Bonus Question

AI1110: Probability and Random Variables
INDIAN INSTITUTE OF TECHNOLOGY, HYDERABAD

Dikshant Khandelwal
CS22BTECH11017

11.16.3.5: Given that a fair coin is marked 1 on one face and 6 on the other and a fair die are tossed.find the probability sum turns up to be 3 and 12

Solution: Let X be the random variable for number on coin and Y be the random variable for number on dice. We know that, for a random variable moment generating function is ,

$$MGF_X(s) = E(e^{sX}) = \sum_{i=0}^{\infty} \Pr(X = i) e^{sX} \tag{1}$$

Here , let us define random variable Z,

$$\begin{aligned} MGF_Z(s) &= E(e^{s(X+Y}) && (2)\\ &= E(e^{sX}e^{sY}) && (3)\\ &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \Pr(X = i, Y = j) e^{s(X+Y)} && (4)\\ & && (5) \end{aligned}$$

Here, X and Y are independent random variables,

$$\begin{aligned} \therefore \Pr(X = i, Y = j) = \Pr(X = i)\Pr(Y = j) && (6)\\ && (7) \end{aligned}$$

So,

$$\begin{aligned} MGF_Z(s) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} \Pr(X = i)\Pr(Y = j) e^{sX}e^{sY} && (8)\\ &= \left(\sum_{i=0}^{\infty} \Pr(X = i) e^{sX}\right)\left(\sum_{j=0}^{\infty} \Pr(Y = j) e^{sY}\right) && (9) \end{aligned}$$

$$\therefore MGF_Z(s) = MGF_X(s)MGF_Y(s) \tag{10}$$

$$MGF_X(s) = \frac{e^s}{2} + \frac{e^{6s}}{2} \tag{11}$$

$$MGF_Y(s) = \frac{e^s + e^{2s} + e^{3s} + e^{4s} + e^{5s} + e^{6s}}{6} \tag{12}$$

$$MGF_Z(s) = \left(\frac{e^s + e^{2s} + e^{3s} + e^{4s} + e^{5s} + e^{6s}}{6}\right)\left(\frac{e^s}{2} + \frac{e^{6s}}{2}\right) \tag{13}$$

Now , coefficient of e^{sz} in MGF represents the probability of $Z = z$

$$\begin{aligned} \therefore \Pr(Z = 3) &= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) && (14)\\ &= \frac{1}{12} && (15)\\ \Pr(Z = 12) &= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) && (16)\\ &= \frac{1}{12} && (17) \end{aligned}$$