05/03/23

DSP - Theory Assignment - 1

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1] Compute the DFT of the sequence, $x[n] = [1, -1, -1, -1, 1, 1, 1, -1]$ using DIF FFT algorithm.

Let $x(k)$ be the DFT of the sequence

$x(k)$ will have 8-samples.

The algorithm involves 3-stages.

The twiddle factors involved are,

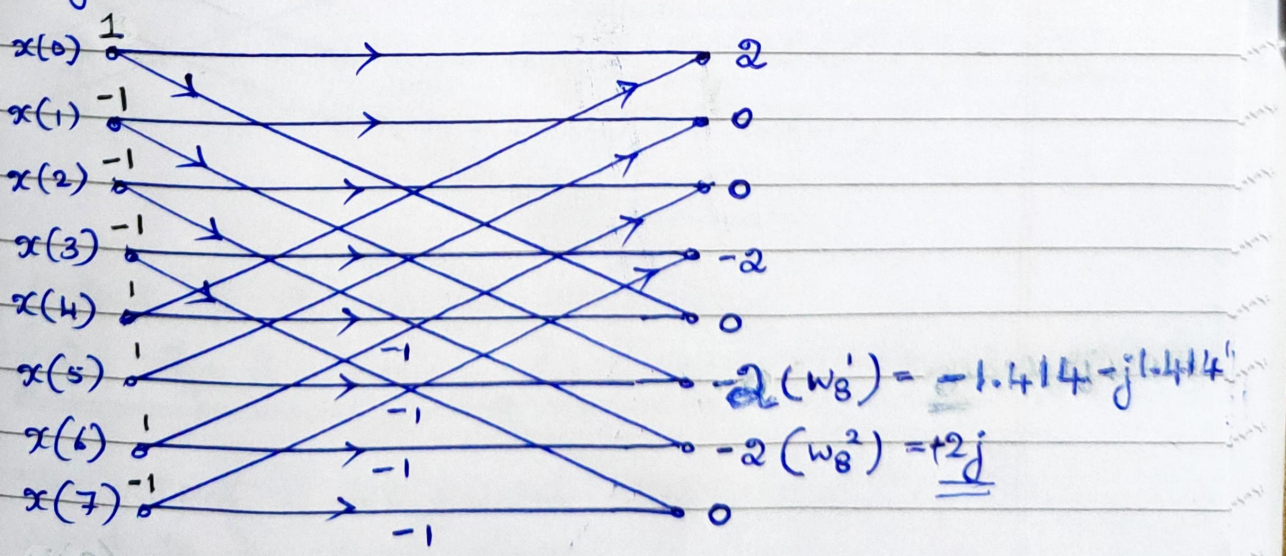
$W_{N=8}^{0} = e^{-j\frac{2\pi}{8} \times 0} = e^{0} = 1$

$W_8^1 = e^{-j\frac{2\pi}{8} \times 1} = e^{-j\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) - j\sin\left(\frac{\pi}{4}\right) = -0.707 - j0.707$

$W_8^2 = e^{-j\frac{2\pi}{8} \times 2} = e^{-j\frac{\pi}{2}} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$

$W_8^3 = e^{-j\frac{2\pi}{8} \times 3} = e^{-j\frac{3\pi}{4}} = \cos\left(\frac{3\pi}{4}\right) - j\sin\left(\frac{3\pi}{4}\right) = -0.707 - j0.707$

Stage 1 :-



Output of first stage :-

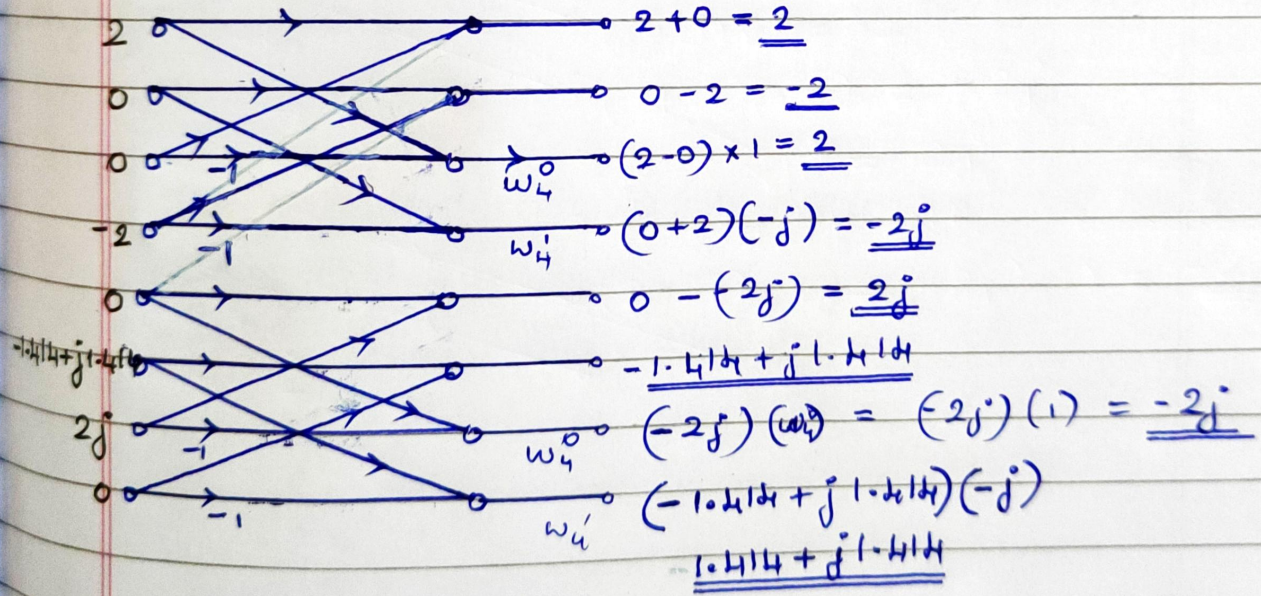
$\{2, 0, 0, -2, 0, -1.414 + j1.414, 2j, 0\}$

The output of the first stage will be the input to the second stage.

$W_4^0 = e^{-j\frac{2\pi}{4} \times 0} = e^0 = 1$

$W_4^1 = e^{-j\frac{2\pi}{4} \times 1} = \cos\left(\frac{\pi}{2}\right) - j\sin\left(\frac{\pi}{2}\right) = -j$

Stage 2 :-



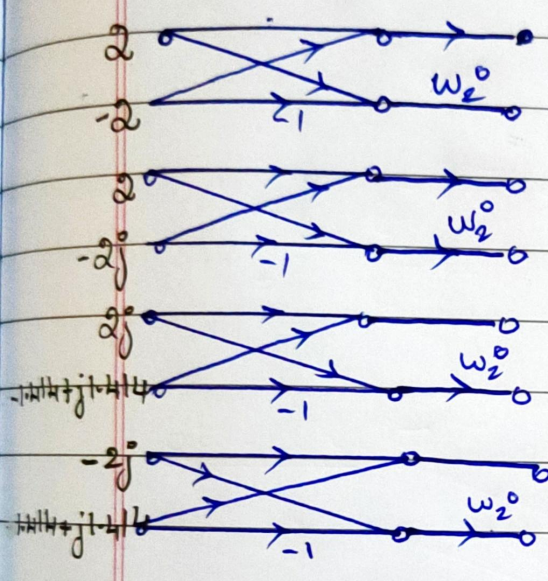
Output of the second stage :-

$\{2, -2, 2, -2j, 2j, -1.414 + j1.414, -2j, 1.414 + j1.414\}$

The output of the second stage will be the input to the third stage.

$$W_2^0 = e^{-j\frac{2\pi}{2}\times 0} = e^{-0} = 1$$

Third Stage :-



$2 - 2 = 0$ $x(0)$

$(2+2)W_2^0 = 4$ $x(4)$

$2 - 2j$ $x(2)$

$2 + 2j$ $x(6)$

$2j - 1.414 + j1.414 = -1.414 + 3.414j$ $x(1)$

$2j + 1.414 - j1.414 = 1.414 + 0.58j$ $x(5)$

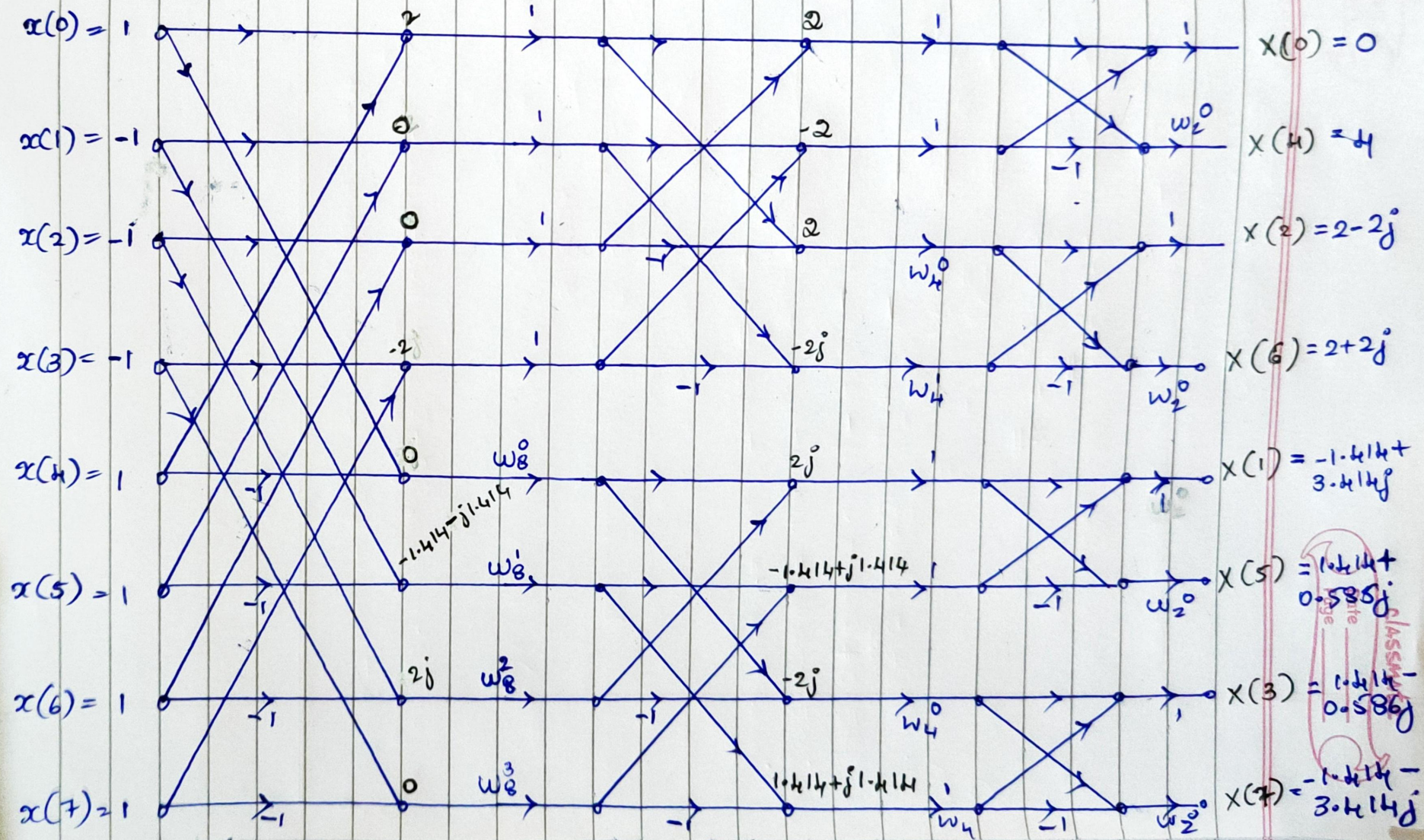
$-2j + 1.414 + j1.414 = 1.414 - 0.586j$ $x(3)$

$-2j - 1.414 - j1.414 = -1.414 - 3.414j$ $x(7)$

The output of the third stage as well as final output DFT sequence

$= \{x(0), x(1), x(2), x(3), x(4), x(5), x(6), x(7)\}$

$x(k) = \{0, -1.414 + 3.414j, 2 - 2j, 1.414 - 0.586j,$
$4, 1.414 + 0.58j, 2 + 2j, -1.414 - 3.414j\}$


x(0) = 1
x(1) = -1
x(2) = -1
x(3) = -1
x(4) = 1
x(5) = 1
x(6) = 1
x(7) = 1
X(0) = 0
X(4) = 4
X(2) = 2-2j
X(6) = 2+2j
X(1) = -1.414+3.414j
X(5) = 1.414+0.585j
X(3) = 1.414-0.586j
X(7) = -1.414-3.414j
-1.414-j1.414
-1.414+j1.414
1.414+j1.414