Proof Practice

$Y = ax + b$, $x, y \in \mathbb{R}$, $a \neq 0$

$$Corr(x, y) = \frac{Cov[x, Y]}{\sqrt{Var[X]\, Var[Y]}}$$

$$= \frac{Cov[X, ax+b]}{\sqrt{Var[X] \cdot Var[ax+b]}} \quad \text{(By substitution)}$$

$$= \frac{Cov[X, ax] + Cov[X, b]}{\sqrt{Var[X] \cdot (Var[aX] + Var[b])}}$$

$$= \frac{a\, Var[X] + 0}{\sqrt{Var[X] \cdot (a^2\, Var[X] + 0)}}$$

$$= \frac{a\, Var[X]}{\sqrt{Var[X] \cdot a^2 \cdot Var[X]}}$$

$$= \frac{a\, Var[X]}{|a|\, Var[X]}, \quad \text{variance is always +ve.}$$

If $a > 0$, then $\frac{a\, Var[X]}{|a|\, Var[X]}$ will be positive and 1.

If $a < 0$, then the above expression will be -1.

Thus $Corr[X, Y] = \{-1, 1\}$ that is $\rho_{XY}(x, y) = \begin{cases} -1, & a < 0 \\ 1, & a > 0 \end{cases}$