



# Knowledge Card

Aptitude | Number System



# Number System

## The Natural Numbers:

The **natural** (or **counting**) **numbers** are 1,2,3,4,5,1,2,3,4,5, etc. There are infinitely many natural numbers. The set of natural numbers,  $\{1,2,3,4,5,\dots\}$ , is sometimes written NN for short.

**The Whole Numbers:** The **whole numbers** are the natural numbers together with 0. The sum of any two natural numbers is also a natural number (for example,  $4+2000=2004$ ), and the product of any two natural numbers is a natural number ( $4 \times 2000=8000$ ). This is not true for subtraction and division, though.

**The Integers:** The **integers** are the set of real numbers consisting of the natural numbers, their additive inverses and zero.

$\{\dots, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \dots\}$

The set of integers is sometimes written JJ or ZZ for short.

The sum, product, and difference of any two integers is also an integer. But this is not true for division... just try  $1 \div 2$ .

**The Rational Numbers:** The **rational numbers** are those numbers which can be expressed as a ratio between two integers. For example, the fractions  $\frac{1}{3}$  and  $-\frac{11}{18}$  are both rational numbers.

All decimals which terminate are rational numbers (since 8.27827 can be written as  $\frac{827100827}{100000000}$ .) Decimals which have a repeating pattern after some point are also rational

**The Irrational Numbers:** An **irrational number** is a number that cannot be written as a ratio (or fraction). In decimal form, it never ends or repeats. The ancient Greeks discovered that not all numbers are rational; there are equations that cannot be solved using ratios of integers.

The first such equation to be studied was  $x^2 = 2$ . What number times itself equals 2?

$\sqrt{2}$  is about 1.414213562, because  $1.4142^2 = 1.999396$ , which is close to 2. But you'll never hit exactly by squaring a fraction (or terminating decimal). The square root of 2 is an irrational number, meaning its decimal equivalent goes on forever, with no repeating pattern:

$\sqrt{2} = 1.41421356237309\dots$

Other famous irrational numbers are **the golden ratio**, a number with great importance to biology:

Irrational numbers can be further subdivided into **algebraic** numbers, which are the solutions of some polynomial equation (like  $\sqrt{2}$  and the golden ratio), and **transcendental** numbers, which are not the solutions of any polynomial equation.  $\pi$  and  $e$  are both transcendental.



## The Real Numbers

The real numbers is the set of numbers containing all of the rational numbers and all of the irrational numbers. The real numbers are “all the numbers” on the number line. There are infinitely many real numbers just as there are infinitely many numbers in each of the other sets of numbers. But, it can be proved that the infinity of the real numbers is a **bigger** infinity.

**The Complex Numbers:** The complex numbers are the set  $\{a+bi \mid a \text{ and } b \text{ are real numbers}\}$ , where  $i$  is the imaginary unit,  $i^2 = -1$ . The complex numbers include the set of real numbers. The real numbers, in the complex system, are written in the form  $a+0i=a$ . This set is sometimes written as  $C$  for short. The set of complex numbers is important because for any polynomial  $p(x)$  with real number coefficients, all the solutions of  $p(x)=0$  will be in  $C$ .

## Divisibility

### IF A Number Divisible

- by 2 End with 0,2,4,6,8 are divisible by 2 Divisible
- by 3 Sum of its digits is divisible by 3 Divisible
- by 4 Last two digit divisible by 4 Divisible
- by 5 Ends with 0 or 5 Divisible
- by 6 Divides by Both 2 & 3 Divisible
- by 8 Last 3 digit divide by 8 Divisible
- by 10 End with 0 Divisible
- by 11 [Sum of its digit in odd places-Sum of its digits in even places= 0 or multiple of 11]
- by 12 [The number must be divisible by 3 and 4]
- by 13 [Multiply last digit with 4 and add it to remaining number in given number, result must be divisible by 13]
- by 14 [The number must be divisible by 2 and 7. Because 2 and 7 are prime factors of 14.]
- by 15 [The number should be divisible by 3 and 5. Because 3 and 5 are prime factors of 15.]
- by 16 [The number formed by last four digits in given number must be divisible by 16.] Divisible by 17 [Multiply last digit with 5 and subtract it from remaining number in given number, result must be divisible by 17]
- by 18 [The number should be divisible by 2 and 9]



- by 19 [Multiply last digit with 2 and add it to remaining number in given number, result must be divisible by 19]
- by 20 [The number formed by last two digits in given number must be divisible by 20.]

### Sum Rules:

- Sum of first  $n$  natural numbers =  $\frac{n(n+1)}{2}$
- Sum of square of first  $n$  natural numbers =  $\frac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first  $n$  natural numbers =  $\left(\frac{n(n+1)}{2}\right)^2$
- Sum of first  $n$  odd numbers =  $n^2$
- Sum of first  $n$  even numbers =  $n(n+1)$
- A very basic formula for division rules is:  
dividend = ( divisor  $\times$  quotient ) + remainder

## Number System Tips and Tricks and Shortcuts

- Tips and Tricks number system help to answer number system problem effectively.  
**Whole Number, Natural number and Integers** are some common example of number system.
- Here, are easy tips and tricks on Number System problems easily, and efficiently in competitive exams.

**For example:** Question:  $\pi$  is rational number or irrational.

Solution: Yes,  $\pi$  is rational number as it can be written in  $p/q$  form.

**Question 1 Which one of the following is not a prime number?**

**Options**

**A. 51**

**B. 23**

**C. 17**



**D. 11**

**Solution:** 51 is divisible by 17. So, it is not a prime number.

**Correct option: A**

**Question 2. How many prime numbers are there from 1 to 50?**

**Options**

**A. 20**

**B. 15**

**C. 21**

**D. 25**

**Solution:** Prime numbers less than 50 are:

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

**Correct option: B**

**Question 3  $1198 * 1198 = ?$**

**Options**

**A. 1435204**

**B. 1432504**

**C. 1453204**

**D. 1435024**

**Solution:**  $(1198)^2$



$$(1200 - 2)^2$$

We can use the identity:  $(a-b)^2 = a^2 + b^2 - 2ab$

$$(1200)^2 + 2^2 - 2 * 1200 * 2$$

$$1440000 + 4 - 4800$$

$$1435204$$

**Correct option: A**

**Question 4. The sum of first five prime numbers is:**

**Options**

**A. 20**

**B. 28**

**C. 30**

**D. 25**

**Solution:** Sum of first five prime numbers is  $= 2+3+5+7+11 = 28$

**Correct option: B**

**Question 5. If x and y are odd numbers, then which of the following is even numbers?**

**Options**

**A.  $x + y$**

**B.  $xy$**



C.  $x - y$

D.  $xy + 2$

**Solution:** The sum of two odd number is always even. So,  $x + y$  is even.

**Correct option: A**

## Formulas for Number System and Basic Concept

1.  $(a - b)(a + b) = (a^2 - b^2).$
2.  $(a + b)^2 = (a^2 + b^2 + 2ab)$
3.  $(a - b)^2 = (a^2 + b^2 - 2ab)$
4.  $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)(a^3 + b^3) = (a + b)(a^2 - ab + b^2)$
5.  $(a^3 - b^3) = (a - b)(a^2 + ab + b^2)$
6.  $(a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ac)$
7. When  $a + b + c = 0$ , then  $a^3 + b^3 + c^3 = 3abc$ .

## Formulas for finding the Squares of a number.

### Squares of numbers 91-100:

- $97^2$

Step 1:  $100 - 97 = 3$

Step 2:  $97 - 3 = 94$

Step 3:  $3^2 = 09$

Final result: From step 2 and

Step 3  $\Rightarrow 97^2 = 9409$

- $91^2$



Step 1:  $100-9 = 91$

Step 2:  $91-9 = 82$

Step 3:  $9^2 = 81$

Final Result: From step 2 and step 3  $\Rightarrow 91^2 = 8281$

### **Squares of numbers 100-109**

- $102^2$

Step 1:  $102-100 = 2$

Step 2:  $102 + 2 = 104$

Step 3:  $2^2 = 04$  Final result:

From step 2 and step 3  $\Rightarrow 102^2 = 10404$

- $107^2$

Step 1:  $107-100 = 7$

Step 2:  $107+7 = 114$

Step 3:  $7^2 = 49$

Final Result: From step 2 and step 3  $\Rightarrow 107^2 = 11449$

### **Squares of numbers 51-60**

- $53^2$

Step 1:  $53-50 = 3$

Step 2:  $25+3 = 28$





Step 3:  $3^2 = 09$

Final result: From step 2 and step 3  $\Rightarrow 53^2 = 2809$ .

- $42^2$

Step 1:  $50-42 = 8$

Step 2:  $25-8 = 17$

Step 3:  $8^2 = 1764$

Final Result From step 2 and step 3  $\Rightarrow 42^2 = 1764$