Assignment 1

AI1110: Probability and Random Variables
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10.13.3.36: Question. Two dice are thrown at the same time. Determine the probability distibution based on n,where n is the difference of the numbers on the two dice

Answer:

Let X be the random variables representing the outcome for a die.

Assuming the die to be fair, the probability mass function (pmf) is expressed as

$$p_X(k) = \begin{cases} \frac{1}{6} & 1 \leq k \leq 6 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

When 2 dice are rolled, each die will have 6 outcomes and the events are independent

Let the probability of event that difference between the numbers on the dice is n be $p_E(n)$.

$n \in \{0, 1, 2, 3, 4, 5\}$

Let us assume X_1 be the outcome of first die and X_2 be the outcome of second die

case (1) for $1 \leq n \leq 5$

$$X_1 - X_2 = n \tag{2}$$

(or)

$$X_2 - X_1 = n \tag{3}$$

Let E_1 be the event satisfing (2) and E_2 be the event satisfing (3)

$$p_E(n) = Pr(E_1 + E_2)$$

events E_1 and E_2 are independent

$$p_E(n) = Pr(E_1) + Pr(E_2)$$

Consider E_1

$$\begin{aligned} Pr(E_1) = Pr(X_1 - X_2 = n) &= Pr(X_1 = X_2 + n) \\ &= \sum_k Pr(X_1 = k + n | X_2 = k) p_{X_2}(k) \end{aligned} \tag{4}$$

after unconditioning. X_1 and X_2 are independent,

$$\begin{aligned} Pr(X_1 = k + n | X_2 = k) &= Pr(X_1 = k + n) \\ &= p_{X_1}(k + n) \end{aligned} \tag{5}$$

From (4) and (5),

$$Pr(E_1) = \sum_k p_{X_1}(k + n) p_{X_2}(k) \quad = p_{X_1}(n) * p_{X_2}(n) \tag{6}$$

where * denotes the convolution operation. Substituting from (1) in (5)

$$Pr(E_1) = \frac{1}{6} \sum_{k=1}^{6} p_{X_1}(k + n) = \frac{1}{6} \sum_{1+n}^{6+n} p_{X_1}(k) \tag{7}$$

Substituting from (1) in (7)

$$\begin{aligned} Pr(E_1) &= \frac{1}{6} \sum_{1+n}^{6+n} p_{X_1}(k) \\ &= \frac{1}{6} (\sum_{1+n}^{6} p_{X_1}(k) + \sum_{7}^{1+6} p_{X_1}(k)) \\ &= \frac{1}{6} (6 - n\frac{1}{6} + 0) \\ &= \frac{6 - n}{36} \end{aligned} \tag{8}$$

Because of symmetry between (2) and (3)

$$Pr(E_2) = P(E_1) = \frac{6-n}{36}$$

Therefore,the probability of $p_E(n)$ is

$$\begin{aligned} p_E(n) &= Pr(E_1) + Pr(E_2) \\ &= \frac{6-n}{36} + \frac{6-n}{36} \\ &= \frac{12-2n}{36} \end{aligned}$$

case (3) for n=0

$$\begin{aligned} X_1 - X_2 &= 0 \\ X_1 &= X_2 \end{aligned} \tag{9}$$

$$\begin{aligned} p_E(0) &= Pr(X_1 - X_2 = 0) = Pr(X_1 = X_2) \\ &= \sum_k Pr(X_1 = k|X_2 = k)p_{X_2}(k) \end{aligned} \tag{10}$$

after unconditioning. X_1 and X_2 are independent,

$$\begin{aligned} Pr(X_1 = k|X_2 = k) &= Pr(X_1 = k) \\ &= p_{X_1}(k) \end{aligned} \tag{11}$$

From (10) and (11),

$$p_E(0) = \sum_k p_{X_1}(k)p_{X_2}(k) \quad = p_{X_1}(n) * p_{X_2}(n) \tag{12}$$

where * denotes the convolution operation. Substituting from (1) in (5)

$$p_E(0) = \frac{1}{6}\sum_{k=1}^{6} p_{X_1}(k) \tag{13}$$

Substituting from (1) in (13)

$$\begin{aligned} p_E(0) &= \frac{1}{6}\sum_{1}^{6} p_{X_1}(k) \\ &= \frac{1}{6}(\sum_{1}^{6} p_{X_1}(k) \\ &= \frac{1}{6}(6\frac{1}{6}) \\ &= \frac{6}{36} \end{aligned}$$

$$p_E(n) = \begin{cases} 0 & n < 0 \\ \frac{6}{36} & n = 0 \\ \frac{12-2n}{36} & 1 \le n \le 5 \\ 0 & n > 5 \end{cases} \tag{14}$$

Difference	Probability
0	6/36
1	10/36
2	8/36
3	6/36
4	4/36
5	2/36