SG2218 Turbulence Lab

Turbulence Measurements in the wake of a circular cylinder

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1. Plane Wake and the self-similarity solution

A plane wake is formed when a uniform flow stream moves over a static object (considered here to be a cylinder) placed in the path of the flow field. With increasing distance from the cylinder, it can be seen that the wake spreads (i.e. $y_{1/2}$ increases) and the decays (U_s/U_c decreases). Figure 1 depicts the plane wake phenomenon behind a cylinder and the parameters that are associated with it.

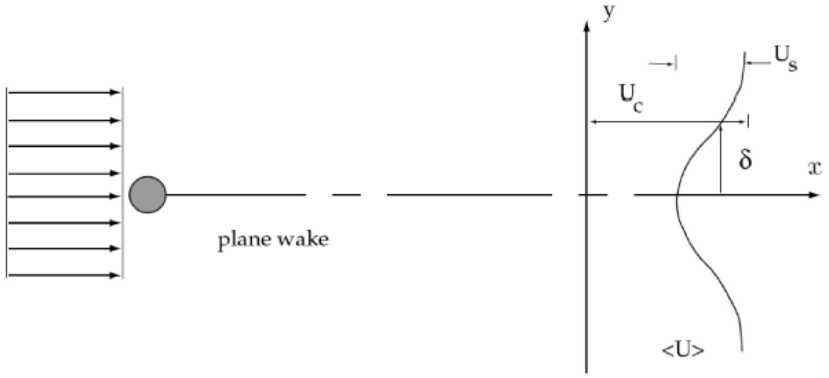

Figure 1: Representation of a plane wake showing the characteristic flow width $\delta = l$, the characteristic convective velocity $U_c = U_o$ and the characteristic velocity difference $U_s = U_d$

For a far wake flow of a 2D cylinder, the mean flow is expressed as:

$$U_0\left(\frac{\partial U_d}{\partial x}\right) = -\frac{\partial < uv >}{\partial y} \qquad (1)$$

Where $U_d = U_0 - U$ is the velocity deficit ($\frac{U_d}{U_0} \ll 1$)

A classical self-similarity hypothesis for all wake flows is formulated by considering the velocity profiles and the Reynolds stresses to be a function of a similarity variable η to analyse such flows.

Self-Similarity is defined as a state of flow in which equilibrium is established in the flow state. In this state, the flow no longer depends on its origin and the mean flows depends on a few geometrical parameters.

The similarity ansatz is given by:

$$\frac{U_d}{U_s} = f(\eta)\ and\ \frac{-< uv >}{U_d^2} = g(\eta) \qquad (2)$$

Where $U_s = U_d(0)$ and $\eta = y/l$. The velocity and length scales U_s and l are functions of x only. From equations (1) and (2) and also considering the momentum thickness (given by (3)) of the flow to be independent of x,

$$\theta = \left(\frac{1}{U_0}\right)\int_{-\infty}^{\infty} U_d \, dy \tag{3}$$

It can be observed that the for $\frac{U_d}{U_0} \ll 1$, the length and velocity scales l and U_s vary as $x^{1/2}$ and $x^{-1/2}$ respectively. For flow over cylinder, the length scale l is defined as:

$$l \sim (x - x_0)^{1/2} \tag{4}$$

Where x_0 is the virtual origin of the wake

The self-similar profile is given by Figure 2, which plots the normalised U_d profile in the self-similar coordinate plane. The similarity solution is given by:

$$f(\xi) = \exp(-\alpha\xi^2) \tag{5}$$

Where, $\xi = y/y_{1/2}$ and α is found to be ln (2)

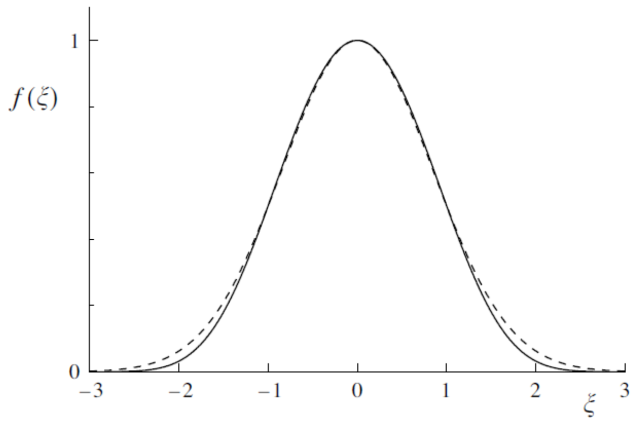

Figure 2: Normalised U_d profile computed from the self-similarity ansatz (From Fig. 5.26 in Pope,2000)

2. <u>Experimental Plots</u>

a. Mean velocity profiles and centreline location of the wake.

This analysis is conducted for the following cases:

- Free Stream velocity $(U_0) = 20\ m/s$
- 3 values of $\frac{x}{d}$: $980, 740\ and\ 500$
- NOTE: The experimental data for groups 1,2 and 3 are considered for this analysis as our group was not able to perform the experiment due to an issue with the HWA probes on the day of the experiment.

Figure 3 gives the plot between the mean velocity profiles for different x/d values with respect to the y position after appropriate corrections such that all of the curves share the same centreline position. From the above plot, it can be seen that the centreline location of flow is approximately 0.9mm

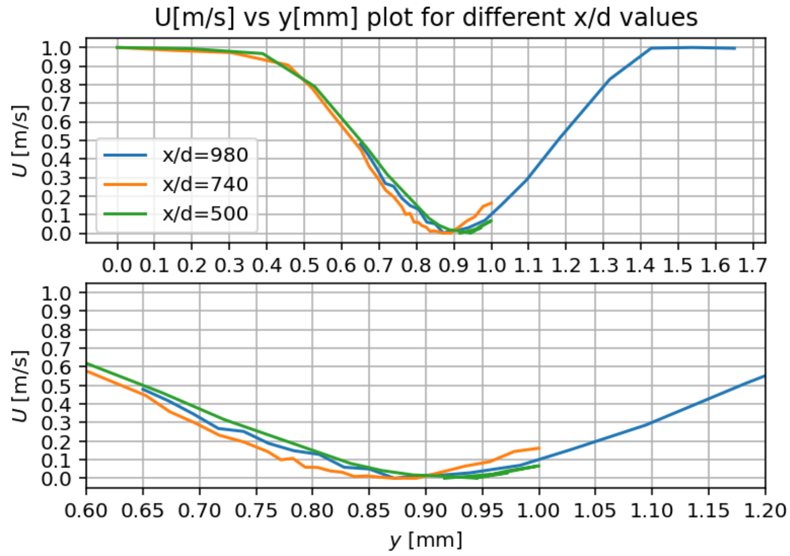

Figure 3: Mean Velocity Profiles for different x/d values

b. Deficit velocity U_d and Velocity Scale U_s

Figure 4 depicts the plot between U_d and y for the same Re but different x/d values. The virtual origin for the x/d=500 case is considered as the common virtual origin for all the cases. We can see that as the value of x/d increase, the centreline of the flow is shifted in the downwards direction. However, trends for U_d cannot be accurately derived from this plot. The velocity scales calculated from the experimental data is given in Table 1, wherein U_s is considered to be the maximum velocity deficit.

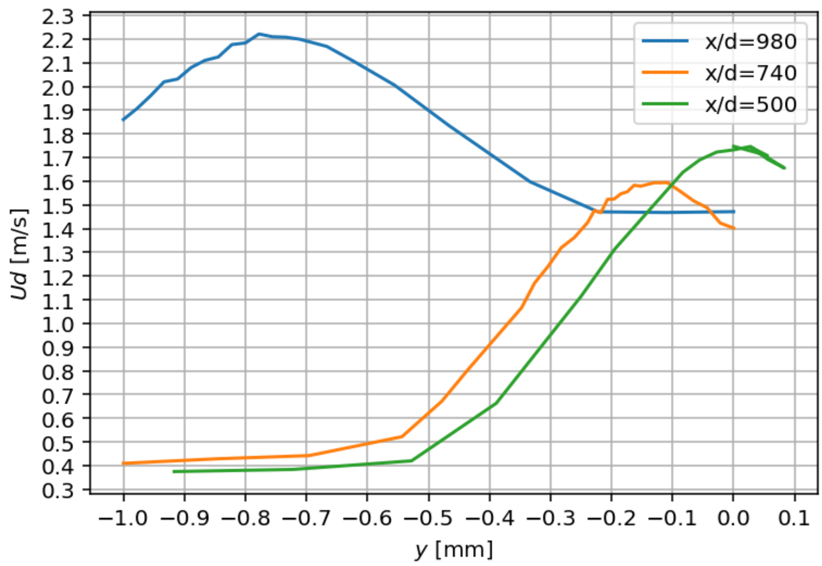

Figure 4: U_d vs y plot for different x/d cases.

Value of x/d	Velocity Scale (U_s)
980	2.221
740	1.594
500	1.747

Table 1: Velocity Scales (U_s) for different x/d values

c. Cross stream length scale ($\delta(x)$)

The Cross-stream length scale $\delta(x)$ is the defined for this case as the distance from the virtual origin from which the Deficit velocity becomes half of the velocity scale. This can be expressed mathematically as:

$$U_d\left(x, y_{\frac{1}{2}}\right) = \frac{U_s}{2} \qquad (5)$$

$$Here, y_{1/2} = \delta(x)$$

$\delta(x)$ in this case is also known as the Half-length of the wake. Table 2 gives the absolute values for $\delta(x)$ for the different x/d values. These values are obtained by interpolating the experimental data and finding the value of y for the point $U_s/2$.

Value of x/d	Length Scale ($\delta(x)$)(mm)
980	1.000
740	0.438
500	0.324

Table 2: Absolute values of length scale ($\delta(x)$) for different values of x/d

d. Scaled Velocity Deficit profiles

Using the length and the velocity scales that were obtained in sections (b) and (c), the U_d and y are scaled with U_s and $\delta(x)$ respectively. Figure 5 gives the plot of U_d in the self-similar coordinate system with the virtual origin for $x/d = 500$ considered as the common origin for all the other x/d cases. As previously observed in section (b), this plot makes it clear that the wake region moves more towards the negative y direction as the x/d values are increased. Therefore, it can be concluded that the geometry/size of the cylinder affects the formation of the wake when the Re is constant.

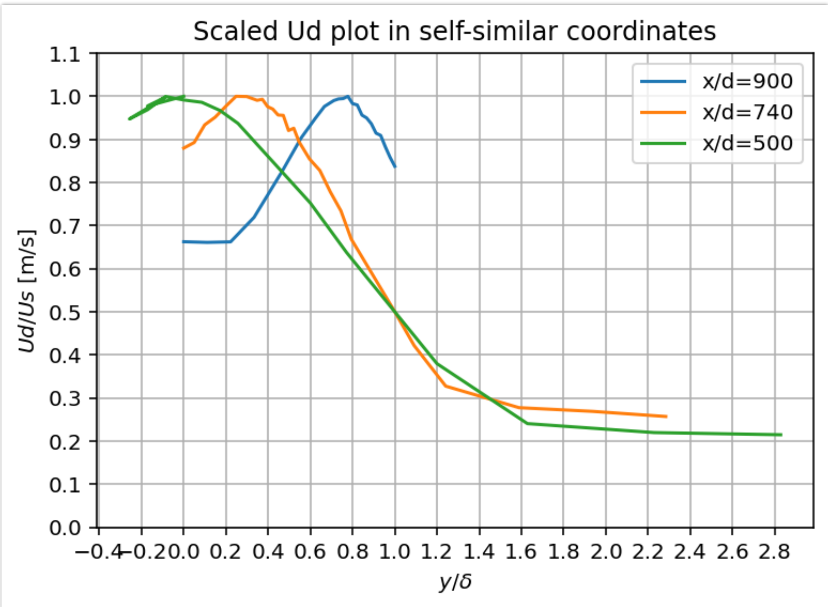

Figure 5: Normalised U_d plot in self-similar coordinate system

e. Comparison of experimental data with self-similar solution

Figure 6, Figure 7 and Figure 8 show the comparison of experimental data with the self-similarity solution for the $x/d = 980$, 740, and 500 respectively.

Figure 6: Normalised U_d plot for $x/d = 980$.

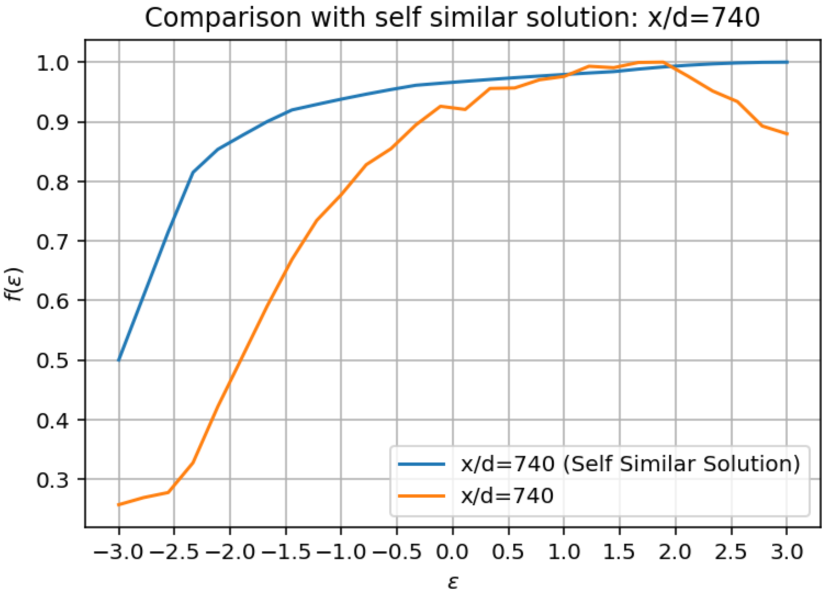

Figure 7: Normalised U_d plot for $x/d = 740$.

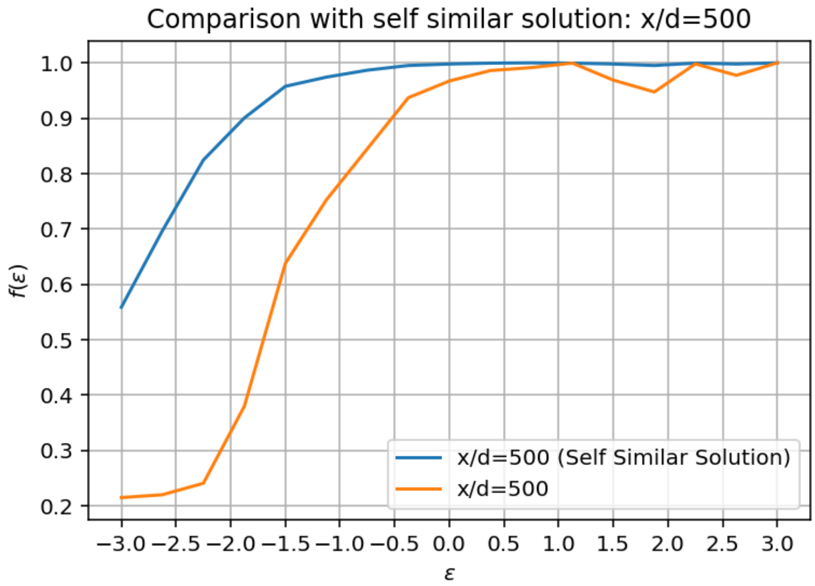

Figure 8: Normalised U_d plot for $x/d = 500$.

Initial observations from the three graphs reveals that there is good correlation between the experimental data and the expected plot from the self-similarity solution close to the origin. This shows that the self-similarity of the turbulent flow behind a cylinder is valid when the flow is fully developed. However, the self-similarity curve shows a different peak as compared to the experimental data. Moreover, the correlation of the curves at the tail region is not satisfactory in all the cases. This can be explained by the variation in the physical flow properties and the ideal flow that is assumed in the self-similarity solution. Some the physical flow properties affecting the results include the anisotropy in the turbulent viscosity, variation of turbulent viscosity with the radius of the cylinder and the downstream position.