281 Live Session

Week 5 — 2023/2/8

Agenda

Questions on Final Projects

Introduction to Frequency Analysis

Exercise - Convolution

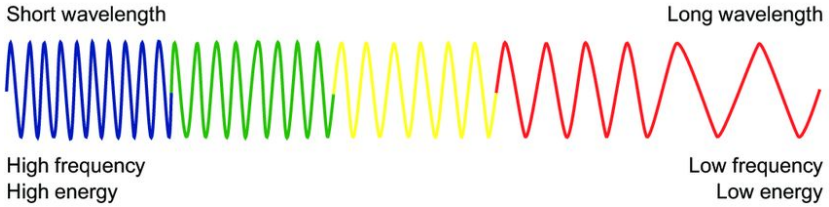
Overview of Frequency Analysis

What is frequency?

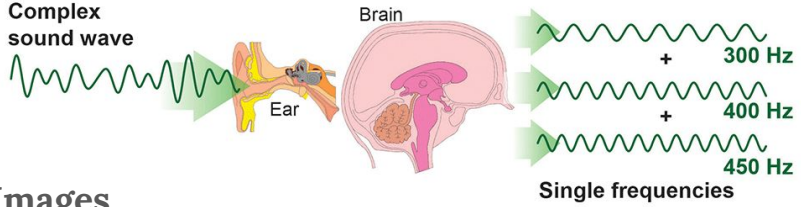
Amplitude change over time or space

For images, frequency is the distance in pixels required for a full cycle of light and dark

Light



Sound



Images

Sources: Principles of Remote Sensing (pp.56-85), pasco.com, futurity.org

Frequency Interacts With Contrast

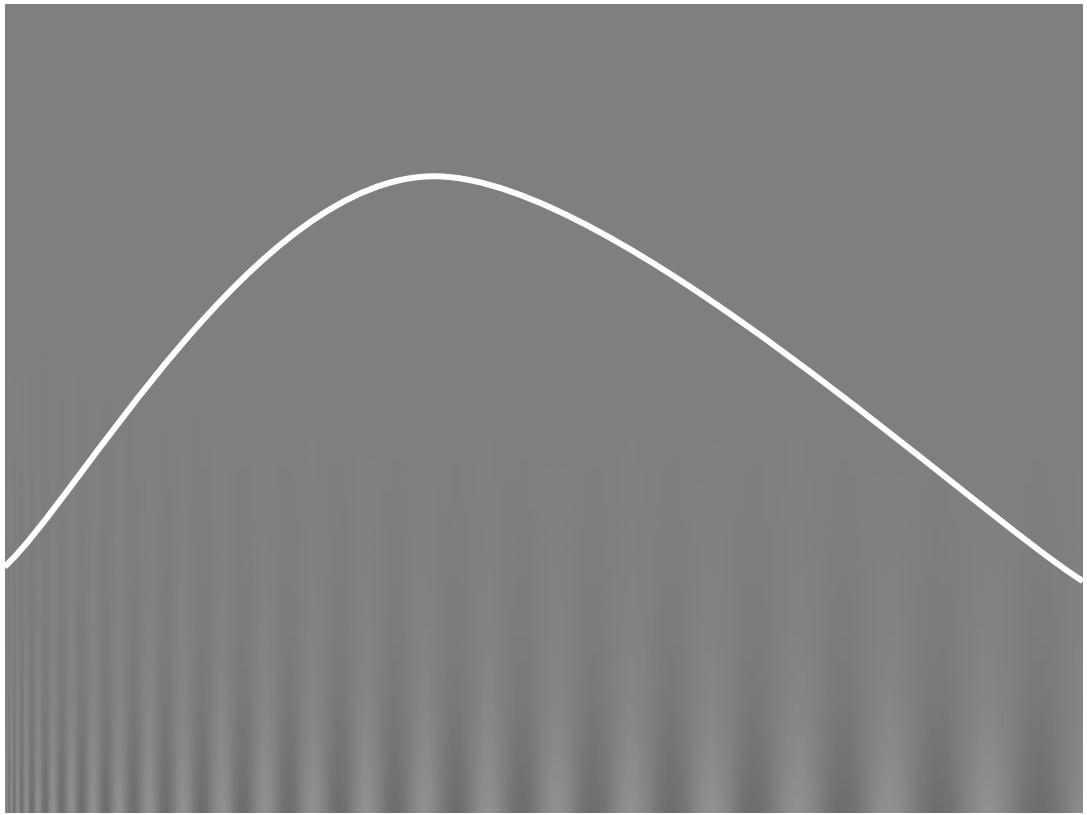
Contrast Sensitivity Function

Visibility is a function of

- Spatial frequency
- Contrast
- Viewing distance

Inverted U shape

(very high and very low spatial frequencies are less visible)



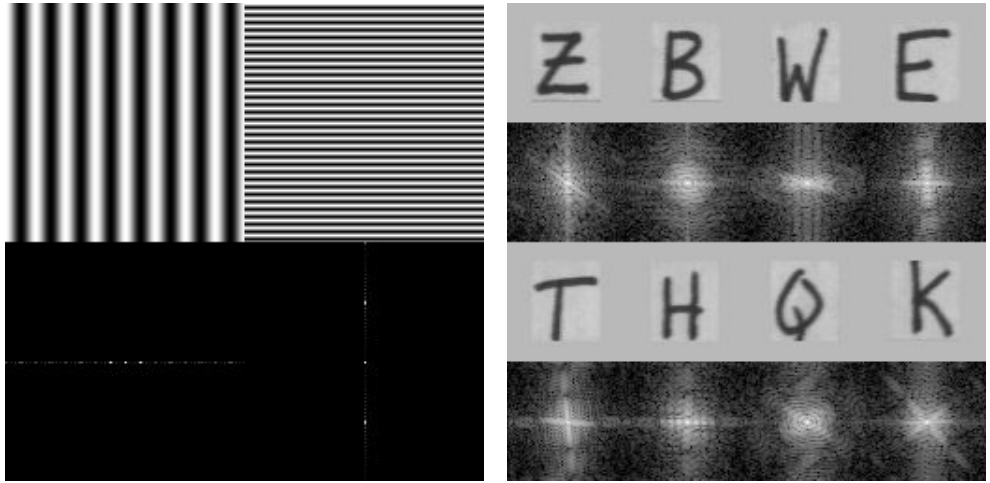
Sources: wikimedia commons

Why Learn About Frequencies?

Image content can be alternately represented by a Fourier Transform

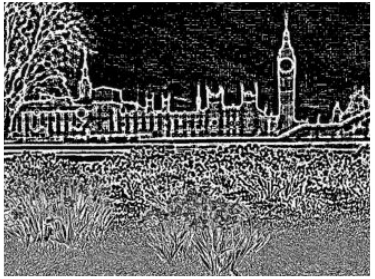
In the frequency domain we can

- Perform faster convolution and filtering
- Isolate features/information at specific scales



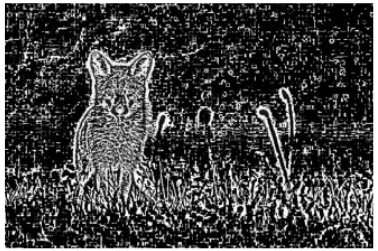
Sources: cs.unm.edu

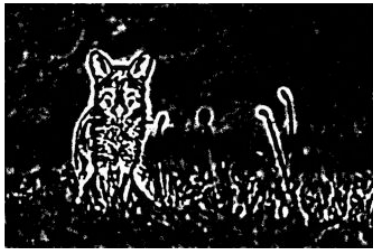
Frequency decomposition examples



Source: medium.com/mindorks

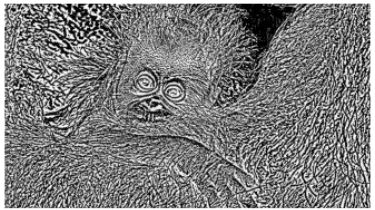
Frequency decomposition examples





Source: naturettl.com

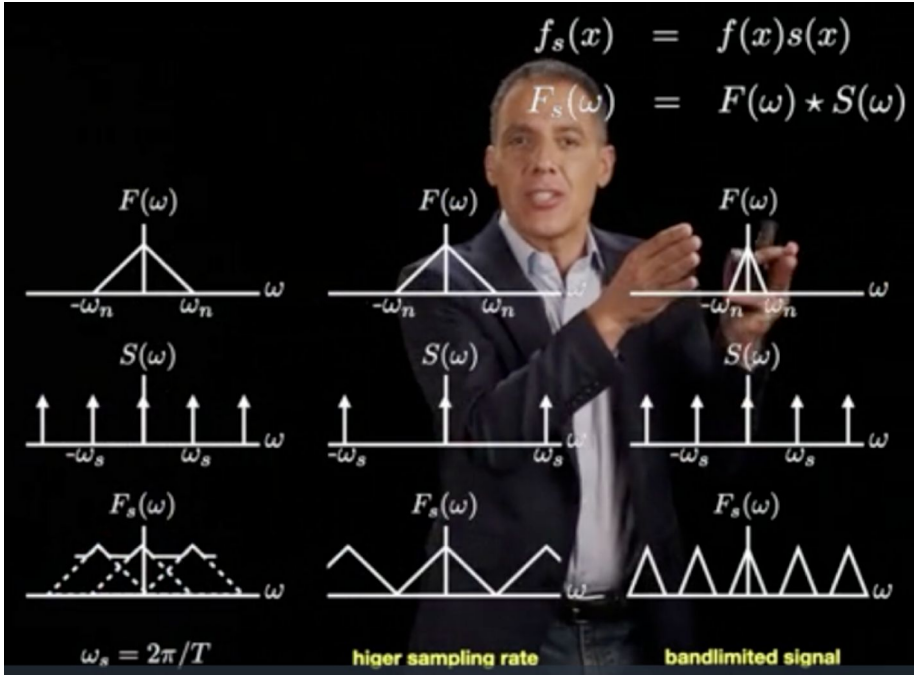
Frequency decomposition examples



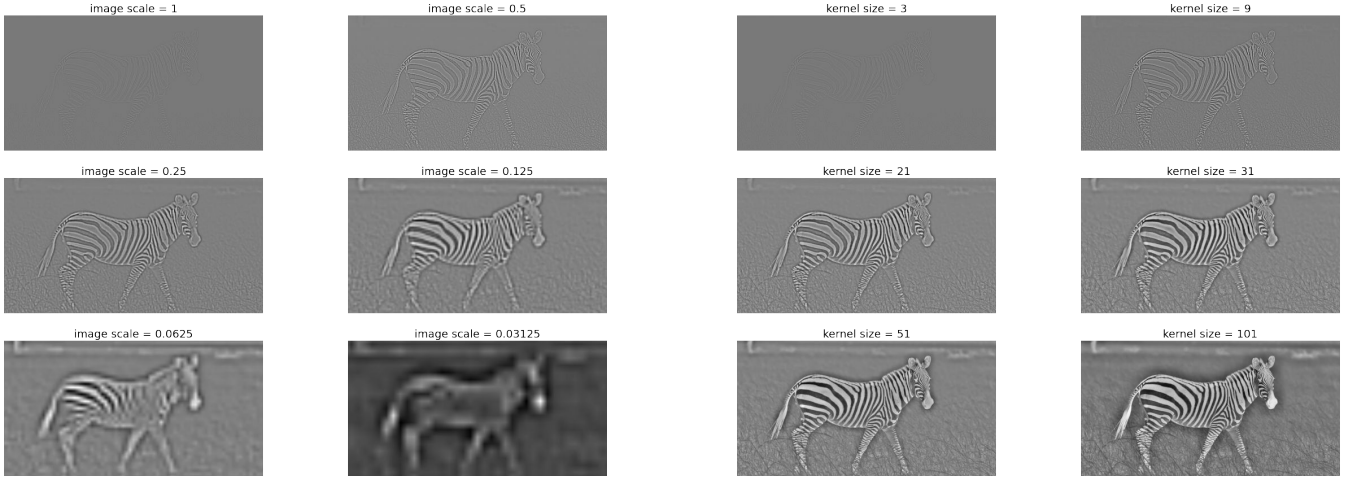
Source: naturettl.com

Using the Impulse Function as a Basis

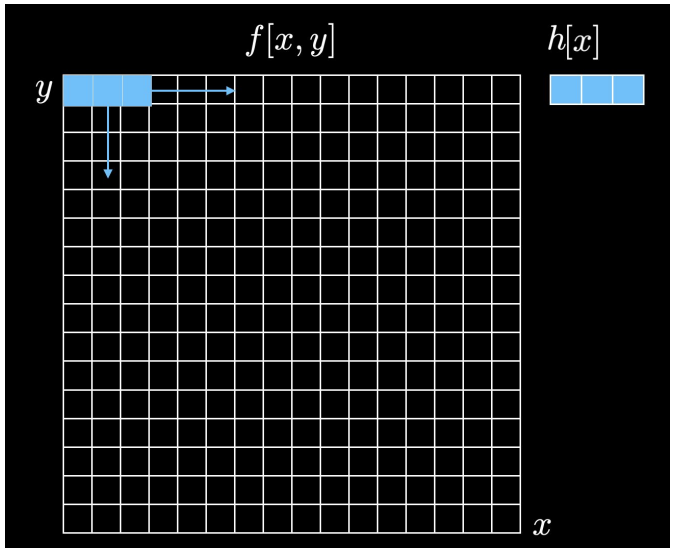
$$
\begin{aligned}
f[x] &= (1\ \ 2\ \ 4\ \ 5\ \ 3\ \ 0\ \ \dots\ \ 7) \\
f[x] &= 1\,(1\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0) \\
&+ 2\,(0\ \ 1\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0) \\
&+ 4\,(0\ \ 0\ \ 1\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 0) \\
&+ \dots \\
&+ 7\,(0\ \ 0\ \ 0\ \ 0\ \ 0\ \ 0\ \ \dots\ \ 1) \\
f[x] &= \sum_{k=0}^{m-1} a_k b_k(x) \\
a_k &= \sum_{l=0}^{m-1} f(l) b_k(l)
\end{aligned}
$$



Group Exercise – Convolution



Constructing the Deconvolution Matrix (A)



$$\begin{pmatrix} g[0] \\ g[1] \\ g[2] \\ \vdots \\ g[n-2] \\ g[n-1] \\ g[n] \end{pmatrix} = \begin{pmatrix} h_0 & h_{-1} & 0 & 0 & \dots & 0 & 0 & 0 & h_1 \\ h_1 & h_0 & h_{-1} & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & h_1 & h_0 & h_{-1} & \dots & 0 & 0 & 0 & 0 \\ \vdots & & & & \ddots & & & & \vdots \\ 0 & 0 & 0 & 0 & \dots & h_1 & h_0 & h_{-1} & 0 \\ 0 & 0 & 0 & 0 & \dots & 0 & h_1 & h_0 & h_{-1} \\ h_{-1} & 0 & 0 & 0 & \dots & 0 & 0 & h_1 & h_0 \end{pmatrix} \begin{pmatrix} f[0] \\ f[1] \\ f[2] \\ \vdots \\ f[n-2] \\ f[n-1] \\ f[n] \end{pmatrix}$$

$$\vec{g} = M\vec{f}$$

$$\vec{f} = M^{-1}\vec{g}$$

Upcoming ToDo's

Find a group and topic for Final Project

Finish Assignments 2 & 3

Watch Async lectures for Unit 5

Extra optional resources for Convolution/Fourier

http://www.cns.nyu.edu/~eero/NOTES/linearSystems.pdf

https://www.cs.unm.edu/~brayer/vision/fourier.html