Assignment 1

AI1110: Probability and Random Variables
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10.13.3.36: Question. Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2

Answer: $\frac{8}{36}$.

Let X be the random variables representing the outcome for a die.

Assuming the die to be fair, the probability mass function (pmf) is expressed as

$$p_X(k) = \begin{cases} \frac{1}{6} & 1 \le k \le 6 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

When 2 dice are rolled, each die will have 6 outcomes and the events are independent

Let the event that difference between the numbers on the dice is 2 be E.

Let us assume X_1 be the outcome of first die and X_2 be the outcome of second die

To satisfy the event E

$$X_1 - X_2 = 2 \tag{2}$$

(or)

$$X_2 - X_1 = 2 \tag{3}$$

Let E_1 be the event satisfing (2) and E_2 be the event satisfing (3)

$$Pr(E) = Pr(E_1 + E_2)$$

events E_1 and E_2 are independent

$$Pr(E) = Pr(E_1) + Pr(E_2)$$

Consider E_1

$$\begin{aligned} Pr(E_1) &= Pr(X_1 - X_2 = 2) = Pr(X_1 = X_2 + 2) \\ &= \sum_k Pr(X_1 = k + 2 | X_2 = k) p_{X_2}(k) \end{aligned} \tag{4}$$

after unconditioning. X_1 and X_2 are independent,

$$\begin{aligned} Pr(X_1 = k + 2 | X_2 = k) &= Pr(X_1 = k + 2) \\ &= p_{X_1}(k + 2) \end{aligned} \tag{5}$$

From (4) and (5),

$$Pr(E_1) = \sum_k p_{X_1}(k + 2) p_{X_2}(k) \quad = p_{X_1}(2) * p_{X_2}(2) \tag{6}$$

where * denotes the convolution operation. Substituting from (1) in (5)

$$Pr(E_1) = \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(k + 2) = \frac{1}{6} \sum_{3}^{8} p_{X_1}(k) \tag{7}$$

Substituting from (1) in (7)

$$\begin{aligned} Pr(E_1) &= \frac{1}{6} \sum_{3}^{8} p_{X_1}(k) \\ &= \frac{1}{6} (\sum_{3}^{6} p_{X_1}(k) + \sum_{7}^{8} p_{X_1}(k)) \\ &= \frac{1}{6} (4\frac{1}{6} + 0) \\ &= \frac{4}{36} \end{aligned} \tag{8}$$

Because of symmetry between (2) and (3)

$$Pr(E_2) = P(E_1) = \frac{4}{36}$$

Therefore,the probability of E is

$$\begin{aligned} Pr(E) &= Pr(E_1) + Pr(E_2) \\ &= \frac{4}{36} + \frac{4}{36} \\ &= \frac{8}{36} \end{aligned}$$

Two dice are thrown at the same time the probability that the difference of the numbers on the two dice is 2 is $\frac{8}{36}$