281 Live Session

Week 9 — 2023/3/8

Agenda

- Overview: Ordinary Least Squares
- Practical applications
- Blobworld paper

Ordinary Least Squares

What's the model?

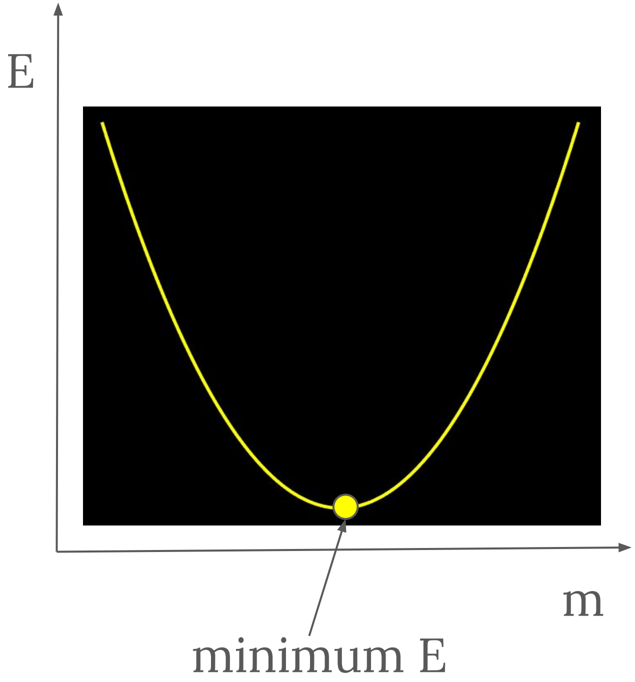
- line → $y = mx + c$

What's the objective?

- minimize → $E(m,c) = \sum[(mx + c) - y]^2$

How we minimize (optimize) the objective?

- least squares



Ordinary Least Squares

What's the model?

- line → y = mx + c

What's the objective?

- minimize → E(m,c) = ∑[(mx + c) - y]^2

How we minimize (optimize) the objective?

- least squares: $$\vec{u} = (X^T X)^{-1} X^T \vec{y}$$

$$\begin{pmatrix} x_1 & 1 \\ x_2 & 1 \\ \vdots & \vdots \\ x_n & 1 \end{pmatrix} \begin{pmatrix} m \\ b \end{pmatrix} - \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix}$$

$$X\vec{u} - \vec{y} = \vec{\Delta}$$

Weighted Least Squares

What's the model?

- line → y = mx + c

What's the objective?

- minimize → E(m,c) = ∑[$\boxed{w}$(mx + c) - y]^2

How we minimize (optimize) the objective?

- weighted least squares

$$\vec{u} = (X^T W_2 X)^{-1} X^T W_2 \vec{y}$$

Practical applications

Feature matching

Camera calibration

Distortion correction

Image stitching

3D reconstruction

Motion estimation

Video stabilization

Practical applications

LSMAT
Least Squares Medial Axis Transform

Daniel Rebain[1,2], Baptiste Angles[1,2], Julien Valentin[2], Nicholas Vining[2], Jiju Peethambaran[1], Shahram Izadi[2], Andrea Tagliasacchi[1,2,3]

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[2]Google LLC, Mountain View, CA, USA
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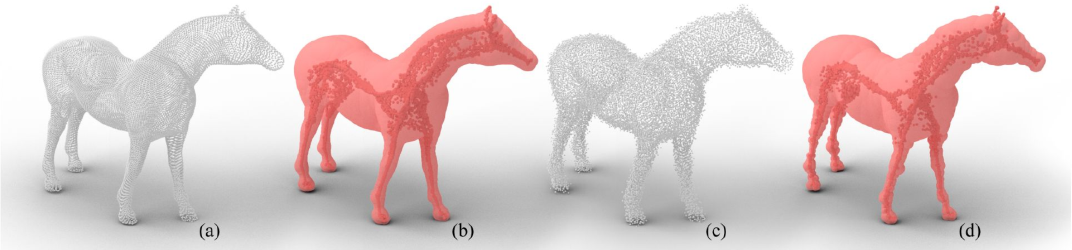


Figure 1: The LSMAT is a novel efficient least-squares formulation of the medial axis transform that operates on unorganized oriented point sets. (a, b) Regardless of input noise, the resulting medial representation is *stable*. (c, d) Its least-squares nature allows it to operate in the presence of heavy noise where most approaches would fail. We visualize the oriented point cloud with oriented splats, draw the union of medial spheres in light red, and their corresponding centers in dark red.

Source: Rebain, Daniel, et al. "LSMAT least squares medial axis transform." *Computer Graphics Forum*. Vol. 38. No. 6. 2019.

Practical applications

Fast Updates for Least-Squares Rotational Alignment

Jiayi Eris Zhang[1] Alec Jacobson[1] Marc Alexa[2]

[1]University of Toronto, Canada [2]TU Berlin, Germany

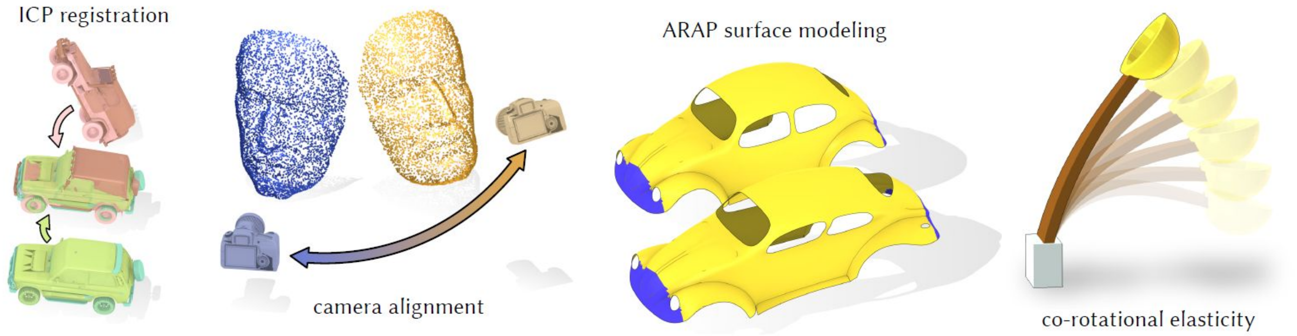


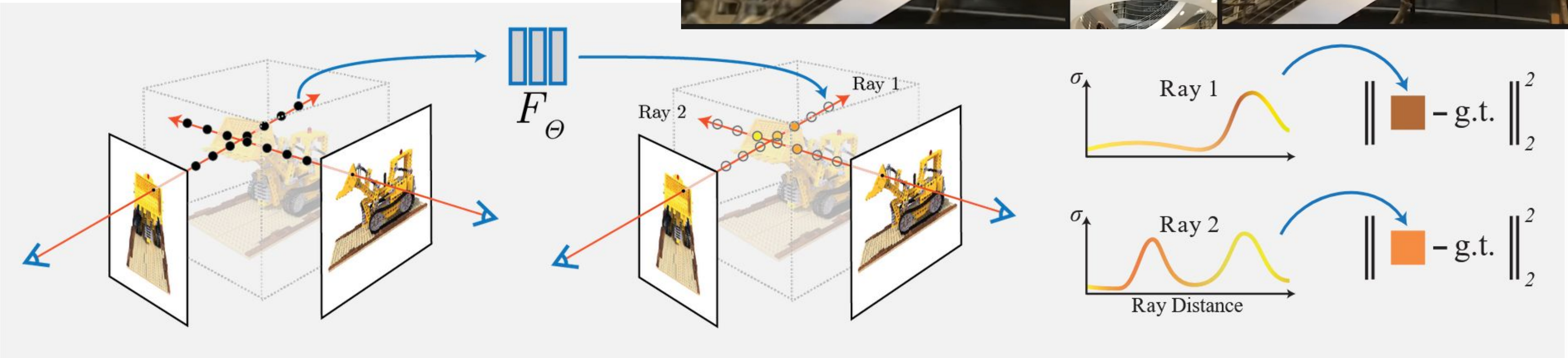
Figure 1: *Least-squares rotation fitting is a core low-level subroutine in a number of important high-level tasks in computer graphics, geometry processing, robotics and computer vision.*

Source: Zhang, Jiayi Eris, Alec Jacobson, and Marc Alexa. "Fast Updates for Least-Squares Rotational Alignment." *Computer Graphics Forum*. Vol. 40. No. 2. 2021.

Practical Applications

NeRFs — Neural Radiance Fields

Multi-view geometry allows reconstruction of complex scenes

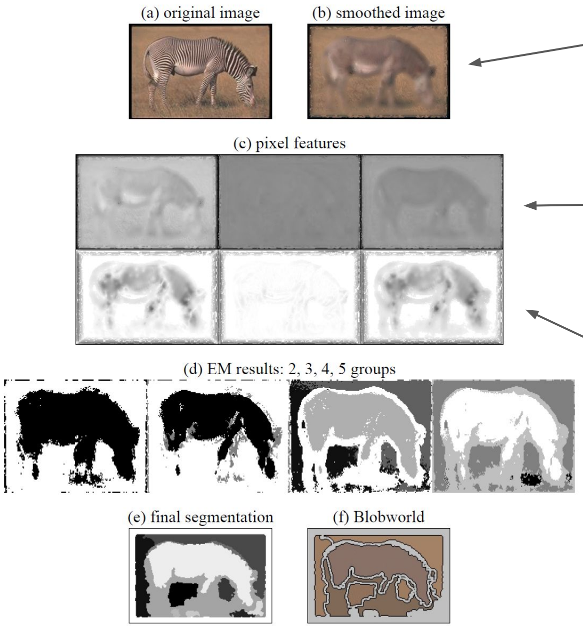


Source: www.matthewtancik.com

Blobworld Paper

Carson, Chad, Serge Belongie, Hayit Greenspan, and Jitendra Malik. "Blobworld: Image segmentation using expectation-maximization and its application to image querying." *IEEE Transactions on pattern analysis and machine intelligence* 24, no. 8 (2002): 1026-1038.

http://www.cse.psu.edu/~rtc12/CSE586/papers/emCarson99blobworld.pdf



Texture smoothing using *scale selection* preserves only important contours

Top row features:
Luminance, Red-Green, Blue-Yellow

Second row features:
Anisotropy, Polarity, Contrast

Blobworld Paper - Choosing a Scale

Sigma → scale of the dominant gradient

Polarity → consistency of regional gradients

Process:

1. Compute polarity at every scale
2. Blur regions with Gaussians of various sigma size
3. Determine at what scale polarity levels off

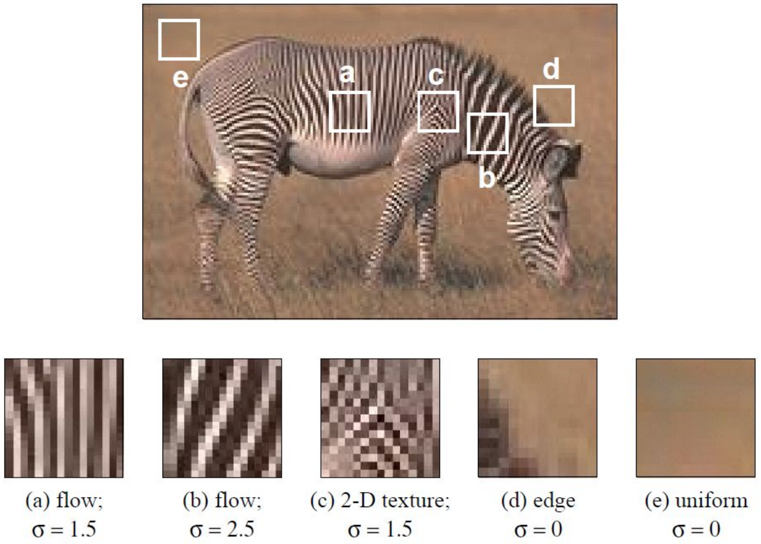


Figure 3. Five sample patches from a zebra image. (a) and (b) have stripes (1-D flow) of different scales and orientations, (c) is a region of 2-D texture, (d) contains an edge, and (e) is a uniform region.

Blobworld Paper - Feature Vector

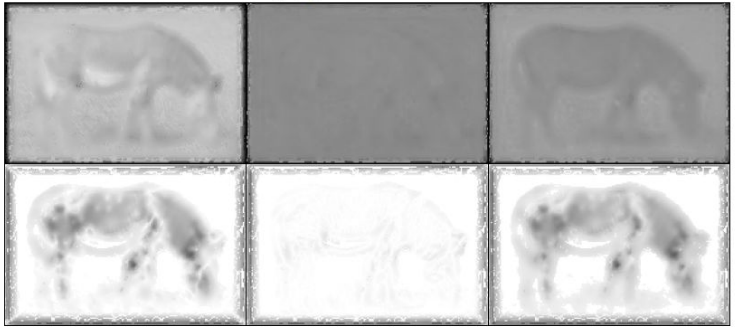
L*a*b color space (Luminance, Red-Green, Blue-Yellow)

Anisotropy → relative strength between gradients

Polarity → neighborhood gradient consistency
(edge vs texture vs uniform)

Contrast → overall strength of all gradients

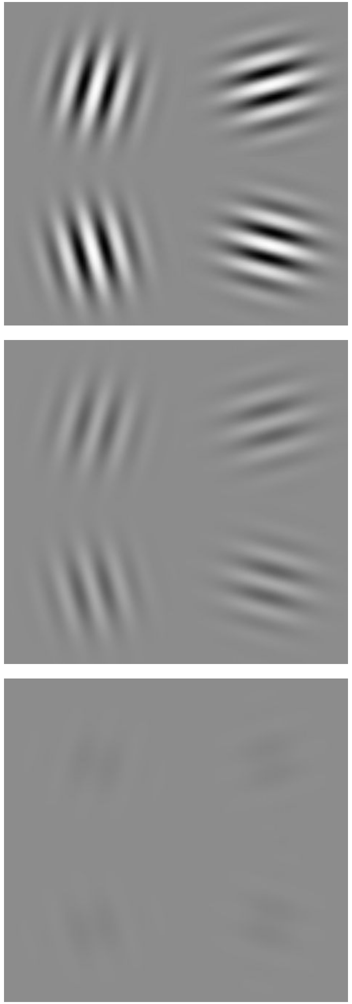
(c) pixel features



Blobworld Paper

“By this process we are performing a soft version of local spatial frequency estimation, since the smoothed polarity tends to stabilize once the scale window encompasses one approximate period.”

“In effect, a given textured patch in an image first has its texture properties extracted and then is replaced by a smooth patch of averaged color (see Figure 2(b)). In this manner, the color and texture properties in a given region are decoupled; for example, a zebra becomes a gray horse plus stripes.”



Blobworld Paper

Take-away ideas:

- Regions of an image can be differentiated by multi-scale, multi-orientation gradients

- Features are just pixel values that have been transformed to a new, more useful basis that better captures the variation in the dataset

Upcoming ToDo's

Complete final project proposal (Due Mar 13th)

Finish Assignment 5 (Due Mar 14th)

Watch Async lectures for Unit 9