The city of Moosonee, Ontario is soliciting bids from 4 companies (A, B, C, and D) for 6 potential bus routes that support the surrounding communities. Each company submits a set of bids which reflect the cost of providing service to a route for a single year. That is, let C_{ij} represent the cost that company i says it will charge to provide service to route j. After all bids are submitted, city council must make two decisions: it must decide whether to open a route, and if so, which company should be assigned to service it. They also have restrictions:

1. Only one company can be assigned to a route if it is opened.
2. Each company can be assigned to at most two bus routes.
3. At least three bus routes must be opened.
4. Route 2 and 5 must both be opened or not at all.
5. Either route 3 must be opened or route 4 must be opened but not both.
6. If company B is assigned to route 1, it cannot also provide service to route 4.
7. If company A is assigned to route 3, it must also be assigned to route 5.
8. Company D must be assigned to at least one route.

The council's objective is to minimize the total cost of providing service to all opened routes. Formulate a binary program to determine which routes to open and the optimal assignment of companies to routes.

Let x_{ij} be a binary variable that equals one if company $i = \{1, 2, 3, 4\} = \{A, B, C, D\}$ is assigned to route $j = \{1, 2, 3, 4, 5, 6\}$ and zero otherwise. Further, let y_j be a binary variable that equals one if route $j = \{1, 2, 3, 4, 5, 6\}$ is opened and zero otherwise.

Minimize	$\sum_{i=1}^{4}\sum_{j=1}^{6} C_{ij}x_{ij}$	
Subject to:	$\sum_{i=1}^{4} x_{ij} = y_j$	$\forall j$
	$\sum_{j=1}^{6} x_{ij} \leq 2$	$\forall i$
	$\sum_{j=1}^{6} y_j \geq 3$	$\forall i$
	$y_2 = y_5$	
	$y_3 + y_4 = 1$	
	$x_{B1} + x_{B4} \leq 1$	
	$x_{A3} \leq x_{A5}$	
	$\sum_{j=1}^{6} x_{Dj} \geq 1$	
	$x_{ij}, y_j \in \{0, 1\}$	$\forall i, j$

McKenna Logistics is a logistics company that ships three types of pallets: heavy, medium, and light. It has recently entered a contract to provide shipping services for septic system suppliers; this will see them transport various septic products from its Toronto warehouse to wholesalers across Ontario. Each day, *exactly* 7 heavy pallets (weighing 4 tons each), 6 medium pallets (weighing 3 tons each), and 5 light pallets (weight 0.5 tons each) of septic products will be shipped out of the Toronto warehouse. McKenna Logistics owns three types of trucks: the first type has a 13.5-ton weight limit and costs $5,500 per trip, the second truck has a 12-ton weight limit and costs $4,700 per trip, and the third truck has a 10-ton weight limit and costs $3,900 per trip. Each day, each truck can complete *exactly* one trip. McKenna Logistics can assign, at most, three of each truck type to the septic business but has promised the suppliers that at least one type of each truck will be used. Formulate a *binary program* to determine which pallet should be assigned to which truck in order to minimize costs.

Let x_{ij} be a binary variable that equals one if heavy pallet $i = 1, \dots, 7$ is assigned to truck $j = 1, \dots, 9$ and zero otherwise. Let y_{ij} be a binary variable that equals one if medium package $i = 1, \dots, 6$ is assigned to truck $j = 1, \dots, 9$ and zero otherwise. Let z_{ij} be a binary variable that equals one if light package $i = 1, \dots, 5$ is assigned to truck $j = 1, \dots, 9$ and zero otherwise. Let v_j be a binary variable that equals one if truck $j = 1, \dots, 9$ is used and zero otherwise.

Minimize $\sum_{j=1}^{3} 5500v_j + \sum_{j=4}^{6} 4700v_j + \sum_{j=7}^{9} 3900v_j$

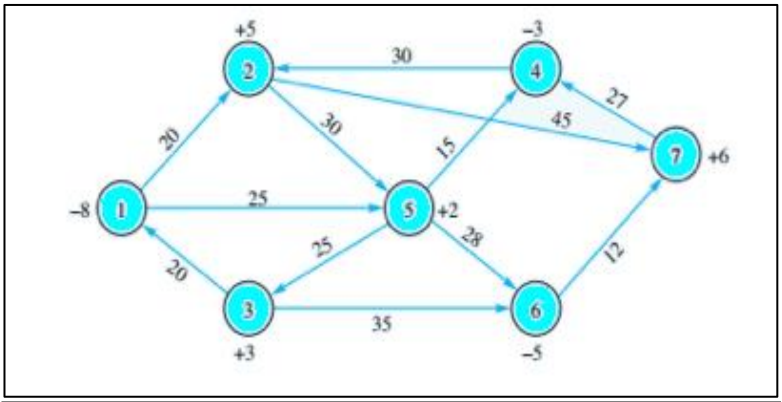
Subject to:

$$
\begin{aligned}
&\sum_{j=1}^{3} v_j \geq 1 \\
&\sum_{j=4}^{6} v_j \geq 1 \\
&\sum_{j=7}^{9} v_j \geq 1 \\
&\sum_{i=1}^{7} 4x_{ij} + \sum_{i=1}^{6} 3y_{ij} + \sum_{i=1}^{5} 0.5z_{ij} \leq 13.5v_j && j = 1, \dots, 3 \\
&\sum_{i=1}^{7} 4x_{ij} + \sum_{i=1}^{6} 3y_{ij} + \sum_{i=1}^{5} 0.5z_{ij} \leq 12.0v_j && j = 4, \dots, 6 \\
&\sum_{i=1}^{7} 4x_{ij} + \sum_{i=1}^{6} 3y_{ij} + \sum_{i=1}^{5} 0.5z_{ij} \leq 10.0v_j && j = 7, \dots, 9 \\
&\sum_{j=1}^{9} x_{ij} = 1 && i = 1, \dots, 7 \\
&\sum_{j=1}^{9} y_{ij} = 1 && i = 1, \dots, 6 \\
&\sum_{j=1}^{9} z_{ij} = 1 && i = 1, \dots, 5 \\
&x_{ij}, y_{ij}, z_{ij}, v_j \in \{0, 1\} && \forall i, j
\end{aligned}
$$

Optimal Solution: 19600 with trucks 1, 3, 4, 7.

Suppose that the network below shows the locations of the 7 bike hubs in the program. The edges/arcs of the network indicate the cost and direction of moving one bike between hubs. The writing next to the nodes show typical hub-bike imbalances after the rush hour period. A positive number indicates an excess supply of bikes and a negative number indicates an excess demand of bikes at a hub. These values are expressed as a percentage of the total demand predicted by the forecasting model (assume the model predicts 1400 bikes). Formulate a linear program to redistribute the bikes at the end of rush hour. More specifically, create a linear programming model to determine what hubs the bikes should be coming from and what hubs the bikes should be going to in order to minimize the cost of redistribution given that the following restrictions must be adhered to:

- The total number of bikes transferred from hubs 2 and 3 to any other hub must not exceed twice the number of bikes transferred from hubs 4 and 5 to any other hub.
- The number of bikes transferred between hubs 1-5 must be between 5% and 50% of predicted demand.



Comment on how the data and/or forecasting model can be improved to better support the inputs to this model? *Hint: For simplicity, define the set of parameters c_{ij} to be the cost of moving a single bike from hub i to hub j.*

Let D be total mean demand as predicted by our forecasting model in Question 1 and $q_i D$ be the excess/deficit at hub location i. Let x_{ij} be the number of bikes moved from hub i to hub j.

Minimize $\sum_{i=1}^{7}\sum_{j=1}^{7} c_{ij}x_{ij}$

Subject to:

$$x_{12} + x_{15} - x_{31} = -112$$
$$x_{25} + x_{27} - x_{12} - x_{42} = 70$$
$$x_{31} + x_{36} - x_{53} = 42$$
$$x_{42} - x_{54} - x_{74} = -42$$
$$x_{25} + x_{27} + x_{31} + x_{36} \le 2(x_{53} + x_{54} + x_{56} + x_{42})$$
$$\sum_{i=1}^{5}\sum_{j=1}^{5} x_{ij} \ge 70$$
$$\sum_{i=1}^{5}\sum_{j=1}^{5} x_{ij} \le 700$$
$$x_{53} + x_{54} + x_{56} - x_{15} - x_{25} = 28$$
$$x_{67} - x_{36} - x_{56} = -70$$
$$x_{74} - x_{27} - x_{67} = 84$$
$$x_{ij} \ge 0 \qquad \textit{for all } i \textit{ and } j$$

Optimal Solution: 12838 where 112 bikes moves from 2 to 5, 112 bikes moves from 3 to 1, 42 bikes moves from 4 to 2, 70 bikes moved from 5 to 3, 70 bikes moved from 5 to 6, and 84 bikes moved from 7 to 4.