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DSP Theory Assignment - 3

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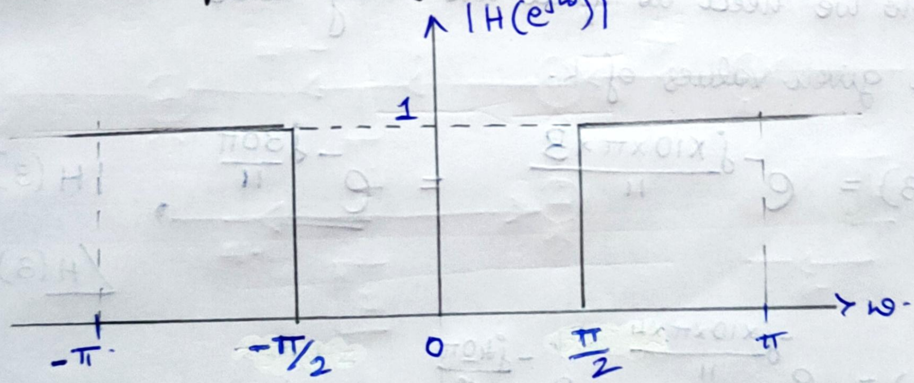
1] The desired amplitude response of a certain FIR filter with linear phase is

$$|H(e^{jf})| = 1 \; ; \quad \text{for } f \geq 500\,\text{Hz}$$
$$|H(e^{jf})| = 0 \; ; \quad \text{elsewhere}$$

The sampling frequency $f_s = 2\,\text{kHz}$ and the impulse response is to be 5.5 ms long. Design & model the filter using frequency-sampling technique.



$$\omega_c = \frac{2\pi f}{f_s} = \frac{2\pi \times 500}{2\times 10^3} = \underline{\underline{\frac{\pi}{2}}}$$

$$\therefore H(e^{j\omega}) = \begin{cases} e^{-j\alpha\omega} & ; \; -\pi \leq \omega_c \leq \pi/2 \\ & \; \frac{\pi}{2} \leq \omega_c \leq \pi \\ 0 & ; \; \text{elsewhere} \end{cases}$$

$\alpha = \frac{N-1}{2}$ but here $N = 5.5\,\text{ms} \times f_s$

$N = 5.5 \times 2 = \underline{\underline{11}}$ (odd)

$\alpha = \frac{11-1}{2} = \underline{5}$ (odd)

$\therefore$ k ranges from $0 \rightarrow N-1$. ie 0 to 10

k	0	1	2	3	4	5	6	7	8	9	10
$\frac{2\pi k}{N}$	0	$\frac{2\pi}{11}$	$\frac{4\pi}{11}$	$\frac{6\pi}{11}$	$\frac{8\pi}{11}$	$\frac{10\pi}{11}$	$\frac{12\pi}{11}$	$\frac{14\pi}{11}$	$\frac{16\pi}{11}$	$\frac{18\pi}{11}$	$\frac{20\pi}{11}$

Since ω_c ranges from $\frac{\pi}{2}$ to π we choose $k = 3, 4, 5$

$$\therefore H(k) = \begin{cases} e^{-j\frac{5\times 2\pi k}{11}} = e^{\frac{-j10\pi k}{11}} ; & k = 3, 4, 5 \\ 0 ; & k = 0, 1, 2, 6, 7, 8, 9, 10 \end{cases}$$

Now we need to find the magnitude and phase of the given values of k.

$$H(3) = e^{-\frac{j\times 10\times\pi\times 3}{11}} = e^{-\frac{j30\pi}{11}}, \quad |H(3)| = 1$$

$$\angle H(3) = \frac{-30\pi}{11}$$

$$H(4) = e^{\frac{-j\times 10\times\pi\times 4}{11}} = e^{-\frac{j40\pi}{11}}, \quad |H(4)| = 1$$

$$\angle H(4) = \frac{-40\pi}{11}$$

$$H(5) = e^{\frac{-j\times 10\times\pi\times 5}{11}} = e^{-\frac{j50\pi}{11}}, \quad |H(5)| = 1$$

$$\angle H(5) = \frac{-50\pi}{11}$$

We know that,

$$H(z) = \frac{1-z^{-N}}{N}\left[\frac{H(0)}{1-z^{-1}} + \sum_{k=1}^{\frac{N-1}{2}} 2\,|H(k)|\,H_k(z)\right] \quad \text{for } N \text{ odd.}$$

$$H(z) = \frac{1-z^{-11}}{11}\left[\frac{H(0)}{1-z^{-11}} + \sum_{k=1}^{5} 2\,|H(k)|\,H_k(z)\right]$$

Here,

$$H_k(z) = \frac{\left(\cos(\theta(k))\right) - z^{-1}\left[\cos\left(\theta(k) - \frac{2\pi k}{N}\right)\right]}{1 - 2z^{-1}\cos\left(\frac{2\pi k}{N}\right) + z^{-2}}$$

$$H_3(z) = \frac{\cos(\theta(3)) - z^{-1}\left[\cos\left(\theta(3) - \left(\frac{2\pi\times 3}{11}\right)\right)\right]}{1 - 2z^{-1}\cos\left(\frac{2\pi\times 3}{11}\right) + z^{-2}}$$

$$H_3(z) = \frac{\cos\left(-\frac{30\pi}{11}\right) - z^{-1}\left[\cos\left(-\frac{30\pi}{11} - \frac{6\pi}{11}\right)\right]}{1 - 2z^{-1}\cos\left(\frac{6\pi}{11}\right) + z^{-2}}$$

$$H_3(z) = \frac{-0.7 - 0.7z^{-1}}{1 + 0.28z^{-1} + z^{-2}}$$

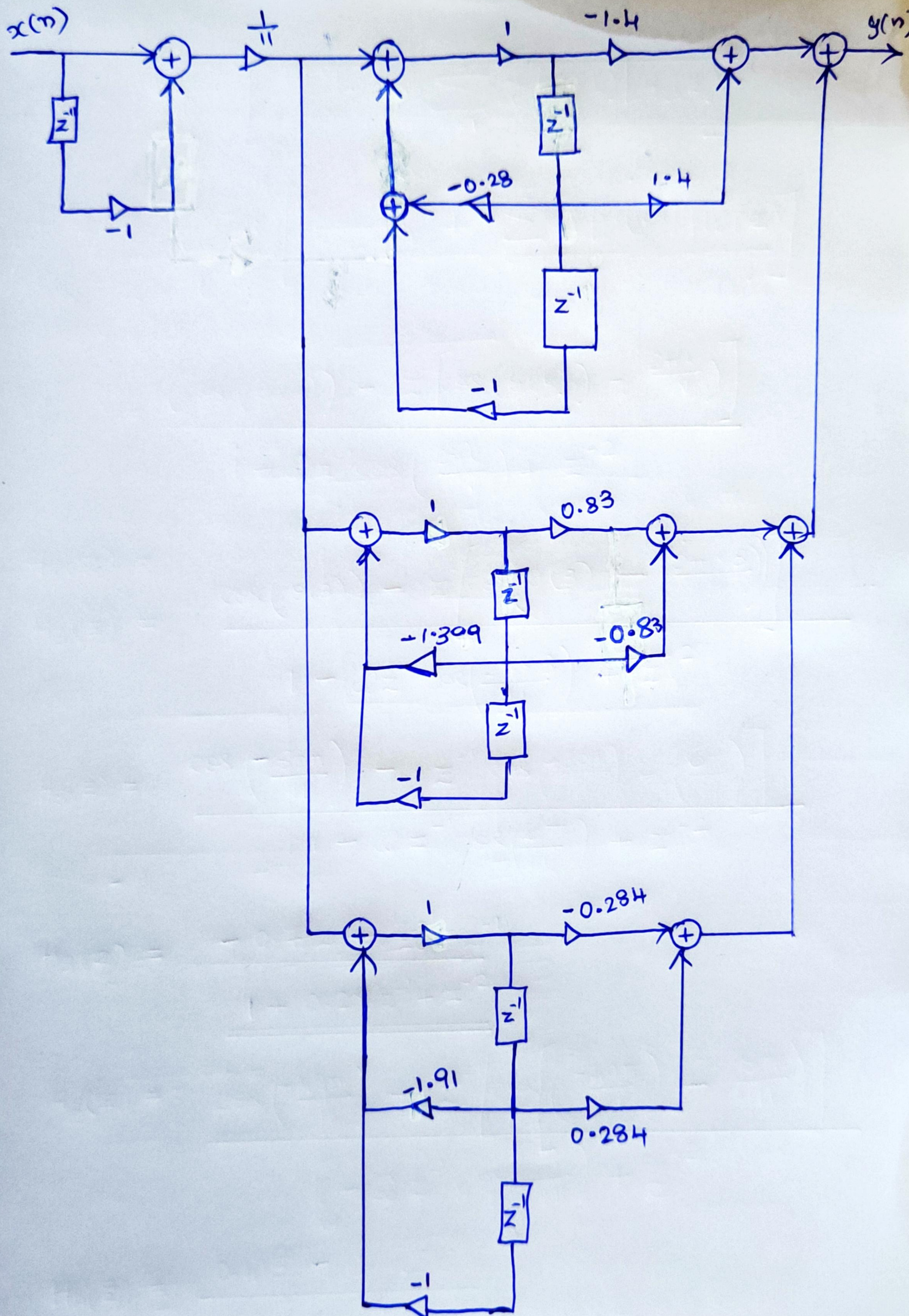
$$H_4(z) = \frac{\cos\left(-\frac{40\pi}{11}\right) - z^{-1}\left[\cos\left(-\frac{40\pi}{11} - \frac{8\pi}{11}\right)\right]}{1 - 2z^{-1}\cos\left(\frac{8\pi}{11}\right) + z^{-2}}$$

$$H_4(z) = \frac{0.415 - 0.415z^{-1}}{1 + 1.309z^{-1} + z^{-2}}$$

$$H_5(z) = \frac{\cos\left(\frac{-50\pi}{11}\right) - z^{-1}\cos\left(\frac{-50\pi}{11} - \frac{1\pi}{11}\right)}{1 - 2z^{-1}\cos\left(\frac{10\pi}{11}\right) + z^{-2}}$$

$$H_5(z) = \frac{-0.142 + 0.142z^{-1}}{1 + 1.91z^{-1} + z^{-2}}$$

$$\therefore\ H(z) = \frac{1-z^{-11}}{11}\left[\frac{-1.4 + 1.4z^{-1}}{1+0.28z^{-1}+z^{-2}} + \frac{0.83 - 0.83z^{-1}}{1+1.309z^{-1}+z^{-2}} + \frac{-0.284 + 0.284z^{-1}}{1+1.91z^{-1}+z^{-2}}\right]$$



x(n)
y(n)
z^-1
-1
1/11
1
-1.4
-0.28
1.4
0.83
-1.309
-0.83
-0.284
-1.91
0.284