281 Live Session

Week 6 — 2023/2/15

Agenda

Final Project Q&A

Intro to Assignment 4

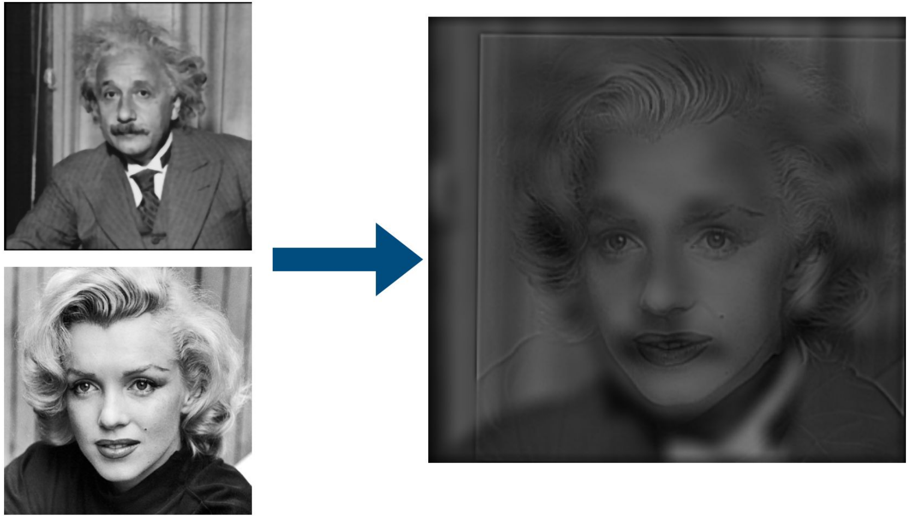
Overview of Fourier Decomposition

Practical applications of frequency analysis

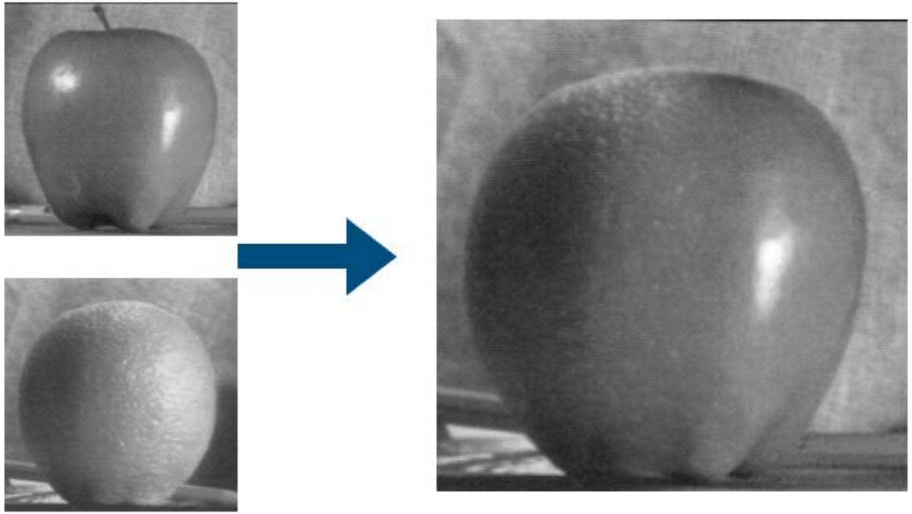
Exercise on Frequencies

Assignment 4 – Frequencies

Part 1: Hybrid Images



Part 2: Multi-Resolution Blending



Discussion Questions

- 1: Perspective Projection
- 2: Image Formation
- 3: Image Artifacts
- 4: Convolution
- 5: Fourier
- 6: Pyramids, Edges, and Features
- 7: Image Analysis
- 8: Least-Squares
- 9: Total and Iterative Least-Squares
- 10: Clustering
- 11: Dimensionality Reduction
- 12: Linear Classifiers
- 13: Nonlinear Classifiers

- What is a linear time invariant system?
- Why are LTIs useful for image processing?
- What is the basis used for Fourier decomposition?
- Why is frequency decomposition useful for image processing?
- What is being plotted in a Fourier diagram? What is omitted?
- What are edge artifacts? How can they be avoided?
- What is a discrete Fourier transform?
- How does sampling rate influence signal reconstruction in a DFT?
- What is the Nyquist limit?

Linear Time Invariant Systems

Image filters can be separated out and recombined independently at lower cost

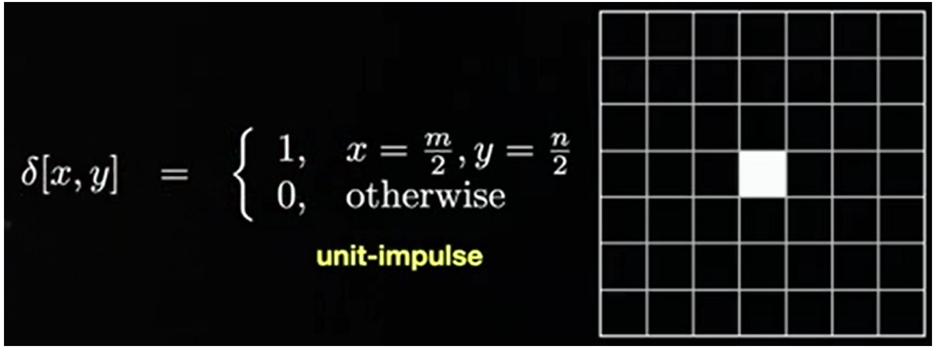
$I_b = k_b * I$

$I_s = I + [\ I - I_b\ ]$

$I_s = 2\ I - I_b$

$I_s = [\ k_{2x} * I\ ] - [\ k_b * I\ ]$

$I_s' = [\ k_{2x} - k_b\ ] * I$



Linear → obeys superposition

Time Invariant → input shifted == output shifted

Fourier Basis

Sine & Cosine are translated into a complex exponential

Phase information is discarded

$$f_{\text{Magnitude}} = \text{sqrt}(f_{\text{Real}}^{\,2} + f_{\text{Imaginary}}^{\,2})$$

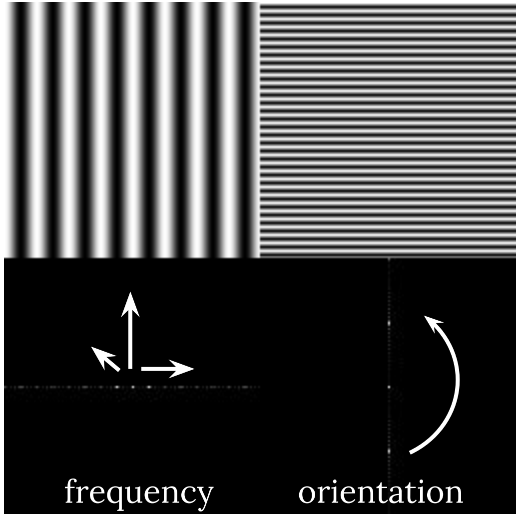
$$f_{\text{Phase}} = \text{atan}(f_{\text{Imaginary}} / f_{\text{Real}})$$

$$f[x] = \frac{1}{m}\sum_{k=0}^{m-1} a_k \cos(\omega_k x) + b_k \sin(\omega_k x) \quad \text{Fourier series}$$

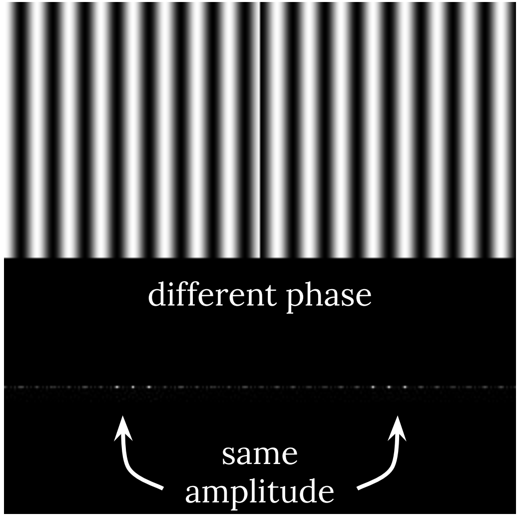
$$a_k = \sum_{l=0}^{m-1} f[l]\cos(\omega_k l) \quad \text{and} \quad b_k = \sum_{l=0}^{m-1} f[l]\sin(\omega_k l)$$

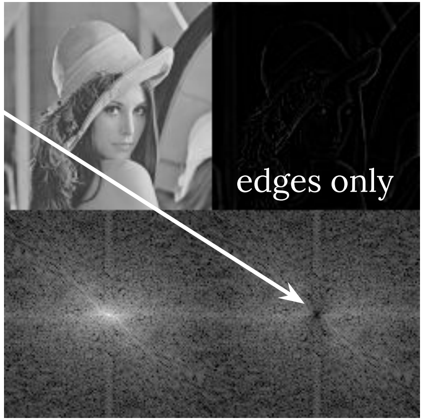
Fourier transform

$$e^{i\omega x} = \cos(\omega x) + i\sin(\omega x)$$



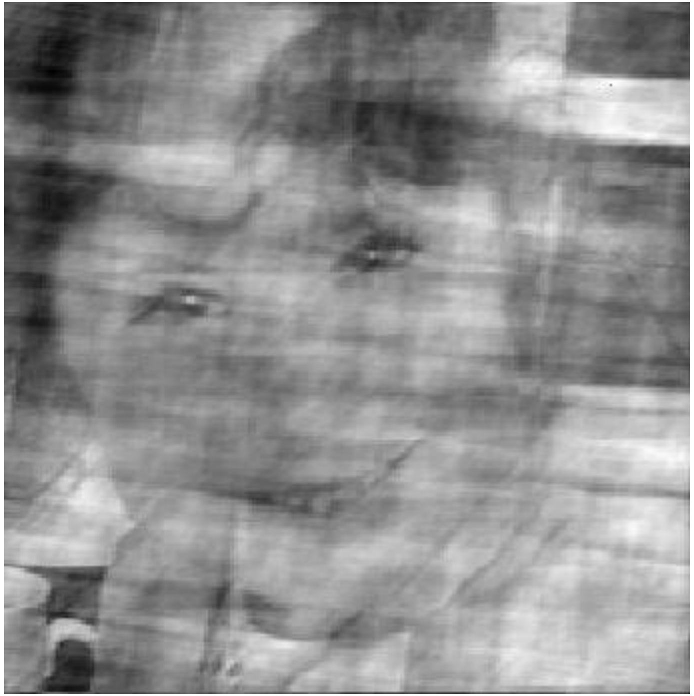
center = lowest freq (average value)





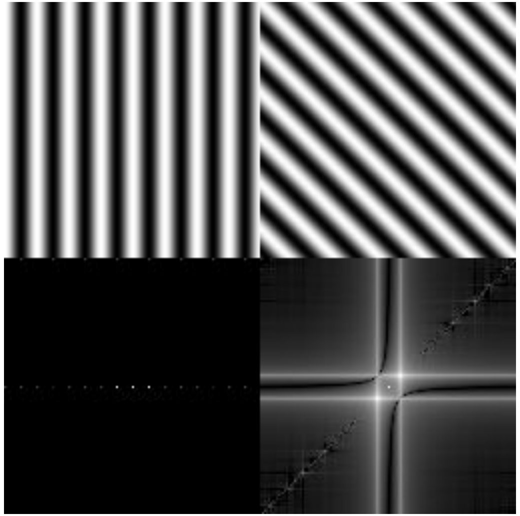
Source: www.cs.unm.edu

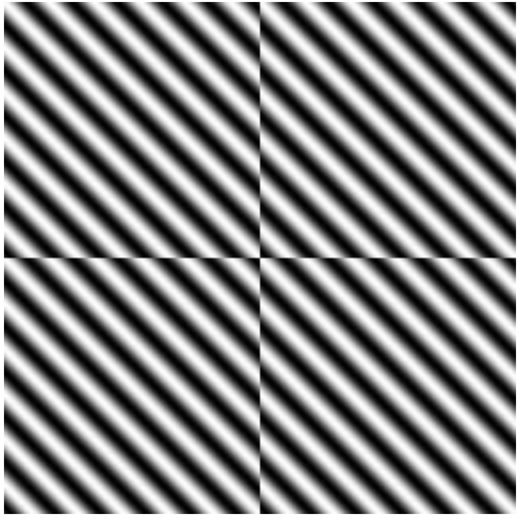
Phase Misalignment

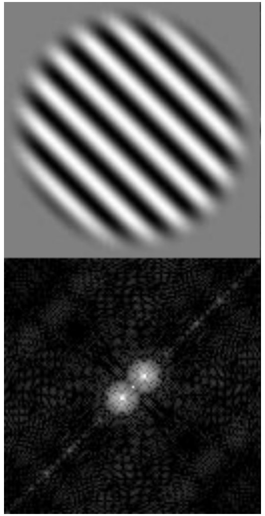


Edge Artifacts

All images are treated as periodic signals across the image boundary
Mismatched boundaries produce false edges
Windowing can reduce artifacts

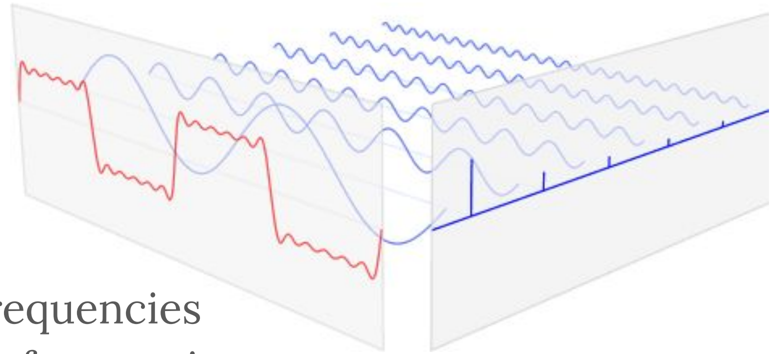




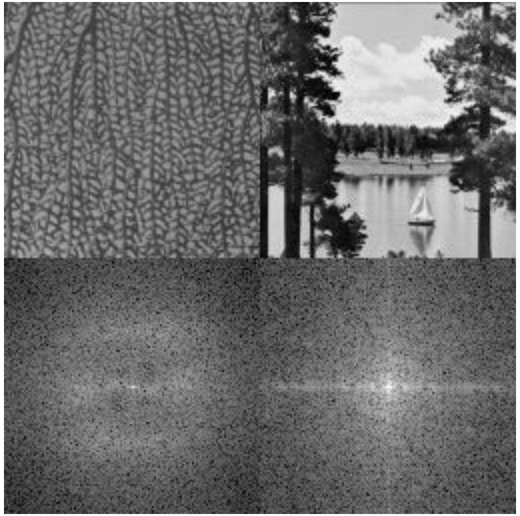


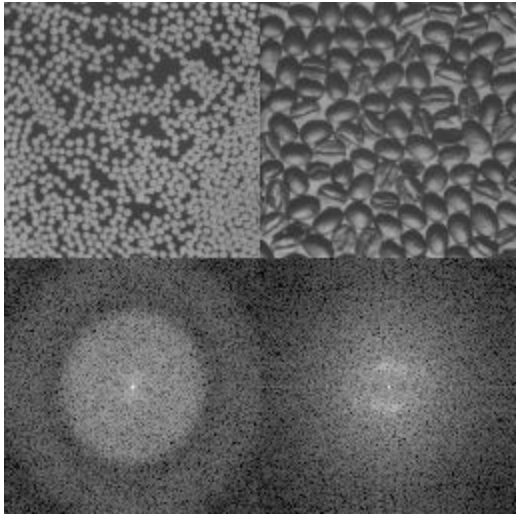
Source: www.cs.unm.edu

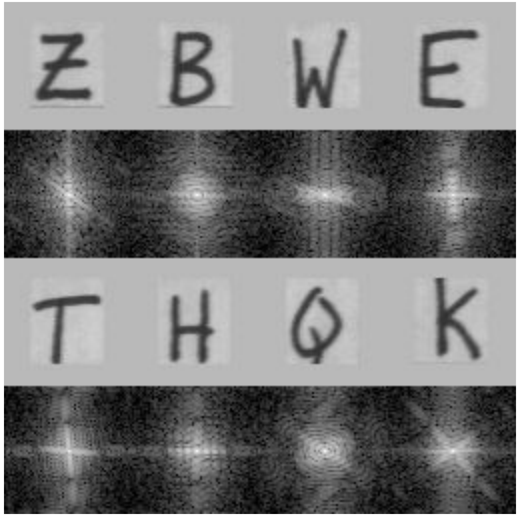
Natural Images



Usually power at many spatial frequencies
Sharp low frequency edges are composed of high frequencies
Specific frequencies can be used to extract features from an image

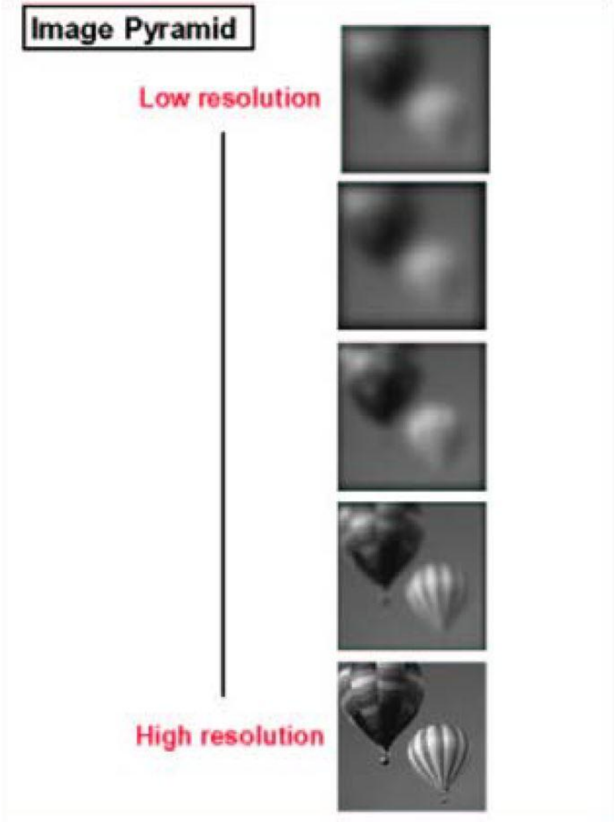


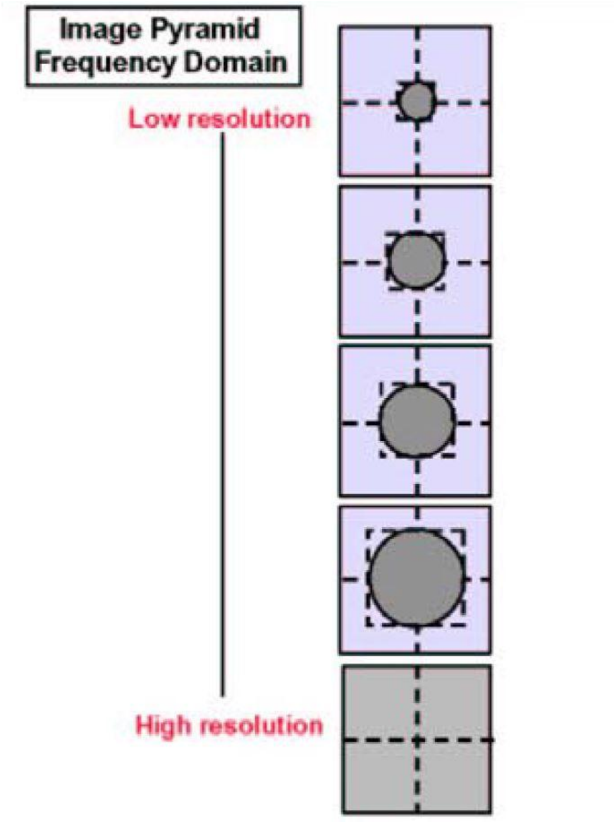




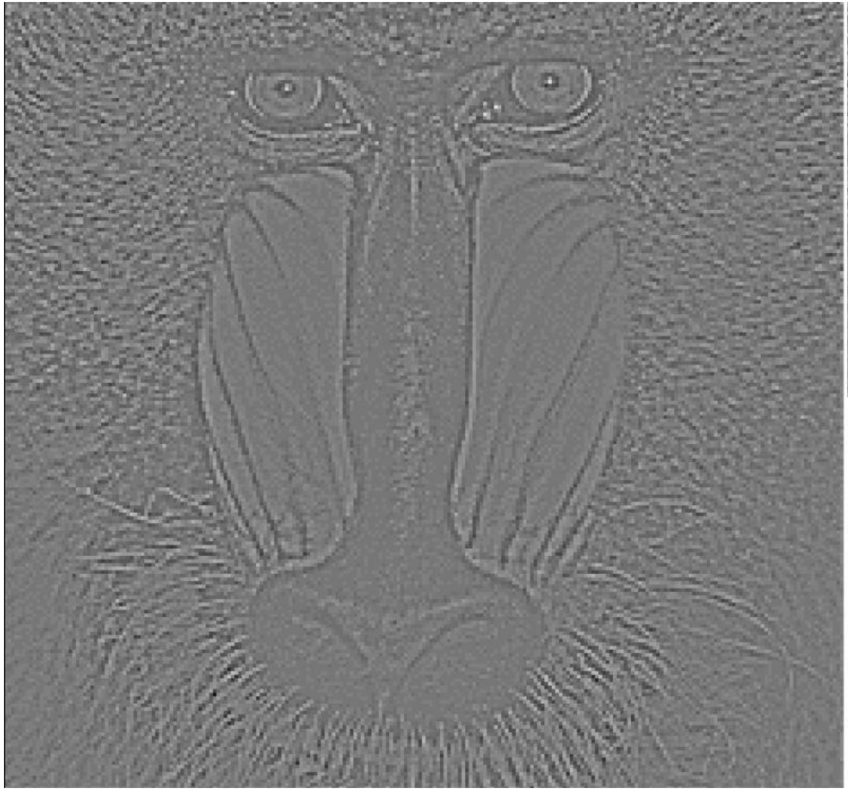
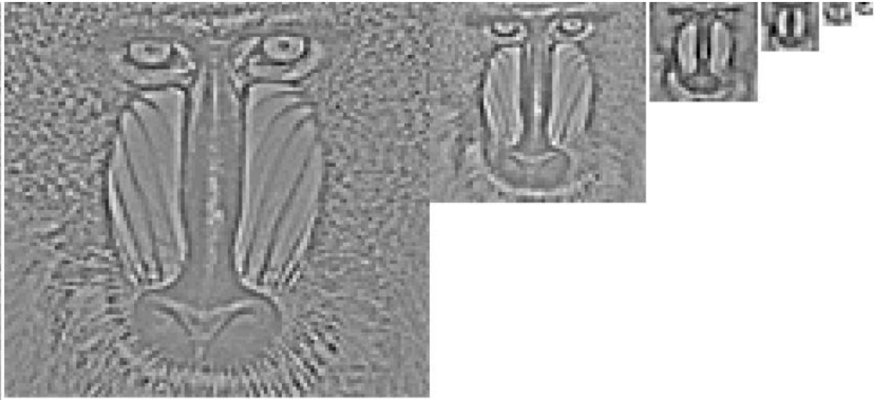
Sources: ritchievink.com, www.cs.unm.edu

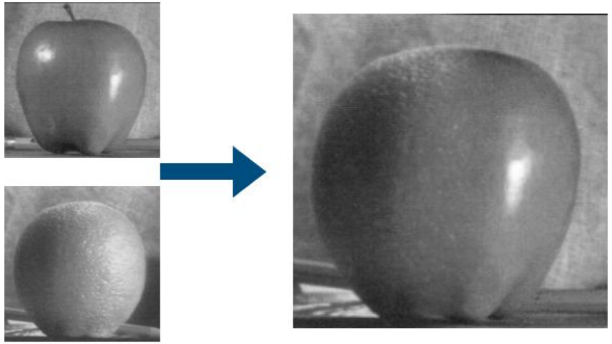
Image vs Frequency Domain





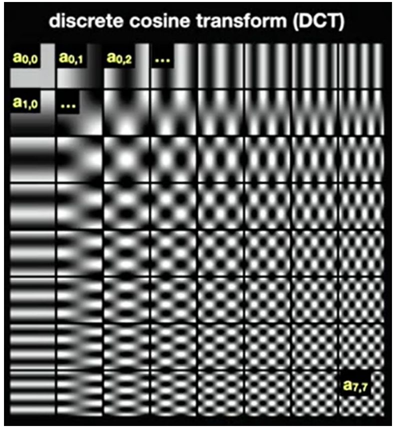
Laplacian Pyramid

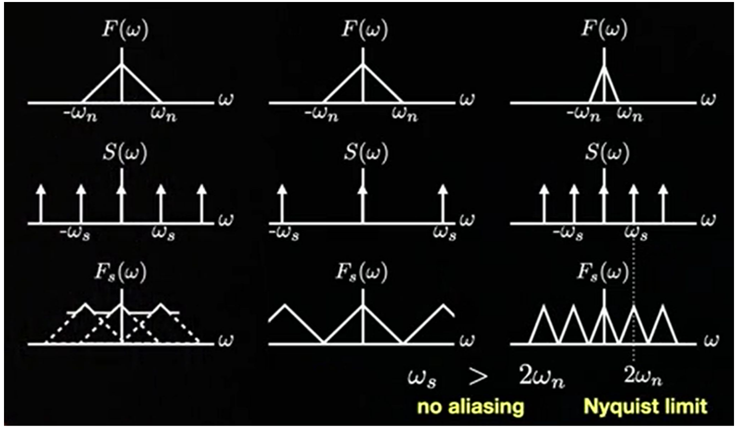



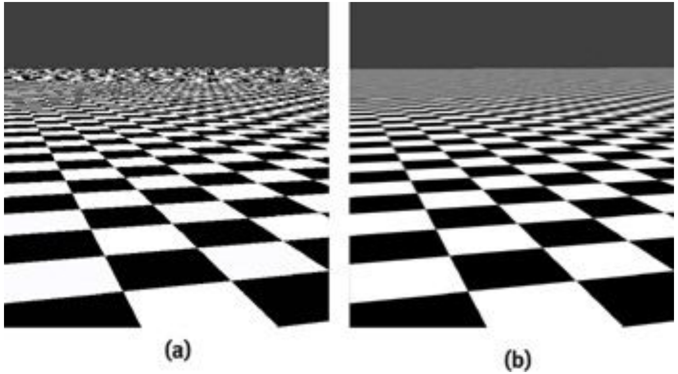


DCT and Nyquist Frequency

Pixels are discrete signals, usually also sampled by discrete frequency decomposition
When sampling frequencies are not aligned, you get aliasing (corrupted signal)
Nyquist limit – maximum frequency that can be represented given a sampling interval
Blurring before sampling (removing high frequencies) can prevent aliasing







Source: nvidia.com

Applications of Fourier Transform

Image Analysis

- detecting images that have a particular frequency distribution

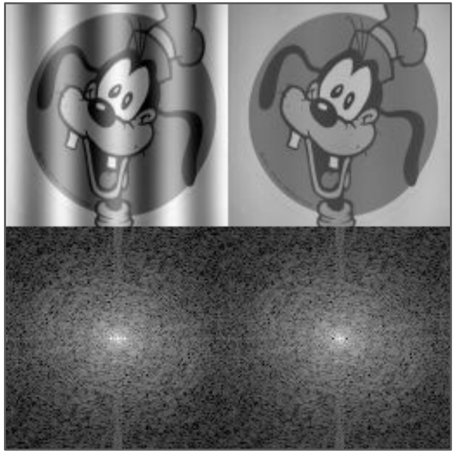
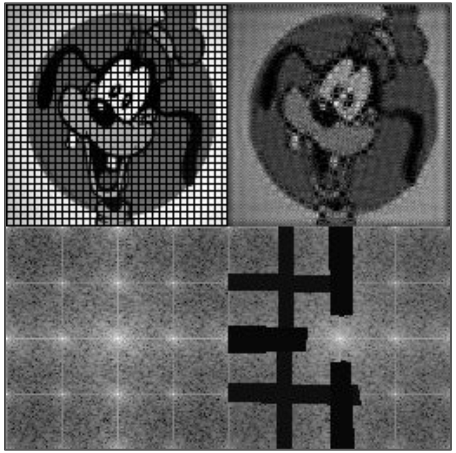


Image Processing

- faster convolution

- filtering out specific frequencies from images



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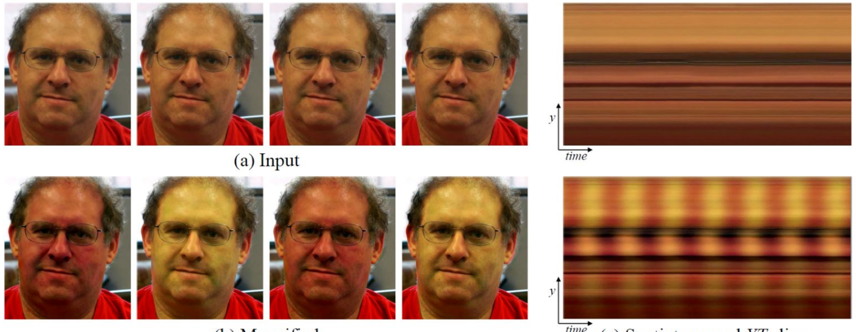
Practical applications of frequency analysis

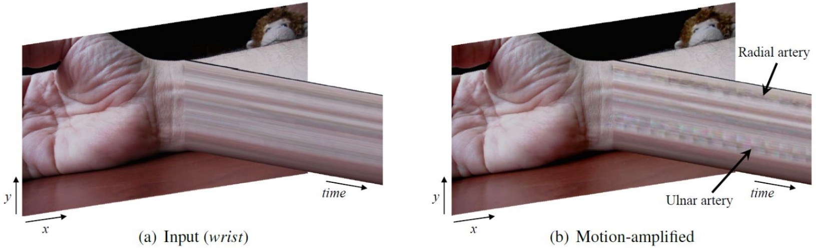
Eulerian Video Magnification for Revealing Subtle Changes in the World

Hao-Yu Wu[1] Michael Rubinstein[1] Eugene Shih[2] John Guttag[1] Frédo Durand[1] William Freeman[1]

[1]MIT CSAIL [2]Quanta Research Cambridge, Inc.

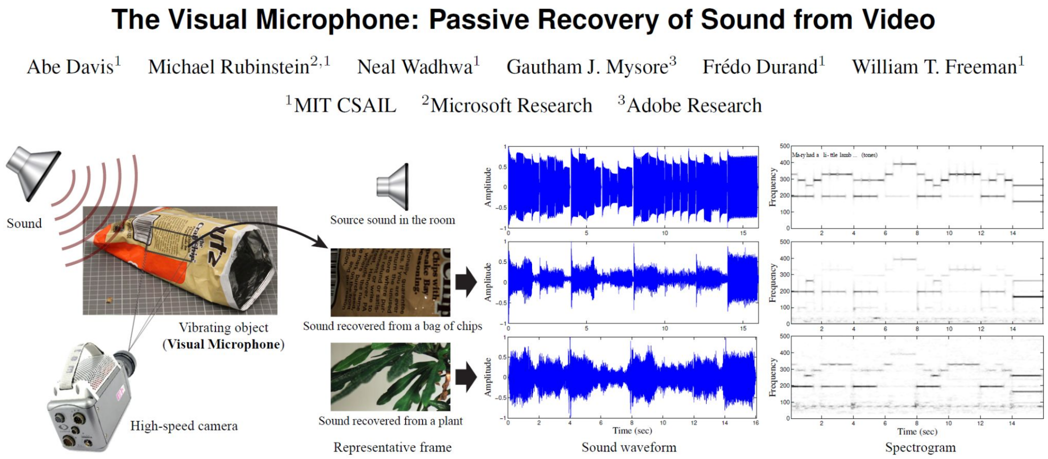


(a) Input
(b) Magnified
(c) Spatiotemporal *YT* slices

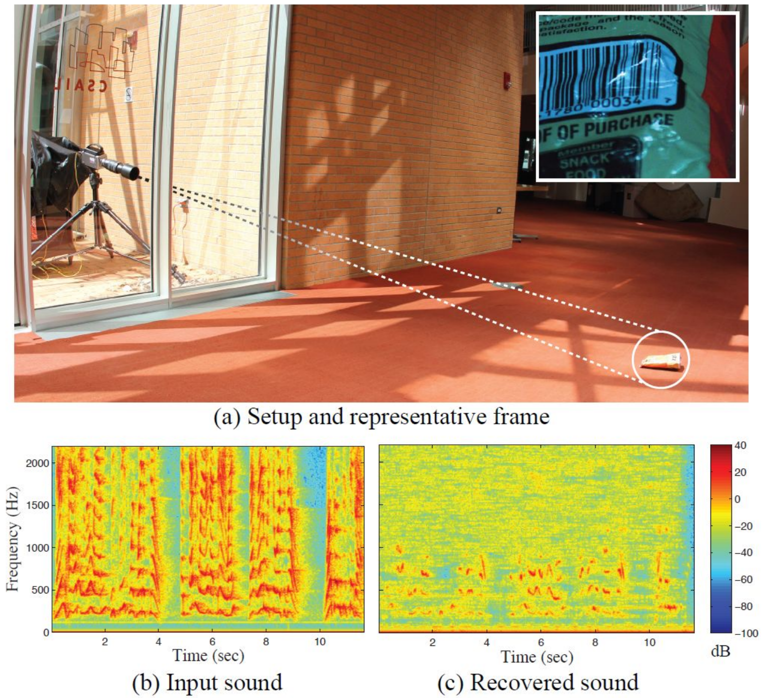


(a) Input (*wrist*)
(b) Motion-amplified

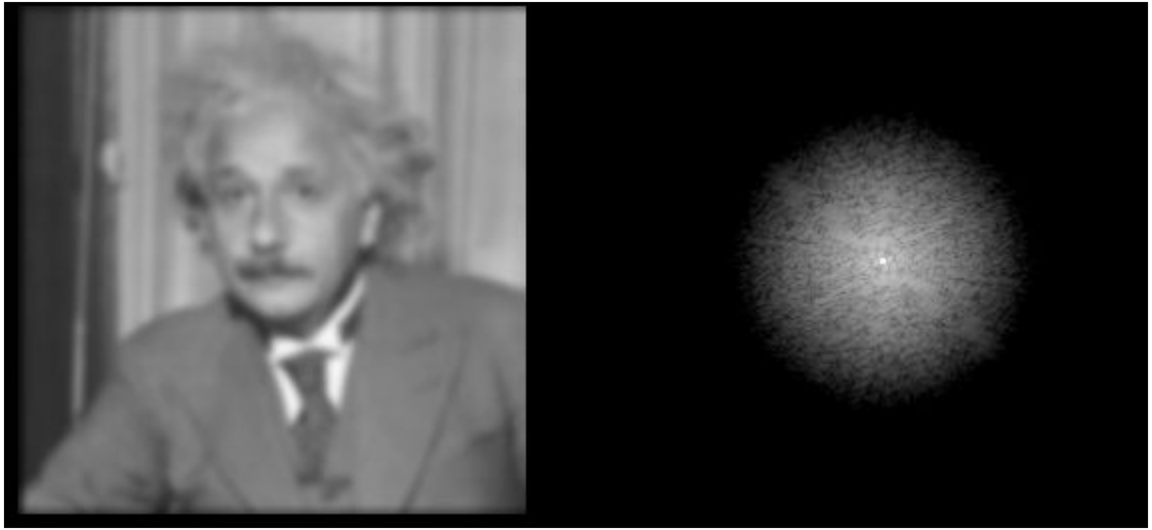
Practical applications of frequency analysis



http://people.csail.mit.edu/mrub/VisualMic/



Group Exercise – Frequencies



Upcoming ToDo's

Assignment 4 due Feb 27th

Watch Async lectures for Unit 6

Continue work on final project proposals