10/04/2026

DSP Theory Assignment - 5

SRILATHA P
221039001

1] Design and Construct a low pass Chebychev digital filter for the following specifications.

$|H(j\Omega)| \geq -0.5\,dB \qquad 0 \leq \Omega \leq 50\,rad/s$

$|H(j\Omega)| \leq -50\,dB \qquad \Omega \geq 500\,rad/s$

Assume the sampling frequency to be 4k rad/s.
Use impulse invariance technique.

$\Omega_{samp} = 4000\,rad/s \Rightarrow T = \frac{2\pi}{4000} = 1.57\,ms$

$A_p = -0.5dB$, $\Omega_p = 50\,rad/s$

$A_s = -50dB$, $\Omega_s = 500\,rad/s$

Chebyshev filter frequency Response,

$$|H(j\Omega)|^2 = \frac{1}{1 + \epsilon^2 V_N^2\left(\frac{\Omega}{\Omega_c}\right)}$$

$\Omega_c = \Omega_p = 50\,rad/s$

From Pass Band Specification,

$$|H(j\Omega)|_{\Omega=\Omega_c} = -0.5dB$$

$$20\log_{10}|H(j\Omega_c)| = -0.5$$

$$10\log_{10}|H(j\Omega_c)|^2 = -0.5$$

$$\log_{10}\left[\frac{1}{1 + \epsilon^2 V_N^2\left(\frac{\Omega}{\Omega_c}\right)}\right] = \underline{\underline{-0.05}}$$

$$V_N^2\left(\frac{\Omega}{\Omega_c}\right)\Big|_{\Omega=\Omega_c} = V_N^2(1) = \frac{1}{2}$$

$$\frac{1}{1+\epsilon^2} = 10^{-0.05}$$

$$1+\epsilon^2 = 10^{0.05}$$

$$\epsilon = \sqrt{10^{0.05} - 1}$$

$$\epsilon = 0.3493$$

From Stop Band Specification,

$$A_s = -20\log_{10}\epsilon - 6(N-1) - 20N\log_{10}\left(\frac{\Omega_s}{\Omega_c}\right)$$

$$-50 = -20\log_{10}(0.3493) - 6N + 6 - 20N\log_{10}\left(\frac{500}{50}\right)$$

$$-50 = 9.136 - 6N + 6 - 20N$$

$$-26N = -65.136$$

$$N = \frac{65.136}{26} = 2.5$$

$N \approx 3$

Let $N=3$

Calculating poles,

$$\alpha = \epsilon^{-1} + \sqrt{1+\epsilon^{-2}}$$

$$\alpha = 0.3493^{-1} + \sqrt{1+0.3493^{-2}}$$

$$\alpha = 5.8954$$

$$a = \frac{1}{2}\left[\alpha^{\frac{1}{N}} - \alpha^{-\frac{1}{N}}\right]$$

$$a = \frac{1}{2}\left[(5.8954)^{\frac{1}{3}} - (5.8954)^{-\frac{1}{3}}\right]$$

$$a = \frac{1}{2}\left[1.8065 - 0.5535\right]$$

$$\underline{a = 0.6265}$$

$$b = \frac{1}{2}\left[\alpha^{\frac{1}{N}} + \alpha^{-\frac{1}{N}}\right]$$

$$b = \frac{1}{2}\left[(5.8954)^{\frac{1}{3}} + (5.8954)^{-\frac{1}{3}}\right]$$

$$b = \frac{1}{2}\left[1.8065 + 0.5535\right]$$

$$\underline{b = 1.1800}$$

Semi-minor axis $= a\Omega_c = 0.6265 \times 50 = \underline{31.325}$

Semi-major axis $= b\Omega_c = 1.18 \times 50 = \underline{59}$

Poles are given by,

$$S_k = -a\Omega_c \sin\left[\frac{(2k+1)\pi}{2N}\right] + jb\Omega_c \cos\left[\frac{(2k+1)\pi}{N}\right]$$

$$S_k = -31.325 \sin\left[\frac{(2k+1)\pi}{6}\right] + j\,59 \cos\left[\frac{(2k+1)\pi}{6}\right]$$

$$S_0 = -15.663 + j51.095 = \underline{53.444 \angle 1.868}$$

$$S_1 = -31.325 + j0 = \underline{31.325 \angle \pi}$$

$$S_2 = S_0^* = -15.663 - j51.095 = \underline{53.444 \angle -1.2733}$$

$|S_0| = |S_2| = 53.44$

$|S_1| = 31.325$

$$H(S) = \frac{|S_0||S_1||S_2|}{(S-S_0)(S-S_1)(S-S_2)}$$

$$H(S) = \left[\frac{(53.44)^2 \times 31.325}{(S+15.663-j51.095)(S+31.325)(S+15.663+j51.095)}\right]$$

$$H(S) = \left[\frac{31.325}{S+31.325}\right]\left[\frac{2855.8336}{S^2+31.325S+2855.8336}\right]$$

Impulse Response $h(t) = L^{-1}\{H(s)\}$

$$h(t) = L^{-1}\left\{\frac{(53.44)^2 \times 31.325}{(S-S_0)(S-S_1)(S-S_2)}\right\}$$

$$h(t) = L^{-1}\left\{\frac{A}{S-S_0} + \frac{B}{S-S_1} + \frac{C}{S-S_2}\right\}$$

$$2855.8 \times 31.325 = A(S-S_1)(S-S_2) + B(S-S_0)(S-S_2) + C(S-S_0)(S-S_1)$$

when $S = S_0 = -15.663 + j51.095$

$$2855.8 \times 31.325 = A\{[(-15.663+j51.095)-(-31.325)]\{-15.663+j51.095-(-15.663-j51.095)\}\}$$

$$= A\{(15.662+j51.095)(0+j102.19)\}$$

$$= A\{1600.5j + j^2 5221.4\}$$

$$= A(-5221.4 + j1600.5)$$

$$A = \frac{2855.8 \times 31.325}{-5221.4 + j1600.5}$$

$$A = \frac{2855.8 \times 31.325}{5461.19 e^{2.844j}} = \frac{16.38\, e^{-2.844j}}{}$$

$$A = -15.66 - j4.8023$$

when $s = s_1 = -31.32$

$$2855.8 \times 31.325 = B(s - s_0)(s - s_2)$$

$$= B\{(-31.32 + 15.66 - j51.095)\}$$

$$\{(-31.32 + j15.66 + j51.095)\}$$

$$= B(-15.66 - j51.095)(-15.66 + j51.095)$$

$$= B(245.2356 + 2610.699)$$

$$B = \frac{89444.708}{2855.9346} = 31.318 = 31.32$$

when $s = s_2 = -15.66 - j51.095$

$$C = A^* = -15.66 + j4.8023$$

$$h(t) = L^{-1}\left[\frac{A}{s - s_0} + \frac{B}{s - s_1} + \frac{C}{s - s_2}\right]$$

$$h(t) = Ae^{s_0 t}u(t) + Be^{s_1 t}u(t) + Ce^{s_2 t}u(t)$$

$$h(n) = h(t)\big|_{t=nT}$$

$$= Ae^{s_0 nT}u(n) + Be^{s_1 nT}u(n) + Ce^{s_2 nT}u(n)$$

$$\therefore H(z) = Z\{h(n)\}$$

$$H(z) = \frac{A}{1-e^{s_0 T}z^{-1}} + \frac{B}{1-e^{s_1 T}z^{-1}} + \frac{C}{1-e^{s_2 T}z^{-1}}$$

$$= \frac{B}{1-e^{s_1 T}z^{-1}} + \frac{(A+C) - z^{-1}\left[Ae^{s_2 T} + Ce^{s_0 T}\right]}{1 - z^{-1}\left[e^{s_0 T} + e^{s_1 T}\right] + e^{(s_0+s_2)T}z^{-2}}$$

$B = 31.32$

$e^{s_1 T} = 0.952$

$A + C = -31.32$

$Ae^{s_2 T} + Ce^{s_0 T} = 16.378 e^{-j2.844} e^{(-15.66 - j51.095)1.571\times10^{-3}}$

$= 16.378 e^{(-15.66)1.571\times10^{-3}} \left[2\cos\left(2.844 + 51.095 \times 1.571\times10^{-3}\right)\right.$

$= 15.979 \times 2 \times (-0.9764)$

$= -31.204$

$e^{s_0 T} + e^{s_2 T} = e^{-15.66 \times 1.571\times10^{-3}} \left\{2\cos\left(51.095 \times 1.57\times10^{-3}\right)\right.$

$= 1.945$

$e^{(s_0+s_2)T} = e^{-2\times15.66\times1.571\times10^{-3}}$

$= 0.952$

$$H(z) = \frac{31.32}{1-0.95z^{-1}} + \frac{-31.32 + 31.204z^{-1}}{1 - 1.945z^{-1} + 0.952z^{-2}}$$

Realization :