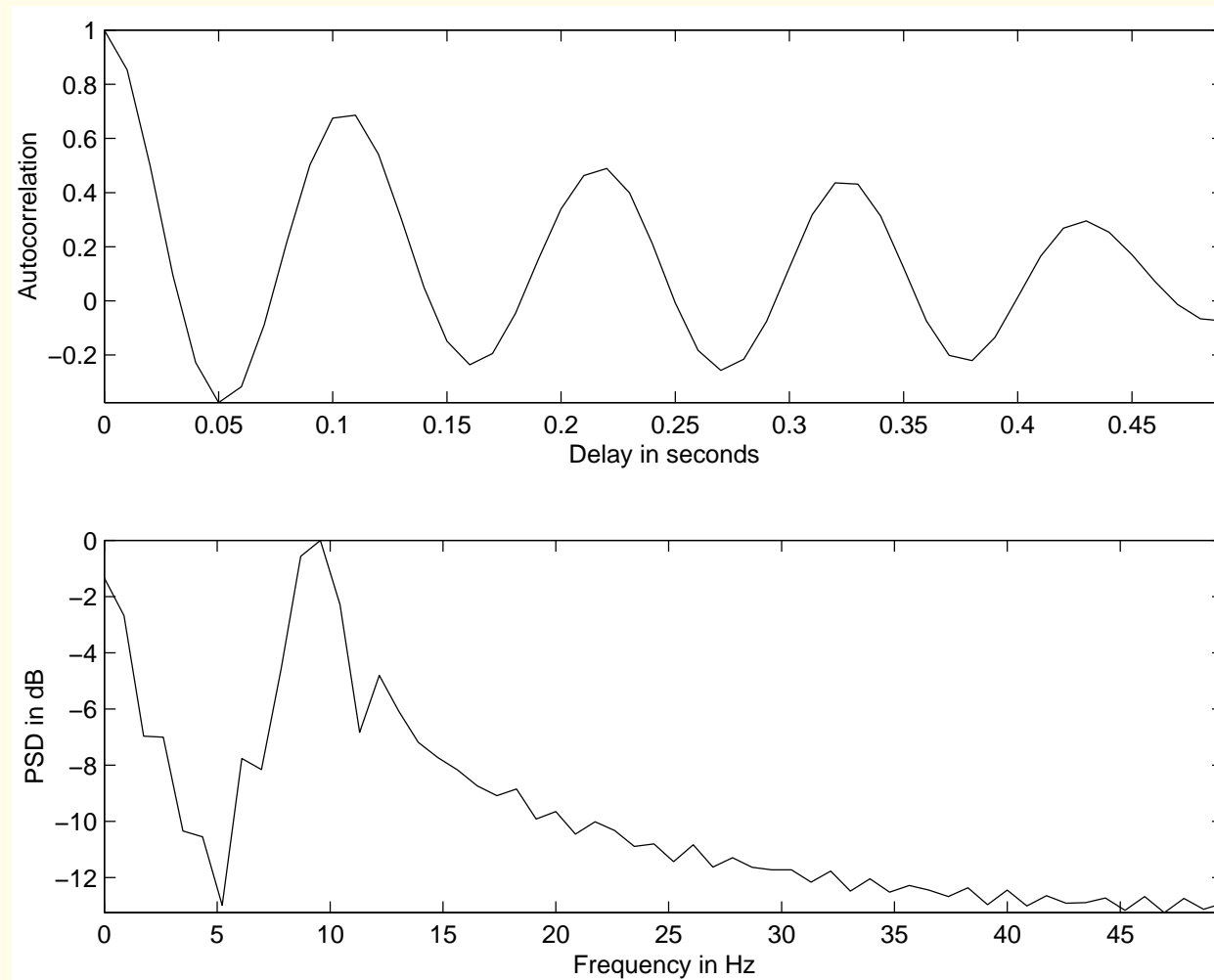


Figure 4.8: Upper trace: ACF of the 4.67–5.81 s portion of the p4 channel of the EEG signal shown in Figure 1.22. Lower trace: The PSD of the signal segment in dB, given by the Fourier transform of the ACF.

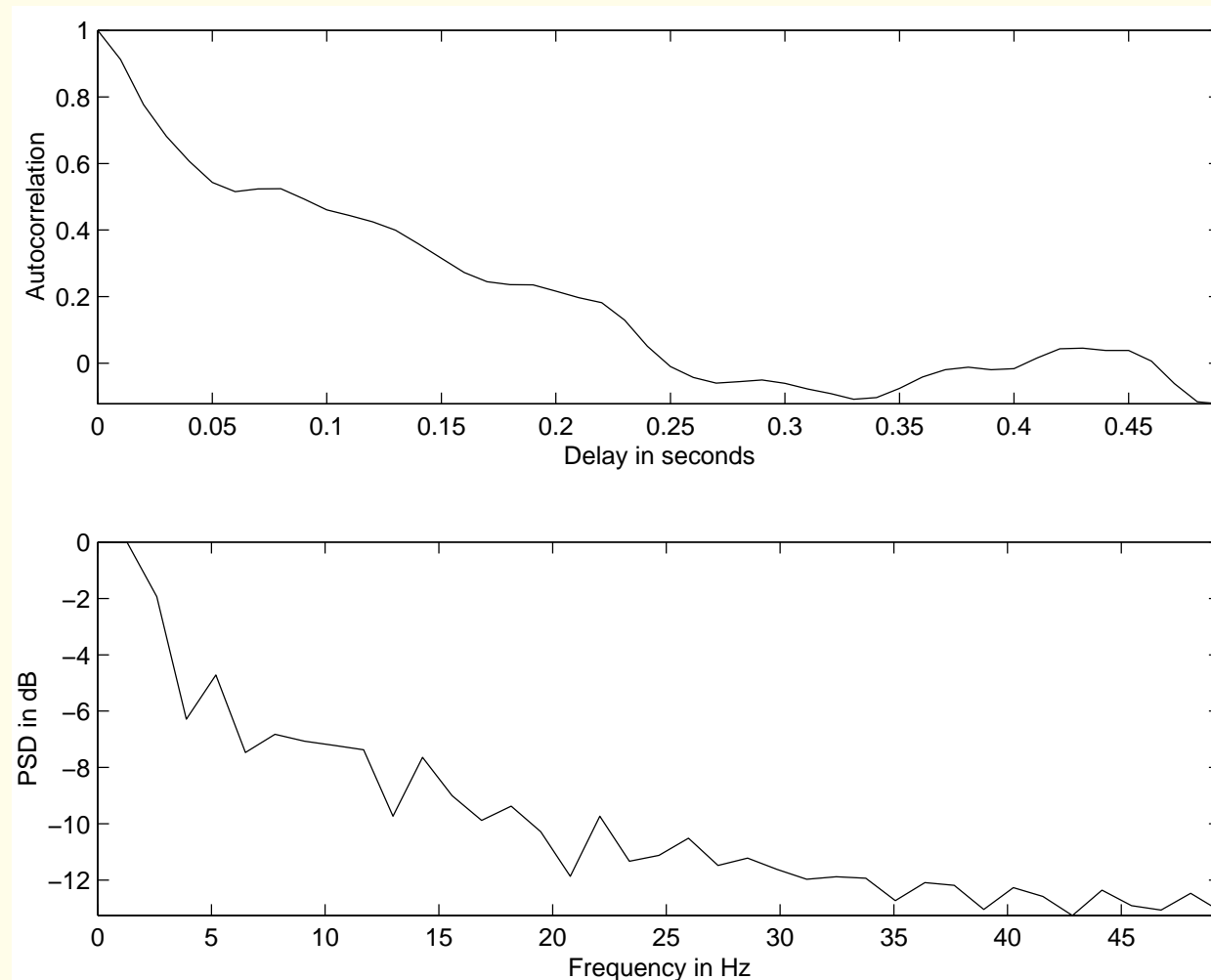


Figure 4.9: Upper trace: ACF of the 4.2–4.96 s portion of the f3 channel of the EEG signal shown in Figure 1.22. Lower trace: The PSD of the signal segment in dB.

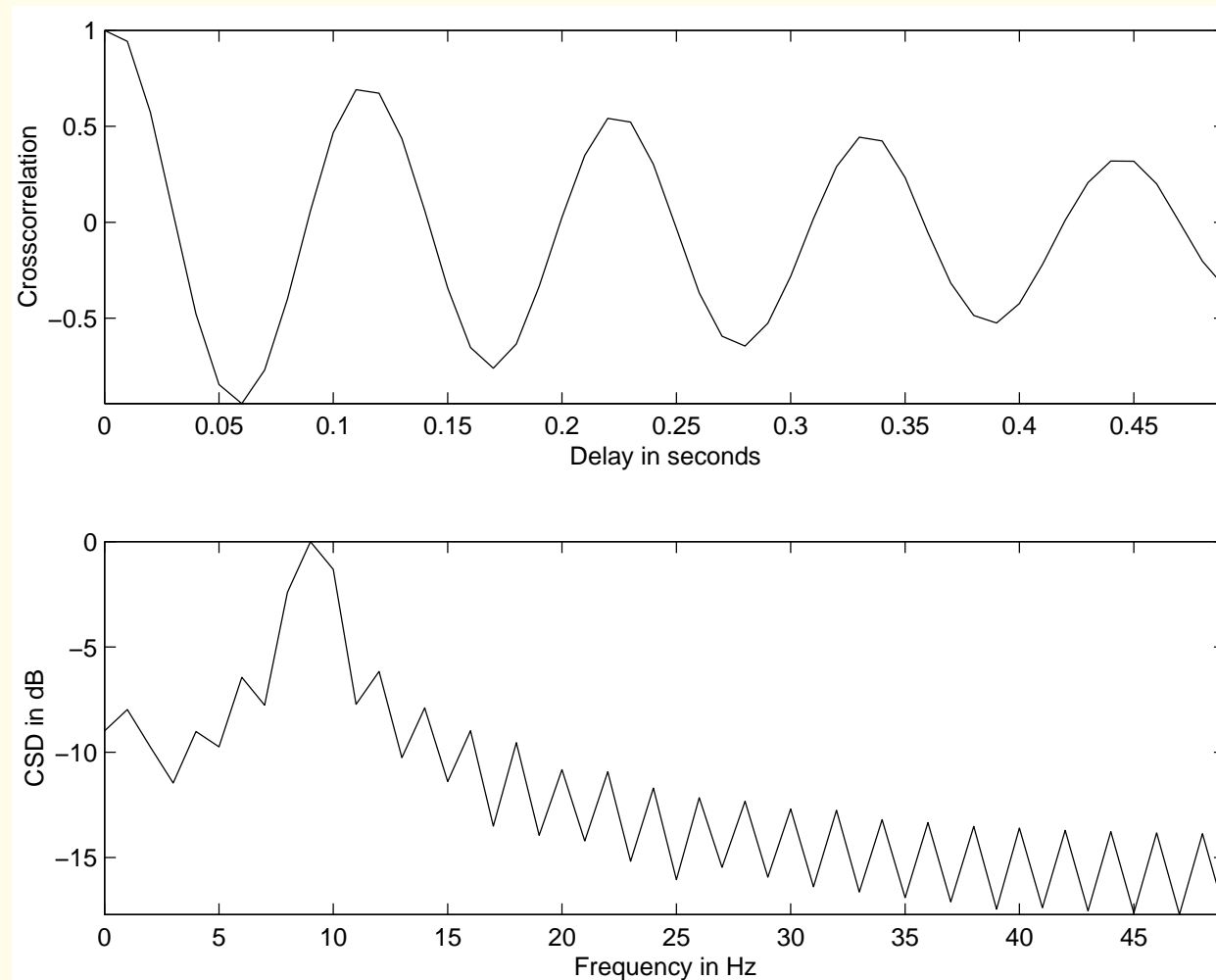


Figure 4.10: Upper trace: CCF between the 4.72 – 5.71 s portions of the p3 and p4 channels of the EEG signal shown in Figure 1.22. Lower trace: The CSD of the signal segments in dB, computed as the Fourier transform of the CCF.

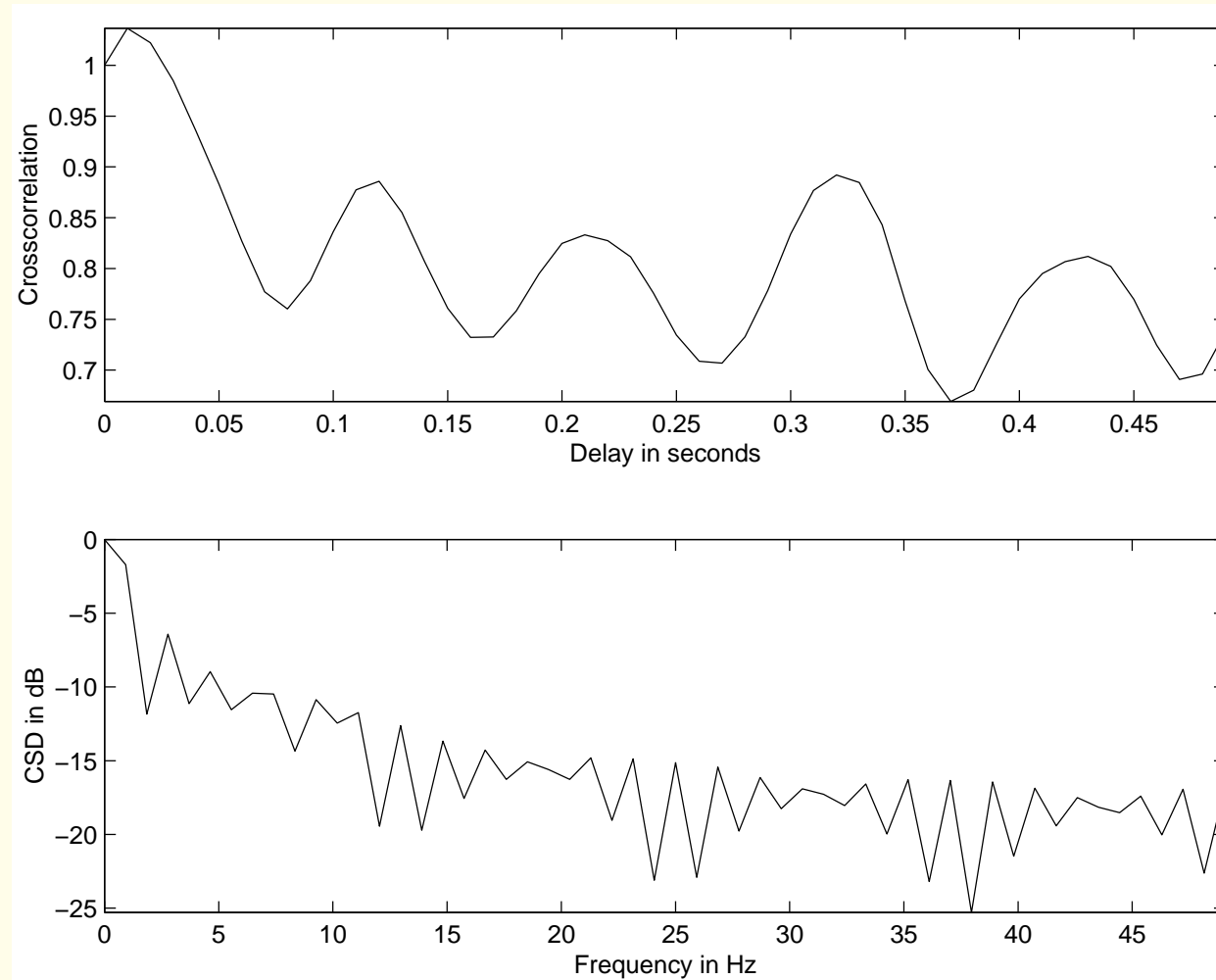


Figure 4.11: Upper trace: CCF between the 5.71 – 6.78 s portions of the o2 and c4 channels of the EEG signal shown in Figure 1.22. Lower trace: The CSD of the signal segments in dB.

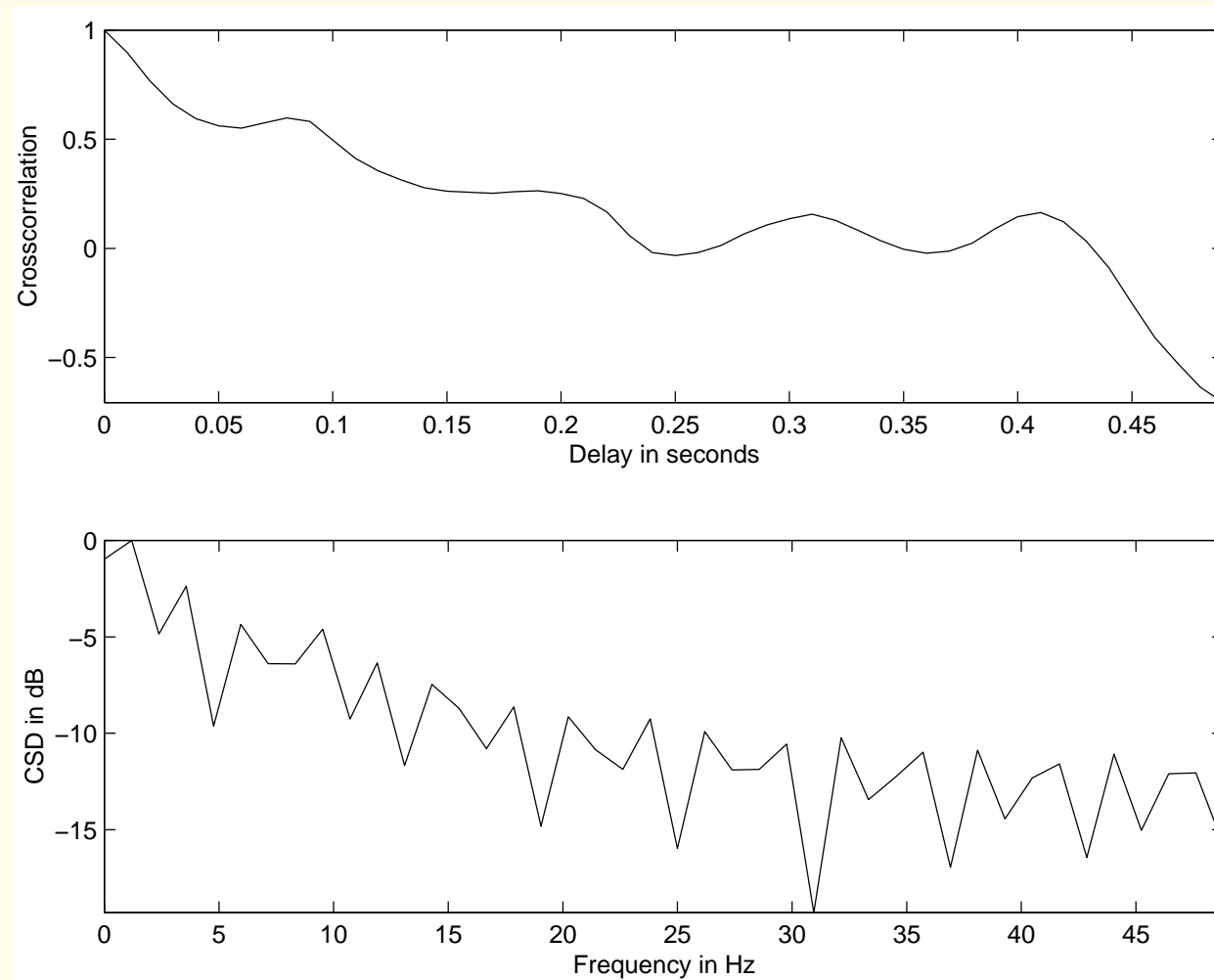


Figure 4.12: Upper trace: CCF between the 4.13 – 4.96 s portions of the f3 and f4 channels of the EEG signal shown in Figure 1.22. Lower trace: The CSD of the signal segments in dB.



4.4.2 Template matching for EEG spike-and-wave detection

Problem: *Propose a method to detect spike-and-wave complexes in an EEG signal.*

You may assume that a sample segment of a spike-and-wave complex is available.



Solution:

A spike-and-wave complex is a well-defined event,

composed of a sharp spike followed by a wave

with a frequency of about 3 Hz ;

the wave may contain a half period or a

full period of an almost-sinusoidal pattern.

Extract an epoch of a spike-and-wave complex

from an EEG channel and use it for template matching.

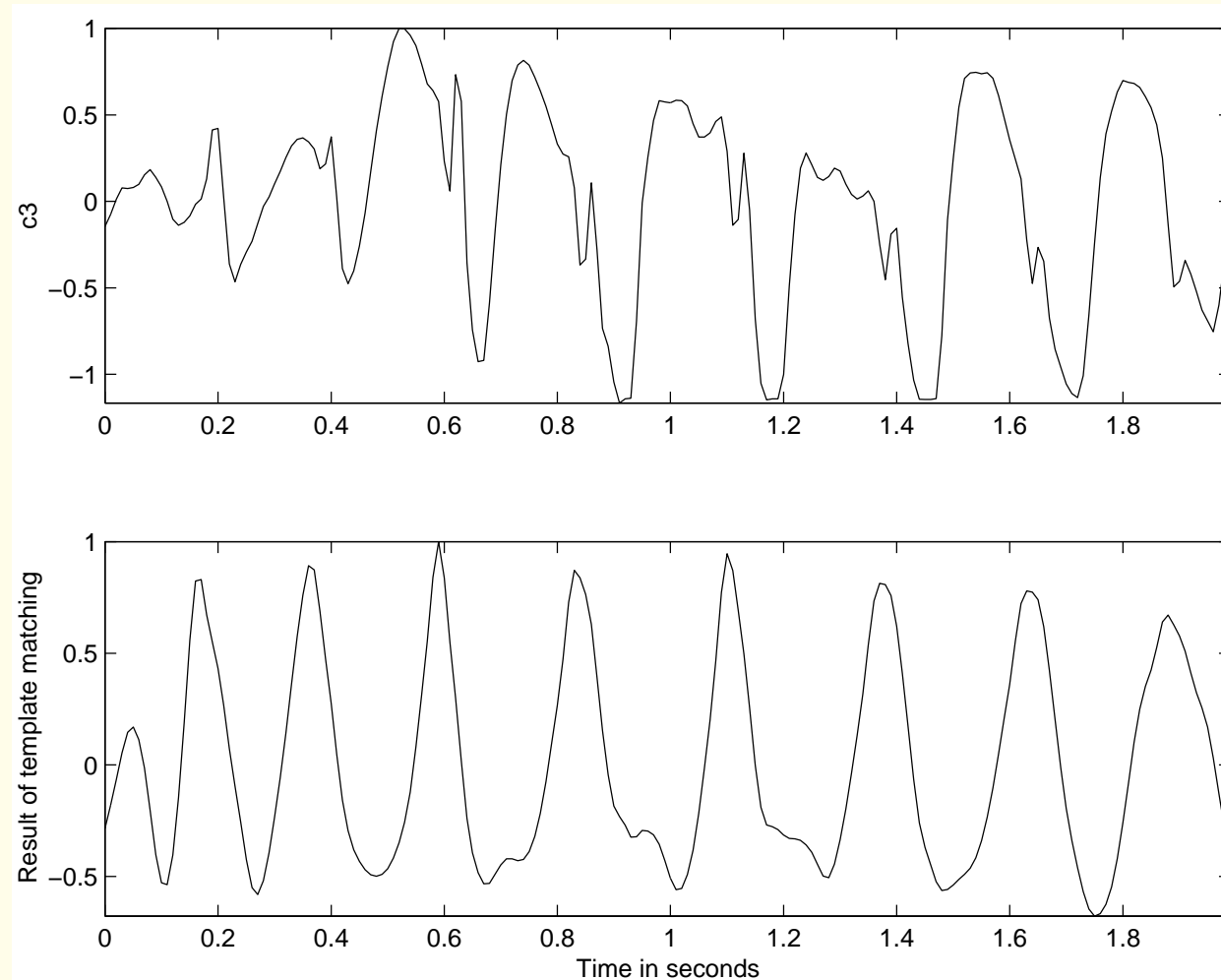


Figure 4.13: Upper trace: the c3 channel of the EEG signal shown in Figure 1.23. Lower trace: result of template matching. The spike-and-wave complex between 0.60 s and 0.82 s in the signal was used as the template.

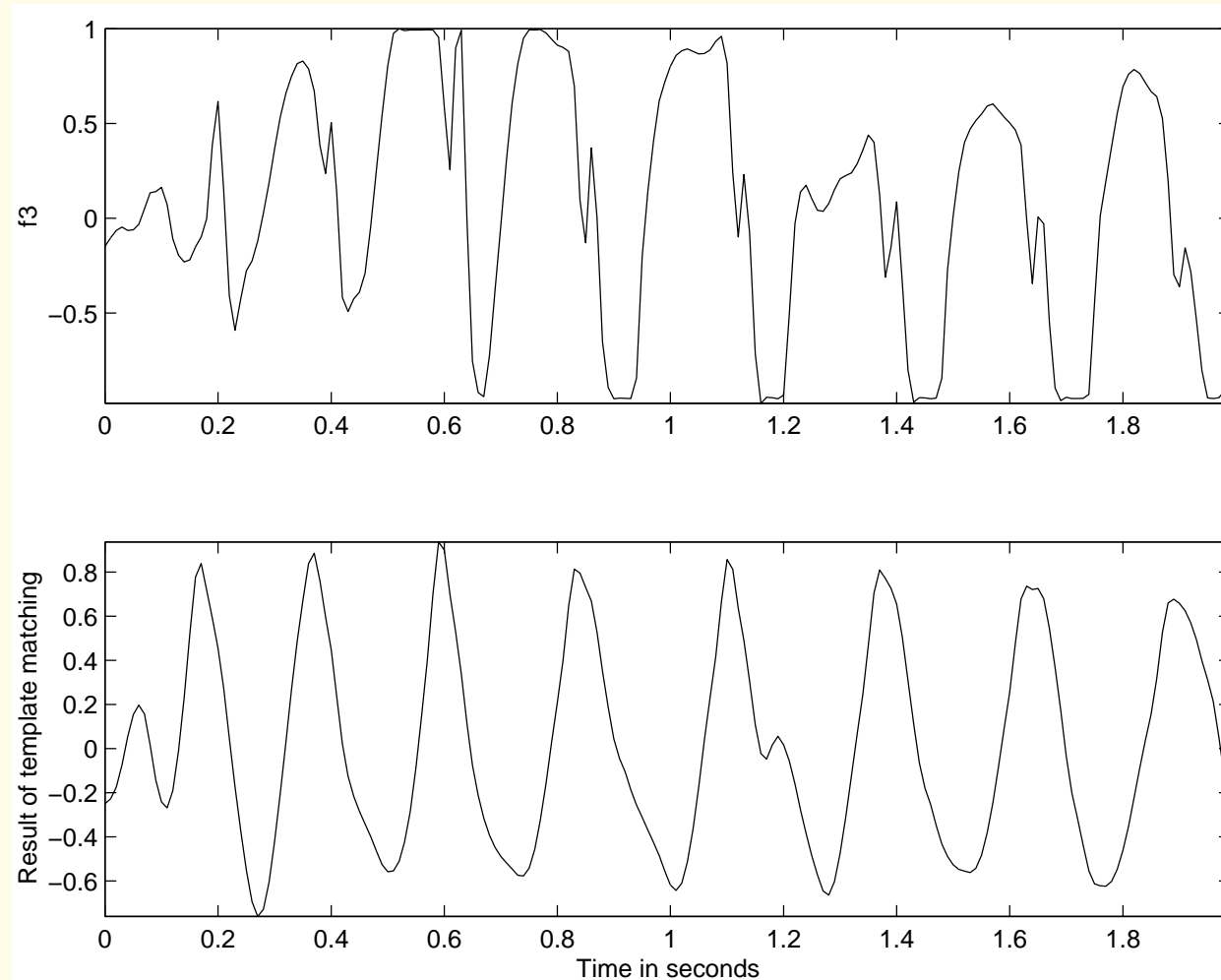


Figure 4.14: Upper trace: the f3 channel of the EEG signal shown in Figure 1.23. Lower trace: result of template matching. The spike-and-wave complex between 0.60 s and 0.82 s in the c3 channel (see Figure 4.13) was used as the template.



4.5 Cross-spectral Techniques

Multiple peaks in ACF or CCF may cause confusion in the detection of rhythms.

Fourier-domain equivalents of the ACF or the CCF — PSD or CSD — permit easier and intuitive analysis in the frequency domain than in the time domain.

Notion of rhythms easier with frequencies in *cps* or *Hz* than with the corresponding inversely related periods.



4.5.1 Coherence analysis of EEG channels

Problem: *Describe a frequency-domain approach to study the presence of rhythms in multiple channels of an EEG.*

Solution: PSD $S_{xx}(f)$ of a signal

$$S_{xx}(f) = FT[\phi_{xx}(\tau)] = X(f) X^*(f) = |X(f)|^2. \quad (4.26)$$



The Fourier transform of the CCF between two signals

gives the CSD:

$$S_{xy}(f) = FT[\theta_{xy}(\tau)] = X^*(f) Y(f). \quad (4.27)$$



PSD displays peaks at frequencies corresponding to periodic activities in the signal.

Facilitates the detection of rhythms in the EEG:

α rhythm indicated by a peak or multiple peaks in the range $8 - 13 \text{ Hz}$.

PSD also shows the presence of activity spread over specific bands of frequencies: formants in speech, murmurs in PCG.



CSD exhibits peaks at frequencies present in

both of the signals being compared.

CSD may be used to detect rhythms present

in common between two channels of the EEG.

Normalized *magnitude coherence spectrum* of two signals:

$$\Gamma_{xy}(f) = \left[\frac{|S_{xy}(f)|^2}{S_{xx}(f) S_{yy}(f)} \right]^{1/2}. \quad (4.28)$$



Phase of the coherence spectrum:

$$\psi_{xy}(f) = \angle S_{xy}(f),$$

represents average phase difference or time delay

between common frequency components in the two signals.



Illustration of application:

Coherence between EEG signals recorded from different positions on the scalp depends upon the structural connectivity or functional coupling between the corresponding parts of the brain — *symmetry*.



4.6 The Matched Filter

When a sample observation or template of a typical version of a signal event is available, it is possible to design a filter that is *matched* to the characteristics of the event.

If a signal that contains repetitions of the event is passed through the *matched filter*, the output should have peaks at the time instants of the events.



Matched filters useful in the detection of signals of known characteristics that are buried in noise.

They are designed to perform correlation between the input signal and the signal template, and hence are known as *correlation filters*.



4.6.1 Detection of EEG spike-and-wave complexes

Problem: *Design a matched filter*

to detect spike-and-wave complexes in an EEG signal.

A reference spike-and-wave complex is available.

Solution:

Let $x(t)$ be the given reference signal,

representing an ideal observation of the event of interest.



Let $X(f)$ be the Fourier transform of $x(t)$.

Consider passing $x(t)$ through a linear time-invariant filter

whose impulse response is $h(t)$;

the transfer function of the filter is $H(f) = FT[h(t)]$.

Output: $y(t) = x(t) * h(t)$ or $Y(f) = X(f) H(f)$.



The output energy is maximized when

$$H(f) = K X^*(f) \exp(-j2\pi f t_0), \quad (4.29)$$

where K is a scale factor and

t_0 is a time instant or delay; see Appendix 1.

Corresponds to the impulse response

$$h(t) = K x(t_0 - t). \quad (4.30)$$



The transfer function of the matched filter is proportional to the complex conjugate of the FT of the signal to be detected.

The impulse response is a *reversed* or *reflected* version of the reference signal that is scaled and delayed.

A delay has to be added to make the filter causal.



Impulse response is a reversed version of $x(t)$; therefore
convolution performed by the matched filter \equiv correlation:
output equal to cross-correlation of input and reference.

If the input contains the reference signal:

$$Y(f) = X(f) H(f) = X(f) X^*(f) = S_{xx}(f). \quad (4.31)$$

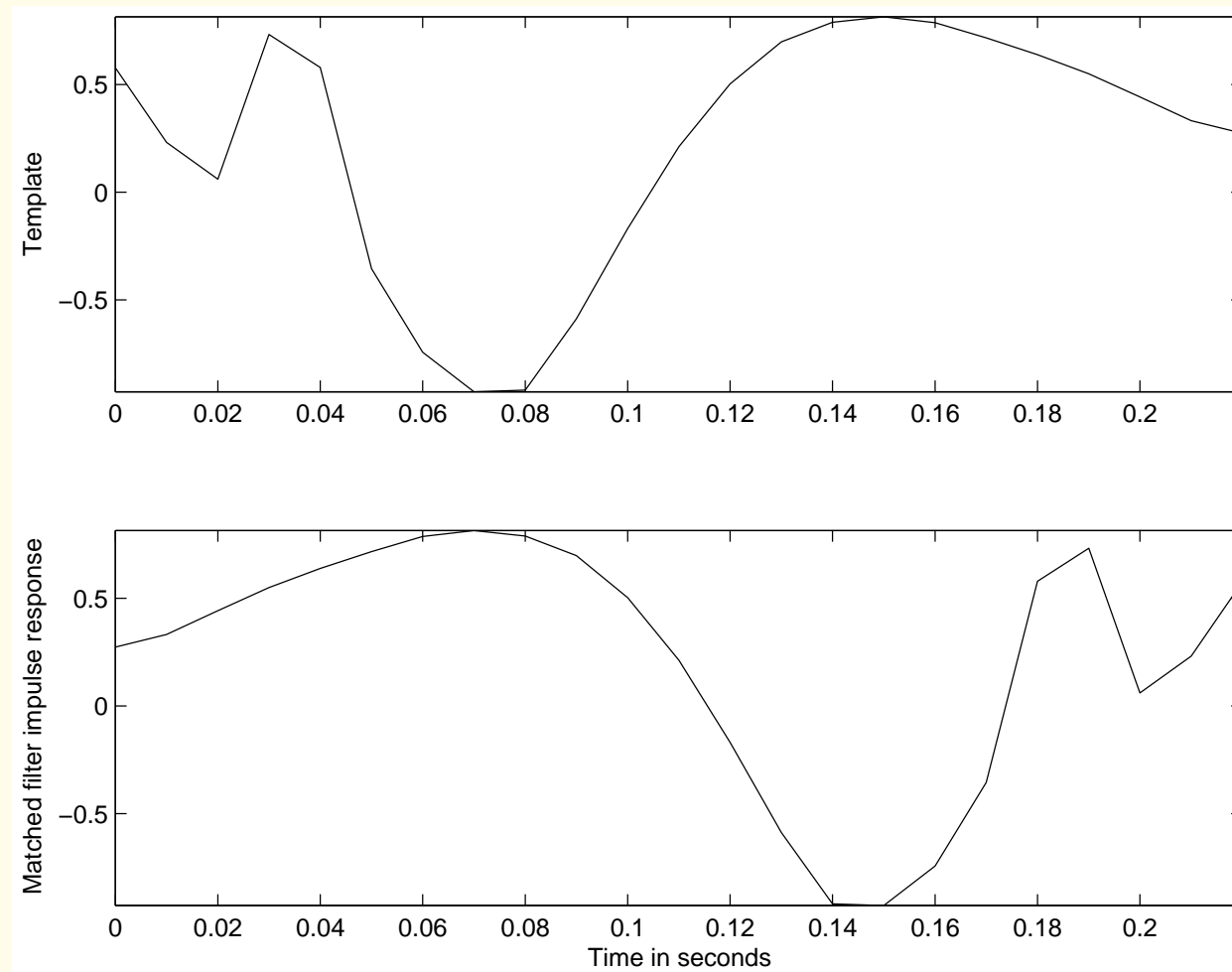


Figure 4.15: Upper trace: The spike-and-wave complex between 0.60 s and 0.82 s in the c3 channel of the EEG signal shown in Figure 1.23. Lower trace: Impulse response of the matched filter derived from the signal segment in the upper trace. Observe that the latter is a time-reversed version of the former.

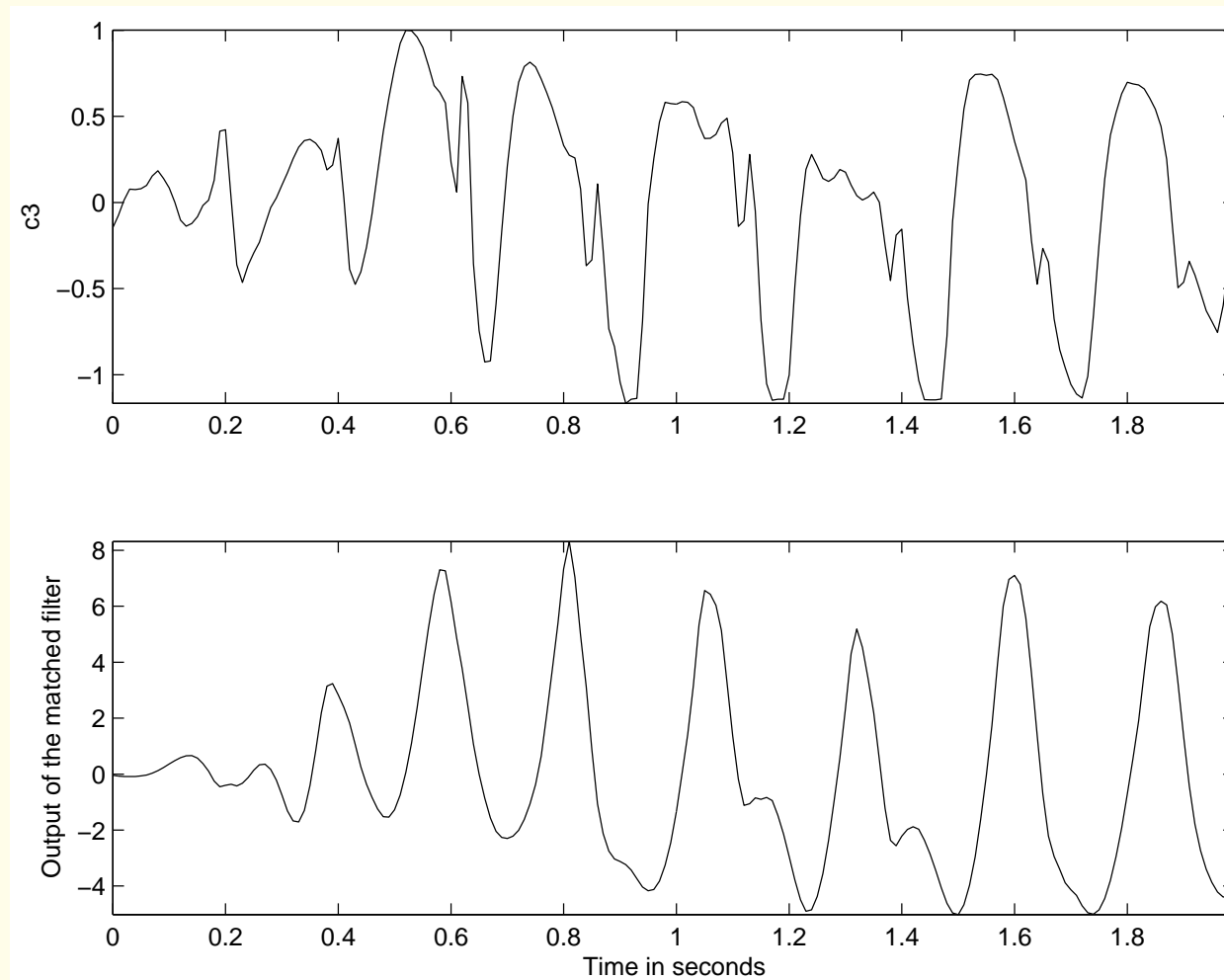


Figure 4.16: Upper trace: The c3 channel of the EEG signal shown in Figure 1.23, used as input to the matched filter in Figure 4.15. Lower trace: Output of the matched filter. See also Figure 4.13.

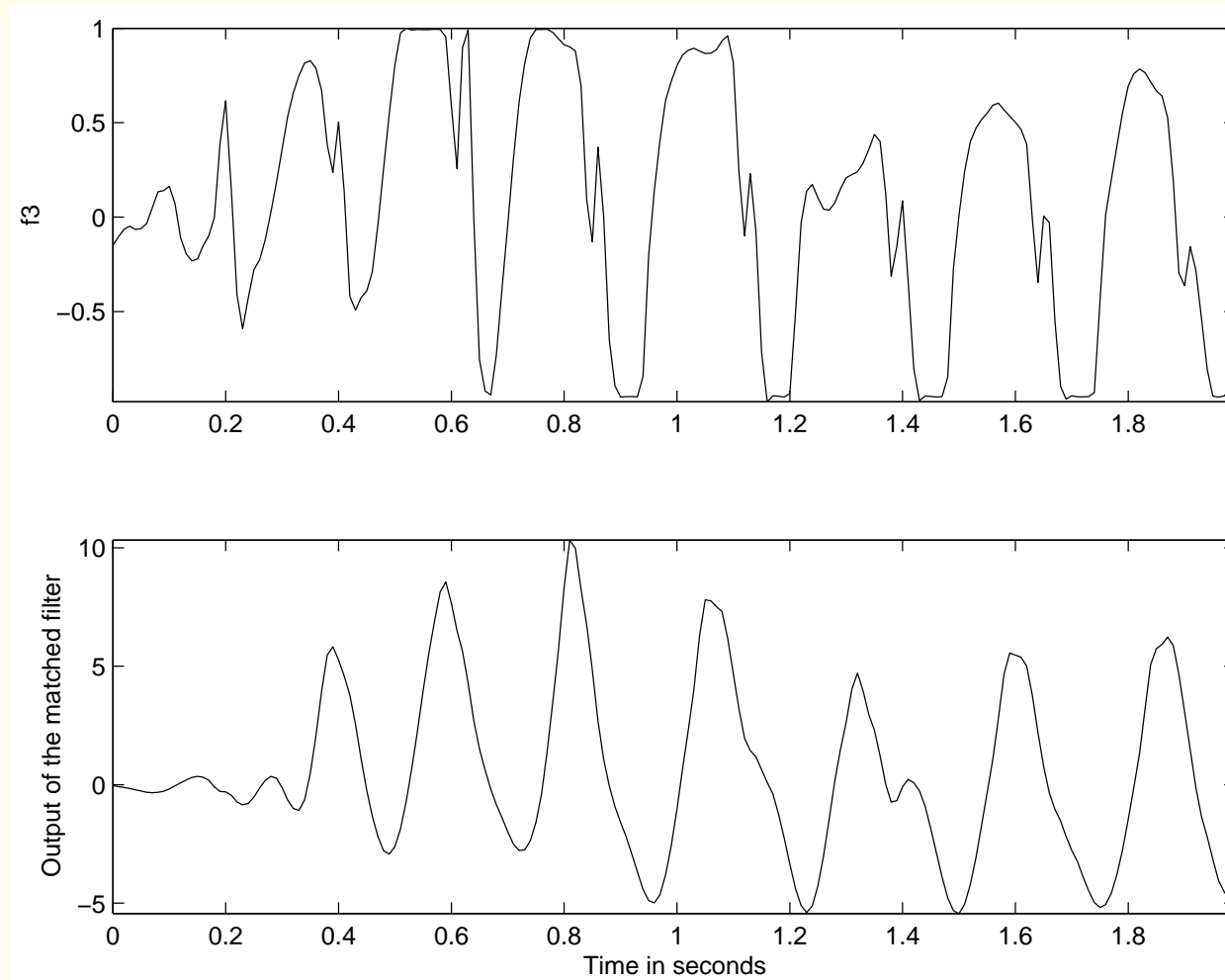


Figure 4.17: Upper trace: The f3 channel of the EEG signal shown in Figure 1.23, used as input to the matched filter in Figure 4.15. Lower trace: Output of the matched filter. See also Figure 4.14.



4.7 Detection of the P Wave

Detection of the P wave is difficult, as it is small, has an ill-defined and variable shape, and could be placed in a background of noise of varying size and origin.

Problem: *Propose an algorithm to detect the P wave in the ECG signal.*



Solution 1: Hengeveld and van Bommel.

1. QRS detected, deleted, and replaced with base-line.
2. Resulting signal bandpass filtered with
 -3 dB points at 3 Hz and 11 Hz .
3. Search interval defined as

$$QT_{\max} = \frac{2}{9}RR + 250\text{ ms}.$$



4. Maximum and minimum values found in all three VCG leads from the end of the preceding T wave to the onset of the QRS.
5. Signal rectified and thresholded at 50% and 75% of the maximum to obtain a ternary (three-level) signal.



6. Cross-correlation of the result computed with a ternary template derived in a manner similar to the previous step from a representative set of P waves.
7. Peak in the cross-correlation corresponds to the P location in the original ECG.



The algorithm overcomes the dominance of the QRS

by first detecting the QRS and then deleting it.

Observe that the cross-correlation is computed

not with an original P wave, which could be obscure,

but with a ternary wave derived from the P wave,

which represents a simplified template of the P wave.

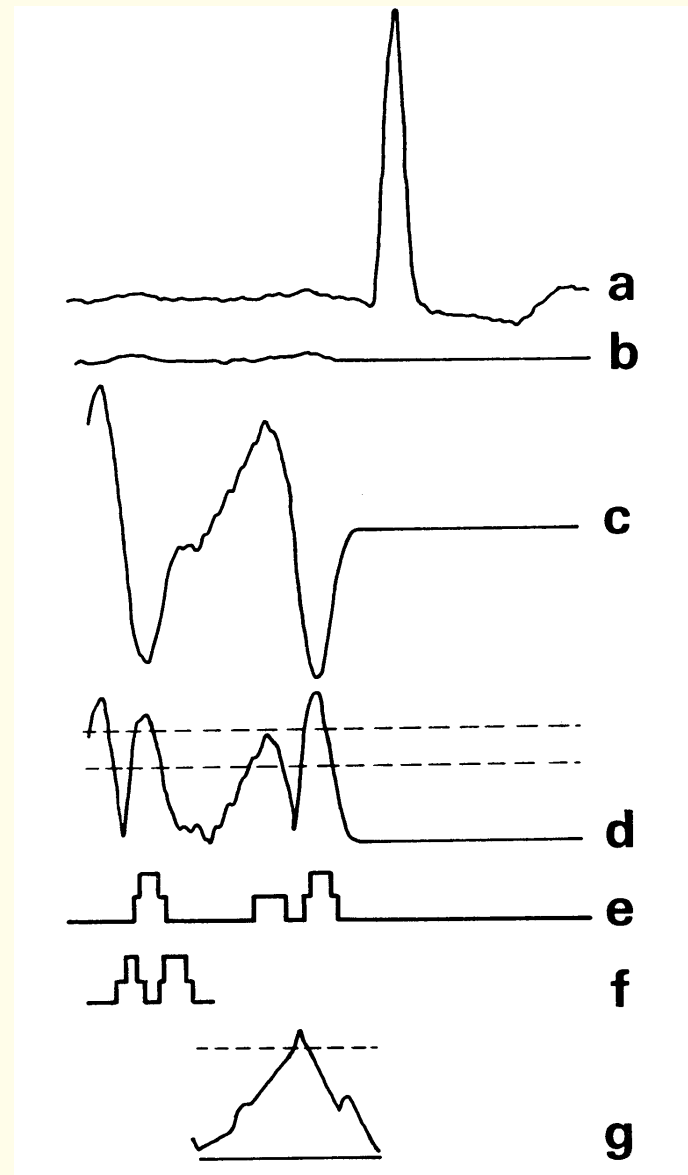


Figure 4.18: Illustration of the results at various stages of the Hengeveld and van Bommel method for P wave detection. From top to bottom: (a) the original ECG signal; (b) after replacement of the QRS with the baseline; (c) after bandpass filtering; (d) after rectification, with the dashed lines indicating the thresholds; (e) the thresholded ternary signal; (f) the ternary P wave template; and (g) result of cross-correlation between the signals in (e) and (f). Reproduced with permission from S.J. Hengeveld and J.H. van Bommel, Computer detection of P waves, *Computers and Biomedical Research*, 9:125–132, 1976. ©Academic Press.



Solution 2: Gritzali et al. proposed a common approach to detect the QRS, T, and P waves in multichannel ECG based upon the “length” transformation.

Given a set of ECG signals from N simultaneous channels

$x_1(t), x_2(t), \dots, x_N(t)$, length transformation defined as

$$L(N, w, t) = \int_t^{t+w} \sqrt{\sum_{j=1}^N \left(\frac{dx_j}{dt} \right)^2} dt, \quad (4.32)$$

w : width of time window for integration.



The procedure computes the total squared derivative of the signals across the various channels available, and integrates the summed quantity over a moving time window. P and T expected to be well-defined in at least one channel.



QRS first detected by applying a threshold to $L(N, w, t)$,

with w set equal to the average QRS width.

End points of QRS marked by a pulse.

QRS complexes replaced by iso-electric base-line.

Procedure repeated with w equal to average T duration:

T waves detected and marked.

Same steps repeated to detect P waves.

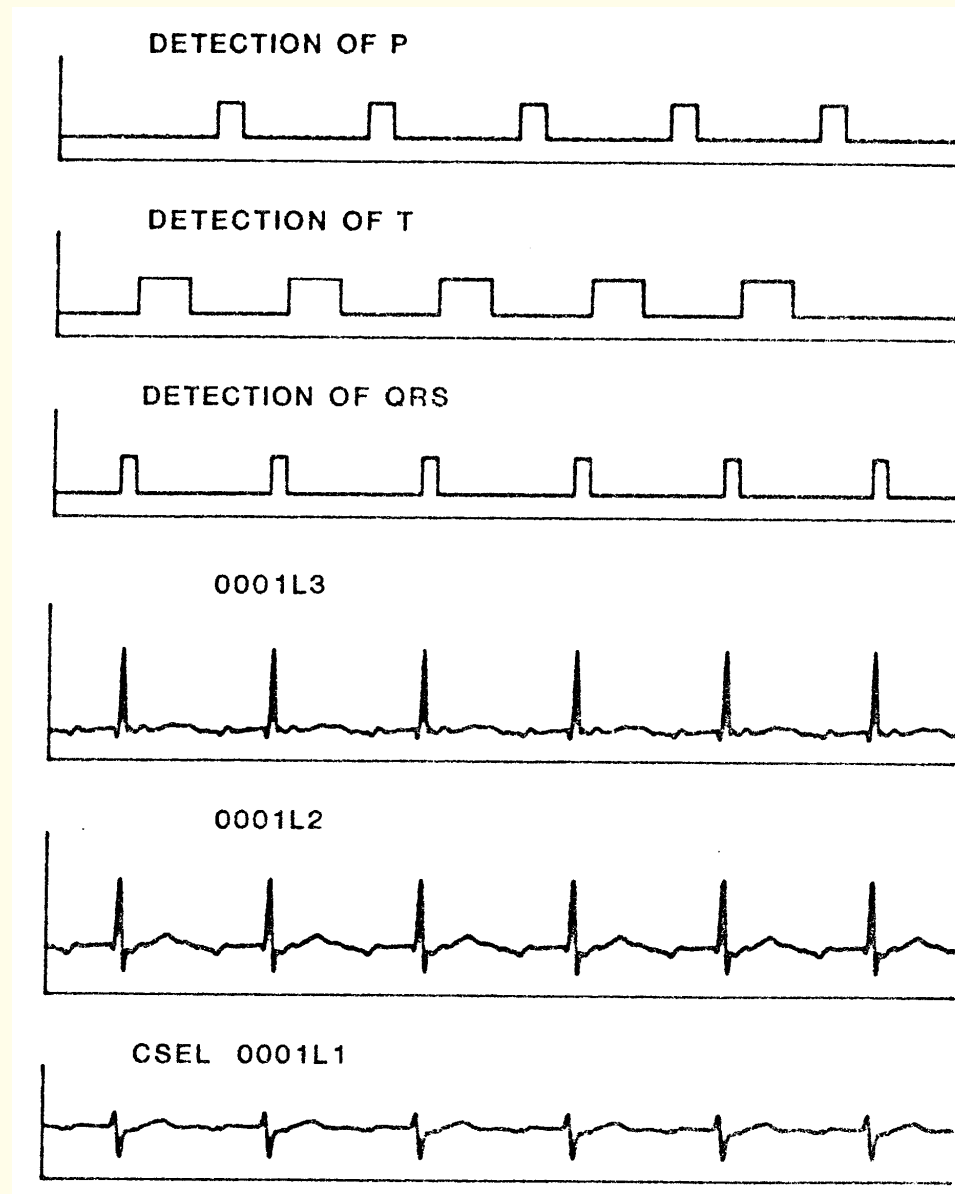


Figure 4.19: Detection of the P, QRS, and T waves in a three-channel ECG signal using the length transformation. The lower three traces show the three ECG channels. The upper three traces indicate the onset and end of the P, QRS, and T waves detected by the procedure in the form of pulse trains. The first P and the last T waves have not been processed. Reproduced with permission from F. Gritzali, G. Frangakis, G. Papakonstantinou, Detection of the P and T waves in an ECG, *Computers and Biomedical Research*, 22:83–91, 1989. ©Academic Press.



4.8 Homomorphic Filtering and the Complex Cepstrum

Linear filters are designed to separate signals that are

added together: given $y(t) = x(t) + \eta(t)$,

how to extract $x(t)$ only.



$$Y(\omega) = X(\omega) + \eta(\omega).$$

A linear filter will facilitate

the separation of $X(\omega)$ and $\eta(\omega)$,

if they have significant portions of their energies in

different frequency bands.



Suppose that we are presented with a signal that is the

product of two signals: $y(t) = x(t) p(t)$.

Multiplication or convolution property of the FT:

$$Y(\omega) = X(\omega) * P(\omega).$$

$*$: convolution in the frequency domain!

How would we be able to separate $x(t)$ from $p(t)$?



Furthermore, suppose we have $y(t) = x(t) * h(t)$:

passage of the glottal pulse train or random excitation $x(t)$

through the vocal-tract system

with the impulse response $h(t)$.

$$Y(\omega) = X(\omega) H(\omega).$$

How would we attempt to separate $x(t)$ and $h(t)$?



4.8.1 *Generalized linear filtering*

Given that linear filters are well established and understood, it is attractive to consider extending their application to signals combined by operations other than addition, especially by multiplication and convolution.



Interesting possibility to achieve this:

conversion of the operation combining the signals into
addition by one or more transformations.



Assume that the transformed signals occupy

different portions of the transform space;

use linear filters to separate them.

Inverses of the transformations take us back

to the original space of the signals.



Homomorphic system for multiplied signals:

$$y(t) = x(t) p(t). \quad (4.33)$$

$$\log[y(t)] = \log[x(t) p(t)] = \log[x(t)] + \log[p(t)];$$

$$x(t) \neq 0, p(t) \neq 0 \quad \forall t. \quad (4.34)$$

$$Y_l(\omega) = X_l(\omega) + P_l(\omega); \quad (4.35)$$

subscript l : FT applied to log-transformed signal.



Assuming that the logarithmic transformation
has not affected the separability of the
Fourier components of the two signals,
a linear filter — lowpass, highpass, or bandpass —
may be applied to $Y_l(\omega)$ to separate them.

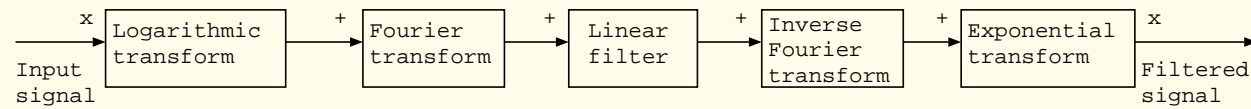


Figure 4.20: Operations involved in a multiplicative homomorphic system or filter. The symbol at the input or output of each block indicates the operation that combines the signal components at the corresponding step.



Useful in image processing: image modeled as the product of an illumination function and a transmittance or reflectance function.

The homomorphic filter facilitates separation of the illumination function and correction for nonuniform lighting: simultaneous dynamic range compression and contrast enhancement.



4.8.2 Homomorphic deconvolution

Problem: *Propose a homomorphic filter to separate two signals that have been combined through convolution.*

Solution: Convert convolution to addition.



$$y(t) = x(t) * h(t). \quad (4.36)$$

From the convolution property of the Fourier transform:

$$Y(\omega) = X(\omega) H(\omega). \quad (4.37)$$

Taking the complex logarithm of $Y(\omega)$:

$$\log[Y(\omega)] = \log[X(\omega)] + \log[H(\omega)]; \quad X(\omega) \neq 0, \quad H(\omega) \neq 0 \quad \forall \omega. \quad (4.38)$$

Note: $\log_e[X(\omega)] = \hat{X}(\omega) = \log_e[|X(\omega)|] + j\angle X(\omega).$

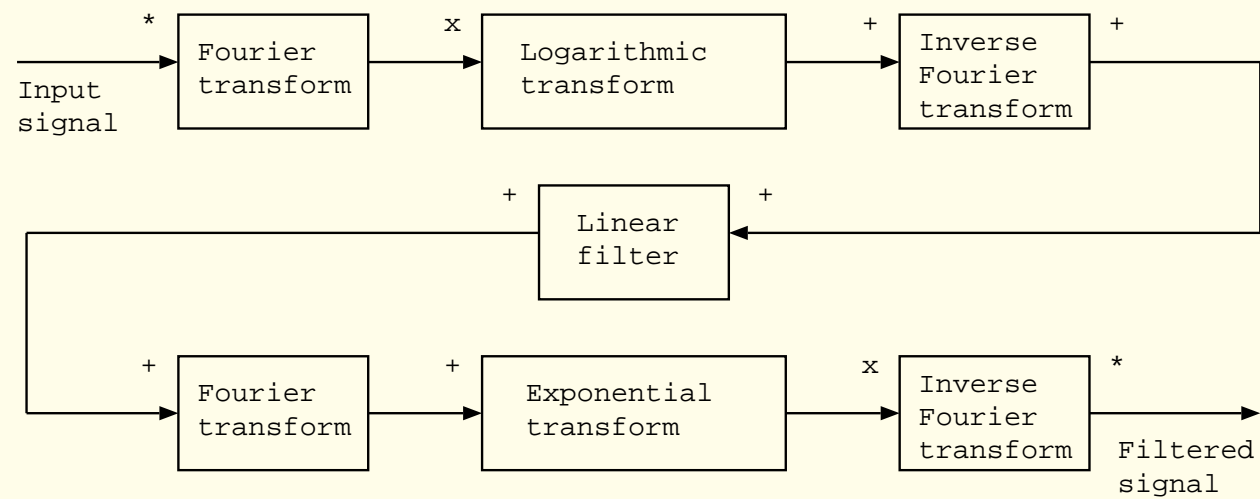


Figure 4.21: Operations involved in a homomorphic filter for convolved signals. The symbol at the input or output of each block indicates the operation that combines the signal components at the corresponding step.

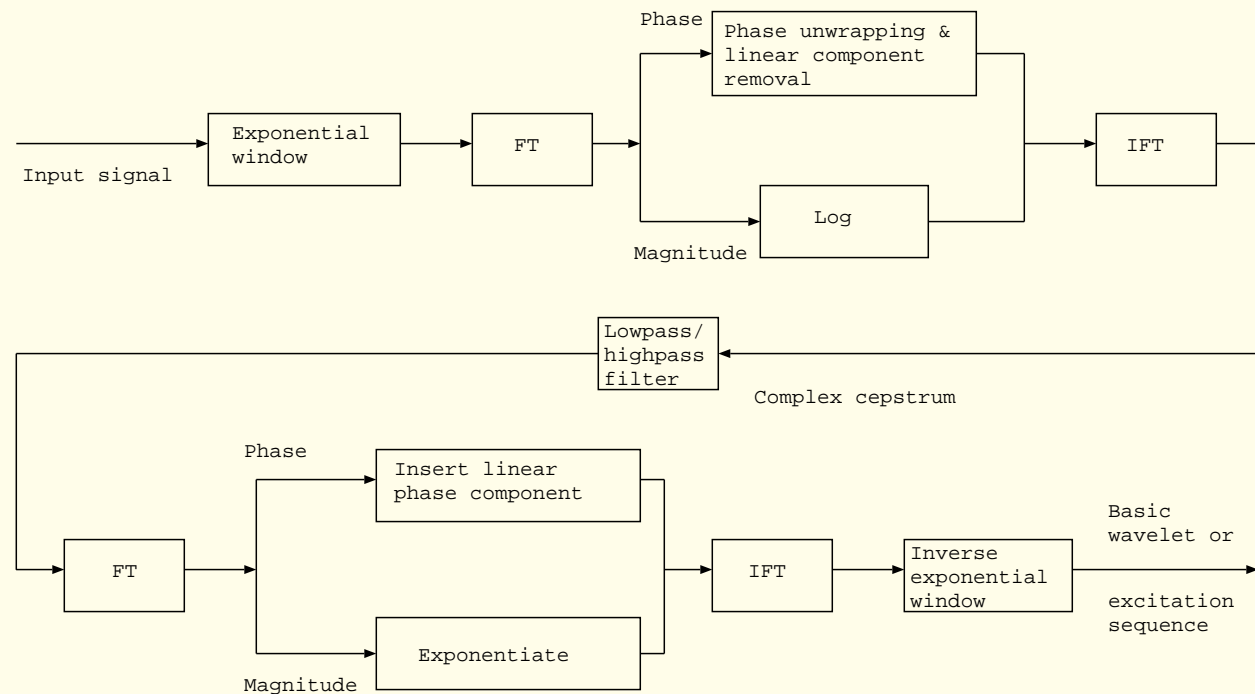


Figure 4.22: Detailed block diagram of the steps involved in deconvolution of signals using the complex cepstrum.



A linear filter may be used to separate the

transformed components of x and h ,

assuming that they are separable in the transform space.

Inverses of the transformations: back to the original domain.

General formulation of the homomorphic filter is

in terms of the z -transform.



4.8.3 Extraction of the vocal-tract response

Problem: *Design a homomorphic filter*

to extract the basic wavelet corresponding to the

vocal-tract response from a voiced-speech signal.

Solution:

Voiced speech is generated by excitation of the vocal tract, as it is held in a particular form, with a glottal waveform that may be approximated as a series of pulses.



Voiced-speech signal: $y(n) = x(n) * h(n)$,

$x(n)$: glottal waveform or excitation sequence,

$h(n)$: impulse response of the vocal tract (basic wavelet).

Glottal excitation sequence: $x(n) = p(n) * g(n)$,

$p(n)$: train of ideal impulses (Dirac delta functions),

$g(n)$: a smoothing function —

this part will be neglected in our discussions.



The complex cepstrum: is the inverse z -transform of the complex logarithm of the z -transform of the input signal.

“cepstrum”: transposing the syllables of “spectrum”.

$y(n)$: input signal; $Y(z)$ its z -transform;

complex cepstrum $\hat{y}(n)$ defined as

$$\hat{y}(n) = \frac{1}{2\pi j} \oint \log[Y(z)] z^{n-1} dz. \quad (4.39)$$



Contour integral performs inverse z -transform;

evaluated within an annular region in the complex z -plane

where $\hat{Y}(z) = \log[Y(z)]$ is single-valued and analytic.

Unit of n in $\hat{y}(n)$ in the cepstral domain:

quefrequency — switching the syllables in the term frequency.



Given $y(n) = x(n) * h(n)$, it follows that

$$\hat{Y}(z) = \hat{X}(z) + \hat{H}(z) \quad \text{or} \quad \hat{Y}(\omega) = \hat{X}(\omega) + \hat{H}(\omega). \quad (4.40)$$

The complex cepstra of the signals are related as

$$\hat{y}(n) = \hat{x}(n) + \hat{h}(n). \quad (4.41)$$



The $\hat{}$ symbol over a function of z or ω :

complex logarithm of the corresponding function of z or ω .

$\hat{}$ symbol over a function of time (n) :

complex cepstrum of the corresponding signal.

If $y(n)$ is real, its complex cepstrum $\hat{y}(n)$ is *real*;

the prefix *complex* indicates that the preceding

z and logarithmic transformations are complex functions.

The complex cepstrum is a function of time.



Important consideration in the evaluation of the complex logarithm of $Y(z)$ or $Y(\omega)$: phase.

Phase spectrum computed as its principal value

in the range $0 - 2\pi$, given by $\tan^{-1} \left[\frac{\text{imaginary}\{Y(\omega)\}}{\text{real}\{Y(\omega)\}} \right]$,

will have discontinuities that will conflict with the

requirements of the inverse z -transformation or IFT.



$Y(\omega)$ needs to be separated into its magnitude and phase components, logarithmic operation applied to the magnitude, the phase corrected to be continuous by adding correction factors of $\pm 2\pi$ at discontinuities larger than π , and the two components combined before inverse transformation.



Correcting phase spectrum as above: *phase unwrapping*.

Linear phase term in the spectrum of the input signal may

cause rapidly decaying oscillations in the cepstrum:

advisable to remove the linear phase term;

may be added to the filtered result, as a time shift.



Exponential signals:

signals that have a rational z -transform:

expressed as ratios of polynomials in z ; Appendix 2.

Effectively represented as weighted sums of exponentials.

Properties of the complex cepstrum of an exponential signal:

- $\hat{y}(n)$ is of infinite duration even if $y(n)$ is of finite duration, and exists for $-\infty < n < \infty$ in general.



- The complex cepstrum $\hat{y}(n)$ decays at least as fast as $\frac{1}{n}$.
- If $y(n)$ is a minimum-phase signal — having all poles or zeros inside the unit circle in the z -plane — then $\hat{y}(n) = 0$ for $n < 0$.
- If $y(n)$ is a maximum-phase signal — having no poles or zeros inside the unit circle in the z -plane — then $\hat{y}(n) = 0$ for $n > 0$.



Limiting ourselves to causal signals of finite energy,

we need not consider the presence of

poles on or outside the unit circle in the z -plane.

However, the z -transform of a finite-energy signal may have

zeros outside the unit circle: composite, mixed-phase signal.



A composite signal may be separated into its minimum-phase component and maximum-phase component by extracting the causal part ($n > 0$) and anti-causal part ($n < 0$) of its complex cepstrum, followed by the inverse procedures.

Composite signal =

minimum-phase component * maximum-phase component.



Effect of echoes or repetitions of a wavelet:

Consider a signal $y(n) = x(n) * h(n)$, where

$$x(n) = \delta(n) + a \delta(n - n_0), \quad (4.42)$$

with a and n_0 being two constants.



$$y(n) = h(n) + a h(n - n_0). \quad (4.43)$$

Signal has two occurrences of the basic wavelet $h(n)$

at $n = 0$ and $n = n_0$.

a : magnitude of the second appearance of the basic wavelet,

called an *echo* in seismic applications,

n_0 : delay of echo, or pitch in a voiced-speech signal.

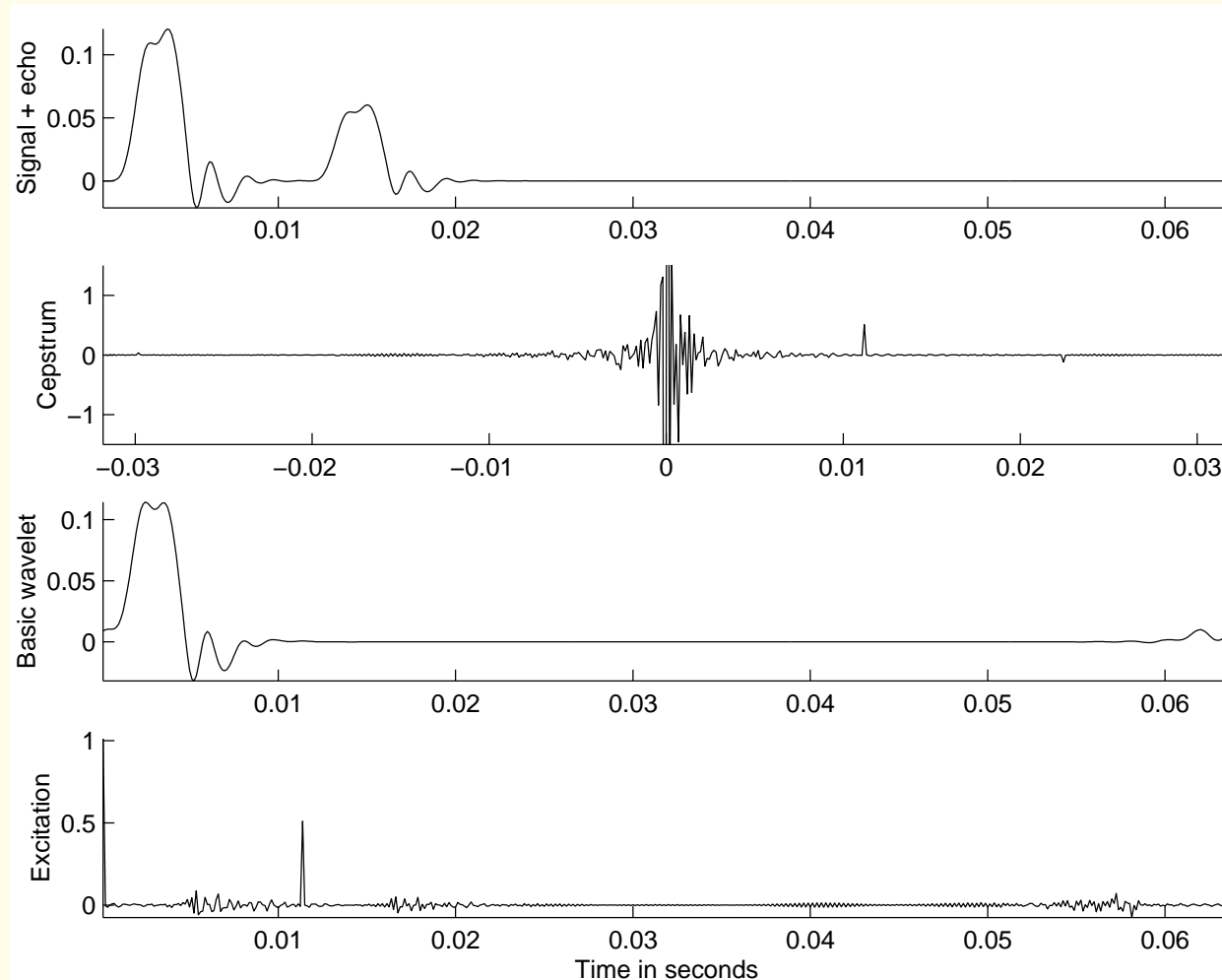


Figure 4.23: From top to bottom: a composite signal with a wavelet and an echo; the complex cepstrum of the signal (the amplitude axis has been stretched to make the peaks at the echo time and its multiples more readily visible; values outside the range ± 1.5 have been clipped); the basic wavelet extracted by shortpass filtering the cepstrum; and the excitation sequence extracted by longpass filtering the cepstrum.

Echo: $a = 0.5$, $n_0 = 0.01125$ s.



Taking the z -transform of the signal, we have

$$Y(z) = (1 + a z^{-n_0}) H(z). \quad (4.44)$$

If the z -transform is evaluated on the unit circle,

we get the Fourier-transform-based expression

$$Y(\omega) = [1 + a \exp(-j\omega n_0)] H(\omega). \quad (4.45)$$

Taking the logarithm, we have

$$\hat{Y}(\omega) = \hat{H}(\omega) + \log[1 + a \exp(-j\omega n_0)]. \quad (4.46)$$



If $a < 1$, the \log term may be expanded in a power series:

$$\begin{aligned}\hat{Y}(\omega) = & \hat{H}(\omega) + a \exp(-j\omega n_0) - \frac{a^2}{2} \exp(-2j\omega n_0) \\ & + \frac{a^3}{3} \exp(-3j\omega n_0) - \dots\end{aligned}\quad (4.47)$$

Taking the inverse Fourier transform:

$$\begin{aligned}\hat{y}(n) = & \hat{h}(n) + a \delta(n - n_0) - \frac{a^2}{2} \delta(n - 2n_0) \\ & + \frac{a^3}{3} \delta(n - 3n_0) - \dots\end{aligned}\quad (4.48)$$



The complex cepstrum of a signal with

a basic wavelet and an echo

= complex cepstrum of the basic wavelet

+ a series of impulses at echo delay and integral multiples.

Amplitudes of impulses proportional to echo amplitude (a)

and decay for the higher-order repetitions (if $a < 1$).



If there are multiple echoes or repetitions of a basic wavelet, the cepstrum will possess multiple impulse trains.

Voiced-speech signal: location of first peak = pitch.



Assuming that the complex cepstrum of the basic wavelet decays to negligible values before the first impulse

$a \delta(n - n_0)$ related to the echo, $\hat{h}(n)$ may be extracted

from the complex cepstrum $\hat{y}(n)$ of the composite signal

by a simple window that has unit value for $|n| < n_c < n_0$,

n_c being the cutoff point.

The inverse procedures will yield $h(n)$.



If $a \geq 1$, the complex cepstrum will have

a train of impulses on its negative time axis:

at $(n + kn_0)$, $k = 1, 2, \dots$

An exponential weighting sequence may be used

to achieve the condition $a < 1$.

If the weighted signal satisfies the minimum-phase

condition, the cepstrum will be causal.



The power cepstrum: square of the inverse z -transform of the logarithm of the squared magnitude of the z -transform of the given signal.

In practice, z -transform replaced by the FFT.

Advantage: phase unwrapping not required.

The power cepstrum does not facilitate separation of the components of the signal.



Evaluating the inverse z -transform on the unit circle

in the z -plane, the power cepstrum $\hat{y}_p(n)$

of a signal $y(n)$ is defined as

$$\hat{y}_p(n) = \left\{ \frac{1}{2\pi j} \oint \log |Y(z)|^2 z^{n-1} dz \right\}^2. \quad (4.49)$$



Consider $y(n) = x(n) * h(n)$.

$$|Y(z)|^2 = |X(z)|^2 |H(z)|^2.$$

$$\log |Y(z)|^2 = \log |X(z)|^2 + \log |H(z)|^2.$$

$$\hat{y}_p(n) = \hat{x}_p(n) + \hat{h}_p(n). \quad (4.50)$$



$\hat{h}_p(n)$: power cepstrum of the basic wavelet.

$\hat{x}_p(n)$: power cepstrum of the excitation signal.

Cross-product term neglected: will be zero if the two component power cepstra occupy non-overlapping ranges in the cepstral domain (quefreny).

Final squaring operation in Equation 4.49 may be omitted: cross-term does not arise, and Equation 4.50 is valid.



Power cepstrum does not retain phase information.

Useful in the identification of the presence of echoes.

Power cepstrum related to complex cepstrum as

$$\hat{y}_p(n) = [\hat{y}(n) + \hat{y}(-n)]^2. \quad (4.51)$$



Consider a signal with two occurrences of a basic wavelet:

$$y(n) = h(n) + a h(n - n_0).$$

$$|Y(z)|^2 = |H(z)|^2 |1 + a z^{-n_0}|^2. \quad (4.52)$$



Taking the logarithm and substituting $z = \exp(j\omega)$:

$$\begin{aligned}\log |Y(\omega)|^2 &= \log |H(\omega)|^2 + \log[1 + a^2 + 2a \cos(\omega n_0)] \\ &= \log |H(\omega)|^2 + \log(1 + a^2) \\ &\quad + \log \left(1 + \frac{2a}{1 + a^2} \cos(\omega n_0) \right).\end{aligned}\tag{4.53}$$



Log (PSD) has sinusoidal components or ripples

due to the echo: difficulties in spectral analysis.

Amplitudes and frequencies of the ripples related to

amplitude a of the echo and its time delay n_0 .



Illustration of application: Voiced-speech =

slow vocal-tract response * fast glottal pulse train.

Deconvolve to estimate vocal-tract response.

Shortpass filter the complex cepstrum; $n_c = 0.003125$ s.

Signal padded with zeros to twice its duration;

exponential weighting with $\alpha = 0.99$.

Facilitates spectral analysis without the

effect of the quasi-periodic repetitions in the speech signal.

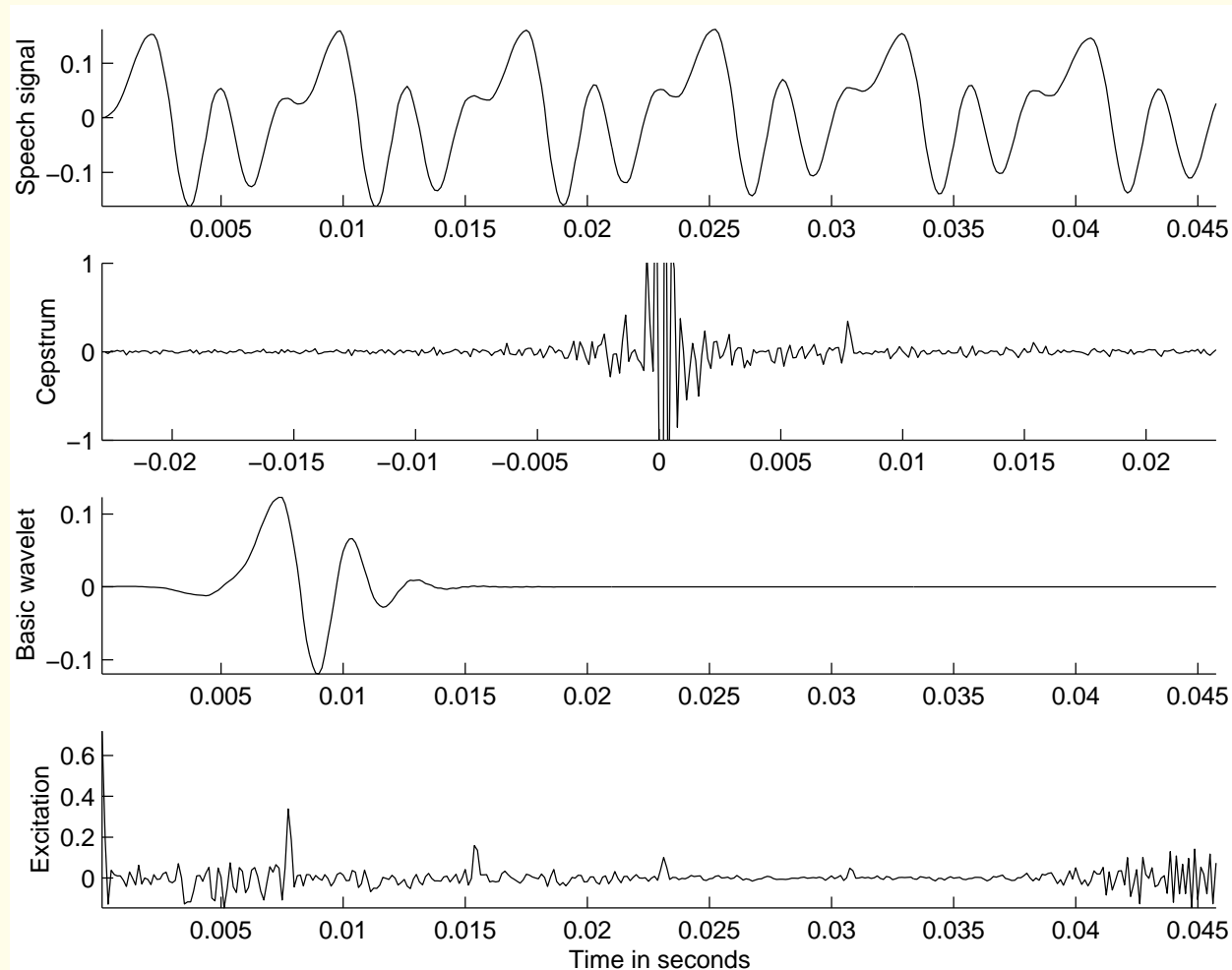


Figure 4.24: From top to bottom: a segment of a voiced-speech signal over six pitch periods (extracted from the signal shown in Figure 1.30 and lowpass filtered); the complex cepstrum of the signal (the amplitude axis has been stretched to make the peaks at the echo time and its multiples more readily visible; values outside the range ± 1.0 have been clipped); the (shifted) basic wavelet extracted by shortpass filtering the cepstrum; and the excitation sequence extracted by longpass filtering the cepstrum.



4.9 Application: ECG Rhythm Analysis

Problem: *Describe a method to measure the heart rate and average RR interval from an ECG signal.*

Solution:

QRS detection using the Pan-Tompkins method.

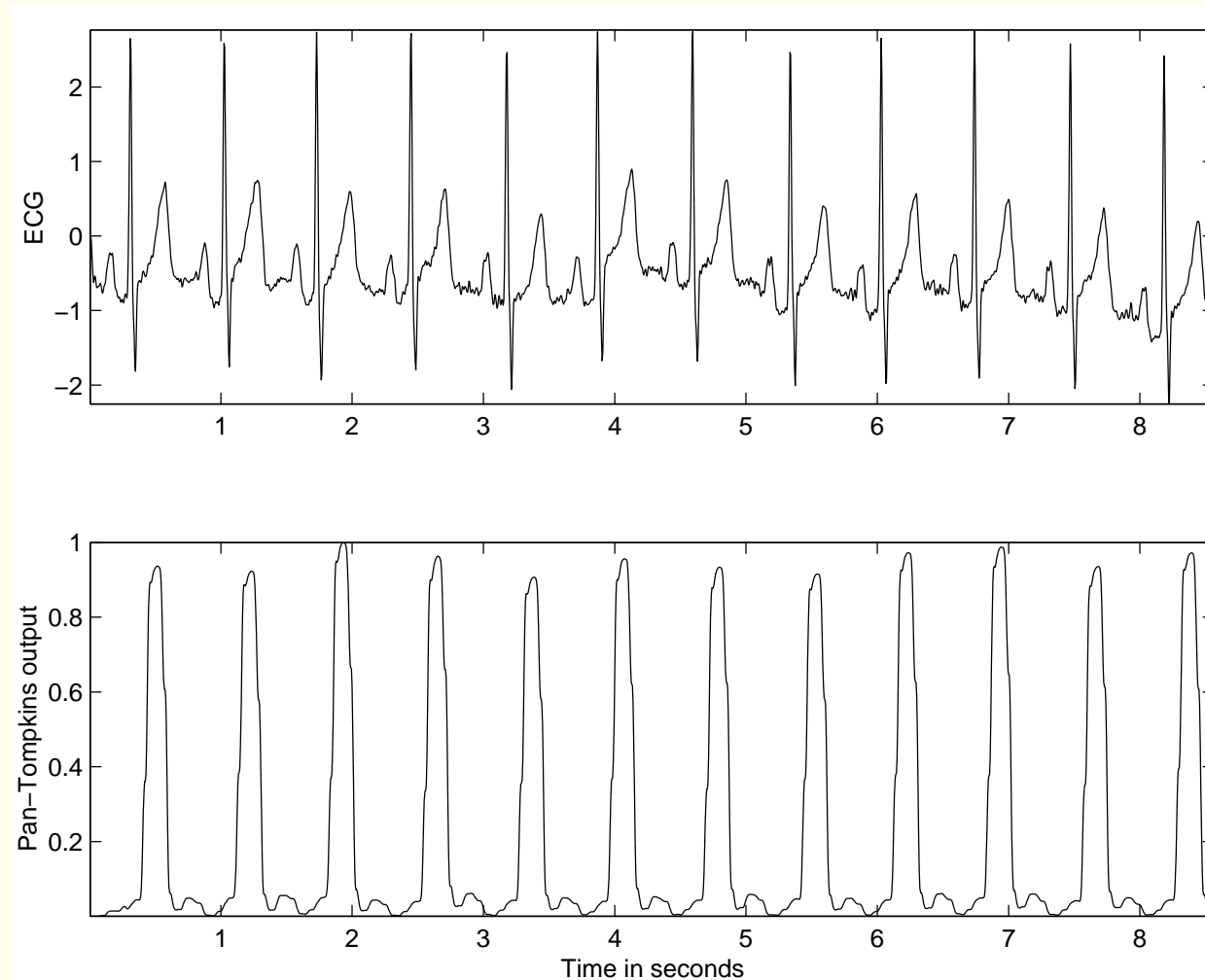


Figure 4.25: Results of the Pan-Tompkins algorithm. Top: lowpass-filtered version of the ECG signal shown in Figure 3.5. Bottom: normalized result of the final integrator.

Average RR interval = 716 ms; effective heart rate = 84 bpm.

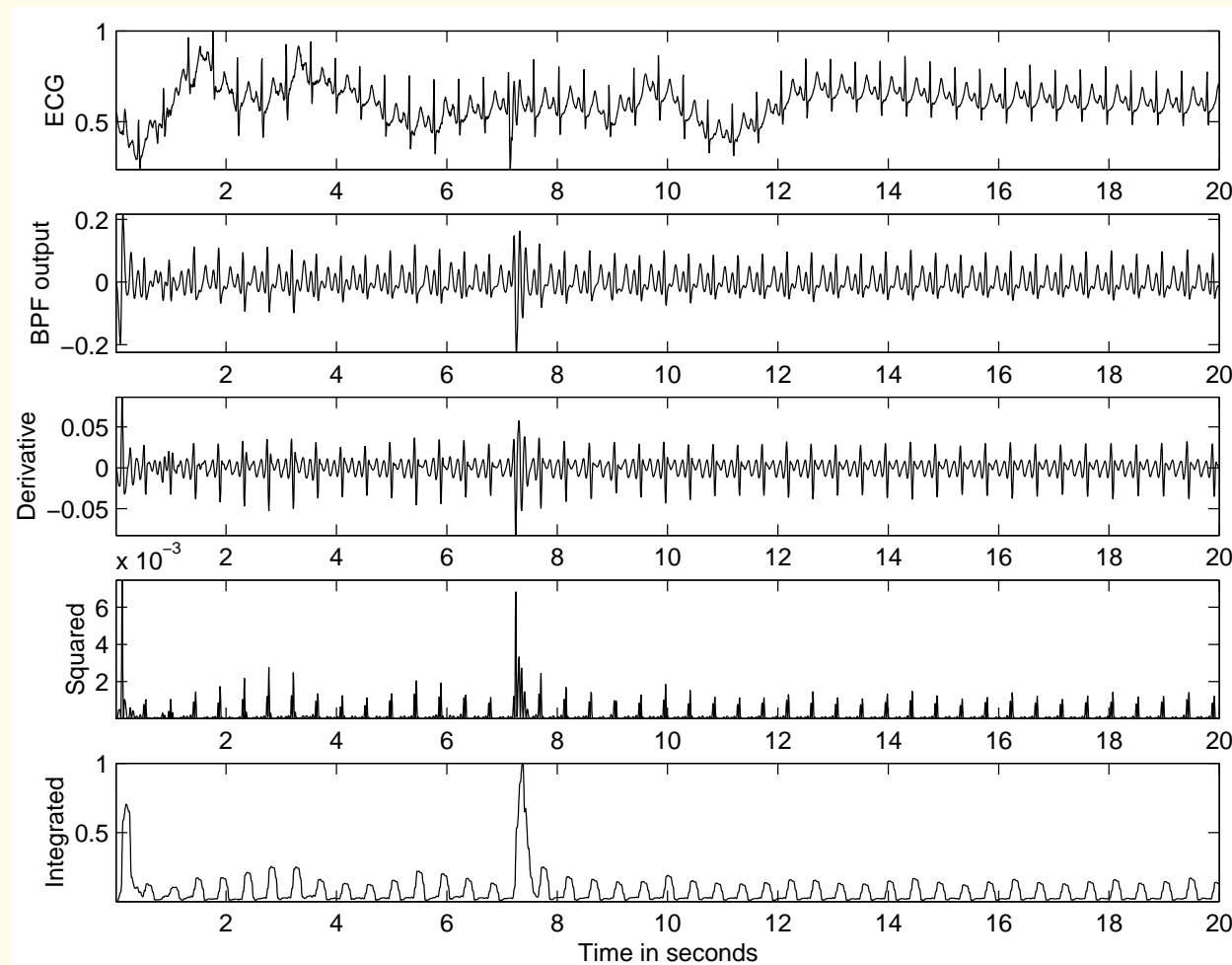


Figure 4.26: Results of the Pan-Tompkins algorithm with a noisy ECG signal. From top to bottom: ECG signal sampled at 200 Hz ; output of the bandpass filter (BPF); output of the derivative-based operator; the result of squaring; and normalized result of the final integrator.

Average RR interval = 446.6 ms ; effective heart rate = 137 bpm .



4.10 Application: Identification of Heart Sounds

Problem: *Outline a signal processing algorithm*

to identify S1 and S2 in a PCG signal, and further

segment the PCG signal into its systolic and diastolic parts.

The ECG and carotid pulse signals are available

for reference.



Solution: Use ECG and carotid pulse signals to

demarcate the onset of S1 and S2 in the PCG.

Beginning of S1 in PCG = beginning of QRS in ECG.

Detect QRS using the Pan-Tompkins method.



Detection of the beginning of S2:

Let the heart rate be HR bpm.

Pre-ejection period PEP = interval from the

beginning of QRS to onset of carotid upstroke.

Rate-corrected PEP : $PEPC = PEP + 0.4 HR$,

with the periods in ms .

$PEPC$: $131 \pm 13 ms$ for normal adults.



Ejection time ET = interval from
onset of carotid upstroke to dicrotic notch.

Rate-corrected ejection time in ms :

$$ETC = ET + 1.6 HR,$$

range of $395 \pm 13 ms$ for normal adult males and

$415 \pm 11 ms$ for normal adult females.



Using $PEPC_{\max} = 144 \text{ ms}$ and $HR_{\min} = 60 \text{ bpm}$,

we get $PEP_{\max} = 120 \text{ ms}$.

With $HR_{\min} = 60 \text{ bpm}$ and $ETC_{\max} = 425 \text{ ms}$,

we get $ET_{\max} = 325 \text{ ms}$.

With these parameters, the maximum interval

between QRS and dicrotic notch is 380 ms .



Procedure proposed by Lehner and Rangayyan

for detection of the dicrotic notch:

search the output of the second-derivative-based method

in a 500 ms interval after the QRS (Section 4.3.3).

After the dicrotic notch is detected, subtract the time delay

between the beginning of S2 and D.

Average S2 – D delay: 42.6 ms ; SD: 5.0 ms .



Procedure to segment a PCG signal into its

systolic and diastolic parts:

1. Use Pan-Tompkins method to locate the QRS complexes in the ECG.
2. Identify one period of PCG as the interval between two successive QRS locations.

Subtract the delay introduced by the filters in the Pan-Tompkins method.



3. Use the Lehner and Rangayyan method to detect the dicrotic notch in the carotid pulse.
4. Standardized S2 – D delay = mean plus two SD = *52.6 ms*.
Subtract the delay from the detected dicrotic notch Location to obtain the onset of S2.
5. S1 – S2 interval is the systolic part of the PCG cycle.
6. Interval between S2 and next S1 is the diastolic part.

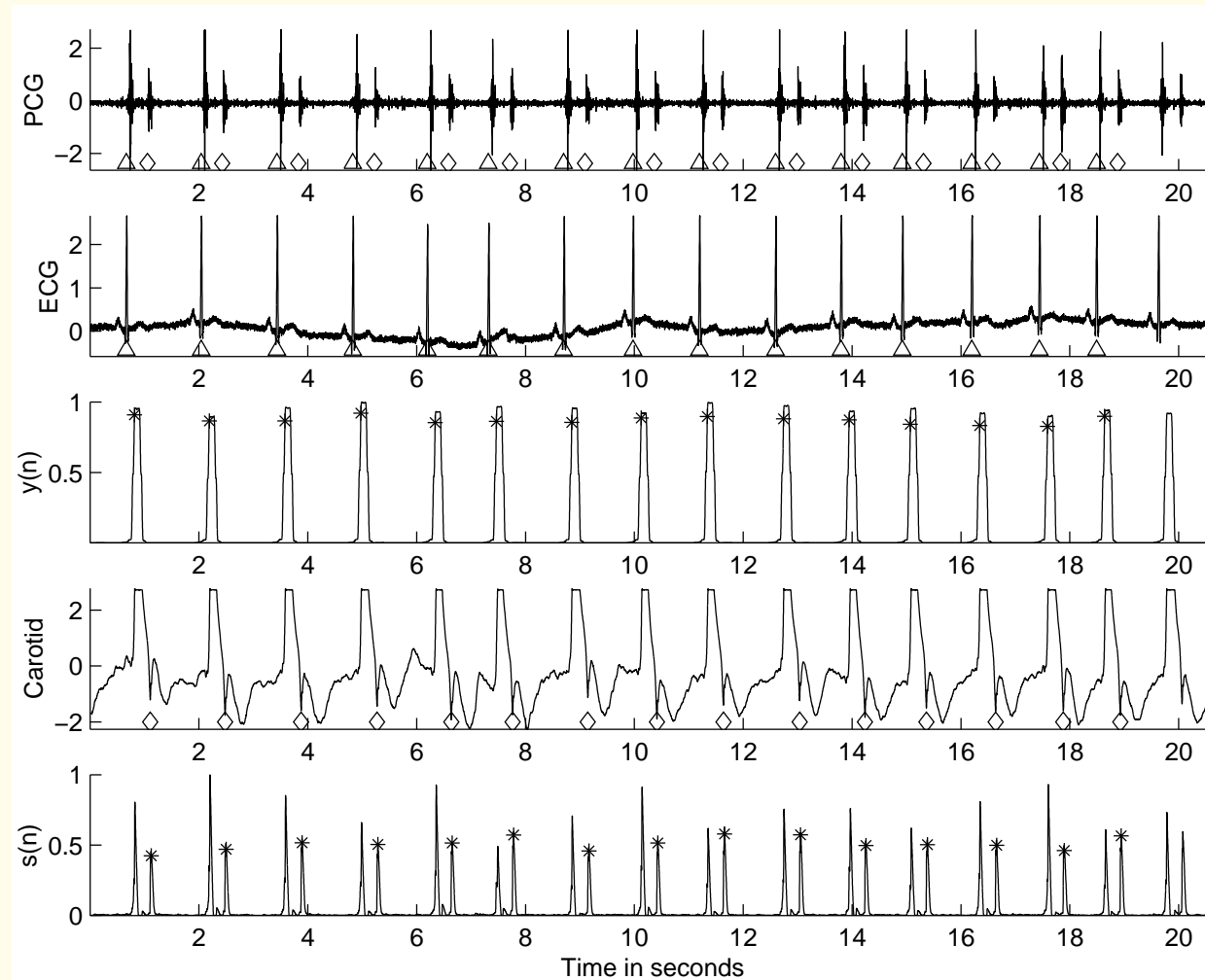


Figure 4.27: Results of segmentation of a PCG signal into systolic and diastolic parts using the ECG and carotid pulse signals for reference. From top to bottom: the PCG signal of a normal subject (male subject, 23 years); the ECG signal; $y(n)$, the output of the Pan-Tompkins method for detection of the QRS after normalization to the range $(0, 1)$; the carotid pulse signal; $s(n)$, the output of the Lehner and Rangayyan method for detection of the dicrotic notch, normalized to the range $(0, 1)$. The peaks detected in the outputs of the two methods have been identified with * marks. The QRS and D positions have been marked with the triangle and diamond symbols, respectively. The S1 and S2 positions are marked on the PCG trace with triangles and diamonds, respectively. The last cardiac cycle was not processed.

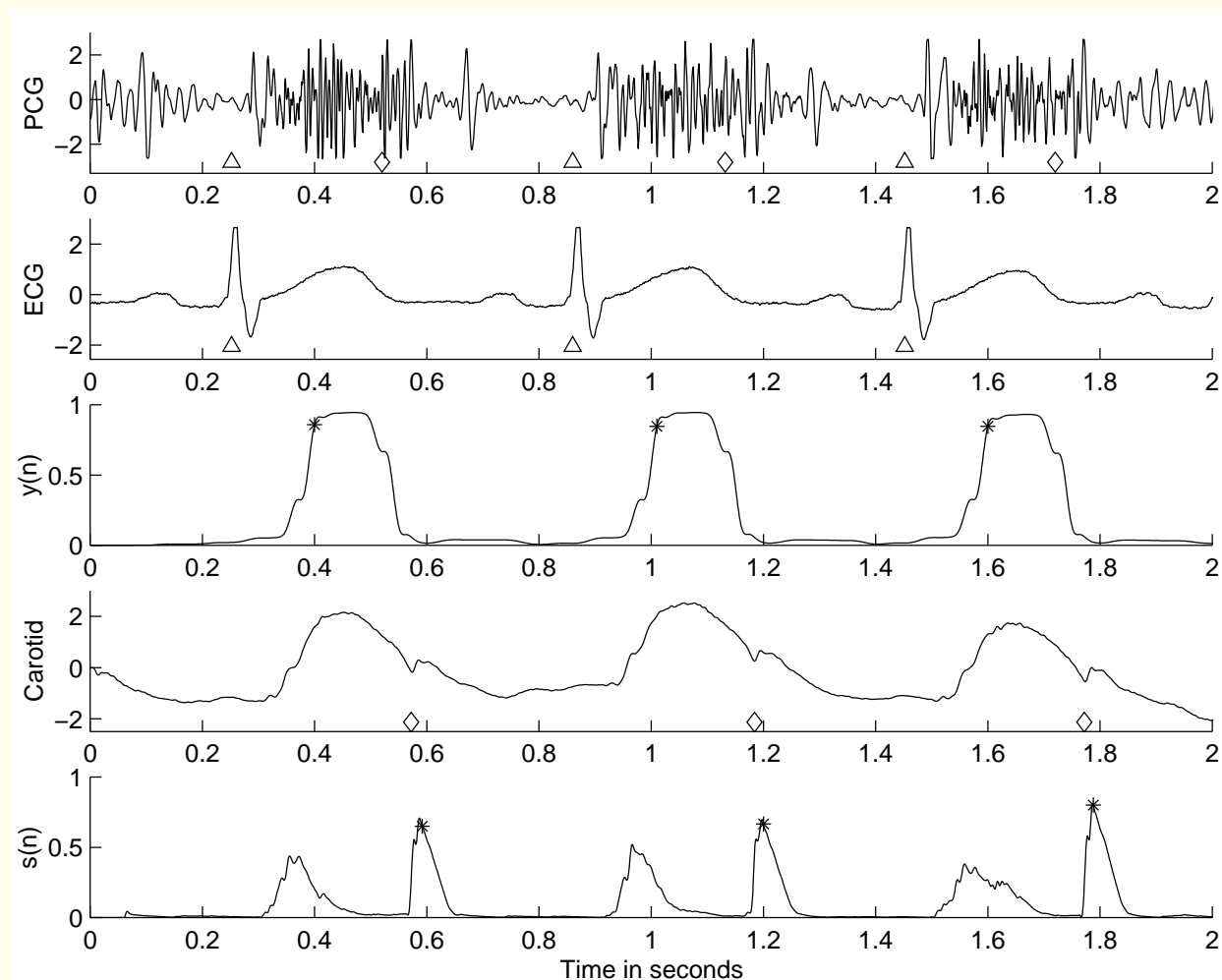


Figure 4.28: Results of segmentation of a PCG signal into systolic and diastolic parts using the ECG and carotid pulse signals for reference. From top to bottom: the PCG signal of a patient with a split S2, systolic ejection murmur, and opening snap of the mitral valve (female patient, 14 months); the ECG signal; $y(n)$, the output of the Pan-Tompkins method for detection of the QRS; the carotid pulse signal; $s(n)$, the output of the Lehner and Rangayyan method for detection of the diastolic notch. The peaks detected in the outputs of the two methods have been identified with * marks. The QRS and D positions have been marked with the triangle and diamond symbols, respectively. The S1 and S2 positions are marked on the PCG trace with triangles and diamonds, respectively.



4.11 Application: Detection of the Aortic Component of the Second Heart Sound

A2 caused by closure of aortic valve at end of systole.

P2 caused by closure of pulmonary valve.

Relative timing of A2 and P2 depends upon the pressure differences across the corresponding valves in the left and right ventricular circulatory systems.



In a normal individual, timing of P2 varies with respiration; the timing of A2 is independent of respiration.

Pulmonary pressure decreased during inspiration,

leading to a delayed closure of the pulmonary valve and

increased (audible and visible) gap between A2 and P2.

Gap closed; A2, P2 overlap during expiration (normally).

A2 and P2 have individual durations of about *50 ms*.



Normal inspiratory A2 - P2 gap $\sim 30 - 40 \text{ ms}$;

abnormal splits as long as 100 ms have been recorded.

Split in S2 $> 40 \text{ ms}$ during sustained expiration: abnormal.

Complete right bundle-branch block could cause

delayed activation of the right ventricle:

delayed pulmonary valve closure, delayed P2, split S2.

Other conditions causing split S2: pulmonary stenosis,

atrial septal defect, ventricular septal defect.



Left bundle-branch block: delayed contraction of the

left ventricle, delayed aortic valve closure

(w.r.t. right ventricle and the pulmonary valve),

A2 appears after P2 — *reversed splitting*.

Aortic insufficiency and abnormally early closure of the

pulmonary valve also cause reversed splitting of S2.

Identification of A2 and P2 could assist in the

diagnosis of several cardiovascular defects and diseases.



Problem: *Given that the second heart sound S_2 is made up of an aortic component A_2 and a pulmonary component P_2 with variable temporal relationships, propose a method to detect only A_2 .*



Solution:

Detect dicrotic notch in the carotid pulse signal.

Detect the beginning of S2.

Direct relationship between aortic valve closure,
aortic incisura, and consequently the dicrotic notch.

Detect and segment S2 over several cardiac and
respiratory cycles.



Perform synchronized averaging of S2:

A2 appears at the same instant in every S2 segment;

strengthened by synchronized averaging.

P2 appears at different times;

cancelled or suppressed by averaging.

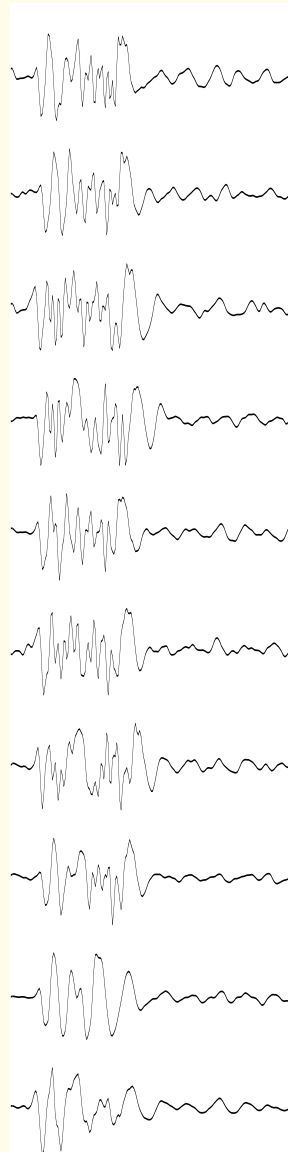


Figure 4.29: Synchronized averaging of S2 to detect A2 and suppress P2. The figure displays nine consecutive segments of S2 (duration = 300 ms) of a patient (female, 7 years) with atrial septal defect leading to a variable split in S2. The trace at the bottom is the average of S2 over 21 consecutive beats.



4.12 Appendix

Derivation of the Transfer Function of the Matched Filter [Schwartz]

Let the signal $x(t)$ be the input to the matched filter.

Fourier transform of $x(t)$:

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \exp(-j\omega t) dt. \quad (4.54)$$



Output of the matched filter:

$$\begin{aligned} s(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) H(\omega) \exp(+j\omega t) d\omega \\ &= \int_{-\infty}^{\infty} X(f) H(f) \exp(+j 2\pi f t) df. \quad (4.55) \end{aligned}$$



Consider the presence of white noise at the input, with PSD

$$G(f) = \frac{n_0}{2}. \quad (4.56)$$

Output noise PSD:

$$G_n(f) = \frac{n_0}{2} |H(f)|^2. \quad (4.57)$$

Average output noise power:

$$N = \frac{n_0}{2} \int_{-\infty}^{\infty} |H(f)|^2 df. \quad (4.58)$$

RMS noise in the absence of any signal = \sqrt{N} .



Magnitude of instantaneous output signal at $t = t_0$:

$$A = |s(t_0)| = \left| \int_{-\infty}^{\infty} X(f) H(f) \exp(+j 2\pi f t_0) df \right|. \quad (4.59)$$

SNR at the output = $\frac{A}{\sqrt{N}}$.

To maximize the SNR, we could maximize

$$\frac{A^2}{N} = \frac{\text{instantaneous peak power of signal}}{\text{noise mean power}} = \text{peak-power SNR.}$$



For a given signal $x(t)$, the total energy is a constant:

$$E = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |X(f)|^2 df. \quad (4.60)$$

Consider the ratio

$$\frac{A^2}{EN} = \frac{\left| \int_{-\infty}^{\infty} X(f) H(f) \exp(+j 2\pi f t_0) df \right|^2}{\frac{n_0}{2} \int_{-\infty}^{\infty} |X(f)|^2 df \int_{-\infty}^{\infty} |H(f)|^2 df}. \quad (4.61)$$

Consider maximizing this quantity.



Schwarz's inequality for complex functions $A(f)$ and $B(f)$:

$$\left| \int_{-\infty}^{\infty} A(f) B(f) df \right|^2 \leq \left[\int_{-\infty}^{\infty} |A(f)|^2 df \right] \left[\int_{-\infty}^{\infty} |B(f)|^2 df \right] . \quad (4.62)$$

For real functions $a(t)$ and $b(t)$:

$$\left[\int_{-\infty}^{\infty} a(t) b(t) dt \right]^2 \leq \left[\int_{-\infty}^{\infty} a^2(t) dt \right] \left[\int_{-\infty}^{\infty} b^2(t) dt \right] . \quad (4.63)$$



For vectors \bar{a} and \bar{b} :

$$|\bar{a} \cdot \bar{b}| \leq |\bar{a}| |\bar{b}|, \quad (4.64)$$

$$|\bar{a} + \bar{b}| \leq |\bar{a}| + |\bar{b}|. \quad (4.65)$$

Equality achieved if $\bar{a} = K \bar{b}$, i.e., \bar{a} and \bar{b} are collinear,

or if $a(t) = K b(t)$, or if $A(f) = K B^*(f)$;

K is a real constant.



The inequality in Equation 4.62 can be applied to the measure in Equation 4.61 by considering

$A(f) = X(f) \exp(+j 2\pi f t_0)$ and $B(f) = H(f)$. Then,

$$\frac{n_0 A^2}{2 E N} \leq 1, \quad (4.66)$$



$$\left| \int_{-\infty}^{\infty} X(f) H(f) \exp(+j 2\pi f t_0) df \right|^2$$

$$\leq \int_{-\infty}^{\infty} |X(f)|^2 df \int_{-\infty}^{\infty} |H(f)|^2 df . \quad (4.67)$$

The left-hand side is the instantaneous output power A^2 .



The ratio in Equation 4.61 is maximized with equality:

$$\begin{aligned} H(f) &= K [X(f) \exp(+j 2\pi f t_0)]^* \\ &= K X^*(f) \exp(-j 2\pi f t_0), \end{aligned} \quad (4.68)$$

which leads to maximal peak output SNR.



Taking the inverse Fourier transform:

$$h(t) = K x[-(t - t_0)]. \quad (4.69)$$

Therefore, the impulse response of the matched filter
= a scaled, reversed, and shifted version
of the signal of interest.



Output of the matched filter:

$$\begin{aligned}
 s(t) &= \int_{-\infty}^{\infty} X(f) H(f) \exp(+j 2\pi f t) df \\
 &= \int_{-\infty}^{\infty} X(f) K X^*(f) \exp(-j 2\pi f t_0) \exp(+j 2\pi f t) df \\
 &= K \int_{-\infty}^{\infty} |X(f)|^2 \exp[+j 2\pi f (t - t_0)] df \\
 &= K \phi(t - t_0).
 \end{aligned} \tag{4.70}$$

The last relationship based on FT [ACF] = PSD.



Note: FT [$x(-t)$] = $X^*(f)$;

FT [$x(t - \tau)$] = $\exp(-j 2\pi f \tau) X(f)$.



Exponential sequences and the complex cepstrum

[Oppenheim and Schafer]

Exponential sequences are sequences that have

rational z -transforms:

$$X(z) = A z^r \frac{\prod_{k=1}^{m_i} (1 - a_k z^{-1}) \prod_{k=1}^{m_o} (1 - b_k z)}{\prod_{k=1}^{p_i} (1 - c_k z^{-1}) \prod_{k=1}^{p_o} (1 - d_k z)}, \quad (4.71)$$

with $|a_k|, |b_k|, |c_k|, |d_k| < 1$.



p_i poles inside the unit circle: c_k

p_o poles outside the unit circle: $1/d_k$

m_i zeros inside the unit circle: a_k

m_o zeros outside the unit circle: $1/b_k$



In the process of computing the complex cepstrum, we get

$$\begin{aligned}
 \hat{X}(z) &= \log[X(z)] & (4.72) \\
 &= \log[A] + \log[z^r] \\
 &\quad + \sum_{k=1}^{m_i} \log(1 - a_k z^{-1}) + \sum_{k=1}^{m_o} \log(1 - b_k z) \\
 &\quad - \sum_{k=1}^{p_i} \log(1 - c_k z^{-1}) - \sum_{k=1}^{p_o} \log(1 - d_k z) .
 \end{aligned}$$



For real sequences, A is real.

If $A > 0$, $\hat{x}(0) = \log[A]$.

z^r : delay or advance by r samples.



$$\log(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \cdots; \quad \text{if } |x| < 1. \quad (4.73)$$

$$\log(1 - \alpha z^{-1}) = - \sum_{n=1}^{\infty} \frac{\alpha^n}{n} z^{-n}; \quad \text{if } |z| > |\alpha|. \quad (4.74)$$

$$\log(1 - \beta z) = - \sum_{n=1}^{\infty} \frac{\beta^n}{n} z^n; \quad \text{if } |z| < |\beta^{-1}|. \quad (4.75)$$



Expanding all the log terms in their equivalent power series

and taking the inverse z -transform, we get

$$\begin{aligned}
 \hat{x}(n) &= \log |A| \text{ for } n = 0; \\
 &= - \sum_{k=1}^{m_i} \frac{a_k^n}{n} + \sum_{k=1}^{p_i} \frac{c_k^n}{n} \text{ for } n > 0; \\
 &= + \sum_{k=1}^{m_o} \frac{b_k^{-n}}{n} - \sum_{k=1}^{p_o} \frac{d_k^{-n}}{n} \text{ for } n < 0. \quad (4.76)
 \end{aligned}$$



Properties of the complex cepstrum from the above:

The complex cepstrum decays at least as fast as $1/n$:

$$|\hat{x}(n)| < K \left| \frac{\alpha^n}{n} \right|, \quad -\infty < n < \infty, \quad (4.77)$$

$\alpha = \max(|a_k|, |b_k|, |c_k|, |d_k|)$; $K = \text{a constant}$.



For a minimum-phase sequence, there are no poles or zeros

outside the unit circle: $b_k = d_k = 0$.

Then, the complex cepstrum is causal:

$$\hat{x}(n) = 0 \text{ for } n < 0.$$



For a maximum-phase sequence, there are no poles or zeros inside the unit circle: $a_k = c_k = 0$.

Then, the complex cepstrum is anticausal:

$$\hat{x}(n) = 0 \text{ for } n > 0.$$

The complex cepstrum is of infinite duration even if $x(n)$ is of finite duration.







5

Analysis of Waveshape and Waveform Complexity

Some biomedical signals, e.g., the ECG and carotid pulse, have simple and recognizable waveshapes, which are modified by abnormal events and pathological processes.



EMG, PCG, VAG: no identifiable waveshapes.

EMG: complex interference patterns of several SMUAPs.

PCG: vibration waves with no specific waveshapes.

The waveform complexity in the EMG and the PCG varies in relation to physiological and pathological phenomena.

Analyzing the waveform complexity of such signals may assist in understanding of the processes they reflect.



5.1 Problem Statement

Explain how waveshapes and waveform complexity in biomedical signals relate to the characteristics of the underlying physiological and pathological phenomena.

Propose techniques to parameterize and analyze the signal features you identify.



5.2 Illustration of the Problem with Case-studies

5.2.1 *The QRS complex in the case of bundle-branch block*

The His bundle and its branches conduct the cardiac excitation pulse from the AV node to the ventricles.

A block in one of the bundle branches causes asynchrony between the contraction of the left and the right ventricles.



This causes a staggered summation of the action potentials of the myocytes of the left and the right ventricles over a longer-than-normal duration.

The result is a longer and possibly jagged QRS complex.



5.2.2 *The effect of myocardial ischemia and infarction on QRS waveshape*

Occlusion of a coronary artery or a branch due to deposition of fat, calcium, etc. results in reduced blood supply to a portion of the cardiac musculature: the part of the myocardium served by the affected artery suffers from ischemia — lack of blood supply.



Prolonged ischemia leads to myocardial infarction:

the deceased myocytes cannot contract any more,

and no longer produce action potentials.

Action potential of an under-nourished ventricular myocyte:

smaller amplitude and shorter duration.

ST segment either elevated or depressed,

T wave may be inverted.



5.2.3 *Ectopic beats*

Ectopic beats generated by cardiac tissue that possess abnormal pacing capabilities.

Ectopic beats originating in the atria: altered P waveshape due to different paths of propagation of the excitation pulse.

QRS complex of atrial ectopic beats will appear normal.



Ectopic beats originating on the ventricles (PVCs):

bizarre waveshapes due to differing paths of conduction.

PVCs typically lack a preceding P wave.

PVCs triggered by ectopic foci close to the AV node

may possess near-normal QRS shape.

RR intervals of preceding beat (short) and

succeeding beat (compensatory pause) play important

roles in determining the nature of ectopic beats.



5.2.4 *EMG interference pattern complexity*

Motor units are recruited by two mechanisms — spatial and temporal recruitment — to produce increasing levels of contraction and muscular force output.

SMUAPs of the active motor units overlap and produce a complex interference pattern.

Increasing complexity of EMG with increasing level of contraction.



5.2.5 PCG intensity patterns

Vibration waves in PCG not amenable to visual analysis.

General intensity pattern of PCG over a cardiac cycle

recognizable by auscultation or visual inspection.

Cardiovascular diseases and defects alter the

relative intensity patterns of S1 and S2,

cause additional sounds or murmurs,

split S2 into two distinct components, etc.



Many diseases may cause systolic murmurs;
intensity pattern or envelope of murmur could assist
in arriving at a specific diagnosis.



5.3 Analysis of Event-related Potentials

Most important parameter in a visual ERP:

timing or latency of the first major positivity P120.

Latencies of the troughs before and after P120,

called N80 and N145, are also of interest.

Amplitudes of ERP features of lesser importance.



5.4 Morphological Analysis of ECG Waves

ECG waveshape changed by many abnormalities:

myocardial ischemia or infarction,

bundle-branch block, and ectopic beats.



5.4.1 Correlation coefficient

Problem: *Propose a general index*

to indicate altered QRS waveshape.

You are given a normal QRS template.



Solution: Jenkins et al. used correlation coefficient

$$\gamma_{xy} = \frac{\sum_{n=0}^{N-1} x(n) y(n)}{\left[\sum_{n=0}^{N-1} x^2(n) \sum_{n=0}^{N-1} y^2(n) \right]^{1/2}} .$$

Normal beat used as template to compute γ_{xy}

for each detected beat; see Figure 2.2.

Most normal beats: $\gamma_{xy} > 0.9$.

PVCs and beats with abnormal shape: lower values of γ_{xy} .



5.4.2 *The minimum-phase correspondent and signal length*

Most of the energy of a normal ECG signal is concentrated within an interval of about 80 ms in the QRS complex.

Normally iso-electric PQ, ST, and TP segments: no energy.

Certain abnormal conditions cause the QRS to widen or the ST segment to bear a nonzero value:

energy of signal spread over a longer duration.



Problem: *Investigate the effect of the*

distribution of energy over the time axis

on a signal's characteristics.

Propose measures to parameterize the effects and

study their use in the classification of ECG beats.



Solution:

Signal $x(t)$: distribution of the amplitude

of a certain variable over the time axis.

$x^2(t)$: instantaneous energy of the signal.

$x^2(t)$, $0 \leq t \leq T$: energy distribution or density function.

Total energy of the signal:

$$E_x = \int_0^T x^2(t) dt. \quad (5.1)$$



Facilitates definition of moments of the energy distribution.

Centroidal time:

$$t_{\bar{x}} = \frac{\int_0^T t x^2(t) dt}{\int_0^T x^2(t) dt}. \quad (5.2)$$

Dispersion of energy about the centroidal time:

$$\sigma_{t_{\bar{x}}}^2 = \frac{\int_0^T (t - t_{\bar{x}})^2 x^2(t) dt}{\int_0^T x^2(t) dt}. \quad (5.3)$$



Similarity between the equations above and

Equations 3.1 and 3.3: normalized function

$$p_x(t) = \frac{x^2(t)}{\int_0^T x^2(t) dt} \quad (5.4)$$

treated as a PDF.

Other moments may also be defined to characterize and

study the distribution of $x^2(t)$ over the time axis.



Minimum-phase signals:

Distribution of energy of a signal over its duration

related to its amplitude spectrum and phase spectrum.

Notion of minimum phase useful in analyzing

the distribution of energy over the time axis.



A signal $x(n)$ is a minimum-phase signal if both the signal and its inverse $x_i(n)$ are one-sided signals — completely causal or anti-causal — with finite energy:

$$\sum_{n=0}^{\infty} x^2(n) < \infty,$$

$$\sum_{n=0}^{\infty} x_i^2(n) < \infty.$$

Note: The inverse of a signal is defined such that

$$x(n) * x_i(n) = \delta(n); \text{ equivalently, } X_i(z) = \frac{1}{X(z)}.$$



Important properties of a minimum-phase signal:

- For a given amplitude spectrum,
there exists one and only one minimum-phase signal.



- Of all finite-energy, one-sided signals with identical amplitude spectra, the energy of the minimum-phase signal is optimally concentrated toward the origin, and the signal has the smallest phase lag and phase-lag derivative at each frequency.



- The z -transform of a minimum-phase signal has all poles and zeros inside the unit circle in the z -plane.
- The complex cepstrum of a minimum-phase signal is causal.



Minimum-phase and maximum-phase components:

A signal $x(n)$ that does not satisfy

the minimum-phase condition,

referred to as a composite signal or a mixed-phase signal,

may be split into its minimum-phase component

and maximum-phase component

by filtering its complex cepstrum $\hat{x}(n)$.



To obtain the minimum-phase component,

the causal part of the complex cepstrum is chosen as:

$$\hat{x}_{\min}(n) = \begin{cases} 0 & n < 0 \\ 0.5 \hat{x}(n) & n = 0 \\ \hat{x}(n) & n > 0 \end{cases} . \quad (5.5)$$

The inverse procedures yield the

minimum-phase component $x_{\min}(n)$.



Maximum-phase component obtained by application of the inverse procedures to the anti-causal part of the cepstrum:

$$\hat{x}_{\max}(n) = \begin{cases} \hat{x}(n) & n < 0 \\ 0.5 \hat{x}(n) & n = 0 \\ 0 & n > 0 \end{cases} . \quad (5.6)$$



The minimum-phase and maximum-phase components

of a signal satisfy the following relationships:

$$\hat{x}(n) = \hat{x}_{\min}(n) + \hat{x}_{\max}(n), \quad (5.7)$$

$$x(n) = x_{\min}(n) * x_{\max}(n). \quad (5.8)$$



The minimum-phase correspondent (MPC):

A mixed-phase signal may be converted to a minimum-phase signal that has the same spectral magnitude as the original signal by filtering the complex cepstrum as

$$\hat{x}_{MPC}(n) = \begin{cases} 0 & n < 0 \\ \hat{x}(n) & n = 0 \\ \hat{x}(n) + \hat{x}(-n) & n > 0 \end{cases} \quad (5.9)$$

and applying the inverse procedures.



The *minimum-phase correspondent* or MPC

possesses optimal concentration of energy around the origin

under the constraint imposed by the magnitude spectrum

of the original mixed-phase signal.



Observe that $\hat{x}_{MPC}(n) = 2 \times$ even part of $\hat{x}(n)$ for $n > 0$.

This leads to a simpler procedure to compute the MPC:

Assume $\hat{X}(z) = \log X(z)$ to be analytic over the unit circle.

$$\hat{X}(\omega) = \hat{X}_R(\omega) + j\hat{X}_I(\omega);$$

R and I indicate the real and imaginary parts.

$\hat{X}_R(\omega)$ and $\hat{X}_I(\omega)$ are the log-magnitude and

phase spectra of $x(n)$.



Now, IFT of $\hat{X}_R(\omega)$ = even part of $\hat{x}(n)$,

defined as $\hat{x}_e(n) = [\hat{x}(n) + \hat{x}(-n)]/2$.

Thus, we have

$$\hat{x}_{MPC}(n) = \begin{cases} 0 & n < 0 \\ \hat{x}_e(n) & n = 0 \\ 2 \hat{x}_e(n) & n > 0 \end{cases} . \quad (5.10)$$



Thus we do not need to compute the complex cepstrum,
which requires the unwrapped phase spectrum of the signal,
but need only to compute a *real cepstrum*
using the log-magnitude spectrum.

Furthermore, given that PSD = FT of ACF, we have

$$\log [FT \{ \phi_{xx}(n) \}] = 2 \hat{X}_R(\omega).$$



It follows that, in the cepstral domain,

$\hat{\phi}_{xx}(n) = 2 \hat{x}_e(n)$, and therefore

$$\hat{x}_{MPC}(n) = \begin{cases} 0 & n < 0 \\ 0.5 \hat{\phi}_{xx}(n) & n = 0 \\ \hat{\phi}_{xx}(n) & n > 0 \end{cases}, \quad (5.11)$$

where $\hat{\phi}_{xx}(n)$ is the cepstrum of the ACF $\phi_{xx}(n)$ of $x(n)$.



Signal length: different from signal duration!

SL relates to how the energy of a signal is

distributed over its duration.

SL depends upon both magnitude and phase spectra.

For one-sided signals, minimum SL implies

minimum phase; the converse is also true.



General definition of SL of a signal $x(n)$:

$$SL = \frac{\sum_{n=0}^{N-1} w(n) x^2(n)}{\sum_{n=0}^{N-1} x^2(n)}. \quad (5.12)$$

$w(n)$: nondecreasing, positive weight function; $w(0) = 0$.

Definition of $w(n)$ depends upon the application and the desired characteristics of SL .

Samples of the signal away from the origin $n = 0$ receive progressively heavier weighting by $w(n)$.



Definition of SL : normalized moment of $x^2(n)$.

If $w(n) = n$: SL = centroidal time instant of $x^2(n)$.

For a given amplitude spectrum and hence total energy,

the minimum-phase signal has its energy

optimally concentrated near the origin \rightarrow lowest SL .

Signals with increasing phase lag have their

energy spread over a longer time duration:

larger SL due to the increased weighting by $w(n)$.



Illustration of application: Normal QRS vs PVCs.

Duration of normal QRS-T waves $\sim 350 - 400 \text{ ms}$.

QRS $\sim 80 \text{ ms}$ due to rapid and coordinated depolarization of the ventricular motor units via the Purkinje fibers.



PVCs have QRS-T complexes that are wider than normal:
energy distributed over longer span within the total duration,
due to slower and disorganized excitation sequences
triggering the ventricular muscle fibers.

Ectopic triggers may not be conducted
via the Purkinje system;

may be conducted through the ventricular muscle cells.

PVCs lack an iso-electric ST segment.



Murthy and Rangaraj proposed the application of SL

to classify ECG beats as normal or ectopic (PVC).

To overcome ambiguities in the detected onset of each beat:

SL of the MPC of segmented ECG signals (P-QRS-T).

208 beats of a patient: 132 out of 155 normals and

48 out of 53 PVCs were correctly classified;

one beat missed by QRS detection algorithm.

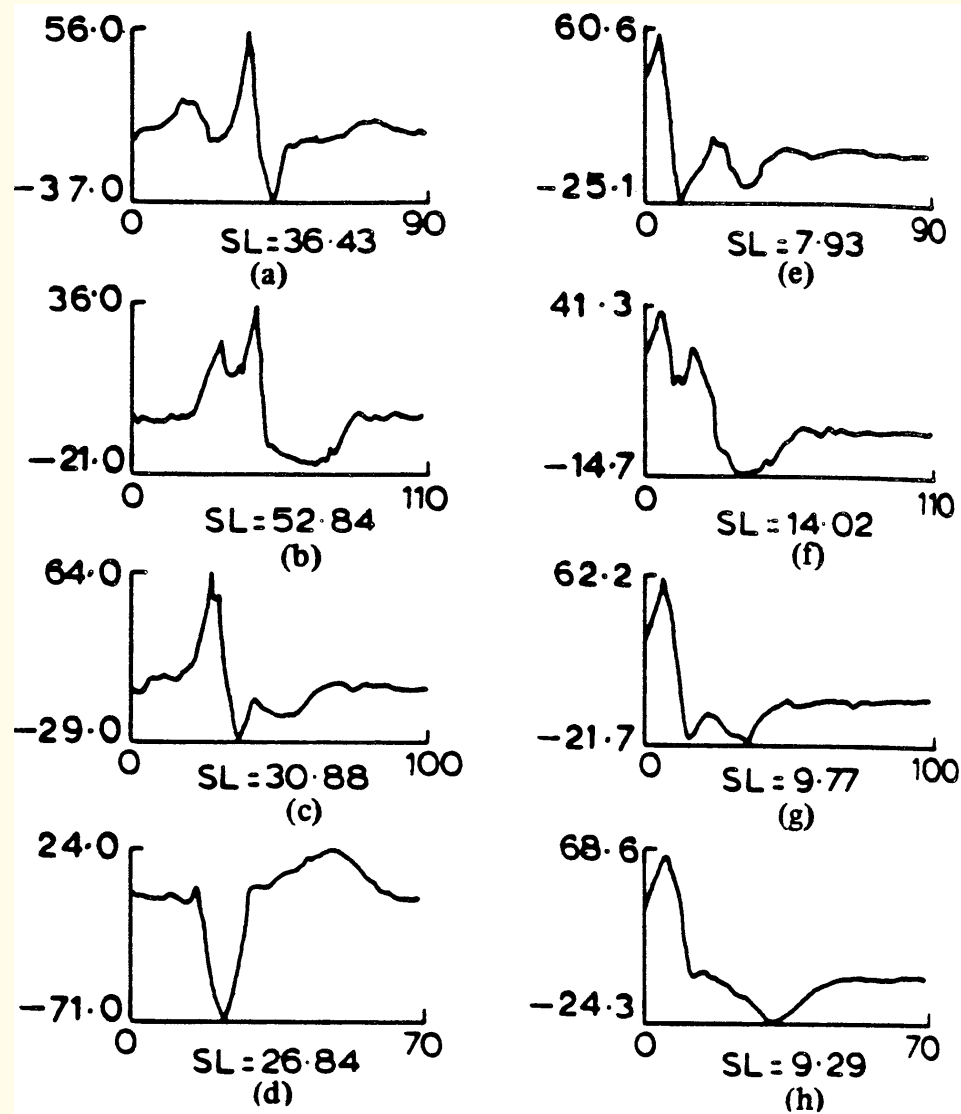
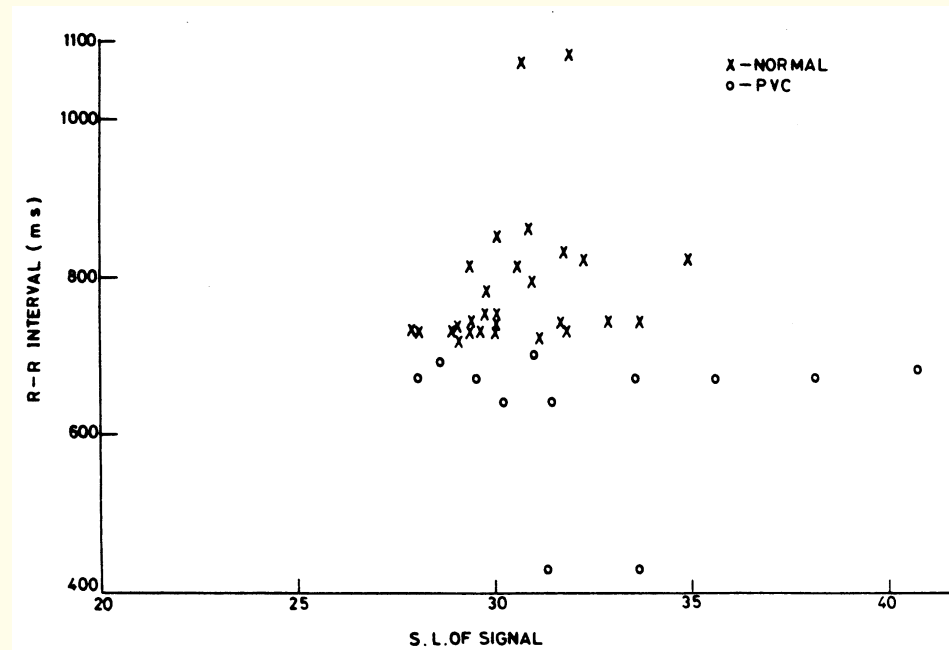


Figure 5.1: (a) A normal ECG beat and (b) – (d) three ectopic beats (PVCs) of a patient with multiple ectopic foci. (e) – (h) MPCs of the signals in (a) – (d). The SL values of the signals are also indicated. Note that the abscissa is labeled in samples, with a sampling interval of 10 ms. The ordinate is not calibrated. The signals have different durations and amplitudes although plotted to the same size. Reproduced with permission from I.S.N. Murthy and M.R. Rangaraj, New concepts for PVC detection, *IEEE Transactions on Biomedical Engineering*, 26(7):409–416, 1979. ©IEEE.



(a)

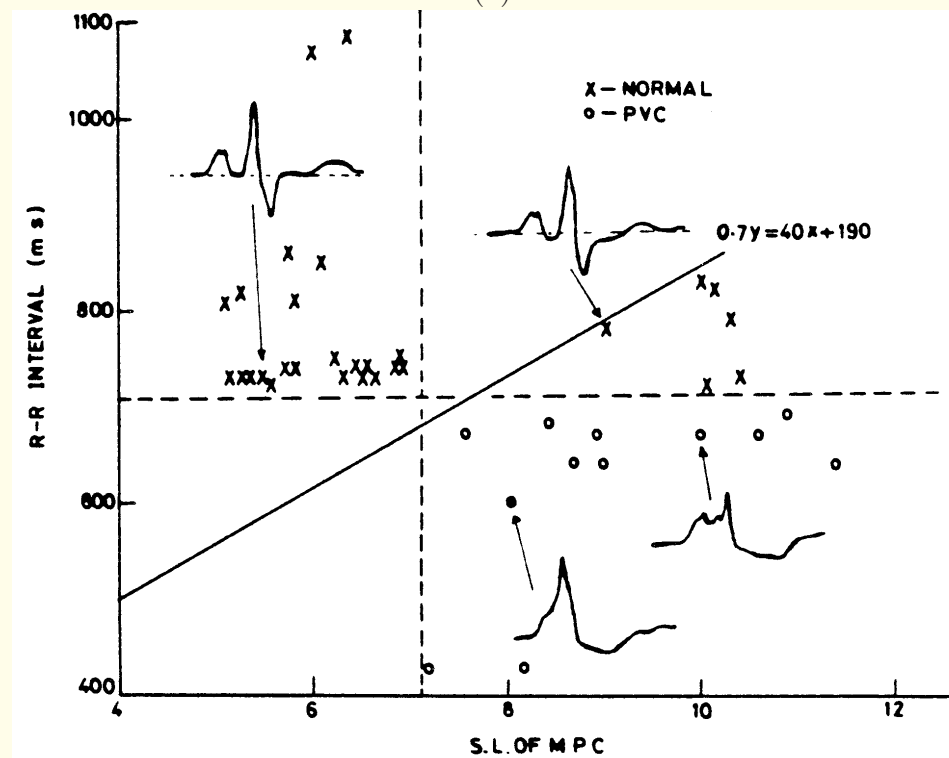




Figure 5.2: (a) Plot of RR and SL values of several beats of a patient with multiple ectopic foci (as in Figure 5.1). (b) Same as (a) but with the SL of the MPCs of the signals. A few representative ECG cycles are illustrated. The linear discriminant (decision) function used to classify the beats is also shown. Reproduced with permission from I.S.N. Murthy and M.R. Rangaraj, New concepts for PVC detection, *IEEE Transactions on Biomedical Engineering*, 26(7):409–416, 1979. ©IEEE.



5.4.3 ECG waveform analysis

Measures such as correlation coefficient and SL provide general parameters to compare waveforms.

Detailed analysis of ECG waveforms requires several features or measurements for categorization of various QRS shapes and correlation with cardiovascular diseases.

ECG waveform depends upon the lead used:

sets of features derived for multiple-lead ECGs.



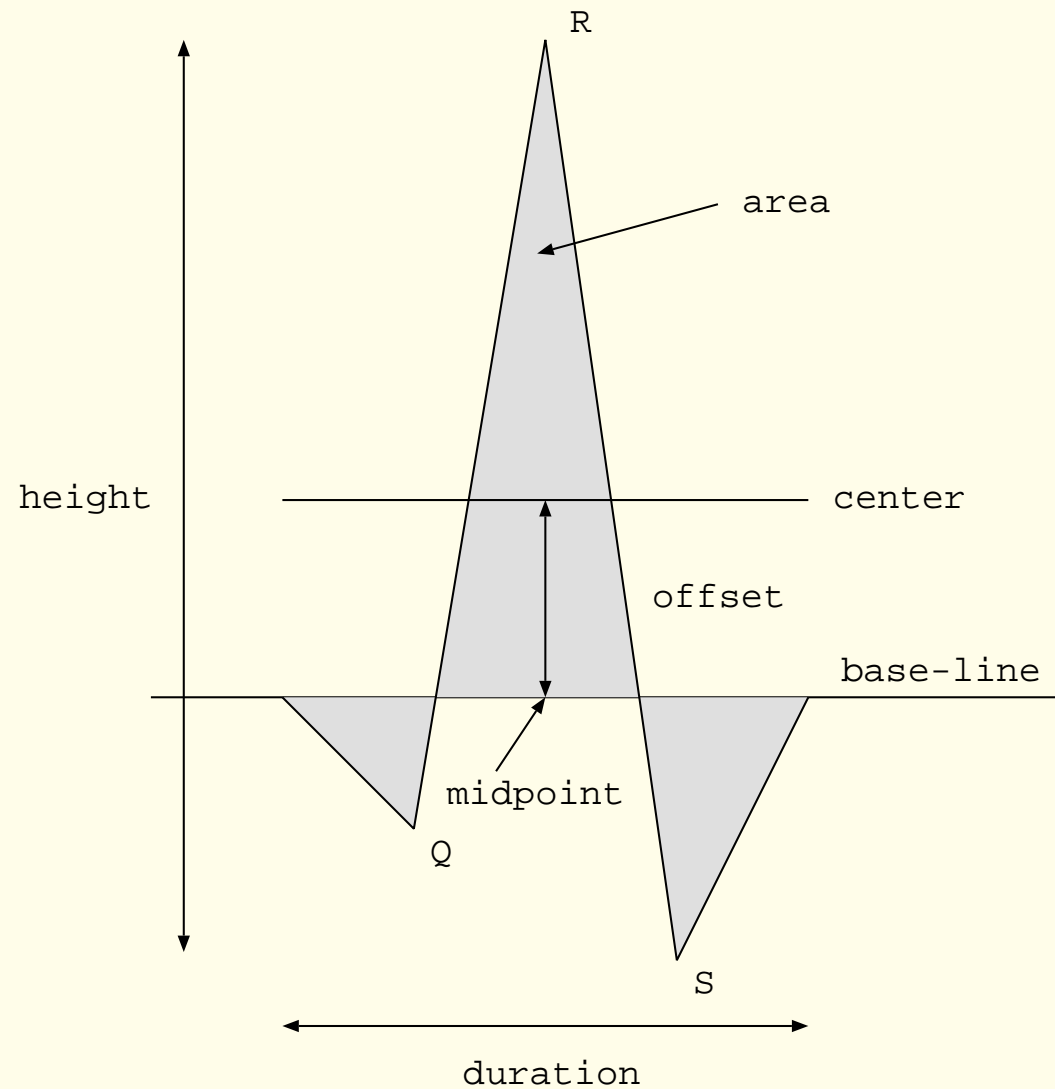
Steps for ECG waveform analysis:

1. Detection of ECG waves, primarily the QRS complex, and possibly the P and T waves.
2. Delimitation of wave boundaries, including the P, QRS, and T waves.
3. Measurement of inter-wave intervals, such as RR, PQ, QT, ST, QQ, and PP intervals.
4. Characterization of the morphology (shape) of the waves.
5. Recognition of iso-electric segments expected (PQ and ST).



Cox et al. proposed four measures to characterize QRS complexes:

1. *Duration* — duration or width of QRS.
2. *Height* — maximum minus minimum amplitude of QRS.
3. *Offset* — positive or negative vertical distance from midpoint of base-line to center of QRS complex.
Base-line: line connecting temporal boundary points of QRS complex.
Center: midpoint between highest and lowest QRS amplitude.
4. *Area* — area under QRS waveform rectified w.r.t. straight line through midpoint of base-line.



Graphical definitions of the duration, height, offset, and area of the QRS complex.



“Argus:” Arrhythmia Guard System.

QRS complexes divided into 16 dynamic families.

Families 00, 01, 02, 04, 06, and 10: normal beats.

Clinical tests of Argus with over 50,000 beats:

85% of 45,364 normal beats detected & classified correctly;

78% of 4,010 PVCs detected & classified correctly;

0.04% of normal beats missed; 5.3% of PVCs missed;

38 normals ($< 0.1\%$ of the beats) falsely labeled as PVCs.

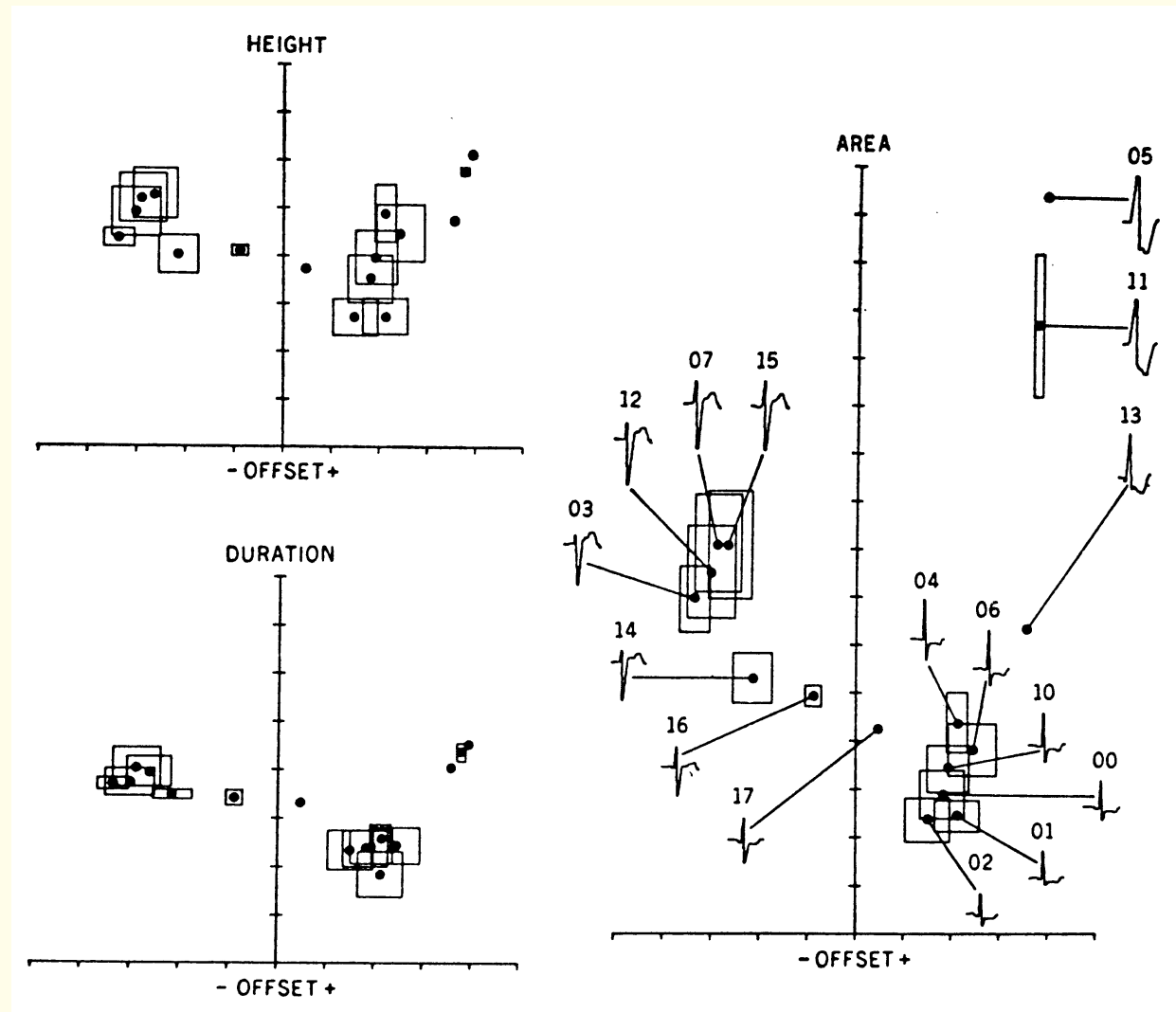


Figure 5.3: Use of four features to catalog QRS complexes into one of 16 dynamic families of similar complexes enclosed by four-dimensional boxes. The waveforms of typical members of each family are shown in the area-versus-offset feature plane. The family numbers displayed are in the octal (base eight) system. The families labeled 00, 01, 02, 04, 06, and 10 were classified as normal beats, with the others being PVCs or border-line beats. Reproduced with permission from J.R. Cox, Jr., F.M. Nolle, and R.M. Arthur, Digital analysis of the electroencephalogram, the blood pressure wave, and the electrocardiogram, *Proceedings of the IEEE*, 60(10):1137–1164, 1972. ©IEEE.



5.5 Envelope Extraction and Analysis

Signals with complex patterns, such as the EMG and PCG, may not permit direct analysis of their waveshape.

Intricate high-frequency variations may not be of interest; general trends in level of the overall activity useful:

the *envelope* of the signal carries important information.



Problem: *Formulate algorithms to extract the envelope of an EMG or PCG signal to facilitate analysis of trends in the level of activity or energy in the signal.*

Solution:

Obtain the absolute value of the signal at each instant:
perform full-wave rectification.



Rectification creates abrupt discontinuities at instants when signal values change sign: at zero-crossings.

Discontinuities create high-frequency components of significant magnitude: need lowpass filter with

low bandwidth in the range of $0 - 10$ or $0 - 50$ Hz

to obtain smooth envelopes of EMG and PCG signals.



Moving-average filter useful for lowpass filtering.

Basic definition of time-averaged envelope:

$$y(t) = \frac{1}{T_a} \int_{t-T_a}^t |x(t)| dt, \quad (5.13)$$

where T_a is the duration of the moving-average window.



Lehner and Rangayyan applied a weighted MA filter to the squared PCG signal: smoothed energy distribution curve

$$E(n) = \sum_{k=1}^M x^2(n - k + 1) w(k), \quad (5.14)$$

where $x(n)$ is the PCG signal,

$w(k) = M - k + 1$, and $M = 32$ with $f_s = 1,024 \text{ Hz}$.

Observe: difference between energy and power is

division by the time interval; scale factor ignored.



Envelope: total averaged activity within averaging window.

Filter: balance between the need to smooth discontinuities

and the requirement to maintain good sensitivity

to represent relevant changes in signal level or amplitude.

Procedure known as envelope detection or

amplitude demodulation.



5.5.1 *Amplitude demodulation*

Amplitude modulation (AM) of signals for

radio transmission:

multiplication of the signal $x(t)$ to be transmitted

by an RF carrier $\cos(\omega_c t)$, where ω_c is the carrier frequency.

AM signal $y(t) = x(t) \cos(\omega_c t)$.



If the exact carrier wave used at the transmitting end were available at the receiving end (including phase), *synchronous demodulation* possible by multiplying the received signal $y(t)$ with the carrier.



Demodulated signal:

$$\begin{aligned}x_d(t) &= y(t) \cos(\omega_c t) = x(t) \cos^2(\omega_c t) \\&= \frac{1}{2} x(t) + \frac{1}{2} x(t) \cos(2\omega_c t).\end{aligned}\quad (5.15)$$

AM component at $2\omega_c$ removed by a lowpass filter,

leaving the desired signal $x(t)$.



If $x(t)$ is always positive, or a DC bias is added,
the envelope of the AM signal is equal to $x(t)$.

Asynchronous demodulation possible —

just need to follow the envelope of $y(t)$;

does not require the carrier.



Carrier frequency ω_c is far greater than

the maximum frequency present in $x(t)$:

positive envelope of $y(t)$ extracted by

half-wave rectification.

Lowpass filter with an appropriate time constant

to “fill the gaps” between the peaks of the carrier wave

gives a good estimate of $x(t)$.



Complex demodulation: The given signal is

demodulated to derive the time-varying amplitude and phase characteristics for each frequency (band) of interest.

$$x(t) = a(t) \cos[\omega_o t + \psi(t)] + x_r(t). \quad (5.16)$$

ω_o : frequency of interest,

$a(t)$ and $\psi(t)$: time-varying amplitude and phase at ω_o ;

$x_r(t)$: remainder after component at ω_o removed.



Assume that $a(t)$ and $\psi(t)$ vary slowly

in relation to the frequencies of interest.

$x(t)$ expressed in terms of complex exponentials:

$$\begin{aligned} x(t) = & \frac{1}{2} a(t) \{ \exp\{j[\omega_o t + \psi(t)]\} \\ & + \exp\{-j[\omega_o t + \psi(t)]\} \} + x_r(t). \end{aligned} \quad (5.17)$$



In complex demodulation, the signal is shifted in frequency

by $-\omega_o$ via multiplication with $2 \exp(-j\omega_o t)$, to obtain

$$\begin{aligned}
 y(t) &= 2 x(t) \exp(-j\omega_o t) & (5.18) \\
 &= a(t) \exp[j\psi(t)] + a(t) \exp\{-j[2\omega_o t + \psi(t)]\} \\
 &\quad + 2 x_r(t) \exp(-j\omega_o t).
 \end{aligned}$$

Second term centered at $2\omega_o$, third term centered at ω_o ;

only first term placed at DC.



A lowpass filter may be used to extract the first term:

$$y_o(t) \approx a(t) \exp[j\psi(t)]. \quad (5.19)$$

The desired entities may then be extracted as

$$a(t) \approx |y_o(t)| \text{ and } \psi(t) \approx \angle y_o(t).$$



Frequency resolution depends upon the bandwidth
of the lowpass filter used.

The procedure may be repeated at every frequency
or frequency band of interest.

Result: envelope of the signal for the specified frequency
or frequency band.



In biomedical signals such as the PCG and the EMG,
there is no underlying RF carrier wave in the signal:
the envelope rides on relatively high-frequency
acoustic or electrical activity with a composite spectrum.



5.5.2 *Synchronized averaging of PCG envelopes*

ECG and PCG: good signal pair for synchronized averaging.

One could average the PCG over several cardiac cycles

with the ECG as the trigger.

However, the PCG is not amenable to direct

synchronized averaging: the vibration waves may

interfere in a destructive manner and cancel one another.



Karpman et al. proposed to first rectify the PCG signal, smooth the result using a lowpass filter, and then perform synchronized averaging of the envelopes using the ECG as the trigger.

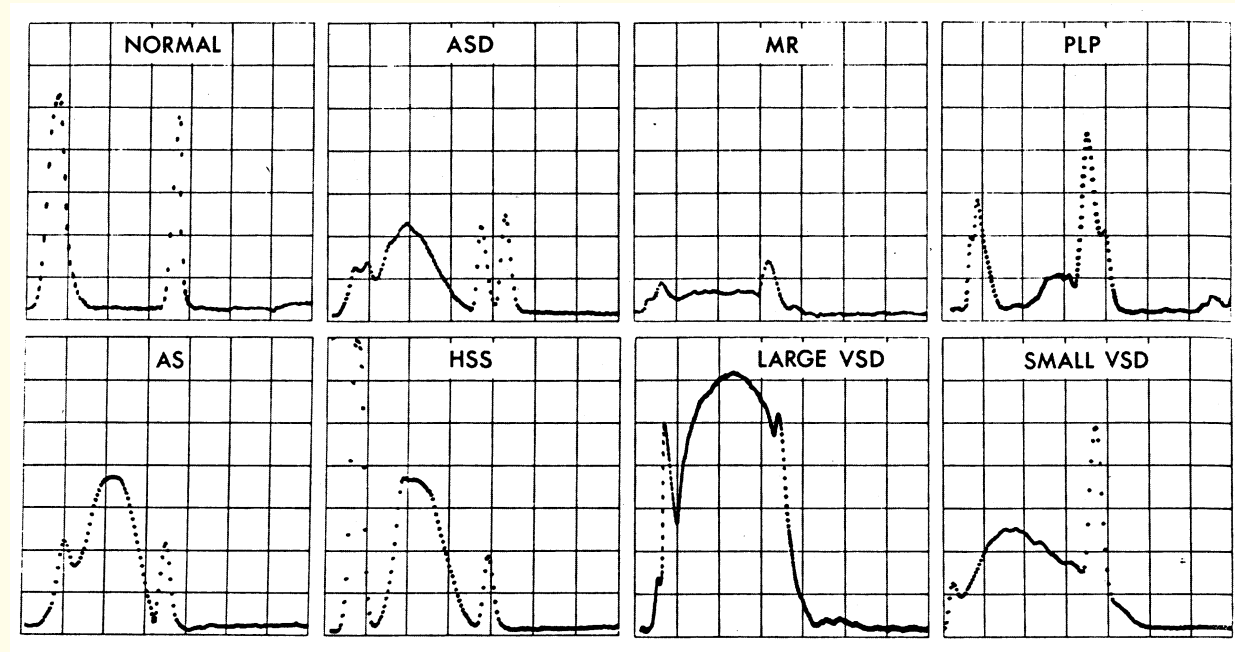


Figure 5.4: Averaged envelopes of the PCG signals of a normal subject and patients with systolic murmur due to aortic stenosis (AS), atrial septal defect (ASD), hypertrophic subaortic stenosis (HSS), rheumatic mitral regurgitation (MR), ventricular septal defect (VSD), and mitral regurgitation with posterior leaflet prolapse (PLP). Reproduced with permission from L. Karpman, J. Cage, C. Hill, A.D. Forbes, V. Karpman, and K. Cohn, Sound envelope averaging and the differential diagnosis of systolic murmurs, *American Heart Journal*, 90(5):600–606, 1975. ©American Heart Association.

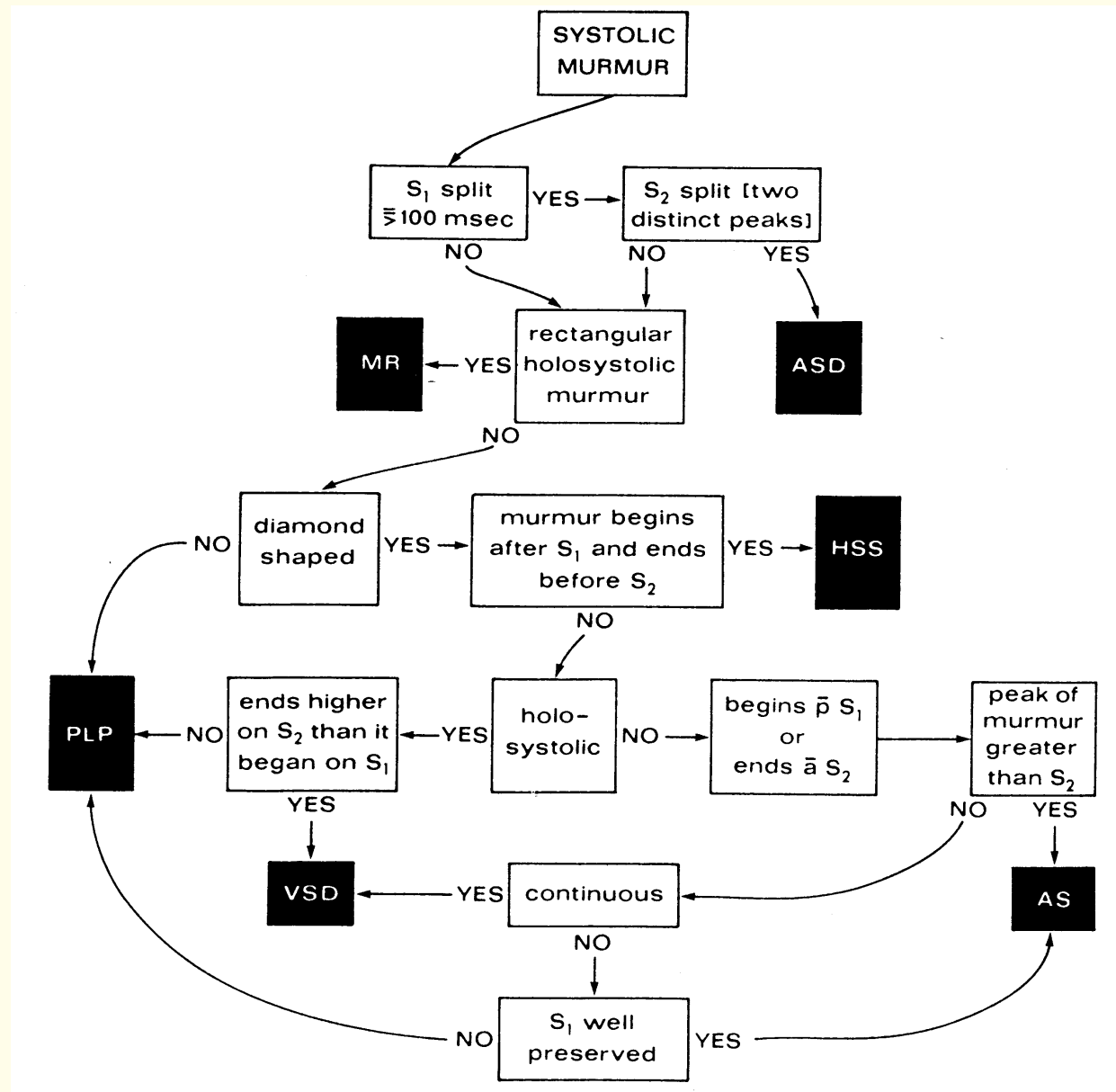


Figure 5.5: Decision tree to classify systolic murmurs based upon envelope analysis. For details on the abbreviations used, refer to the text or the caption of Figure 5.4. $\bar{p}S_1$: after S_1 ; $\bar{a}S_2$: before S_2 . Reproduced with permission from L. Karpman, J. Cage, C. Hill, A.D. Forbes, V. Karpman, and K. Cohn, Sound envelope averaging and the differential diagnosis of systolic murmurs, *American Heart Journal*, 90(5):600–606, 1975. ©American Heart Association.



5.5.3 The envelopogram

Sarkady et al.: *envelopogram estimate* —

magnitude of the analytic signal $y(t)$ formed using the

PCG $x(t)$ and its Hilbert transform $x_H(t)$ as

$$y(t) = x(t) + jx_H(t). \quad (5.20)$$



An analytic function is a complex function of time

having a Fourier transform that is zero for $f < 0$.

The Hilbert transform of a signal is defined as

the convolution of the signal with $\frac{1}{\pi t}$:

$$x_H(t) = \int_{-\infty}^{\infty} \frac{x(\tau)}{\pi(t - \tau)} d\tau. \quad (5.21)$$



The Fourier transform of $\frac{1}{\pi t}$ is $-j \operatorname{sgn}(\omega)$, where

$$\operatorname{sgn}(\omega) = \begin{cases} -1 & \omega < 0 \\ 0 & \omega = 0 \\ 1 & \omega > 0 \end{cases} . \quad (5.22)$$

Then, $Y(\omega) = X(\omega)[1 + \operatorname{sgn}(\omega)]$.

$Y(\omega)$ is a one-sided or single-sideband function of ω

containing positive-frequency terms only.



Algorithm of Sarkady et al. to obtain

the envelopogram estimate:

1. Compute the DFT of the PCG signal.
2. Set the negative-frequency terms to zero;

$$X(k) = 0 \text{ for } \frac{N}{2} + 2 \leq k \leq N,$$

with the DFT indexed $1 \leq k \leq N$ as in MATLAB.



3. Multiply the positive-frequency terms,
 $X(k)$ for $2 \leq k \leq \frac{N}{2} + 1$, by 2;
the DC term $X(1)$ remains unchanged.
4. Compute the inverse DFT of the result.
5. The magnitude of the result gives the
envelopogram estimate.



The complex demodulation procedure of Sarkady et al.

yields a high-resolution envelope of the input signal.

Envelopograms and PSDs of PCG signals over single cycles

tend to be noisy; affected by respiration and muscle noise.

Sarkady et al.: synchronized averaging of envelopograms

and PSDs of PCGs over several cardiac cycles.

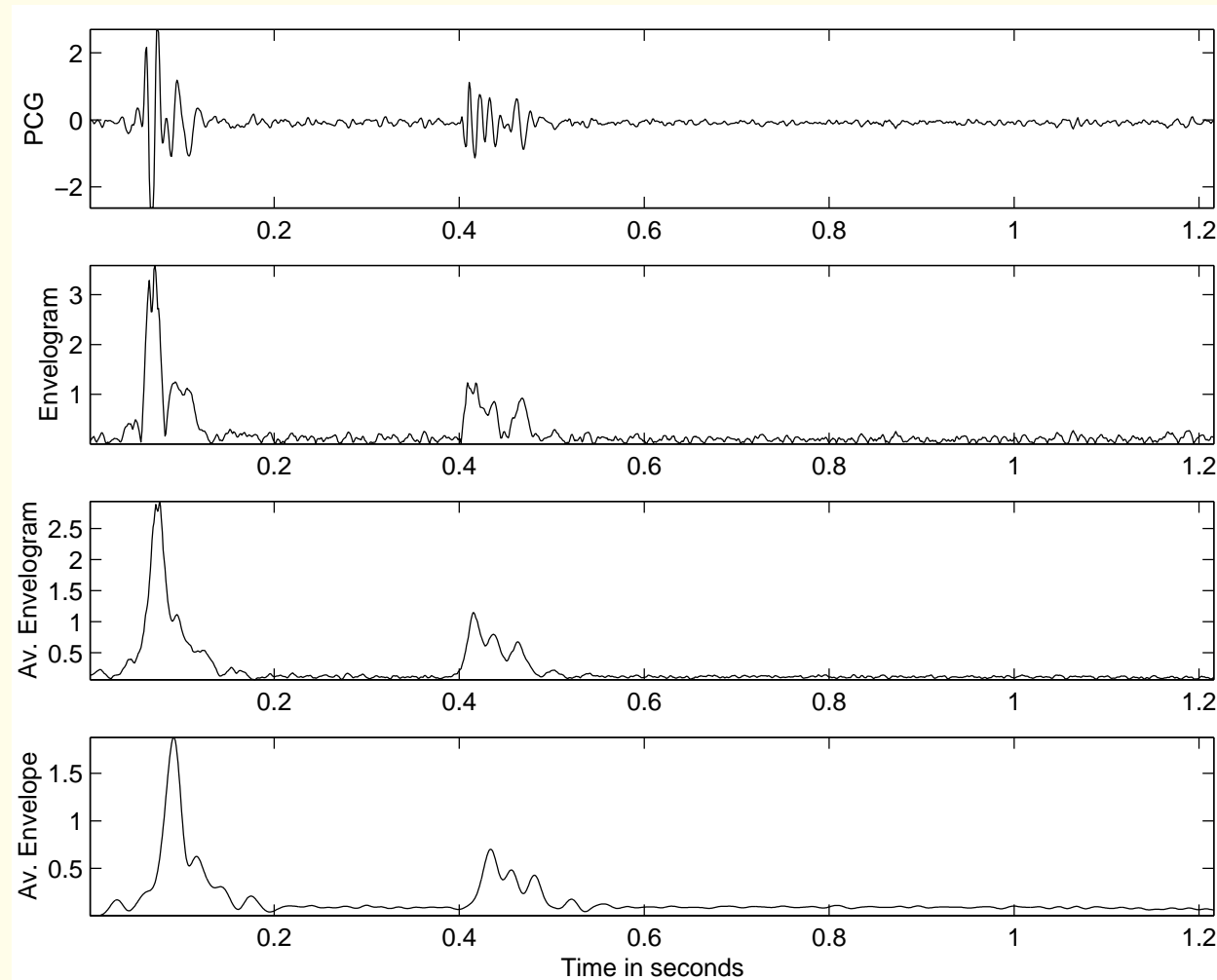


Figure 5.6: Top to bottom: PCG signal of a normal subject (male, 23 years); envelopogram estimate of the signal shown; averaged envelopogram over 16 cardiac cycles; averaged envelope over 16 cardiac cycles. The PCG signal starts with S1. See Figure 4.27 for an illustration of segmentation of the same signal.

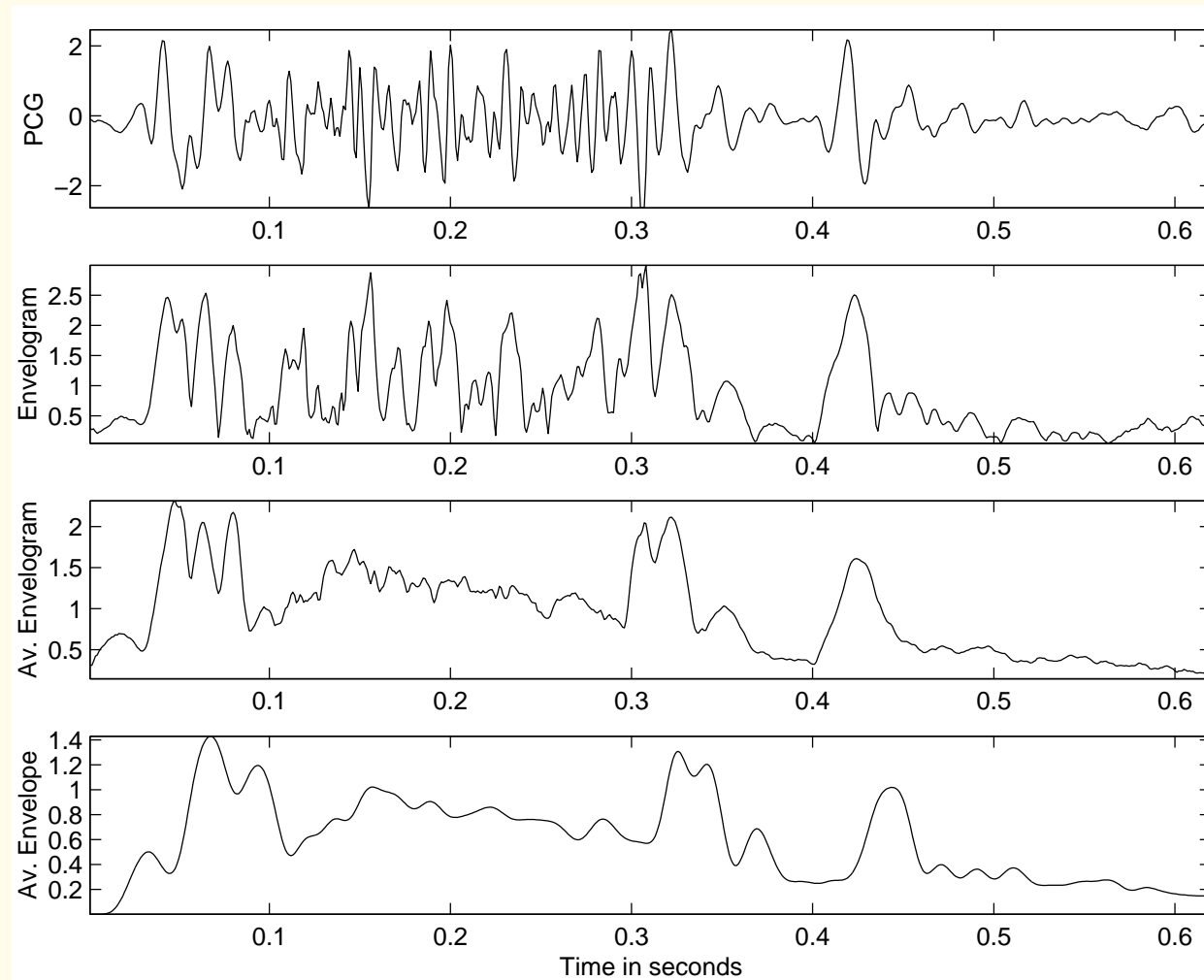


Figure 5.7: Top to bottom: PCG signal of a patient (female, 14 months) with systolic murmur (approximately 0.1 – 0.3 s), split S2 (0.3 – 0.4 s), and opening snap of the mitral valve (0.4 – 0.43 s); envelopogram estimate of the signal shown; averaged envelopogram over 26 cardiac cycles; averaged envelope over 26 cardiac cycles. The PCG signal starts with S1. See Figure 4.28 for an illustration of segmentation of the same signal.



5.6 Analysis of Activity

Problem: *Propose measures of waveform complexity or activity to analyze the extent of variability in signals such as the PCG and EMG.*

Solution: Samples of a given EMG or PCG signal may be treated as a random variable x .



Variance $\sigma_x^2 = E[(x - \mu_x)^2]$:

averaged measure of the variability or *activity*

of the signal about its mean.

If the signal has zero mean, or is so preprocessed,

$\sigma_x^2 = E[x^2]$: variance = average power.

Standard deviation = root mean-squared (RMS) value.

RMS value: indicator of the level of activity about the mean.



5.6.1 The root mean-squared value

RMS value of $x(n)$ over total duration of N samples:

$$RMS = \left[\frac{1}{N} \sum_{n=0}^{N-1} x^2(n) \right]^{\frac{1}{2}}. \quad (5.23)$$

Global measure of signal level (related to power):

not useful for the analysis of trends in nonstationary signals.



Running estimate of RMS value over a

causal window of M samples:

$$RMS(n) = \left[\frac{1}{M} \sum_{k=0}^{M-1} x^2(n-k) \right]^{\frac{1}{2}}. \quad (5.24)$$

Useful indicator of average power as a function of time:

short-time analysis of nonstationary signals.

Duration of the window M needs to be chosen

in accordance with the bandwidth of the signal; $M \ll N$.



Gerbarg et al.: power versus time curves of PCG signals;

average power in contiguous segments of duration 10 ms .

Used to identify systolic and diastolic segments of the PCG:

diastolic segments expected to be

longer than systolic segments.



Gerbarg et al. also computed ratios of the mean power of the

last third of systole to the mean power of systole

and also to a certain “standard” noise level.

Ratio also computed of mean energy of systole to

mean energy of PCG over the complete cardiac cycle.

78 – 91% agreement between computer classification

and clinical diagnosis of mitral regurgitation.



5.6.2 Zero-crossing rate

Intuitive indication of “busy-ness” of a signal provided by the number of times it crosses the zero-activity line or some other reference level.

ZCR: number of times the signal crosses the reference within a specified interval.



ZCR increases as the high-frequency content

of the signal increases;

Affected by DC bias, base-line wander,

low-frequency artifacts.

Advisable to measure ZCR of the derivative of the signal;

similar to turning points in the test for randomness.



ZCR used in practical applications:

Saltzberg and Burch — relationship between ZCR and moments of PSDs, application to EEG analysis.

Speech signal analysis — speech versus silence decision; to discriminate between voiced and unvoiced sounds.

PCG analysis — detection of murmurs.



Jacobs et al.:

ZCR for normal versus abnormal classification

of PCG signals using the ECG as a trigger.

Decision limit of 20 zero-crossings in a cardiac cycle.

Correct-classification rates of 95% for normals (58/61) and

94% for abnormals (77/82).



Yokoi et al.: maximum amplitude and ZCR

in 8 ms segments of PCG signals sampled at 2 kHz .

Correct-classification rates of

98% with $4,809$ normal subjects;

76% with $1,217$ patients with murmurs.



5.6.3 Turns count

Willison: analyze level of activity in EMG signals by determining number of spikes in interference pattern.

Instead of counting zero-crossings, Willison's method investigates the significance of every change in phase — direction or slope — of the EMG signal called a *turn*.

Turns greater than $100 \mu V$ are counted; threshold avoids counting insignificant fluctuations due to noise.



Turns count similar to counting turning points

in the test for randomness, but

robust in the presence of noise due to the threshold.

Not directly sensitive to SMUAPs, but significant

phase changes caused by superimposed SMUAPs counted.

EMG signals of subjects with myopathy:

higher turns counts than those of normal subjects

at comparable levels of volitional effort.

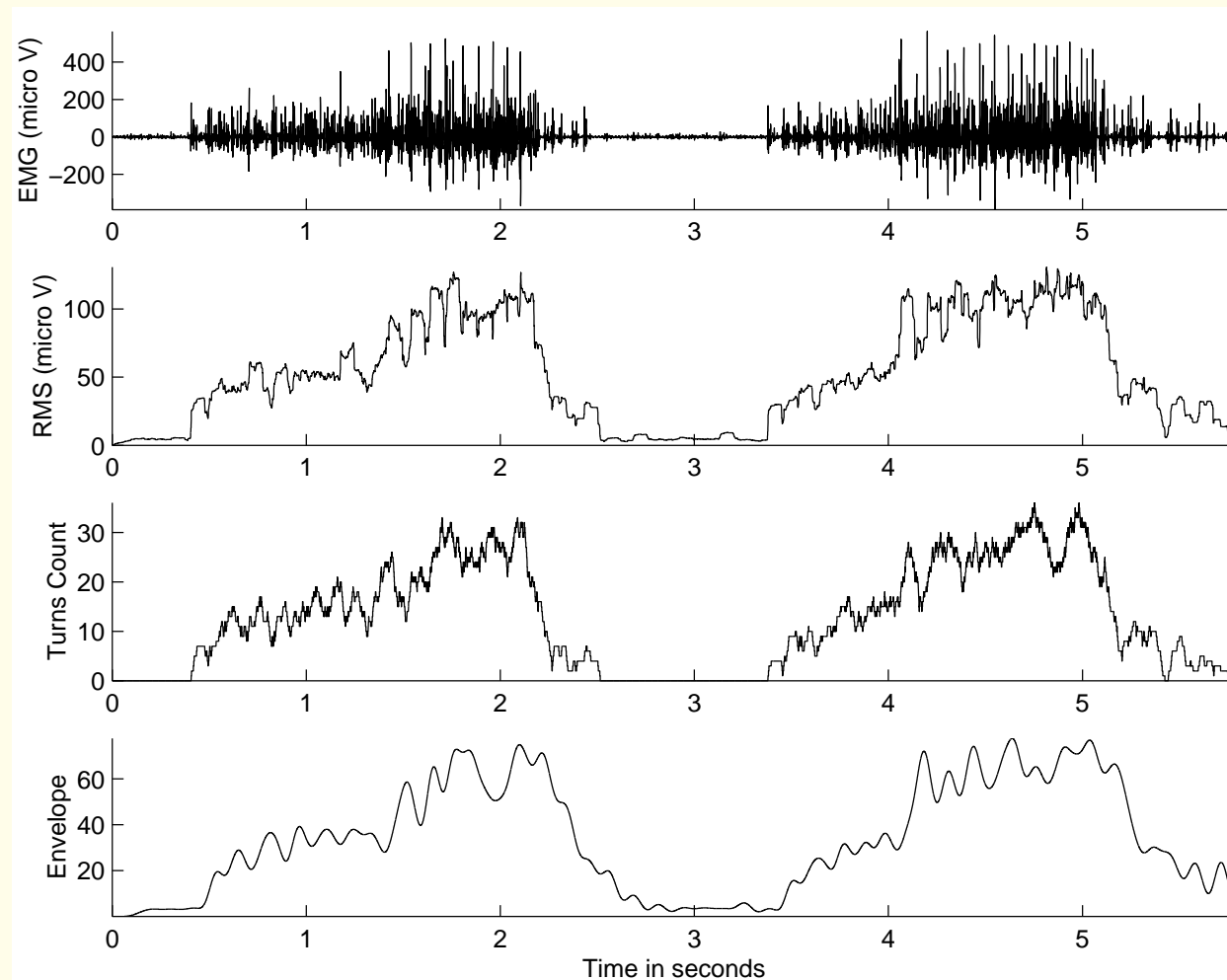


Figure 5.8: Top to bottom: EMG signal over two breath cycles from the crural diaphragm of a dog recorded via implanted fine-wire electrodes; short-time RMS values; turns count using Willison's procedure; and smoothed envelope of the signal. The RMS and turns count values were computed using a causal moving window of 70 ms duration. EMG signal courtesy of R.S. Platt and P.A. Easton, Department of Clinical Neurosciences, University of Calgary.

Envelope: absolute value of the signal (equivalent to full-wave rectification) followed by a Butterworth lowpass filter of order $N = 8$ and cutoff frequency $f_c = 8\text{ Hz}$.

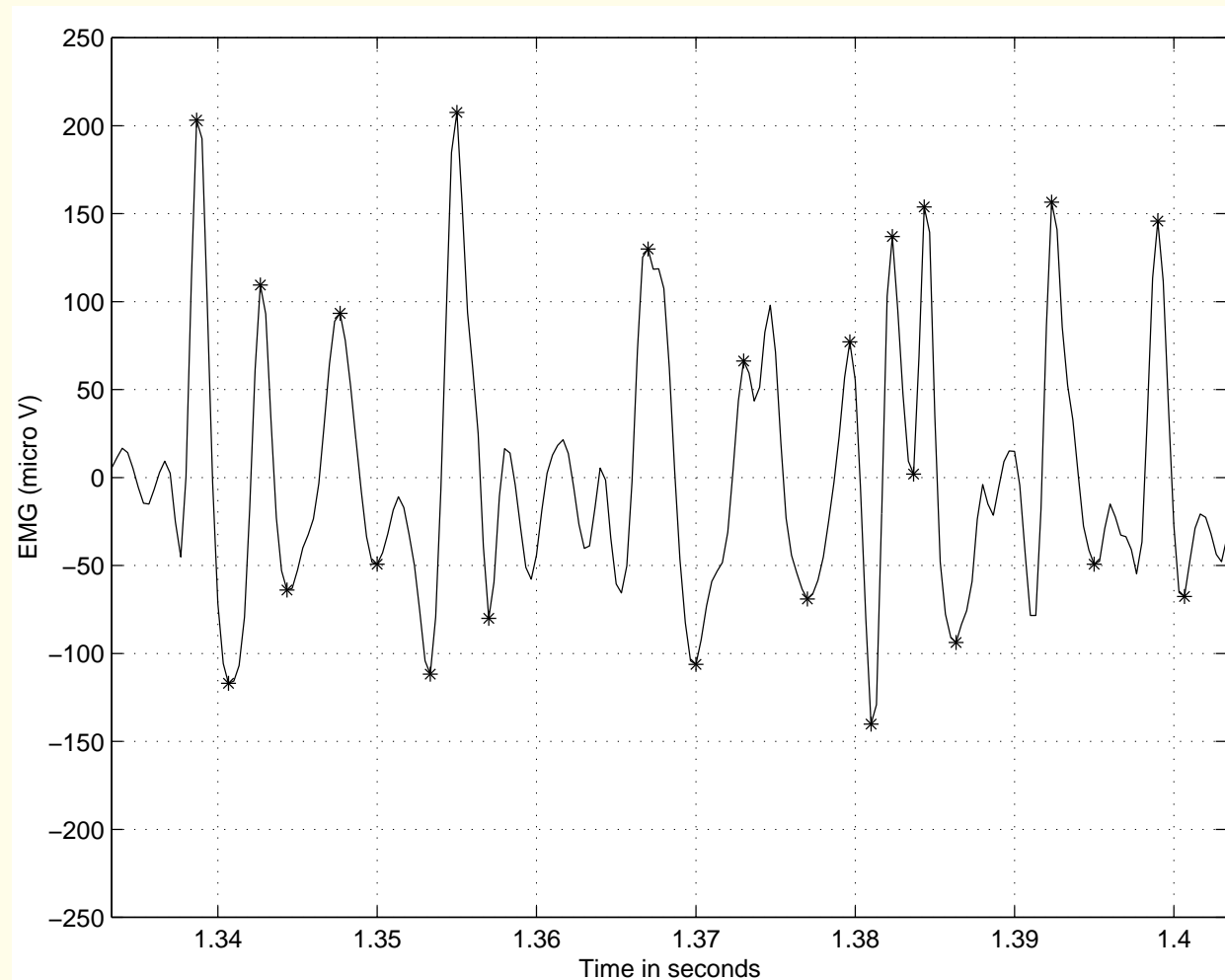


Figure 5.9: Illustration of the detection of turns in a 70 ms window of the EMG signal in Figure 5.8. Threshold $= 100\text{ }\mu\text{V}$. The segments of the signal between pairs of ‘*’ marks have been identified as significant turns.



5.6.4 Form factor

Based upon variance as a measure of signal activity,

Hjorth proposed a method for the analysis of EEG waves.

Segments of duration ~ 1 s analyzed using

three parameters:

Activity = variance σ_x^2 of signal segment x .



Mobility M_x :

$$M_x = \left[\frac{\sigma_{x'}^2}{\sigma_x^2} \right]^{\frac{1}{2}} = \frac{\sigma_{x'}}{\sigma_x}. \quad (5.25)$$

x' : first derivative of x .



Complexity or *form factor* FF :

$$FF = \frac{M_{x'}}{M_x} = \frac{\sigma_{x''}/\sigma_{x'}}{\sigma_{x'}/\sigma_x}. \quad (5.26)$$

x'' : second derivative of the signal.

Complexity or FF of a sinusoidal wave = unity.

Complexity values increase with the extent of variations in the signal.



Hjorth described the mathematical relationships between activity, mobility, complexity, and PSD of a signal; applied them to model EEG signal generation.

Binnie et al.: application of FF and spectrum analysis to EEG analysis for the detection of epilepsy.

FF based upon the first and second derivatives of the signal and their variances: sensitive to noise.



5.7 Application: Parameterization of Normal and Ectopic ECG Beats

Problem: *Develop a parameter to discriminate between normal ECG waveforms and ectopic beats (PVCs).*

Solution:

Ectopic beats have bizarre and complex waveshapes.

Form factor FF parameterizes waveform complexity:

a value that increases with complexity.

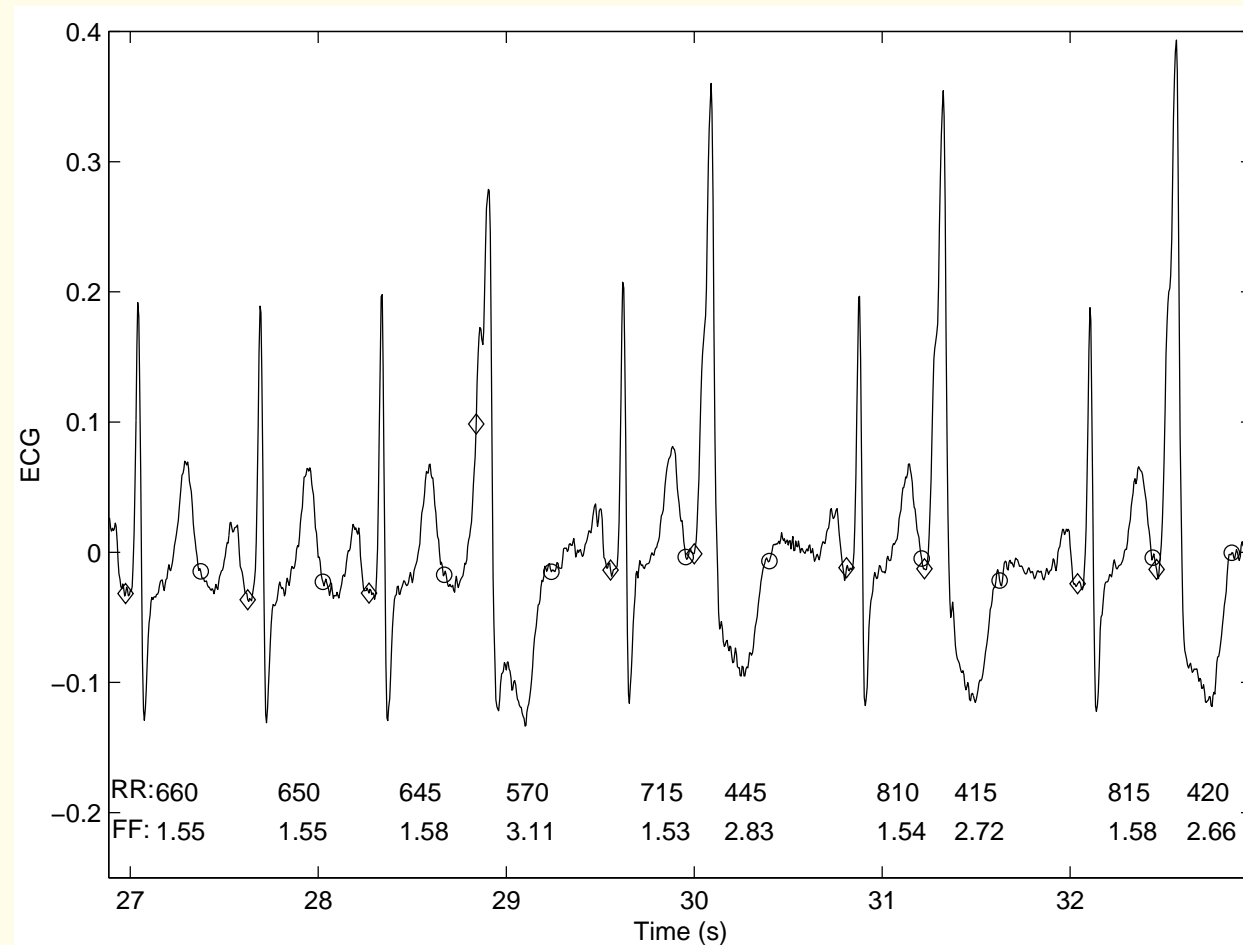
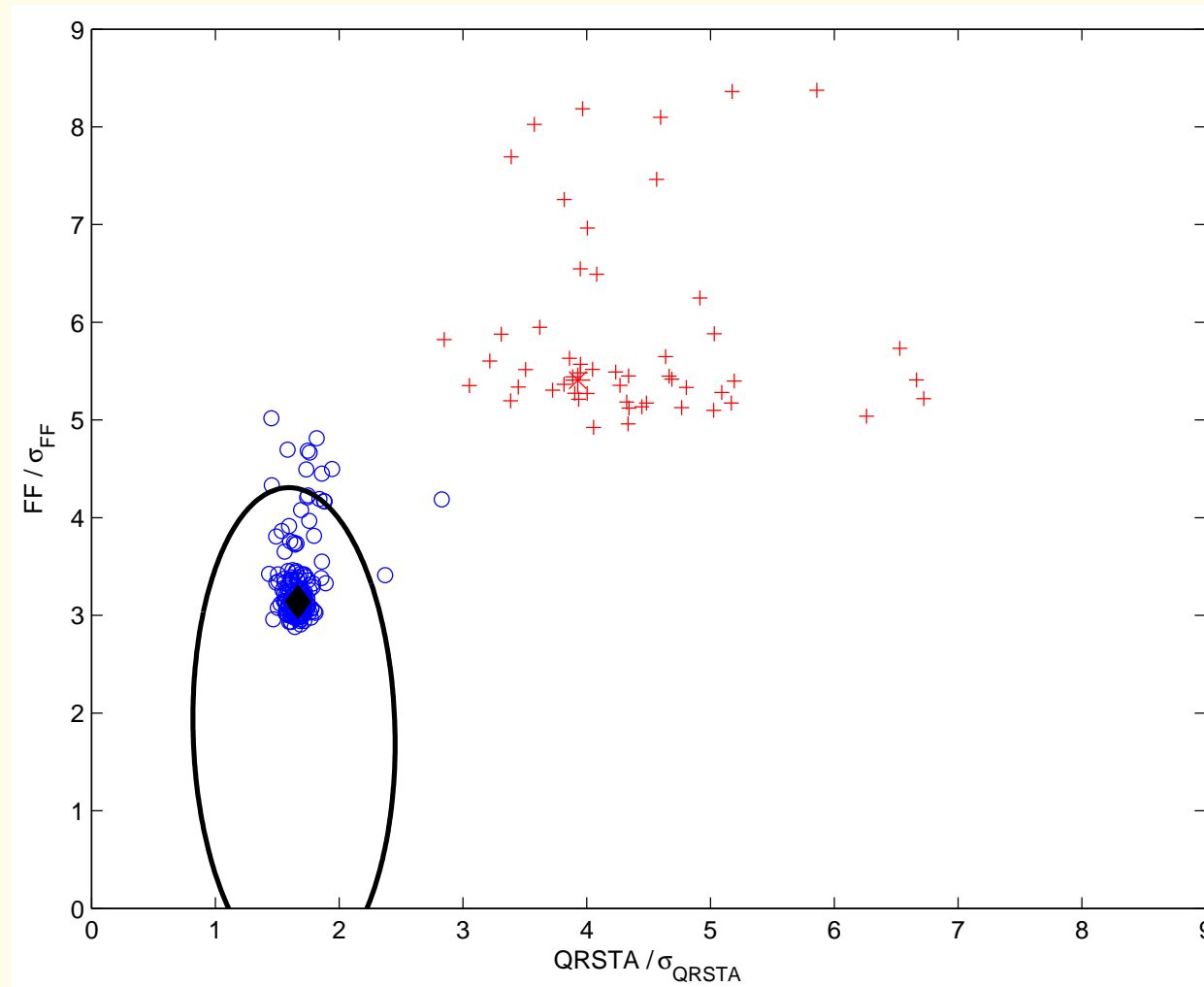


Figure 5.10: Segment of the ECG of a patient (male, 65 years) with ectopic beats. The diamond and circle symbols indicate the starting and ending points, respectively, of each beat obtained using the Pan-Tompkins algorithm for QRS detection. The RR interval (in ms) and form factor FF values are printed for each beat.

Each beat segmented at points 160 ms before and 240 ms after the detected marker point.



Normalized FF and QRST area for 236 ECG beats of a patient, including 183 normal beats and 53 PVCs. The black oval represent the decision boundary provided by the Bayes classifier, which was trained using a different set of 162 of the same patient, including 123 normal beats and 39 PVCs.



5.8 Application: Analysis of Exercise ECG

Problem: *Develop an algorithm to analyze*

changes in the ST segment of the ECG during exercise.

Solution:

Hsia et al.: ECG analysis performed as part of
radionuclide ventriculography (gated blood-pool imaging).



Nuclear medicine images obtained of the left ventricle before and after exercise on a treadmill or bicycle ergometer. Images obtained at different phases of the cardiac cycle by gating the radionuclide (gamma ray) emission data with the ECG; image data for each phase averaged over several cardiac cycles.



Analysis of exercise ECG complicated: base-line artifacts

caused by the effects of respiration,

skin resistance changes due to perspiration, and

soft tissue movement affecting electrode contact.

Detection of changes in ST segment in the presence of

such artifacts poses a major challenge.



Main parameter used by Hsia et al.: correlation coefficient.

$$\gamma_{xy} = \frac{\sum_{n=0}^{N-1} [x(n)] [y(n) - \Delta]}{\sqrt{\sum_{n=0}^{N-1} [x(n)]^2 \sum_{n=0}^{N-1} [y(n) - \Delta]^2}}. \quad (5.27)$$

$x(n)$: template; $y(n)$: ECG signal being analyzed;

Δ : base-line correction factor =

difference between base-line of $y(n)$ and base-line of $x(n)$;

N = duration (number of samples) of template and signal.



Template generated by averaging up to 20 QRS complexes that met a specified RR interval constraint.

$\gamma_{xy} < 0.85$: abnormal beat.



Beats with abnormal morphology, such as PVCs rejected.

ST reference point defined as

$$R + 64 \text{ ms} + \max\left(4, \frac{200 - HR}{16}\right) \times 4 \text{ ms}$$

$$\text{or } S + 44 \text{ ms} + \max\left(4, \frac{200 - HR}{16}\right) \times 4 \text{ ms}.$$

R or S : position of R or S of the present beat in ms ,

HR : heart rate in bpm .

Elevation or depression of the ST segment by more than

0.1 mV with reference to the baseline reported.



5.9 Application: Analysis of Respiration

Problem: *Propose a method to*

relate EMG activity to airflow during inspiration.

Solution: Platt et al. recorded EMG signals from the parasternal intercostal and crural diaphragm muscles of dogs.



EMG signal obtained from a pair of electrodes mounted
at a fixed distance of 2 mm placed between
fibers in the third left parasternal intercostal muscle
about 2 cm from the edge of the sternum.



Crural diaphragm EMG obtained via fine-wire electrodes

sewn in-line with the muscle fibers, placed 10 mm apart.

Dog breathed through a snout mask;

pneumo-tachograph used to measure airflow.

Envelope obtained by smoothing full-wave-rectified EMG.

Modified Bessel filter: severely attenuated frequencies

beyond 20 Hz with gain $< -70\text{ dB}$.

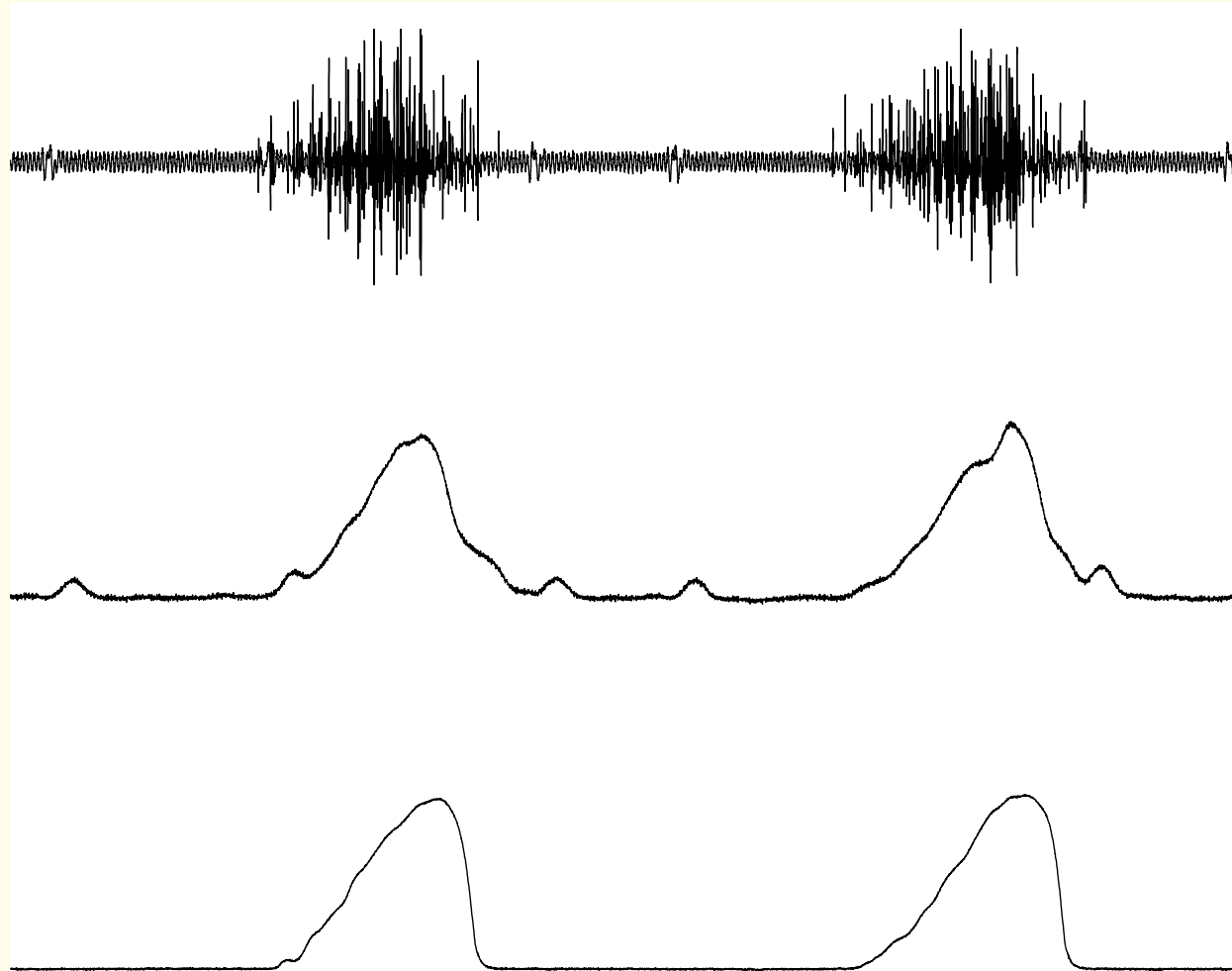


Figure 5.11: Top to bottom: EMG signal over two breath cycles from the parasternal intercostal muscle of a dog recorded via implanted electrodes; EMG envelope obtained with the modified Bessel filter with a time constant of 100 ms ; and inspiratory airflow. The duration of the signals plotted is 5 s . The several minor peaks appearing in the envelope are related to the ECG which appears as an artifact in the EMG signal. Data courtesy of R.S. Platt and P.A. Easton, Department of Clinical Neurosciences, University of Calgary.

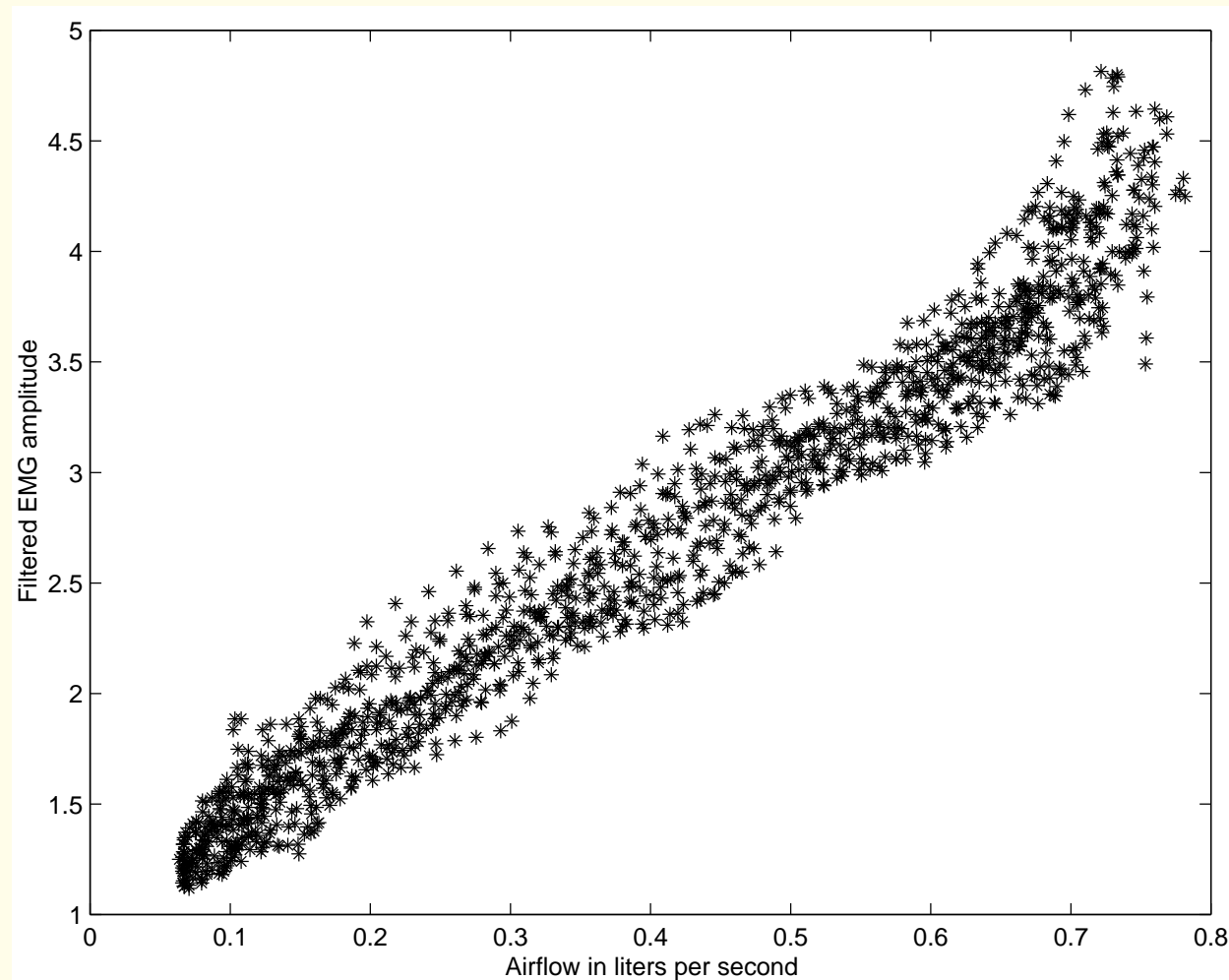


Figure 5.12: Correlation between EMG amplitude from Bessel-filtered envelope versus inspiratory airflow. The EMG envelope was filtered using a modified Bessel filter with a time constant of 100 ms . Data courtesy of R.S. Platt and P.A. Easton, Department of Clinical Neurosciences, University of Calgary.



5.10 Application: Electrical and Mechanical Correlates of Muscular Contraction

Problem: *Derive parameters from the electrical and mechanical manifestations of muscular activity that correlate with the level of contraction or force.*

Solution: Zhang et al. studied the usefulness of simultaneously recorded EMG and VMG signals in the analysis of muscular force.



Subjects performed isometric contraction
of the rectus femoris (thigh) muscle
(with no movement of the associated leg)
to different levels of torque with a Cybex II dynamometer.
Four levels of contraction: 20%, 40%, 60%, and 80%
of the maximal voluntary contraction (MVC) level;
at three knee-joint angles of 30°, 60°, and 90°.



Each contraction held for a duration of about 6 s ;

rest between experiments to prevent muscle fatigue.

VMG signal recorded using a Dytran 3115a accelerometer;

surface EMG signals recorded using $Ag - AgCl$ electrodes.

VMG signals filtered to $3 - 100\text{ Hz}$;

EMG signals filtered to $10 - 300\text{ Hz}$.

VMG and EMG sampled at 250 Hz and $1,000\text{ Hz}$.



RMS values computed for each contraction level over 5 s.

Almost-linear trends of RMS values of

both EMG and VMG with muscular contraction:

useful in analysis of muscular activity.

EMG RMS vs force relationships vary

from muscle to muscle.

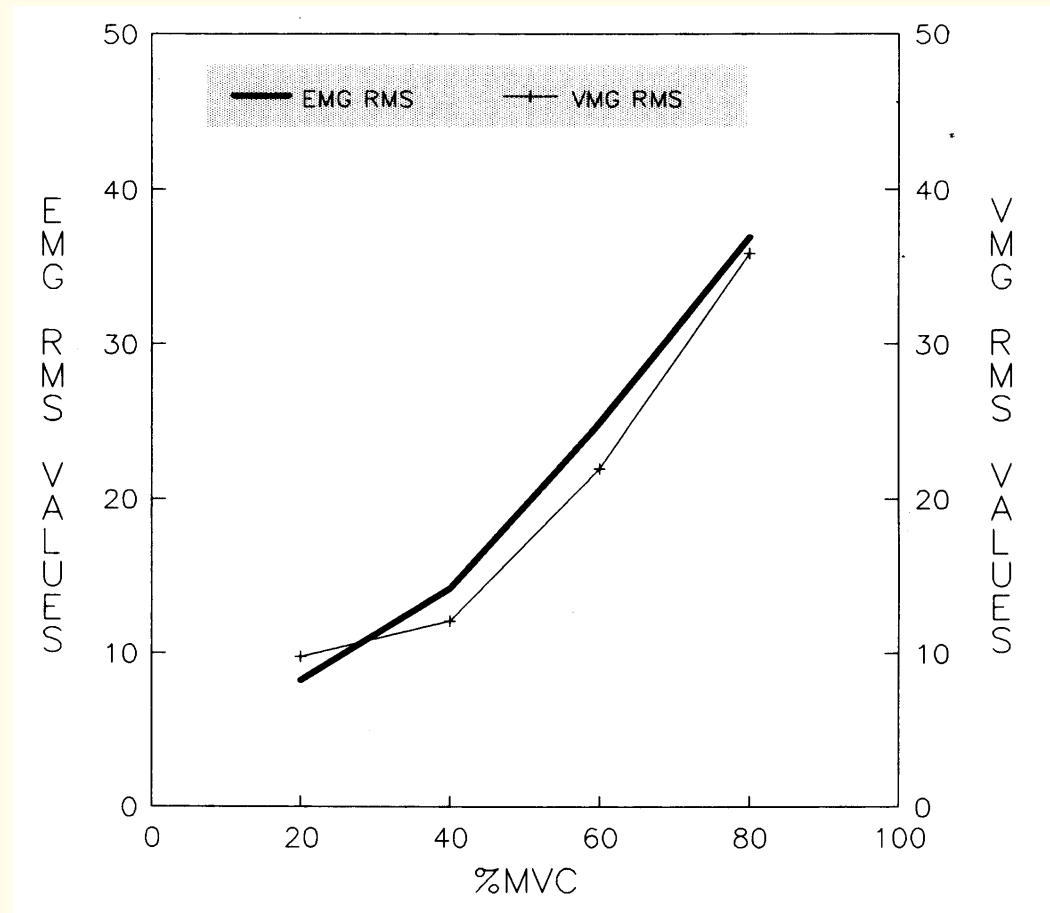


Figure 5.13: RMS values of the VMG and EMG signals for four levels of contraction of the rectus femoris muscle at 60° knee-joint angle averaged over four subjects. Reproduced with permission from Y.T. Zhang, C.B. Frank, R.M. Rangayyan, and G.D. Bell, Relationships of the vibromyogram to the surface electromyogram of the human rectus femoris muscle during voluntary isometric contraction, *Journal of Rehabilitation Research and Development*, 33(4): 395–403, 1996. ©Department of Veterans Affairs.

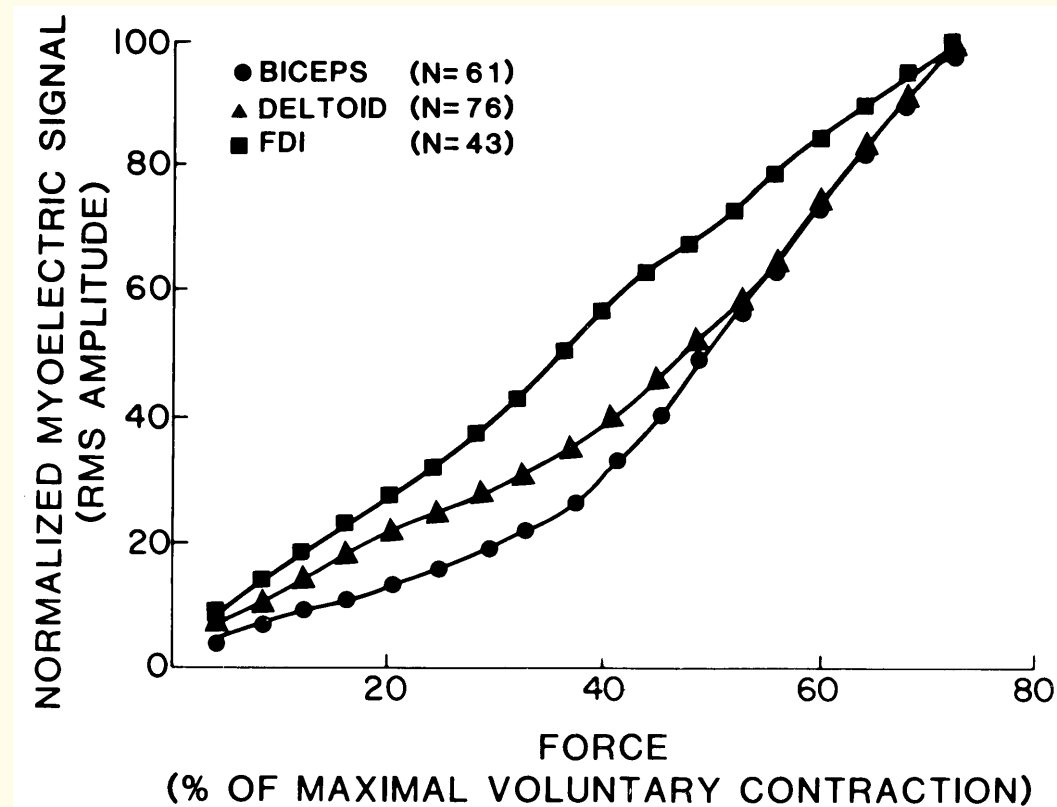


Figure 5.14: EMG RMS value versus level of muscle contraction expressed as a percentage of the maximal voluntary contraction level (%MVC) for each subject. The relationship is displayed for three muscles. FDI: first dorsal interosseus. N: number of muscles in the study. Reproduced with permission from J.H. Lawrence and C.J. de Luca, Myoelectric signal versus force relationship in different human muscles, *Journal of Applied Physiology*, 54(6):1653–1659, 1983. ©American Physiological Society.





6

Frequency-domain Characterization of Signals and Systems

Many biomedical systems exhibit innate rhythms and periodicity: more readily expressed and appreciated in terms of frequency than time units.



Cardiac rhythm more conveniently expressed in

beats per minute,

a measure of frequency of occurrence or rate of repetition,

than in terms of duration of a beat or

interval between beats in seconds (*RR* interval).

Cardiac rhythm expressed as *72 bpm* more easily

understood than the corresponding *RR* interval as *0.833 s*.



EEG rhythm conveyed more readily by description in

cycles per second or in *Hertz (Hz)*

than a time-domain description:

alpha rhythm having a frequency of 11.5 Hz

versus the equivalent period of 0.087 s .



PCG: interesting example of a signal with

multiple frequency-domain features.

Beat-to-beat periodicity or rhythm.

Heart sounds within a cardiac cycle exhibit *resonance*.

Multi-compartmental nature of the cardiac system:

multiple resonance frequencies: composite *spectrum* of

several dominant or resonance frequencies.



Constrained flow of blood through an orifice (septal defect)

or across a stenosed valve acting as a baffle

leads to turbulence: *wide-band noise*.

Need to consider the distribution of the signal's energy or

power over a wide band of frequencies:

leads to the notion of the power *spectral density* function.



6.1 Problem Statement

Investigate the potential use of the Fourier spectrum and parameters thereof in the analysis of biomedical signals.

Identify physiological and pathological processes that could modify the frequency content of the corresponding signals.

Outline the signal processing tasks needed to perform spectral analysis of biomedical signals and systems.



6.2 Illustration of the Problem with Case-studies

6.2.1 *The effect of myocardial elasticity on heart sound spectra*

The first and second heart sounds — S1 and S2 —
are typically composed of low-frequency components,
due to the fluid-filled and elastic nature
of the cardiohemic system.



Sakai et al. processed recorded heart sound signals by using

tunable bandpass filters, with a bandwidth of 20 Hz ,

tuned over the range $20 - 40\text{ Hz}$ to $400 - 420\text{ Hz}$, and

estimated the frequency distributions of S1 and S2.

Heart sound spectra maximum in the $20 - 40\text{ Hz}$ band;

S1: peaks at lower frequencies than those of S2;

S2: “gentle peaking” between 60 Hz and 220 Hz .



Gerbarg et al.: computer program to simulate a filter bank.

Obtained averaged power spectra of S1 and S2 of

1,000 adult males,

32 high-school children, and 75 patients in a hospital.

Averaged PSDs of S1 and S2:

peak power in the range $60 - 70 \text{ Hz}$,

power levels lower than -10 dB beyond 150 Hz .

PSD of S2: more high-frequency energy than S1.



Yoganathan et al.: FFT analysis of S1 and S2.

Spectra of 250 ms windows with S1 averaged over 15 beats.

Spectrum of S1:

peaks in a low-frequency range ($10 - 50\text{ Hz}$)

and a medium-frequency range ($50 - 140\text{ Hz}$).



Spectrum of S2: peaks in low-frequency ($10 - 80 \text{ Hz}$),
medium-frequency ($80 - 220 \text{ Hz}$),
and high-frequency ranges ($220 - 400 \text{ Hz}$).

Spectral peaks related to resonance and the
elastic properties of heart muscles.



Adolph et al.: dynamic spectrum analyzer to study frequency content of S1 during iso-volumic contraction.

Center frequency of filter with 20 Hz bandwidth varied from 30 Hz to 70 Hz , in steps of 10 Hz .

Outputs of filters averaged over 10 consecutive beats.

Ratios of the average peak voltage of the filtered outputs to that of full S1 during iso-volumic contraction computed.



Adolph et al. hypothesized that frequency content of S1 during iso-volumic contraction should depend on relative contributions of mass and elasticity of left ventricle.

Mass of left ventricle with blood content remains constant during iso-volumic contraction.

Frequency content of S1 decreases in case of diseases that reduce ventricular elasticity: myocardial infarction.

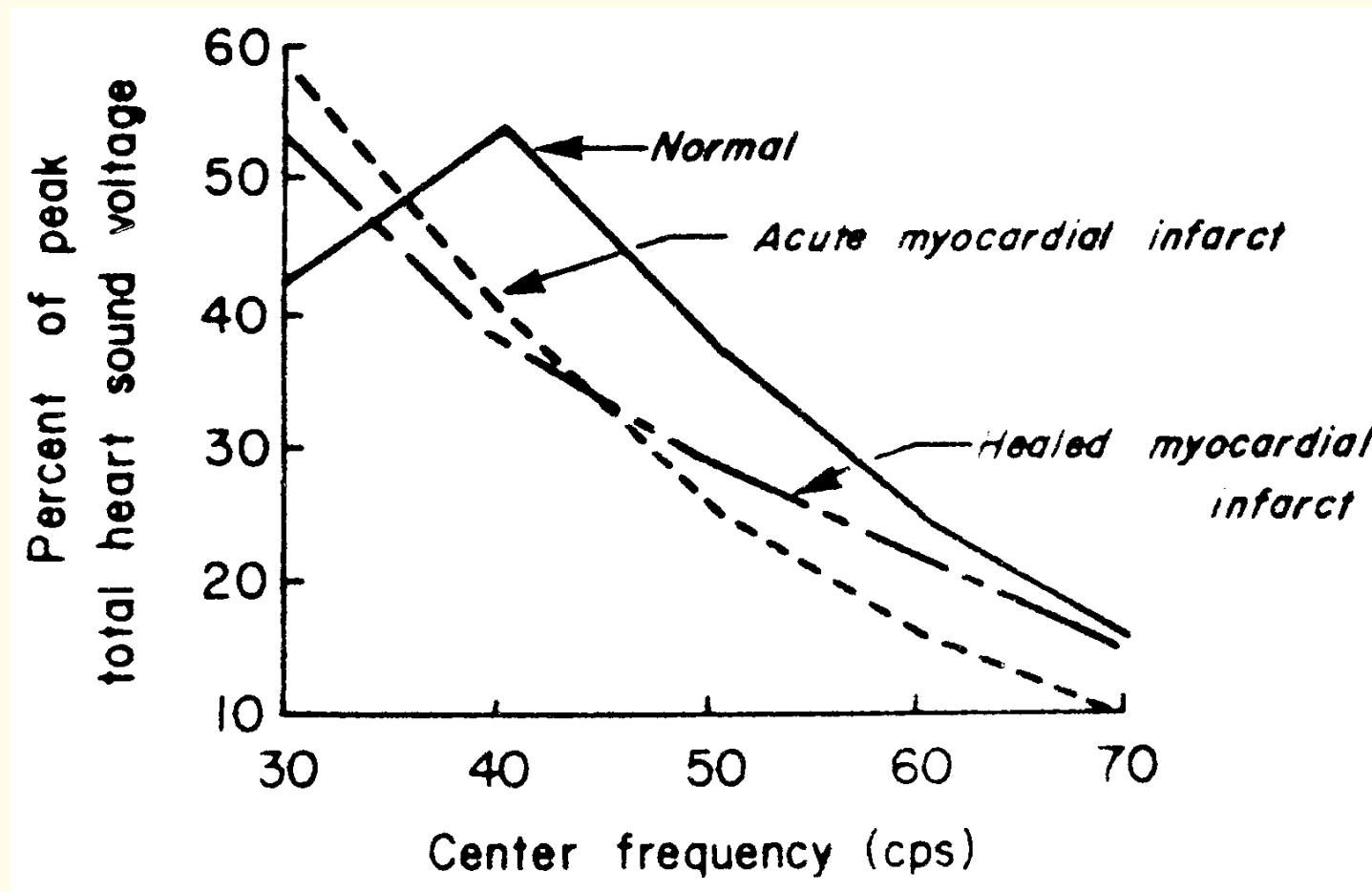


Figure 6.1: First heart sound spectra for normal, acute myocardial infarct, and healed myocardial infarct cases. The latter two cases exhibit an increased percentage of low-frequency components. Reproduced with permission from R.J. Adolph, J.F. Stephens, and K. Tanaka, The clinical value of frequency analysis of the first heart sound in myocardial infarction, *Circulation*, 41:1003–1014, 1970. ©American Heart Association.



6.2.2 Frequency analysis of murmurs to diagnose valvular defects

Cardiovascular valvular defects and diseases cause high-frequency, noise-like sounds known as murmurs.

Yoshimura et al.: tunable bandpass filter with cutoff

$18 - 1,425 \text{ Hz}$ to process recorded PCG signals.

Diastolic *rumble* of mitral stenosis: $20 - 200 \text{ Hz}$.

Diastolic *blow* of aortic regurgitation: $200 - 1,600 \text{ Hz}$.



van Vollenhoven et al.:

tunable bandpass filter, 50 Hz bandwidth,

center frequency tunable in steps of 50 Hz .

100 ms window in diastolic phase of recorded PCG signals.

Murmur of mitral stenosis: limited to less than 400 Hz .

Aortic insufficiency combined with mitral stenosis:

more high-frequency energy in the range $300 - 1,000\text{ Hz}$.



Sarkady et al.:

synchronized averaging of PSDs of PCG signals

over several cardiac cycles computed using the FFT.



Johnson et al.:

FFT-based PSDs of systolic murmur (aortic stenosis).

PSDs of systolic windows of duration 86, 170, and 341 *ms*,

averaged over 10 cardiac cycles.

Hypothesis: higher murmur frequencies generated

as the severity of aortic stenosis increases.



Trans-aortic-valvular systolic pressure gradient measured

with catheterization and cardiac fluoroscopy:

$10 - 140 \text{ mm of Hg}$.

Spectral power ratios computed with the bands

$25 - 75 \text{ Hz}$: constant area (CA) related to normal sounds,

$75 - 150 \text{ Hz}$: predictive area (PA) related to murmurs.

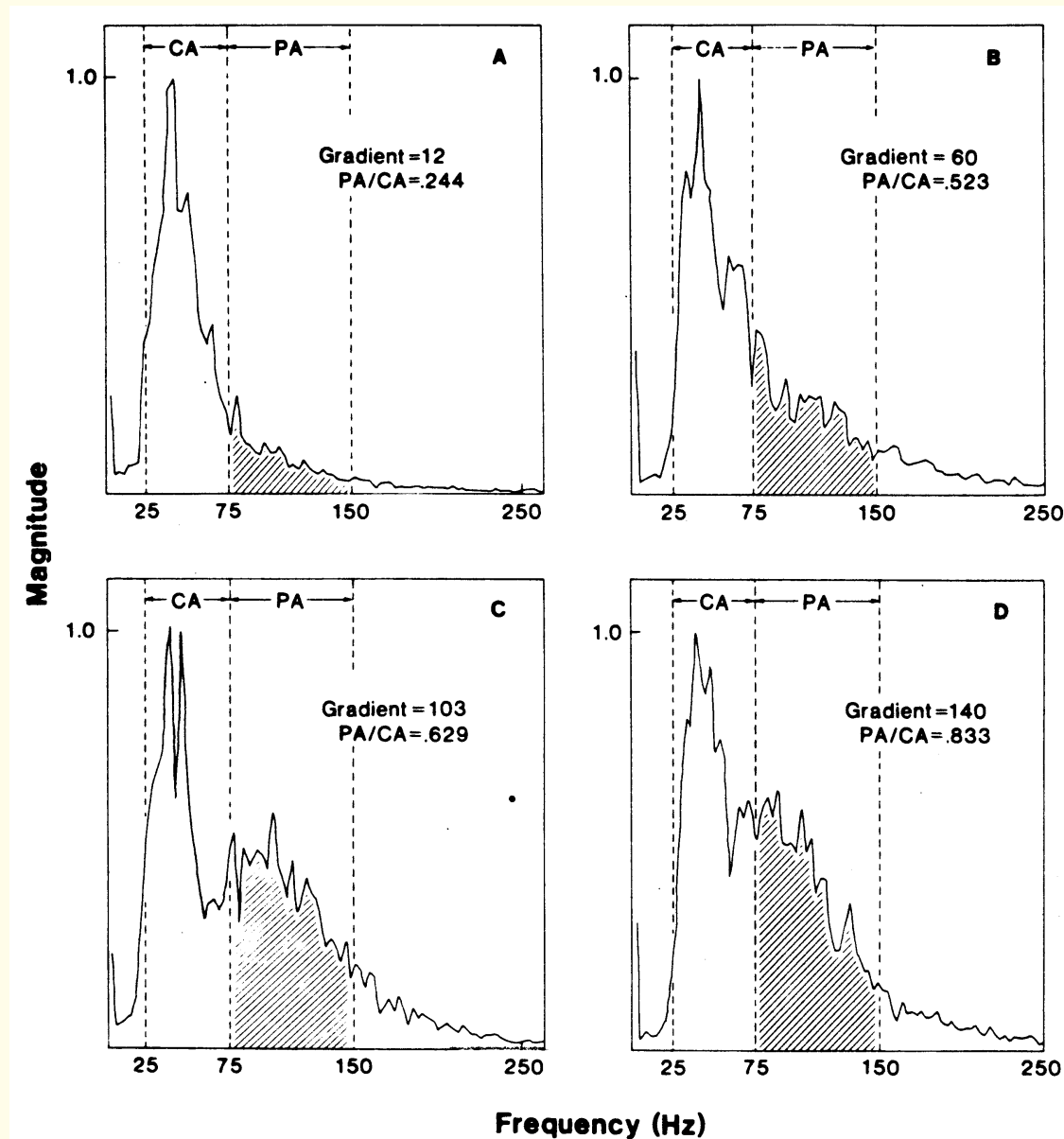


Figure 6.2: Averaged and normalized PSDs of four patients with aortic stenosis of different levels of severity. Each PSD is segmented into two parts: a constant area *CA* and a predictive area *PA*. The trans-valvular systolic pressure gradient (measured via catheterization in *mm of Hg*) and the *PA/CA* spectral power ratio are shown for each case. Reproduced with permission from the American College of Cardiology: G.R. Johnson, R.J. Adolph, and D.J. Campbell, Estimation of the severity of aortic valve stenosis by frequency analysis of the murmur, *Journal of the American College of Cardiology*, 1(5):1315–1323, 1983 ©Elsevier Science.



6.3 The Fourier Spectrum

Fourier transform: most commonly used transform

to study the frequency-domain characteristics of signals.

Fourier transform uses sinusoidal functions

as its basis functions.



Projections computed of the given signal $x(t)$ onto the complex exponential basis function of frequency ω in *radians/s*:

$$\exp(j\omega t) = \cos(\omega t) + j \sin(\omega t).$$

$$X(\omega) = \int_{-\infty}^{\infty} x(t) \exp(-j\omega t) dt, \quad (6.1)$$



or in the frequency variable f in Hz as

$$X(f) = \int_{-\infty}^{\infty} x(t) \exp(-j2\pi ft) dt. \quad (6.2)$$

This is the *analysis* step of the transform.



Inverse transformation: *synthesis* of the signal $x(t)$ as a
weighted combination of the complex exponential
basis functions:

$$\begin{aligned} x(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) \exp(j\omega t) d\omega \\ &= \int_{-\infty}^{\infty} X(f) \exp(j2\pi ft) df. \end{aligned} \quad (6.3)$$



Discrete-time signal $x(n)$, continuous frequency variable ω :

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n) \exp(-j\omega n). \quad (6.4)$$

Normalized-frequency: $0 \leq \omega \leq 2\pi$ or $0 \leq f \leq 1$.

Lower limit of the summation: 0 if the signal is causal.

Upper limit: index $(N - 1)$ of the last sample

in the case of a finite-duration signal with N samples.



Frequency variable ω may also be defined over $0 \leq \omega \leq \omega_s$,

equivalent to $0 \leq f \leq f_s$;

then n multiplied by the sampling interval T in seconds.

Fourier transform equivalent to z -transform evaluated

on the unit circle with $z = \exp(j\omega)$.

Fourier transform of a sampled signal is periodic:

period = sampling frequency ω_s or

2π (normalized frequency).



When processing digital signals on a computer,

frequency variable ω is sampled, as $\omega = 2\pi \frac{f_s}{N} k$,

or in normalized frequency as $\omega = \frac{2\pi}{N} k$.

k : frequency sample index.

N : number of samples over one period of spectrum $X(\omega)$.



DFT (analysis) relationship:

$$X(k) = \sum_{n=0}^{N-1} x(n) \exp\left(-j \frac{2\pi}{N} k n\right), \quad k = 0, 1, 2, \dots, N-1. \quad (6.5)$$

Assumed that the given signal has N samples.

FT of a discrete-time signal with N samples is

completely determined by N samples of its FT

equally spaced around the unit circle in the z -plane.



Inverse DFT (synthesis) relationship:

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) \exp \left(j \frac{2\pi}{N} k n \right), \quad n = 0, 1, 2, \dots, N-1. \quad (6.6)$$

Sampling the frequency variable causes the signal to become periodic in the time domain.

The equations above define forward and inverse DFTs over one period.



$$\exp\left(j\frac{2\pi}{N}kn\right) = \cos\left(\frac{2\pi}{N}kn\right) + j \sin\left(\frac{2\pi}{N}kn\right) \quad (6.7)$$

Sine and cosine functions of normalized frequency

$$f = \frac{1}{N}k, \quad k = 0, 1, 2, \dots, N - 1.$$

Normalized frequency in the range $0 \leq f \leq 1$;

converted to real frequency in Hz

by multiplication with $f_s Hz$.



DFT Equation 6.5:

dot product or projection of the given signal $x(n)$ onto each complex exponential or sinusoid $\exp(-j\frac{2\pi}{N}kn)$.

Inverse DFT Equation 6.6:

synthesis of the signal $x(n)$ as a linear combination of the complex exponential basis functions, with the weights being the DFT coefficients $X(k)$.



Important properties of the DFT and their implications:

- A signal $x(n)$ and its DFT $X(k)$ are both periodic.
- If a signal $x(n)$ has N samples, its DFT $X(k)$ must be computed with at least N samples equally spaced over the normalized-frequency range $0 \leq \omega \leq 2\pi$ (equivalently, around the unit circle in the z -plane) for complete representation and determination of $X(\omega)$, and exact reconstruction of $x(n)$ = inverse DFT of $X(k)$.



One may use more than N samples to compute $X(k)$ to employ an FFT algorithm with $L = 2^M \geq N$ samples, or to obtain $X(\omega)$ with finer frequency sampling than $\frac{2\pi}{N}$.

- The DFT is linear:

$$\text{DFT} [a x(n) + b y(n)] = a X(k) + b Y(k).$$

$X(k)$ and $Y(k)$: DFTs of $x(n)$ and $y(n)$.



- DFT $[x(n - n_o)] = \exp(-j \frac{2\pi}{N} k n_o) X(k)$.

A time shift leads to a linear component being added to the phase of the original signal.

All sequences in DFT relationships are periodic: shift operation defined as a circular or periodic shift. If at least n_o zeros are present or are padded at the end of the signal before the shift operation, a circular shift will be equivalent to a linear shift.



- DFT $[x(n) * h(n)] = X(k) H(k)$.

Inverse DFT $[X(k) H(k)] = x(n) * h(n)$.

Similarly, $x(n) h(n)$ and $X(k) * H(k)$ form a DFT pair.

Convolution in one domain is equivalent to multiplication in the other.

All signals must have the same number of samples N .



All sequences in DFT relationships are periodic:
convolution operations in the DFT relationships are
periodic convolution and not linear convolution.
Circular or periodic convolution defined for
periodic signals having the same period:
result will also be periodic with the same period.



The result of linear convolution of two signals

$x(n)$ and $h(n)$

with different durations N_x and N_h samples, respectively, will have a duration of $N_x + N_h - 1$ samples.

If linear convolution is desired via the inverse DFT of $[X(k) H(k)]$, the DFTs must be computed with $L \geq N_x + N_h - 1$ samples.

The signals should be padded with zeros at the end to make their effective durations equal for DFT computation and multiplication.

All signals and DFTs are periodic with the augmented period L .



- DFT of a real signal $x(n)$ possesses conjugate symmetry:
 $X(-k) = X^*(k)$.

Real part and magnitude of $X(k)$: even sequences;
imaginary part and phase of $X(k)$: odd sequences.



- Parseval's theorem: total energy of the signal remains the same before and after Fourier transformation.

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(\omega)|^2 d\omega, \quad (6.8)$$

$$\sum_{n=-\infty}^{\infty} |x(n)|^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega,$$

$$\sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2.$$



Integral of $|X(\omega)|^2$ over all ω or the
sum of $|X(k)|^2$ over all k : total energy of the signal,
or average power if divided by the duration of the signal.
 $|X(\omega)|^2$ and $|X(k)|^2$ — power spectral *density*
as a function of frequency.



6.4 Estimation of the Power Spectral Density Function

Consider a signal with N samples:

$$x(n), n = 0, 1, 2, \dots, N - 1.$$

To compute the time-averaged ACF

$\phi_{xx}(m)$ for a delay of m samples,

we need to form the product $x(n) x(n \pm m)$

and sum over the available range of data samples.

True ACF: $\phi_{xx}(m) = E[x(n) x(n + m)]$.



We may sum from $n = 0$ to $n = N - 1$ when computing

$\phi_{xx}(0)$ with $x(n) x(n) = x^2(n)$.

However, when computing $\phi_{xx}(1)$ with $x(n) x(n + 1)$,

we can only sum from $n = 0$ to $n = N - 2$.

Linear shift of m samples to compute $\phi_{xx}(\pm m)$:

m samples of one signal drop out of the window of analysis

indicated by the overlap between the

two versions of the signal.



Only $N - |m|$ pairs of data samples available to estimate

ACF for delay of $\pm m$ samples.

Sample-mean estimate of ACF:

$$\phi_1(m) = \frac{1}{N - |m|} \sum_{n=0}^{N-|m|-1} x(n) x(n + m). \quad (6.9)$$



Oppenheim and Schafer:

$\phi_1(m)$ is a consistent estimate of $\phi_{xx}(m)$.

Zero bias and variance that tends to zero as $N \rightarrow \infty$.

Variance of estimate exceptionally large as $m \rightarrow N$:

very few non-zero pairs of samples available

to compute the ACF.



Alternative definition of ACF:

ignore lack of $|m|$ non-zero pairs of samples,

apply same scale factor for all delays:

$$\phi_2(m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} x(n) x(n+m). \quad (6.10)$$



Upper limit of summation could be $N - 1$:

no effect on the result.

First or last $|m|$ samples of $x(n)$

will not overlap with $x(n + m)$:

result in zero product terms.



Oppenheim and Schafer: $\phi_2(m)$ has bias $= \frac{m}{N} \phi_{xx}(m)$.

Bias tends to actual value being estimated as $m \rightarrow N$.

Variance almost independent of m ,

tends to zero as $N \rightarrow \infty$.

Both ACF estimates asymptotically unbiased;

bias of $\phi_2(m)$ tends to zero as $N \rightarrow \infty$.

Good estimates of ACF if N is large and $m \ll N$.



ACF estimates $\phi_1(m)$ and $\phi_2(m)$ inter-related:

$$\phi_2(m) = \frac{N - |m|}{N} \phi_1(m). \quad (6.11)$$

$\phi_2(m)$ is a scaled version of $\phi_1(m)$.

Scaling factor is a function of m : referred to as a *window*.



6.4.1 The periodogram

PSD and ACF: Fourier transform pair.

$$S_2(\omega) = \sum_{m=-(N-1)}^{N-1} \phi_2(m) \exp(-j\omega m). \quad (6.12)$$

Fourier transform of $x(n)$, $n = 0, 1, 2, \dots, N - 1$:

$$X(\omega) = \sum_{n=0}^{N-1} x(n) \exp(-j\omega n). \quad (6.13)$$



$$S_2(\omega) = \frac{1}{N} |X(\omega)|^2. \quad (6.14)$$

PSD estimate $S_2(\omega)$: *periodogram* of $x(n)$.



Oppenheim and Schafer:

$S_2(\omega)$ is a biased estimate of the PSD, with

$$E[S_2(\omega)] = \sum_{m=-(N-1)}^{N-1} \frac{N - |m|}{N} \phi_{xx}(m) \exp(-j\omega m). \quad (6.15)$$



Fourier transform of $\phi_1(m)$: a different estimate of PSD.

$$S_1(\omega) = \sum_{m=-(N-1)}^{N-1} \phi_1(m) \exp(-j\omega m). \quad (6.16)$$

$$E[S_1(\omega)] = \sum_{m=-(N-1)}^{N-1} \phi_{xx}(m) \exp(-j\omega m). \quad (6.17)$$

$S_1(\omega)$ is a biased estimate of the PSD.



The two estimates $S_2(\omega)$ and $S_1(\omega)$ may be seen as the

Fourier transforms of windowed ACFs.

Window function for $\phi_2(m)$:

triangular function or Bartlett window.

$$w_B(m) = \begin{cases} \frac{N-|m|}{N}, & |m| < N \\ 0, & \text{otherwise} \end{cases} . \quad (6.18)$$



Window function for $\phi_1(m)$: rectangular function.

$$w_R(m) = \begin{cases} 1 & |m| < N \\ 0, & otherwise \end{cases} . \quad (6.19)$$

The windows have a duration of $(2N - 1)$ samples.



ACF multiplied with window \rightarrow

PSD convolved with FT of window:

leads to spectral leakage and loss of resolution.

Fourier transforms of Bartlett and rectangular windows:

$$W_B(\omega) = \frac{1}{N} \left[\frac{\sin(\omega N/2)}{\sin(\omega/2)} \right]^2, \quad (6.20)$$

$$W_R(\omega) = \frac{\sin[\omega(2N - 1)/2]}{\sin(\omega/2)}. \quad (6.21)$$



Oppenheim and Schafer: periodogram has a

variance that does not approach zero as $N \rightarrow \infty$;

variance of the order of σ_x^4 regardless of N .

The periodogram is not a consistent estimate of the PSD.



6.4.2 *The need for averaging*

Common approach to reduce the variance of an estimate:

average over a number of statistically independent estimates.

A number of periodograms may be computed

over multiple observations of a signal and

averaged to obtain a better estimate of the PSD.



It is necessary for the process to be stationary,
at least over the period over which the
periodograms are computed and averaged.



Problem: *How can we obtain an averaged periodogram*

when we are given only one signal record of finite duration?

Solution: Bartlett method to average

periodograms of segments of the given signal record.



1. Divide given data $x(n)$, $n = 0, 1, 2, \dots, N - 1$, into K segments of M samples each:

$$x_i(n) = x(n + (i-1)M), \quad 0 \leq n \leq M-1, \quad 1 \leq i \leq K. \quad (6.22)$$

2. Compute the periodogram of each segment as

$$S_i(\omega) = \frac{1}{M} \left| \sum_{n=0}^{M-1} x_i(n) \exp(-j\omega n) \right|^2, \quad 1 \leq i \leq K. \quad (6.23)$$

Fourier transform: DFT using the FFT.



3. If ACF $\phi_{xx}(m)$ negligible for $|m| > M$, the periodograms of the K segments with M samples each may be assumed to be mutually independent. Bartlett estimate $S_B(\omega)$ of the PSD = sample mean of the K independent observations of the periodogram:

$$S_B(\omega) = \frac{1}{K} \sum_{i=1}^K S_i(\omega). \quad (6.24)$$



Oppenheim and Schafer: expected value of the

Bartlett estimate $S_B(\omega)$ = convolution of true PSD $S_{xx}(\omega)$

with the FT of Bartlett window.

Bias, spectral smearing, and leakage:

loss of spectral resolution.

Variance tends to zero as number of segments K increases:

consistent estimate of the PSD.



Stationary signal of fixed duration of N samples: limitations

on the number of segments K that we may obtain.

Variance of PSD estimate decreases as K is increased:

but as M decreases, the main lobe of the FT of the

Bartlett window widens; frequency resolution lost.



Cyclo-stationary signals such as the PCG offer a unique and interesting approach to synchronized averaging of periodograms over a number of cycles, without trade-off between reduction of variance and loss of spectral resolution.



6.4.3 *The use of windows: Spectral resolution and leakage*

Bartlett procedure: ensemble averaging approach to reduce the variance of the periodogram.

Another approach: to obtain a smooth spectrum, convolve periodogram $S(\omega)$ with filter or smoothing function $W(\omega)$ in the frequency domain (similar to the use of an MA filter in the time domain).



Smoothed estimate $S_s(\omega)$:

$$S_s(\omega) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S(\nu) W(\omega - \nu) d\nu. \quad (6.25)$$

ν : temporary variable for integration.

PSD: nonnegative function.

Condition on the smoothing function $W(\omega)$:

$$W(\omega) \geq 0, -\pi \leq \omega \leq \pi.$$



Oppenheim and Schafer:

variance of smoothed periodogram reduced by the factor

$$\frac{1}{N} \sum_{m=-(M-1)}^{M-1} w^2(m) = \frac{1}{2\pi N} \int_{-\pi}^{\pi} W^2(\omega) d\omega, \quad (6.26)$$

with reference to variance of original periodogram.

N : total number of samples in signal.

$(2M - 1)$: number of samples in smoothing window.



Rectangular window: variance-reduction factor of $\sim \frac{2M}{N}$.

Bartlett window: variance-reduction factor of $\sim \frac{2M}{3N}$.

Smoothing of spectrum and reduction of variance achieved
at the price of loss of frequency resolution.



Periodogram = FT of ACF estimate $\phi(m)$:

convolution operation in frequency domain in Equation 6.25

equivalent to multiplying $\phi(m)$ with $w(m) = \text{IFT} [W(\omega)]$.

Same PSD estimate as $S_s(\omega)$ obtained by

applying a window to the ACF estimate and

taking the Fourier transform of the result.

ACF is an even function: window should also be even.



Welch's method to average modified periodograms:

Given signal is segmented as in the Bartlett procedure,

but a window is applied to the original signal segments.

Periodograms of windowed segments:

$$S_{W_i}(\omega) = \frac{1}{ME_w} \left| \sum_{n=0}^{M-1} x_i(n) w(n) \exp(-j\omega n) \right|^2, \quad (6.27)$$

$$i = 1, 2, \dots, K.$$



E_w = average power of window, duration = M samples:

$$E_w = \frac{1}{M} \sum_{n=0}^{M-1} w^2(n). \quad (6.28)$$

Welch PSD estimate $S_W(\omega)$ =

average of modified periodograms:

$$S_W(\omega) = \frac{1}{K} \sum_{i=1}^K S_{W_i}(\omega). \quad (6.29)$$



Welch showed that, if the segments are not overlapping, the variance of the averaged modified periodogram is inversely proportional to K , the number of segments used.

Spectral window convolved with the PSD in the frequency domain is proportional to the squared magnitude of the

Fourier transform of the time-domain data window applied:

any type of data window may be used.



Commonly used data windows:

causal, length = N samples, defined for $0 \leq n \leq N - 1$.

Rectangular:

$$w(n) = 1. \quad (6.30)$$

Bartlett (triangular):

$$w(n) = \begin{cases} \frac{2n}{N-1}, & 0 \leq n \leq \frac{N-1}{2}, \\ 2 - \frac{2n}{N-1}, & \frac{N-1}{2} \leq n \leq N-1. \end{cases} \quad (6.31)$$



Hamming:

$$w(n) = 0.54 - 0.46 \cos \left(\frac{2\pi n}{N-1} \right). \quad (6.32)$$

Hanning (von Hann):

$$w(n) = \frac{1}{2} \left[1 - \cos \left(\frac{2\pi n}{N-1} \right) \right]. \quad (6.33)$$

Blackman:

$$w(n) = 0.42 - 0.5 \cos \left(\frac{2\pi n}{N-1} \right) + 0.08 \cos \left(\frac{4\pi n}{N-1} \right). \quad (6.34)$$

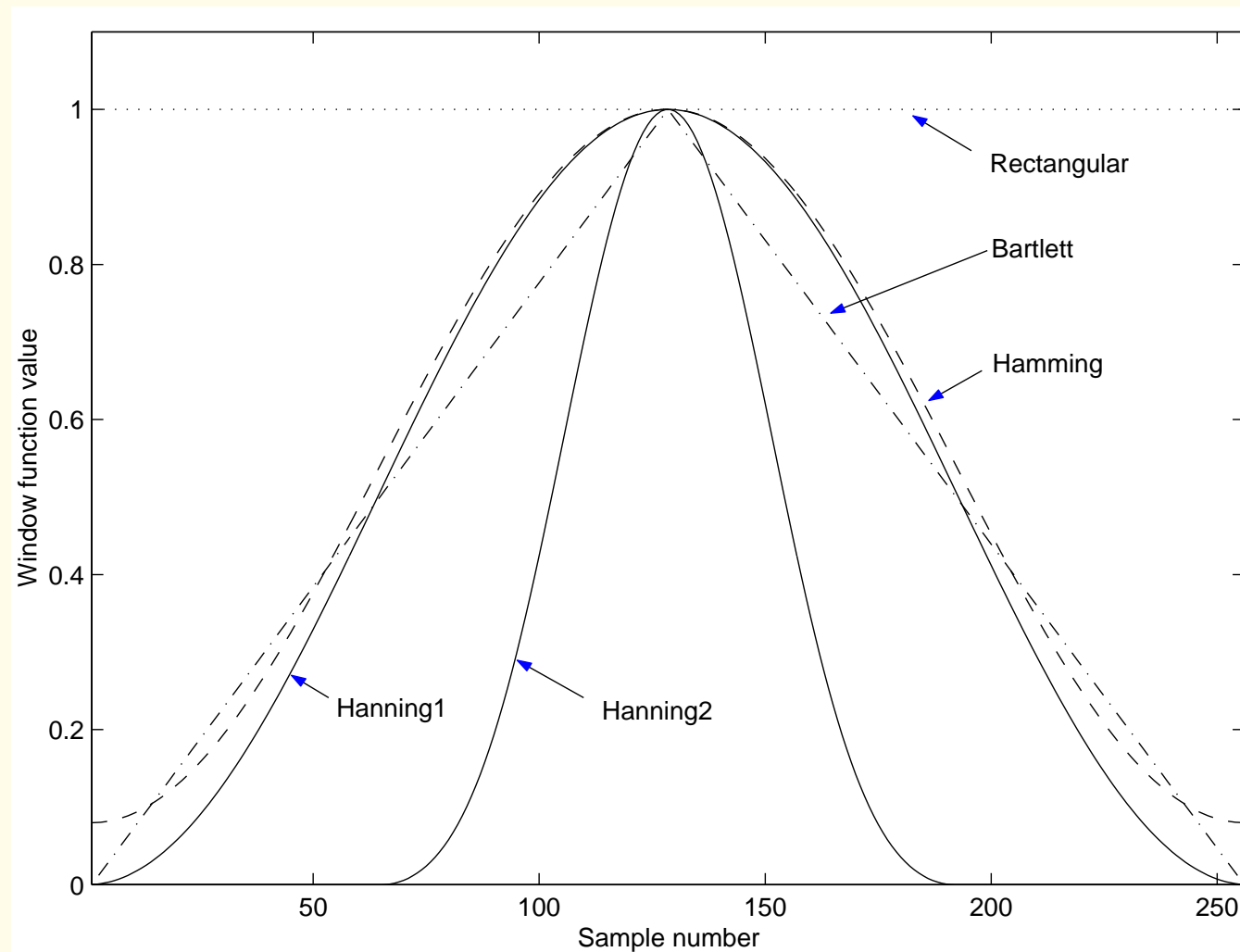


Figure 6.3: Commonly used window functions: rectangular, Bartlett, Hamming, and Hanning windows with $N = 256$ (Hanning1), and Hanning window with $N = 128$ samples (Hanning2). All windows are centered at the 128^{th} sample.



Advantage of tapered windows

(all of the above, except rectangular):

ends of given signal reduced to zero (except Hamming).

No discontinuities in the periodic version of the signal

encountered in DFT-based procedures.

All window functions above are symmetric (even) functions:

linear phase; zero phase if window centered at the origin.

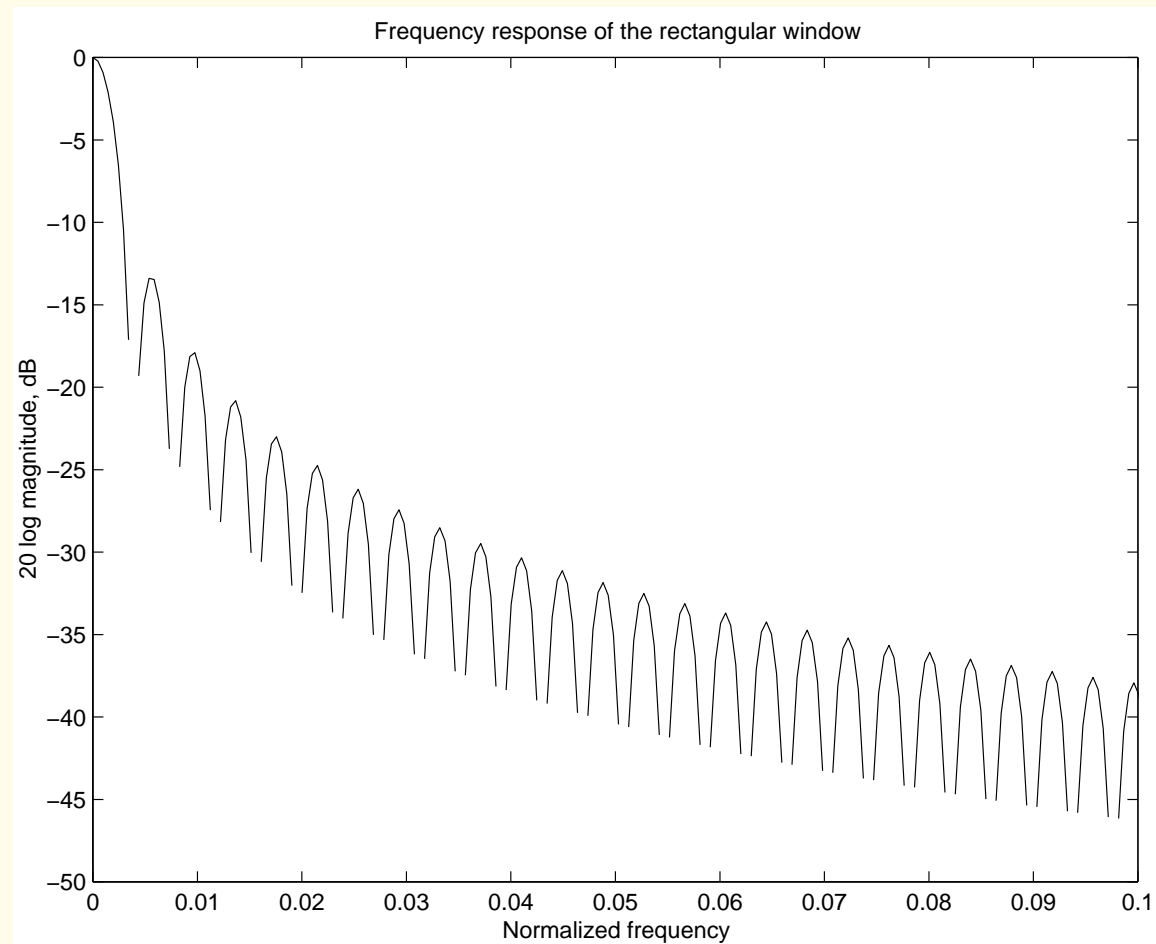


Figure 6.4: Log-magnitude frequency response of the rectangular window illustrated in Figure 6.3. The window width is $N = 256$ samples.

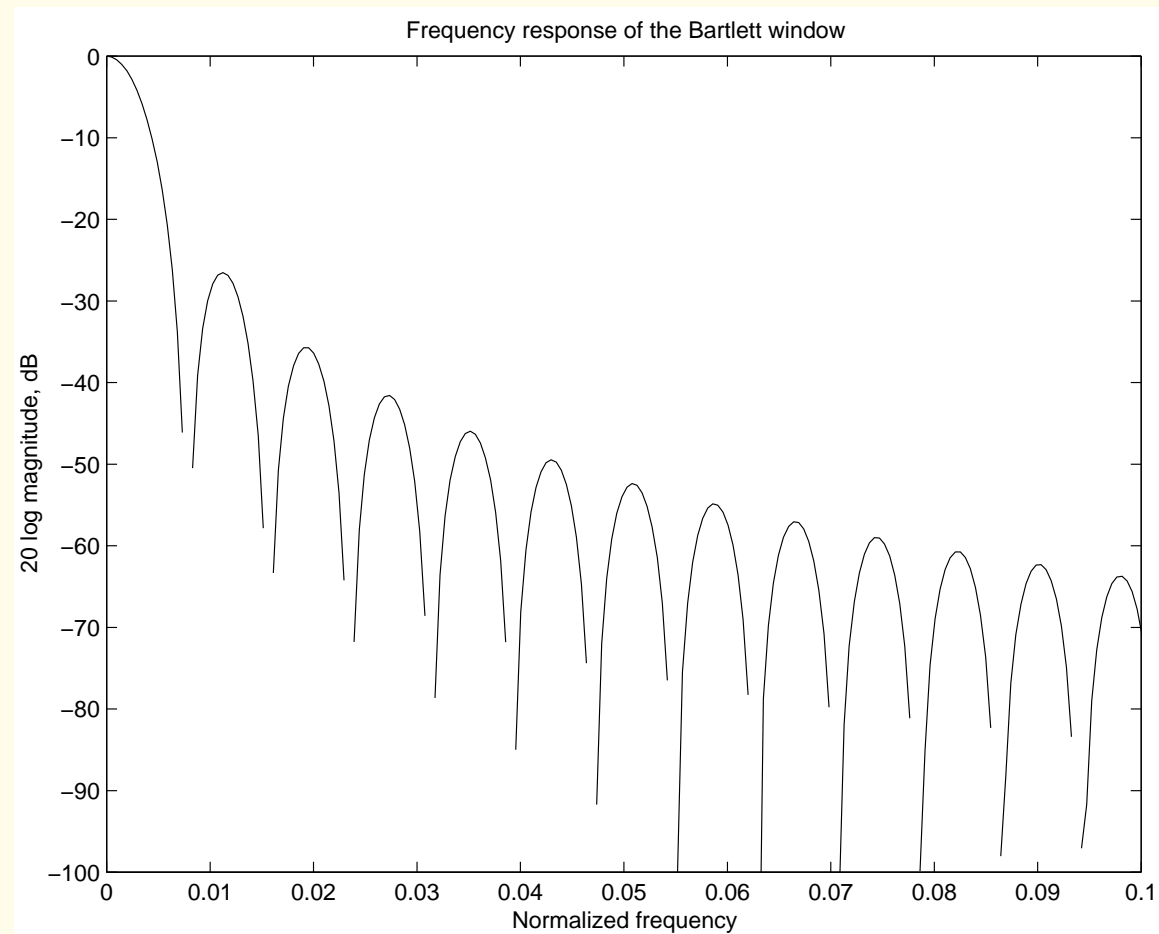


Figure 6.5: Log-magnitude frequency response of the Bartlett window illustrated in Figure 6.3. The window width is $N = 256$ samples.

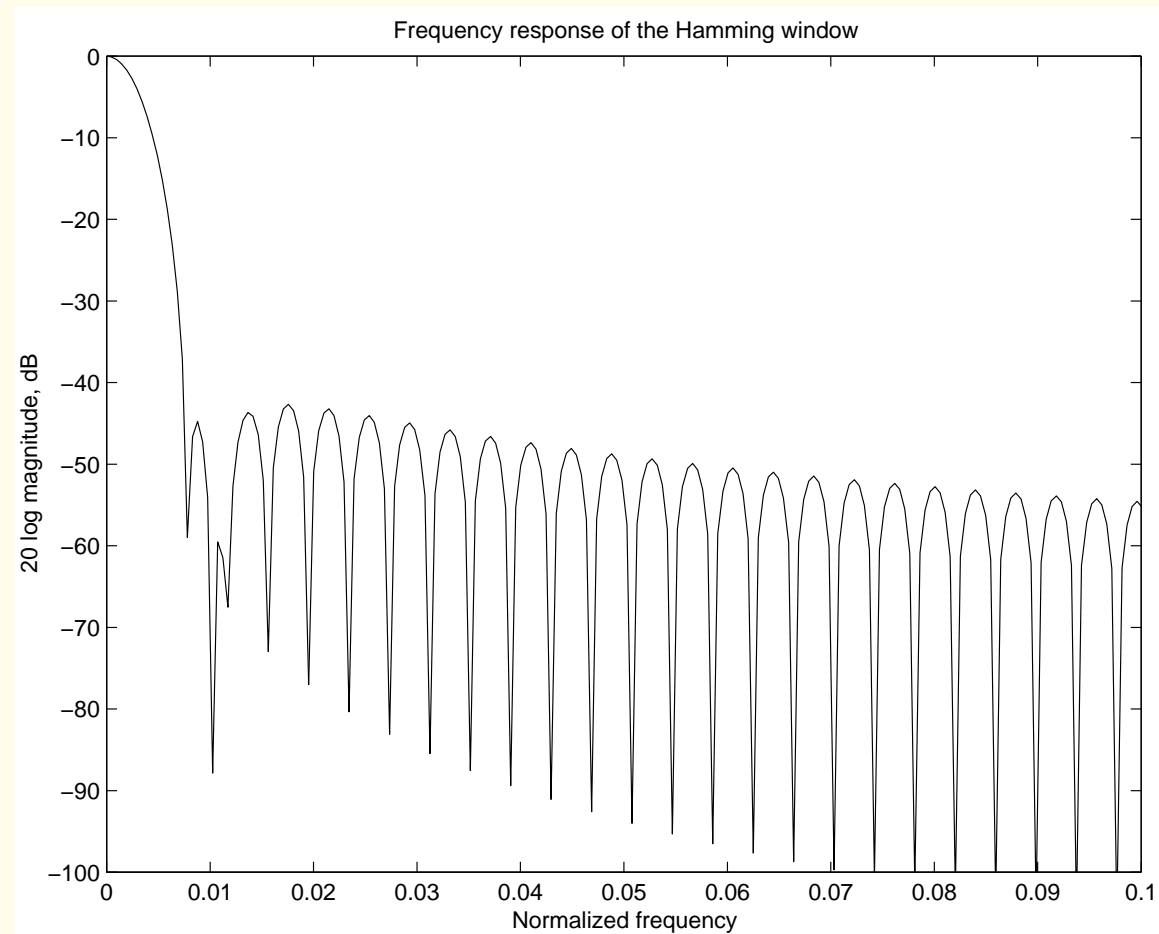


Figure 6.6: Log-magnitude frequency response of the Hamming window illustrated in Figure 6.3. The window width is $N = 256$ samples.

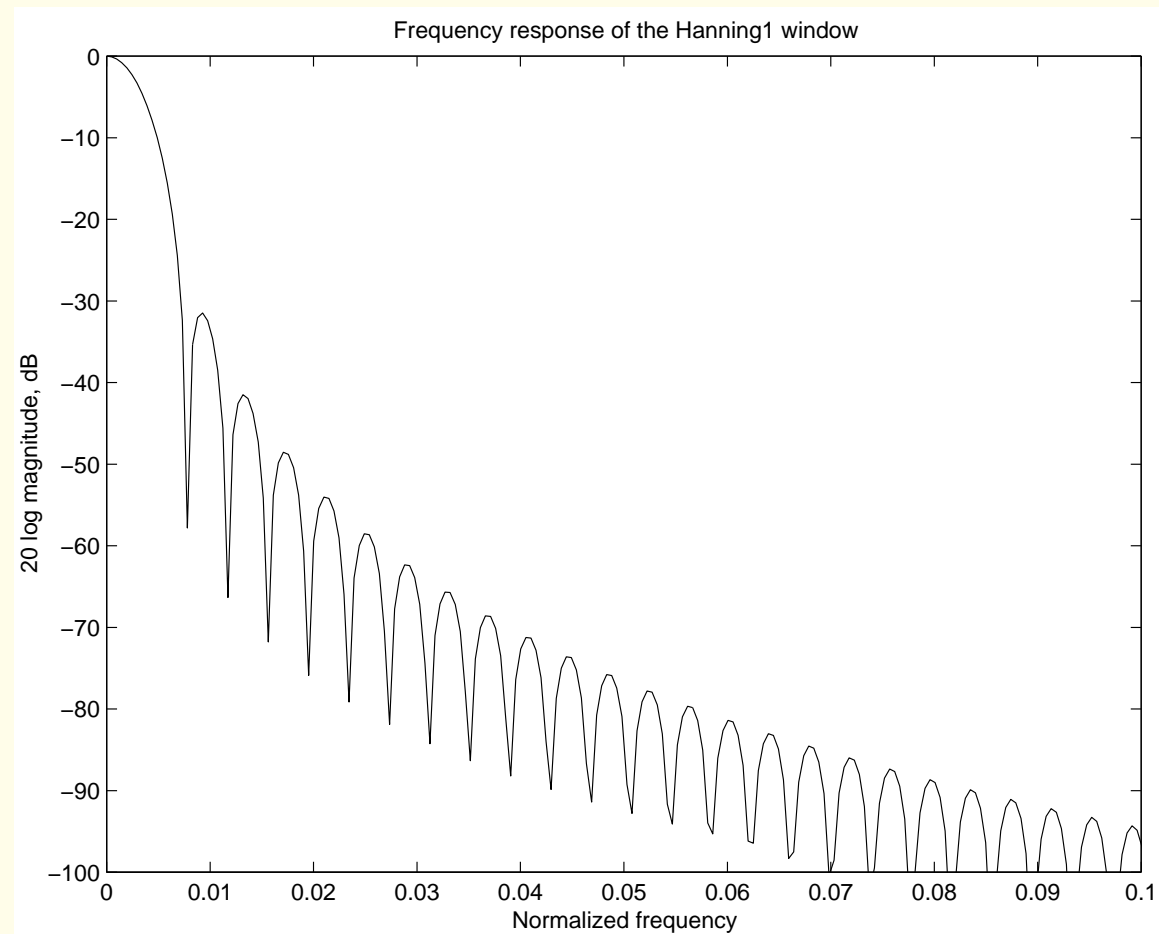


Figure 6.7: Log-magnitude frequency response of the Hanning1 window illustrated in Figure 6.3. The window width is $N = 256$ samples.

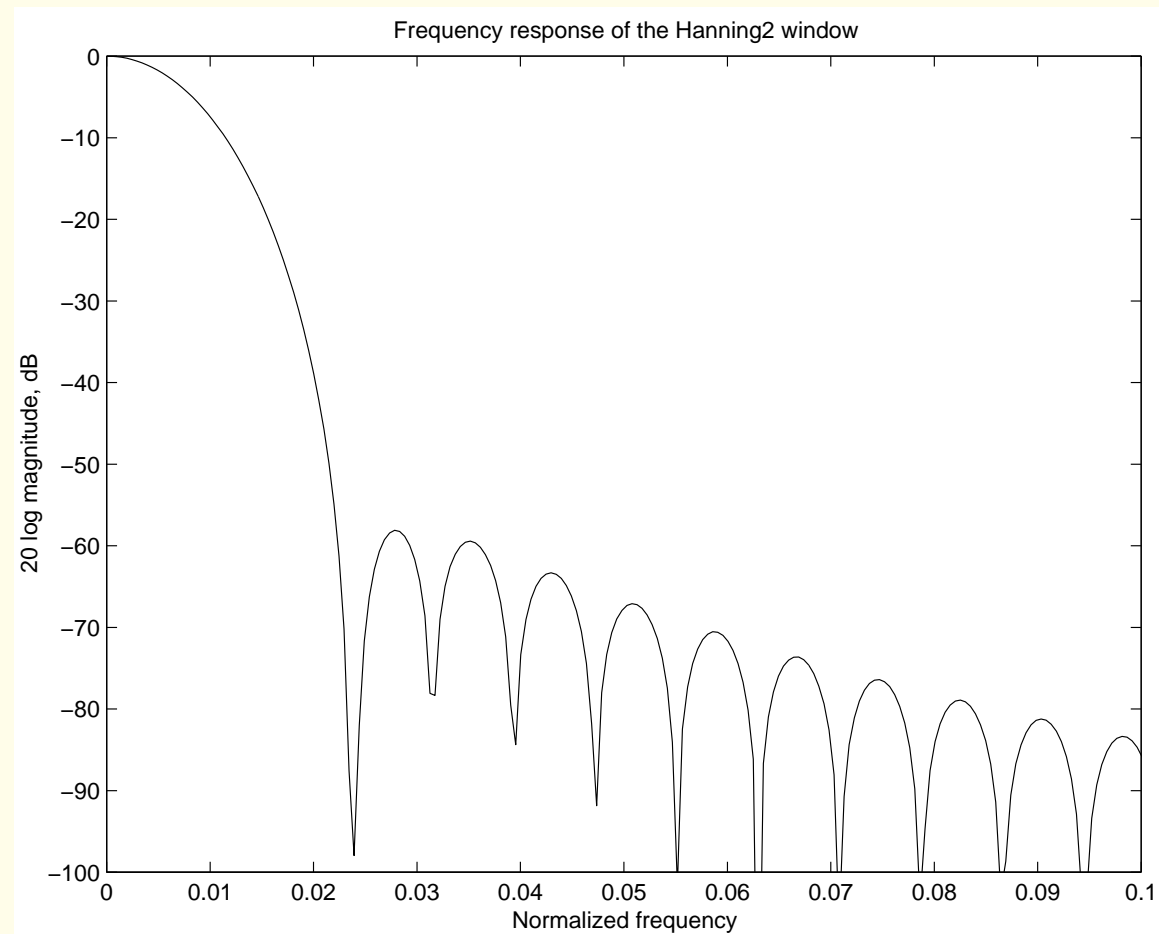


Figure 6.8: Log-magnitude frequency response of the Hanning2 window illustrated in Figure 6.3. The window width is $N = 128$ samples.



Rectangular window: narrowest main lobe of width $\frac{4\pi}{N}$.

Main lobe wider = $\frac{8\pi}{N}$ for Bartlett, Hanning, and Hamming

windows; widest for Blackman window = $\frac{12\pi}{N}$.

Reduction in window width leads to

increase in the main-lobe width.

The wider the main lobe, the greater is the

spectral smoothing: severe loss of spectral resolution.



Rectangular window: highest peak side-lobe = -13 dB .

Bartlett, Hamming, Hanning, and Blackman windows:

peak side-lobe -25 dB , -31 dB , -41 dB , and -57 dB .

Higher side-lobes cause more spectral leakage,

result in a distorted spectrum.

Reduction of leakage with tapered windows comes at the

price of increased main-lobe width: more smoothing and

loss of spectral resolution.

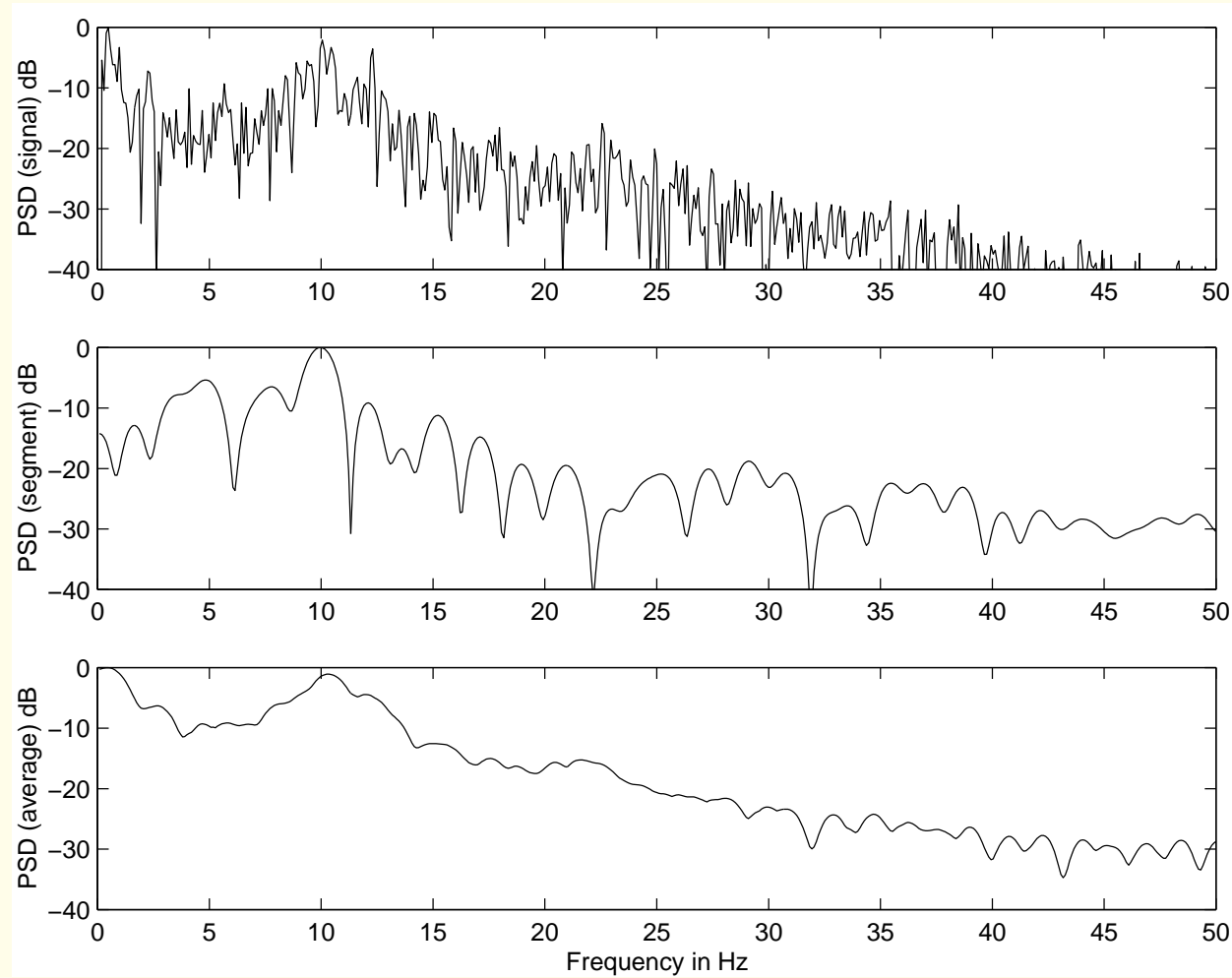


Figure 6.9: Bartlett PSD estimate of the o2 channel of the EEG signal in Figure 1.22. Top trace: PSD of the entire signal. Middle trace: PSD of the 11th segment. Bottom trace: Averaged PSD using $K = 11$ segments of the signal. The rectangular window was (implicitly) used in all cases. Number of samples in the entire signal: $N = 750$. Number of samples in each segment: $M = 64$. All FFT arrays were computed with $L = 1,024$ samples. Sampling frequency $f_s = 100 \text{ Hz}$.

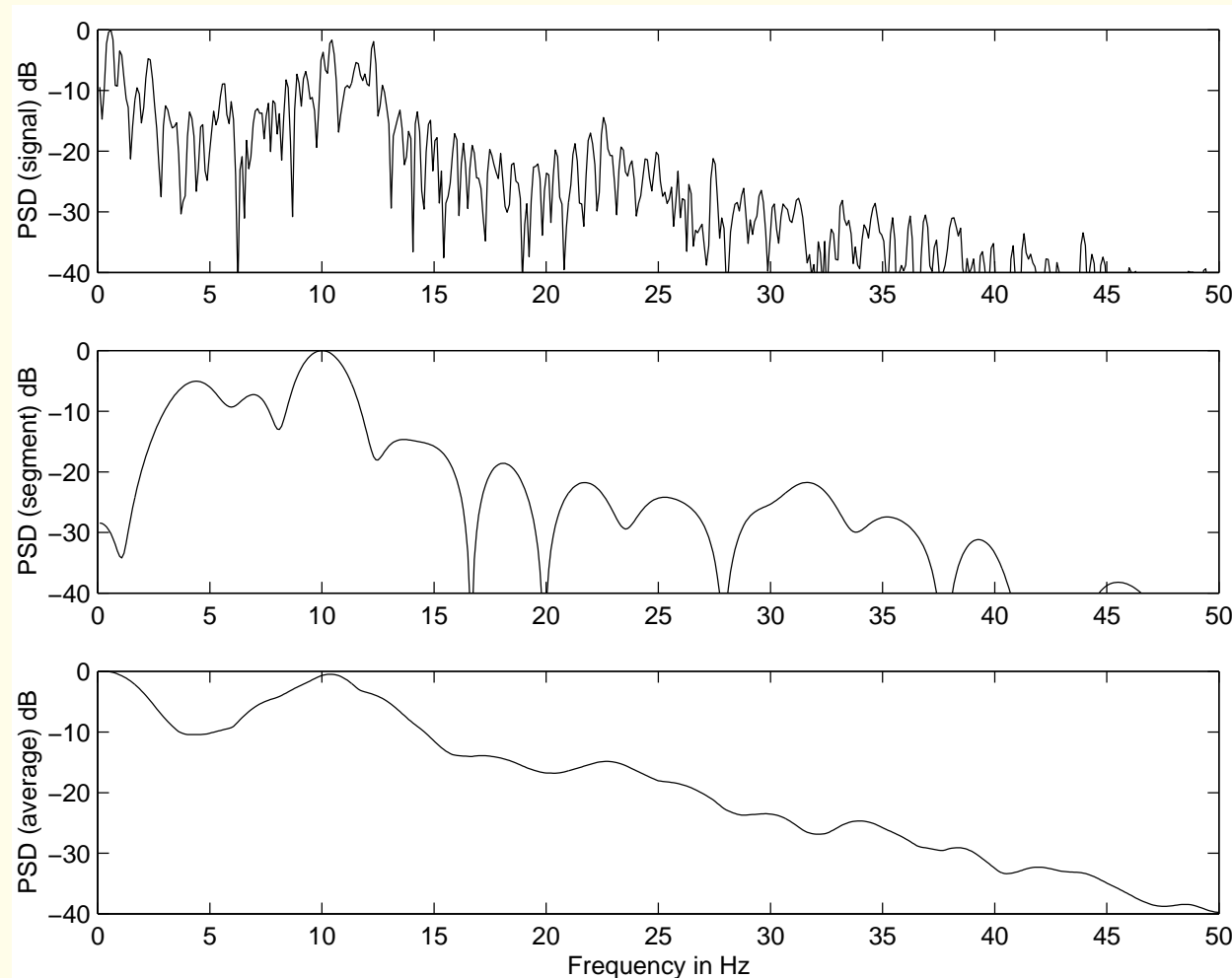


Figure 6.10: Welch PSD estimate of the o2 channel of the EEG signal in Figure 1.22. Top trace: PSD of the entire signal. Middle trace: PSD of the 11th segment. Bottom trace: Averaged PSD using $K = 11$ segments of the signal. The Hanning window was used in all cases. Number of samples in the entire signal and the size of the Hanning window used in computing the PSD of the entire signal: $N = 750$. Number of samples in each segment and the size of the Hanning window used in the averaged periodogram method: $M = 64$. All FFT arrays were computed with $L = 1,024$ samples. Sampling frequency $f_s = 100 \text{ Hz}$.



Hanning window: smoothing of spurious peaks in PSD.

Wider main-lobe of Hanning window's frequency response:

more severe loss of frequency resolution (smoothing).

Averaged PSD in the lowest trace of Figure 6.10 illustrates

benefit of Hanning window: reduced power for $f > 30 \text{ Hz}$.

Lower side-lobe levels of Hanning window: less leakage

(as compared to the rectangular window).



6.4.4 *Estimation of the autocorrelation function*

Good estimates of ACF required in applications such as design of the optimal Wiener filter and estimation of the statistics of stochastic processes.

After a PSD estimate has been obtained (Bartlett or Welch) take the inverse Fourier transform \rightarrow estimate of ACF.

We may also fit a smooth curve or a parametric model — Gaussian, Laplacian, etc. — to the PSD or ACF.



$$\phi_2(m) = \frac{1}{N} \sum_{n=0}^{N-|m|-1} x(n) x(n+m). \quad (6.35)$$

ACF is an even function: need to compute only for $m > 0$.

ACF estimate = $\frac{1}{N} \times$ linear convolution of $x(n)$ with $x(-n)$.

DFT $[x(n)] = X(k) \rightarrow$ DFT $[x(-n)] = X^*(k)$.

Convolution in time \equiv multiplication in frequency:

obtain $X(k) X^*(k) = |X(k)|^2$, take its inverse DFT.



DFT: circular convolution, not linear convolution.

Pad $x(n)$ with at least $M - 1$ zeros;

M : largest lag for which the ACF is desired.

Compute DFT with at least $L = N + M - 1$ samples;

N : number of samples in the original signal.

Include the above steps in averaged periodogram procedure,

take inverse DFT of final PSD estimate \rightarrow estimate of ACF.

Divide by $\frac{1}{N}$, or divide by $\phi_{xx}(0)$ to get normalized ACF.



6.4.5 Synchronized averaging of PCG spectra

PCG: nonstationary signal.

Amount of blood in each cardiac chamber and

state of contraction of cardiac muscles

change continuously during each cardiac cycle.

S2 usually has more high-frequency content than S1.

PSD of normal PCG changes within about 300 ms .



Valve opening or closing sounds: short duration $\sim 10\text{ ms}$;

transients with high-frequency character.

Murmurs add another dimension of nonstationarity:

frequency content well beyond that of normal heart sounds.

PSD of abnormal PCG could change within 100 ms .

Individual epochs of S1, S2, valve snaps, and murmurs of

limited durations of $\sim 10 - 300\text{ ms}$.

Cannot perform segmented averaging (Bartlett or Welch).



The transmission characteristics of the chest wall
change during breathing.

PCG at various locations on the chest are subject to
different transmission-path effects.

Adult subjects may cooperate in PCG signal acquisition by
holding their breath or performing other maneuvers:

no possible with infants and young children in poor health.



Problem: *Propose a method to obtain averaged*

PSD estimates of the systolic and diastolic heart sounds.

Solution: Cyclo-stationarity of PCG — unique approach to average functions of PCG segments corresponding to the same phase of the cardiac cycle cut from multiple beats.

Nonstationarity of the signal within a cardiac cycle:

need to segment phases of PCG cycles.

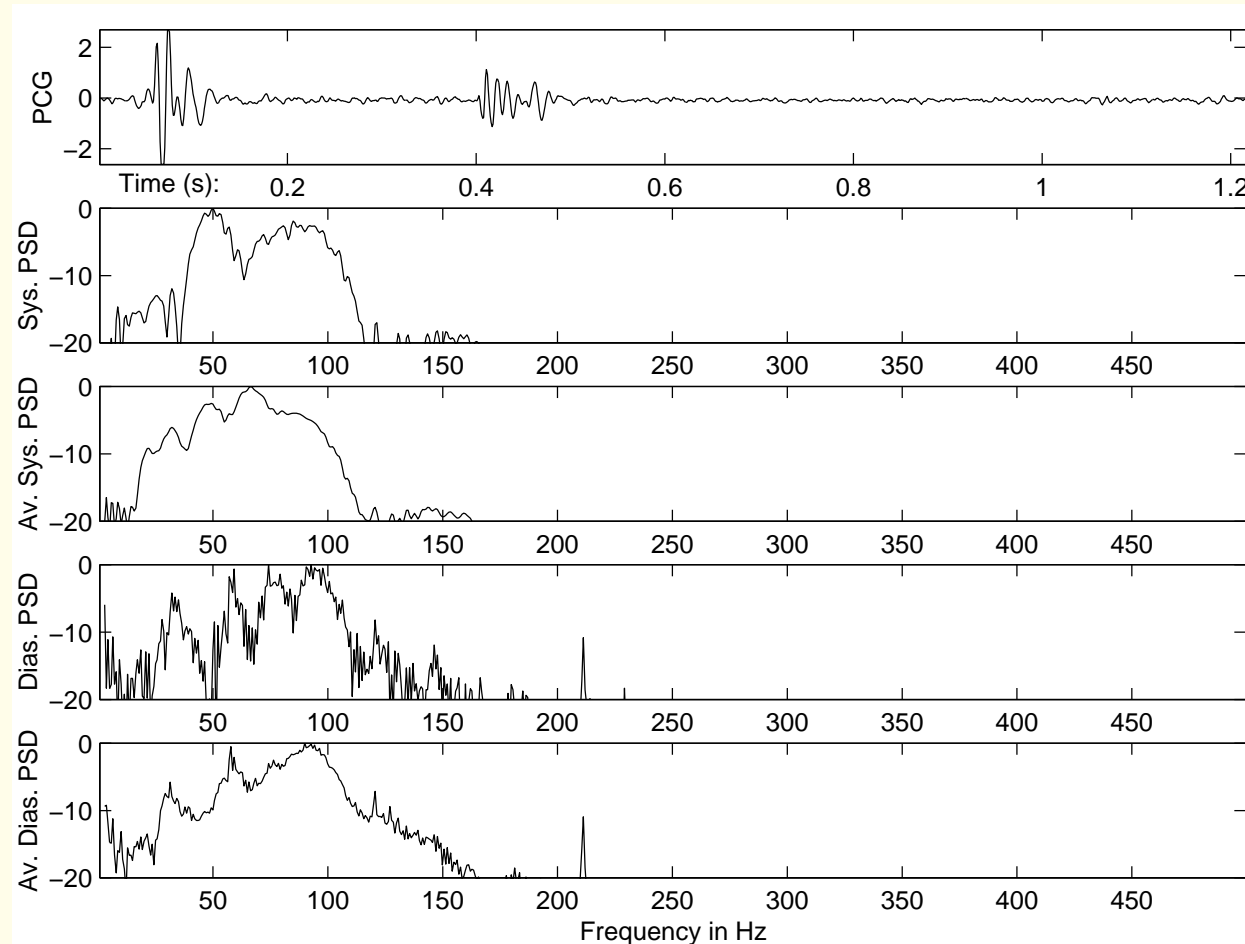


Figure 6.11: Top to bottom: A sample PCG signal over one cardiac cycle of a normal subject (male, 23 years; see also Figures 4.27 and 5.6); periodogram of the systolic portion of the signal (approximately 0 – 0.4 s); averaged periodogram of the systolic parts of 16 cardiac cycles segmented as illustrated in Figure 4.27; periodogram of the diastolic portion of the signal shown in the first plot (approximately 0.4 – 1.2 s); averaged periodogram of the diastolic parts of 16 cardiac cycles. The periodograms are on a log scale (*dB*).

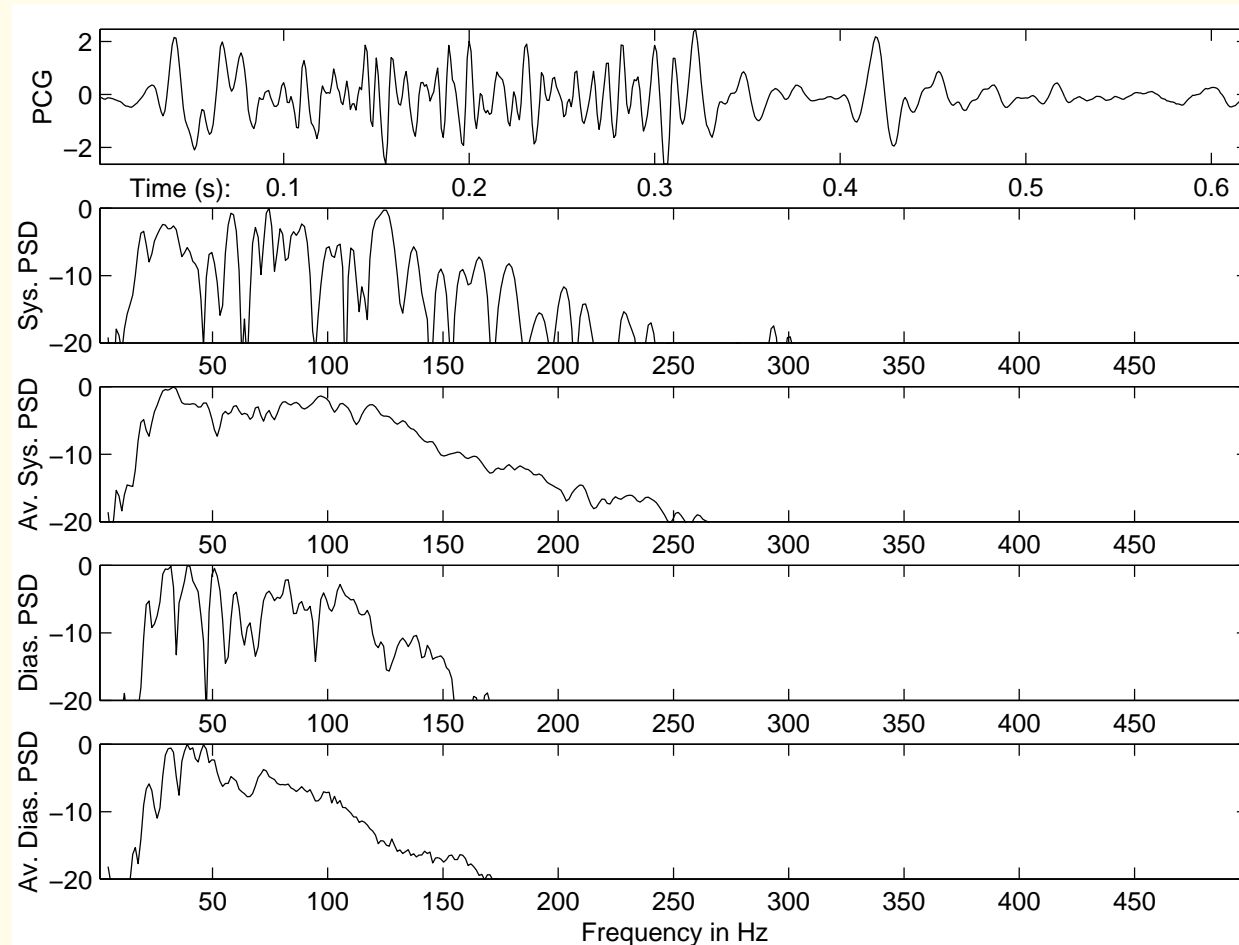


Figure 6.12: Top to bottom: A sample PCG signal over one cardiac cycle of a patient with systolic murmur, split S2, and opening snap of the mitral valve (female, 14 months; see also Figures 4.28 and 5.7); periodogram of the systolic portion of the signal (approximately 0 – 0.28 s); averaged periodogram of the systolic parts of 26 cardiac cycles segmented as illustrated in Figure 4.28; periodogram of the diastolic portion of the signal shown in the first plot (approximately 0.28 – 0.62 s); averaged periodogram of the diastolic parts of 26 cardiac cycles. The periodograms are on a log scale (dB).



6.5 Measures Derived from Power Spectral Density Functions

PSD: density function of signal amplitude

or power versus frequency.

Facilitates *nonparametric* and general spectral analysis.

Problem: *Derive parameters or measures from a Fourier spectrum or PSD that can help in the characterization of the spectral variations or features contained therein.*



Solution:

PSD is a nonnegative function as well as a density function:

Treat it as a PDF and compute statistics using moments.

Detect peaks corresponding to resonance,

measure their bandwidth or quality factor,

derive measures of concentration of power in

specific frequency bands of interest or concern.



PSD is nonparametric:

but we may derive several parameters that,

while not completely representing the entire PSD,

may facilitate the characterization of

physiological and/or pathological phenomena.



6.5.1 Moments of PSD functions

Area under the PSD = total signal power or energy $\neq 1$.

Normalize all moments by total energy of the signal E_x :

$$\begin{aligned}
 E_x &= \sum_{n=0}^{N-1} |x(n)|^2 = \frac{1}{N} \sum_{k=0}^{N-1} |X(k)|^2 & (6.36) \\
 &= \frac{1}{2\pi} \int_0^{2\pi} |X(\omega)|^2 d\omega = \int_{f_n=0}^1 |X(f_n)|^2 df_n.
 \end{aligned}$$



Frequency f_n : normalized as $f_n = f/f_s$; $0 \leq f_n \leq 1$.

Replace $|X(\cdot)|^2$ in the above expressions by $S_{xx}(\cdot)$.

In all definitions of moments of PSDs:

$$S_{xx}(k) = \frac{1}{N} |X(k)|^2.$$

$X(k)$: DFT of $x(n)$; $S_{xx}(k)$: PSD of $x(n)$.



Measure of concentration of signal power over its

frequency range: mean frequency \overline{f} = first-order moment

$$\overline{f_n} = \frac{2}{E_x} \int_{f_n=0}^{0.5} f_n S_{xx}(f_n) df_n. \quad (6.37)$$

$$\overline{f_{Hz}} = \frac{f_s}{N} \frac{2}{E_x} \sum_{k=0}^{N/2} k S_{xx}(k). \quad (6.38)$$



Median frequency $f_{med} =$

frequency that splits the PSD in half:

$$f_{med} = \frac{m}{N} f_s \text{ with the largest } m \text{ such that} \quad (6.39)$$

$$\frac{2}{N E_x} \sum_{k=0}^m S_{xx}(k) < \frac{1}{2}; \quad 0 \leq m \leq \frac{N}{2}.$$



Higher-order statistics:

- Variance f_{m2} = second-order moment using $(f_n - \overline{f_n})^2$:

$$f_{m2} = \frac{2}{E_x} \int_{f_n=0}^{0.5} (f_n - \overline{f_n})^2 S_{xx}(f_n) df_n. \quad (6.40)$$

$$f_{m2} = \left(\frac{f_s}{N} \right)^2 \frac{2}{E_x} \sum_{k=0}^{N/2} (k - \overline{k})^2 S_{xx}(k). \quad (6.41)$$

$\overline{k} = N \overline{f_{Hz}} / f_s$ = frequency sample index
corresponding to $\overline{f_{Hz}}$.



▪ Skewness:

$$skewness = \frac{f_{m3}}{(f_{m2})^{3/2}}. \quad (6.42)$$

Third-order moment f_{m3} computed with $(f_n - \overline{f_n})^3$:

$$f_{m3} = \frac{2}{E_x} \int_{f_n=0}^{0.5} (f_n - \overline{f_n})^3 S_{xx}(f_n) df_n. \quad (6.43)$$

$$f_{m3} = \left(\frac{f_s}{N}\right)^3 \frac{2}{E_x} \sum_{k=0}^{N/2} (k - \overline{k})^3 S_{xx}(k). \quad (6.44)$$



- Kurtosis:

$$kurtosis = \frac{f_{m4}}{(f_{m2})^2}. \quad (6.45)$$

Fourth-order moment f_{m4} computed with $(f_n - \overline{f_n})^4$:

$$f_{m4} = \frac{2}{E_x} \int_{f_n=0}^{0.5} (f_n - \overline{f_n})^4 S_{xx}(f_n) df_n. \quad (6.46)$$

$$f_{m4} = \left(\frac{f_s}{N} \right)^4 \frac{2}{E_x} \sum_{k=0}^{N/2} (k - \overline{k})^4 S_{xx}(k). \quad (6.47)$$



Mean frequency: measure of concentration of signal power;
could indicate resonance frequency
of unimodal distributions.

Multiple resonance frequencies: mean frequency not useful.

Multimodal PSDs: series of peak frequencies,
with measures of relative level, bandwidth, or quality factor.



Square-root of f_{m2} : measure of spectral spread.

Skewness = zero if PSD symmetric about mean frequency;

otherwise, indicates extent of asymmetry of distribution.

Kurtosis indicates if PSD is a long-tailed function.



6.5.2 Spectral power ratios

Fraction of signal power in a

frequency band of interest $(f_1 : f_2)$

$$E_{(f_1:f_2)} = \frac{2}{E_x} \int_{f=f_1}^{f_2} |X(f)|^2 df = \frac{2}{N E_x} \sum_{k=k_1}^{k_2} |X(k)|^2. \quad (6.48)$$

k_1 and k_2 : DFT indices corresponding to f_1 and f_2 .

May be computed for several frequency bands.



Johnson et al.: compared integral of magnitude spectrum of
systolic murmurs due to aortic stenosis

over the band $75 : 150 \text{ Hz}$ to that over the band $25 : 75 \text{ Hz}$.

Higher-frequency band: *predictive area* (PA)

of spectrum related to aortic stenosis.

Lower-frequency band: *constant area* (CA)

common to all systolic PCG signal segments.



$$\frac{PA}{CA} = \frac{\int_{f=f_2}^{f_3} |X(f)| df}{\int_{f=f_1}^{f_2} |X(f)| df}. \quad (6.49)$$

$$f_1 = 25 \text{ Hz}, f_2 = 75 \text{ Hz}, f_3 = 150 \text{ Hz}.$$

Figure 6.2:

$\frac{PA}{CA}$ ratio correlates well with severity of aortic stenosis.



Binnie et al.:

spectrum analysis of EEG for detection of epilepsy.

Partitioning or banding of EEG spectrum into not only the

traditional δ , θ , α , and β bands,

but also seven nonuniform bands:

$1 - 2$, $2 - 4$, $4 - 6$, $6 - 8$, $8 - 11$, $11 - 14$, and > 14 Hz .

Additional features related to form factor used.



Study with 275 patients with suspected epilepsy:

90% of signals of patients with epilepsy

classified as abnormal;

86% of patients with EEGs classified as abnormal

had epilepsy.



6.6 Application: Evaluation of Prosthetic Heart Valves

Efficient opening and closing actions of cardiac valves of paramount importance for proper pumping of blood by the heart.

When native valves fail, they may be replaced by mechanical prosthetic valves or by bioprosthetic valves.



Mechanical prosthetic valves prone to sudden failure

due to fracture of their components.

Bioprosthetic valves fail gradually due to tissue

degeneration and calcification: last for 7 – 12 years.

Follow-up of the health of patients with prosthetic valves

requires periodic, noninvasive assessment of the

functional integrity of the valves.



Problem: *Deposition of calcium causes the normally pliant and elastic bioprosthetic valve leaflets to become stiff.*

Propose a method to assess the functional integrity of bioprosthetic valves.



Solution: Increased stiffness of a valve expected to lead to higher-frequency components in its opening or closing sounds.

Durand et al. studied spectra of S1 to evaluate sounds contributed by closure of porcine (pig) bioprosthetic valves implanted in the mitral position in humans.



Normal S1 spectra: limited in bandwidth to about 100 Hz .

Degenerated bioprosthetic valves: significant spectral

energy in the range $100 - 250\text{ Hz}$.



Spectral parameters used by Durand et al.:

first and second dominant peak frequencies;

median frequency;

bandwidth and quality factor of dominant peak;

integrated mean area above -20 dB ;

highest frequency at -3 dB ;



total area and RMS value of spectrum;

area and RMS value in the bands

$20 - 100$, $100 - 200$, $200 - 300$ Hz .

Normal versus degenerated valve classification

accuracy: 98%.

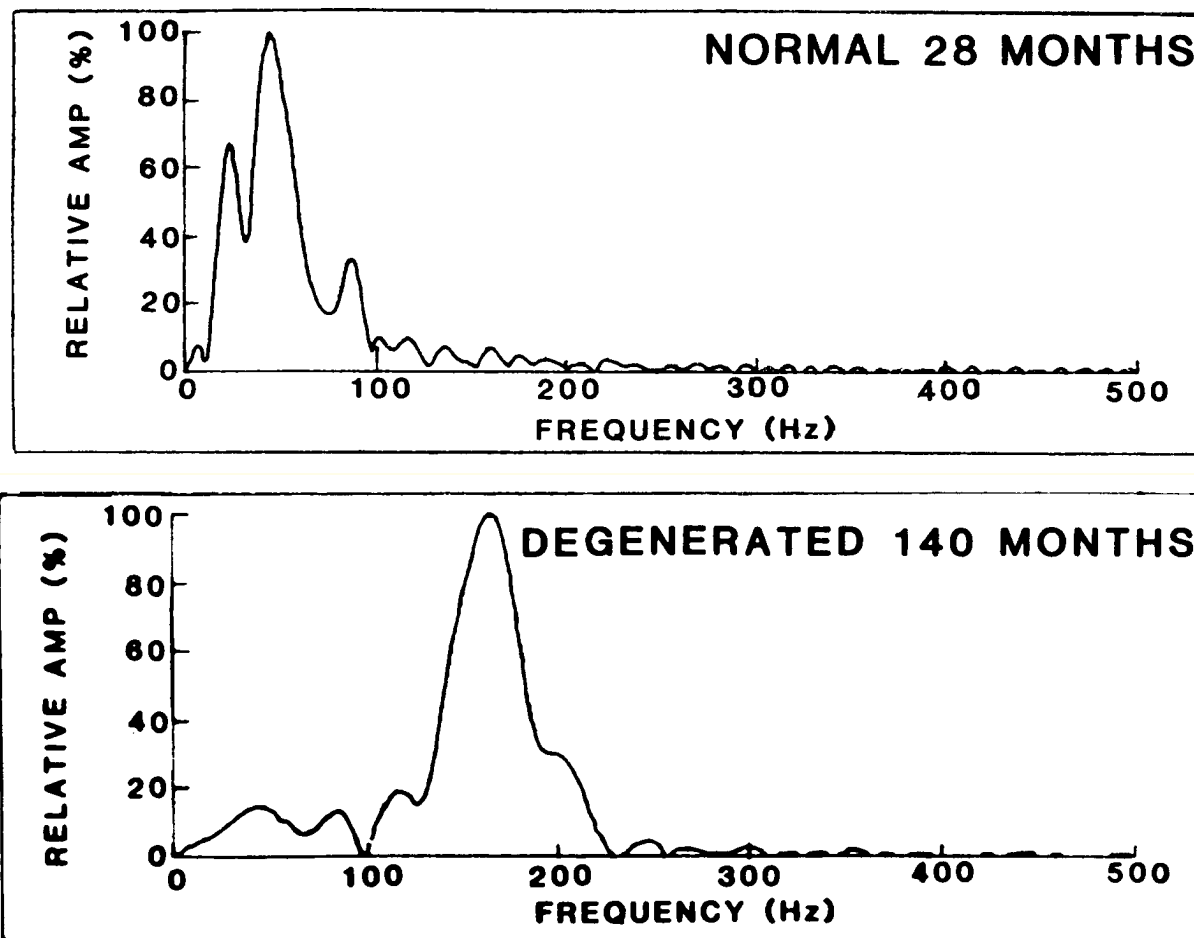


Figure 6.13: First heart sound spectra in the case of normal and degenerated porcine bioprosthetic valves implanted in the mitral position. Reproduced with permission from L.G. Durand, M. Blanchard, G. Cloutier, H.N. Sabbah, and P.D. Stein, Comparison of pattern recognition methods for computer-assisted classification of spectra of heart sounds in patients with a porcine bioprosthetic valve implanted in the mitral position, *IEEE Transactions on Biomedical Engineering*, 37(12):1121–1129, 1990 ©IEEE.