

$$(6) (a) P(X|C) = \frac{P(X, C)}{P(C)} = \frac{P(X, E, C) + P(X, E', C)}{P(C)}$$

$$= \frac{P(X|E, C) \times P(E, C) + P(X|E', C) \times P(E', C)}{P(C)}$$

$$= \frac{(0.8) (P(E, C, B) + P(E, C, B')) + 0.7 (P(E', C, B) + P(E', C, B'))}{P(C)}$$

$$= \frac{(0.8) (P(E|C, B) \cdot P(C, B) + P(E|C, B') \cdot P(C, B')) + (0.7) (P(E'|C, B) \cdot P(C, B) + P(E'|C, B') \cdot P(C, B'))}{P(C)}$$

$$\begin{aligned} P(C, B) &= P(C, B, A) + P(C, B, A') \\ &= P(B|A, C) \cdot P(A, C) + P(B|A', C) \cdot P(A', C) \\ &= (0.4) (0.05) + (0.9) (0.45) \\ &= 0.425, \end{aligned}$$

$$\begin{aligned} P(C, B') &= P(B'|A, C) \cdot P(A, C) + P(B'|A', C) \cdot P(A', C) \\ &= (0.6) (0.05) + (0.1) (0.45) \\ &= 0.075, \end{aligned}$$

$$P(X|C) = \frac{0.8 (0.1 \times 0.425 + 0.2 \times 0.075) + 0.7 (0.9 \times 0.425 + 0.8 \times 0.075)}{0.5}$$

$$= \frac{0.046 + 0.30975}{0.5}$$

$$= \underline{\underline{0.7115}}$$

$$(b) \quad P(E|D) = \frac{P(E, D)}{P(D)}$$

$$\begin{aligned} P(ED) &= P(ED) + P(ED') \\ &= P(E|B) P(BD) + P(E|B') P(B'D) \\ &= (0.1) P(B, D) + (0.2) P(B', D) \\ &= (0.1) (P(BAD) + P(BD'A)) + 0.2 (P(B'AD) + P(B'A'D)) \\ &= (0.1) (P(B|A) P(A, D) + P(B|A') P(A'D)) \\ &\quad + 0.2 (P(B'|A) P(A, D) + P(B'|A') P(A'D)) \end{aligned}$$

$$\begin{aligned} P(AD) &= P(AD|C) + P(AD|C') \\ &= P(D|C) P(C) + P(D|C') P(C') \\ &= 0.7 \times 0.05 + 0.4 \times 0.05 \\ &= \underline{0.055} \end{aligned}$$

$$\begin{aligned} P(A'D) &= P(A'D|C) + P(A'D|C') \\ &= P(D|C) P(A, C) + P(D|C') P(A', C') \\ &= 0.7 \times 0.45 + 0.4 \times 0.45 \\ &= \underline{0.495} \end{aligned}$$

$$\begin{aligned} P(E, D) &= 0.1 \times (0.4 \times 0.055 + 0.9 \times 0.495) + \\ &\quad 0.2 (0.6 \times 0.055 + 0.1 \times 0.495) \\ &= 0.06325 \end{aligned}$$

$$\begin{aligned} P(D) &= P(D|C) + P(D|C') = P(D|C) P(C) + P(D|C') P(C') \\ &= 0.7 \times 0.5 + 0.4 \times 0.5 = \underline{0.55} \end{aligned}$$

$$P(E|D) = \frac{P(E, D)}{P(D)} = \frac{0.06325}{0.55} = \underline{0.115}$$

(c)

$$P(G|B, \sim D) = \frac{P(GBD')}{P(BD')}$$

$$P(GBD') = P(GFB D') + P(GF' B D')$$

$$= P(G|F) P(F B D') + P(G|F') P(F' B D')$$

$$= 0.9 (P(F|B D') \cdot P(B D') + 0.1 (P(F'|B D') \cdot P(B D')))$$

$$= 0.9 (0.4 \times P(B D')) + 0.1 (0.6 \times P(B D'))$$

$$P(B D') = P(B D' A) + P(B D' A')$$

$$= P(B|A) P(D'A) + P(B|A') P(D'A')$$

$$= 0.4 \times P(D'A) + 0.9 \times P(D'A')$$

$$= 0.4 (P(D'|C) P(A C) + P(D'|C') P(A C'))$$

$$+ 0.9 (P(D'|C) P(A' C) + P(D'|C') P(A' C'))$$

$$= 0.4 \times 0.055 + 0.9 \times 0.495$$

$$= 0.4675$$

$$P(G|B, \sim D) = \frac{0.9 (0.4 \times 0.4675) + 0.1 (0.6 \times 0.4675)}{0.4675}$$

$$= \frac{0.1683 + 0.02805}{0.4675} = \frac{0.19635}{0.4675}$$

$$= \underline{0.42}$$