

Assignment 1

AI1110: Probability and Random Variables
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10.13.3.36: Question. Two dice are thrown at the same time. Determine the probability distribution based on n , where n is the difference of the numbers on the two dice

Answer:

Let X be the random variables representing the outcome for a die.

Assuming the die to be fair, the probability mass function (pmf) is expressed as

$$p_X(k) = \begin{cases} \frac{1}{6} & 1 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

When 2 dice are rolled, each die will have 6 outcomes and the events are independent

Let the probability of event that difference between the numbers on the dice is n be $p_E(n)$.

$$n \in \{0, 1, 2, 3, 4, 5\}$$

Let us assume X_1 be the outcome of first die and X_2 be the outcome of second die

$$\begin{aligned} p_E(n) &= Pr(|X_1 - X_2| = n) \\ &= Pr((X_1 - X_2 = n) + (X_1 - X_2 = -n)) \end{aligned}$$

Let E_1 be the event satisfying $X_1 - X_2 = n$ and E_2 be the event satisfying $X_1 - X_2 = -n$

$$p_E(n) = Pr(E_1 + E_2)$$

for $n=0$

Since events E_1 and E_2 are identical

$$Pr(E_1 + E_2) = Pr(E_1)$$

$$\begin{aligned} p_E(0) &= Pr(X_1 - X_2 = 0) = Pr(X_1 = X_2) \\ &= \sum_k Pr(X_1 = k | X_2 = k) p_{X_2}(k) \end{aligned} \quad (2)$$

after unconditioning. X_1 and X_2 are independent,

$$\begin{aligned} Pr(X_1 = k | X_2 = k) &= Pr(X_1 = k) \\ &= p_{X_1}(k) \end{aligned} \quad (3)$$

From (2) and (3),

$$p_E(0) = \sum_k p_{X_1}(k) p_{X_2}(k) = p_{X_1}(n) * p_{X_2}(n) \quad (4)$$

where $*$ denotes the convolution operation. Substituting from (1) in (7)

$$p_E(0) = \frac{1}{6} \sum_{k=1}^6 p_{X_1}(k) \quad (5)$$

Substituting from (1) in (5)

$$\begin{aligned} p_E(0) &= \frac{1}{6} \sum_1^6 p_{X_1}(k) \\ &= \frac{1}{6} \left(\sum_1^6 p_{X_1}(k) \right) \\ &= \frac{1}{6} \left(6 \cdot \frac{1}{6} \right) \\ &= \frac{6}{36} \end{aligned}$$

for $1 \leq n \leq 6$

since events E_1 and E_2 are disjoint

$$\begin{aligned} p_E(n) &= Pr(E_1 + E_2) \\ &= Pr(E_1) + Pr(E_2) \end{aligned}$$

Because of symmetry between E_1 and E_2

$$\begin{aligned} Pr(E_1) &= Pr(E_2) \\ p_E(n) &= 2 * Pr(E_1) \end{aligned}$$

$$\begin{aligned} Pr(E_1) &= Pr(X_1 - X_2 = n) = Pr(X_1 = X_2 + n) \\ &= \sum_k Pr(X_1 = k + n | X_2 = k) p_{X_2}(k) \quad (6) \end{aligned}$$

after unconditioning. X_1 and X_2 are independent,

$$\begin{aligned} Pr(X_1 = k + n | X_2 = k) &= Pr(X_1 = k + n) \\ &= p_{X_1}(k + n) \quad (7) \end{aligned}$$

From (6) and (7),

$$Pr(E_1) = \sum_k p_{X_1}(k + n) p_{X_2}(k) = p_{X_1}(n) * p_{X_2}(n) \quad (8)$$

where $*$ denotes the convolution operation.
Substituting from (1) in (7)

$$Pr(E_1) = \frac{1}{6} \sum_{k=1}^6 p_{X_1}(k + n) = \frac{1}{6} \sum_{1+n}^{6+n} p_{X_1}(k) \quad (9)$$

Substituting from (1) in (9)

$$\begin{aligned} Pr(E_1) &= \frac{1}{6} \sum_{1+n}^{6+n} p_{X_1}(k) \\ &= \frac{1}{6} \left(\sum_{1+n}^6 p_{X_1}(k) + \sum_7^{1+6} p_{X_1}(k) \right) \\ &= \frac{1}{6} \left(6 - n \frac{1}{6} + 0 \right) \\ &= \frac{6 - n}{36} \end{aligned}$$

Therefore, the probability of $p_E(n)$ is

$$\begin{aligned} p_E(n) &= 2 * Pr(E_1) \\ &= 2 * \frac{6 - n}{36} \\ &= \frac{12 - 2n}{36} \end{aligned}$$

$$p_E(n) = \begin{cases} 0 & n < 0 \\ \frac{6}{36} & n = 0 \\ \frac{12-2n}{36} & 1 \leq n \leq 5 \\ 0 & n > 5 \end{cases} \quad (10)$$

Difference	Probability
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36