Verification of calculated and model values of altitude of optical depth

Theoritical Calculation of altitude of unit optical depth- Table data

$$\tau(z)=1=\sigma^a H n(z)$$

$$Exponential\ density: n(z)=n_0 \exp\left(\frac{-z}{H}\right)$$

$$Altitude\ of\ unit\ optical\ depth: z=H \ln\left(H n_0 \sigma^a\right)$$

$$Assuming: Scale\ Height\ H=7\,km, n_0=2.7e19\,cm^{-3}, \sigma^a\ for\ N_2\ as\ given,$$

$$we\ find\ the\ value\ for\ z\ for\ optical\ depth\ \tau=1\ by\ using\ the\ above\ formula$$

Calculated	Results				
Bin #	λ min (A)	λ max (A)	Mean λ (A)	Mean σN2abs (mega-barns)	Alt of tau=1 (km)
1	0.5	1	0.75	2.48e-5	43
2	1	1.5	1.25	1.47e-4	55.5
3	1.5	2	1.75	4.57e-4	63.5
4	2	2.5	2.25	1.02e-3	69.1
5	2.5	3	2.75	1.9e-3	73.3
6	3	4	3.5	4.0e-3	78.7
7	4	5	4.5	8.55e-3	84
8	5	6	5.5	1.55e-2	88.1
9	6	8	7	3.12e-2	93
10	8	10	9	6.32e-2	98
11	10	14	12	0.14	103.5
12	14	18	16	0.30	109

Effect of altitude bin Δz

1. If we take the altitude bin in the model to be Δz= 0.5 to 1 km, the deviation in calculating the altitude of unit optical depth is minimum, so Δz= 1 km in the model.

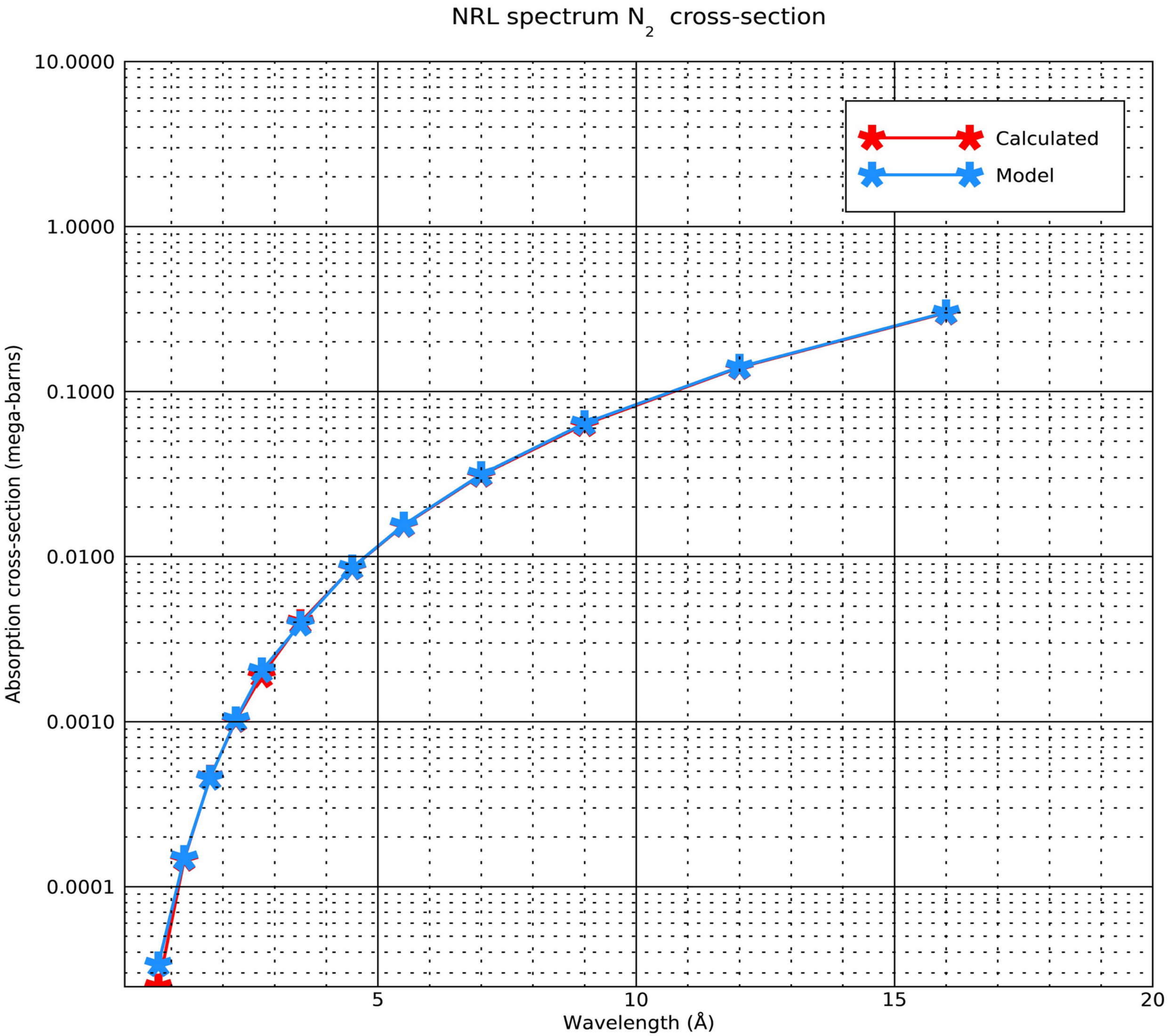
Calculated	Results				
Bin #	λ min (A)	λ max (A)	Mean λ (A)	Mean σN2abs (mega-barns)	Alt of tau=1 (km)
1	0.5	1	0.75	2.48e-5	43
2	1	1.5	1.25	1.47e-4	55.5
3	1.5	2	1.75	4.57e-4	63.5
4	2	2.5	2.25	1.02e-3	69.1
5	2.5	3	2.75	1.9e-3	73.3
6	3	4	3.5	4.0e-3	78.7
7	4	5	4.5	8.55e-3	84
8	5	6	5.5	1.55e-2	88.1
9	6	8	7	3.12e-2	93
10	8	10	9	6.32e-2	98
11	10	14	12	0.14	103.5
12	14	18	16	0.30	109

Model Results		
σN2abs (mega-barns)	Alt of tau=1 (km)	Exact value of optical depth tau
3.38E-05	46	1.12
1.49E-04	58	1.02
4.55E-04	65	1.32
1.03E-03	71	1.21
2.04E-03	75	1.26
3.91E-03	79	1.25
8.59E-03	83	1.39
1.56E-02	87	1.22
3.15E-02	90	1.17
6.43E-02	94	1.15
1.41E-01	99	1.24
3.02E-01	103	1.31

Altitued Difference (km)
3
2.5
1.5
1.9
1.7
0.3
1
1.1
3
4
4.5
6

N_2 Cross-sections

1. I changed the cross-sections of N_2 in the model to the one from the table.
2. The cross-section of the first bin is different in the model and the table.
3. Changing the cross-sections only changes the altitude of unit optical depth in the first bin.
4. I checked the Henke + Fennelly data, and the model value is more accurate than the table value.



N_2 Cross-sections

Calculated	Results							
Bin #	λ min (A)	λ max (A)	Mean λ (A)	Mean σN2abs (mega-barns)	Alt of tau=1 (km)	Alt of tau=1 (km)	Exact value of optical depth tau	Altitued Difference (km)
1	0.5	1	0.75	2.48e-5	43	45	1.12	2
2	1	1.5	1.25	1.47e-4	55.5	58	1.02	2.5
3	1.5	2	1.75	4.57e-4	63.5	65	1.32	1.5
4	2	2.5	2.25	1.02e-3	69.1	71	1.21	1.9
5	2.5	3	2.75	1.9e-3	73.3	75	1.26	1.7
6	3	4	3.5	4.0e-3	78.7	79	1.25	0.3
7	4	5	4.5	8.55e-3	84	83	1.39	1
8	5	6	5.5	1.55e-2	88.1	87	1.22	1.1
9	6	8	7	3.12e-2	93	90	1.17	3
10	8	10	9	6.32e-2	98	94	1.15	4
11	10	14	12	0.14	103.5	99	1.24	4.5
12	14	18	16	0.30	109	103	1.31	6

This bin changed

Estimation of n_0 from the model

1. In the table (slide 2), it was assumed n_0= 2.7E19 cm^3
2. I used the formula from slide 2, to get an estimation of n_0 of the model.
3. This combination of scale height and density gives the same value of altitude of unit optical depth as in slide 5

n_0 (cm^3)	Height of unit optical depth (km)	Scale Height (km)
1.63e+19	45	7.82
3.01e+19	58	7.19
5.72e+19	65.5	6.70
1.13e+20	71	6.34
1.86e+20	75	5.95
2.45e+20	79	5.60
6.31e+20	83.5	5.27
1.84e+21	87	5.27
1.58e+21	90.5	5.30
9.81e+20	94.5	5.45
1.19e+21	99	5.40
5.36e+20	103	5.62

Estimation of altitude of unit optical depth using constant n_0

1. However, if I use n_0= 2.7E19 cm^3 , the model's scale height, and the formula for altitude of unit optical height derived in slide 2, the altitude is very different (column2 of table on the right)
2. So the correct scale height and density is required to get the correct altitude.

n_0 (cm^3)	Height of unit optical depth (km)	Scale Height (km)
2.7e+19	48.9	7.82
2.7e+19	57	7.19
2.7e+19	60.5	6.70
2.7e+19	62	6.34
2.7e+19	63	5.95
2.7e+19	65.9	5.60
2.7e+19	65.9	5.27
2.7e+19	64.8	5.30
2.7e+19	68.9	5.30
2.7e+19	74.9	5.45
2.7e+19	78.5	5.40
2.7e+19	86.2	5.62

Calculation of altitudes with MSIS values of n_0

1. The MSIS value for n_0= 1.9 E19 cm^3
2. Taking scale height H= 7km and the formula for altitude of unit optical height derived in slide 2, and calculating the altitude of unit optical depth.
3. The altitudes are different than the ones calculated in the table with n_0= 2.7E19 cm^3

Calculated	Results					Alt of tau=1 (km) (n0= 1.9 E19)	Model Results
Bin #	λ min (A)	λ max (A)	Mean λ (A)	Mean σN2abs (mega-barns)	Alt of tau=1 (km) n0= 2.7 E19		Alt of tau=1 (km)
1	0.5	1	0.75	2.48e-5	43	40.6	46
2	1	1.5	1.25	1.47e-4	55.5	53.	58
3	1.5	2	1.75	4.57e-4	63.5	61	65
4	2	2.5	2.25	1.02e-3	69.1	66.6	71
5	2.5	3	2.75	1.9e-3	73.3	71	75
6	3	4	3.5	4.0e-3	78.7	76	79
7	4	5	4.5	8.55e-3	84	81.5	83
8	5	6	5.5	1.55e-2	88.1	85.6	87
9	6	8	7	3.12e-2	93	90.6	90
10	8	10	9	6.32e-2	98	95.5	94
11	10	14	12	0.14	103.5	101	99
12	14	18	16	0.30	109	106	103

Conclusion

1. The difference in altitude of unit optical depth between the model data and the calculated data is due to the difference in the assumptions made in its calculations and the different values calculated by the model.
2. An altitude binning of $\Delta z = 0.5$ to 1 km is good for getting the correct altitude of optical depth.
3. The N_2 cross-section of the first bin in the model is more accurate than that given in the theoretical table.