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DSP Theory Assignment - 4

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1] Design a low-pass filter using windows that will have 3-dB cut off at 30π rad/sec and an attenuation of 50dB at 45π rad/sec. This filter is required to have linear phase characteristics and the system employs a sampling frequency of 100Hz.

Cut off frequency, $\omega_c = 30\pi$ rad/s

$\therefore$ Passband edge frequency, $\omega_p = \frac{30\pi}{100} = \underline{0.3\pi}$

Attenuation frequency = 45 rad/s.

$\therefore$ Stopband corner frequency, $\omega_s = \frac{45\pi}{100} = \underline{\underline{0.45\pi}}$

Transition width, $\omega_t = |\omega_s - \omega_p|$

$\omega_t = |0.45\pi - 0.3\pi|$

$\underline{\underline{\omega_t = 0.15\pi}}$

Since the stop band attenuation is 50dB we consider Hamming Window which has a minimum stopband attenuation of 54.5dB.

$\omega_t = \frac{3.32\pi}{\frac{N-1}{2}}$

$\frac{0.15\pi \times N-1}{2} = 3.32\pi$

$$N = \frac{6.64}{0.15} + 1$$

$$N \cong 47$$

Since the filter has a linear phase characteristics

$$\alpha = \frac{N-1}{2} = \frac{47-1}{2}$$

$$\alpha = \frac{46}{2} = 23$$

$$H_d(e^{j\omega}) = \begin{cases} e^{-j23\omega} & ; \ 0 \le |\omega| \le 0.3\pi \\ 0 & ; \ \text{elsewhere} \end{cases}$$

$$h_d(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} H_d(e^{j\omega}) e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-0.3\pi}^{0.3\pi} e^{-j23\omega} \cdot e^{j\omega n} \, d\omega$$

$$= \frac{1}{2\pi} \int_{-0.3\pi}^{0.3\pi} e^{j\omega(n-23)} \, d\omega$$

$$h_d(n) = \frac{\sin\left(0.3\pi(n-23)\right)}{\pi(n-23)}$$

Since we are using Hamming window we will be using the Hamming window function, $w(n)$.

$$w(n) = 0.54 - 0.46 \cos \frac{2\pi n}{N-1}$$

$$\therefore w(n) = 0.54 - 0.46 \cos \frac{2\pi n}{46}$$

∴ The impulse response of the filter, $h(n)$

$$h(n) = h_d(n)\, w(n)$$

$$h(n) = \frac{\sin(0.3\pi(n-23))}{\pi(n-23)} \left\{ 0.54 - 0.46 \cos\frac{2\pi n}{46} \right\}; \quad 0 \le n \le 46$$

When $n = \alpha$,

$h(n) = \frac{\omega_c}{\pi}$. In this case it's $n = 23$

$$\therefore h(23) = \frac{0.3\cancel{\pi}}{\cancel{\pi}} = \underline{0.3}$$

Realization :-