Formula Sheet

1. **Predictions for user a on item i (user user CF):**

$$p_{a,i} = \bar{r}_a + \frac{\sum_{i=1}^{k} (r_{u,i} - \bar{r}_u) \times \mathrm{sim}\ (a,u)}{\sum_{u=1}^{k} \mathrm{sim}\ (a,u)},$$

- $\bar{r}_a$ is the mean rating for user a,
- u1, u2...uk are the k-nearest neighbours of a,
- $r_{u,i}$ is the rating of user u on item i,
- $\mathrm{sim}\ (a, u)$ is the similarity between users a and u

2. **Predictions for user a on item i (item item CF):**

$$P_{u,i} = \frac{\sum_{\text{all similar items, N}} (s_{i,N} * R_{u,N})}{\sum_{\text{all similar items, N}} (|s_{i,N}|)}$$

3. **Pearson similarity measure:**

$$pearson\mathrm{Sim}\ (u, v) = \frac{\sum_{i \in C} (r_{\mathbf{u},i} - \bar{r}_{\mathbf{u}})(r_{\mathbf{v},i} - \bar{r}_{\mathbf{v}})}{\sqrt{\sum_{i \in C} (r_{\mathbf{u},i} - \bar{r}_{\mathbf{u}})^2} \sqrt{\sum_{i \in C} (r_{\mathbf{v},i} - \bar{r}_{\mathbf{v}})^2}},$$

C, common items rated by both u and v

4. **Weighted pearson similarity measure:**

$$weightedPearson(u, v) = pearsonSim(u, v) * \frac{\min\ (50, C)}{50}$$

5. **Constrained pearson similarity measure:**

$$constrainedPearson(u, v) = cosineSim(u - r_{med}, v - r_{med})$$

where r_{med} is the median value in the rating scale,

e.g. if the rating scale ranges from 1 to 5, $r_{med} = 3$

6. **Dot product similarity measure:**

$$dotSim\ (u, v) = \vec{u} \cdot \vec{v}$$

7. **Cosine similarity measure:**

$$cosineSim\ (u, v) = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\|_2 * \|\vec{v}\|_2}$$

8. **Adjusted cosine similarity measure:**

$$\text{sim}\ (\mathbf{i}, \mathbf{j}) = \frac{\sum_{u \in U} (r_{u,\mathbf{i}} - \bar{r}_u)(r_{u,\mathbf{j}} - \bar{r}_u)}{\sqrt{\sum_{u \in U} (r_{u,\mathbf{i}} - \bar{r}_u)^2} \sqrt{\sum_{u \in U} (r_{u,\mathbf{j}} - \bar{r}_u)^2}}$$

9. **Manhattan (or city block) distance:**

$$L_1(x, y) = |x_1 - y_1| + |x_2 - y_2| + \cdots + |x_d - y_d| = \sum_{i=1}^{d} |x_i - y_i|$$

10. **Euclidean distance:** $d(x, y) = \sqrt{(|x_1 - y_1|^2 + |x_2 - y_2|^2 + \cdots + |x_d - y_d|^2)}$

11. **Jaccard similarity:** $\text{JSim}\ (X, Y) = \frac{X \cap Y}{X \cup Y}$

12. **Accuracy:** (TP + TN) / (TP + TN + FP + FN)
    TP – True positives; FP – False positives
    TN – True negatives; FN – False negatives

13. **Precision:** TP / (TP + FP)

14. **Recall:** TP / (TP + FN)

15. $\boldsymbol{F_1}, \boldsymbol{F_\beta}$ **measure:**

$$F_1 = 2 \cdot \frac{precision \cdot recall}{precision + recall} \quad F_\beta = (1 + \beta^2) \cdot \frac{\text{precision} \cdot \text{recall}}{(\beta^2 \cdot \text{precision}) + \text{recall}}$$

16. **Mean Absolute Error (MAE):**

$$MAE \quad = \quad \frac{1}{n}\sum_{i=1}^{n} | p_i - r_i |$$

where p, r are the predicted and actual ratings

17. **Root Mean Square Error (RMSE):**

$$RMSE \quad = \quad \sqrt{\frac{1}{n}\sum_{i=1}^{n} (p_i - r_i)^2}$$

18. **Discounted Cumulative Gain (DCG):**

$$DCG_{pos} = rel_1 + \sum\nolimits_{i=2}^{pos} \frac{rel_i}{\log_2 i}$$

where:

- *pos* denotes the position up to which relevance is accumulated
- rel_i returns the relevance of recommendation at position *i*

19. **Idealized Discounted Cumulative Gain (IDCG):**

$$IDCG_{pos} = rel_1 + \sum_{i=2}^{|h|} \frac{rel_i}{\log_2 i}$$

where h is the number of hits

20. **Normalized Discounted Cumulative Gain (IDCG):**

$$nDCG_{pos} = \frac{DCG_{pos}}{IDCG_{pos}}$$

21. **Bayes formula:**

$$P(C|X) = \frac{P(X|C)P(C)}{P(X)}$$

posterior = likelihood * prior / evidence

$P(C_i) = (n_i+1) / (n + |C|)$,
n_i is the number of training samples in class C_i
n is the total number of training samples

$P(x_k|C_i) = (n_{ik}+1) / (n_i + |V|)$
n_{ik} is the no. of training samples in class C_i having the value x_k for attribute A_k and
n_i is the no. of training samples belonging to class C_i

22. **TF-IDF measure:**

$$w_{ik} = tf_{ik} * \log(N / n_k)$$

tf_{ik} – Term frequency of word k in document i
N – Total number of documents in the collection
n_k- Number of documents that contain the work k

23. **0-1 Loss:**

$$0 - 1 \text{ Loss} = \frac{1}{N} \sum_{\forall i} I[\hat{y}_i \neq y_i]$$

where $\hat{y}_i$ *and* y_i are the predicted and actual class labels

24. **Hamming Loss:**

$$\text{Hamming Loss} = \frac{1}{N.L} \sum_{\forall i} \sum_{\forall j} I[\hat{y}_{ij} \neq y_{ij}]$$

where $\hat{y}_{ij}$ *and* y_{ij} are the predicted and actual class labels of example i and label j

25. **Accuracy:**

$$\text{Accuracy} = \frac{1}{N} \sum_{\forall i} \frac{|\hat{y}_i \cap y_i|}{|\hat{y}_i \cup y_i|}$$

where $\hat{y}_i$ *and* y_i are the predicted and actual class labels

26. **Coherence:**

$$\text{Coh}_{\text{umass}} = \frac{2}{N(N-1)} \sum_{i=2}^{N} \sum_{j=1}^{i-1} \log \frac{P(w_i, w_j)}{P(w_j)}$$